α_s from lattice QCD

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MITP workshops, High precision fundamental constants at the TeV scale,
March 10–21, 2014, JGU Campus Mainz

Plan of this talk

- Introduction
- Lattice calculation
- Recent update (preliminary)
- Summary

Strong coupling constant: α_s

- High accuracy of α_s is required from the precise SM test and Beyond the SM (BSM) prediction.
  - LHC (8TeV)

    $\Delta\Gamma(H\rightarrow bb)$: ~2.3% for $\Delta\alpha_s$ (and ~3% for Δm_b and ~2% for 2loop EW)

	Scale	PDF+α_s	Total (linear)
$\sigma_{H\rightarrow gg}$	7.1 -- 7.8 %	± 4.0--3.1 %	~11--12 %

1101.0593

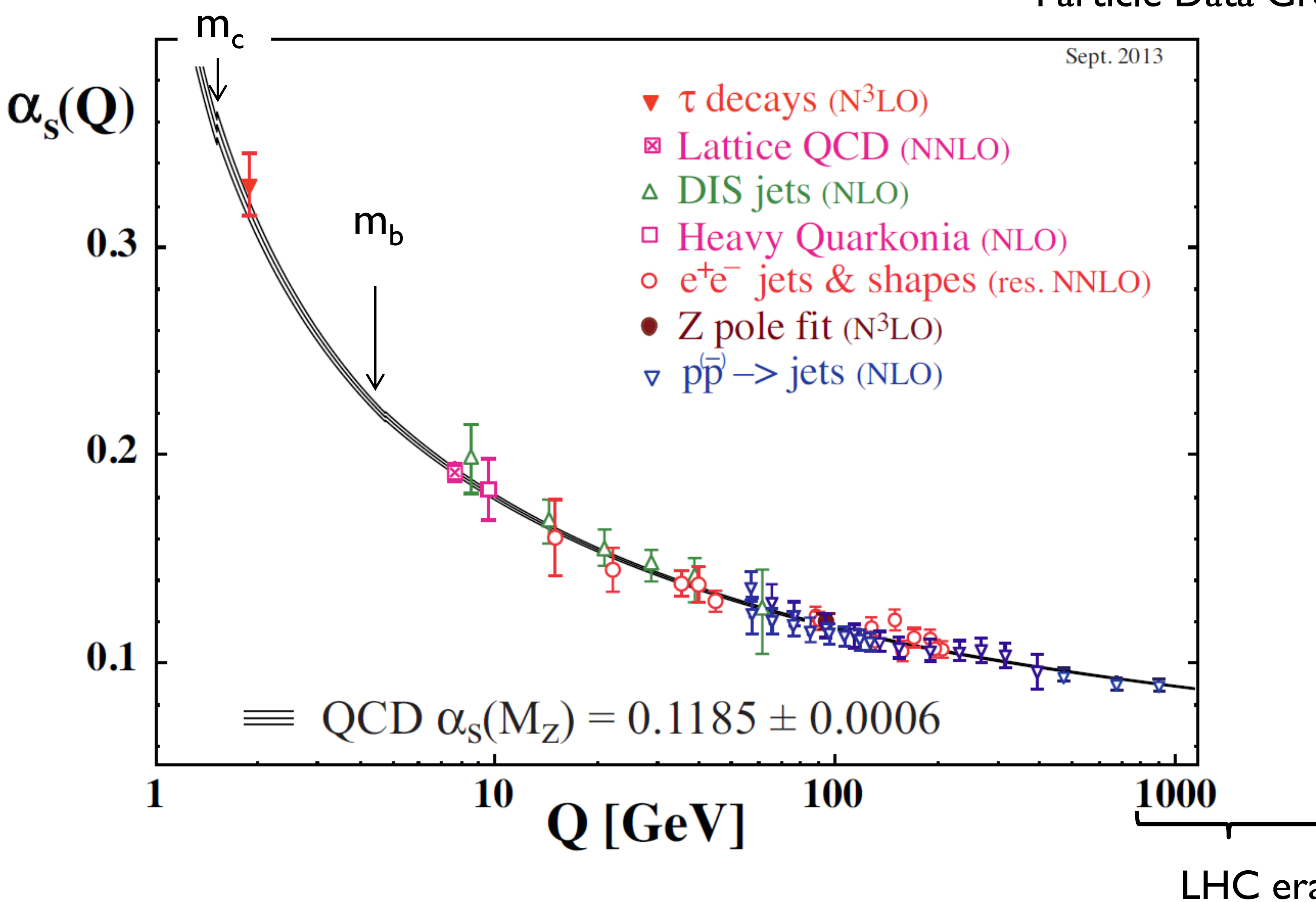
NNLO and higher order of pQCD
$\sigma \sim \alpha_s^2 f_g$ and $O(\alpha_s^3)$ is relatively large.

$\Delta\alpha_s$ and uncertainties of distribution function

For BSM, accuracy of α_s is required to be below EW 2-loop order, which corresponds to < 0.5 %. (and also for m_b)

1. Introduction
Running of α_s

Particle Data Group 2013



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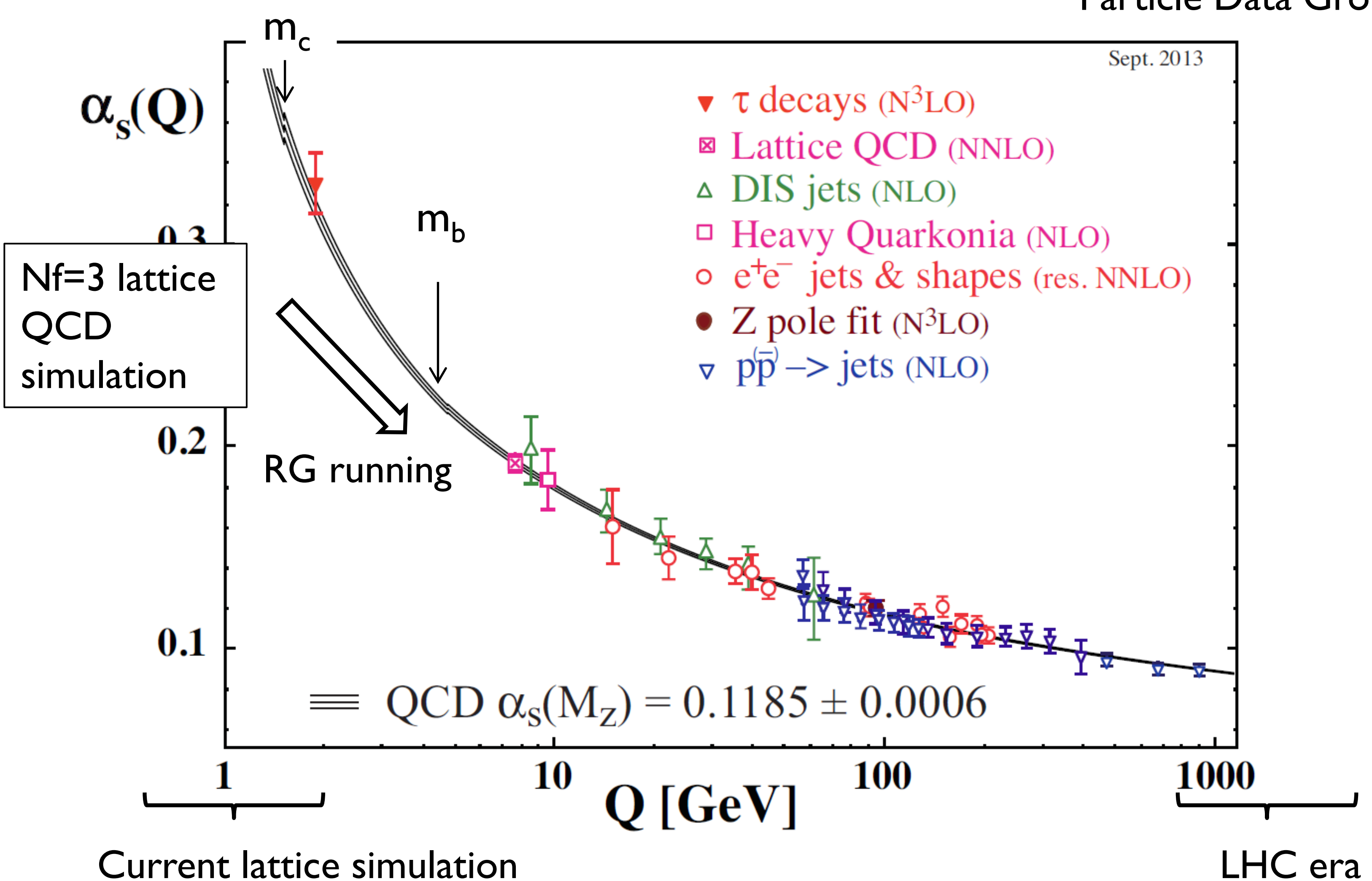
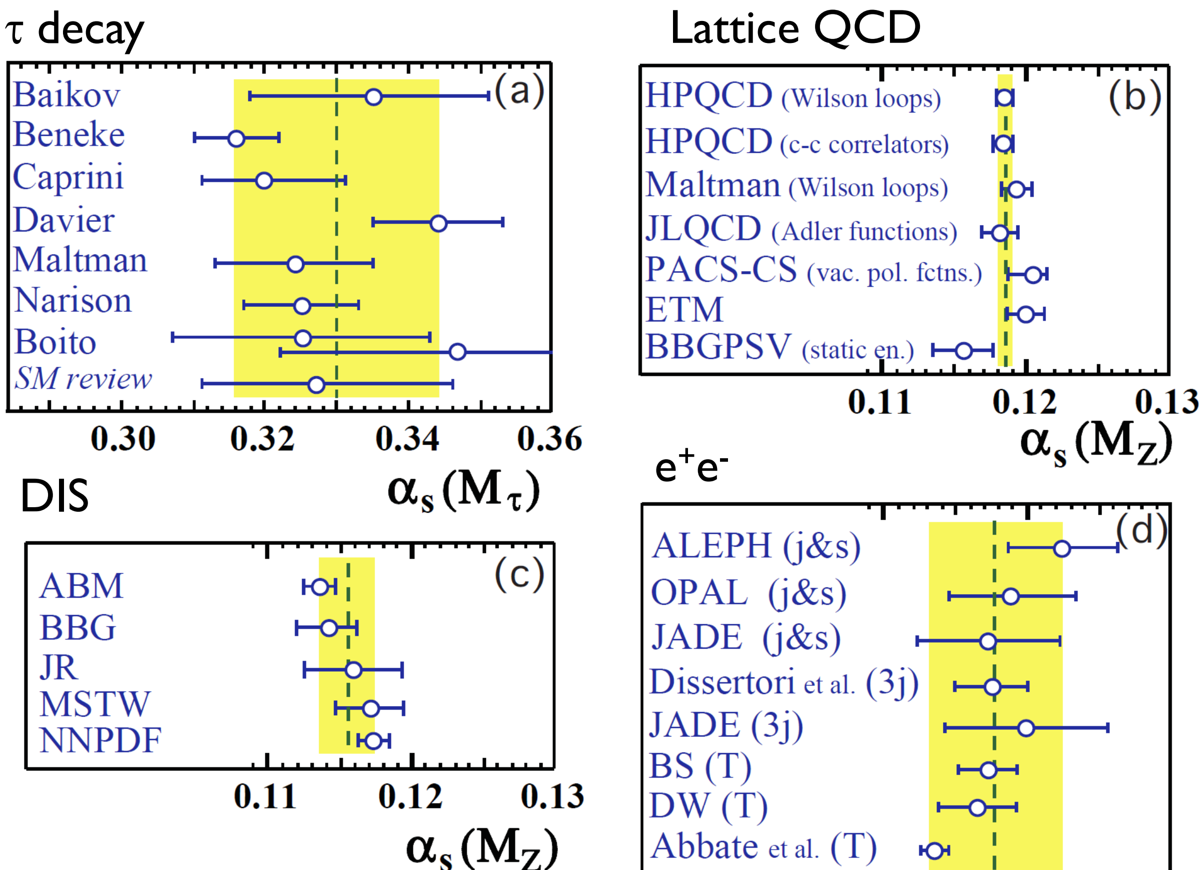


1. Introduction

World average



Particle Data Group 2013

World average: 0.1185(6)

0.5 % relative error

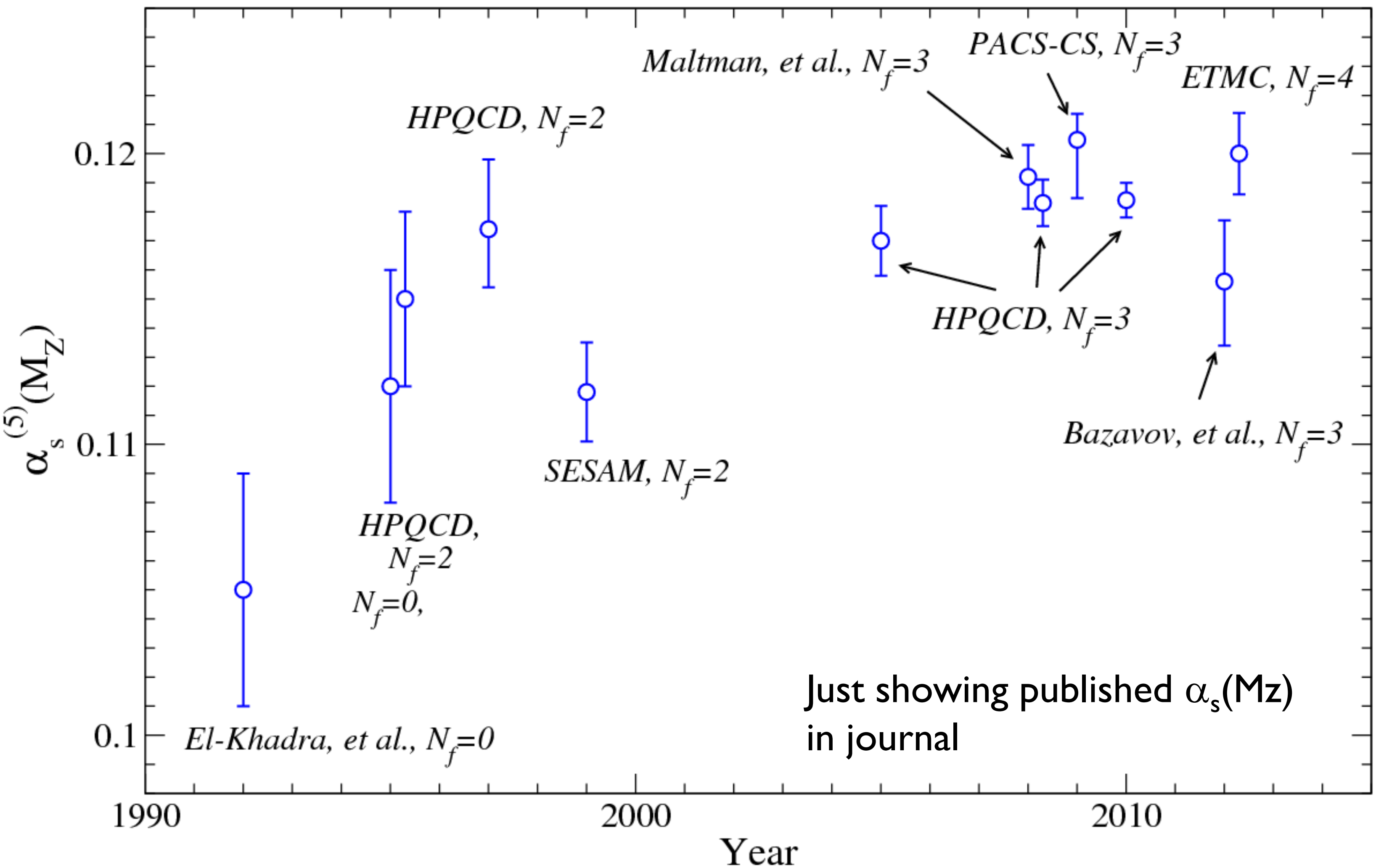
Omitting lattice → 0.1183(12)

50% error increase

⇒ lattice calculation is leading the precision.

Confirm and pursue further precision

History of $\alpha_s(M_z)$ from lattice QCD



Just showing published α_s(Mz) in journal

1. Introduction

Recent lattice efforts

- Wilson loop

  HPQCD [Davies et al. (2008)](Nf=2+1, KS)

- Heavy current correlator (Moments)

  HPQCD [Allison et al. (2008), McNeile et al. (2010)](Nf=2+1, KS)

- Light current correlator (Adler function)

  JLQCD [ES et al.(2009)](Nf=2+1, Overlap)

  RBC/UKQCD [ES et al, lattice 2013] (Nf=2+1, Domain-wall) : recent update

- Schrödinger functional

  ALPHA [DellaMorte et al. (2005)] (Nf=2, Clover)

  PACS-CS [Aoki et al. (2009)](Nf=2+1, Clover)

- Gluon-gluon (-ghost) vertex

  ETMC [Blossier et al. (2012,2014)](Nf=2+1+1, Twisted mass)

- Static energy

  Bazavov et al. (2012) (Nf=2+1, KS)

Strategy

Two ingredients required as “input”

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- For determination of scale on the lattice

$F_{obs}\, a = F_{lattice}$ F: observable like omega masses, pion decay constant or static potential (or force) …

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2. Lattice calculation

Strategy

Two ingredients required as "input"

- For determination of scale on the lattice

$F_{\mathrm{obs}}\, a = F_{\mathrm{lattice}}$ F: observable like omega masses, pion decay constant or static potential (or force) …

Precise determination of *a* is necessary to match a running points with MSbar scheme

- Choose target quantity and scheme
  - Known as perturbative expansion (NNLO, NNNLO, …)

$$\mathcal{O}_{\mathrm{PT}}(\alpha_s) = c_0 + c_1\alpha_s + c_2\alpha_s^2 + \cdots$$

$c_{0,1,2,\ldots}$: analytic function

  - Lattice quantity

$$\mathcal{O}_{\mathrm{lat}} = \sum_n C_n^{\mathrm{renom\,scheme}} (1/\beta^r)^n$$

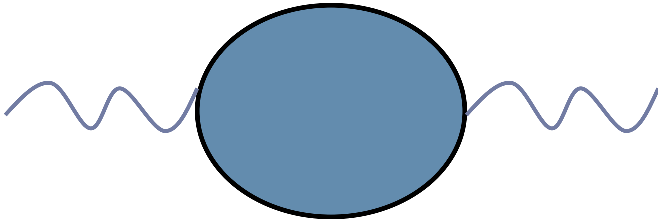
heavy quark potential, small Wilson loops, vacuum polarization function, …

Matching at good convergent region, evaluated α_s with any scheme.
Lattice PT, expanded by bare lattice coupling β^{-1}, is bad convergence, so the quantity should be given by appropriately renormalized coupling.

Legape and Mackenzie (1993)

Vacuum polarization function (VPF)

- Adler function, which is given by derivative of Q^2, is physical quantity.



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- PT expansion (MSbar) up to NNNLO has been known.
- OPE describes non-perturbative effect as the expansion of multiple dimension operator condensate.

2. Lattice calculation

Vacuum polarization function (VPF)

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- OPE describes non-perturbative effect as the expansion of multiple dimension operator condensate.
- Current-current correlator

$$\int d^4x e^{iQx} \langle 0 | J^a_\mu(x) J^{b\dagger}_\nu(0) | 0 \rangle = \delta^{ab} \left[(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu) \Pi^{(1)}_J - Q_\mu Q_\nu \Pi^{(0)}_J(Q) \right]$$

- J : vector (V) or axial-vector (A) current
  (conserved and local current correlator is better for implimentation)
- $\Pi_J^{(1)}(Q)$: VPF (transverse part), $\Pi_J^{(0)}(Q)$: VPF (longitudinal part)
- Using $\Pi_J^{(0)}(Q)+\Pi_J^{(1)}(Q)$
- Q: Euclidean (= space-like, $Q^2 < 0$) momentum

Perturbative expansion

- OPE for VPF at Q^2:

$$\Pi_J^{0+1}(Q) = d + C_0(Q^2,\mu^2,\alpha_s) + C_m^J(Q^2,\mu^2,\alpha_s)\frac{\bar{m}^2(Q)}{Q^2} + \sum_{q=u,d,s} C_{\bar{q}q}^J \frac{\langle m_q \bar{q}q\rangle}{Q^4} + C_{GG}(Q^2,\alpha_s)\frac{\langle \alpha_s/\pi GG\rangle}{Q^4} + \mathcal{O}(Q^{-6})$$

  - *d* is scheme dependent divergence, depending on cut-off scale.
  - Perturbative expansion in the continuum scheme (MSbar)
    → α_s in MSbar directly
  - 4 free parameters : d, <qq>, <GG>, α_s (or Λ_{QCD})
  - C_0 up to $O(\alpha_s^4)$ [Baikov et al. (2008)]
    C_m, C_{qq} up to $O(\alpha_s^2)$ [Chetyrkin et al. (1997), Chetyrkin et al. (1985)]
    C_{GG} up to $O(\alpha_s)$ [Chetyrkin et al. (1985)]

Lattice QCD

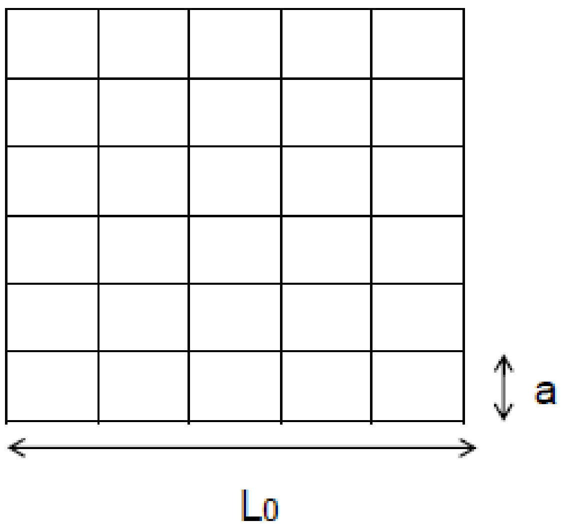
The path integral is computed by Monte-Carlo integral in the lattice regularization

$$\langle O \rangle = Z^{-1} \int D\Psi \, O(\Psi) e^{-S(\Psi)} \rightarrow \sum_{i=1}^{N} p[\Psi_i] O(\Psi_i) + O(1/\sqrt{N})$$

- Exact QCD calculation (but need enough large number of sampling N)
- Gauge invariant
- Translational invariant
- Ultraviolet cut-off *a* (lattice spacing)

  Infrared cut-off $V = L_0^D$ (lattice volume)
- Continuum limit, and infinite volume are needed.
- The development of machine (BG, GPGPU, …)

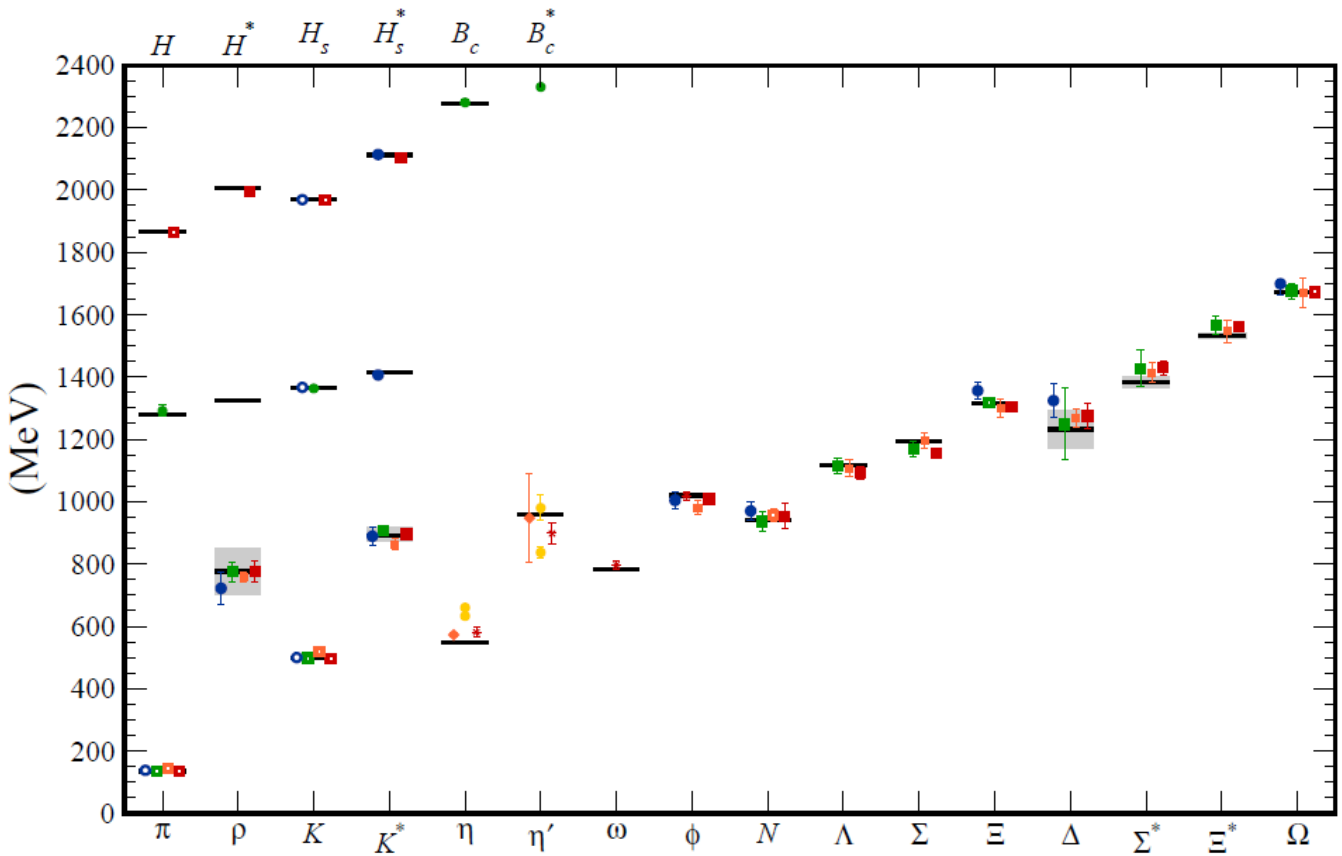
  and algorithm, which make much progress.



Hadron spectrum in lattice QCD

- Good agreement with various lattice action and fermion with experimental results !

Kronfeld, 1209.3468

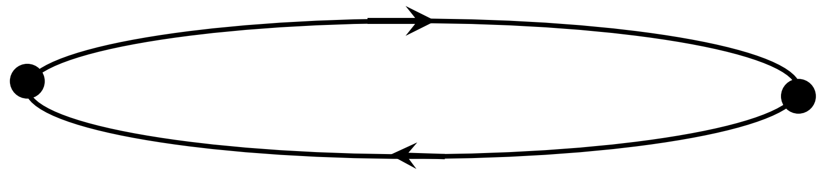


2. Lattice calculation

Method

- Current-current correlator is computed by contraction of two quark propagators between (0,0) ⇔ (x,t)

$$\sum_{x,t} e^{i\vec{Q}\vec{x}+iQ_t t}\Big[J_\mu(0,0)$$



$$J_\nu(x,t)\Big]$$

- Background gluonic and sea quark contribution is included in gauge ensemble $p(\mathrm{U})$. RBC-UKQCD, Phys.Rev.D83.074508(2011)

How To

1. Lattice calculation of current-current correlator, and extracts VPF with tuning parameters, L, a^{-1}, $m_{u,d,s}$ to physical points.
2. Fitting lattice result at Nf=3 (or 4) with OPE formula, $\alpha_s^{(3\text{ or }4)}$ (or $\Lambda_{QCD}^{(3\text{ or }4)}$) is determined at renormalization scale μ
3. RG running to threshold m_b (and also m_c), and switching on 5 flavor
4. Scale to M_z

Systematic uncertainties

- Lattice artifact in large Q^2

$$\langle V_\mu^{\rm cv} V_\nu^{\rm lc}\rangle(Q) = \Pi_J(Q)\Big(Q^2\delta_{\mu\nu} - Q_\mu Q_\nu\Big) + \mathcal{O}((aQ)^3)$$

Lattice artifact

- partially satisfy Ward-Takahashi identity for conserved $V^{\rm cv}$ but not local $V^{\rm lc}$

$$\sum_\mu Q_\mu \langle V_\mu^{\rm cv} V_\nu^{\rm lc}\rangle = 0, \quad \sum_\nu Q_\nu \langle V_\mu^{\rm cv} V_\nu^{\rm lc}\rangle \neq 0$$

- Π_J (Q) also has effect of breaking Lonrenz symmetry on the lattice.

2. Lattice calculation
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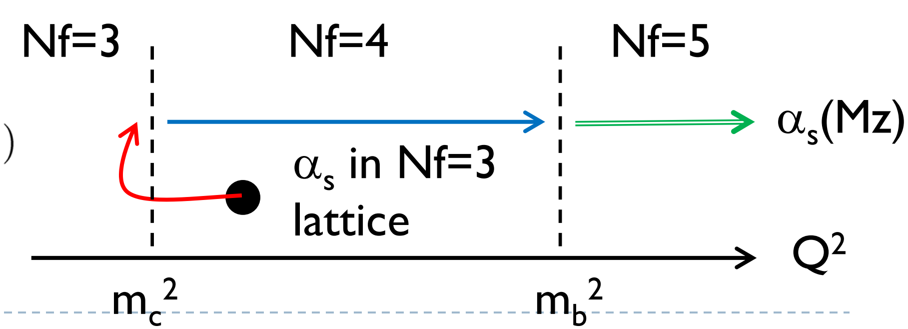
  - Π_J (Q) also has effect of breaking Lonrenz symmetry on the lattice.

- Truncation error
  - Higher order of PT expansion, and higher dimension operator
- Threshold effect
  - using RGE for conversion

$$\alpha_s^{(N_f+1)} = \alpha_s^{(N_f)}\left(1 + \ln\left(\frac{\mu^2}{m_{b,c}^2}\right)\frac{\alpha_s^{(N_f)}}{6\pi^2} + \cdots\right)$$

  - Truncation effect in threshold



Choice of lattice fermion

- There are several kinds of fermion definition on the lattice.

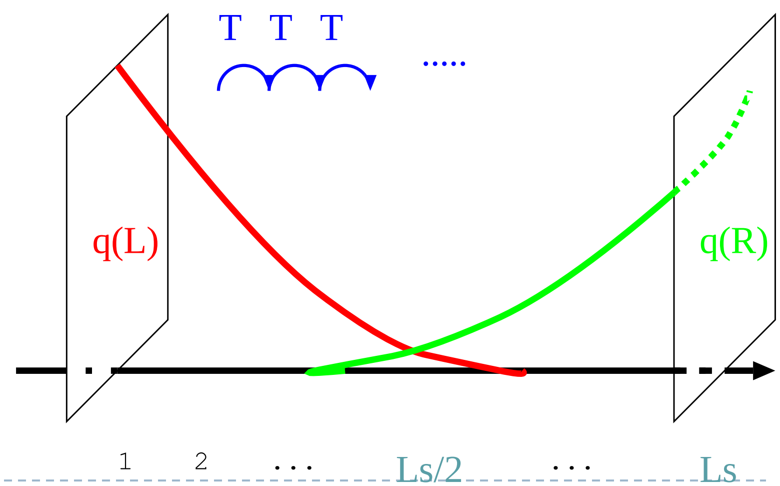
  Wilson(-clover), staggered, domain-wall, overlap, …
- Require "realistic" fermion for the precise calculation
  - good approximated chiral symmetry on the lattice.
  - suppression of O(a) effect, but trading computational time
  - Domain-wall fermion is appropriate selection.
- Domain-Wall fermion (DWF) [Blum Soni, (97), RBC/UKQCD. (05 --)]

  - L, R fermion are localized on boundaries

    Exact chiral symmetry is realized if $L_s \to \infty$.
  - In finite L_s

    Violation of chiral symm. is suppressed as

    $am_{res} \sim \exp(-L_s) \ll 1$.

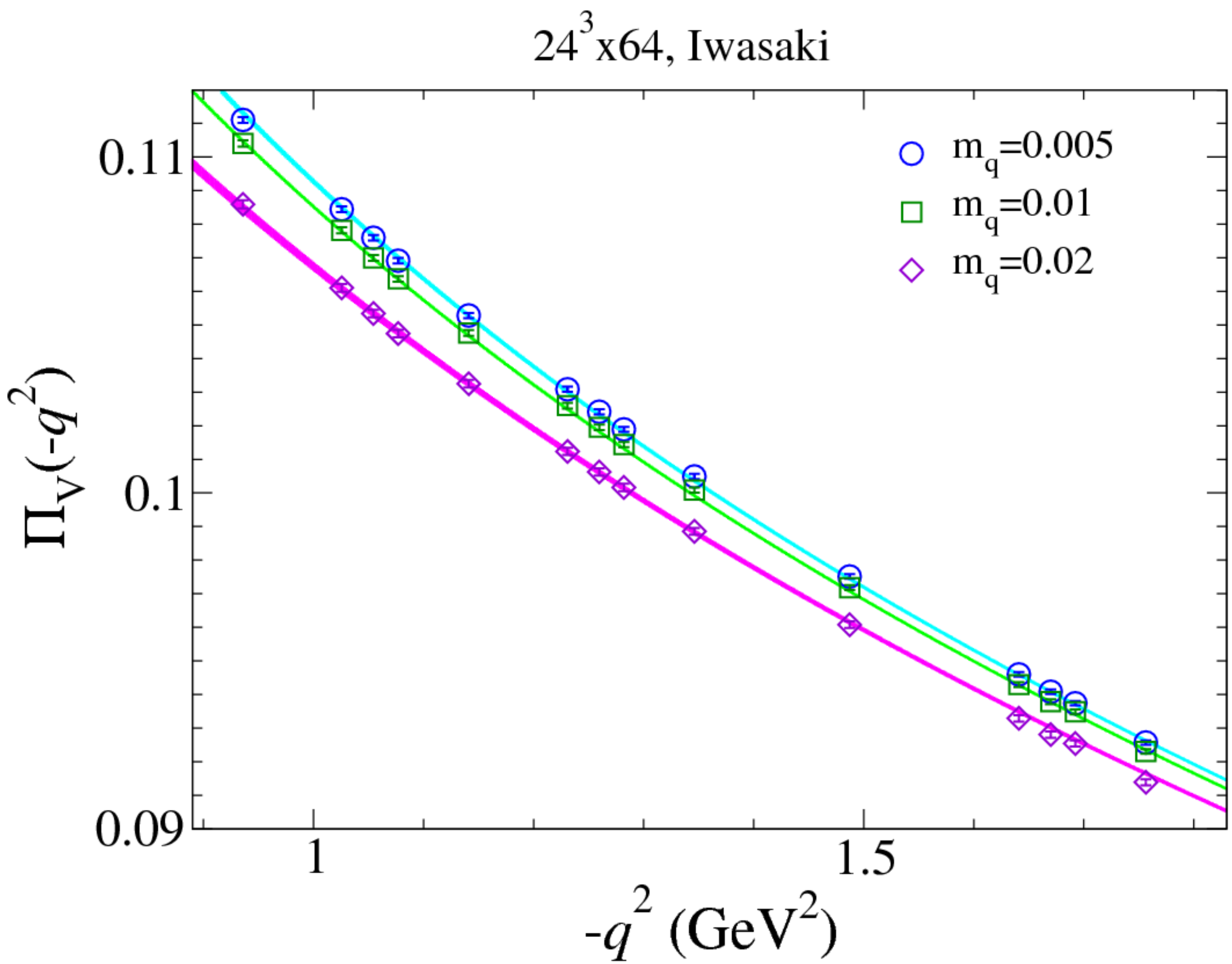


3. Recent update (preliminary)
Tuning parameters

- DWF
  - $24^3 \times 64$ lattice, a^{-1} = 1.73 GeV (~2.7 fm^3 lattice)
    $32^3 \times 64$ lattice, a^{-1} = 2.28 GeV (~2.8 fm^3 lattice) — Continuum limit
  - L_s = 16 and am_{res} = 0.003 (< 1% pion mass shift) → negligible
  - m_π = 0.28 -- 0.42 GeV → chiral extrapolation
- All-mode-averaging (AMA) Blum, Izubuchi, ES (2013)
  - Statistical error is significantly reduced by using truncated solver
  - Low-mode preconditioning
  - # of low-mode : N_λ = 400 (m=0.005), 180 (m=0.01)
  - Stopping condition, 180 CG iteration
  - N_G = 32 (2 for spatial $\times$ 4 for temporal direction of source location)
  - More than 5 times speed-up,

3. Recent update (preliminary)

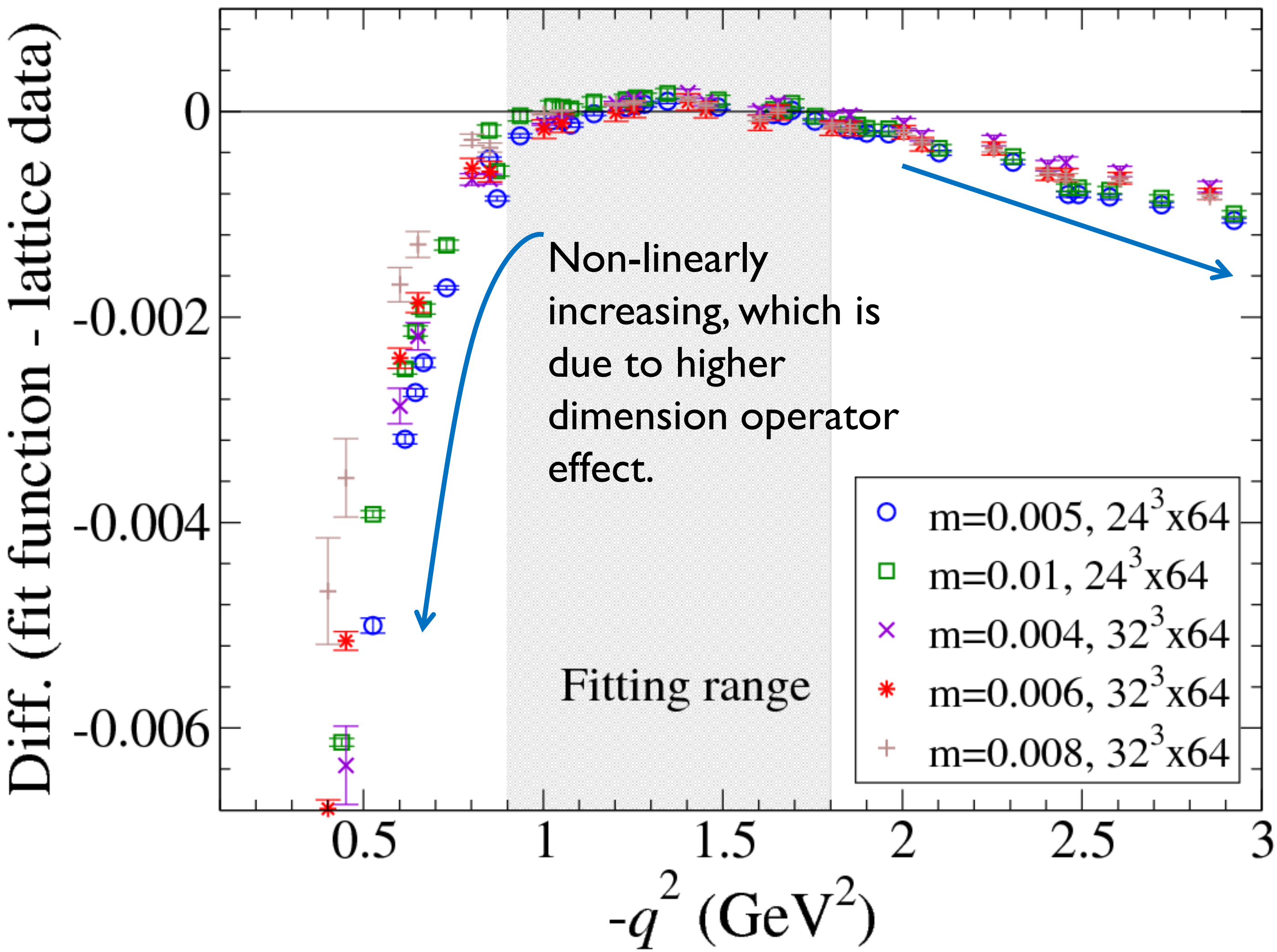
VPF from 1 GeV^2 to 2 GeV^2



- High precise lattice data, less than sub-percent level.
- Lattice data is discretized because of lattice momentum.
- Fitting quality with NNLO PT+OPE(4dim) is good, and chi^2/dof is less than 2.
- Quark mass dependence is described by 4dim term in OPE.

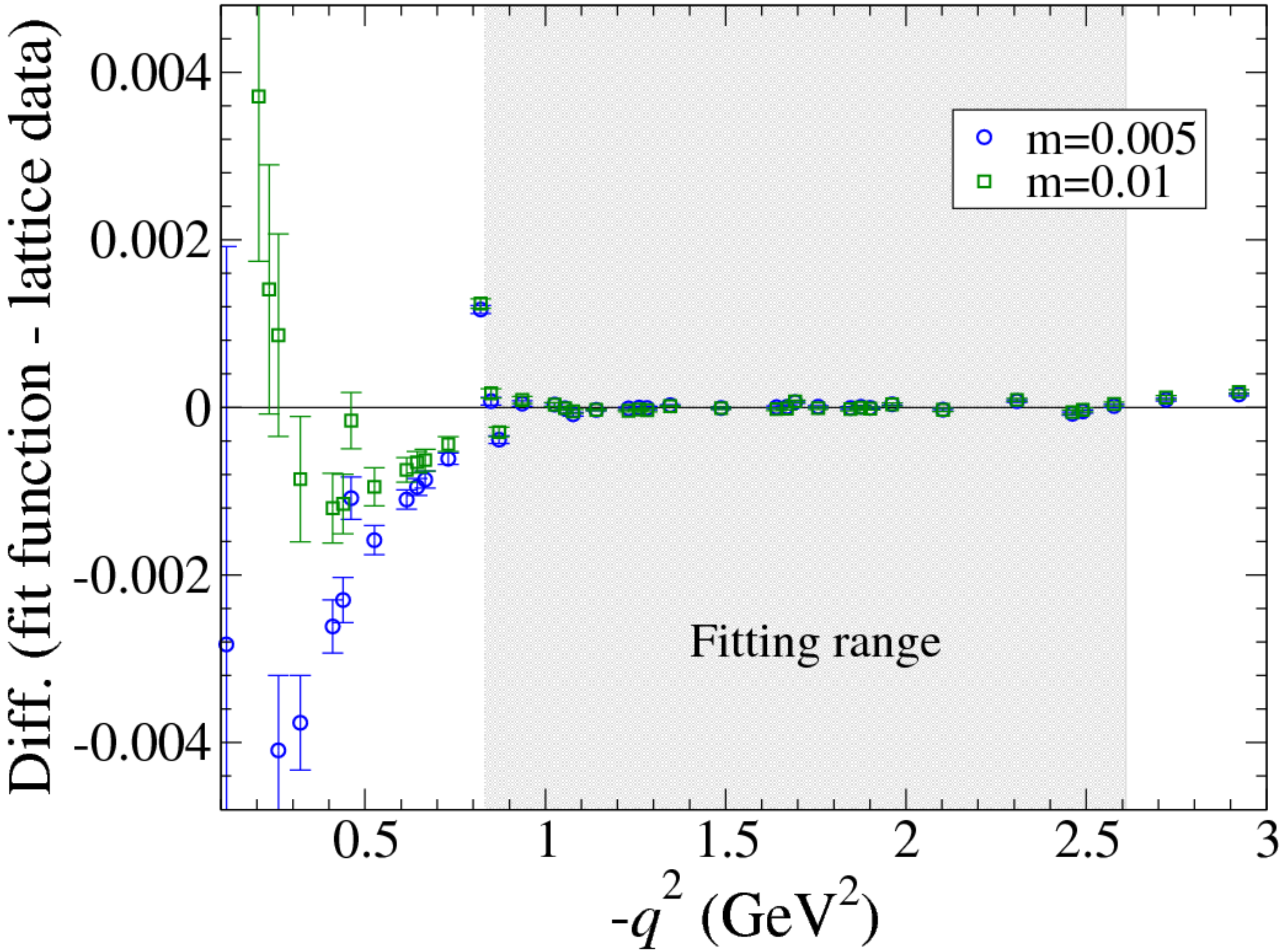
3. Recent update (preliminary)

Discrepancy from perturbation + OPE



Discrepancy is linearly increasing, which is $O((aQ)^2)$ lattice artifact.

3. Recent update (preliminary)
Lattice artifacts

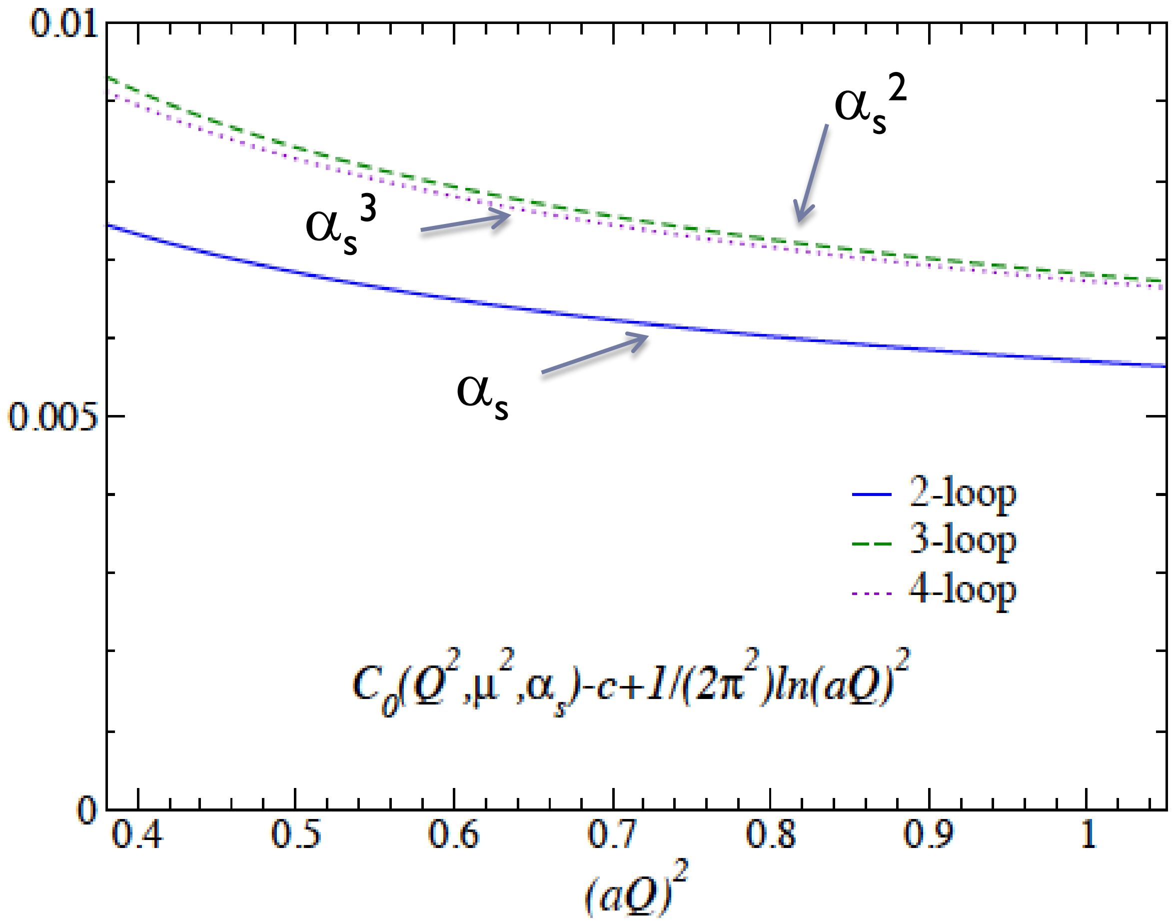


- Including the additional fitting parameter corresponding to lattice artifact.
- Fitting becomes better in high Q^2 region.

3. Recent update (preliminary)

Convergence

- Correction to C_0 from higher orders than NNLO



- The contribution of $O(\alpha_s{}^3)$ is small.
- The correction of $O(\alpha_s{}^4)$[5-loop] is about the same size.

  Baikov, Chetyrkin, Kuhn (2008)

3. Recent update (preliminary)

Table of fitting result

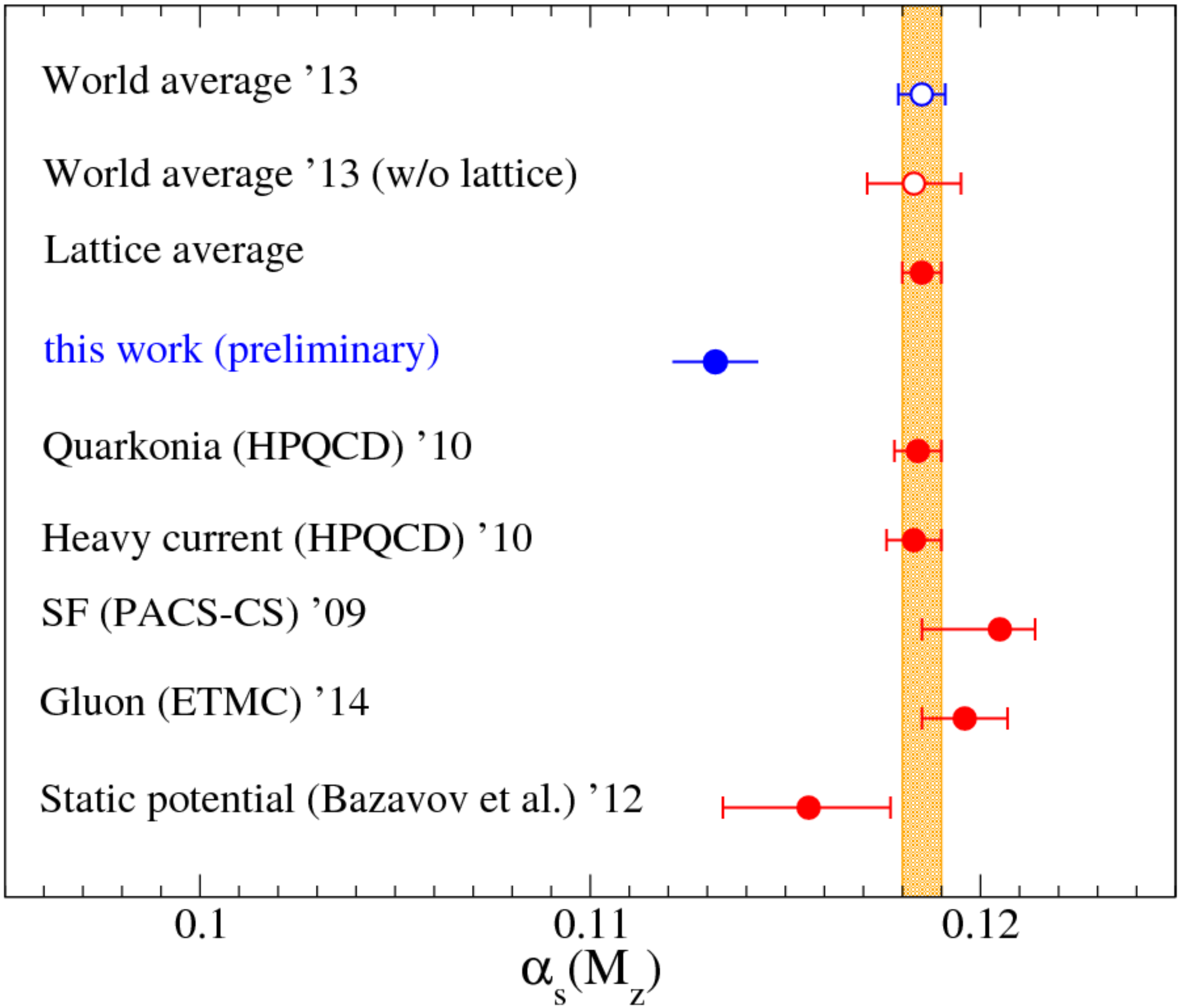
a^{-1}(GeV)	α_s/π(2GeV)	$\Lambda^{(3)}_{\overline{\rm MS}}$ (GeV)	$\langle q\bar{q}\rangle^{1/3}_{\overline{\rm MS}}$ (GeV)	$\langle \frac{\alpha_s}{\pi} GG\rangle$ (GeV^4)	χ^2/dof
1.73	0.0819(3)	0.2486(24)	-0.256(fix)	0.205(3)	2.5
2.23	0.0820(3)	0.2489(19)	-0.256(fix)	0.474(10)	2.7
1.83[1]	0.0817(6)	0.247(5)	-0.242(fix)	-0.020 (2)	2.8
1.73	0.0819(4)	0.2486(27)	-0.276(11)	0.237(18)	1.8
2.23	0.0830(8)	0.2557(56)	-0.325(29)	0.744(144)	1.3
Cont.	0.0844(20)				

[1] overlap fermion, [JLQCD (2010)]

- More accurate statistics
- Chiral condensate is in good agreement with result obtained from Hadron spectroscopy. ⇒ consistency with OPE formula
- Strong coupling constant in Nf=3 in physical point is 2.4% accuracy. (but it is statistical error only)

3. Recent update (preliminary)

$\alpha_s(M_Z)$ from lattice QCD



- Precision of statistics is 1 % level.
- Our result is smaller value than world average, discrepancy over 2 sigma.
- Careful estimate of systematic error is needed (under way)

Summary

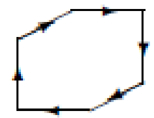
- Lattice QCD calculation of α_s is important to lead the precision.
- Several groups pursue very high precision, ~0.5% accuracy.
- Starting the computation including sea charm quark, which is able to avoid the m_c threshold uncertainty.
- Physical pion simulation is also under way.

Backup

Wilson loop results

HPQCD (2008)

- Target: Wilson loop, PT expansion → MSbar alpha$_s$ (M_Z)

$$W_{mn},\quad \Big\langle \mathrm{BR} = $$  $$\Big\rangle_{\mathrm{lat}} \quad \Big\langle \mathrm{CC} = $$  $$\Big\rangle_{\mathrm{lat}}$$

$$\Rightarrow \sum_{n=1}^{5} c_n \alpha_v^n(d/a)$$

- Lattice PT with improved coupling constant for $c_{n=1,2,3}$ and multi cut-off fit for $c_{n=4,5}$
- Fitting $\alpha_V(d/a)$ at different cut-off and several W loops

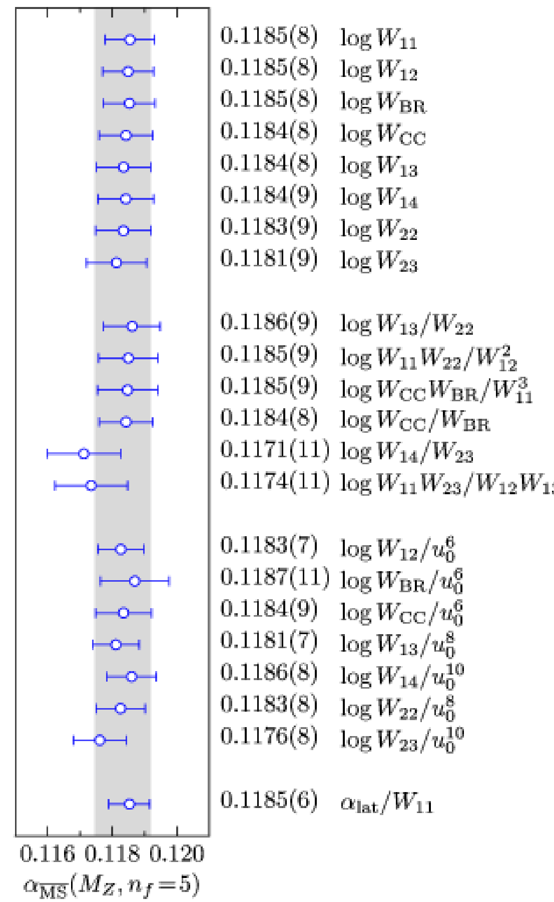
$$q^2 \frac{d\alpha_V(q)}{dq^2} = -\beta_0 \alpha_V^2 - \beta_1 \alpha_V^3 - \beta_2 \alpha_V^4 - \beta_3 \alpha_V^5$$

$$\Rightarrow \alpha_V(7.5\mathrm{GeV}, n_f = 3) = 0.2120(28)$$

Convert to MSbar in N^{3}LO and b, c scale with PT

$$\Downarrow$$

$$\alpha_s^{\overline{MS}}(M_Z, n_f = 5) = 0.1183(8)$$



Heavy quark correlator

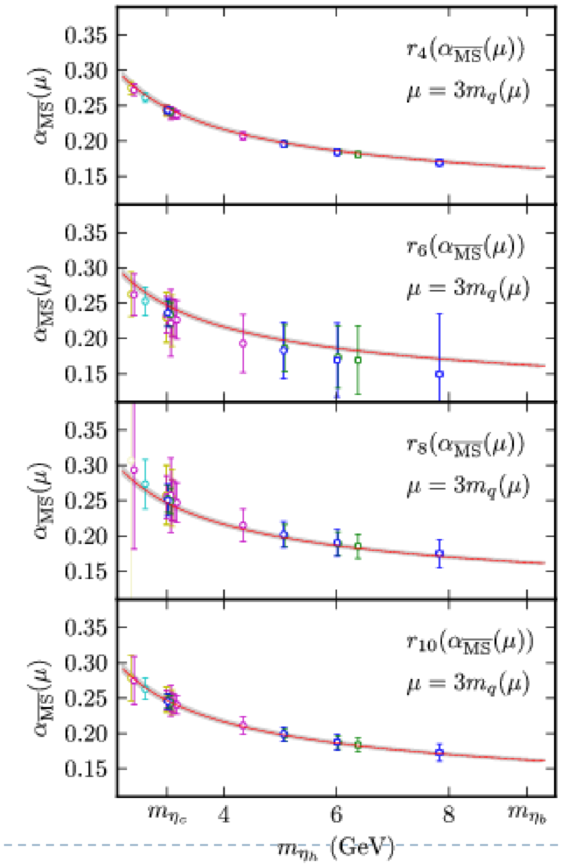
HPQCD (2009, 2010)

- Target : the moments of Heavy-heavy current correlator

$$G(t) = a^6 \sum_{\vec{x}} (am_h)^2 \langle j_5^h(\vec{x},t) j_5^h(0,0) \rangle, \quad G_n = \sum_t (t/a)^n G(t)$$

and ratio to the tree level

$$R_n = \begin{cases} G_4(t)/G_4(0) & (n=4), \\ \frac{am_{\eta_h}}{2am_h} (G_n/G_n^{(0)})^{1/(n-4)} & (n \geq 6) \end{cases}$$



Fitting $R_{4\text{--}18}$ with PT form:

$$r_n(\alpha_s^{\overline{MS}}, \mu) = 1 + \sum_{j=1}^{N_{\rm ph}=6} r_{n\,j}(\mu/m_h)(\alpha_s^{\overline{MS}})^j(\mu)$$

to obtain $\alpha_s^{(5)}(M_Z) = 0.1183(7)$

Also they obtained
m_c(3 GeV, Nf=4) = 0.986(6) GeV,
m_b (10 GeV, Nf=5) = 3.62(3) GeV

Schrödinger functional scheme

- Target : Dirichlet BC

$$\left.\frac{\partial \Gamma}{\partial \eta}\right|_{\eta=\nu=0} = \frac{k}{\bar{g}^2(L)}$$

Differential of $\Gamma[\eta,\nu]$ = ln Z with respect to BC field η, ν gives renormalized coupling. k determined by PT.

Non-perturbative running coupling with step scaling method

$$\Sigma(u, \frac{a}{L}) = \bar{g}^2(2L)\Big|_{u=\bar{g}^2(L)}, \quad \sigma(u) = \lim_{a/L \to 0} \Sigma(u, \frac{a}{L})$$

continuum limit with L=4,6,8

$\bar{g}^2(L)$ is run to high scale using $\sigma(u)$,

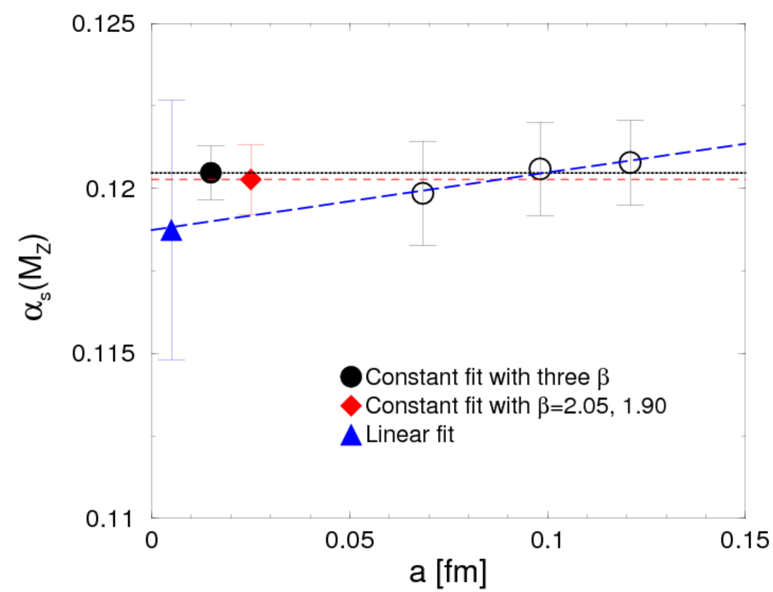
and then converted to MSbar with

NNLO PT

$$\alpha_s^{\overline{MS}}(sq) = \sum_{i=0}^{2} c_i(s) \alpha_{SF}^{i+1}(q)$$

$\Downarrow$ matching at physical m_c and m_b

$$\alpha_s^{(5)}(M_z) = 0.1205(^{+9}_{-16})$$



Analytic form of C

$$
\begin{aligned}
C_0(Q^2,\mu^2) &= \frac{1}{16\pi^2}\Big\{\frac{20}{3}+4\ln\frac{\mu^2}{Q^2}+\frac{\alpha_s(\mu^2)}{\pi}\Big[\frac{55}{3}-16\zeta(3)+4\ln\frac{\mu^2}{Q^2}\Big] \\
&+ \Big(\frac{\alpha_s(\mu^2)}{\pi}\Big)^2\Big[\frac{41927}{216}-\frac{3701}{324}N_f-\Big(\frac{1658}{9}-\frac{76}{9}N_f\Big)\zeta(3)+\frac{100}{3}\zeta(5) \\
&+ \Big\{\frac{356}{6}-\frac{11}{3}N_f-\Big(44-\frac{8}{3}N_f\Big)\zeta(3)+\Big(\frac{11}{2}-\frac{1}{3}N_f\Big)\ln\frac{\mu^2}{Q^2}\Big\}\ln\frac{\mu^2}{Q^2}\Big]\Big\} \\
C_m^V(Q^2,\mu^2) &= \frac{1}{4\pi^2}\Big[-6+\frac{\alpha_s(\mu^2)}{\pi}\Big(-16-12\ln\frac{\mu^2}{Q^2}\Big) \\
&+ \Big(\frac{\alpha_s(\mu^2)}{\pi}\Big)^2\Big\{-\frac{19691}{72}+\frac{95}{12}N_f-\frac{124}{9}\zeta(3)+\frac{1045}{9}\zeta(5) \\
&- \Big(55+12\ln\frac{\mu^2}{Q^2}\Big)\ln\frac{\mu^2}{Q^2}-\Big(11-\frac{2}{3}N_f\Big)\Big(\frac{13}{2}+\frac{3}{2}\ln\frac{\mu^2}{Q^2}\Big)\ln\frac{\mu^2}{Q^2}\Big\}\Big] \\
&+ \frac{N_f}{16\pi^2}\Big(\frac{\alpha_s(\mu^2)}{\pi}\Big)^2\Big[\frac{128}{3}-32\zeta(3)\Big] \\
C_m^A(Q^2,\mu^2) &= \frac{1}{4\pi^2}\Big[-6+\frac{\alpha_s(\mu^2)}{\pi}\Big(-12-12\ln\frac{\mu^2}{Q^2}\Big) \\
&+ \Big(\frac{\alpha_s(\mu^2)}{\pi}\Big)^2\Big\{-\frac{4681}{24}+\frac{55}{12}N_f-\Big(34-\frac{8}{3}N_f\Big)\zeta(3)+115\zeta(5) \\
&- \Big(47+12\ln\frac{\mu^2}{Q^2}\Big)\ln\frac{\mu^2}{Q^2}-\Big(11-\frac{2}{3}N_f\Big)\Big(\frac{11}{2}+\frac{3}{2}\ln\frac{\mu^2}{Q^2}\Big)\ln\frac{\mu^2}{Q^2}\Big\}\Big] \\
&+ \frac{N_f}{16\pi^2}\Big(\frac{\alpha_s(\mu^2)}{\pi}\Big)^2\Big[\frac{128}{3}-32\zeta(3)\Big]
\end{aligned}
$$

Analytic form of C

$$
\begin{aligned}
C^{V/A}_{\bar{q}q}(Q^2,\mu^2) \quad &= \quad -2\frac{\alpha_s(\mu^2)}{\pi}\Big[1+\frac{1}{24}\frac{\alpha_s(\mu^2)}{\pi}\Big\{116-4N_f+(66-4N_f)\ln\frac{\mu^2}{Q^2}\Big\}\Big] \\
+/- \quad & 2\Big[1+\frac{4}{3}\frac{\alpha_s(\mu^2)}{\pi}+\frac{4}{3}\Big(\frac{\alpha_s(\mu^2)}{\pi}\Big)^2\Big\{\frac{191}{24}-\frac{7}{36}N_f+\Big(\frac{11}{4}-\frac{1}{6}N_f\Big)\ln\frac{\mu^2}{Q^2}\Big\}\Big] \\
+ \quad & \frac{N_f}{3}\Big(\frac{\alpha_s(\mu^2)}{\pi}\Big)^2\Big(4\zeta(3)-3+\ln\frac{\mu^2}{Q^2}\Big)+0/4
\end{aligned}
$$

$$
C_{GG}(Q^2,\alpha_s)=\frac{1}{12}\Big[1-\frac{11}{18}\frac{\alpha_s(Q)}{\pi}\Big]
$$