$\alpha_s(M_Z^2)$ at NNLO and N^{3}LO

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Contents

Introduction

- Precision determinations of parton distribution functions (PDFs) and $\alpha_s(M_Z^2)$ are currently being performed at NNLO (α_s^3).
- NLO fits suffer from scale uncertainties being of $O(\pm 5\%)$ and therefore too large.
- The heavy flavor corrections are available in NLO + threshold corrections and the calculation of the NNLO corrections is making progress.
- Sensitive data, capable to constrain the known PDFs better, have to be selected, rather than performing global analyses using data with problematic systematics.
- Current data: DIS World data (including H1+ZEUS combined); Di-muon data (s); Drell-Yan data ($\bar{d} - \bar{u}$); pp-jet data (Tevatron); LHC data on $W^{\pm}, Z$ off-resonance Drell-Yan. Very soon: LHC Jet data.
- α_s and HT terms have to be fitted along with the non-pert. parameters of the PDFs.

Introduction

- Among the hard processes, the next important analyses will be those of the LHC jet data: single jet; in particular two-jet data and later 3-jet/2-jet ratios. Accuracy needed: NNLO.
- For the more distant future: Giga-Z at a future ILC to measure $\alpha_s(M_Z)$ precisely from $R(s)$.
- τ-decay data are very precise. Do we yet understand their theoretical description ?
- Lattice determinations: various results at present. m_π still lowers, 2+1+1 flavors, non-perturbative renormalization needed in all analyses; final values still ahead.
- In what follows I will mainly discuss the situation for DIS+DY+Dimuon and touch on the next steps in hard scattering and other α_s-measurements more briefly.

The Data: DIS and related

	Experiment	NDP	χ^2(NNLO)	χ^2(NLO)
DIS inclusive	H1&ZEUS	486	537	531
	H1	130	137	132
	BCDMS	605	705	695
	NMC	490	665	661
	SLAC-E-49a	118	63	63
	SLAC-E-49b	299	357	357
	SLAC-E-87	218	210	219
	SLAC-E-89a	148	219	215
	SLAC-E-89b	162	133	132
	SLAC-E-139	17	11	11
	SLAC-E-140	26	28	29
Drell-Yan	FNAL-E-605	119	167	167
	FNAL-E-866	39	52	55
DIS di-muon	NuTeV	89	46	49
	CCFR	89	61	62
Total		3036	3391	3378

Tabelle 2: The value of χ^2 obtained in the NNLO and NLO fits for different data sets.

$\alpha_s(M_Z^2)$: ABM11 Analysis

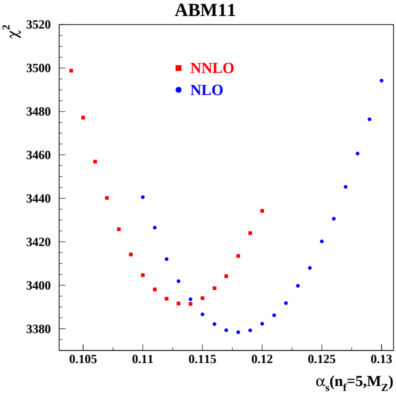


Figure 17: The χ^2-profile as a function of $\alpha_s(M_Z)$ in the present analysis. At NLO (circles) and NNLO (squares).

$\alpha_s(M_Z^2)$

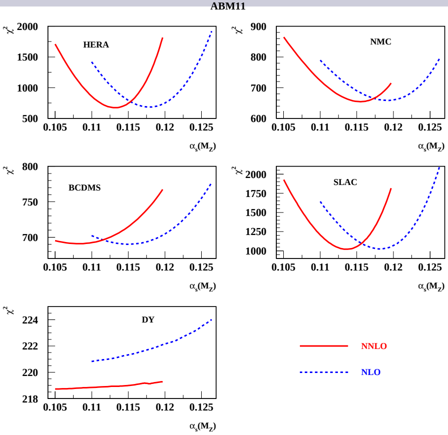


Figure 19: The χ^2-profile versus the value of $\alpha_s(M_Z)$ for the data sets used, all calculated with the PDF and HT parameters fixed at the values obtained from the fits with $\alpha_s(M_Z)$ released (solid lines: NNLO fit, dashes: NLO one).

$\alpha_s(M_Z^2)$

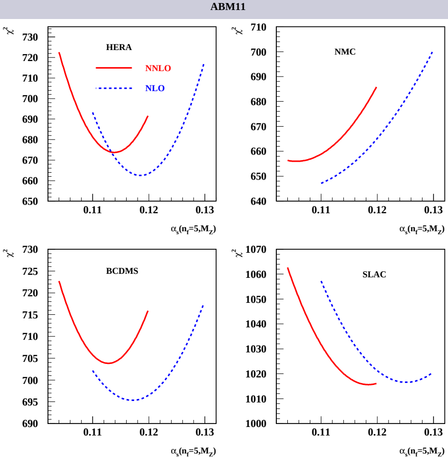


Figure 18: The χ^2-profile versus the value of $\alpha_s(M_Z)$ for the data sets used all obtained in variants of the present analysis with the value of α_s fixed and all other parameters fitted (solid lines: NNLO fit, dashes: NLO fit).

$\alpha_s(M_Z^2)$: ABM11

Experiment	$\alpha_s(M_Z)$		
	NLO$_{exp}$	NLO	NNLO
BCDMS	0.1111 ± 0.0018	0.1150 ± 0.0012	0.1084 ± 0.0013
NMC	$0.117 \; {}^{+\,0.011}_{-\,0.016}$	0.1182 ± 0.0007	0.1152 ± 0.0007
SLAC		0.1173 ± 0.0003	0.1128 ± 0.0003
HERA comb.		0.1174 ± 0.0003	0.1126 ± 0.0002
DY		0.108 ± 0.010	0.101 ± 0.025
ABM11		0.1180 ± 0.0012	0.1134 ± 0.0011

Tabelle 4: Comparison of the values of $\alpha_s(M_Z)$ obtained by BCDMS and NMC at NLO with the individual results of the fit in the present analysis at NLO and NNLO for the HERA data the NMC data the BCDMS data the SLAC data and the DY data.

$\alpha_s(M_Z^2)$: ABM11 + Tevatron jets

Experiment	$\alpha_s(M_Z)$		
	NLO_{exp}	NLO	NNLO*
D0 1 jet	$0.1161 \begin{smallmatrix} +\ 0.0041 \\ -\ 0.0048 \end{smallmatrix}$	0.1190 ± 0.0011	0.1149 ± 0.0012
D0 2 jet		0.1174 ± 0.0009	0.1145 ± 0.0009
CDF 1 jet (cone)		0.1181 ± 0.0009	0.1134 ± 0.0009
CDF 1 jet ($k_\perp$)		0.1181 ± 0.0010	0.1143 ± 0.0009
ABM11		0.1180 ± 0.0012	0.1134 ± 0.0011

Tabelle 5: Comparison of the values of $\alpha_s(M_Z)$ obtained by D0 with the ones based on including individual data sets of Tevatron jet data into the analysis at NLO. The NNLO* fit refers to the NNLO analysis of the DIS and DY data together with the NLO and soft gluon resummation corrections (next-to-leading logarithmic accuracy) for the 1 jet inclusive data.

$\alpha_s(M_Z^2)$: NNPDF

Experiment	$\alpha_s(M_Z)$		
	NLO$_{exp}$	NLO	NNLO
BCDMS	0.1111 ± 0.0018	0.1204 ± 0.0015	0.1158 ± 0.0015
NMC$_p$		0.1192 ± 0.0018	0.1150 ± 0.0020
NMC$_{pd}$	$0.117 \, {}^{+0.011}_{-0.016}$		0.1146 ± 0.0107
SLAC		> 0.124	> 0.124
HERA I		0.1223 ± 0.0018	0.1199 ± 0.0019
ZEUS H2		0.1170 ± 0.0027	0.1231 ± 0.0030
ZEUS F2C		0.1144 ± 0.0060	
NuTeV		0.1252 ± 0.0068	0.1177 ± 0.0039
E605		0.1168 ± 0.0100	
E866		0.1135 ± 0.0029	
CDF Wasy		0.1181 ± 0.0060	
CDF Zrap		0.1150 ± 0.0034	0.1205 ± 0.0081
D0 Zrap		0.1227 ± 0.0067	
CDF R2KT		0.1228 ± 0.0021	0.1225 ± 0.0021
D0 R2CON	$0.1161 \, {}^{+0.0041}_{-0.0048}$	0.1141 ± 0.0031	0.1111 ± 0.0029
NN21		0.1191 ± 0.0006	0.1173 ± 0.0007

Tabelle 7: Comparison of the values of $\alpha_s(M_Z)$ obtained by BCDMS, NMC, and D0 at NLO with the results of NN21 for the fits to DIS and other hard scattering data at NLO and NNLO and the corresponding response of the different data sets analysed.

$\alpha_s(M_Z^2)$: MSTW08/09

Experiment	$\alpha_s(M_Z)$		
	NLO$_{exp}$	NLO	NNLO
BCDMS $\mu p, F_2$	0.1111 ± 0.0018	–	0.1085 ± 0.0095
BCDMS $\mu d, F_2$		0.1135 ± 0.0155	0.1117 ± 0.0093
NMC $\mu p, F_2$	$0.117 \ ^{+0.011}_{-0.016}$	0.1275 ± 0.0105	0.1217 ± 0.0077
NMC $\mu d, F_2$		0.1265 ± 0.0115	0.1215 ± 0.0070
NMC $\mu n/\mu p$		0.1280	0.1160
E665 $\mu p, F_2$		0.1203	–
E665 $\mu d, F_2$		–	–
SLAC ep, F_2		0.1180 ± 0.0060	0.1140 ± 0.0060
SLAC ed, F_2		0.1270 ± 0.0090	0.1220 ± 0.0060
NMC,BCDMS,SLAC, F_L		0.1285 ± 0.0115	0.1200 ± 0.0060
E886/NuSea pp, DY %citeWebb:2003bj		–	0.1132 ± 0.0088
E886/NuSea pd/pp, DY		0.1173 ± 0.107	0.1140 ± 0.0110
NuTeV $\nu N, F_2$		0.1207 ± 0.0067	0.1170 ± 0.0060
CHORUS $\nu N, F_2$		0.1230 ± 0.0110	0.1150 ± 0.0090
NuTeV $\nu N, xF_3$		0.1270 ± 0.0090	0.1225 ± 0.0075
CHORUS $\nu N, xF_3$		0.1215 ± 0.0105	0.1185 ± 0.0075
CCFR		0.1190	–
NuTeV $\nu N \to \mu\mu X$		0.1150 ± 0.0170	–
H1 ep 97-00, $\sigma_r^{\rm NC}$		0.1250 ± 0.0070	0.1205 ± 0.0055
ZEUS ep 95-00, $\sigma_r^{\rm NC}$		0.1235 ± 0.0065	0.1210 ± 0.0060
H1 ep 99-00, $\sigma_r^{\rm CC}$		0.1285 ± 0.0225	0.1270 ± 0.0200
ZEUS ep 99-00, $\sigma_r^{\rm CC}$		0.1125 ± 0.0195	0.1165 ± 0.0095
H1/ZEUS ep, $F_2^{\rm charm}$		–	0.1165 ± 0.0095
H1 ep 99-00 incl. jets	$0.1168 \ ^{+0.0049}_{-0.0034}$	0.1127 ± 0.0093	
ZEUS ep 96-00 incl. jets	$0.1208 \ ^{+0.0048}_{-0.0040}$	0.1175 ± 0.0055	
D0 II $p\bar{p}$ incl. jets	$0.1161 \ ^{+0.0041}_{-0.0048}$	0.1185 ± 0.0055	0.1133 ± 0.0063
CDF II $p\bar{p}$ incl. jets		0.1205 ± 0.0045	0.1165 ± 0.0025
D0 II $W \to l\nu$ asym.		–	–
CDF II $W \to l\nu$ asym.		–	–
D0 II Z rap.		0.1125 ± 0.0100	0.1136 ± 0.0084
CDF II Z rap.		0.1160 ± 0.0070	0.1157 ± 0.0067
MSTW		$0.1202 \ ^{+0.0012}_{-0.0015}$	0.1171 ± 0.0014

Table 8: Comparison of the values of $\alpha_s(M_Z)$ obtained by BCDMS, NMC, HERA-jet and D0 at NLO with the results of the MSTW fits to DIS and other hard scattering data at NLO and NNLO and the corresponding response of the different data sets analysed, cf. Figs. 7a and 7b in MSTW08. Entries not given correspond to $\alpha_s(M_Z)$ central values below 0.110 or above 0.130; in case no errors are assigned these are larger than the bounds provided in form of the plots in MSTW08.

Higher Twist Contributions

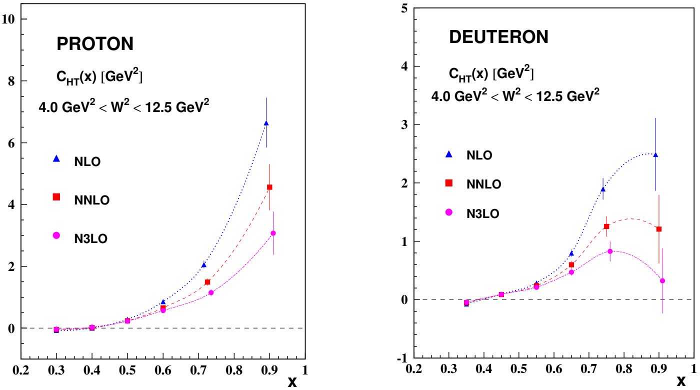
ABM11 :

To completely remove the HT-terms the cut $W^2 > 12.5$ GeV^2, $Q^2 > 10$ GeV^2 is necessary!

	$H_2^{\rm p}(x)/\mathrm{GeV}^2$	$H_2^{\rm ns}(x)/\mathrm{GeV}^2$	$H_T^{\rm p}(x)/\mathrm{GeV}^2$
$x = 0.1$	-0.036 $\pm$ 0.012	-0.034 $\pm$ 0.023	-0.091 $\pm$ 0.017
$x = 0.3$	-0.016 $\pm$ 0.008	0.006 $\pm$ 0.017	-0.061 $\pm$ 0.012
$x = 0.5$	0.026 $\pm$ 0.007	-0.0020 $\pm$ 0.0094	0.0276 $\pm$ 0.0081
$x = 0.7$	0.053 $\pm$ 0.005	-0.029 $\pm$ 0.006	0.031 $\pm$ 0.006
$x = 0.9$	0.0071 $\pm$ 0.0026	0.0009 $\pm$ 0.0041	0.0002 $\pm$ 0.0015

Tabelle 3: The parameters of the twist-4 contribution to the DIS structure functions in for the fit to NNLO accuracy in QCD.

Extracting the Higher Twist Contributions to $F_2(x, Q^2)$



$$F_2(x, Q^2) = F_2^{\tau=2}(x, Q^2)\left[1 + \frac{C(\langle Q^2\rangle, x)}{Q^2}\right]$$

JB & H. Böttcher, 2012

$\alpha_s(M_Z^2)$: Correlation with HT

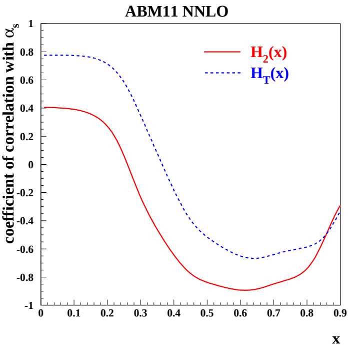


Figure 20: The correlation coefficient of $\alpha_s(M_Z)$ with the nucleon twist-4 coefficients H_2 (solid line) and H_T (dashes) versus x as obtained in our NNLO fit.

$\alpha_s(M_Z^2)$: JR14 analysis

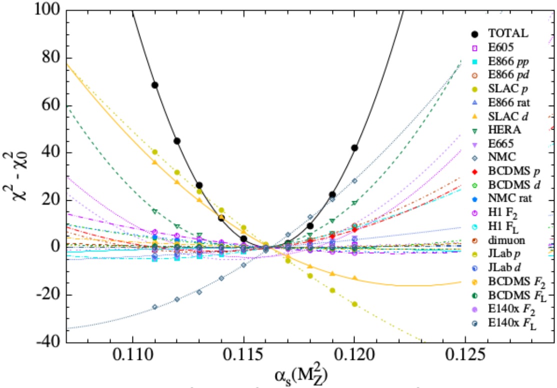


Figure 3: The dependence of χ^2 on $\alpha_s(M_Z^2)$ around the minimum χ_0^2 for our standard NNLO analysis.

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$$\alpha_s^{val}(M_Z^2) = 0.1136 \pm 0.0004 \qquad \alpha_s^{std}(M_Z^2) = 0.1162 \pm 0.0006$$

$\alpha_s(M_Z^2)$: Global DIS Comparison

Data Set	ABM11	BBG	NN21	MSTW
BCDMS	0.1048 ± 0.0013	0.1126 ± 0.0007	0.1158 ± 0.0015	0.1101 ± 0.0094
NMC	0.1152 ± 0.0007	0.1153 ± 0.0039	0.1150 ± 0.0020	0.1216 ± 0.0074
SLAC	0.1128 ± 0.0003	0.1158 ± 0.0034	> 0.124	$\begin{cases} 0.1140 \pm 0.0060 & \text{ep} \\ 0.1220 \pm 0.0060 & \text{ed} \end{cases}$
HERA	0.1126 ± 0.0002		$\begin{cases} 0.1199 \pm 0.0019 \\ 0.1231 \pm 0.0030 \end{cases}$	0.1208 ± 0.0058
DY	0.101 ± 0.025	–	–	0.1136 ± 0.0100
	0.1134 ± 0.0011	0.1134 ± 0.0020	0.1173 ± 0.0007	0.1171 ± 0.0014

Table 9: Comparison of the pulls in $\alpha_s(M_Z)$ per data set between the ABM11, BBG, NN21, MSTW analyses at NNLO.

Use: $W^2 > 12.5$ GeV^2, $Q^2 > 2.5$ GeV^2 and no HT: $\alpha_s(M_Z^2) = 0.1191 \pm 0.0016$
Use: $W^2 > 12.5$ GeV^2, $Q^2 > 10$ GeV^2 and no HT: $\alpha_s(M_Z^2) = 0.1134 \pm 0.0008$
MSTW08 and NNPDF: no HT corrections.
Taking into account error correlations is of importance.
2000: First MSTW NNLO analysis; 2008: $\alpha_s^{NNLO,\overline{MS}}$;
2014: $\alpha_s^{NNLO,DIS}(M_Z) = 0.1136$

$\alpha_s(M_Z^2)$ from NNLO DIS(+) analyses [from ABM13 and update]

	$\alpha_s(M_Z^2)$	
Alekhin [2001]	0.1143 ± 0.013	DIS
BBG [2004]	$0.1134 \ ^{+0.0019}_{-0.0021}$	valence analysis, NNLO
GRS	0.112	valence analysis, NNLO
ABKM	0.1135 ± 0.0014	HQ: FFNS $N_f = 3$
JR14	0.1136 ± 0.0004	dynamical approach
JR14	0.1162 ± 0.0006	including NLO-jets
MSTW	0.1171 ± 0.0014	(2009)
Thorne	0.1136	[DIS+DY, HT*] (2014)
ABM11_J	$0.1134 - 0.1149 \pm 0.0012$	Tevatron jets (NLO) incl.
ABM13	0.1133 ± 0.0011	
ABM13	0.1132 ± 0.0011	(without jets)
CTEQ	0.1159..0.1162	
CTEQ	0.1140	(without jets)
NN21	$0.1174 \pm 0.0006 \pm 0.0001$	
Gehrmann et al.	$0.1131 \ ^{+\ 0.0028}_{-\ 0.0022}$	e^+e^- thrust
Abbate et al.	0.1140 ± 0.0015	e^+e^- thrust
ATLAS/CMS	0.1151 ± 0.0033	$t\bar{t}$
BBG [2004]	$0.1141 \ ^{+0.0020}_{-0.0022}$	valence analysis, N^{3}LO

$$\Delta_{\rm TH}\alpha_s = \alpha_s(\rm N^3LO) - \alpha_s(\rm NNLO) + \Delta_{\rm HQ} = +0.0009 \pm 0.0006_{\rm HQ}$$

NNLO accuracy is needed to analyze the world data. $\Longrightarrow$ NNLO HQ corrections needed.

$\alpha_s(M_Z^2)$: further determinations

	$\alpha_s(M_Z^2)$	
3-jet rate	0.1175 ± 0.0025	Dissertori et al. 2009
Z-decay rate	0.1189 ± 0.0026	BCK 2008/12 (N^{3}LO)
τ-decay rate	0.1212 ± 0.0019	BCK 2008 (N^{3}LO)
τ-decay rate	0.1204 ± 0.0016	Pich 2011
τ-decay rate	0.325 ± 0.018 (at m_τ)	FOTP: Jamin 2013
τ-decay rate	0.374 ± 0.025 $(at(m_\tau)$	CIPT: Jamin 2013
Lattice	0.1205 ± 0.0010	PACS-CS 2009 (2+1 fl.)
Lattice	0.1184 ± 0.0006	HPQCD 2010
Lattice	0.1200 ± 0.0014	ETMC 2012 (2+1+1 fl.)
Lattice	0.1156 ± 0.0022	Brambilla et al. (2+1 fl.)
Lattice	$0.1130 \pm 0.0010(stat)$	RBC-UKQCD
world average	0.1183 ± 0.0010	(2011)

Conclusions

- The DIS+DY analyses lead to a low value of $\alpha_s(M_Z^2) = 0.1132 \pm 0.0011(NNLO)$ and in the NS-case of $\alpha_s(M_Z^2) = 0.1141 \pm 0.0022(N^3LO)$.
- Other groups have obtained comparable quite values by now (except of NNPDF).
- e^+e^- thrust analyses lead to low values of $\alpha_s(M_Z^2)$.
- LHC jet data seem to lead to low values of $\alpha_s(M_Z^2)$ as well.
- τ-decay analyses and inclusive e^+e^- analyses lead to higher values.
- Lattice results have improved recently and are expected to deliver final values soon.
- Errors on $\alpha_s(M_Z^2)$ below 1% cannot be obtained at present due to different theory-uncertainties.