Variable Flavor Number Scheme (VFNS) for Final State Jets

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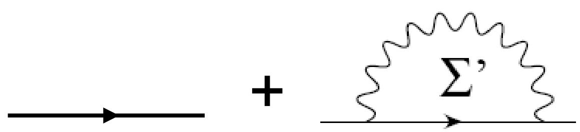
Motivation

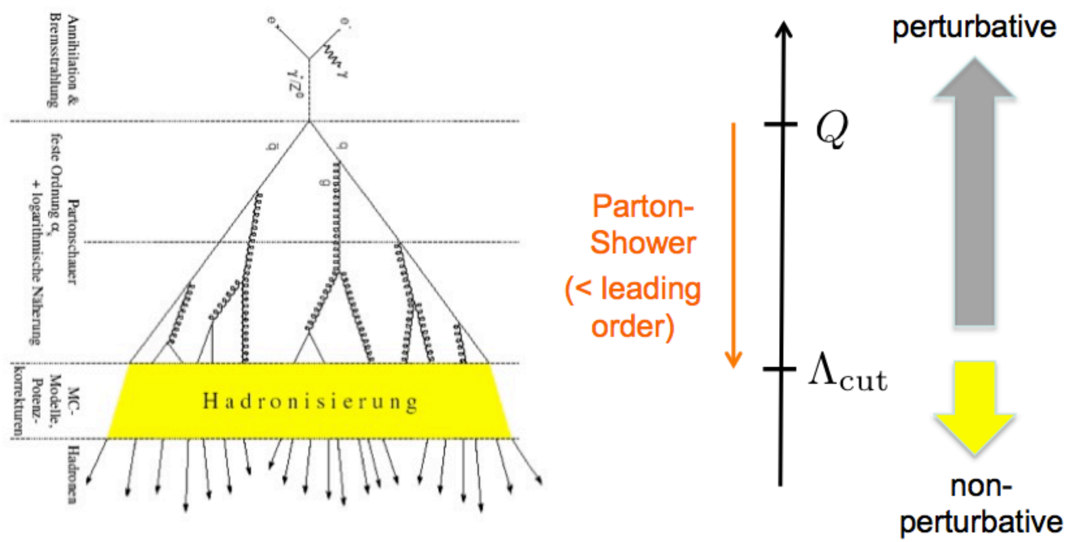
- Heavy quark masses: $\mathcal{L}_{\text{classic}} = -\frac{1}{4} F^{A}_{\alpha\beta} F^{\alpha\beta}_{A} + \sum_{\text{flavors } q} \bar{q}_{\alpha} (iD - m_q)_{\alpha\beta}\, q_b$

  Fields, couplings, masses in classic action are bare quantities that need to be renormalized to have (any) physical relevance



$$= \not{p} - m^0 + \Sigma(p, m^0)$$

- Monte-Carlo Mass
  - Most versatile and flexible tool
  - Can be applied for any observable
  - Mass scheme unclear
  - BUT: short-distance mass with scale R=Λ_{cut}

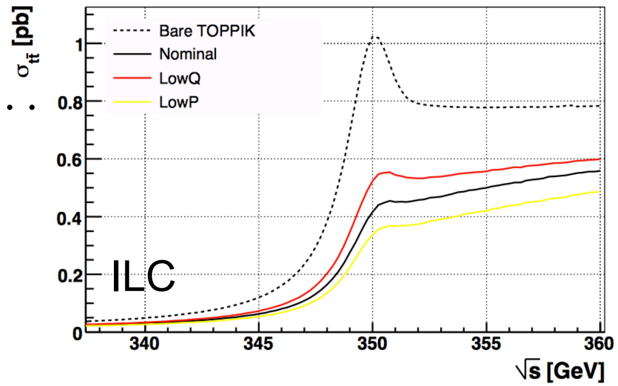


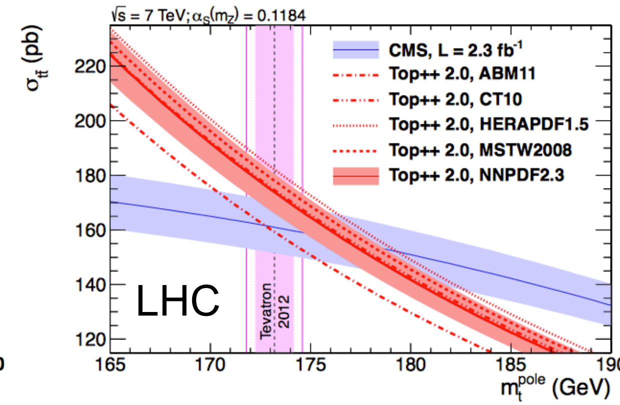
Motivation

- Total cross sections:

$$\sigma(t\bar{t} + X)\,,\ \mathrm{d}\sigma/\mathrm{d}p_T\,,\ \ldots$$

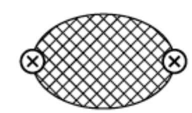
  - totally inclusive (OPE)
  - physically clean
  - theoretically clean
  - normalization dependent





→ Cancellation of IR sensitivity in self energy and interaction between quarks.

$$\mathbf{O}_{\mathbf{p}} = C(\mu)\cdot(\psi^\dagger_{\mathbf{p}}\,\boldsymbol{\sigma}\,\tilde{\chi}^*_{-\mathbf{p}}) + \cdots \qquad t\bar{t}\ (^3S_1)$$



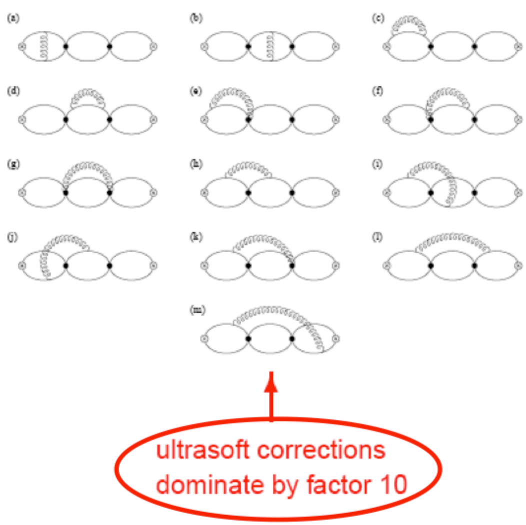
$$\sigma_{\rm tot} \;\propto\; \mathrm{Im}\left[\int \mathrm{d}^4\mathrm{x}\,\mathrm{e}^{-\mathrm{i}\hat{\mathrm{q}}\mathrm{x}}\left\langle 0\left|\mathrm{T}\,\mathbf{O}^\dagger_{\mathbf{p}}(0)\,\mathbf{O}_{\mathbf{p}'}(\mathrm{x})\right|0\right\rangle\right]$$

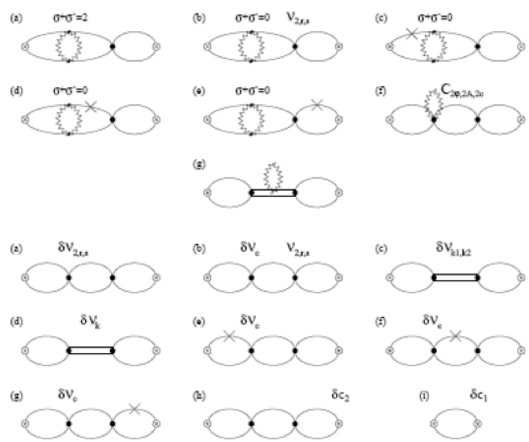
$$\propto\; \mathrm{Im}\left[\mathrm{C}(\mu)^2\,\mathrm{G}(0,0,\sqrt{\mathrm{s}})\right]$$

Total ttbar Cross Section (ILC)

Status of NNLL (QCD) predictions:

- All NNLL QCD effects known since 2000 except for NNLL RG-evolution of leading S-wave production current Wilson coefficient c_1.
- Non-mixing NNLL corrections to anom.dim. known since 2006 (apparently only computable in vNRQCD — Hoang (2006)
- Mixing usoft NNLL corrections to anom.dim. since 2011 — Hoang, Stahlhofen (2007,2011) Pineda (2011)





Non-mixing: from UV-div's of 3-loop vertex corrections

Total ttbar Cross Section (ILC)

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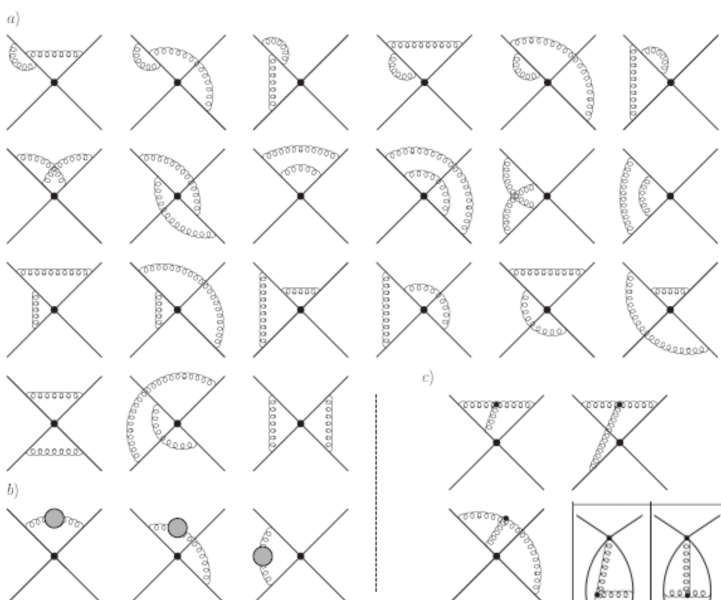
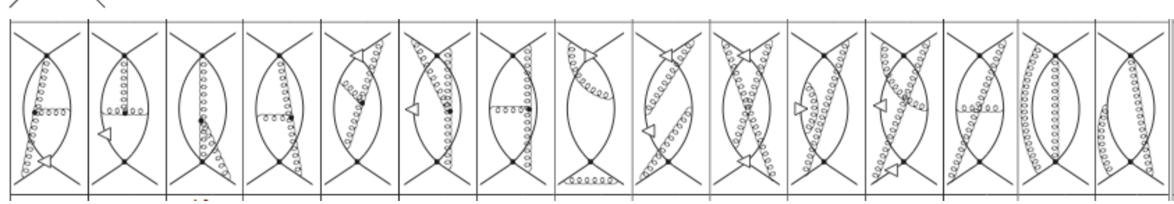
- Mixing usoft NNLL corrections to anom.dim. since 2011 — Hoang, Stahlhofen (2007,2011) Pineda (2011) [pNRQCD]



Mixing: 2-loop anom. dim.'s of coefficients in NLL anom.dim. of c_1.

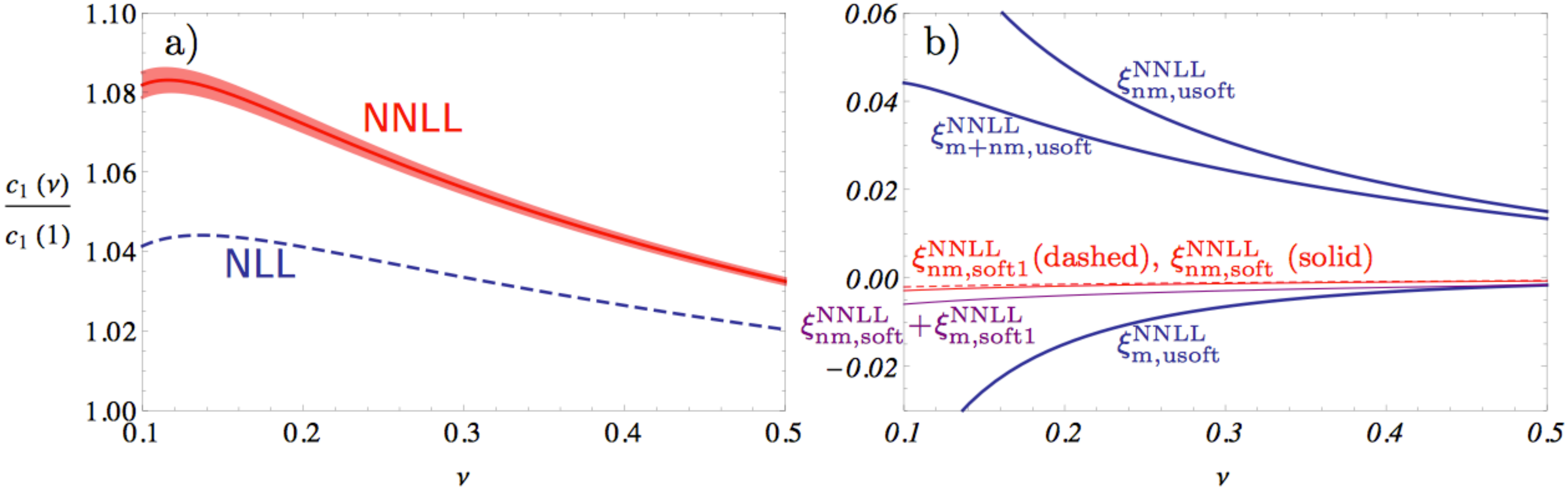
Known soft corrections (spin-dependent) tiny.



Total ttbar Cross Section (ILC)

Status of NNLL (QCD) predictions:

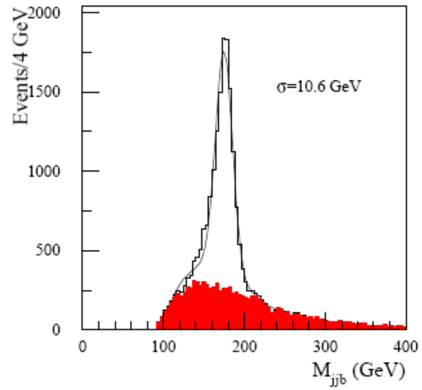
Hoang, Stahlhofen (2013)



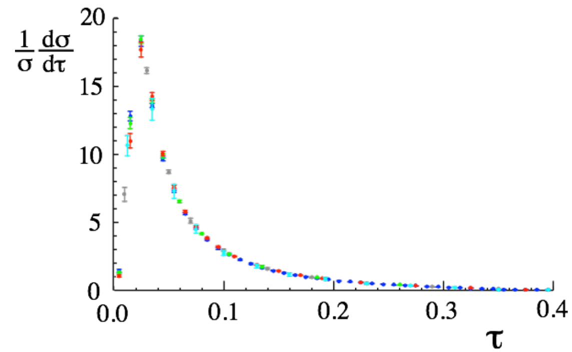
- Uncertainty in NNLL evolution due to missing soft mixing corrections small
- Evolution of c_1 stable
- Huge cancellations between mixing and non-mixing corrections
- Non-mixing corrections contain logs from NNNNLO fixed order !!

Motivation

- Jet cross sections with massive quarks:
  - invariant mass distribution
  - event shapes
  - jettiness
  - end-point reconstruction

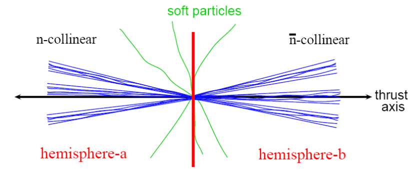


→ mass reconstruction for a single quark

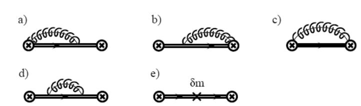


→ color neutralization (collinear): jet functions

→ color neutralization (soft): soft functions



$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m)\, H_m\!\left(m,\frac{Q}{m},\mu_m,\mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\!\left(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\right) B_-\!\left(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\right) S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

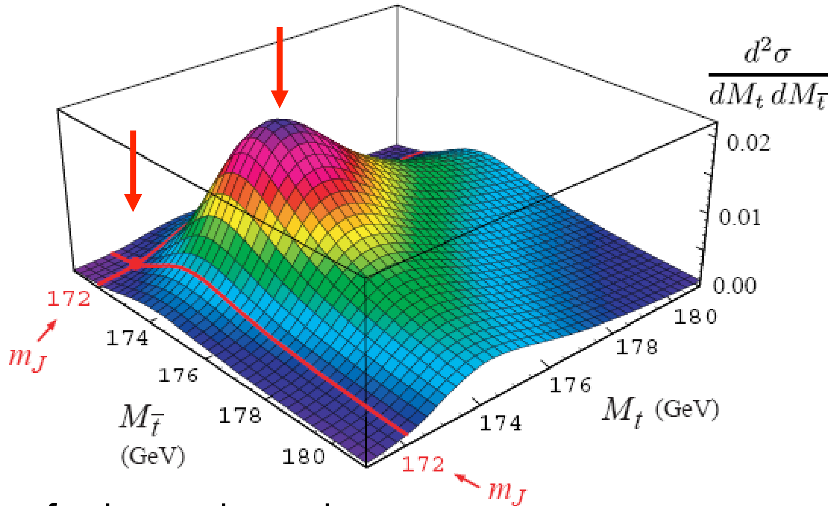


Fleming, Mantry, Stewart, AH (2008)

$$B_+(2v_+\cdot k) = \frac{-1}{8\pi N_c m}\,\mathrm{Disc}\int d^4x\, e^{ik\cdot x}\,\langle 0|\mathrm{T}\{\bar h_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}|0\rangle$$

Motivation / Status for Top

- Top invariant mass distribution for e^+e^- : Fleming, Mantry, Stewart, AH (2008)
  - Top peak region for <u>hemisphere jets</u>
  - Full NLO calculation (+summation at NNLL)
  - Jet mass definition $m_J(R=\Gamma_t)$
  - Relation to the MSbar mass — AH, Stewart
  - Two-Loop B-jet function — Jain, Scimemi, Stewart
  - Two-loop soft function — Schwartz etal, Gritschacher etal.,



Control of scheme dependence

Contr. of scheme dependence

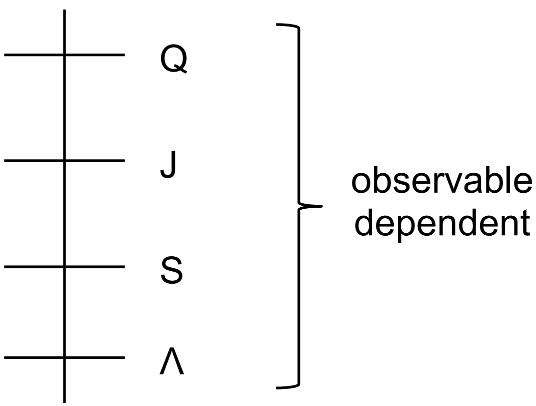
$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{P_T}{m_t}(\alpha_s + \alpha_s^2 + \ldots) + \frac{P_T}{m_t}\Omega_1 + \mathcal{O}\left(\frac{m_t\Lambda_{\rm QCD}}{P_T}\right)$$

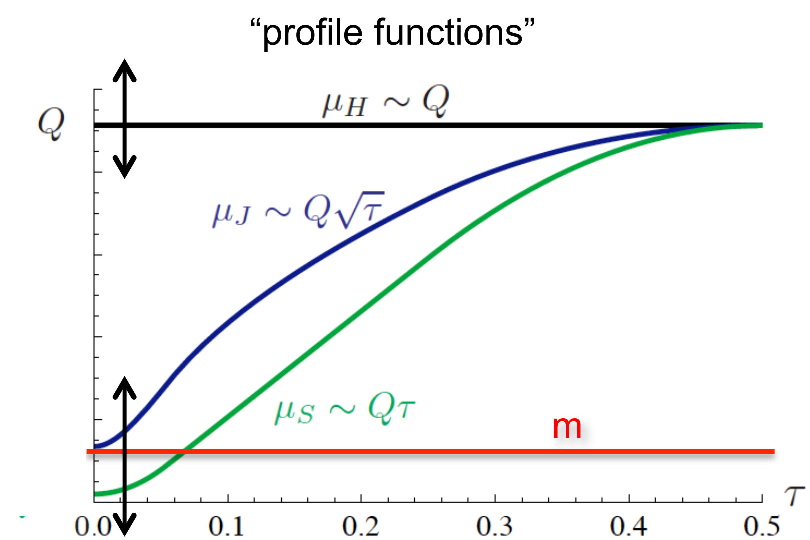
- $\mathrm{NNLO_{full}}$ to be published soon — Pathak, Pietrulewicz, Stewart, AH
- Generalization to LHC: w.i.p. — Mantry, Pathak,, Stewart, AH

Prelude

This talk:
- → Role of massive quarks ($m_q > \Lambda_{QCD}$, for all cases)
- → Full systematics of massive quarks in jets
- → Account for mass-dependent and ALL OTHER logarithms
- → Can we "measure" the MC mass?

$$\sigma \sim H \times J \otimes S \otimes \Lambda$$



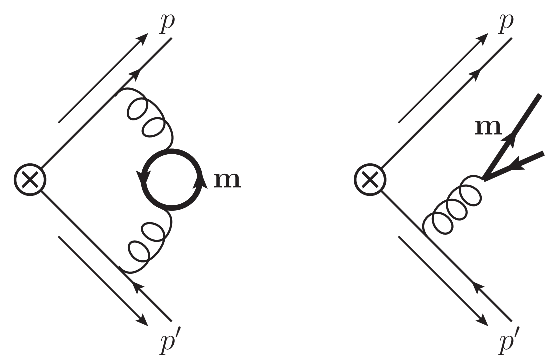


Outline

- Plain fixed-order vs. RG-improved: R-ratio for massless quarks
- R-ratio with a massive quark → CWZ (VFN) scheme for α_s
- ACOT (VFN) scheme for parton distribution functions

  (initial state jets)
- VFN scheme for final state jets:

  Simplest non-trivial case:
  - Massless primary
  - Massive secondary
  - e^+e^-



- Outlook and Conclusions

CWZ: Collins - Wilczek - Zee
ACOT: Aivazis - Collins - Olness - Tung

* In collaboration with: P. Pietrulewicz, I. Jemos, S. Gritschacher

arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
More papers to come

Fixed-Order vs. RGI

R-ratio for massless quarks: $\rightarrow$ valid up to term $O(m^2_{\text{light}}/s)$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \quad \sim \; \text{Im}\left[-i \int dx\, e^{ix.q} \left\langle 0 \middle| T j^\mu(x) j_\mu(0) \middle| 0 \right\rangle \right]$$

$\rightarrow$ vector current conserved: not renormalized

$\rightarrow$ UV divergences only related to strong coupling + field renorm.

$\rightarrow$ MSbar result for any scale μ_0

$$= N_c \sum_q e_q^2 \left\{ 1 + \frac{\alpha_s(\mu_0)}{\pi} + \frac{\alpha_s^2(\mu_0)}{\pi^2} \left[f_3 - \frac{\beta_0}{4} \ln\left(\frac{s}{\mu_0^2}\right) \right] + \ldots \right\}$$

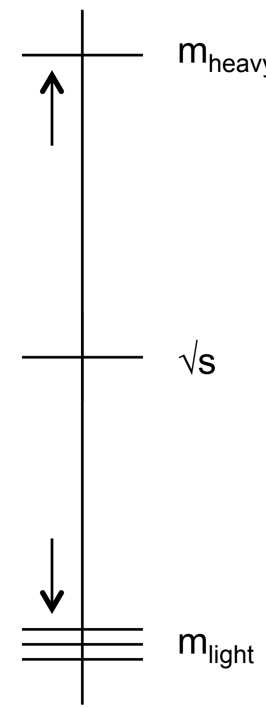
$$\frac{d\alpha_s(\mu)}{d\ln\mu^2} = -\beta_0 \frac{\alpha_s^2(\mu)}{(4\pi)} + \ldots$$

$$\beta_0 = 11 - \frac{2}{3} n_{\text{light}}$$

$\rightarrow$ no large logarithms for $\mu_0 \sim \sqrt{s}$

$\rightarrow$ $\sqrt{s}$ characteristic scale

$$= N_c \sum_q e_q^2 \left\{ 1 + \frac{\alpha_s(\sqrt{s})}{\pi} + \frac{\alpha_s^2(\sqrt{s})}{\pi^2} f_3 + \ldots \right\}$$



$\rightarrow$ Same calculation applies also if there is an ultramassive quark with $m_{\text{heavy}} \gg \sqrt{s}$ (up to terms $O(s/m^2_{\text{heavy}})$)

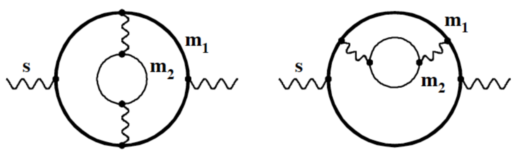
$\rightarrow$ Decoupling of very heavy degrees of freedom

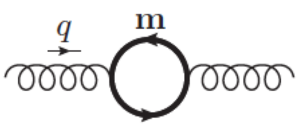
R-ratio with a Massive Quark

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

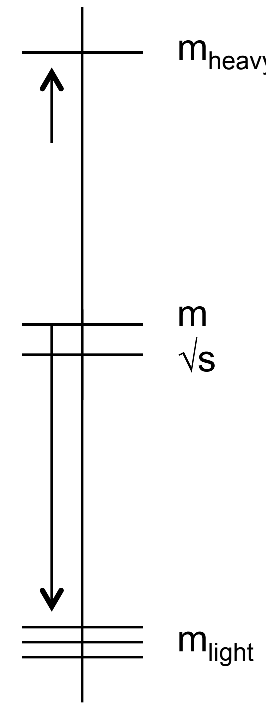
→ no hierarchy between m and √s
→ approximations m≪√s or m≫√s not applicable
→ full mass-dependent matrix elements and phase space
→ renormalization scheme for the massive quark

Virtual quarks:





$$= i(q_\mu q_\nu - g_{\mu\nu} q^2)\,\Pi(q^2)$$
$$= \frac{T_f \alpha_s}{\pi}\left[\frac{1}{3\epsilon} - 2\int_0^1 dx\, x(1-x)\ln\frac{m_2^2 - x(1-x)(q^2+i\epsilon)}{\mu^2}\right]$$



<u>Choice 1:</u> → MSbar for n_{light} quarks and massive quark
→ strong coupling $\alpha_s^{(n_l+1)}(\mu)$ with $\beta_0 = 11 - 2/3(n_l+1)$

- R calculation stable for m~√s but also for m≪√s (calculation smoothly approaches the massless result)
- R calculation with large logarithm for m≫√s

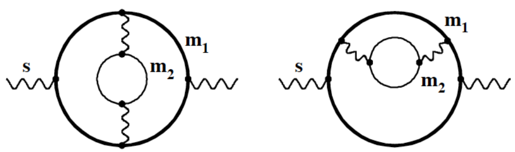
$$R(s) = R^{(0)} + \frac{\alpha_s^{(n_l+1)}(s)}{\pi} R^{(1)} + \left(\frac{\alpha_s^{(n_l+1)}(s)}{\pi}\right)^2 \left[R^{(2)} + \frac{T_f R^{(1)}}{3}\ln\frac{m^2}{s}\right] + \ldots$$

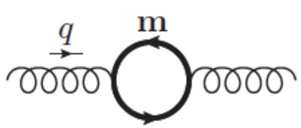
R-ratio with a Massive Quark

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

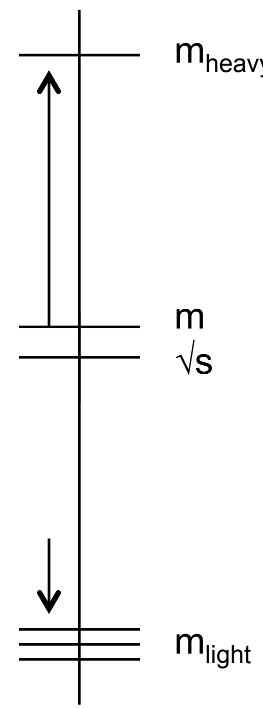
→ no hierarchy between m and √s
→ approximations m≪√s or m≫√s not applicable
→ full mass-dependent matrix elements and phase space
→ renormalization scheme for the massive quark

Virtual quarks:





$$= i(q_\mu q_\nu - g_{\mu\nu} q^2)\,\Pi(q^2)$$
$$= \frac{T_f \alpha_s}{\pi}\left[\frac{1}{3\epsilon} - 2\int_0^1 dx\, x(1-x)\ln\frac{m_2^2 - x(1-x)(q^2+i\epsilon)}{\mu^2}\right]$$



Choice 2: → MSbar for n_{light} quarks, on-shell for massive quark (subtract $\Pi(0)$)
→ strong coupling $\alpha_s^{(n_l)}(\mu)$ with $\beta_0 = 11 - 2/3n_l$

- R calculation stable for m~√s but also for m≫√s
- (calculation smoothly approaches the decoupling result)
- R calculation with large logarithm for m≪√s

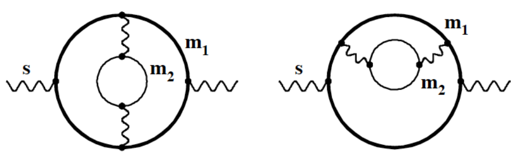
$$R(s) = R^{(0)} + \frac{\alpha_s^{(n_l+1)}(s)}{\pi} R^{(1)} + \left(\frac{\alpha_s^{(n_l+1)}(s)}{\pi}\right)^2 \left[\tilde{R}^{(2)} - \frac{T_f R^{(1)}}{3}\ln\frac{m^2}{s}\right] + \ldots$$

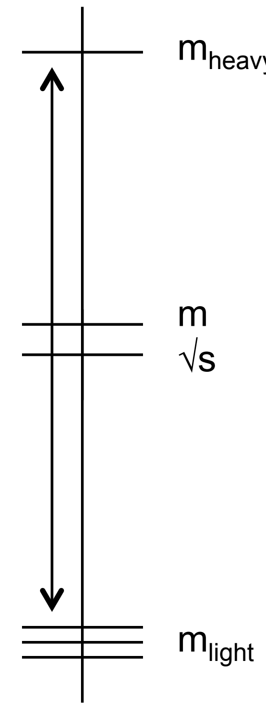
R-ratio with a Massive Quark

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

→ no hierarchy between m and √s
→ approximations m≪√s or m≫√s not applicable
→ full mass-dependent matrix elements and phase space
→ renormalization scheme for the massive quark

Virtual quarks:





→ Choice 1 and choice 2 are equally good for μ ~ √s ~ m

→ Scheme relation for the strong coupling:

$$\alpha_s^{(n_l)}(\mu) = \alpha_s^{(n_l+1)}(\mu)\left(1 + \frac{T_f \alpha_s^{(n_l+1)}(\mu)}{3\pi} \ln\frac{m^2}{\mu^2} + \ldots\right)$$

→ Variable flavor number scheme: (VFN)
Choice 1 for μ ~ √s ≳ m
Choice 2 for μ ~ √s ≲ m
Swap 1↔2 at √s ~ μ_m ~ m

→ Full m^2/s dependence without approximations and w.o. any large logarithms

Collins - Wilczek - Zee (CWZ) scheme

→ comes at the cost of additional μ_m-dependence

R-ratio with a Massive Quark

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

→ no hierarchy between m and √s
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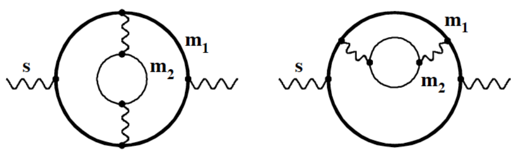
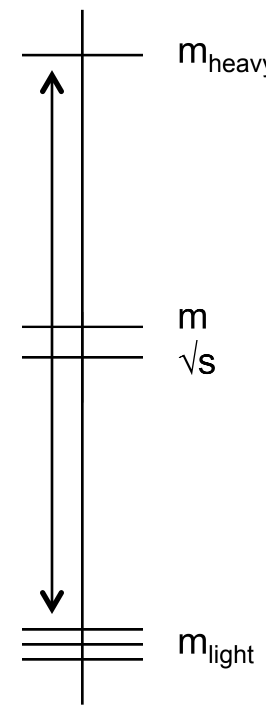
Virtual quarks:





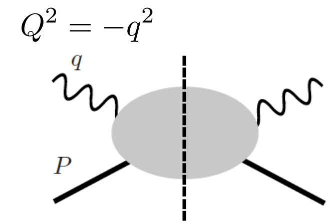
→ Zero-Mass Variable flavor number: (ZM-VFN)

Use $\alpha_s^{(n_l+1)}(\mu)$ for μ ~ √s ≳ m ⊕ massive quark treated massless
Use $\alpha_s^{(n_l)}(\mu)$ for μ ~ √s ≲ m ⊕ massive quark decoupled

→ Very simple implementation (in lack of full information)
→ Gap in the description for μ ~ √s ~ m
→ Useful as long as kinematic region √s ~ m not crucial

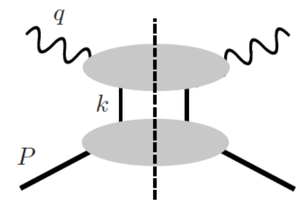
ACOT Scheme for Hadron Collisions

e.g. Deep Inelastic Scattering: $\dfrac{d\sigma(e^- p \to e^- + X)}{dQ\,dx}$



→ consider all quarks as as light ($m_q < \Lambda$)

→ quark number operators with an anomalous dimension between proton states → DGLAP equations



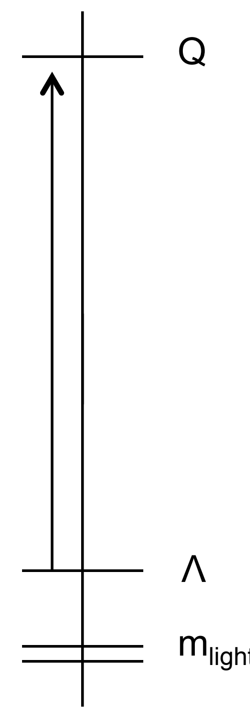
→ Hadronic tensor:

$$W_{\mu\nu}(Q,x) \sim \sum_{\text{partons a}} f_a(\mu) \otimes w_{\mu\nu}(Q,x,\mu)$$

→ μ-dependence with DGLAP equations for (light) parton distribution functions

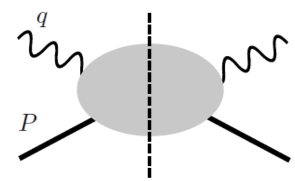
$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\sum_j \int_x^1 \frac{\mathrm{d}\xi}{\xi} \times \begin{pmatrix} P_{q_i q_j}\left(\frac{x}{\xi},\alpha_s(Q^2)\right) & P_{q_i g}\left(\frac{x}{\xi},\alpha_s(Q^2)\right) \\ P_{g q_j}\left(\frac{x}{\xi},\alpha_s(Q^2)\right) & P_{gg}\left(\frac{x}{\xi},\alpha_s(Q^2)\right) \end{pmatrix} \begin{pmatrix} q_j(\xi,Q^2) \\ g(\xi,Q^2) \end{pmatrix}, \quad (11)$$

$$\frac{d\alpha_s(Q)}{d\ln Q^2} = -\beta_0 \frac{\alpha_s^2(Q)}{(4\pi)} + \ldots \qquad \beta_0 = 11 - \frac{2}{3} n_{\text{light}}$$



ACOT Scheme for Hadron Collisions

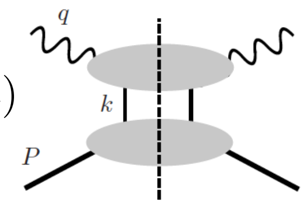
e.g. Deep Inelastic Scattering: $\dfrac{d\sigma(e^- p \to e^- + X)}{dQ\, dx}$



→ realistic case: massive quarks with Q > m > Λ
(charm, bottom [top])

→ Hadronic tensor:

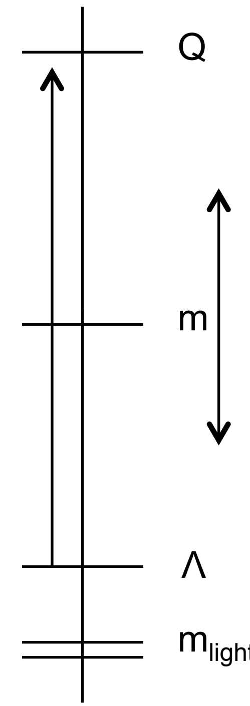
$$W_{\mu\nu}(m,Q,x) \sim \sum_{a=q,g,Q} f_a^{(n_l+1)}(\mu) \otimes w_{\mu\nu}(m,Q,x,\mu)$$



ACOT scheme:

- DGLAP evolution for n_l flavors for μ ≲ m (only light quarks)
- DGLAP evolution for n_l+1 flavors for μ ≳ m (light quarks + massive quark)
- Flavor matching for α_s and the pdfs at μ_m ~ m

$$f_{q,g,Q}^{(n_l+1)}(\mu_m) = \sum_{a=q,g} F_{q,g,Q|a}(m,\mu_m) \otimes f_a^{(n_l)}(\mu_m)$$

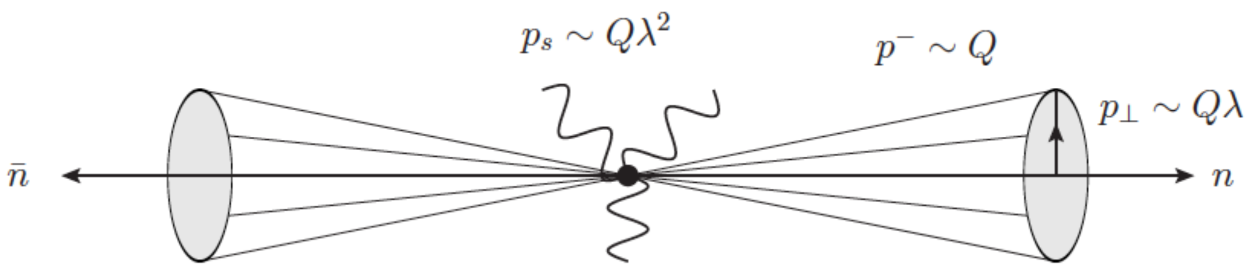


→ hard coefficient $w_{\mu\nu}$(m,Q,x) approaches massless $w_{\mu\nu}$(Q,x) for m→0
→ calculations of $w_{\mu\nu}$(m,Q,x) involves subtraction of pdf IR mass singularities
→ full dependence on m/Q without any large logarithms

Final State Jets in SCET$_1$

Bauer, Fleming, Luke
Bauer, Fleming, Pirjol, Stewart

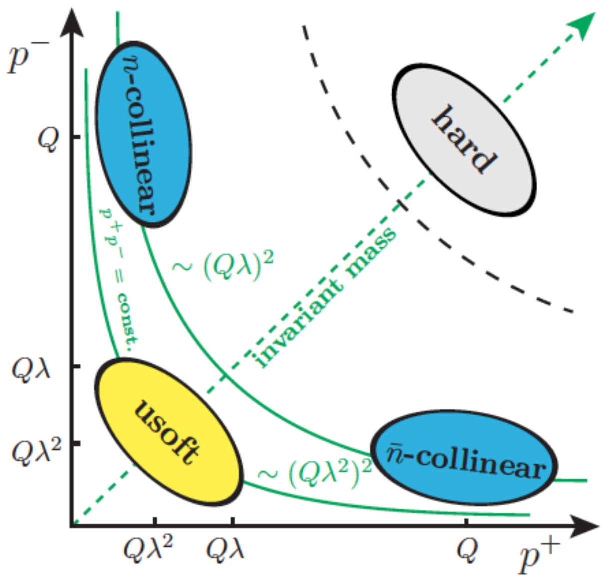
→ consider: dijet in e$^+$e$^-$ annihilation, all quarks are light ($m_q < \Lambda$)



$$n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$$

$$p^\mu = p^- \frac{n^\mu}{2} + p^+ \frac{\bar{n}^\mu}{2} + p_\perp$$

$$p^2 = p^- p^+ + p_\perp^2$$



mode	$p^\mu = (+,-,\perp)$	p^2	fields
hard	$Q(1,1,1)$	Q^2	–
n-collinear	$Q(\lambda^2,1,\lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
$\bar{n}$-collinear	$Q(1,\lambda^2,\lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}}$, $A_{\bar{n}}^\mu$
ultrasoft	$Q(\lambda^2,\lambda^2,\lambda^2)$	$Q^2\lambda^4$	q_{us}, A_{us}^μ

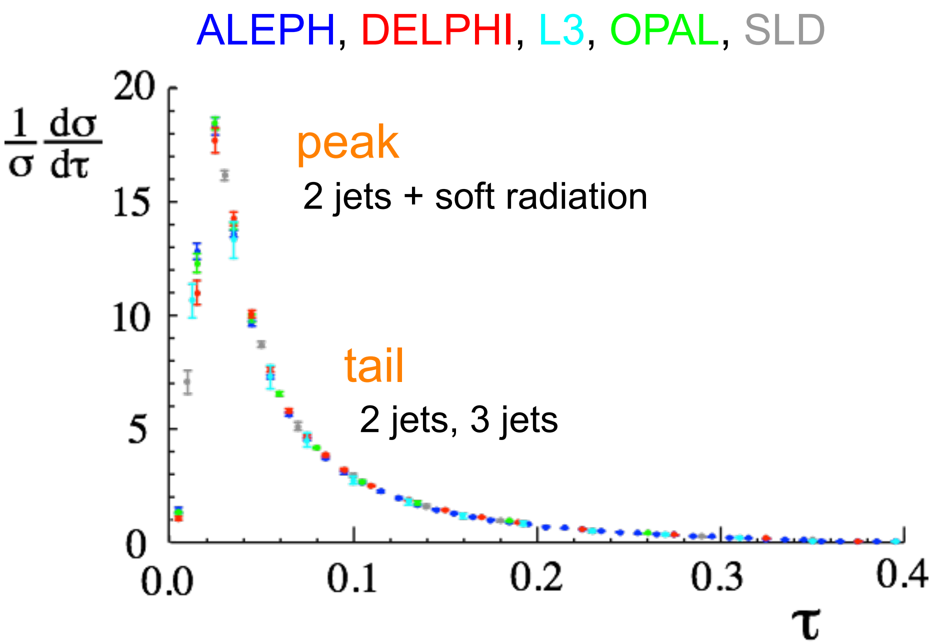
$$\Rightarrow \quad d\sigma = \mathcal{H} \cdot \mathcal{J} \otimes \mathcal{S}$$

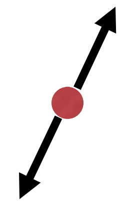
Final State Jets in SCET_1

→ consider: dijet in e^+e^- annihilation, all quarks are light ($m_q < \Lambda$)

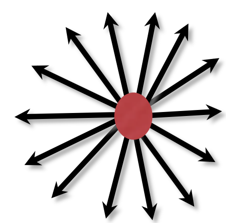
e.g. Thrust:

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \qquad \tau = 1 - T$$





$\tau = 0$



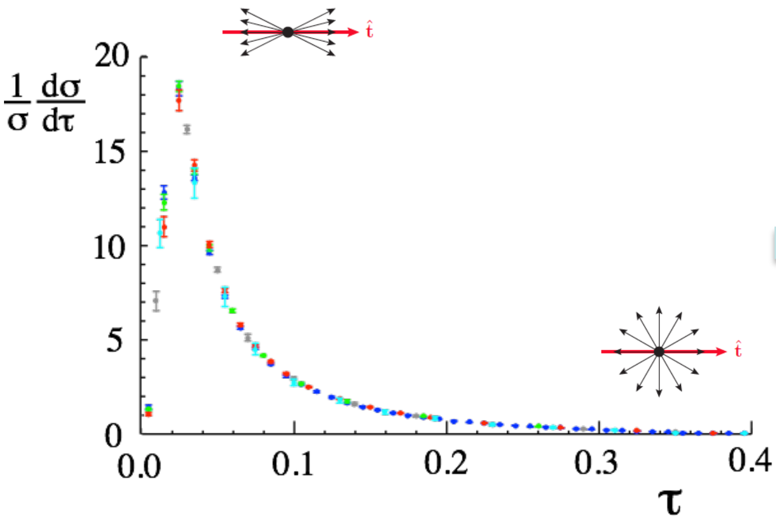
$\tau = 0.5$

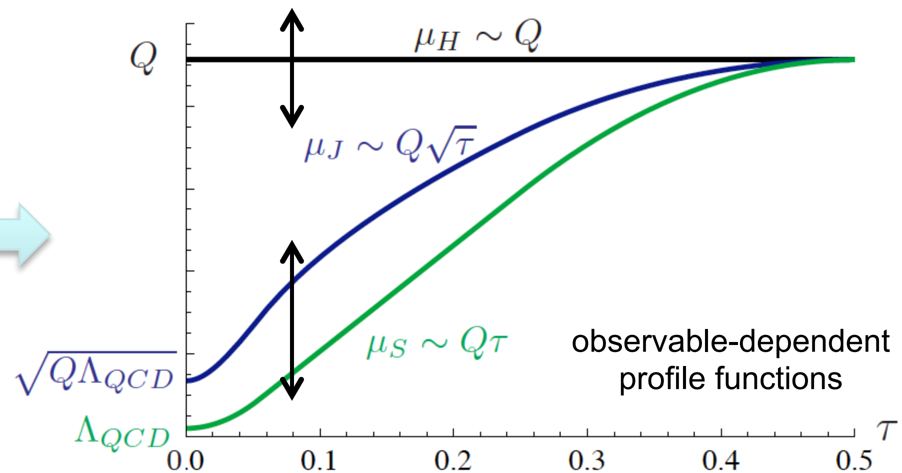
$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q,\mu) \int d\ell \, J_0(Q\ell,\mu)\, S_0\left(Q\tau - \ell, \mu\right)$$

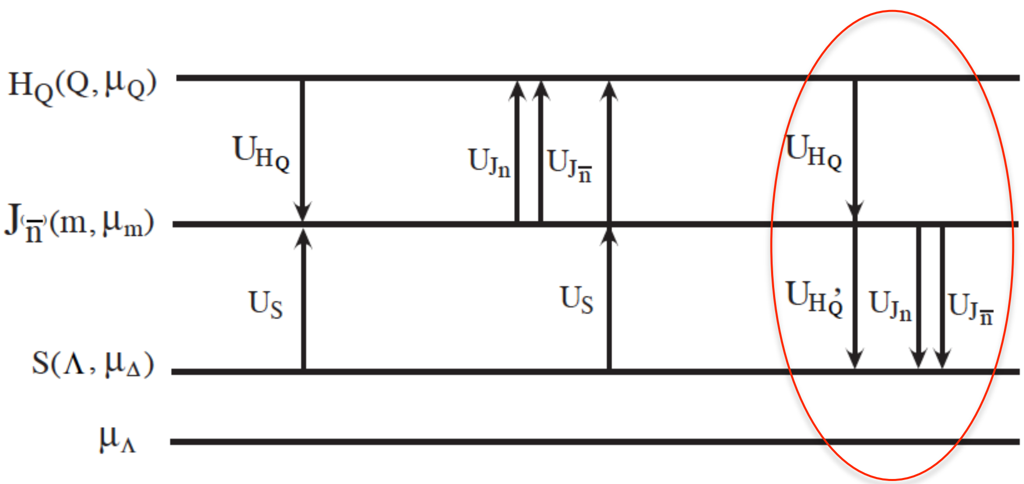
Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman

Final State Jets in SCET$_1$

$$\frac{d\sigma}{d\tau} = Q^2\sigma_0 H_0(Q,\mu)\int d\ell\, J_0(Q\ell,\mu)\, S_0\,(Q\tau-\ell,\mu)$$







→ evolution with n_l light quark flavors

→ consistency conditions w.r.t. to different evolution choices

→ top-down evolution considered in the following

VFN Scheme for Final State Jets

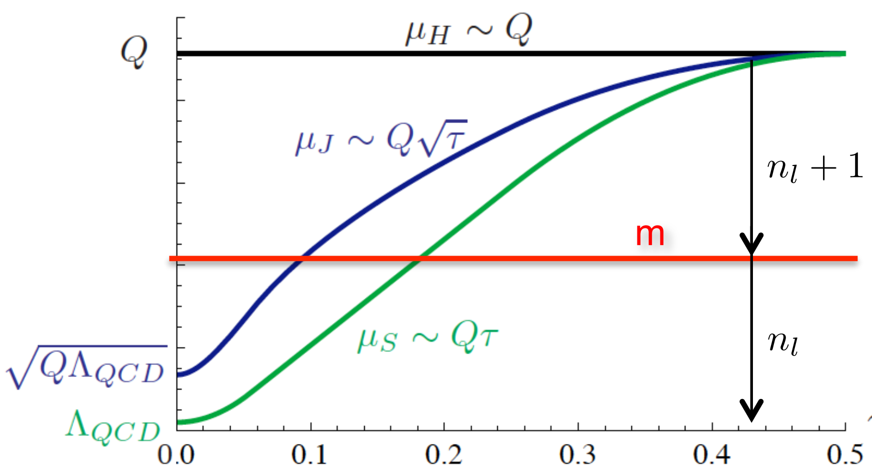
Gritschacher, AH, Jemos, Pietrulewicz

→ consider: dijet in e^+e^- annihilation, n_l light quarks ⊕ one massive quark

→ obvious: (n_l+1)-evolution for $\mu \gtrsim m$ and (n_l)-evolution for $\mu \lesssim m$

→ obvious: different EFT scenarios w.r.t. to mass vs. Q – J – S scales

"profile functions"



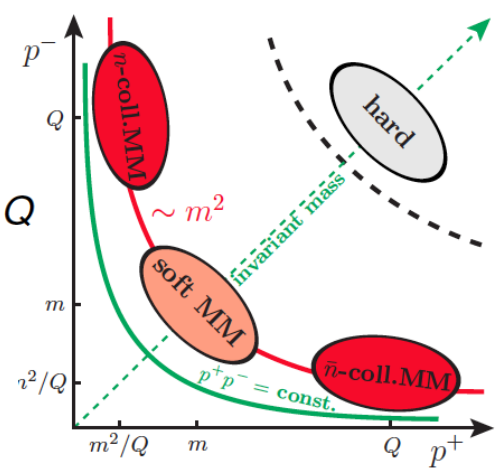
Aims:

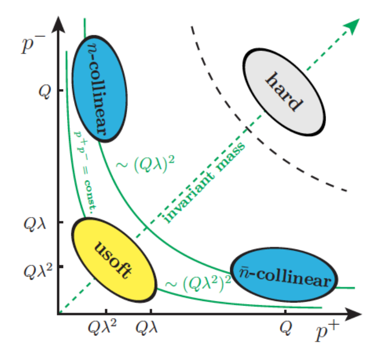
- Full mass dependence (little room for any strong hierarchies): decoupling, massless limit
- Smooth connections between different EFTs
- Determination of flavor matching for current-, jet- and soft-evolution
- Reconcile problem of $SCET_2$-type rapidity divergences

→ Deal with collinear and soft "mass modes"

→ Additional power counting parameter $\lambda_m = m/Q$

mode	$p^\mu = (+,-,\perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2

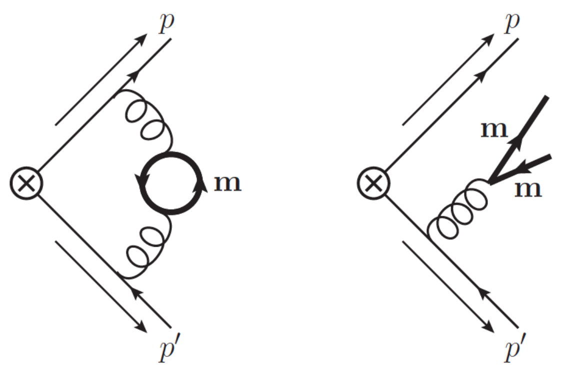




VFN Scheme for Final State Jets

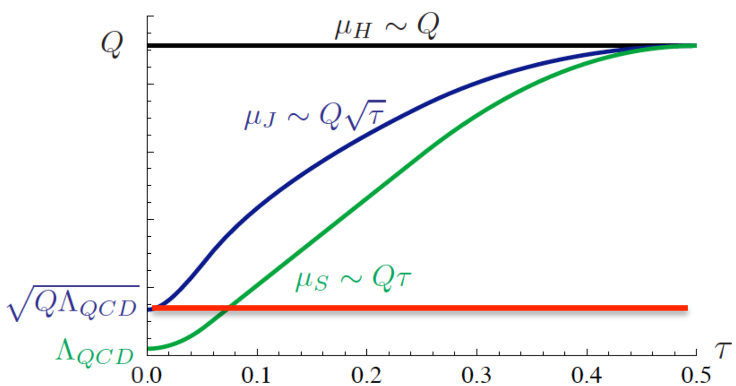
Simplest non-trivial case to study:

→ massless primary quark dijet production in e^+e^- annihilation:
n_l light quarks ⊕ one massive quark arise only through secondary production



→ does not lead to bHQET-type theory when the jet scale approaches the quark mass

→ only SCET-type theories



VFN Scheme for Final State Jets

Simplest non-trivial case to study:

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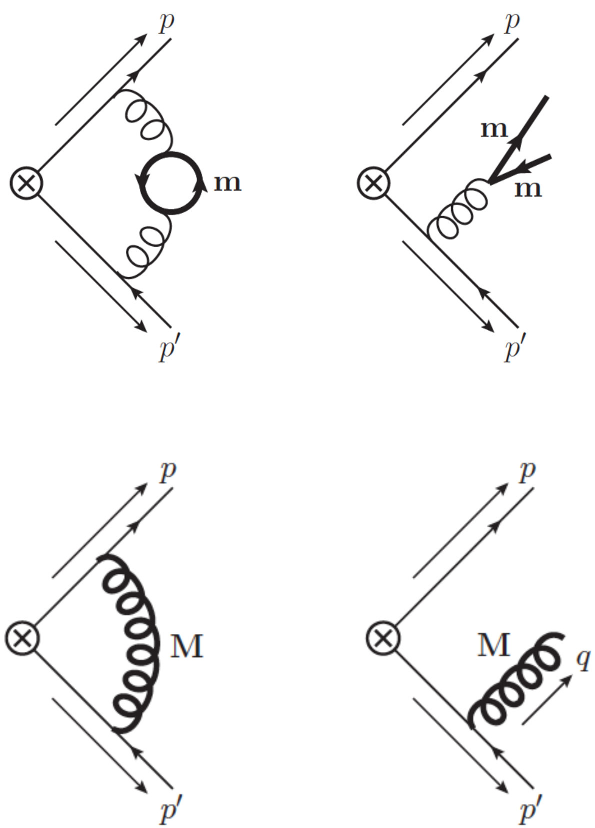
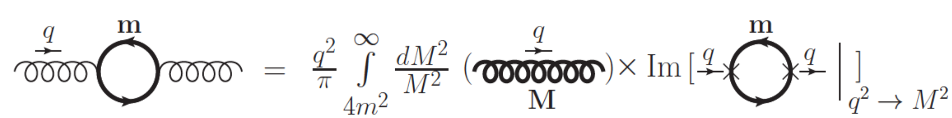


→ field theory: close relation to the problem of massive gauge boson radiation

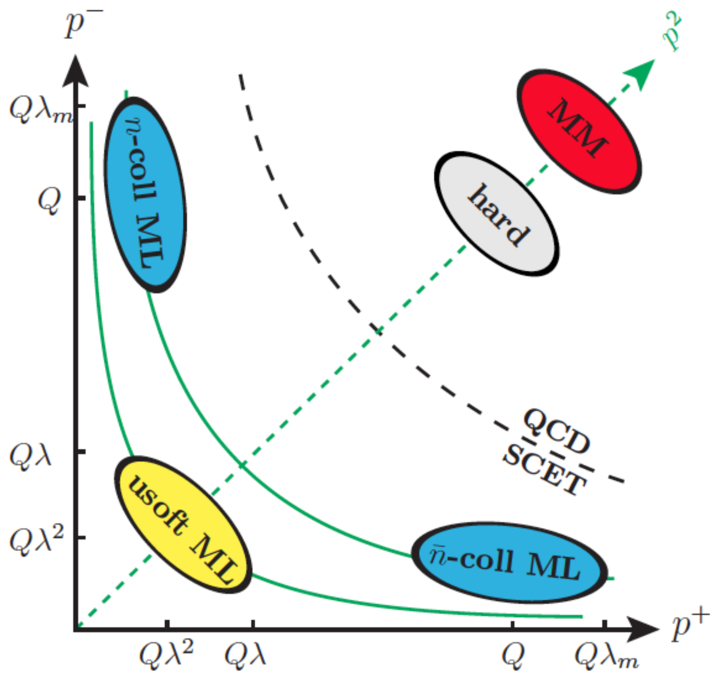
→ dispersion relation: massive quark results can be obtained directly from massive gluon calculations when quark pair treated inclusively (e.g. hard coefficient, jet function)



→ separation of conceptual issues to be resolved and calculations issues related to gluon splitting.

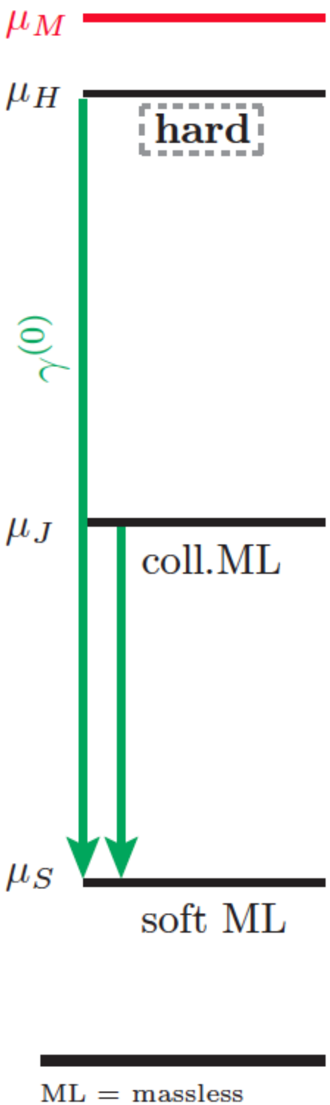
VFN Scheme for Final State Jets

Scenario 1: $\lambda_m > 1 > \lambda > \lambda^2$ (m > Q > J > S)



- EFT only contains light quarks
- Massive quark only in current matching coeff.
- Decoupling for $m/Q \to \infty$

VFN Scheme for Final State Jets



integrate out mass modes at QCD level

$$\frac{d\sigma}{d\tau} \sim \left|C^{I}(\mu_H)\right|^2 U_H^{(0)}(\mu_H,\mu_S) \times \int d\ell \int ds\, J_0(s)\, U_J^{(0)}(Q\ell - s, \mu_S, \mu_J)\, S_0\,(Q\tau - \ell, \mu_S)$$

$U_H^{(0)}$, $U_J^{(0)}$: massless evolution factors

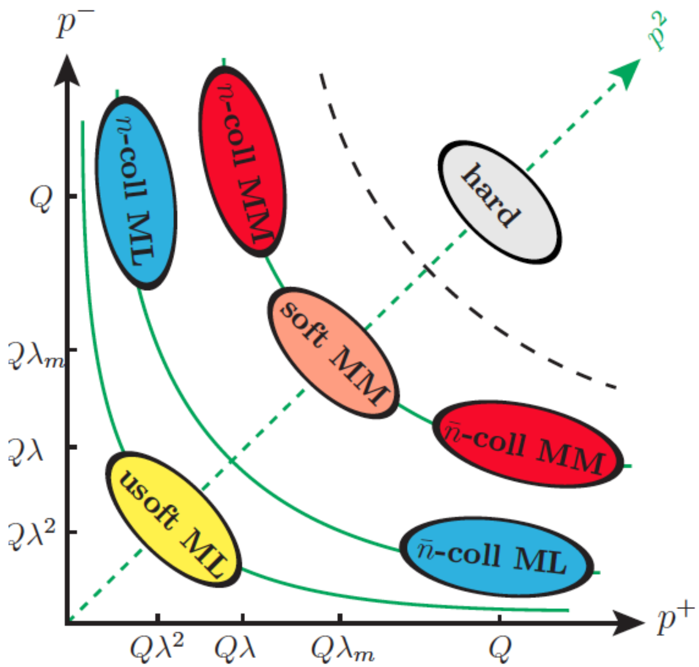
$$C^{I}(\mu_H) = C_0(\mu_H) + \delta F_m^{\rm QCD}$$

C_0: massless matching coefficient
$\delta F_m^{\rm QCD}$: massive full theory contribution (OS)
$\rightarrow$ decoupling for $M/Q \rightarrow \infty$
$\rightarrow$ IR-divergent expression for $M/Q \rightarrow 0$

$U_i^{(0)}$ stands for: (a) massive gluon integrated out
(b) (n_l)-evolution

VFN Scheme for Final State Jets

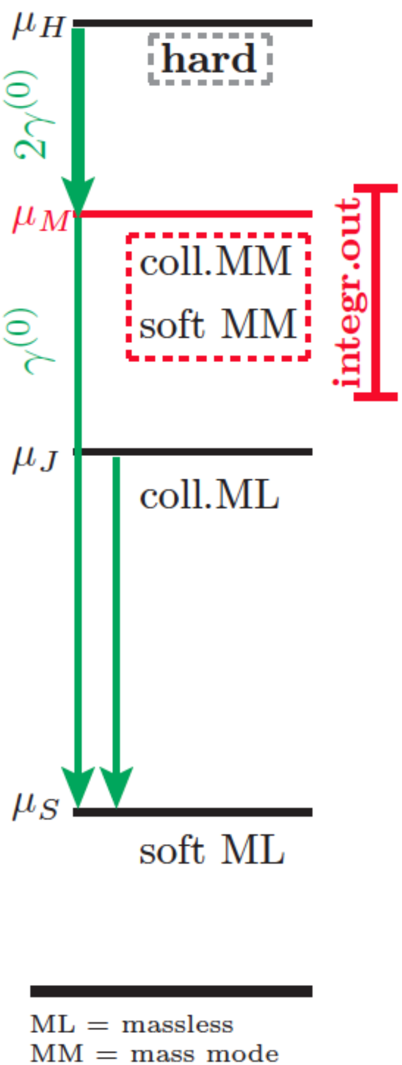
Scenario 2: 1> λ_m > λ > λ^2 (Q > m > J > S)



- Massive modes only virtual
- Jet and soft function as in massless case
- Hard coefficient must have massless limit
- Known Sudakov problem for massive gauge boson

Chiu, Golf, Kelley, Manohar
Chiu, Führer, Hoang, Kelley

VFN Scheme for Final State Jets



mass modes enter SCET, but integrated out before the jet scale

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S)$$
$$\times \int d\ell \int ds\, J_0(s,\mu_J)\, U_J^{(0)}(Q\ell - s,\mu_J,\mu_S)\, S_0(Q\tau-\ell,\mu_S)$$

$U_H^{(1)}$: evolution factor ($\gamma_H^{(1)} = 2\gamma_H^{(0)}$)

$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\rm eff}(\mu_H)$

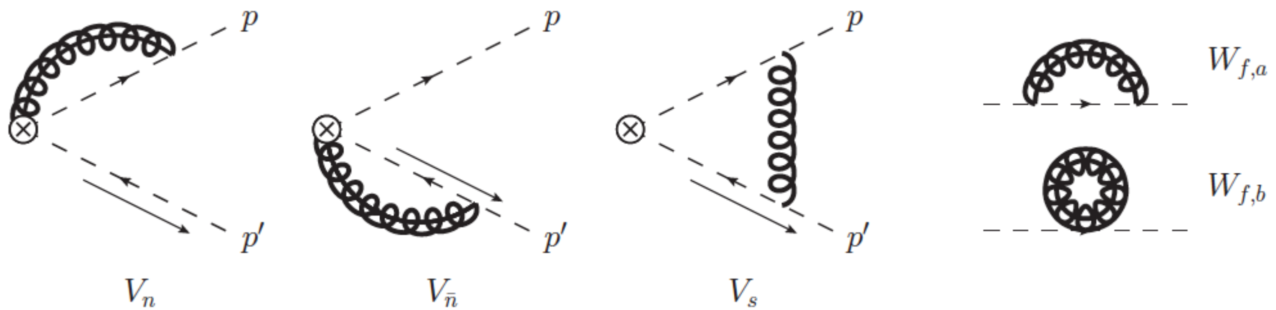
$\delta F_m^{\rm eff}$: massive SCET contribution

$U_i^{(0)}$ stands for: (a) massive gluon integrated out
(b) (n_l)-evolution

$U_i^{(1)}$ stands for: (a) massive gluon dynamical
(b) (n_l+1)-evolution

VFN Scheme for Final State Jets

Scenario 2: mass mode SCET calculation



$$\delta F_m^{\rm eff}(Q,M,\mu) = \frac{\alpha_s C_F}{4\pi}\left\{\ln\left(\frac{M^2}{\mu^2}\right)\left[2\ln\left(\frac{-Q^2}{\mu^2}\right) - \ln\left(\frac{M^2}{\mu^2}\right) - 3\right] - \frac{5\pi^2}{6} + \frac{9}{2}\right\}$$

Chiu, Golf, Kelley, Manohar (2008)
Chiu, Fuhrer, Hoang, Kelley, Manohar (2009)

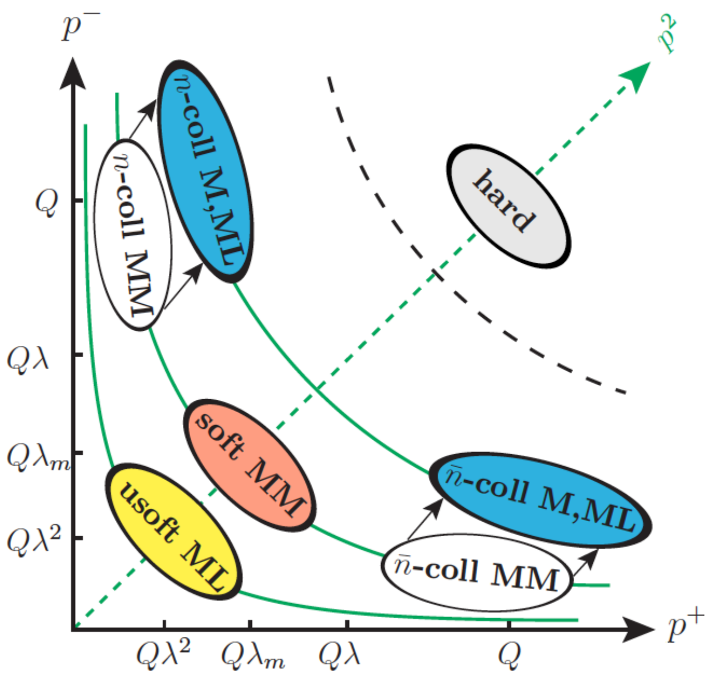
large logarithm $\ln\left(\frac{M^2}{\mu_H^2}\right)$ cancels between $\mathcal{C}^l$ and $\delta F_m^{\rm eff}$

correct massless limit for $\mathcal{C}^{ll}(\mu_H)$:

$$\mathcal{C}^{ll}(Q,M,\mu_H) = \mathcal{C}^l(Q,M,\mu_H) - \delta F_m^{\rm eff}(Q,M,\mu_H) \stackrel{M\to 0}{\longrightarrow} 2C_0(Q,\mu_H)$$

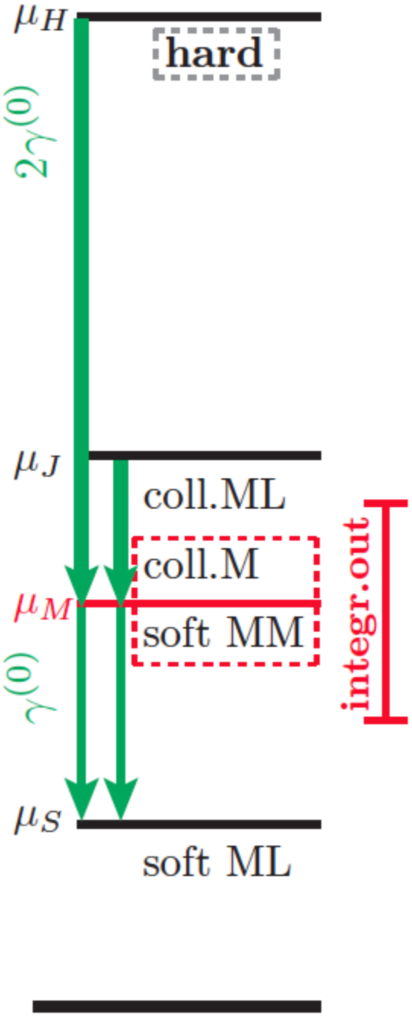
VFN Scheme for Final State Jets

Scenario 3: $1 > \lambda > \lambda_m > \lambda^2$ (Q > J > m > S)



- Current evolution unchanged w.r. to Scen. 2
- Hard coefficient must have massless limit
- Jet function must have massless limit
- Massive and massless collinear in same sector
- Collinear mass modes integrated out at m

VFN Scheme for Final State Jets



ML = massless
MM = mass mode
M = massive

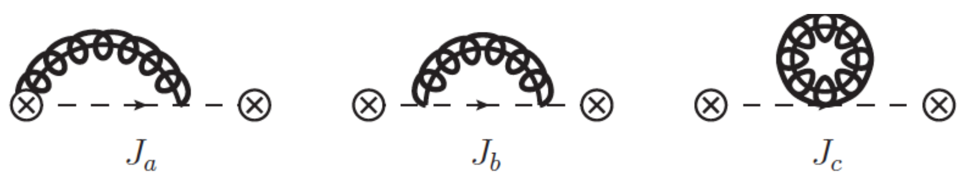
mass modes enter jet sector,
but integrated out before the soft scale

$$
\begin{aligned}
\frac{d\sigma}{d\tau} \sim \; & \left|\mathcal{C}^{ll}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\,|\mathcal{M}_H(\mu_M)|^2\, U_H^{(0)}(\mu_M,\mu_S) \\
& \times \int d\ell \int ds \int ds' \int ds''\, J_{0+m}(s,\mu_J)\, U_J^{(1)}(s'-s,\mu_J,\mu_M) \\
& \times \mathcal{M}_J(s''-s',\mu_M)\, U_J^{(0)}(s''-Q\ell,\mu_M,\mu_S)\, S_0\,(Q\tau-\ell,\mu_S)
\end{aligned}
$$

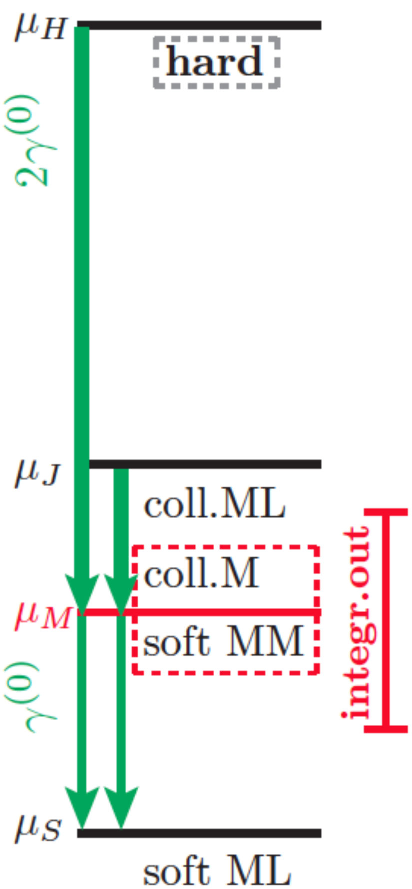
$$J_{0+m}(s,\mu_J) = J_0(s,\mu_J) + \delta J_m^{\mathrm{virt}}(s,\mu_J) + \theta(s-M^2)\,\delta J_m^{\mathrm{real}}(s)$$

$\delta J_m^{\mathrm{virt}}$: virtual piece of jet function (distributive structure)

$\delta J_m^{\mathrm{real}}$: real radiation piece of jet function (function)



VFN Scheme for Final State Jets



ML = massless
MM = mass mode
M = massive

mass modes enter jet sector,
but integrated out before the soft scale

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S)$$
$$\times \int d\ell \int ds \int ds' \int ds''\, J_{0+m}(s,\mu_J)\, U_J^{(1)}(s'-s,\mu_J,\mu_M)$$
$$\times\, \mathcal{M}_J(s''-s',\mu_M)\, U_J^{(0)}(s''-Q\ell,\mu_M,\mu_S)\, S_0(Q\tau-\ell,\mu_S)$$

$$J_{0+m}(s,\mu_J) = J_0(s,\mu_J) + \delta J_m^{\mathrm{virt}}(s,\mu_J) + \theta(s-M^2)\,\delta J_m^{\mathrm{real}}(s)$$

$\delta J_m^{\mathrm{virt}}$: virtual piece of jet function (distributive structure)

$\delta J_m^{\mathrm{real}}$: real radiation piece of jet function (function)

<u>Matching to full theory:</u>

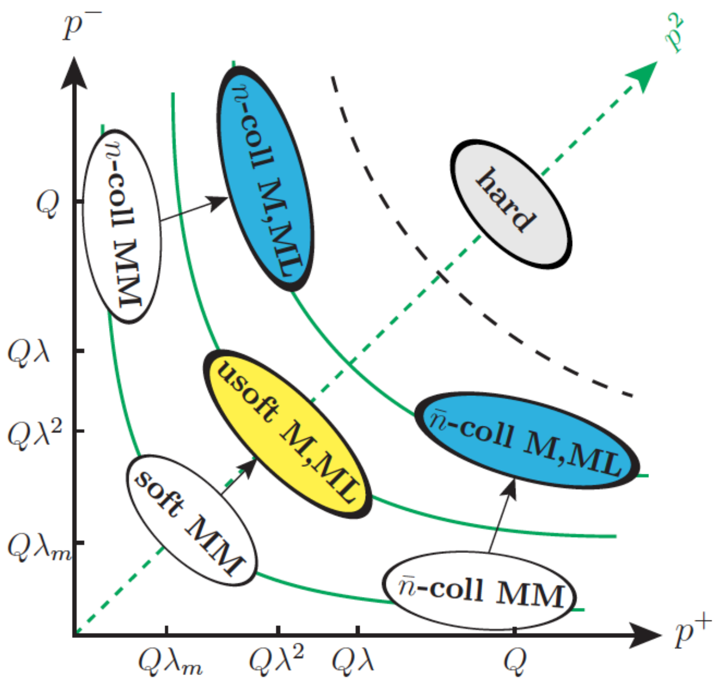
$$\rightarrow \mathcal{M}_J^{(1)}(Q^2\tau,M,\mu) = Q^2\delta(\tau) - \delta J_m^{\mathrm{virt}}(Q^2\tau,M,\mu)$$

<u>Continuity Scenario 2 ↔ Scenario 3:</u> (“consistency condition”)

$$\rightarrow\ J_{0+m}(s,\mu_J)\,\mathcal{M}_J(s,\mu_J) = J_0(s,\mu_J)\quad (\text{for } s < m^2)$$

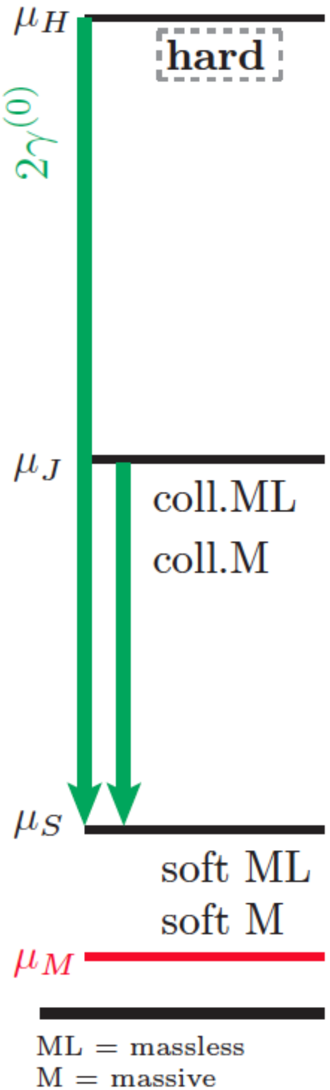
VFN Scheme for Final State Jets

Scenario 4: $1 > \lambda > \lambda^2 > \lambda_m$ (Q > J > S > m)



- Current evolution unchanged w.r. to Scen. 2
- Jet function and evolution as in Scen. 2
- Massive and massless coll. modes same sector
- Massive and massless soft modes same sector
- Hard coefficient, jet and soft function must have massless limit
- All RG-evolution for (n_l+1) flavors

VFN Scheme for Final State Jets



mass modes enter all sectors

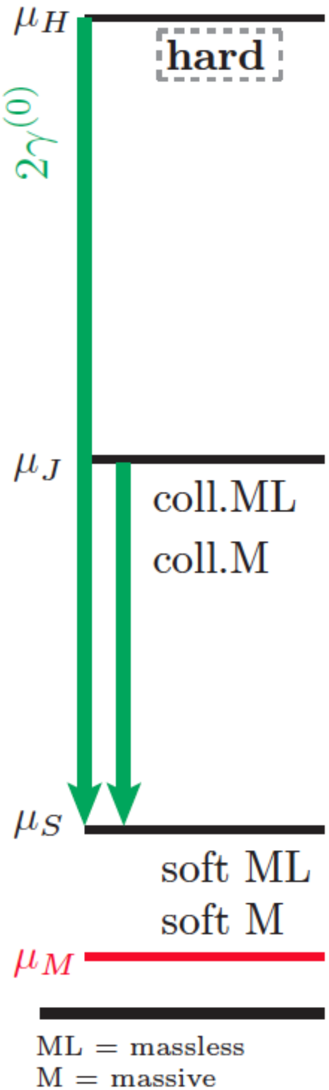
$$\frac{d\sigma}{d\tau} \sim \left|C^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_{0+m}(s, \mu_J)\, U_J^{(1)}(Q\ell - s, \mu_J, \mu_S)\, S_{0+m}(Q\tau - \ell, \mu_S)$$

$$S_{0+m}(\ell, \mu_S) = S_0(\ell, \mu_S) + \delta S_m^{\rm virt}(\ell, \mu_S) + \theta(\ell - M)\, \delta S_m^{\rm real}(\ell)$$

$\delta S_m^{\rm virt}$: virtual piece of massive soft function (distributive structure)

$\delta S_m^{\rm real}$: real radiation piece of massive soft function (function)

VFN Scheme for Final State Jets



mass modes enter all sectors

$$\frac{d\sigma}{d\tau} \sim \left|C^{ll}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_{0+m}(s, \mu_J)\, U_J^{(1)}(Q\ell - s, \mu_J, \mu_S)\, S_{0+m}(Q\tau - \ell, \mu_S)$$

$$S_{0+m}(\ell, \mu_S) = S_0(\ell, \mu_S) + \delta S_m^{\rm virt}(\ell, \mu_S) + \theta(\ell - M)\, \delta S_m^{\rm real}(\ell)$$

$\delta S_m^{\rm virt}$: virtual piece of massive soft function (distributive structure)

$\delta S_m^{\rm real}$: real radiation piece of massive soft function (function)

<u>No matching correction:</u>

$\rightarrow$ agreement with expanded full theory result at fixed order

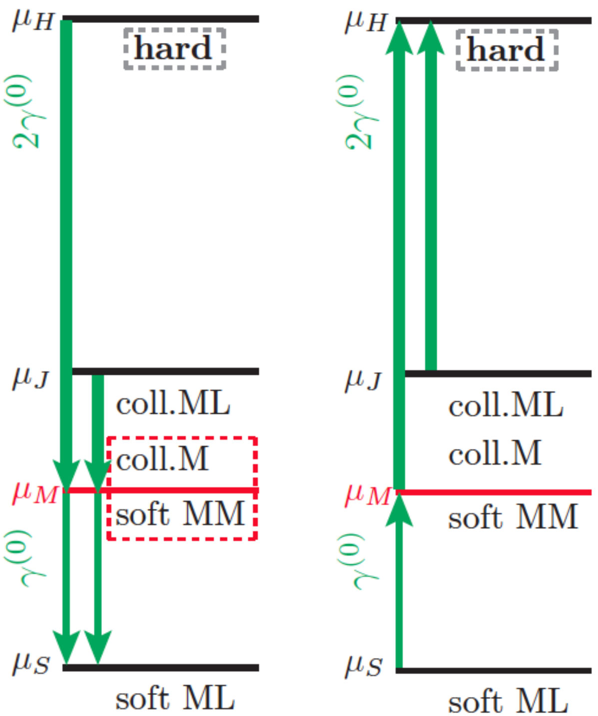
<u>Continuity Scenario 3 $\leftrightarrow$ Scenario 4:</u> (“consistency condition”)

$$2\,{\rm Re}\left[\delta F_m^{\rm eff}(Q, \mu)\right] \delta(\tau) - Q^2 \delta J_m^{\rm virt}(Q^2\tau, \mu) - Q\, \delta S_m^{\rm virt}(Q\tau, \mu) = 0$$

$\rightarrow$ consistency relation between virtual modes

VFN Scheme for Final State Jets

Important role of consistency relation: soft – jet – hard for scenario III



alternative description in bottom-up running ($\mu \sim \mu_H$):

$$\frac{d\sigma}{d\tau} \sim \left|C^{II}(\mu_H)\right|^2 \int d\ell \int d\ell' \int d\ell'' \int ds \int ds' \\ \times U_J^{(1)}(s-s',\mu_J,\mu_H)\, J_0(s',\mu_J)\, U_S^{(1)}(\ell''-s/Q,\mu_M,\mu_H) \\ \times \mathcal{M}_S(\ell'-\ell'',\mu_M)\; U_S^{(0)}(\ell-\ell',\mu_S,\mu_M)\, S_0\left(Q\tau-\ell,\mu_S\right)$$

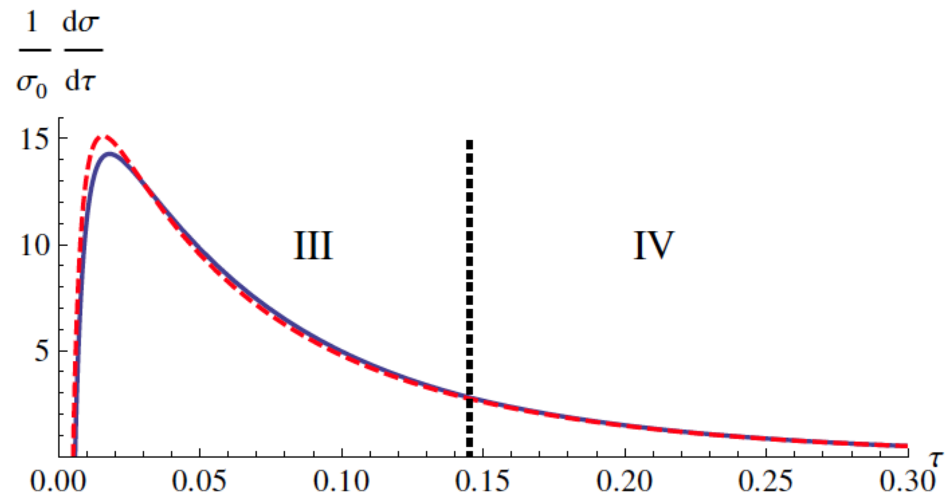
$$\mathcal{M}_S(\ell,\mu_M) = \delta(\ell) + \delta S_m^{\mathrm{virt}}(\ell,\mu_M)$$

consistency relation: $\boxed{\mathcal{M}_S(\ell,\mu_M) = Q\,\left|\mathcal{M}_H(\mu_M)\right|^2 \mathcal{M}_J(Q\ell,\mu_M)}$

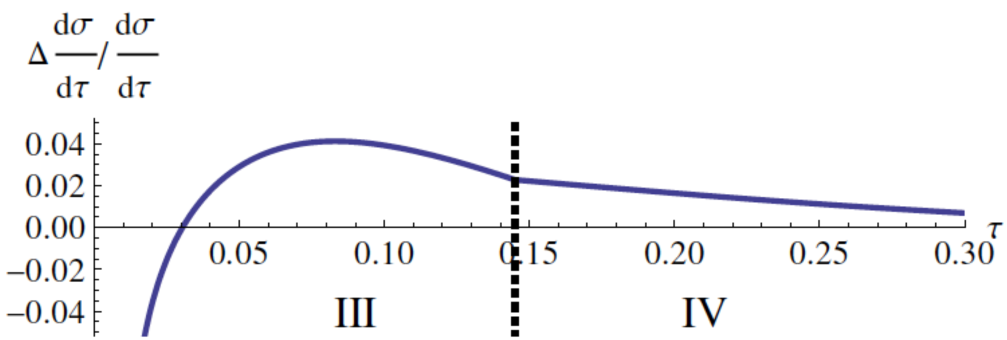
similarly: $U_S^{(1)}(\ell,\mu_S,\mu_M) = Q\,U_H^{(1)}(\mu_M,\mu_S)\,U_J^{(1)}(Q\ell,\mu_M,\mu_S)$

VFN Scheme for Final State Jets

Numerical results: secondary bottom effects (Q=14 GeV)



relative deviation massive vs. massless

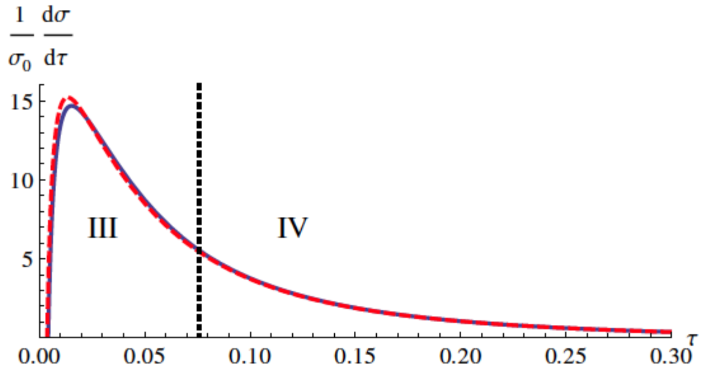


VFN Scheme for Final State Jets

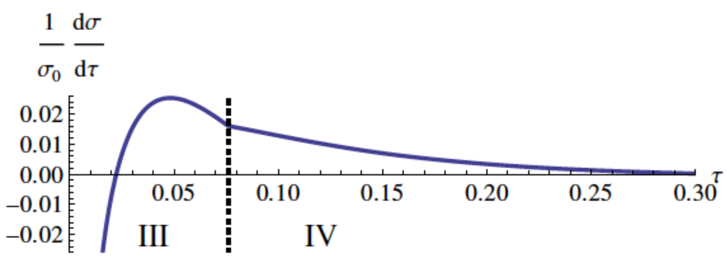
Numerical results: secondary bottom effects (Q=35 GeV)

Possible effect on α_s-determination

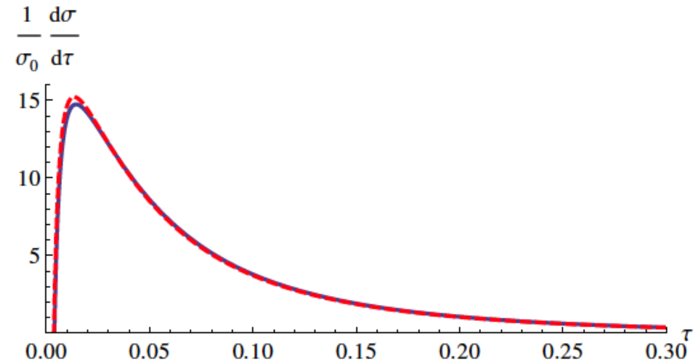
massive vs. massless



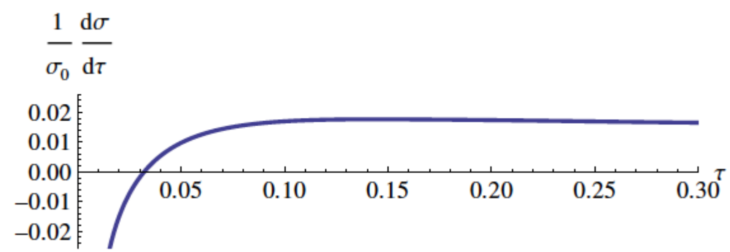
relative deviation
massive vs. massless



$\alpha_s(M_Z) = 0.119$ vs. $\alpha_s(M_Z) = 0.118$



relative deviation
$\alpha_s(M_Z) = 0.119$ vs. $\alpha_s(M_Z) = 0.118$

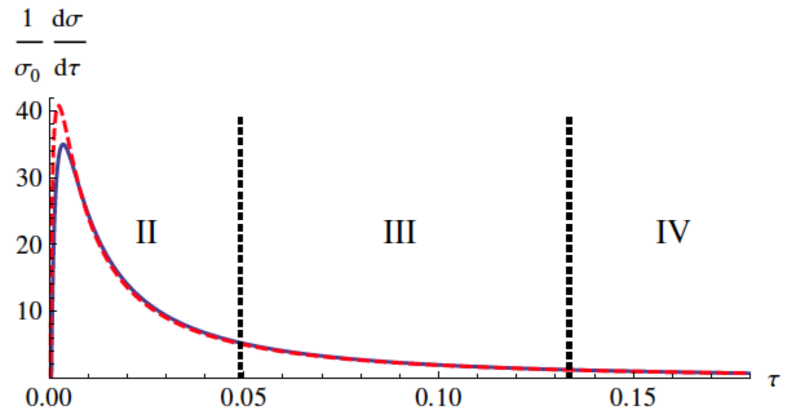


VFN Scheme for Final State Jets

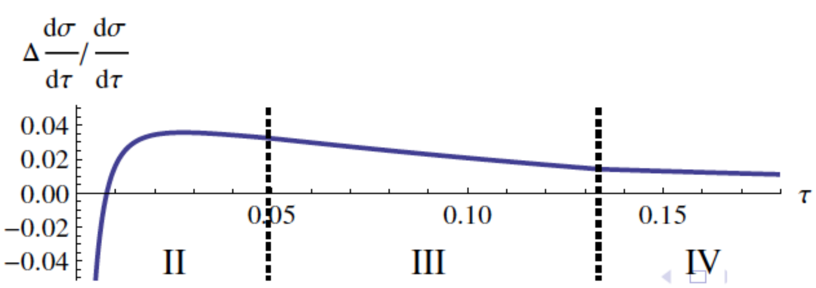
Numerical results: secondary top quark effects (Q=500 GeV)

- $Q = 500$ GeV $\leftrightarrow$ ILC
- comparison of ML (6 light q) and M (5 light q + massive t) thrust distribution
- default values: $\alpha_s(M_Z) = 0.118$, $m_t = 175$ GeV

Thrust distribution: massive vs. massless



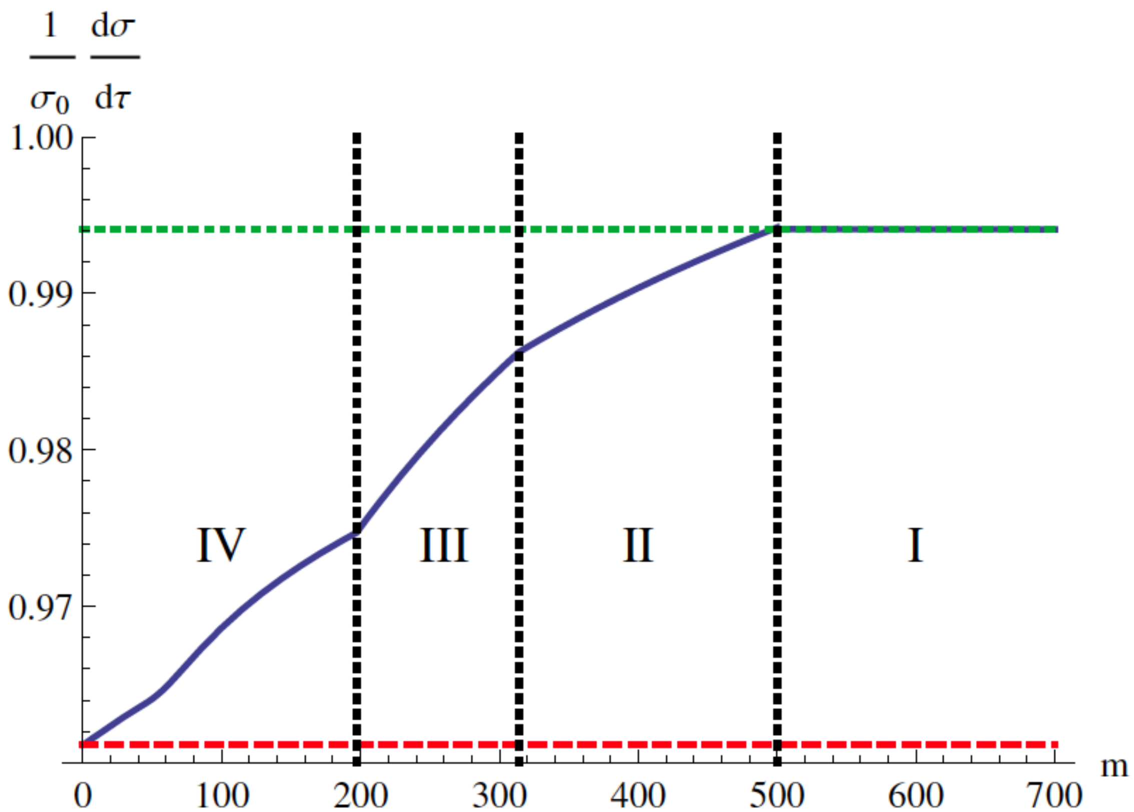
relative deviation massive-massless



VFN Scheme for Final State Jets

Consistency check: continuous transition and correct limiting behaviour

Thrust distribution: $Q = 500$ GeV, $\tau = 0.15$ fixed, vary mass
massless limit (6 flavors): dashed
decoupling limit (5 flavors): dotted



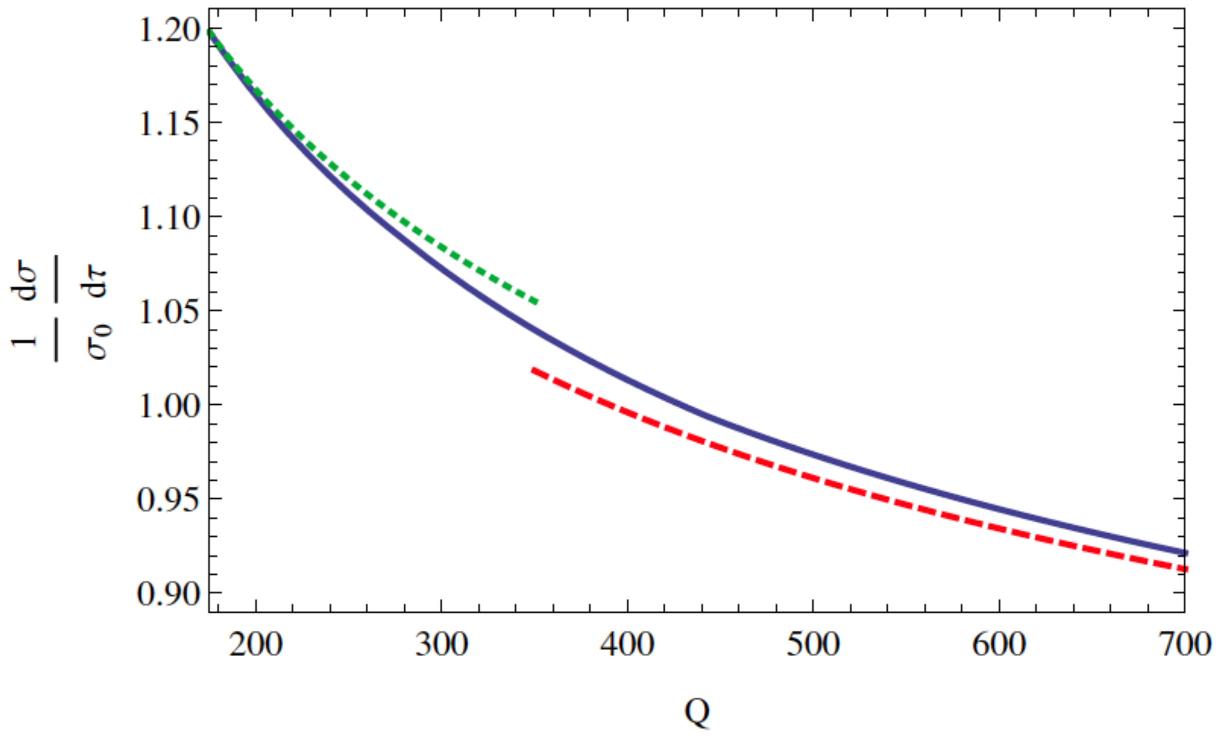
VFN Scheme for Final State Jets

Comparison with Zero-Mass VFN scheme:

Thrust distribution: $\tau = 0.15$, $m_t = 175$ GeV fixed, vary Q

massless limit (6 flavours): dashed

decoupling limit (5 flavours): dotted

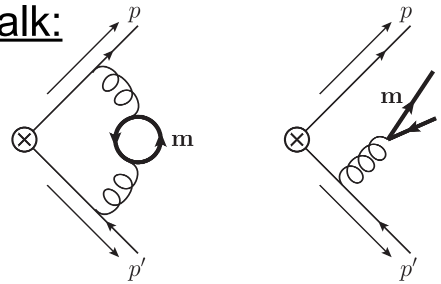


Outlook & Conclusion

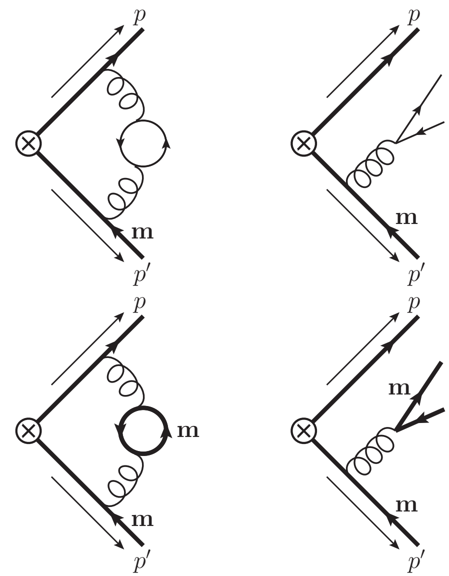
Conclusion:

→ VFN Scheme for final state jets

This talk:



Upcoming:



→ Upcoming:

- → Combination with ACOT scheme for PDFs (DIS)
- → beam functions
- → etc.

→ Conceptually important.

→ Relevant issues where VFN scheme for jets is important:

- → (top) mass measurement from jets (reconstruction)
- → MC mass systematics (Is the MC a more model OR more QCD?
- → intrinsic charm and charm mass determinations (e.g. DIS)

VFN Scheme for Final State Jets

Scenario 3: Jet function

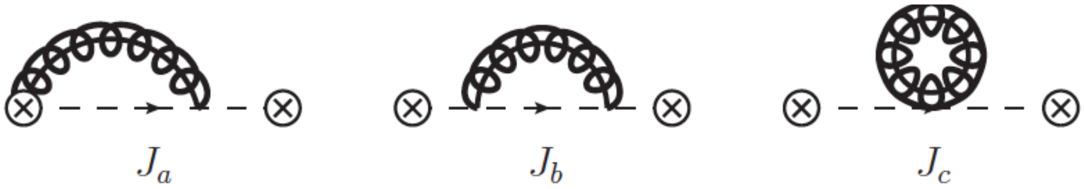


diagram J_a individually not well-defined $\rightarrow$ soft-bin subtractions are crucial!

$$\delta J_m \sim J_a - J_{a,0M} + J_b + J_c$$

$$J_{0+m}(s, M, \mu) = J_0(s, \mu) + \delta J_m^{\rm virt}(s, M, \mu) + \theta(s - M^2)\, \delta J_m^{\rm real}(s, M)$$

$$\mu^2 \delta J_m^{\rm virt}(s, M, \mu) = \frac{\alpha_s C_F}{4\pi} \left\{ \delta(\bar{s}) \left[-4 \ln^2\left(\frac{M^2}{\mu^2}\right) - 6 \ln\left(\frac{M^2}{\mu^2}\right) + 9 - 2\pi^2 \right] + 8 \ln\left(\frac{M^2}{\mu^2}\right) \left[\frac{\theta(\bar{s})}{\bar{s}}\right]_+ \right\}$$

$$\delta J_m^{\rm real}(s, M) = \frac{\alpha_s C_F}{4\pi} \left\{ \frac{2(M^2 - s)(3s + M^2)}{s^3} + \frac{8}{s} \ln\left(\frac{s}{M^2}\right) \right\}$$

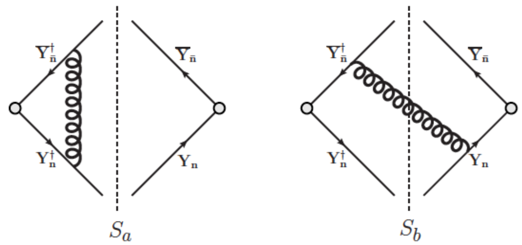
$\rightarrow$ $\delta J_m^{\rm virt}$ = virtual radiation ($\bar{s} \equiv s/\mu^2$)

$\rightarrow$ $\delta J_m^{\rm real}$ = real radiation for $s > M^2$, continuous: $\delta J_m^{\rm real}(s = M^2, M) = 0$

$\rightarrow$ correct massless limit: $J_{0+m}(s, M, \mu_J) \stackrel{M\rightarrow 0}{\longrightarrow} 2J_0(s, \mu_J)$

VFN Scheme for Final State Jets

Scenario 4: Soft function



rapidity divergences in S_a and S_b (not regularized by DIMREG)
$\rightarrow$ we use an analytic regulator ($\int dk^- \rightarrow \int dk^- \left(\frac{\nu}{k^-}\right)^\alpha$) Smirnov (1995)
$\rightarrow$ $S_a = 0$, $\delta S_m = S_b$

$$S_{0+m}(\ell, M, \mu) = S_0(\ell, \mu) + \delta S_m^{\rm virt}(\ell, M, \mu) + \theta(\ell - M)\, \delta S_m^{\rm real}(\ell, M)$$

$$\mu\, \delta S_m^{\rm virt}(\ell, M, \mu) = \frac{\alpha_s C_F}{4\pi} \left\{ \delta(\bar\ell) \left[2 \ln^2\left(\frac{M^2}{\mu^2}\right) + \frac{\pi^2}{3} \right] - 8 \ln\left(\frac{M^2}{\mu^2}\right) \left[\frac{\theta(\bar\ell)}{\bar\ell}\right]_+ \right\}$$

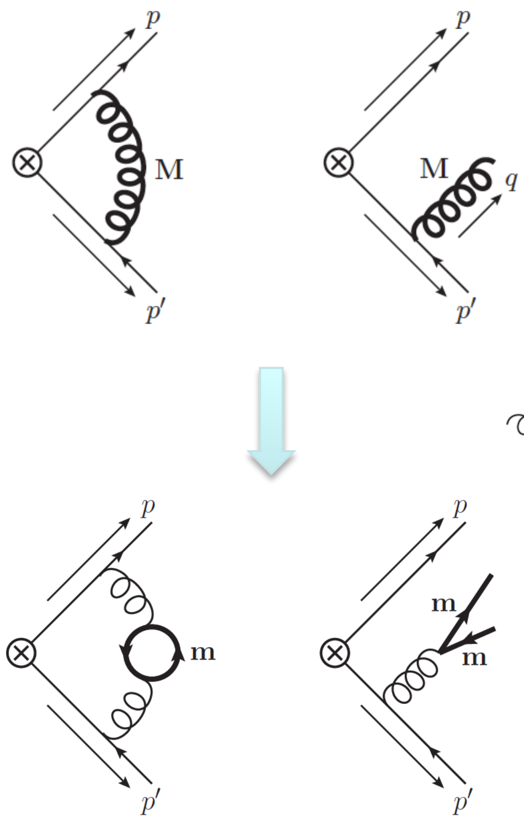
$$\delta S_m^{\rm real}(\ell, M) = \frac{\alpha_s C_F}{4\pi} \left\{ -\frac{8}{\ell} \ln\left(\frac{\ell^2}{M^2}\right) \right\}$$

$\rightarrow$ $\delta S_m^{\rm virt}$ = virtual radiation ($\bar\ell \equiv \ell/\mu$)
$\rightarrow$ $\delta S_m^{\rm real}$ = real radiation for $\ell > M$, continuous: $\delta S_m^{\rm real}(\ell = M, M) = 0$

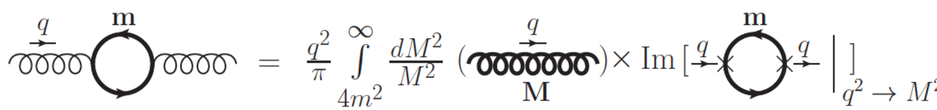
$\rightarrow$ correct massless limit: $S_{0+m}(\ell, M, \mu_S) \xrightarrow{M \to 0} 2 S_0(\ell, \mu_S)$

VFN Scheme for Final State Jets

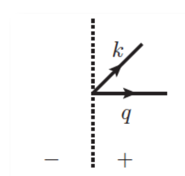
From massive gluons to secondary quarks:

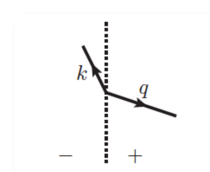


→ dispersion relation: massive quark results can be obtained directly from massive gluon calculations when quark pair treated inclusively (e.g. hard coefficient, jet function)



→ explicit two-loop calculation needed when quarks are treated exclusively (e.g. soft function → hemisphere prescription)





non-global logs

Gritschacher, AH, Jemos, Pietrulewicz
arXiv:1309.6251