6d CFTs and nilpotent orbits

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Introduction

We will consider M5s at ADE singularities

with M2s, this strategy led to [Aharony, Bergman, Jafferis, Maldacena '08]

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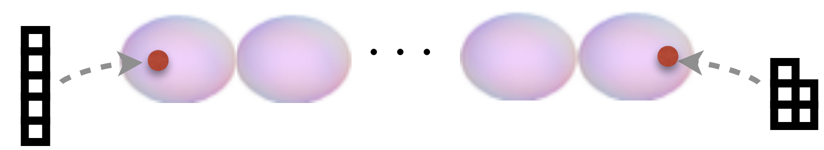
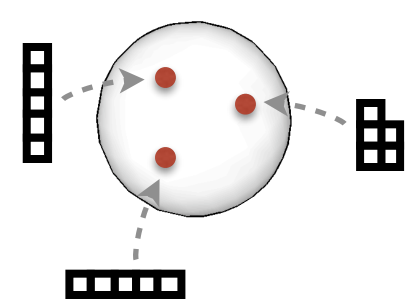
superconformal theories

interesting 'daughter' SCFTs

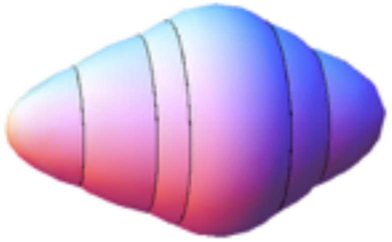
parameterized by 2 nilpotent elements in the ADE Lie group



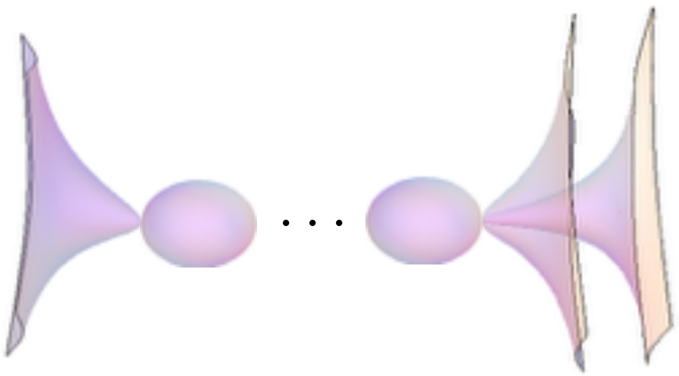
[4d 'class S' theories: 3 nilpotent elements ("punctures")]



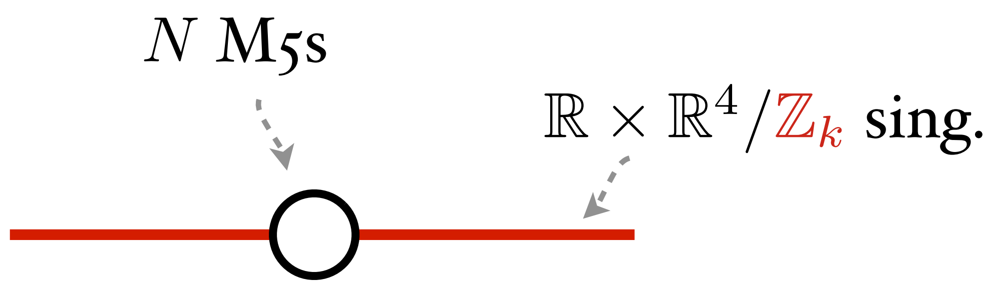
I. 'A' case: IIA realization, AdS7 duals



II. 'D, E' cases: F-theory

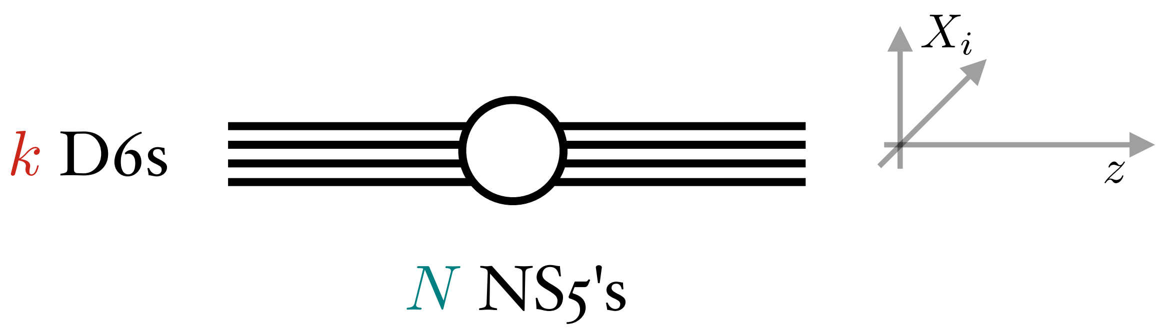


I. M5 branes at a $\mathbb{Z}_k$ singularity



- superconformal
- $\mathcal{N} = (1,0)$ supersymmetry
- number of dof: $a \sim k^2 N^3$

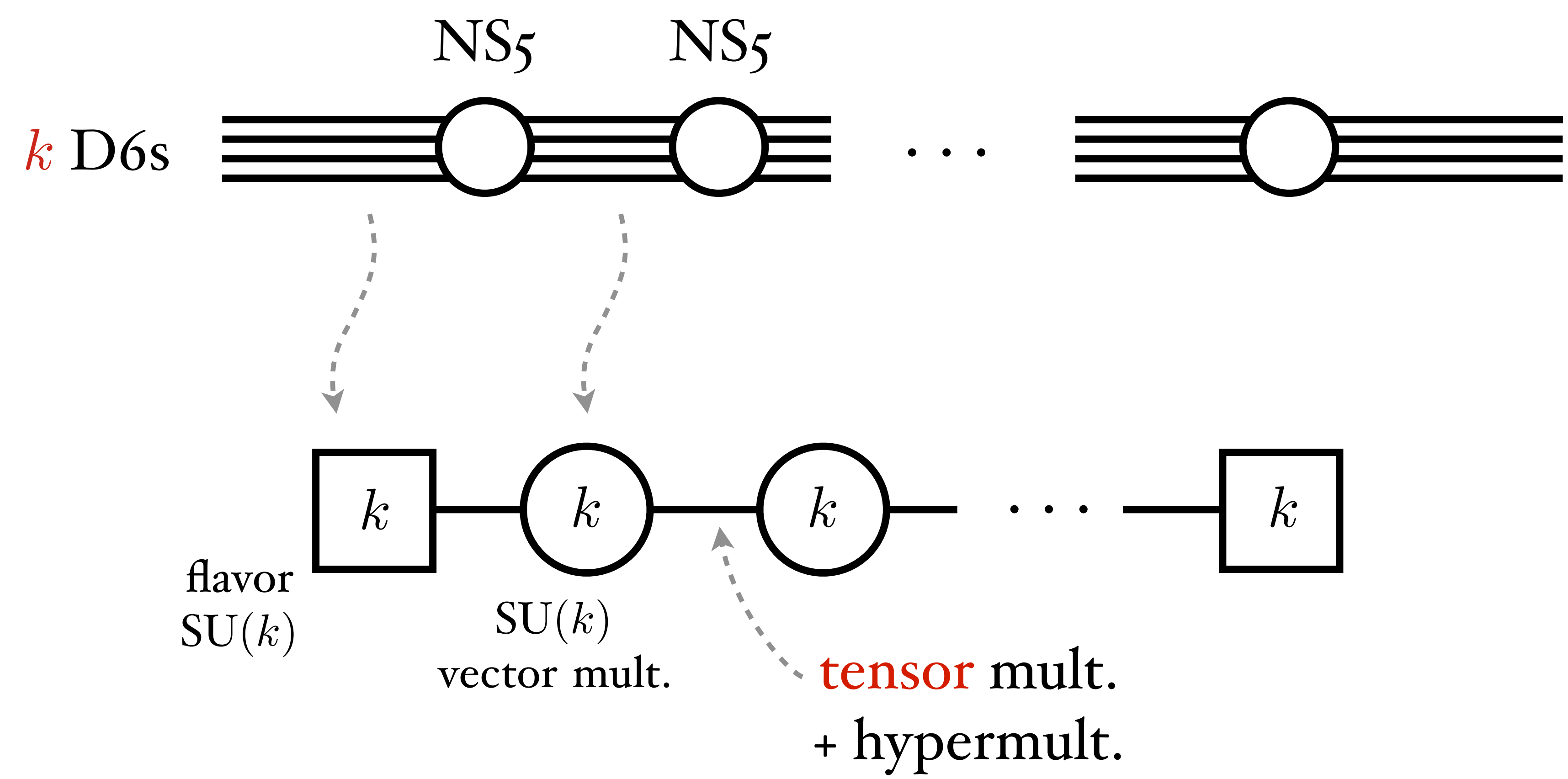
- If we reduce to IIA:



BPS equations on D6:
Nahm equations

$$\partial_z X_i = \epsilon_{ijk}[X_j, X_k]$$

- ‘Effective description’ with gauge groups by separating NS5s



vector mult. (λ_+, A_μ)

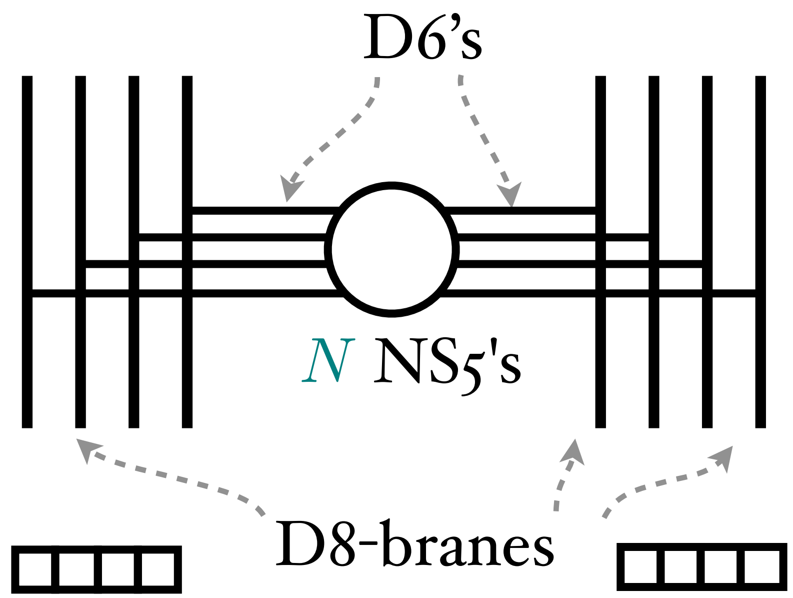
hypermult. (q, ψ_-)

tensor mult. $(\phi, \chi_-, b^+_{\mu\nu})$

$\mathcal{L} \supset (\phi_{i+1} - \phi_i)\mathrm{Tr}F^2$ $\phi_i = x^6$ positions of NS5's

coincident NS5s =
strong coupling point: CFT

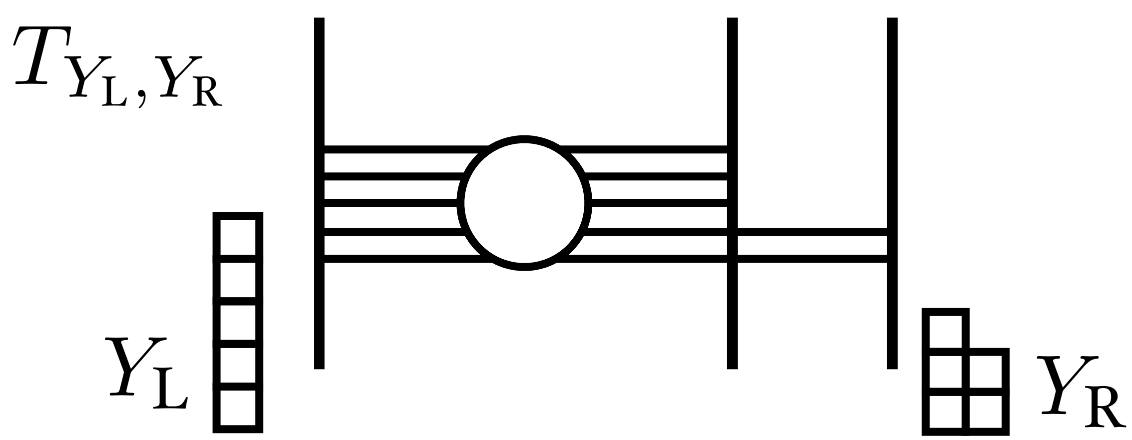
- alternative realization: each D6 ends on a single D8 [Gaiotto, Witten '08; Gaiotto, AT '14]



D8-branes
||
Dirichlet boundary cond. for X_i

- more general conformal theories obtained by

[(Blum,)Intriligator '97, Hanany, Zaffaroni '97, Brunner, Karch '97...]
[Gaiotto, Witten '08; Gaiotto, AT '14]



for ex.

$$t^1 + it^2 = \left(\begin{array}{c|c} \begin{matrix} & 1 & \\ & & 1 \\ & & \end{matrix} & \\ \hline & \begin{matrix} & 1 \\ & \end{matrix} \end{array}\right)$$

Nahm pole boundary conditions:

$$X^i \sim \frac{t_i}{z} \dashrightarrow \mathrm{su}(2) \text{ subalgebra of } \mathrm{su}(k)$$

$\wr||$

partition

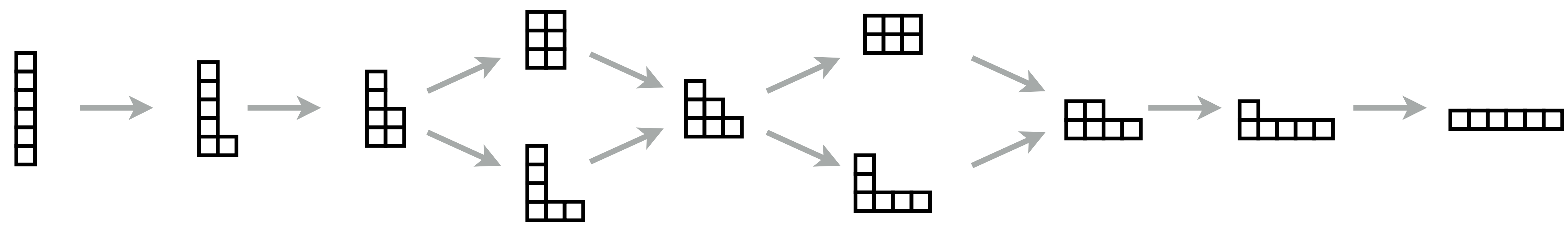
- One can argue

$$T_{Y_{\rm L},Y_{\rm R}} \xrightarrow{\text{RG flow}} T_{Y'_{\rm L},Y_{\rm R}}$$

$$\Updownarrow$$

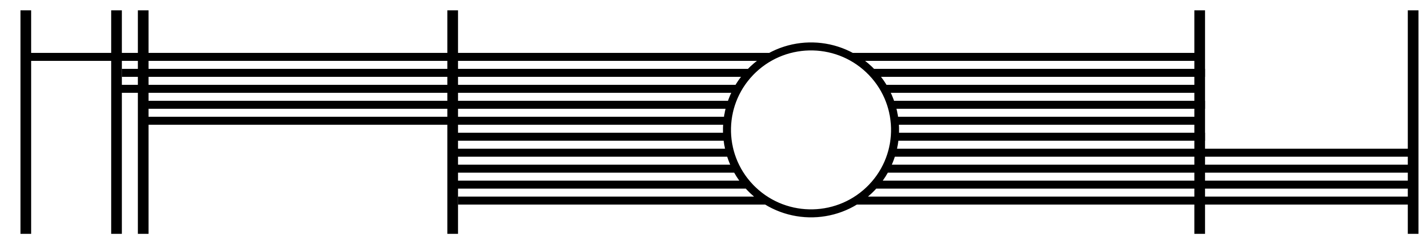
$$Y_{\rm L} < Y'_{\rm L}$$

under the partial ordering of partitions:

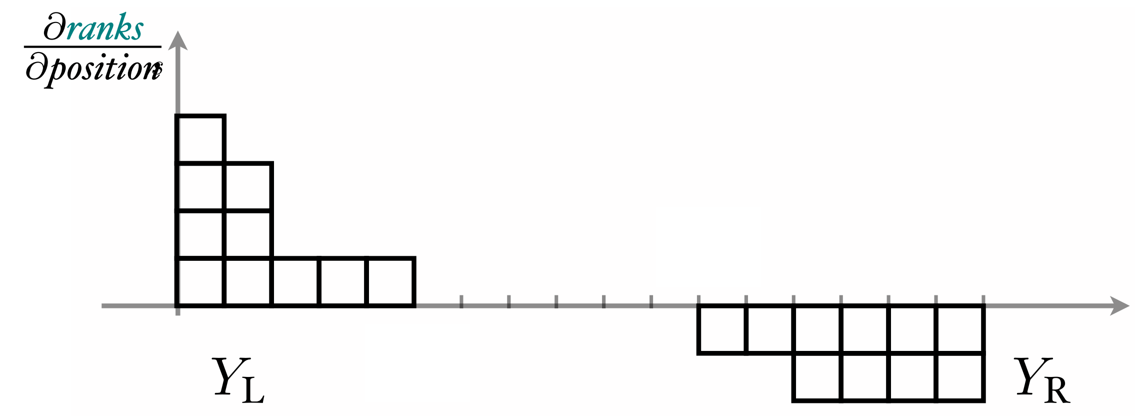


AdS7 gravity dual also available

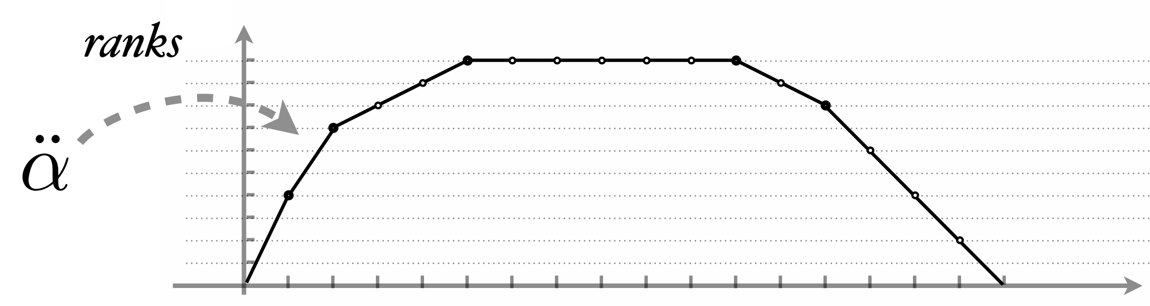
[Apruzzi,Fazzi,Rosa,AT'13; Gaiotto, AT '14; Apruzzi,Fazzi,Passias,Rota'15;Cremonesi,AT'15]

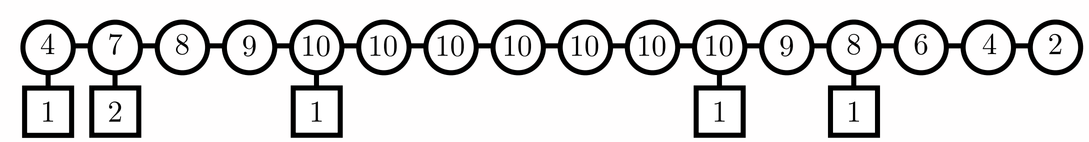


- Start with partitions:



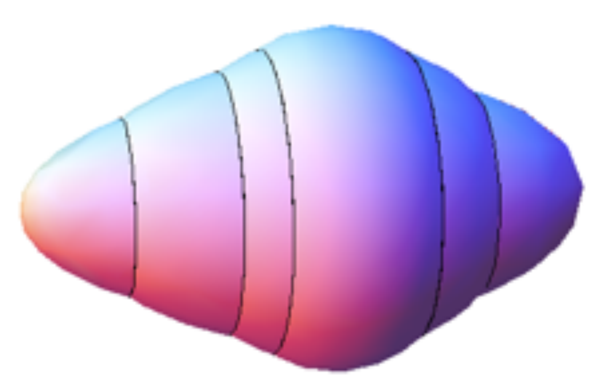
- 'integrate': gauge groups.



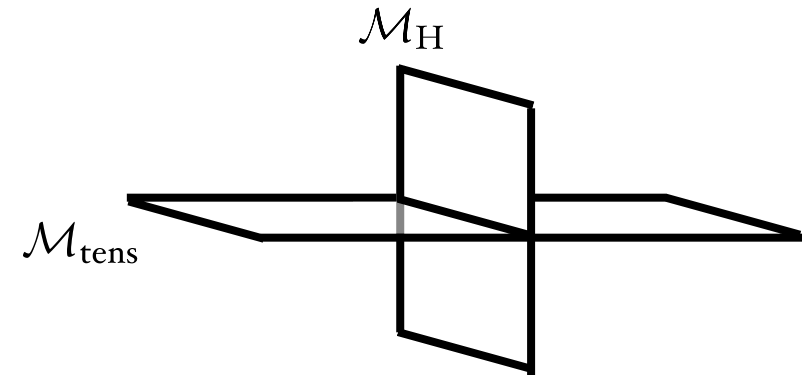


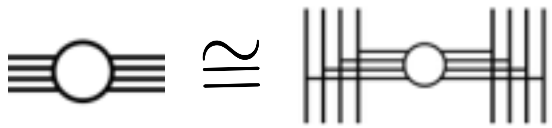
$$ds^2 = 8\sqrt{-\frac{\ddot\alpha}{\alpha}}\,ds^2_{\mathrm{AdS}_7} + \underbrace{\sqrt{-\frac{\alpha}{\ddot\alpha}}\,dz^2 + \frac{\alpha^{3/2}(-\ddot\alpha)^{1/2}}{\sqrt{2\alpha\ddot\alpha-\dot\alpha^2}}\,ds^2_{S^2}}$$

'3d generalized Sasaki–Einsten'

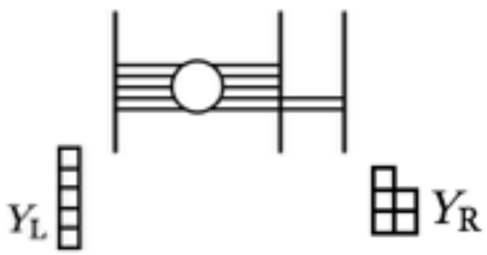


- Higgs moduli space dimension





$$\dim_{\text{Higgs}} = \overbrace{N-1}^{\text{NS5 transverse motions}} + k^2$$



$$\dim_{\text{Higgs}} = N - 1 + k^2 - \dim_{\mathcal{O}_{\mathrm{L}}} - \dim_{\mathcal{O}_{\mathrm{R}}}$$

dimensions nilpotent orbits
associated to partitions

[natural conjecture:

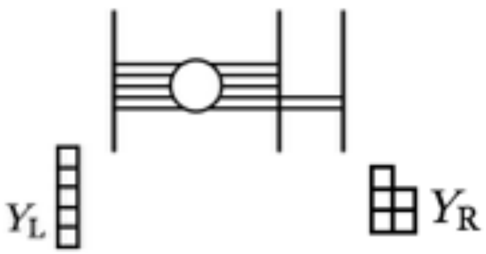
$\mathcal{S}_{\mathcal{O}_{\mathrm{L}}} \cap \mathcal{S}_{\mathcal{O}_{\mathrm{R}}} \subset \mathfrak{su}(k)_{\mathbb{C}}$

'Slodowy slices']

Summary so far

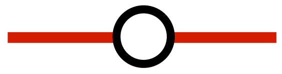
$T^{N}_{Y_{\rm L},Y_{\rm R}}$

- superconformal field theories



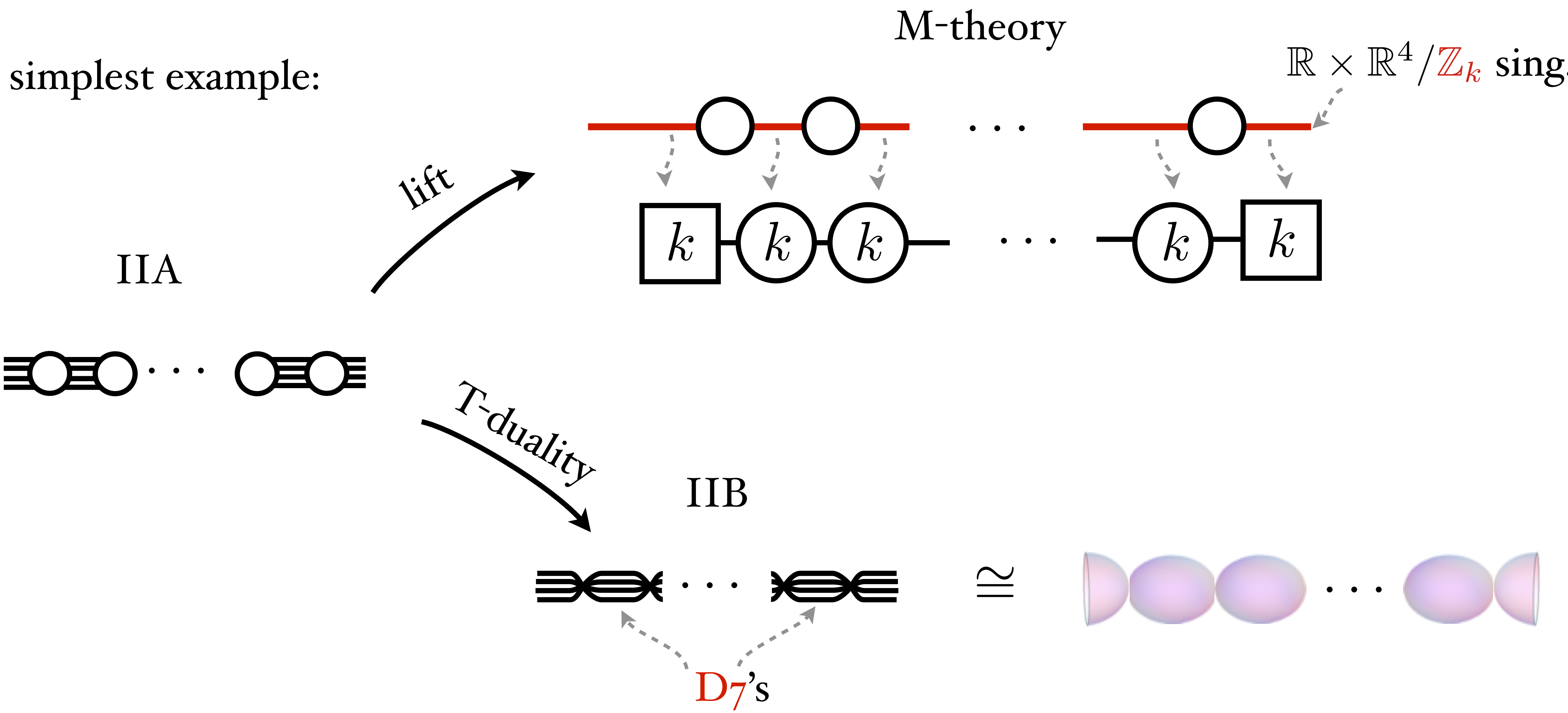
- appear on brane intersections in IIA

- related by RG flow to M5s on

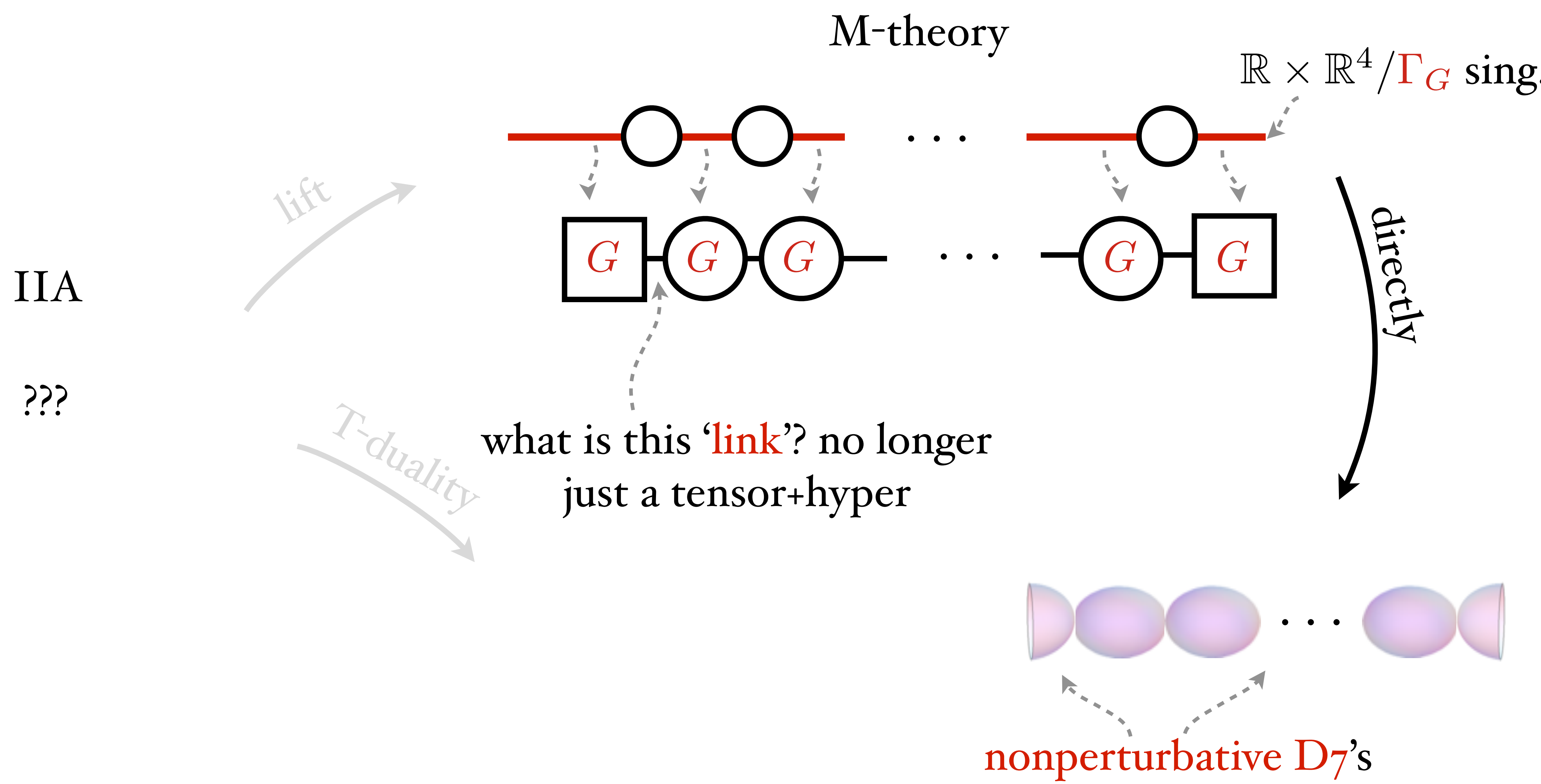


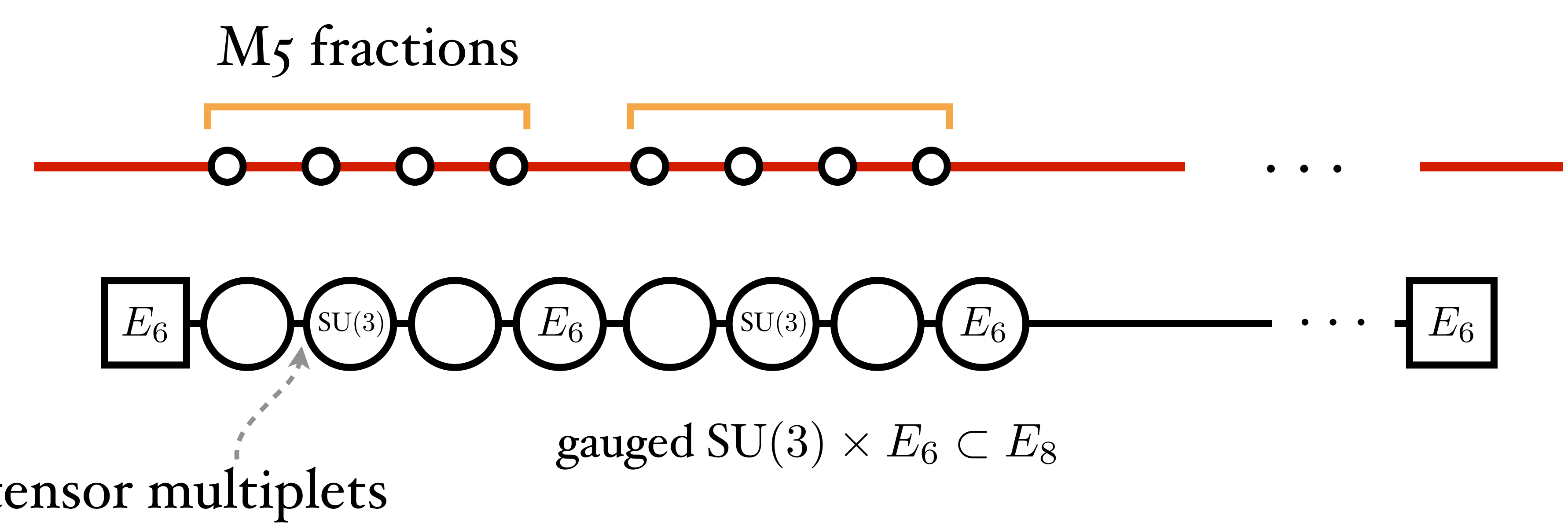
II. T-brane theories

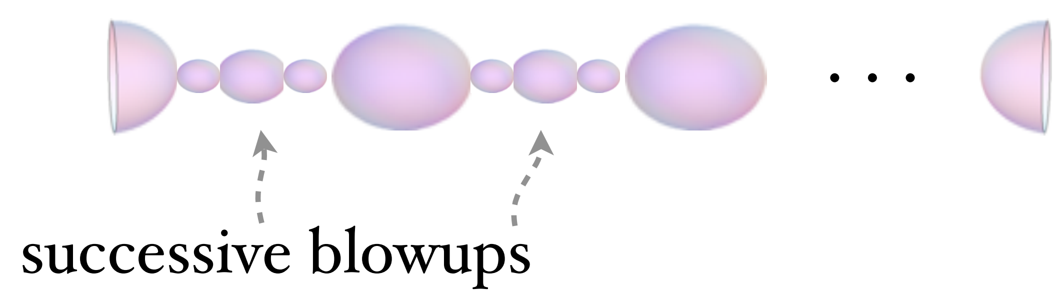
So far we have seen chains of $SU(N)$ gauge groups

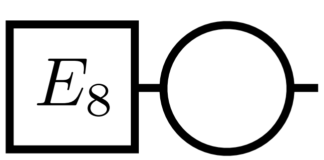


- F-theory allows to include more general gauge groups







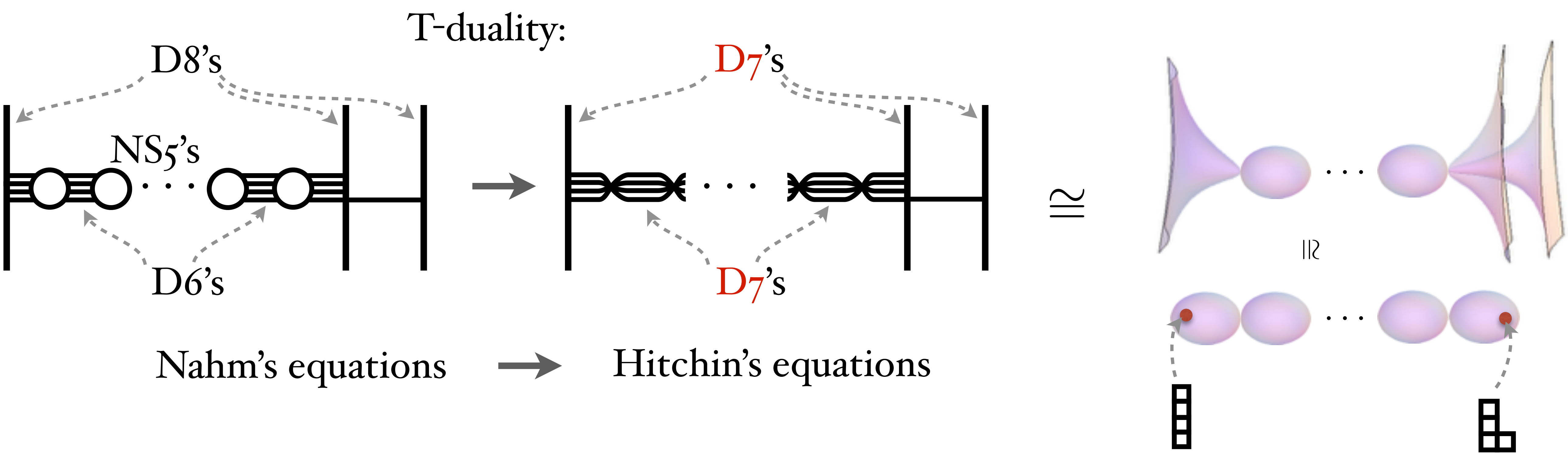


= “E-string”
(no gauge group)
E_8 flavor symmetry

It also appears for M5’s near M9

What about D8s?

[del Zotto, Heckman, AT, Vafa '14]



$$\phi = X^1 + iX^2 \sim \frac{t}{z - z_0}$$

residue is nilpotent

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"T-brane"

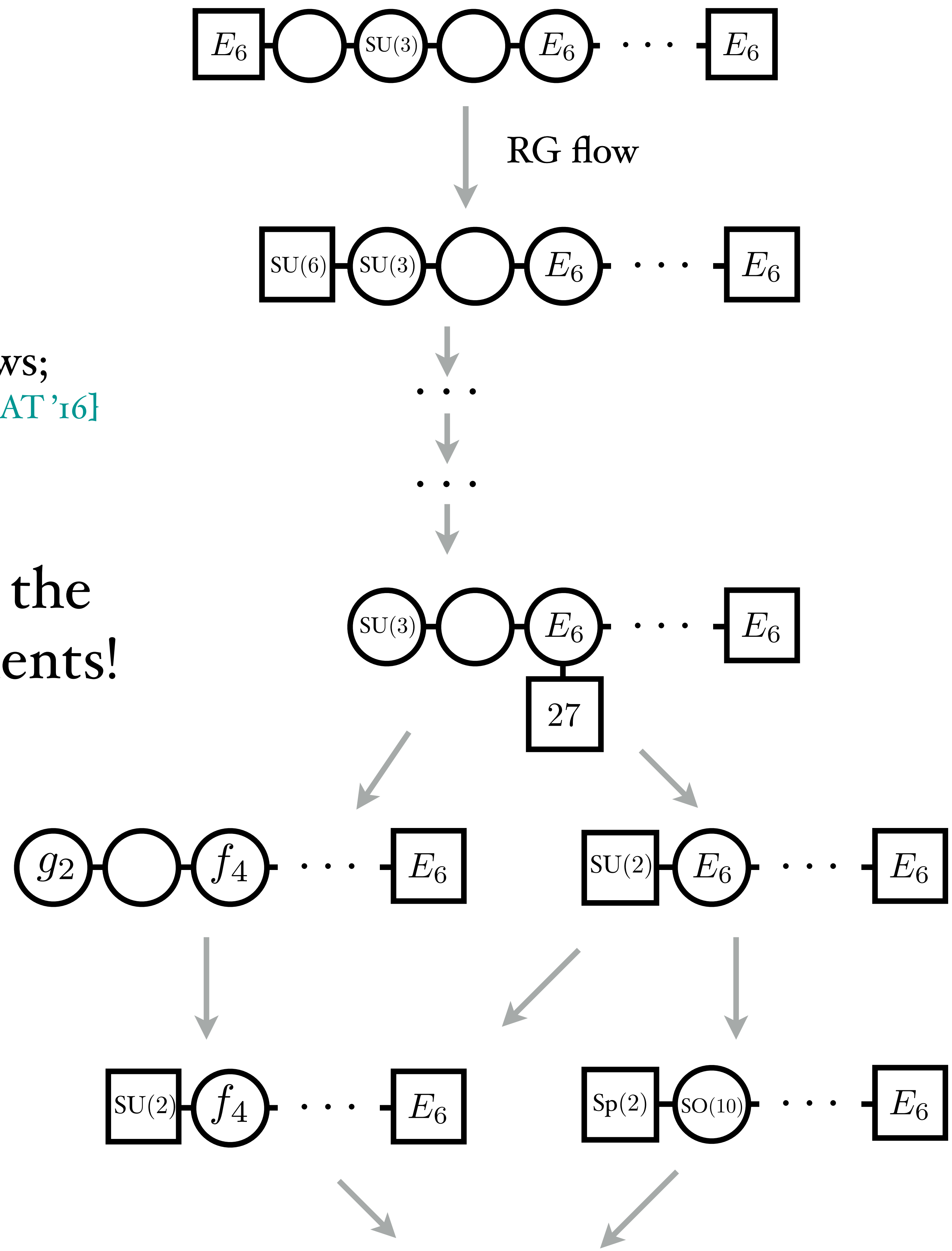
So there should be 6d SCFTs $T^{N}{}_{\rho_{\rm L},\rho_{\rm R}}$

nilpotent ADE elements

how do we find them?

Using F-theory,
we worked out the web of RG flows;
[Heckman, Rudelius, AT '16]

it coincides precisely with the hierarchy of nilpotent elements!



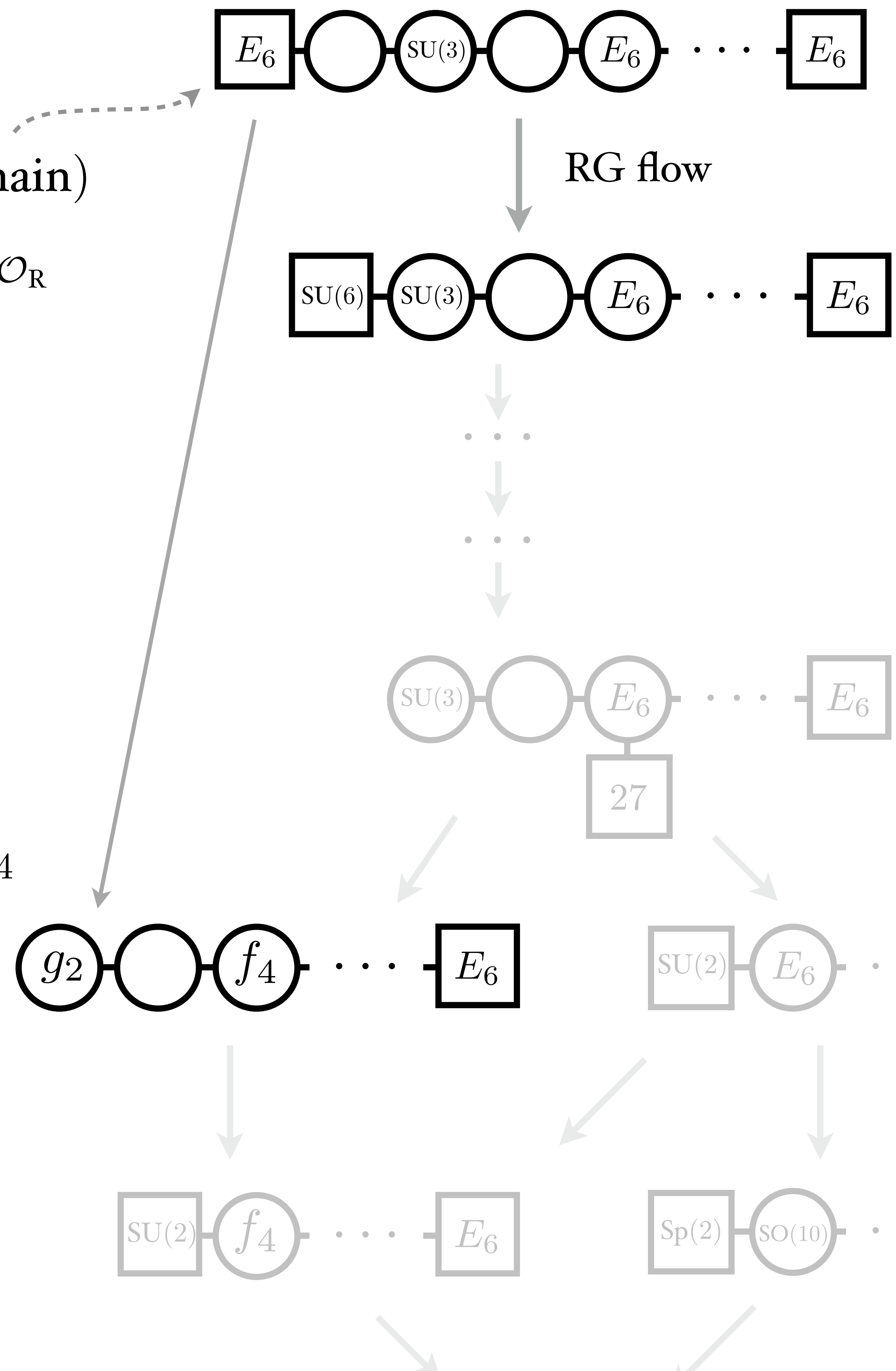
we also computed Higgs moduli space dim.

[Mekareeya, Rudelius, AT '16]
[Mekareeya, Ohmori, Shimizu, AT, in progress]

conjecture:
$$\dim_{\mathrm{H}} = \dim_{\mathrm{H}}(\text{c.m. chain}) - \dim_{\mathcal{O}_{\mathrm{L}}} - \dim_{\mathcal{O}_{\mathrm{R}}}$$

let's check! difference of
[anomaly arguments]

$$29 n_{\mathrm{T}} + n_{\mathrm{H}} - n_{\mathrm{V}}$$



we lose one tensor,
gain 6 fundam. of $\mathfrak{su}_3$
$29 \times 1 - 18 = 11$

$d_{\mathcal{O}} = 11$ ✓

$$\begin{array}{c} \qquad\quad \mathfrak{su}_3 \quad\; \mathfrak{e}_6 \quad\;\; \mathfrak{f}_4 \\ 29 \times 2 - (8 + 78 - 52) = 24 \end{array}$$

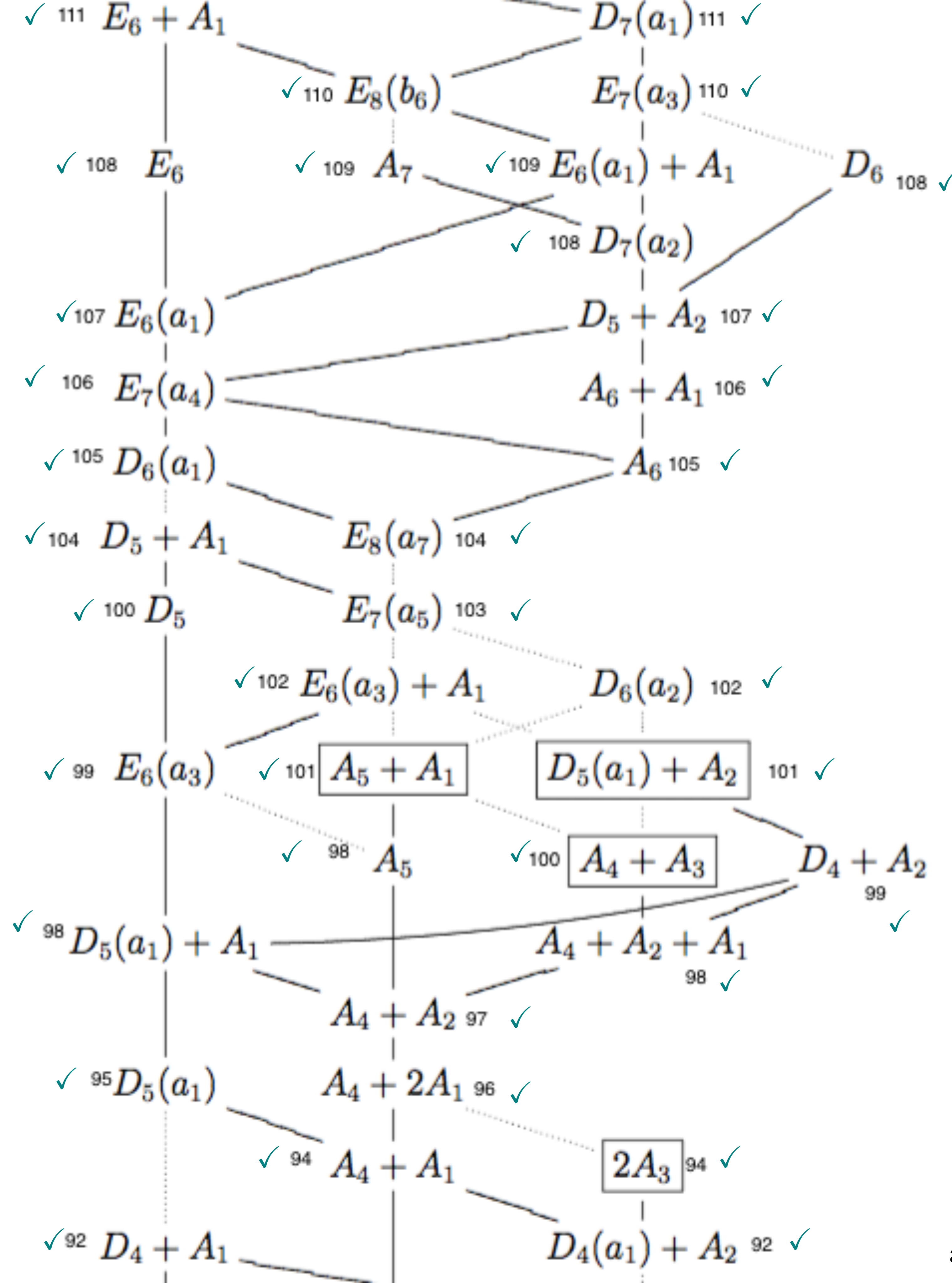
$d_{\mathcal{O}} = 24$ ✓

...it always works
in fun ways

[all dimensions
are quaternionic]

...really!

so we think we have found the $T^N{}_{\rho_L,\rho_R}$



[all dimensions are quaternionic]

Conclusions

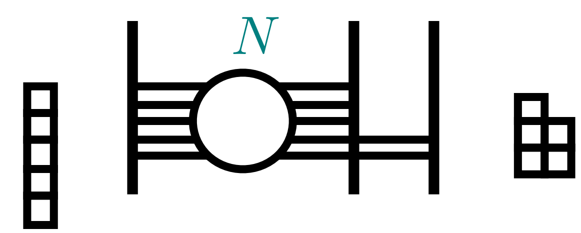
- Hierarchies of superconformal theories $T^{N}_{Y_{\rm L}, Y_{\rm R}}$

  obtained by 'Higgsing' M5s at singularities

  nilpotent
  ADE elements

- In the A_k case:

  - nilpotent = pattern of D6s ending on D8s



- In the D_k, E_k cases:

  - F-theory establishes similar overall picture
  - more exotic ingredients
  - what implications for M5-dynamics?