Determination of the running top quark mass

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Top quark mass

Experimental result CDF & D0 coll. 1305.3929

$$m_t = 173.20 \pm 0.87\,\text{GeV}$$

Top quark mass

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Which is the value of the top quark mass ?

$$m_t = ?$$

Top quark mass

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$$m_t = 173.20 \pm 0.87\,\text{GeV}$$

Which is the value of the top quark mass ?

$$m_t = ?$$

Which top quark mass has this value ?

$$? = 173.20 \pm 0.87\,\text{GeV}$$

Top quark mass

Some Answers

All in all I believe that it is justified to assume that MC mass parameter is interpreted as mpole, within the ambiguity intrinsic in the definition of mpole, thus at the level of $\sim 250 - 500$ MeV.

M. Mangano @ Top2103

That is, we can state as the final result for the likely relation between the top quark mass measured using a given Monte Carlo event generator ("MC") and the pole mass as

$$m_{\text{pole}} = m_{\text{MC}} + Q_0 \left[\alpha_s(Q_0) c_1 + \ldots\right]$$

where $Q_0 \sim 1$ GeV and c_1 is unknown, but presumed to be of order 1 and, according to the argument above, presumed to be positive.

A. Buckley et al. arXiv:1101.2599

Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top quark mass m_t

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_b + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i} \not{D} - m_q\right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a\right)_{ij} A^a_\mu$

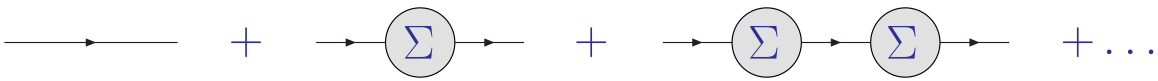
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q

Challenge

- Suitable observables for measurements of α_s, m_q, . . .
  - comparison of theory predictions and experimental data

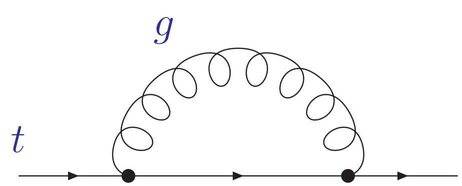
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

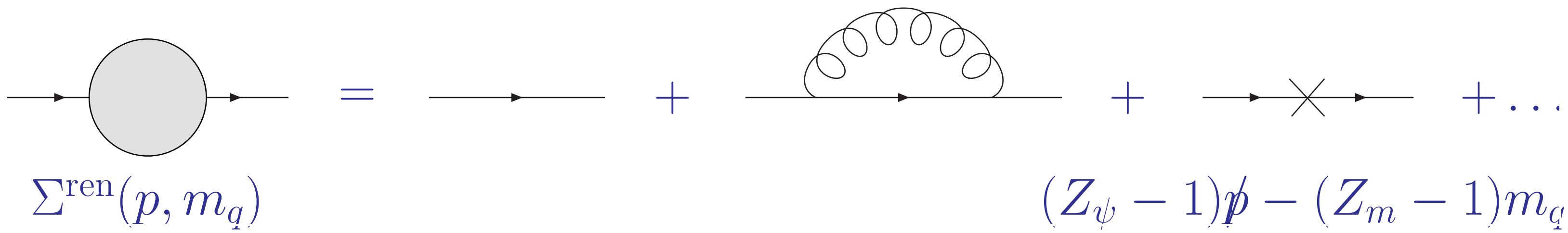
QCD



- QCD corrections to self-energy $\Sigma(p, m_q)$
  - dimensional regularization $D = 4 - 2\epsilon$
  - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)

$$\Sigma^{(1),\mathrm{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_q^2}\right)^{\epsilon} \left\{ (\not{p} - m_q) \left(-C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left(3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}$$

- Relate bare and renormalized mass parameter $m_q^{\mathrm{bare}} = m_q^{\mathrm{ren}} + \delta m_q$



Mass renormalization scheme

Pole mass

- Based on (unphysical) concept of top quark being a free parton
  - m_q^{ren} coincides with pole of propagator at each order

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{\not{p}=m_q} \rightarrow \not{p} - m_q^{\mathrm{pole}}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$

$\overline{MS}$ *scheme*

- $\overline{MS}$ mass definition
  - one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$$

- $\overline{MS}$ scheme induces scale dependence: $m(\mu)$

Scheme transformations

- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and $\overline{MS}$ mass
  - known to three loops in QCD Gray, Broadhurst, Gräfe, Schilcher ‘90; Chetyrkin, Steinhauser ‘99; Melnikov, v. Ritbergen ‘99
  - example: one-loop QCD

$$m^{\rm pole} = m(\mu) \left\{ 1 + \frac{\alpha_s(\mu)}{4\pi} \left(\frac{4}{3} + \ln\left(\frac{\mu^2}{m(\mu)^2} \right) \right) + \dots \right\}$$

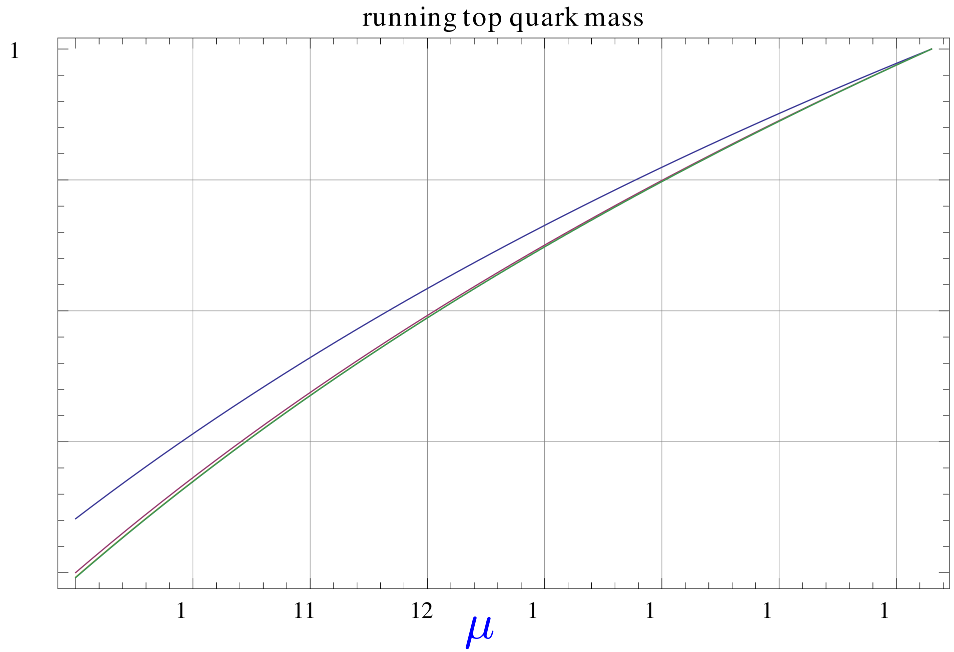
Running quark mass

Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to four loops
    Chetyrkin '97; Larin, van Ritbergen, Vermaseren '97

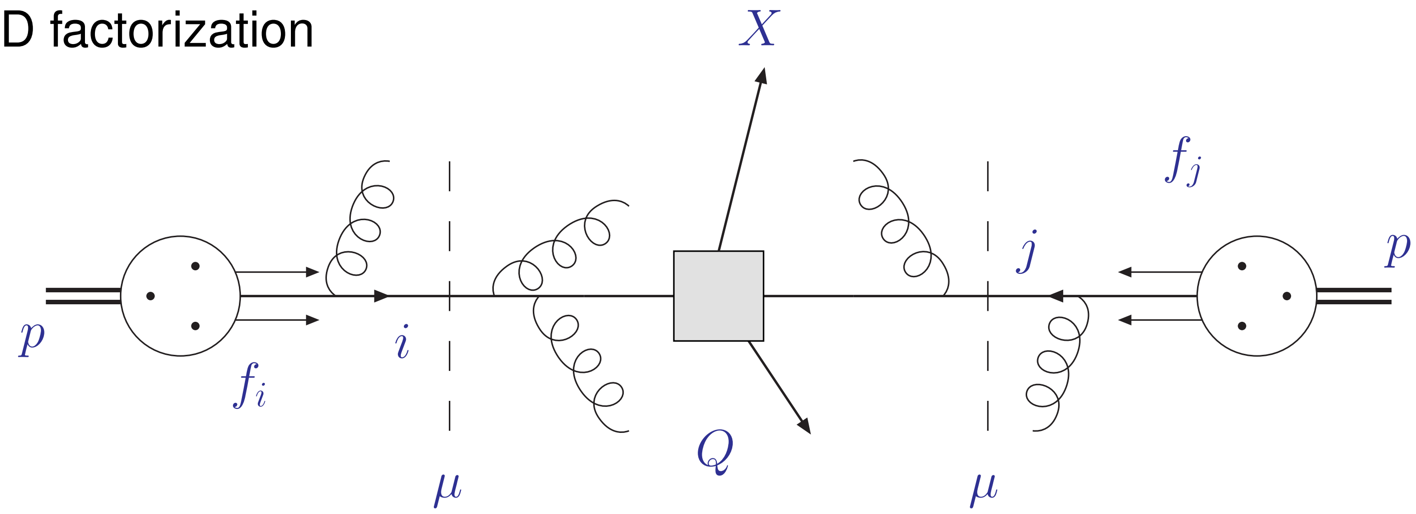
$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right) m(\mu) \;=\; \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



QCD factorization

- QCD factorization



$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X} \left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, ... ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, ...

Non-perturbative parameters

Input for collider phenomenology

- Non-perturbative parameters are universal
- Determination from comparision to experimental data
  - masses of heavy quarks m_c, m_b, m_t
  - parton distribution functions $f_i(x, \mu^2)$
  - strong coupling constant $\alpha_s(M_Z)$

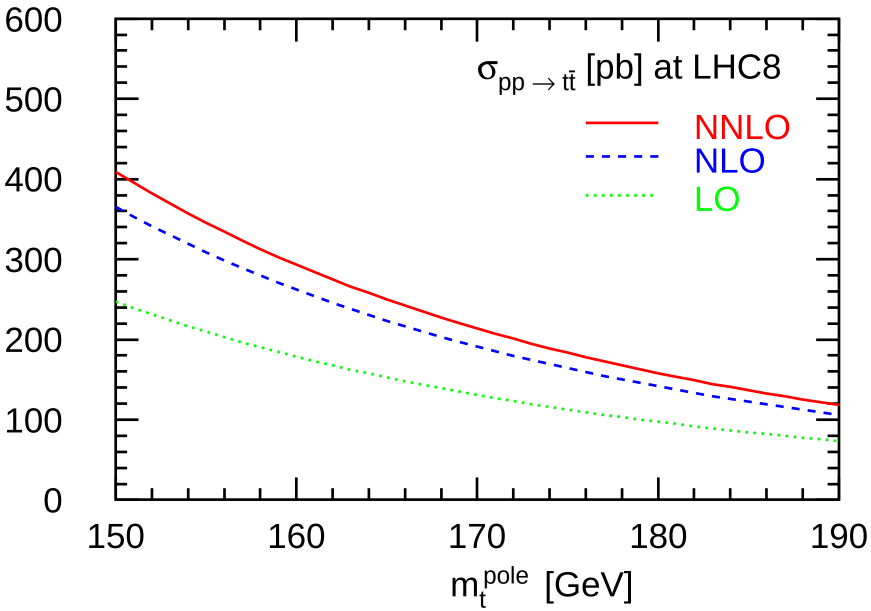
Interplay with perturbation theory

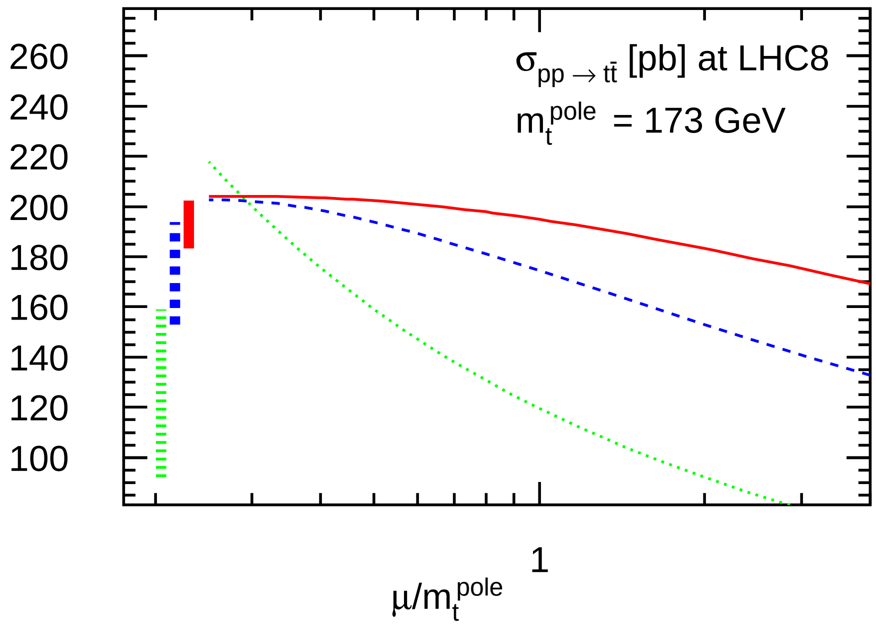
- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
  - radiative corrections at higher orders
  - renormalization and factorization scales μ_R, μ_F
  - chosen scheme (e.g., $\overline{MS}$ scheme)
  - ...

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13

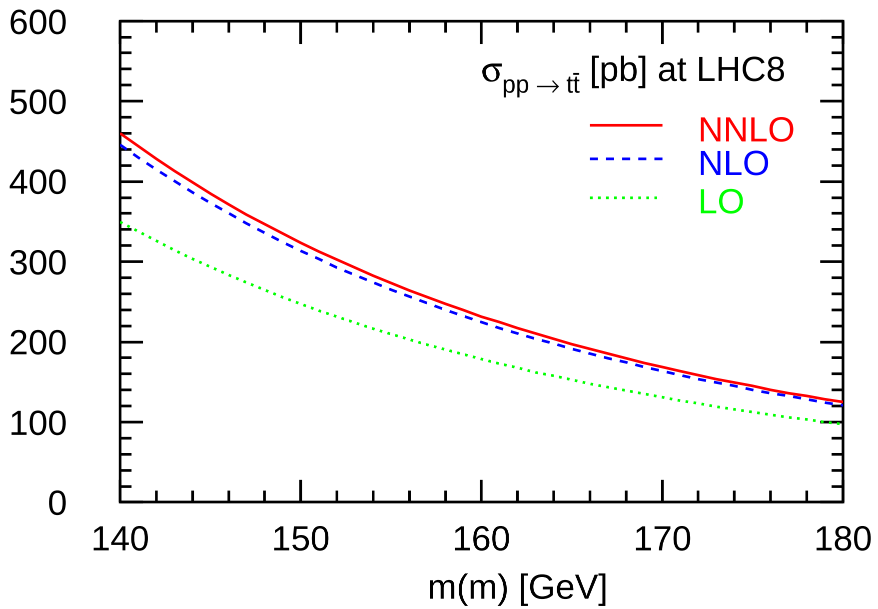


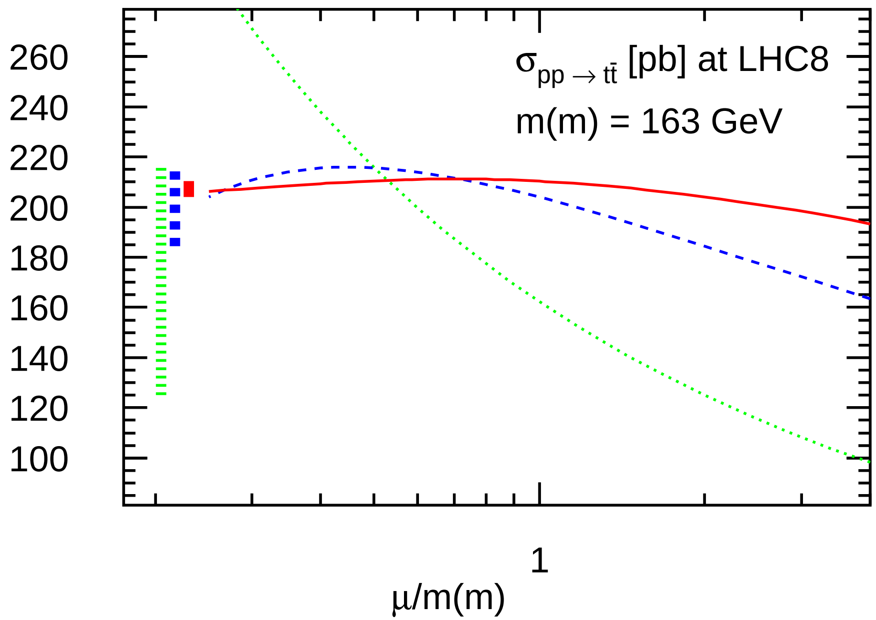


- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Total cross section with $\overline{MS}$ mass

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
  - conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory: $m_t = m(\mu_R)\left(1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)}\right)$
- Good apparent convergence of perturbative expansion
- Small theoretical uncertainity form scale variation





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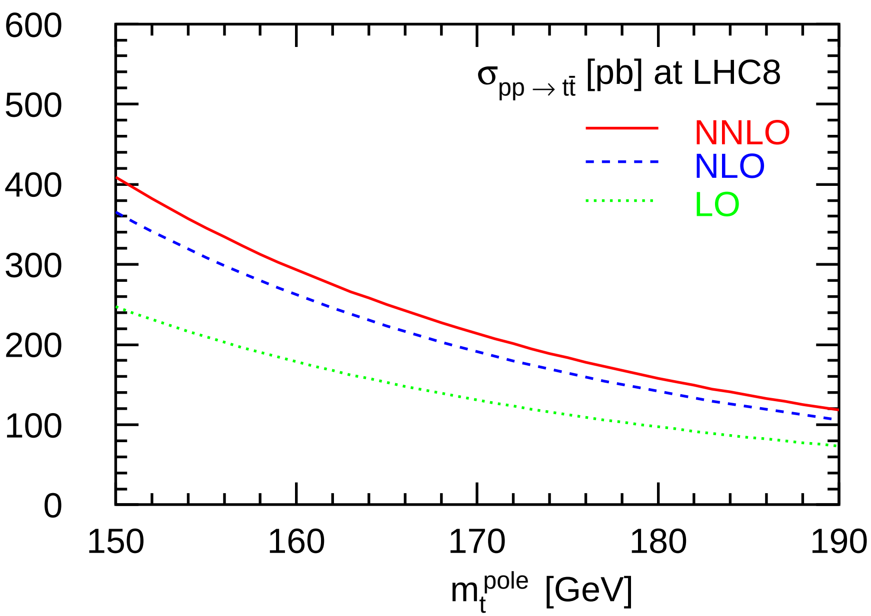
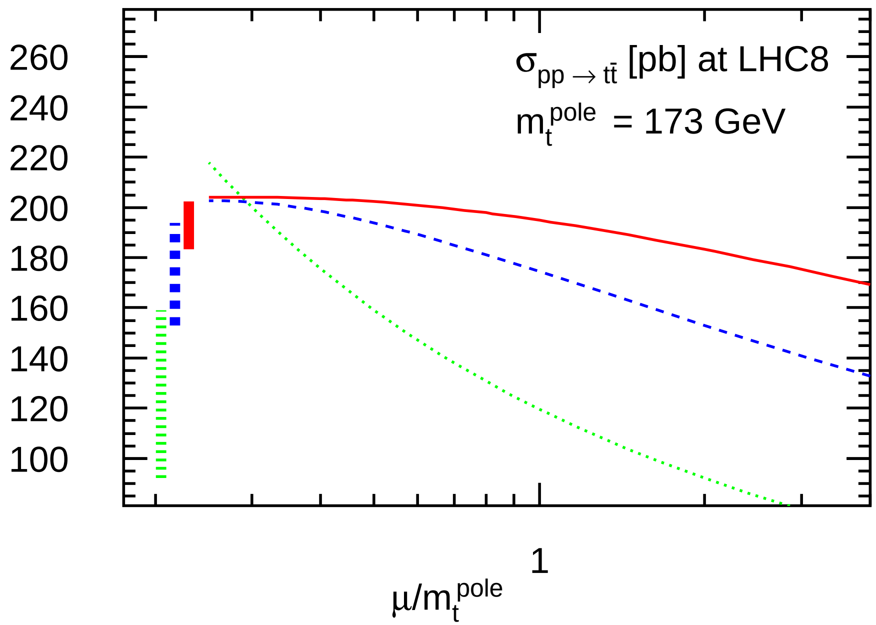
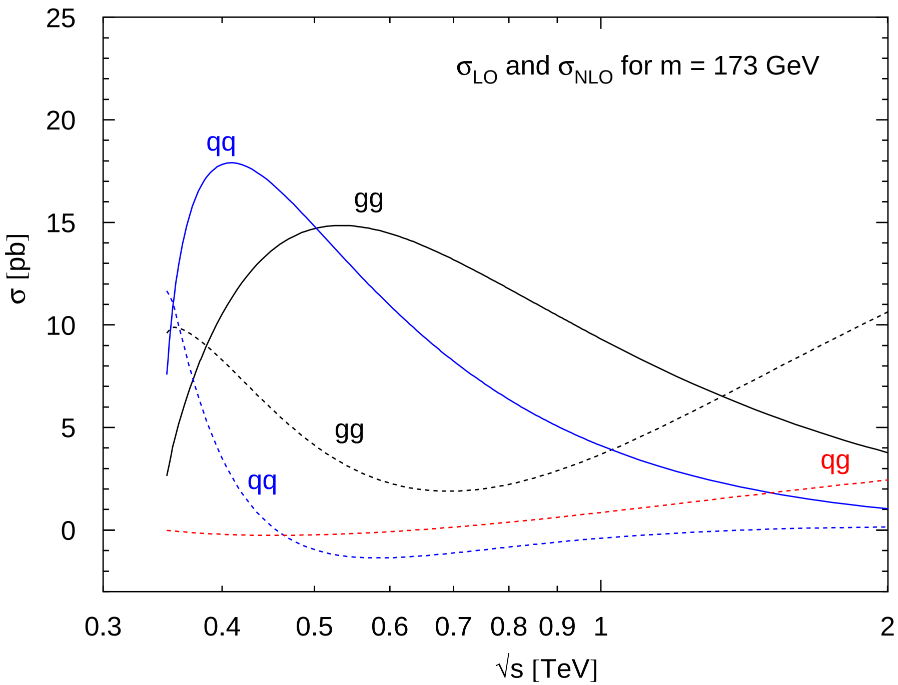
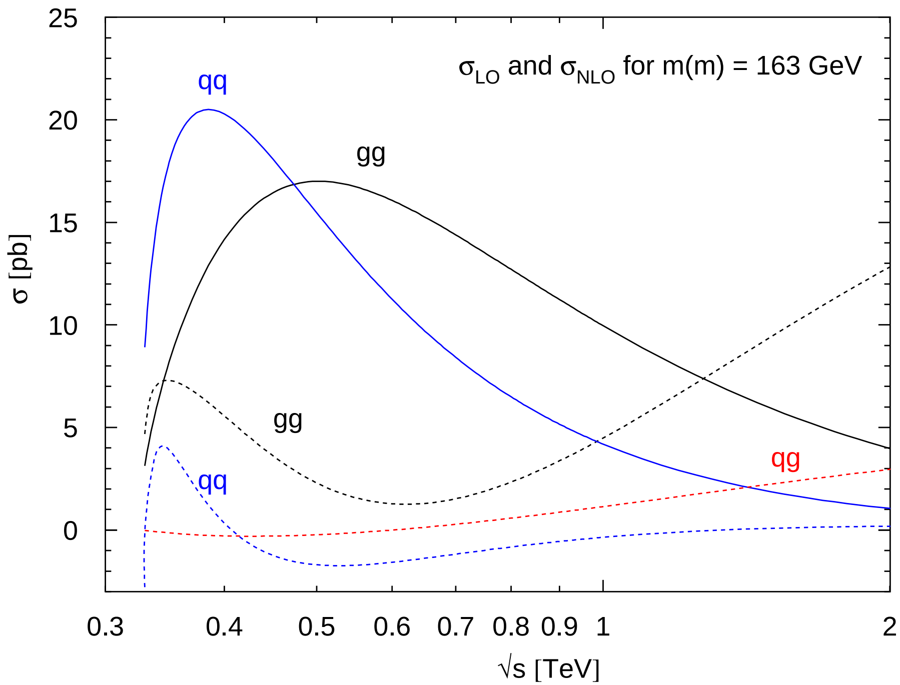
- Pole mass scheme for comparison





- Perturbative stability of predictions with $\overline{MS}$ mass definition
- Parton cross section for channels $q\bar{q}$, gg and qg
  - on-shell scheme for $m_t = 173$ GeV (left)
  - $\overline{MS}$ scheme for $m(m) = 163$ GeV (right)





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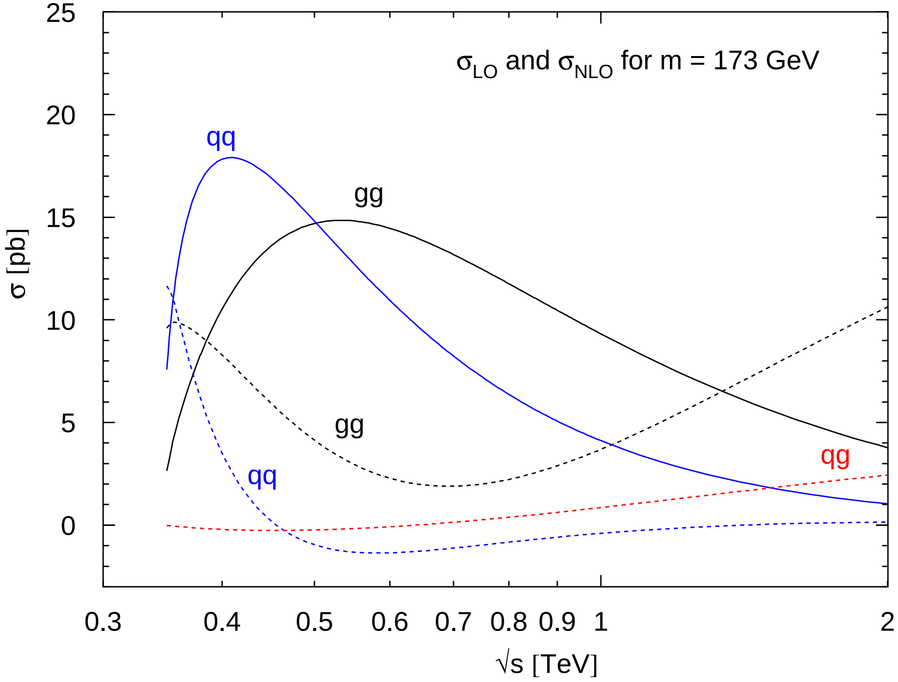
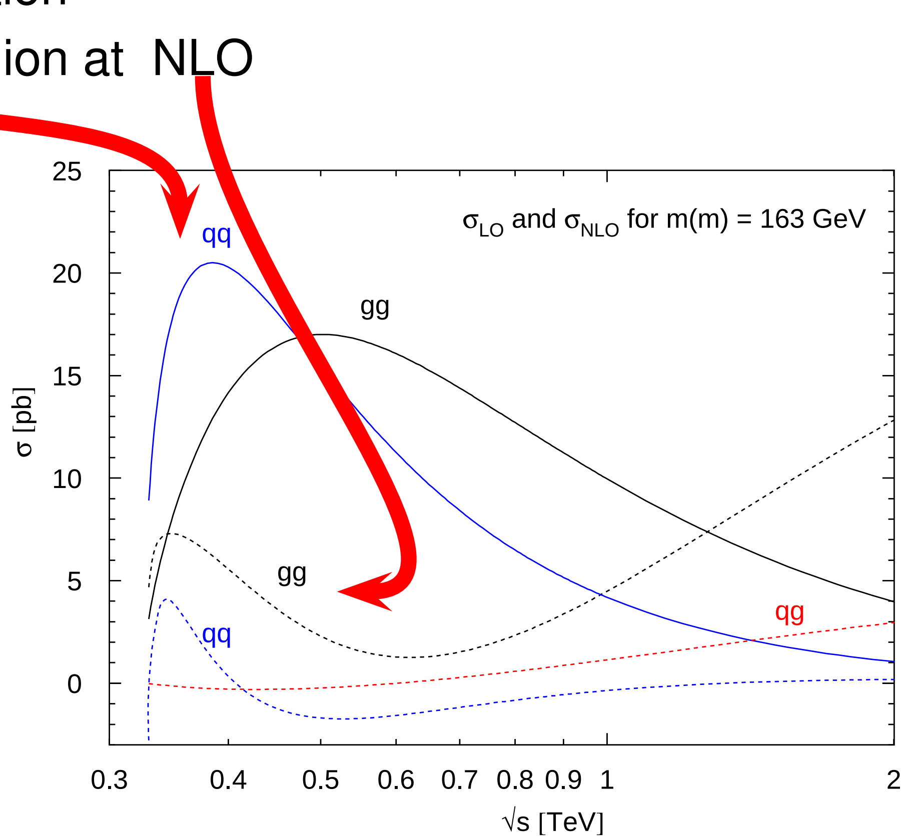
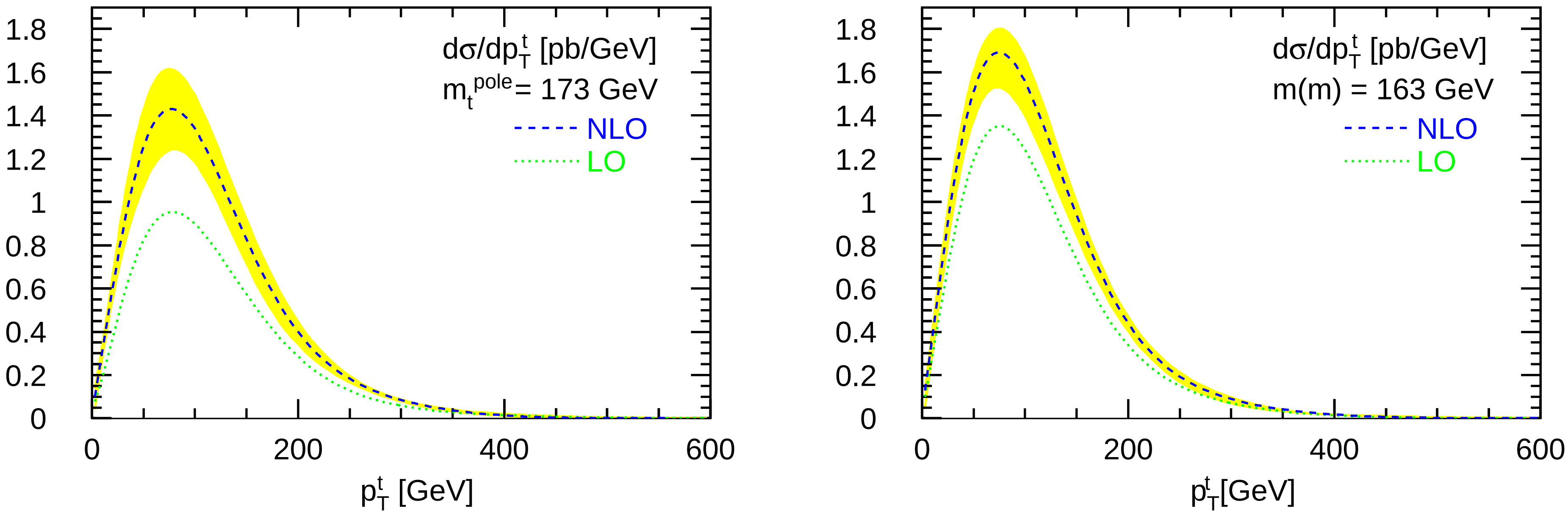
  - $\overline{MS}$ scheme for $m(m) = 163$ GeV (right)
- $\overline{MS}$ scheme
  - more emphasis on LO contribution
  - less significance to threshold region at NLO





Differential cross sections

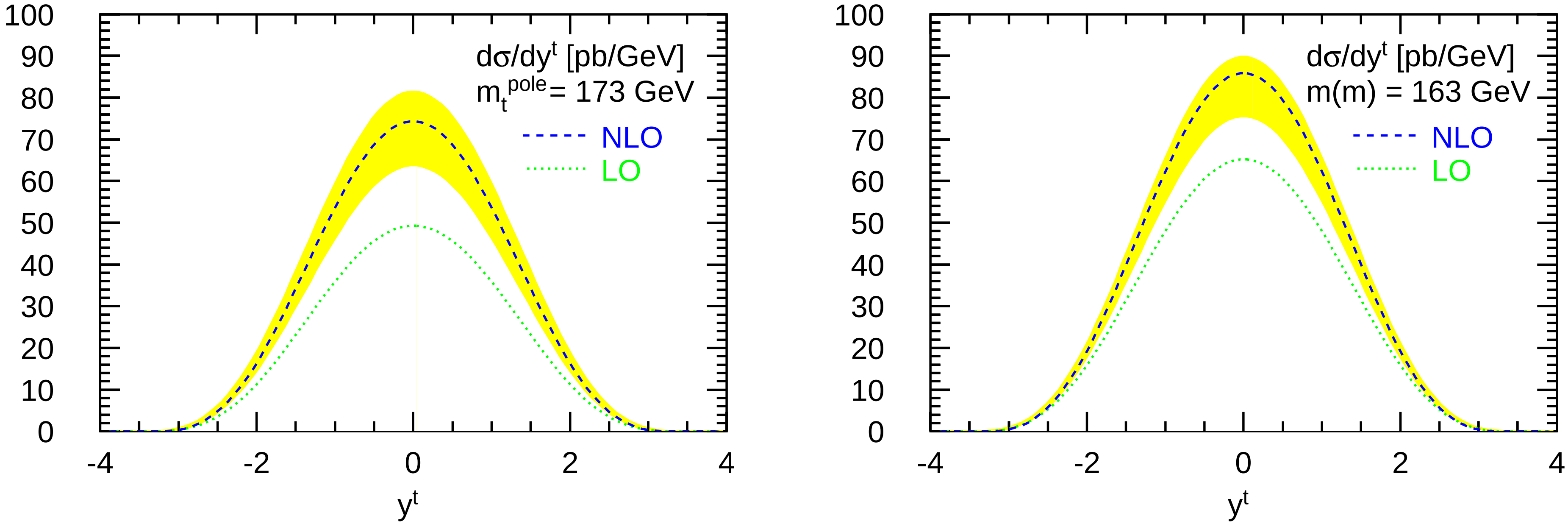
NLO in QCD



- Running mass for differential distributions shows same features, e.g. p_T^t-distribution   Dowling, S.M. '13

Differential cross sections

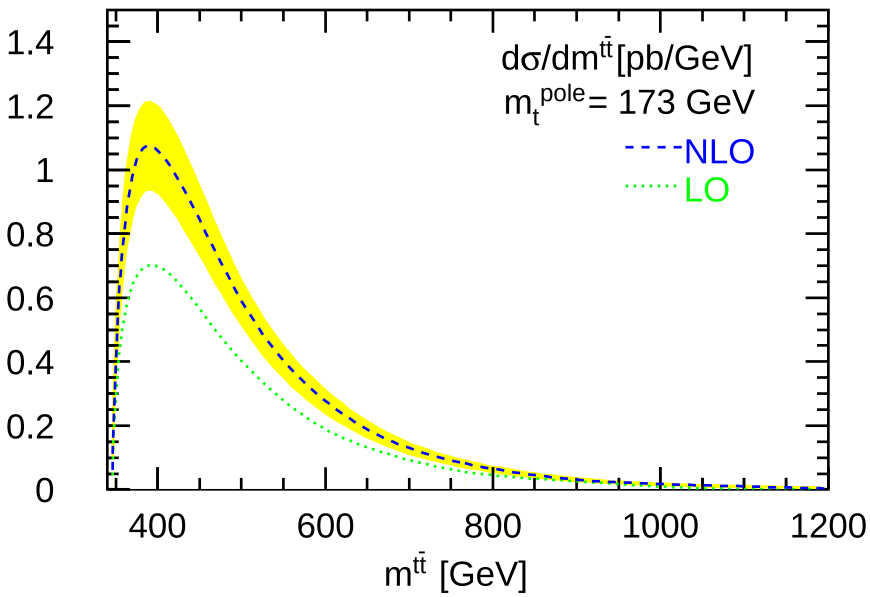
NLO in QCD

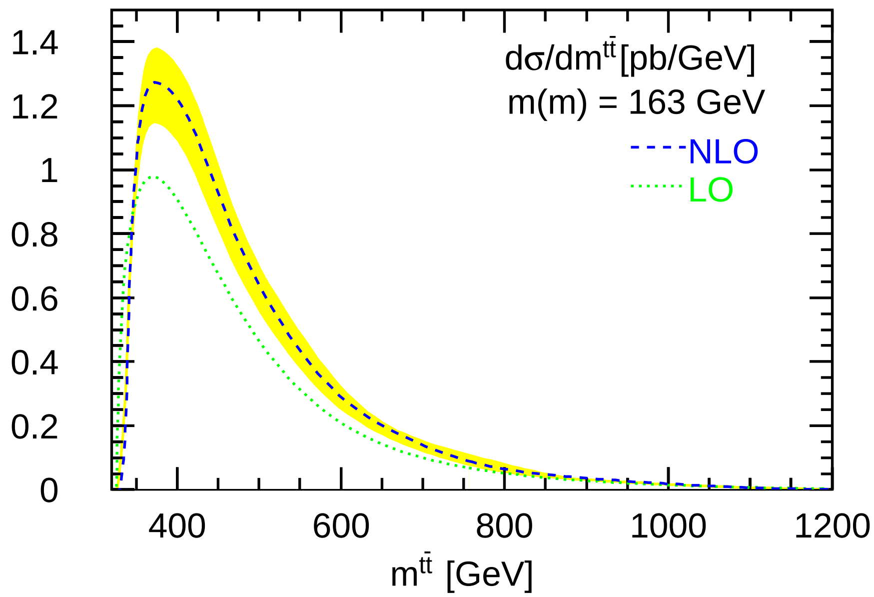


- Running mass for differential distributions shows same features, e.g. y^t-distribution   Dowling, S.M. '13

Differential cross sections

NLO in QCD





- Running mass for differential distributions shows same features, e.g. $m_{t\bar{t}}$-distribution Dowling, S.M. '13

The fine print

- Intrinsic limitation of sensitivity in total cross section

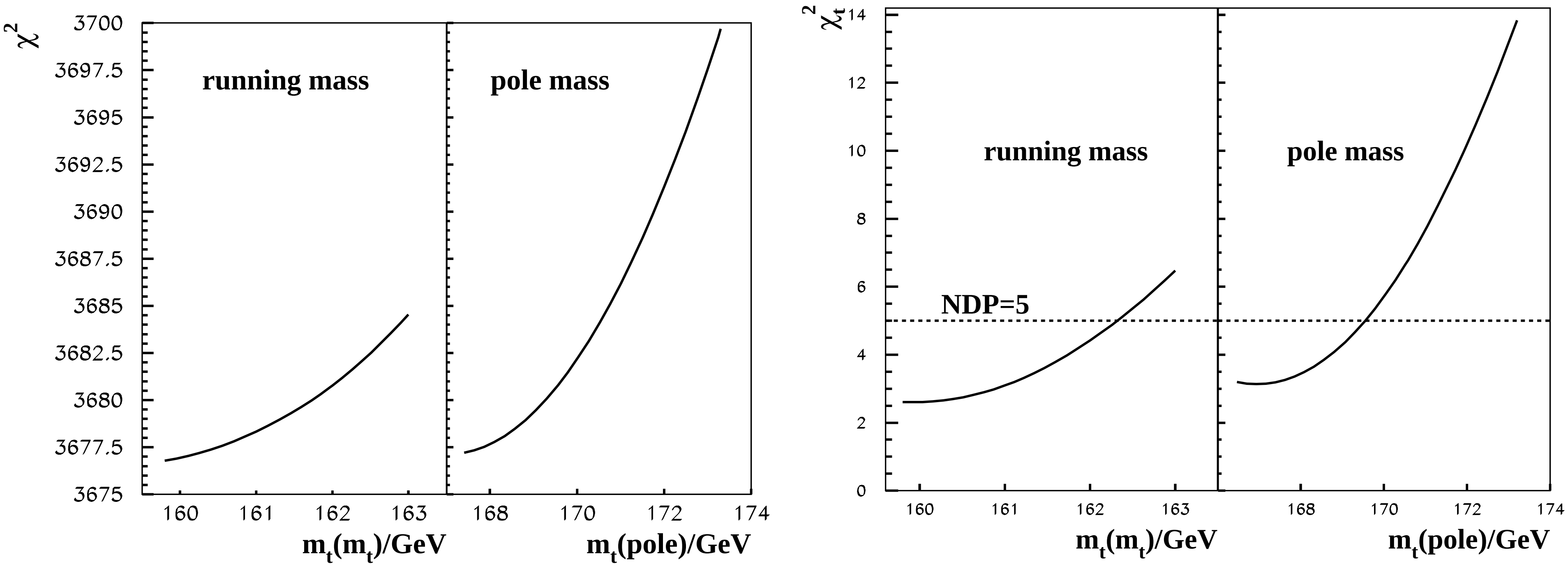
$$\left| \frac{\Delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \right| \simeq 5 \times \left| \frac{\Delta m_t}{m_t} \right|$$

Correlations are essential

- Cross section at LHC has correlation of m_t, $\alpha_S(M_Z)$ and gluon PDF
  $\sigma_{t\bar{t}} \sim \alpha_s^2 \, m_t^2 \, g(x) \otimes g(x)$
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2}$
  - fit with fixed values of m_t and $\alpha_S(M_Z)$ carries significant bias
    Czakon, Mangano, Mitov, Rojo '13
  - likewise, fit with PDF re-weighting and for fixed values of m_t insufficient Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan '12

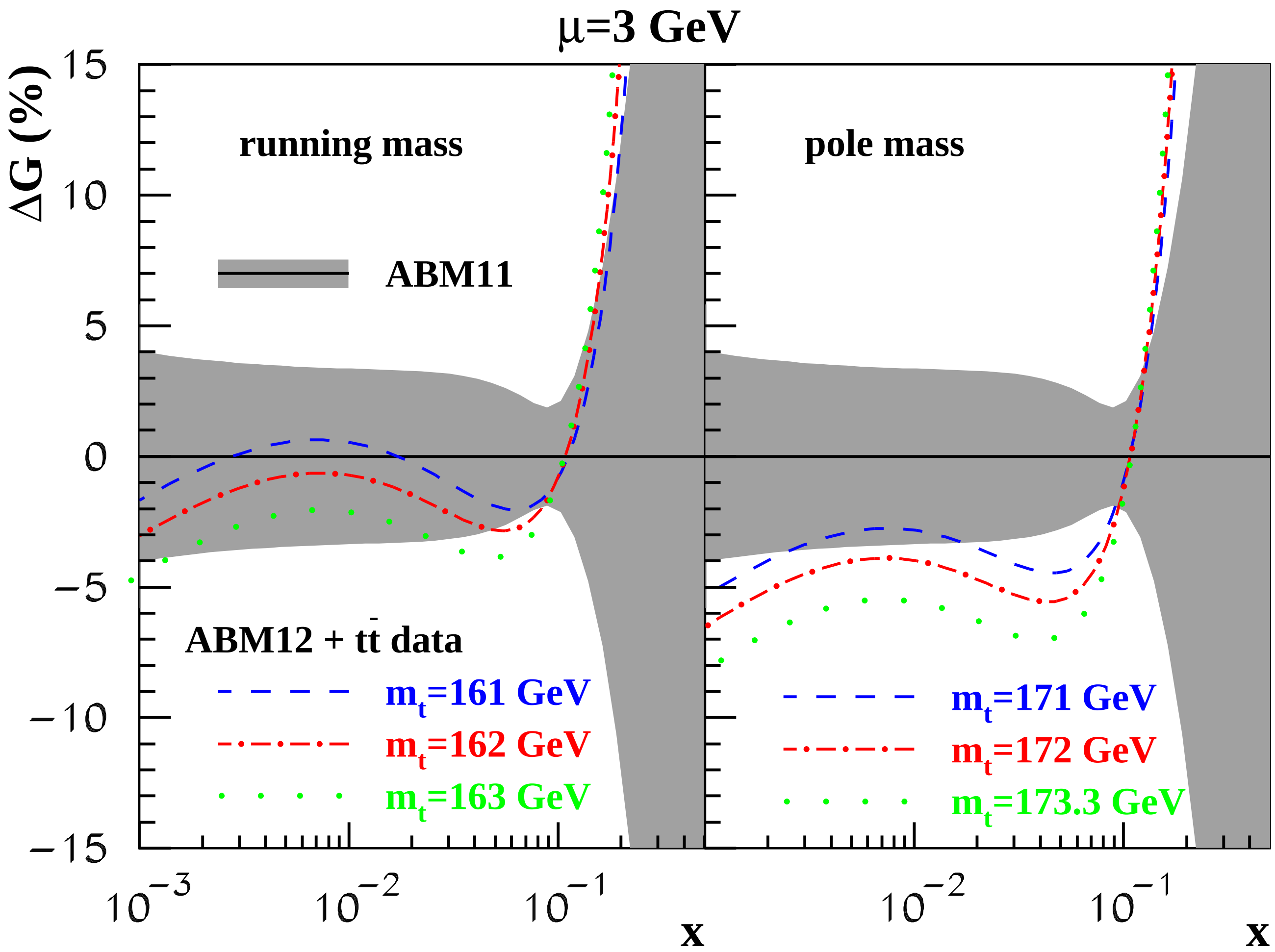
Top cross section data in ABM12 fit

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162.3 \pm 2.3$ GeV (equivalent to pole mass $m_t = 171.2 \pm 2.4$ GeV) Alekhin, Blümlein, S.M. '13
  - χ^2-profile steeper for pole mass (bigger impact of top-quark data and greater sensitivity to theoretical uncertainty at NNLO)



Top cross section data in ABM12 fit

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - correlation of gluon PDF with value of m_t
    (illustration of bias in recent analysis Czakon, Mangano, Mitov, Rojo '13)



Summary

Top quark mass

- Running mass ($\overline{\mathrm{MS}}$ scheme) at NNLO in QCD

$$m_t(m_t) = 163.3 \pm 2.3\mathrm{GeV}$$

- On-shell scheme (pole mass) at NNLO in QCD

$$m_t = 171.2 \pm 2.4\mathrm{GeV}$$

Summary

Top quark mass

- Top quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Radiative corrections at higher orders mandatory for scheme definition
- Correlations in data analysis are important, e.g. with α_s and PDFs

LHC measurements

- Inclusive and differential observables
- Very precise data and well-defined renormalization scheme definition
- $\overline{\mathrm{MS}}$ scheme for mass exhibits better convergence

Future challenge

- Joint effort theory and experiment