Determination of m_t from threshold at the ILC

Jan Piclum

based on (ongoing) work in collaboration with

M. Beneke, Y. Kiyo, P. Marquard,

A. Penin, T. Rauh, D. Seidel, M. Steinhauser direct reconstruction:

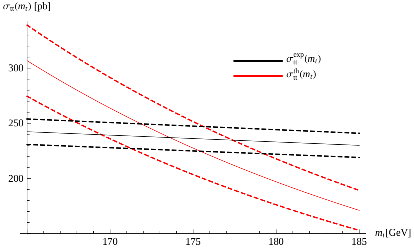
$$\text{Tevatron:} \quad m_t = 173.20 \pm 0.51 \pm 0.71 \text{ GeV}$$

$$\text{LHC:} \quad m_t = 173.29 \pm 0.23 \pm 0.92 \text{ GeV}$$

extract mass from PDF fit:

$\bar{m}_t(\bar{m}_t) = 162.6 \pm 2.3$ GeV $\rightsquigarrow m_t = 171.2 \pm 2.4$ GeV [Alekhin, Blümlein, Moch 2013]

extract mass from cross section:

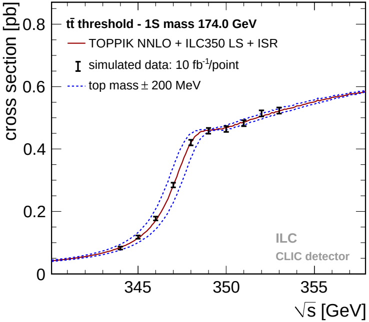


experimental cross section from ATLAS compared with theoretical cross section computed with TOPIXS 2.0 (NNLO/MSTW08)

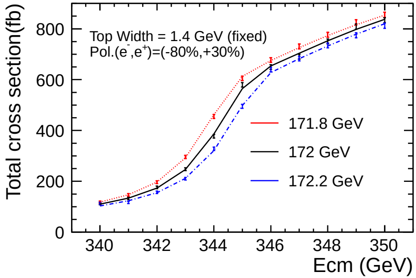
$$\rightsquigarrow m_t = 174.0^{+4.1}_{-4.5} \text{ GeV}$$

perform threshold scan

⇝ comparison with theoretical prediction in well-defined mass scheme (e.g. PS or 1S)

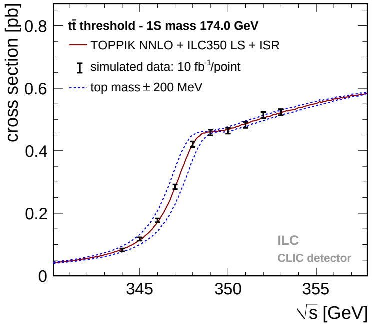


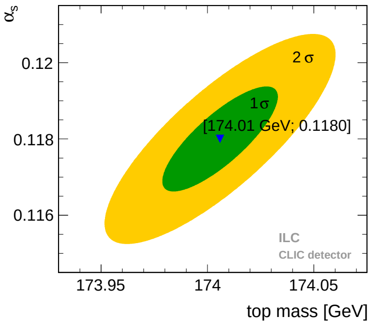
[Seidel, Simon, Tesar, Poss 2013]



[Horiguchi et al. 2013]

[Seidel, Simon, Tesar, Poss 2013]





for 10 fb^{-1} per point:

	δm_t	$\delta \alpha_s$
stat. error	27 MeV	0.0008
theory (3%)	9 MeV	0.0022

Threshold Production

relative velocity of quark-antiquark pair is small:

$$\sqrt{s} = E + 2m_t \approx 2m_t \quad \Rightarrow \quad v = \sqrt{\frac{E}{m_t}} \ll 1\,; \quad v \sim \alpha_s$$

- perturbation theory breaks down due to terms proportional to $\frac{\alpha_s}{v}$
  $\rightsquigarrow$ Coulomb resummation
- formation of bound states below threshold

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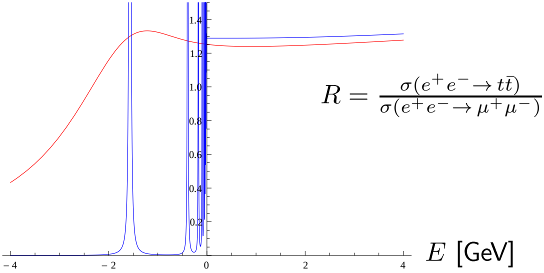
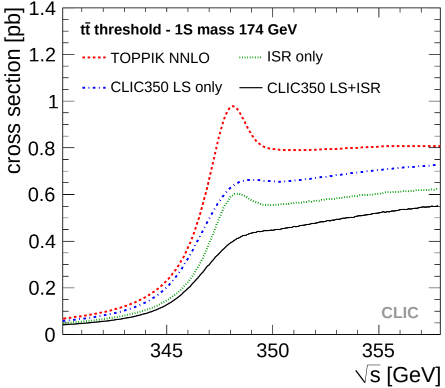
- $b\bar{b}$: bound-state resonances
- $t\bar{t}$: large width prevents existence of bound states



Luminosity Spectrum and Initial-State Radiation



[Seidel, Simon, Tesar, Poss 2013]

- significant influence on cross section shape
- no visible peak

Effective Theories

scale hierarchy: $m_t \gg m_t v \gg m_t v^2 \gg \Lambda_{\mathrm{QCD}}$

QCD full theory

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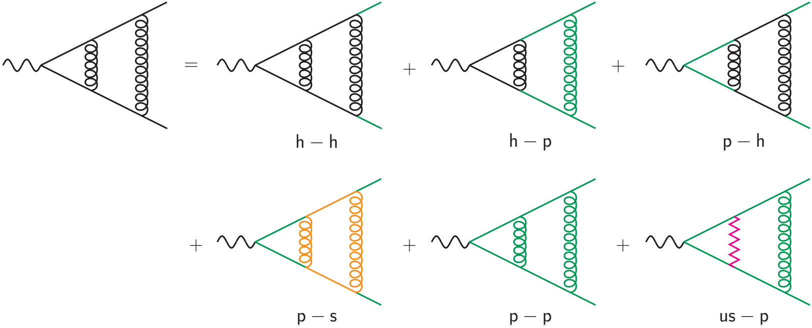
↓ integrate out soft modes: $k^0 \sim k^i \sim m_t v$
soft subgraphs become instantaneous, non-local vertices

pNRQCD — contains potential quarks and ultrasoft gluons

alternative framework: direct matching of QCD to vNRQCD

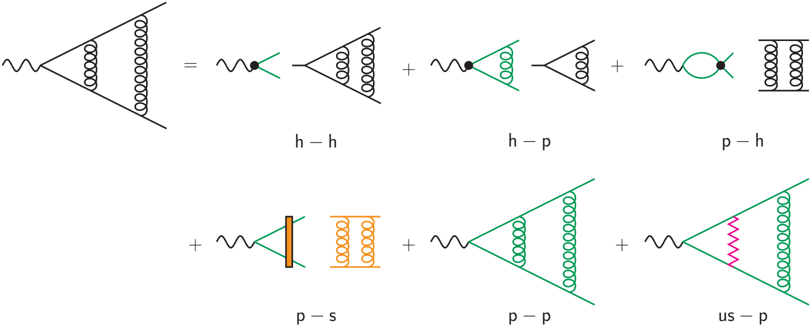
Threshold Expansion



- hard modes: $k \sim m_t$
- potential modes: $k^0 \sim m_t v^2$, $k^i \sim m_t v$
- soft modes: $k \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$

[Beneke, Smirnov 1998]

Threshold Expansion



- hard loops: local operator insertions
- potential loops: Coulomb resummation
- soft loops: instantaneous, non-local operators
- ultrasoft gluons remain dynamical degrees of freedom

integrating out the hard region $\rightsquigarrow$ NRQCD

[Caswell, Lepage 1986; Lepage et al. 1992; Bodwin, Braaten, Lepage 1994; Kinoshita, Nio 1996]

$$\begin{aligned}\mathcal{L}_{\text{NRQCD}} &= \psi^\dagger\left(iD^0 + \frac{\mathbf{D}^2}{2m_t} + \dots\right)\psi - \frac{d_1\, g_s}{2m_t}\psi^\dagger\sigma\cdot\mathbf{B}\psi \\ &+\psi^\dagger\left[\frac{d_2\, g_s}{8m_t^2}\left(\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D}\right) + \frac{d_3\, g_s}{8m_t^2}\sigma\cdot\left(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D}\right)\right]\psi \\ &+\dots + \mathcal{L}_{\text{antiquark}} + \mathcal{L}_{4-\text{quark}} + \mathcal{L}_{\text{light}}\end{aligned}$$

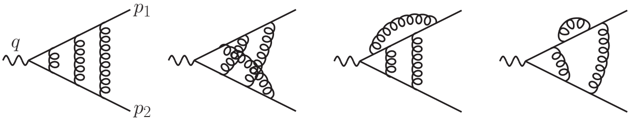
- expansion in top-quark mass
- coefficients have to be determined by matching to QCD

The Vector Current

coupling to external photon:

$$\bar{Q}\gamma^i Q = c_v\,\psi^\dagger\sigma^i\chi + d_v\,\psi^\dagger\frac{\mathbf{D}^2}{6m_t^2}\sigma^i\chi + \ldots$$

- determine c_v and d_v by onshell matching of NRQCD to QCD
- NNLO requires 3-loop result for c_v and 1-loop result for d_v
- only hard region of QCD diagrams contributes



$$\begin{aligned} q^2 &= 4m_t^2 \\ p_i^2 &= m_t^2 \end{aligned}$$

Potential NRQCD

integrating out the soft region $\rightsquigarrow$ pNRQCD

[Pineda, Soto 1997; Beneke, Signer, Smirnov 1999; Brambilla et al. 1999]

$$
\begin{aligned}
\mathcal{L}_{\text{pNRQCD}} &= \psi^\dagger \left(i\partial^0 + g_s A^0(t, \mathbf{0}) + \frac{\partial^2}{2m_t} + \dots \right) \psi \\
&+ \int \mathrm{d}^3\mathbf{r}\, [\psi^\dagger\psi](x+\mathbf{r}) \left(-\frac{\alpha_s C_F}{r} + \delta V(\mathbf{r}) \right) [\chi^\dagger\chi](x) \\
&- g_s \psi^\dagger \mathbf{x}\cdot\mathbf{E}(t, \mathbf{0})\psi + \text{antiquark terms} + \dots
\end{aligned}
$$

- definite power-counting in v
- contains non-local interactions $\rightsquigarrow$ potentials
- LO Coulomb potential is part of LO Lagrangian $\rightsquigarrow$ Coulomb propagator
- ultrasoft fields are multipole expanded

Corrections to the Coulomb potential

the potential in momentum space ($\mathbf{q} = \mathbf{p}' - \mathbf{p}$):

$$\tilde{V}(\mathbf{p}, \mathbf{p}') =$$

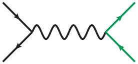
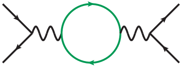
$$-\frac{4\pi\alpha_s C_F}{\mathbf{q}^2}\,\mathcal{V}_C + \frac{4\pi^3\alpha_s C_F}{m_t|\mathbf{q}|}\,\mathcal{V}_{1/m} + \frac{2\pi\alpha_s C_F}{m_t^2}\,\mathcal{V}_\delta$$

$$-\frac{2\pi\alpha_s C_F}{m_t^2}\frac{\mathbf{p^2} + \mathbf{p'^2}}{\mathbf{q}^2}\mathcal{V}_p - \frac{3\pi\alpha_s C_F}{2m_t^2\mathbf{q}^2}\left([\sigma^i, \sigma^j]q^i p^j \otimes 1 - 1 \otimes [\sigma^i, \sigma^j]q^i p^j\right)\mathcal{V}_{so}$$

$$+\frac{\pi\alpha_s C_F}{4m_t^2\mathbf{q}^2}[\sigma^i, \sigma^j]q^j \otimes [\sigma^i, \sigma^k]q^k\,\mathcal{V}_{hf} - \frac{\pi\alpha_s C_F}{4m_t^2}[\sigma^i, \sigma^j] \otimes [\sigma^i, \sigma^j]\,\mathcal{V}_s + \ldots$$

$\mathcal{V}_i$ are series in α_s and depend on NRQCD matching coefficients d_i

use optical theorem:

$\int d\Phi_{\mathrm{PS}} \Big|$  $\Big|^2 \sim \mathrm{Im}$  $\sim \mathrm{Im}\Pi(q)$

compute polarisation function in pNRQCD: $\Pi(q) \sim c_v^2\, G(\vec{0},\vec{0};E)$

- compute hard matching coefficients: d_i, c_v, d_v
- compute potential: $\mathcal{V}_i$
- solve Schrödinger equation
- compute ultrasoft corrections

power counting:

$$\sum_n \left(\frac{\alpha_s}{v}\right)^n \times \Big\{1;\ \underbrace{\alpha_s, v}_{\text{NLO}};\ \underbrace{\alpha_s^2, \alpha_s v, v^2}_{\text{NNLO}};\ \dots\Big\}$$

Theory Status

- NNLO [Hoang, Teubner; Melnikov, Yelkhovsky; Yakovlev; Beneke, Signer, Smirnov; Nagano, Ota, Sumino; Penin, Pivovarov]

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- most recent analysis: $\frac{\delta\sigma}{\sigma}\Big|_{\text{NNLL}} = \pm 5\%$ [Hoang, Stahlhofen 2013]

Finite-Width Divergences

- $E \to E + i\Gamma_t$ at LO $\rightsquigarrow$ Γ_t/ϵ at NNLO
- finite-width divergences lead to scale dependence
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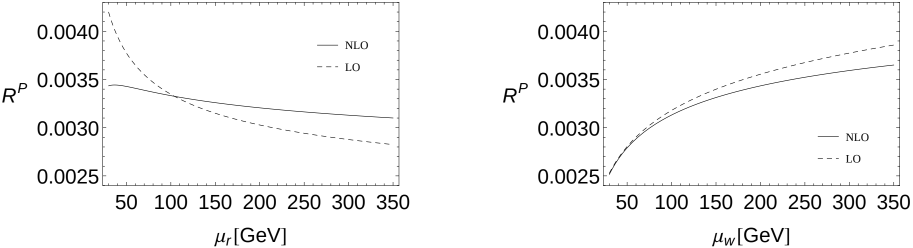
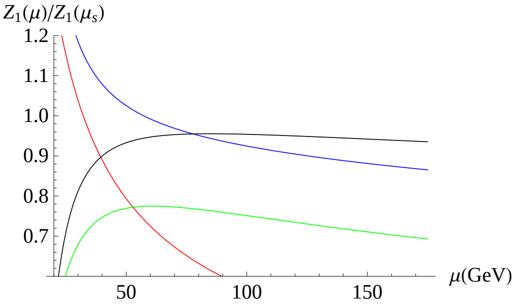
example: P-wave contribution close to threshold [Beneke, JP, Rauh, 2013]



(effect is less pronounced for S-wave)

$$\Pi(q^2) \overset{E \to E_n}{\sim} \frac{Z_n}{E_n - E - i\varepsilon}$$

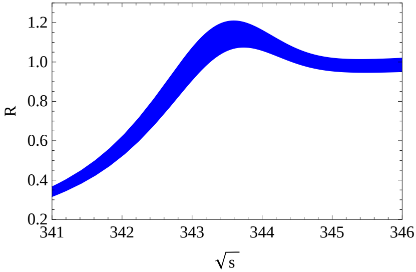
$$Z_1 = \left[c_v^2 - \frac{E_1}{m_t} c_v \left(c_v + \frac{d_v}{3} \right) + \ldots \right] |\psi_1(0)|^2$$



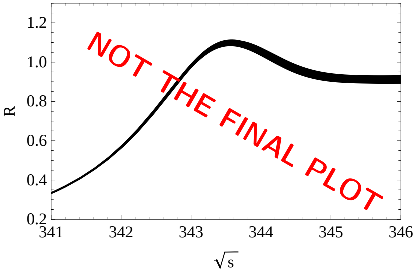
PS scheme

- LO
- NLO
- NNLO
- NNNLO

NNLO:



NNNLO:



PS scheme: $m_t^{\mathrm{PS}} = 171.3$ GeV, $\Gamma_t = 1.4$ GeV, $\alpha_s(M_Z) = 0.1184$
$\mu = 50 - 175$ GeV

final cross checks and analysis of (separate) scale dependence are under way

Summary

- threshold production allows for very precise and theoretically clean determination of top-quark mass
- very high theoretical precision is required
- NNNLO calculation is almost done

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lots of work still to be done, e.g.:

- electroweak/finite-width corrections
- combination of fixed-order and resummed calculations