Killing spinor identities, spinorial geometry and off-shell supergravities
or
Equations of motion off-shell

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Off-shell Killing spinor identities.

Killing spinor identities of $\mathcal{N} = 2$, $d = 5$ off-shell supergravity

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Introduction

- Supersymmetric solutions of a particular supergravity have to solve two sets of equations:
  - Killing spinor equations: "Supersymmetric"
  - Equations of motion of the theory: "Solution"

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- Supersymmetric solutions of a particular supergravity have to solve two sets of equations:
  - Killing spinor equations: "Supersymmetric"
  - Equations of motion of the theory: "Solution"

- Killing spinor identities relate components of equations of motion to each other for supersymmetric configurations in supergravity theories
- Tell us which equations of motion are automatically solved by the supersymmetric geometries
- If the supersymmetry in a theory is realised off-shell then we don't even need to know the action, i.e. the specific theory under consideration.
- Can prove all orders results in effective supergravity description of string theory.
- Apply equally to any higher derivative supergravity.

Introduction

- Normally horrible to try and supersymmetrize higher derivate (string correction or not) actions on-shell, as we need to change the susy transformations and the action.
- Move to off-shell formulation: We will use the superconformal formalism.
  - Superconformal group biggest possible for S-matrix, supersymmetry is realized off-shell.
  - Matter couplings easier to find.
  - Contains super-Poincaré, so the super-Poincaré theories can always be obtained by a suitable gauge fixing.
  - Supersymmetric completions of most curvature squared terms are known.

Off-shell Killing spinor identities

- In work of Ortin & Kallosh and Ortin & Bellorin the Killing spinor identities were derived.
- The derivation does not require that the supersymmetric action is known, just that the action is supersymmetric under the given supersymmetry variations of the fields.
- In work of Meessen (2007) the Killing spinor identities were used in the off-shell $\mathcal{N}=2$ $d=5$ superconformal theory to show that the maximally supersymmetric vacuua of the two derivative theory are the vacua of arbitrarily higher derivative corrected theories, up to a generalization of the very special geometry condition. However an on-shell compensator was used.
- Here we will be interested in what they have to say about solutions with less supersymmetry and later with an off-shell compensator.

Off-shell Killing spinor identities

Lets derive the Killing spinor identities. Let $S[\phi_b, \phi_f]$ be any supergravity action, constructed in terms of bosonic fields ϕ_b and fermionic fields ϕ_f. Let us further assume $S[\phi_b, \phi_f]$ is the spacetime integral of a Lagrangian density:

$$S[\phi_b, \phi_f] = \int d^d x \sqrt{g} \mathcal{L}[\phi_b, \phi_f] \, .$$

The invariance under supersymmetry transformations of the action can be written

$$\delta_Q S[\phi_b, \phi_f] = \int d^d x \sqrt{g} \, \{\mathcal{L}_b[\phi_b, \phi_f] \delta_Q \phi_b[\phi_b, \phi_f] + \mathcal{L}_f[\phi_b, \phi_f] \delta_Q \phi_f[\phi_b, \phi_f]\} = 0 \, ,$$

where δ_Q denotes a local supersymmetry transformation of arbitrary parameter, $\mathcal{L}_b, \mathcal{L}_f$, denote functional derivative of the Lagrangian with respect to ϕ_b, ϕ_f respectively, and a sum over fields is understood.

Off-shell Killing spinor identities

Next consider a second variation of the action functional by varying $\delta_Q S[\phi_b, \phi_f]$ with respect to fermionic fields only. Since $\delta_Q S[\phi_b, \phi_f]$ is identically zero for arbitrary ϕ_b, ϕ_f, we have

$$\delta_Q S[\phi_b, \phi_f + \delta_F \phi_f] = 0$$

and we set the fermions to zero after the variation. Hence we get

$$\begin{aligned}
&\delta_F \delta_Q S|_{\phi_f=0} = 0 \\
&= \int d^d x \sqrt{|g|} \Big[(\delta_F \mathcal{L}_b)(\delta_Q \phi_b) \\
&\quad + \mathcal{L}_b(\delta_F \delta_Q \phi_b) + (\delta_F \mathcal{L}_f)(\delta_Q \phi_f) + \mathcal{L}_f(\delta_F \delta_Q \phi_f) \Big]_{\phi_f=0} .
\end{aligned}$$

Since $\delta_Q \phi_b$ and $\mathcal{L}_f$ are odd in fermions we are left with

$$\int d^d x \sqrt{|g|} \left[(\mathcal{L}_b(\delta_F \delta_Q \phi_b) + (\delta_F \mathcal{L}_f)(\delta_Q \phi_f) \right]_{\phi_f=0} = 0 .$$

Off-shell Killing spinor identities

Calculating $(\delta_F \mathcal{L}_f)_{\phi_f=0}$ requires knowledge of the entire Lagrangian, not only its bosonic truncation. However if we restrict ourselves to supersymmetry transformations having Killing spinors as parameters, δ_K, we have

$$(\delta_K \phi_f)_{\phi_f=0} = 0 \ .$$

Note that

$$\mathcal{L}_b := \frac{1}{\sqrt{|g|}} \frac{\delta S[\phi_b, \phi_f]}{\delta \phi_b} = \frac{1}{\sqrt{|g|}} \frac{\delta S_B[\phi_b]}{\delta \phi_b} + \frac{1}{\sqrt{|g|}} \frac{\delta S_F[\phi_b, \phi_f]}{\delta \phi_b} \ ,$$

where the last term vanishes if $\phi_f = 0$.

Off-shell Killing spinor identities

We are thus led to define

$$\mathcal{E}_b := \frac{1}{\sqrt{|g|}} \frac{\delta S_B[\phi_b]}{\delta \phi_b} ,$$

so that bosonic equations of motion take the form

$$\mathcal{E}_b = 0 .$$

Thus the Killing spinor identities may be written as

$$\int d^d x \sqrt{|g|} \, \mathcal{E}_b (\delta_F \delta_K \phi_b)_{\phi_f = 0} = 0 .$$

$\mathcal{N} = 2$, $d = 5$ off-shell KSIs

To derive the Ksis we need the off-shell variations of the fields under supersymmetry. We shall make use of the Standard Weyl multiplet $(e_\mu^a, \psi_\mu^{\mathbf{i}}, v_{ab}, D, V_\mu^{\mathbf{ij}}, b_\mu, \chi^{\mathbf{i}})$, Abelian vector multiplets $(A_\mu, Y^{\mathbf{ij}}, \Omega^{\mathbf{i}}, M)$ and a linear multiplet $(L^{\mathbf{ij}}, \varphi^{\mathbf{i}}, N, E_a)$ which we use to break the superconformal invariance. After gauge fixing the superconformal theory to super-Poincaré these are for the Weyl multiplet

$$
\begin{aligned}
\delta e_\mu^a &= -2i\bar{\epsilon}\gamma^a\psi_\mu \ , \\
\delta v_{ab} &= -\tfrac{1}{8}i\bar{\epsilon}\gamma_{ab}\chi + \cdots \ , \\
\delta D &= -\tfrac{1}{3}i\bar{\epsilon}\gamma^{\mu\nu}\chi v_{\mu\nu} - i\bar{\epsilon}\gamma^\mu\nabla_\mu\chi + i\bar{\epsilon}^{\mathbf{i}}\gamma^\mu V_{\mathbf{ij}\mu}\chi^{\mathbf{j}} \\
&\quad - \tfrac{i}{6}\bar{\epsilon}^{\mathbf{i}}(\gamma^a E_a + N)L_{\mathbf{ij}}\chi^{\mathbf{j}} + \tfrac{i}{3}\bar{\epsilon}^{\mathbf{i}}\gamma^a V'{}_{a\mathbf{ij}}\chi^{\mathbf{j}} + \cdots \ , \\
\delta V_\mu^{\mathbf{ij}} &= -\tfrac{i}{4}\bar{\epsilon}^{(\mathbf{i}}\gamma_\mu\chi^{\mathbf{j})} + \cdots \ ,
\end{aligned}
$$

$\mathcal{N}=2$, $d=5$ off-shell KSIs

for the vector multiplet and linear multiplet we have

$$\delta A_\mu^I = -2i\bar{\epsilon}\gamma_\mu\Omega^I + \cdots \ ,$$
$$\delta M^I = 2i\bar{\epsilon}\Omega^I \ ,$$
$$\delta Y^{I\mathbf{ij}} = 2i\bar{\epsilon}^{(\mathbf{i}}\gamma^a\nabla_a\Omega^{\mathbf{j})I} - 2i\bar{\epsilon}^{(\mathbf{i}}\gamma^a V_a{}^{\mathbf{j})}{}_{\mathbf{k}}\Omega^{\mathbf{k}I} - \frac{2i}{3}V_a^{\mathbf{k}(\mathbf{i}}\bar{\epsilon}_{\mathbf{k}}\gamma_a\Omega^{\mathbf{j})} - \frac{i}{3}\bar{\epsilon}^{(\mathbf{i}}\gamma_{ab}v^{ab}\Omega^{\mathbf{j})I}$$
$$\delta P_a = \cdots \ ,$$
$$\delta N = \frac{i}{2}L_{\mathbf{ij}}\bar{\epsilon}^{\mathbf{i}}\chi^{\mathbf{j}} \ .$$

$\mathcal{N} = 2$, $d = 5$ off-shell KSIs

- We ignored terms involving the gravitino in the variations, apart from in the veilbein variation.
- This is because we shall choose to solve the Einstein equation last. - It is usually the most involved in any case.
- In particular, if we assume that when we look to solve the Einstein equation that all other eoms have been solved first, we can ignore the ... terms above.

So if we set

$$\mathcal{E}(e)_a^\mu := \frac{1}{\sqrt{|g|}} \frac{\delta S}{\delta e_\mu^a}$$

we get

$$\mathcal{E}(e)_a^\mu \gamma^a \epsilon^{\mathbf{i}} \Big|_{\text{other bosons on-shell}} = 0 \ .$$

$\mathcal{N} = 2$, $d = 5$ off-shell KSIs

To proceed we will need one more ingredient, the gravitino variation which reads

$$\begin{aligned}\delta\psi_\mu^{\mathbf{i}} &= \nabla_\mu\epsilon^{\mathbf{i}} + \tfrac{1}{2}\gamma_{\mu ab}v^{ab}\epsilon^{\mathbf{i}} - \tfrac{1}{3}\gamma_\mu\gamma_{ab}v^{ab} \\ &\quad + V_\mu^{\mathbf{ij}}\epsilon_{\mathbf{j}} + \tfrac{1}{6}\gamma_\mu(\gamma^a P_a + N)L^{\mathbf{ij}}\epsilon_{\mathbf{j}} - \tfrac{1}{3}\gamma_\mu\gamma^a V'^{\mathbf{ij}}_a\epsilon_{\mathbf{j}} = 0\ ,\end{aligned}$$

where $V_\mu^{\mathbf{ij}} = V_\mu L^{\mathbf{ij}} + V'^{\mathbf{ij}}_\mu$ so that $V'^{\mathbf{ij}}_\mu L_{\mathbf{ij}} = 0$.
Let us now write the KSI associated to a variation of gauginos. We obtain

$$\begin{aligned}0 = \quad & \int d^5x\sqrt{|g|}\Big[\mathcal{E}(A)_I^\mu\left(-2i\bar{\epsilon}^{\mathbf{i}}\gamma_\mu\right) + \mathcal{E}(M)_I(2i\bar{\epsilon}^{\mathbf{i}}) \\ & \qquad + \mathcal{E}(Y)_{I\mathbf{jk}}(2i\bar{\epsilon}^{\mathbf{j}})\gamma^a V_a^{\mathbf{ki}} \\ & + \tfrac{2i}{3}\mathcal{E}(Y)^{\mathbf{i}}_{I\mathbf{k}}V_a^{\mathbf{jk}}\bar{\epsilon}_{\mathbf{j}}\gamma_a - \mathcal{E}(Y)_I^{\mathbf{ij}}(\tfrac{i}{3}\bar{\epsilon}_{\mathbf{j}}\gamma^{ab}v_{ab})\Big]\,\delta\Omega^I_{\mathbf{i}} \\ & \qquad + \mathcal{E}(Y)_I^{\mathbf{ij}}(2i\bar{\epsilon}_{\mathbf{j}}\gamma^a)\nabla_a\delta\Omega^I_{\mathbf{i}}\ .\end{aligned}$$

$\mathcal{N}=2$, $d=5$ off-shell KSIs

Integrating by parts and using the fact that the gravitino Killing spinor equation implies

$$\gamma^a \nabla_a \epsilon^{\mathbf{i}} = \tfrac{5}{6}(v\cdot\gamma)\epsilon^{\mathbf{i}} - \gamma^a V_a L^{\mathbf{ij}}\epsilon_{\mathbf{j}} + \tfrac{2}{3} V'^{a\mathbf{ij}}\gamma_a \epsilon_{\mathbf{j}} - \tfrac{5}{6}(\gamma^a P_a + N)L^{\mathbf{ij}}\epsilon_{\mathbf{j}} \ ,$$

we obtain

$$\begin{aligned} 0 = &\left[\mathcal{E}(A)_I^{\mu}\gamma_\mu - \mathcal{E}(M)_I \right.\\ &\left. +\tfrac{5}{12}\mathcal{E}(Y)(\gamma^a(P_a + 2V_a) + N)\right]\epsilon^{\mathbf{i}} \\ &+\left[\left(\nabla^a \mathcal{E}(Y)_I{}^{\mathbf{ij}}\right)\gamma_a - \tfrac{5}{6}\mathcal{E}(Y')_I^{\mathbf{ik}}(\gamma^a(P_a + 2V_a) + N)L_{\mathbf{k}}^{\mathbf{j}} \right.\\ &\left. -\mathcal{E}(Y)_I{}^{\mathbf{ij}}\gamma^{ab}v_{ab}\right]\epsilon_{\mathbf{j}} \ . \end{aligned}$$

$\mathcal{N} = 2$, $d = 5$ off-shell KSIs

Finally we consider the KSI associated with the auxiliary fermion.

$$
\begin{aligned}
0 \;&=\; \int d^5x\sqrt{|g|}\Big[-\tfrac{i}{8}\mathcal{E}(v)^{ab}\bar{\epsilon}^{\mathbf{i}}\gamma_{ab} - i\mathcal{E}(D)\bar{\epsilon}^{\mathbf{j}}\gamma_a V^a L_{\mathbf{j}}^{\mathbf{i}} - \tfrac{i}{3}\mathcal{E}(D)v^{ab}\bar{\epsilon}^{\mathbf{i}}\gamma_{ab} \\
&+\tfrac{i}{6}\mathcal{E}(D)\bar{\epsilon}^{\mathbf{j}}(\gamma^a P_a + N)L_{\mathbf{j}}^{\mathbf{i}} - \mathcal{E}(D)\tfrac{4i}{3}\bar{\epsilon}^{\mathbf{j}}V'^{\mathbf{i}}{}_{a\mathbf{j}}\gamma^a + \tfrac{i}{4}\mathcal{E}(V)^{\mu\mathbf{i}}{}_{\mathbf{j}}\bar{\epsilon}^{\mathbf{j}}\gamma_\mu \\
&+\tfrac{i}{4}\mathcal{E}(Y)^{\mathbf{i}}_{I\mathbf{j}}\bar{\epsilon}^{\mathbf{j}}M^I - \tfrac{i}{2}\mathcal{E}(N)L_{\mathbf{j}}^{\mathbf{i}}\Big]\,\delta\chi_{\mathbf{i}} + \left[-i\bar{\epsilon}\mathcal{E}(D)\gamma^\mu\right]\nabla_\mu\delta\chi\,.
\end{aligned}
$$

Integrating the last term by parts, discarding the total derivative and making use of the gravitino Killing spinor equation we obtain

$$
\begin{aligned}
0 \;&=\; \left[\tfrac{1}{8}\mathcal{E}(v)^{ab} + \tfrac{1}{2}\mathcal{E}(D)v^{ab}\right]\gamma_{ab}\epsilon^{\mathbf{i}} + \nabla^a\mathcal{E}(D)\gamma_a\epsilon^{\mathbf{i}} - \tfrac{1}{4}\mathcal{E}(V)_a^{\mathbf{ij}}\gamma^a\epsilon_{\mathbf{j}} \\
&-\tfrac{1}{4}\mathcal{E}(Y)_I^{\mathbf{ij}}M^I\epsilon_{\mathbf{j}} + 2\mathcal{E}(D)V'^{\mathbf{ij}}_a\gamma^a\epsilon_{\mathbf{j}} + \tfrac{1}{2}\mathcal{E}(N)L^{\mathbf{ij}}\epsilon_{\mathbf{j}} \\
&-\mathcal{E}(D)(\gamma^a P_a + N))L^{\mathbf{ij}}\epsilon_{\mathbf{j}}\,.
\end{aligned}
$$

Spinorial Geometry and classifying supersymmetric configurations

- Supersymmetric configurations solve the Killing spinor equations, i.e. the vanishing of the supersymmetry variations of the fermions on purely bosonic backgrounds.
- We shall truncate the fields $V_\mu^{\mathbf{ij}}$, $Y^{I\mathbf{ij}}$, N and P_a for now, so we are sure we stay in an ungauged theory, as was done in the work of Castro et al.
- We shall have more to say about this later.

Spinorial Geometry and classifying supersymmetric configurations

- Demanding the vanishing of the gravitino variation for a bosonic background implies

$$\delta\psi^i_\mu = \left[\nabla_\mu + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}v^{ab}\gamma_\mu\gamma_{ab}\right]\epsilon^i = 0 \ .$$

- From the vanishing of the gaugino variation for a bosonic background one has

$$\delta\Omega^{Ii} = \left[-\frac{1}{4}F^I_{ab}\gamma^{ab} - \frac{1}{2}\gamma^\mu\partial_\mu M^I - \frac{1}{3}M^I v^{ab}\gamma_{ab}\right]\epsilon^i = 0 \ .$$

- the vanishing of the auxilary fermion variation for a bosonic background we get

$$\delta\chi^i = \left[D - 2\gamma^c\gamma^{ab}\nabla_a v_{bc} - 2\gamma^a\epsilon_{abcde}v^{bc}v^{de} + \frac{4}{3}(v\cdot\gamma)^2\right]\epsilon^i = 0 \, .$$

Spinorial Geometry and classifying supersymmetric configurations

- In order to solve these equations, one may use the bilinears method. Form a bilinear out of spinors, then demand that the equations above hold - restricts the form of the spin connection and matter fields.
- Sometimes awkward to solve explicitly for the spinors ϵ.
- Gillard Gran & Papadopoulos introduced the spinorial geometry techniques:
  - Use Clifford isomorphim to write the space of spinors in terms of an exterior algebra and the action of the gamma matrices as a combination of wedge and interior products.
  - Choose a particular basis of gamma matrices so they act as creation or anihilation matrices - wedge or interior product.
  - Use the Spin(1,4) gauge freedom in the above equations to write representatives for the spinors - up to local lorentz transformations on the bosonic fields

Spinorial Geometry and classifying supersymmetric configurations

- Now we have explicit representatives for the spinors. Can solve the equations "easily", and more importantly systematically.
- If we leave in all the auxiliary fields the result will be true to all orders - but some may not occur due to the imposition of the equations of motion. Eoms imply less general geometry. Supersymmetric configurations are often more general than supersymmetric solutions to a given theory.
- Lets see an example.

Weyl squared corrected $\mathcal{N} = 2$, $D = 5$ supergravity coupled to Abelian vector multiplets

As discussed by Castro et al. We consistently truncate the fields $V_\mu^{\mathbf{ij}}, Y^{I\mathbf{ij}}, N$ and P_a. We take the Lagrangian

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 \ .$$

where at two derivative level we have

$$\begin{aligned}\mathcal{L}_2 = \mathcal{L}_V + \mathcal{L}_H = & \tfrac{1}{2}D(\mathcal{N} - 1) - \tfrac{1}{4}R(\mathcal{N} + 3) + v^2(3\mathcal{N} + 1) \\ & + 2\mathcal{N}_I v^{ab} F^I_{ab} + \mathcal{N}_{IJ}\left(\tfrac{1}{4}F^I_{ab}F^{Iab} - \tfrac{1}{2}\nabla_a M^I \nabla^a M^J\right) \\ & + \tfrac{1}{24} c_{IJK} e^{-1} \epsilon^{abcde} A^I_a F^J_{bc} F^K_{de} \ .\end{aligned}$$

Weyl squared corrected $\mathcal{N} = 2$, $D = 5$ supergravity coupled to Abelian vector multiplets

As far as the four derivative Lagrangian is concerned we will take

$$\begin{aligned}
\mathcal{L}_4 = \tfrac{c_{2I}}{24} \Big\{ & \tfrac{1}{16} e^{-1} \epsilon^{abcde} A^I_a C_{bcfg} C_{de}{}^{fg} + \tfrac{1}{8} M^I C^{abcd} C_{abcd} + \\
& + \tfrac{1}{12} M^I D^2 + \tfrac{1}{6} D v^{ab} F^I_{ab} + \tfrac{1}{3} M^I C_{abcd} v^{ab} v^{cd} + \tfrac{1}{2} C_{abcd} F^{Iab} v^{cd} + \\
& + \tfrac{8}{3} M^I v_{ab} \nabla^b \nabla_c v^{ac} - \tfrac{16}{9} M^I v^{ab} v_{bc} R_a{}^c - \tfrac{2}{9} M^I v^2 R + \\
& + \tfrac{4}{3} M^I \nabla_a v_{bc} \nabla^a v^{bc} + \tfrac{4}{3} M^I \nabla_a v_{bc} \nabla^b v^{ca} + \\
& - \tfrac{2}{3} M^I e^{-1} \epsilon^{abcde} v_{ab} v_{cd} \nabla^f v_{ef} + \tfrac{2}{3} e^{-1} \epsilon^{abcde} F^I_{ab} v_{cf} \nabla^f v_{de} + \\
& + \epsilon^{abcde} F^I_{ab} v_{cf} \nabla_d v_e{}^f - \tfrac{4}{3} F^I_{ab} v^{ac} v_{cd} v^{db} - \tfrac{1}{3} F^I_{ab} v^{ab} v_{cd} v^{cd} + \\
& + 4 M^I v_{ab} v^{bc} v_{cd} v^{da} - M^I v_{ab} v^{ab} v_{cd} v^{cd} \Big\} ,
\end{aligned}$$

EOMs

$$
\begin{aligned}
\frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta D} &= \tfrac{1}{2}(\mathcal{N}-1)\,, \qquad \frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta v_{\mu\nu}} = 2(\mathcal{N}_I F^{I\mu\nu} + (3\mathcal{N}+1)v^{\mu\nu}),\\
\frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta M^I} &= (\tfrac{1}{2}D - \tfrac{1}{4}R + 3v^2)\mathcal{N}_I + c_{IJK}(\tfrac{1}{4}F^J\cdot F^K + \tfrac{1}{2}\nabla M^J\cdot\nabla M^K)\\
&\quad +\mathcal{N}_{IJ}(2F^J_{ab}v^{ab} + \nabla^2 M^J)\\
\frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta A^I_\mu} &= c_{IJK}(\tfrac{1}{8}\epsilon^{\mu abcd}F^J_{ab}F^K_{cd} + F^{J\mu a}\nabla_a M^K) + 4\mathcal{N}_I\nabla_a v^{\mu a}\\
&\quad +\mathcal{N}_{IJ}(4v^{\mu a}\nabla_a M^J + \nabla_a F^{J\mu a})
\end{aligned}
$$

EOMs

$$
\begin{aligned}
\frac{1}{\sqrt{|g|}}\frac{\delta S_2}{\delta g^{\mu\nu}} &= -\tfrac{1}{4}(\mathcal{N}+3)E_{\mu\nu} - \tfrac{1}{4}D(\mathcal{N}-1)g_{\mu\nu} \\
&\quad +2(1+3\mathcal{N})(v_{a\mu}v^{a}{}_{\nu} - \tfrac{1}{4}v^2 g_{\mu\nu}) \\
&\quad +\mathcal{N}_{IJ}(\tfrac{1}{2}F^{I}_{a\mu}F^{Ja}{}_{\nu} + 4F^{I}_{a(\mu}v^{a}{}_{\nu)} - \tfrac{1}{2}\nabla_{\mu}M^{I}\nabla_{\nu}M^{J}) \\
&\quad -\mathcal{N}_{IJ}(\tfrac{1}{8}F^{I}\cdot F^{J} + F^{I}\cdot v - \tfrac{1}{4}\nabla M^{I}\cdot\nabla M^{J})g_{\mu\nu} \\
&\quad +\tfrac{1}{4}(\nabla_{\mu}\nabla_{\nu}\mathcal{N} - \nabla^2\mathcal{N}g_{\mu\nu})\ .
\end{aligned}
$$

EOMs

$$\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta D} = \frac{c_{2I}}{144}\left\{DM^I + v\cdot F^I\right\}$$

$$\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta M^I} = \frac{c_{2I}}{24}\Big\{\tfrac{1}{8}C^{abcd}C_{abcd} + \tfrac{1}{12}D^2 + \tfrac{1}{3}C_{abcd}v^{ab}v^{cd}$$

$$+\tfrac{8}{3}v_{ab}\nabla^b\nabla_c v^{ac} - \tfrac{16}{9}v^{ab}v_{bc}R_a{}^c - \tfrac{2}{9}v^2 R$$

$$+\tfrac{4}{3}(\nabla_a v_{bc})(\nabla^a v^{bc}) + \tfrac{4}{3}(\nabla_a v_{bc})(\nabla^b v^{ca})$$

$$-\tfrac{2}{3}e^{-1}\epsilon^{abcde}v_{ab}v_{cd}\nabla^f v_{ef} + 4v_{ab}v^{bc}v_{cd}v^{da} - (v^2)^2\Big\}$$

EOMs

$$
\begin{aligned}
\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta v_{\mu\nu}} = \frac{c_{2I}}{24}\Big\{ & \tfrac{1}{6} DF^{I\mu\nu} + \tfrac{2}{3} M^I C^{\mu\nu}{}_{ab} v^{ab} + \tfrac{1}{2} C^{\mu\nu}{}_{ab} F^{Iab} \\
& + \tfrac{8}{3} M^I \nabla^{[\mu|}\nabla_a v^{|\nu]a} - \tfrac{8}{3} \nabla^{[\mu|}\nabla_a M^I v^{|\nu]a} \\
& + \tfrac{32}{9} M^I v^{[\mu}{}_a R^{\nu]a} - \tfrac{4}{9} M^I R v^{\mu\nu} \\
& - \tfrac{8}{3} \nabla_a M^I \nabla^a v^{\mu\nu} - \tfrac{8}{3} \nabla_a M^I \nabla^{[\mu} v^{\nu]a} \\
& - \tfrac{4}{3} M^I \epsilon^{\mu\nu abc} v_{ab} \nabla^d v_{cd} + \tfrac{2}{3} \epsilon^{abcd[\mu} \nabla^{\nu]} M^I v_{ab} v_{cd} \\
& + \tfrac{2}{3} \epsilon^{abcd[\mu} F^I_{ab} \nabla^{\nu]} v_{cd} - \tfrac{2}{3} \epsilon^{abc\mu\nu} \nabla^d F^I_{ab} v_{cd} \\
& + \epsilon^{abcd[\mu} F^I_{ab} \nabla_c v_d{}^{\nu]} + \epsilon^{abcd[\mu} \nabla_c F^I_{ab} v_d{}^{\nu]} \\
& + \tfrac{8}{3} F^{I[\mu}{}_a v^{\nu]}{}_b v^{ab} - \tfrac{4}{3} F^I_{ab} v^{a\mu} v^{\nu b} - \tfrac{1}{3} v^2 F^{I\mu\nu} \\
& - \tfrac{2}{3}\left(F^I \cdot V\right) v^{\mu\nu} - 16 M^I v_{ab} v^{a\mu} v^{\nu b} - 4 M^I v^2 v^{\mu\nu} \Big\}
\end{aligned}
$$

EOMs

$$\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta A^I_\mu} = \tfrac{c_{2I}}{24}\Big\{\tfrac{1}{16}\epsilon^{\mu abcd}C_{abef}C_{cd}{}^{ef} - \tfrac{1}{3}\nabla_a D v^{a\mu}$$

$$-\nabla_a C^{a\mu}{}_{bc}v^{bc} + \tfrac{4}{3}\epsilon^{\mu abcd}\nabla_a v_{be}\nabla^e v_{cd}$$

$$+2\epsilon^{\mu abcd}\nabla_a v_{be}\nabla_c v_d{}^e + \tfrac{8}{3}\nabla_a v^{ab}v_{bc}v^{c\mu} + \tfrac{2}{3}\nabla_a v^{a\mu}v^2\Big\}$$

EOMs

$$
\begin{aligned}
\frac{1}{\sqrt{|g|}}\frac{\delta S_4}{\delta g^{\mu\nu}} = \frac{c_{2I}}{24}\Bigg\{ \quad & -\tfrac{1}{8}\left[\epsilon^{abcd}{}_{(\mu|}\nabla_e F^I_{ab} R_{cd}{}^{e}{}_{|\nu)}\right] \\
& + \tfrac{1}{4}\Big[M^I\Big(-C_{abc(\mu|}R^{abc}{}_{|\nu)} + \tfrac{4}{3}R_{ab}C_{\mu}{}^{a}{}_{\nu}{}^{b} + 2C_{\mu}{}^{bcd}C_{\nu bcd} \\
& -\tfrac{1}{4}g_{\mu\nu}C^{abcd}C_{abcd}\Big) + 2\nabla_a\nabla_b M^I C^{a}{}_{\mu}{}^{b}{}_{\nu}\Big] \\
& - \tfrac{1}{24}\left[g_{\mu\nu}M^I D^2\right] + \tfrac{1}{3}\left[Dv_{(\mu}{}^{a}F^I_{\nu)a} - \tfrac{1}{4}g_{\mu\nu}Dv^{ab}F^I_{ab}\right] \\
& + \tfrac{1}{3}\Big[M^I\Big(\left(R_{abc(\mu} - 4C_{abc(\mu}\right)v^{ab}v_{\nu)}{}^{c} + \tfrac{4}{3}R_{ab}v_{\mu}{}^{a}v_{\nu}{}^{b} - \tfrac{1}{3}Rv_{\mu}{}^{a}v_{\nu a} \\
& + \tfrac{1}{6}R_{\mu\nu}v^{ab}v_{ab} - \tfrac{1}{2}g_{\mu\nu}C_{abcd}v^{ab}v^{cd}\Big) \\
& \qquad + 2\nabla_a\nabla_b v_{\mu}{}^{a}v_{\nu}{}^{b}M^I + \tfrac{4}{3}\nabla_a\nabla_{(\mu}v_{\nu)b}v^{ab}M^I \\
& -\tfrac{2}{3}\nabla^2 v_{\mu}{}^{a}v_{\nu a}M^I + \tfrac{2}{3}g_{\mu\nu}\nabla_a\nabla_b v^{ac}v_{c}{}^{b}M^I \\
& \qquad + \tfrac{1}{6}\left(g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu\right)v^{ab}v_{ab}M^I\Big]
\end{aligned}
$$

EOMs

$$
\begin{aligned}
&+\Big[\tfrac{1}{2}R_{abc(\mu}v_{\nu)}{}^{c}F^{lab}+\nabla_a\nabla_b v_{(\mu}{}^{a}F^{l}_{\nu)}{}^{b}+\tfrac{1}{3}\nabla_a\nabla_{(\mu|}v_{|\nu)b}F^{lab}\\
&+\tfrac{1}{3}\nabla_a\nabla_{(\mu}F^{lb}{}_{\nu)}v_b{}^{a}+\tfrac{1}{3}\nabla^2F^{la}{}_{(\mu}v_{\nu)a}\\
&\quad-\tfrac{1}{3}g_{\mu\nu}\nabla_a\nabla_b v^{a}{}_{c}F^{lbc}+\tfrac{2}{3}R_{ab}F^{la}{}_{(\mu}v^{b}{}_{\nu)}\\
&+\tfrac{1}{12}\left(R_{\mu\nu}-\nabla_\mu\nabla_\nu+g_{\mu\nu}\nabla^2\right)v_{ab}F^{lab}\\
&\quad+\tfrac{1}{6}RF^{la}{}_{(\mu}v_{\nu)a}-\left(F^{la}{}_{(\mu}v^{bc}+v^{a}_{(\mu}F^{lbc}\right)C_{|\nu)abc}\\
&-\tfrac{1}{4}g_{\mu\nu}F^{lab}v^{cd}C_{abcd}\Big]\\
&+\tfrac{8}{3}\Big[M^l\Big(v_{a(\mu}\nabla_{\nu)}\nabla_b v^{ab}+v_{ab}\nabla^b\nabla_{(\mu}v^{a}{}_{\nu)}+v_{(\mu|}{}^{a}\nabla_a\nabla_b v_{|\nu)}{}^{b}\\
&-\tfrac{1}{2}g_{\mu\nu}v_{ab}\nabla^b\nabla_c v^{ac}\Big)+\nabla_a v_{(\mu|}{}^{a}\nabla_b M^l v_{|\nu)}{}^{b}\\
&\quad-\nabla_{(\mu}v_{\nu)a}\nabla_b M^l v^{ab}+\tfrac{1}{2}g_{\mu\nu}\nabla_a v^{a}{}_{b}\nabla_c M^l v^{bc}-\nabla_a M^l v^{ab}\nabla_{(\mu}v_{\nu)b}\Big]\\
&-\tfrac{16}{9}\Big[M^l\Big(v^{a}{}_{\mu}v_{\nu}{}^{b}R_{ab}-2v^{ab}v_{a(\mu}R_{\nu)b}-\tfrac{1}{2}g_{\mu\nu}v^{ab}v_b{}^{c}R_{ac}\Big)
\end{aligned}
$$

EOMs

$$+\tfrac{1}{2}\nabla^2 M^I v_{(\mu|}{}^a v_{a|\nu)}$$

$$+\tfrac{1}{2}g_{\mu\nu}\nabla_a\nabla_b M^I v^{ac}v_c{}^b - \nabla_a\nabla_{(\mu|}M^I v^{ab}v_{b|\nu)}\Big]$$

$$-\tfrac{2}{9}\Big[M^I\Big(2v_\mu{}^a v_{\nu a}R + v_{ab}v^{ab}R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}Rv_{ab}v^{ab}\Big)$$

$$-\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^2\right)M^I v_{ab}v^{ab}\Big]$$

$$+\tfrac{4}{3}\Big[M^I\Big((\nabla_\mu v_{ab})(\nabla_\nu v^{ab}) + 2(\nabla_a v_{b\mu})(\nabla^a v^b{}_\nu) - \tfrac{1}{2}g_{\mu\nu}(\nabla_a v_{bc})(\nabla^a v^{bc})\Big)$$

$$+2\nabla_a M^I(\nabla^a v_{(\mu|}{}^b)v_{b|\nu)} + 2\nabla_a M^I(\nabla_{(\mu|}v^{ab})v_{b|\nu)}$$

$$-2\nabla_a M^I(\nabla_{(\mu|}v_{b|\nu)}))v^{ab}\Big]$$

$$+\tfrac{4}{3}\Big[M^I\Big(2(\nabla_{(\mu|}v^{ab})(\nabla_a v_{b|\nu)}) + (\nabla_a v_{b(\mu|})(\nabla^b v_{|\nu)}{}^a)$$

$$-\tfrac{1}{2}g_{\mu\nu}(\nabla_a v_{bc})(\nabla^b v^{ca})\Big)$$

$$+\nabla_a\Big(M^I v_{b(\mu}\nabla_{\nu)}v^{ba} + M^I v_{b(\mu}\nabla^a v^b{}_{\nu)} - M^I v^{ba}\nabla_{(\mu|}v_{b|\nu)}\Big)\Big]$$

EOMs

$$
\begin{aligned}
&- \tfrac{2}{3}\Big[M^I \epsilon^{abcde} v_{ab} v_{cd} \nabla_{(\mu|} v_{e|\nu)} - \epsilon^{abcde} \nabla_{(\mu|} M^I v_{ab} v_{cd} v_{e|\nu)} \\
&\qquad - \epsilon^{abcd}{}_{(\mu|} \nabla_e M^I v_{ab} v_{cd} v_{|\nu)}{}^e + \tfrac{1}{2} g_{\mu\nu} \epsilon^{abcde} \nabla^f M^I v_{ab} v_{cd} v_{ef}\Big] \\
&+ \tfrac{2}{3}\Big[\epsilon^{abcde} F^I_{ab} v_{c(\mu} \nabla_{\nu)} v_{de} - 2\epsilon^{abcd}{}_{(\mu|} \nabla_e F^I_{ab} v_c{}^e v_{d|\nu)}\Big] \\
&+ \Big[\epsilon^{abcde} F^I_{ab} v_{c(\mu|} \nabla_d v_{e|\nu)} + \epsilon^{abcd}{}_{(\mu|} \nabla_e F^I_{ab} v_c{}^e v_{d|\nu)}\Big] \\
&- \tfrac{4}{3}\Big[2F^I_{a(\mu} v_{\nu)}{}^b v_{bc} v^{ac} - 2F^I_{ab} v^a{}_{(\mu} v_{\nu)c} v^{bc} - \tfrac{1}{2} g_{\mu\nu} F^I_{ab} v^{ac} v_{cd} v^{db}\Big] \\
&- \tfrac{1}{3}\Big[2F^I_{a(\mu} v^a{}_{\nu)} v_{bc} v^{bc} + 2F^{Iab} v_{ab} v_{c\mu} v^c{}_\nu - \tfrac{1}{2} g_{\mu\nu} F^{Iab} v_{ab} v^{cd} v_{cd}\Big] \\
&+ \Big[16 M^I v_{ab} v^b{}_{(\mu} v_{\nu)c} v^{ca} - 2 g_{\mu\nu} M^I v_{ab} v^{bc} v_{cd} v^{da}\Big] \\
&+ \Big[4 M^I v_{ab} v^{ab} v_{c\mu} v_\nu{}^c + \tfrac{1}{2} g_{\mu\nu} M^I v_{ab} v^{ab} v_{cd} v^{cd}\Big]\Big\}
\end{aligned}
$$

Conditions for timelike supersymmetry

- For a timelike orbit we can take $\epsilon = (\epsilon^1, \epsilon^2) = (e^{\phi}1, -ie^{\phi}e^{12})$

The Killing spinor equations imply

- $ds^2 = e^{4\phi}(dt + \Omega)^2 - e^{-2\phi}\hat{g}_{mn}dx^m dx^n$
- $\hat{g}$ is metric for Hyper-Kahler "base space".
- $F^I = e^{-2\phi}e^0 \wedge d(M^I e^{2\phi}) - M^I G^{(-)} + F^{I(+)} = -d(M^I e^0) + \Theta^I$
- Θ^I harmonic.
- $v_{\mu\nu}$ completely determined.
- $D = \frac{3}{2}e^{4\phi}\hat{G}^{(-)}\cdot\hat{G}^{(-)} + \frac{1}{2}e^{4\phi}\hat{G}^{(+)}\cdot\hat{G}^{(+)} + 3e^{2\phi}\hat{\nabla}^2\phi - 18e^{2\phi}(\hat{\nabla}\phi)^2$

EOMs and KSIs

Normally to solve the eoms we plug this data into the equations then try and simplify. However for our representative we obtain for the Killing spinor identities

$$
\begin{aligned}
\mathcal{E}(A)_I^0 - \mathcal{E}(M)_I = 0\ , \qquad \mathcal{E}(A)_I^i &= 0\ , \\
\left(\tfrac{1}{4}\mathcal{E}(v) + \mathcal{E}(D)v\right)^{\alpha}{}_{\alpha} + \nabla^0\mathcal{E}(D) &= 0\ , \\
\left(\tfrac{1}{4}\mathcal{E}(v) + \mathcal{E}(D)v\right)^{0i} - \nabla^i\mathcal{E}(D) &= 0\ , \\
\left(\tfrac{1}{4}\mathcal{E}(v) + \mathcal{E}(D)v\right)^{12} = 0\ , \qquad \mathcal{E}(e)_a^{\mu} &= 0\ .
\end{aligned}
$$

off-shell KSI are valid for all higher order corrections that can be added to the theory with the same field content, i.e. for any similar consistent truncation. In particular for any such corrected action it is sufficient to impose the equations of motion

$$
\mathcal{E}(D) = 0\ , \qquad \mathcal{E}(v)^{(+)ij} = 0\ , \qquad \mathcal{E}(M)_I = 0\ .
$$

Simplified eoms

The equation of motion for D is

$$0 = \frac{1}{2}(\mathcal{N} - 1) + \frac{c_{2I}}{48} e^{2\phi} \left[\frac{1}{4} e^{2\phi} M^I \left(\frac{1}{3} \hat{G}^{(+)} \cdot \hat{G}^{(+)} + \hat{G}^{(-)} \cdot \hat{G}^{(-)} \right) \right.$$
$$\left. + \frac{1}{12} e^{2\phi} \hat{G}^{(+)} \cdot \hat{\Theta}^{(+)I} + M^I \hat{\nabla}^2 \phi + \hat{\nabla}\phi \cdot \hat{\nabla} M^I - 4 M^I \hat{\nabla}\phi \cdot \hat{\nabla}\phi \right] ,$$

The M^I equation is more involved, but we find

$$0 = e^{4\phi} \left[\frac{1}{4} c_{IJK} \hat{\Theta}^{(+)J} \cdot \hat{\Theta}^{(+)K} - \hat{\nabla}^2 \left(e^{-2\phi} \mathcal{N}_I \right) \right] +$$
$$+ \frac{c_{2I}}{24} e^{4\phi} \left\{ \hat{\nabla}^2 \left(3 \hat{\nabla}\phi \cdot \hat{\nabla}\phi - \frac{1}{12} e^{2\phi} \hat{G}^2_{(+)} - \frac{1}{4} e^{2\phi} \hat{G}^2_{(-)} \right) + \frac{1}{8} \hat{R}_{ijkl} \hat{R}^{ijkl} \right\} ,$$

This computation has been checked in Mathematica using the package xAct, and the two equations above are in agreement with [Castro et al].

Simplified eoms

Finally, after a very long calculation and making extensive use of duality identities we find the equation of motion for v yeilds

$$
\begin{aligned}
0 = \quad & -4e^{2\phi}\hat{G}_{ij}^{(+)} + 2e^{2\phi}\mathcal{N}_I\hat{\Theta}_{ij}^{I(+)} \\
& + \frac{c_{2I}}{24}\left\{\frac{1}{2}e^{6\phi}\left(\frac{1}{3}\hat{G}_{(+)}^2 + \hat{G}_{(-)}^2\right)\hat{\Theta}_{ij}^{(+)I}\right. \\
- \quad & \frac{1}{3}e^{4\phi}\left(M^I\hat{G}_{kl}^{(+)} + 2\hat{\Theta}_{kl}^{I(+)}\right)\hat{R}_{ij}{}^{kl} \\
& + e^{4\phi}\hat{\nabla}^2\left[M^I(G_{ij}^{(-)} - \frac{1}{3}G_{ij}^{(+)})\right] - \frac{1}{6}e^{-2\phi}\hat{\nabla}^2[e^{6\phi}\hat{\Theta}_{ij}^{I(+)}] \\
- \quad & \left. 4e^{4\phi}\hat{\nabla}_{[i}\hat{\nabla}_k[M^I G^{(-)k}{}_{j]}]\right\} ,
\end{aligned}
$$

Ricci scalar squared invariant

Now let us consider adding the Ricci scalar squared invariant constructed by Ozkan and Yi Pang to the theory above. After gauge fixing this invariant reads

$$
\begin{aligned}
e^{-1}\mathcal{L} =& \mathcal{E}\,\Big[N^2(\tfrac{1}{4}D - \tfrac{1}{8}R + \tfrac{3}{2}v^2) + 2Nv\cdot(dP - 2dV) + \tfrac{1}{4}(dP - 2dV)^2 \\
& -\tfrac{1}{2}(dN)^2 - \tfrac{1}{16}(P^2 + 4V\cdot P - N^2 - 2v^2 + D + 6V'^{\mathbf{ij}}_{a}V'^{a}_{\mathbf{ij}} + \tfrac{3}{2}R)^2 \\
& +2\nabla^a V'^{\mathbf{ij}}_{a}\nabla_b V'^{b}_{\mathbf{ij}}\Big] \\
& + e_I\,\Big[N^2F^I\cdot v + \tfrac{N}{2}F^I\cdot(dP - 2dV) - NdN\cdot dM^I \\
& -\tfrac{1}{2}NY^I(P^2 + 4V\cdot P - N^2 - 2v^2 + D + 6V'^{\mathbf{kl}}_{a}V'^{a}_{\mathbf{kl}} + \tfrac{3}{2}R) \\
& -4NY'^{I}_{\mathbf{ij}}\nabla^\mu V'^{(\mathbf{i}}_{\mu\mathbf{k}}L^{\mathbf{j})\mathbf{k}} \\
& +\tfrac{1}{8}e^{-1}\epsilon^{abcde}A^I_a(dP - 2dV)_{bc}(dP - 2dV)_{de}\Big]\ ,
\end{aligned}
$$

Ricci scalar squared invariant

After truncating the auxiliary fields this leads to the following additions to the eoms

$$
\begin{aligned}
\frac{1}{\sqrt{|g|}}\frac{\delta S_{Rs^2}}{\delta D} &= \tfrac{4}{3}\mathcal{E} D(\tfrac{2}{3}D - \tfrac{4}{3}v^2 + R)\ , \\
\frac{1}{\sqrt{|g|}}\frac{\delta S_{Rs^2}}{\delta M^I} &= e_I(\tfrac{2}{3}D - \tfrac{4}{3}v^2 + R)^2\ , \\
\frac{1}{\sqrt{|g|}}\frac{\delta S_{Rs^2}}{\delta v_{\mu\nu}} &= -\tfrac{16}{3}\mathcal{E}(\tfrac{2}{3}D - \tfrac{4}{3}v^2 + R)v^{\mu\nu}\ , \\
\frac{1}{\sqrt{|g|}}\frac{\delta S_{Rs^2}}{\delta A^I_\mu} &= 0\ , \\
\frac{1}{\sqrt{|g|}}\frac{\delta S_{Rs^2}}{\delta g^{\mu\nu}} &= \mathcal{E}\,\{2(\tfrac{2}{3}D - \tfrac{4}{3}v^2 + R)(R_{\mu\nu} - \tfrac{8}{3}v_{\mu a}v_\nu{}^a) \\
&\quad -\tfrac{1}{2}g_{\mu\nu}(\tfrac{2}{3}D - \tfrac{4}{3}v^2 + R)^2\} \\
&\quad + 2(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^2)\mathcal{E}(\tfrac{2}{3}D - \tfrac{4}{3}v^2 + R)^2\ .
\end{aligned}
$$

Ricci scalar squared invariant - timelike case

However for our time-like representative the gaugino Killing spinor identity yields

$$\mathcal{E}(A)_{I0} = \mathcal{E}(M)_I \ .$$

This implies

$$(\tfrac{2}{3}D - \tfrac{4}{3}v^2 + R) = 0$$

which can be checked using the explicit form of D and v and calculating R, using the fact that the base is Ricci flat. This implies that all of the contributions to the equations of motion for this invariant vanish for supersymmetric backgrounds in the time-like class.

Ricci scalar squared invariant - Null case

In the null case our representative has first component $\epsilon = 1 + e^1$. Using a suitable "null" basis we obtain a linear system for the KSIs which imply

$$\mathcal{E}(M)_I = 0$$

This can only be true if the coupling e_I vanishes, or again $R = \frac{4}{3}v^2 - \frac{2}{3}D$. Since the KSIs are off-shell they do not know the value of e_I, so we may conclude that $R = \frac{4}{3}v^2 - \frac{2}{3}D = 0$ also for the null cases, and this invariant therefore does not contribute to any equation of motion for any supersymmetric background.

Maximally supersymmetric solutions in the general case

- We made a consitent truncation of some auxiliary fields in order to ensure we were discussing an ungauged supergravity, but what is the story in the general case?
- Let us consider maximal supersymmetry and generalize the work of Meessen to the case of an off-shell compensating multiplet.
- As we are off-shell, we do not (and cannot) make any distinction between the (U(1)) gauged and ungauged supergravities. i.e. we do not know whether the coupling to the cosmological constant density (0-derivative) is non-zero.

Maximally supersymmetric solutions in the general case

- As expected we find only the known solutions of the ungauged case and AdS_5.
- In the case of Minkowski and AdS_5 the equation of motion of $Y^{\mathbf{ij}}$ remains to be solved, but corrections to it are always constant.
- The eom of D always remains to be solved, but corrections to it are always constant. The very special geometry condition is just renormalized.
- However each solution has different components of the P_a equation that remain to be solved, generating constraints.
- The simplest example is already very well known. Solving the P_a equation of motion at 2 derivative level where we demand the coupling to the cosmological constant density is non-zero in the case of Minkowski space leads to a contradiction - Minkowski space is not a maximally supersymmetric solution of the gauged supergravity.

Outlook

Wish list

- Riemann tensor squared or Ricci tensor squared invariants in a form we can use.
- Understand the freedom to make field redefinitions in string theory to choose the cooeffients of the invariants - can we simplify things all the way to the truncated case order by order? Do we want to?
- If not, progress on the full untruncated spectrum- related to the classification of the two derivative general matter coupled case.
- Cases in 3-6d where we have the superconformal off-shell formalism (and matter to perform the gauge fixing).
- Non-Abelian case.

Outlook

Applications

- Generalize classification of order α' near horizon geometries given in "Small Horizons" [Gutowksi,Klemm, P.S., Sabra]
- Make progress on more general order α' full black hole solutions, and black rings (and things).
- Stringy corrections to ADS/CFT calculations in the gauged case.