Recent Advances in Heterotic Moduli

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arXiv:1402.1725, 1409.3347, 1509.08724,

1607.03473, 1704.xxxxx, 17xx.xxxx, etc.

April 28 2017, Workshop on Geometry, Gravity and Supersymmetry,

Mainz Institute for Theoretical Physics

Introduction

Motivation and Summary

Long History of heterotic string compactifications.

- Minkowski 4d $N = 1$ vacua with good particle physics from compactifications on Calabi-Yau manifolds with vector bundles.
- α'-corrections induce torsional geometries.
- Loose toolbox of algebraic geometry and Kähler geometry.
- Harder to understand moduli.

This talk: Heterotic compactifications on $SU(3)$-structure and G_2-structure manifolds.

- Review of infinitesimal (massless) heterotic moduli of $SU(3)$-structure compactifications to Mink_4.
- Some comments on the Geometric properties of moduli space (Superpotential, Kähler potential).
- Infinitesimal moduli of heterotic G_2 system.
- Higher order deformations of $SU(3)$ system, heterotic DGLA.
- Comments on work in progress..

General String Compactifications

String theory is ten-dimensional:

$$\mathcal{M}_{10} = \mathcal{M}_d \times X_{10-d} \,,$$

where $\mathcal{M}_d$ is the maximally symmetric (often Minkowski) d-dimensional spacetime, and X is the internal geometry, often assumed compact.

Supersymmetry: Puts conditions on X.

- $d = 4$, $\mathcal{O}(\alpha'^0) \Rightarrow X$ torsion-free, Calabi-Yau.
- $d = 3$, $\mathcal{O}(\alpha'^0) \Rightarrow X$ torsion-free for Minkowski compactifications, G_2-Holonomy.
- Note: Zeroth order AdS_3 compactification requires torsion.

Deformations $\delta X \Leftrightarrow$ Moduli fields in external geometry.

String Phenomenology: Find compact geometries whose moduli contains the Standard Model.

Heterotic: Don't fully understand moduli of generic compactifications yet. Special cases known (Standard Embedding, etc).

Heterotic supergravity is a 10d $N = 1$ supergravity coupled to an $E_8 \times E_8$ Yang-Mills theory.

- Good for phenomenology.
- Can also be useful for describing geometries with more fibration structures.
- Mathematically interesting. Natural generalisations of torsion free geometries when bundles can back-react.

Complications:

- Torsional geometries not well understood.
- Few "non-trivial" examples [Dasgupta et al. 99, Becker et al 06, Halmagyi-Israel-EES 16,..].
- Complicated equations to deal with, e.g. Bianchi Identity:

$$\mathrm{d}H = \frac{\alpha'}{4}(\mathrm{tr}\, F^2 - \mathrm{tr}\, R^2)\ .$$

Need a "nicer" description to deal with moduli [Anderson et al 10, Anderson et al 14, delaOssa-EES 14, Garcia-Fernandez et al 15, Candelas et al 16, ..].

Steps in Understanding Moduli

3 Steps in understanding moduli of a string compactification:

- *Step 1:* Infinitesimal *massless* spectrum $T\mathcal{M}$. Derive differential $\mathcal{D}$ and associated cohomology giving moduli

$$T\mathcal{M} = H^1_{\mathcal{D}}(\mathcal{Q}) \; .$$

  The fields can usually be taken to be one-forms $\mathcal{X}$ with values in a bundle $\mathcal{Q}$ (or sheaf more generally), naturally associated to the given moduli problem.

- *Step 2:* Understand geometry of $\mathcal{M}$ (Kähler metric, etc). Higher order deformations, obstructions (Yukawa couplings). Maurer-Cartan elements,

$$\mathcal{D}\mathcal{X} + \tfrac{1}{2}[\mathcal{X}, \mathcal{X}] = 0 \; ,$$

  and associated differentially graded Lie algebra (or L_∞-algebra more generally).

- *Step 3:* Understand *quantum cohomology ring*. Include non perturbative effects such as world-sheet instantons, and quantum corrections (higher genus effects). Construct topological theory of corresponding structures?

The Heterotic $SU(3)$-system

The Superpotential

Compactification on six dimensional compact $SU(3)$-structure manifold X results in a 4d supergravity coupled to Yang-Mills.

Four-dimensional heterotic theory has GVW-superpotential [Becker et al 03, Cardoso et al 03, Lukas et al 05, McOrist 16, ..]

$$W = \int_X (H + i\mathrm{d}\omega) \wedge \Omega \,,$$

where (Ω, ω) form the $SU(3)$-structure, and

$$H = \mathrm{d}B + \frac{\alpha'}{4}\omega_{CS}(A) \,.$$

F-term (BPS) conditions:

$$\delta W = W = 0 \,.$$

- $\mathrm{d}\Omega = 0$ and so X is a *complex manifold*.
- $F \wedge \Omega = 0$ and so the bundle given by A is *holomorphic*.
- $H = i(\partial - \overline{\partial})\omega$, flux is identified with internal torsion.

There are also “D-term” conditions (conformally balances, Yang-Mills conditions).

Infinitesimal Moduli and Massless Fields

Note: Will assume "technical assumption" such as the $\partial\overline{\partial}$-lemma, or $h^{(0,1)} = 0$.

Infinitesimal moduli preserving BPS conditions $\Leftrightarrow$ Massless fields in 4d theory

Assumption for massless spectrum (infinitesimal moduli):

$$T\mathcal{M} \subseteq H^{(0,1)}(T^{*(1,0)}X) \oplus H^{(0,1)}(T^{(1,0)}X) \oplus H^{(0,1)}(\text{End}(V)) \,.$$

Massless fields:

Let δ_2 be massless, while δ_1 is a generic KK mode. Must then require

$$\delta_2\delta_1 W = 0 \quad \forall \quad \delta_1 \,.$$

From this we get that

$$\text{d}\delta_2\Omega = 0 \;\Rightarrow \delta_2\Omega \in H^{(2,1)}(X) \Leftrightarrow \mu_2 \in H^{(0,1)}(T^{(1,0)}X) \,,$$

where μ_2 is tangent bundle valued. Also get

$$\delta_2(F\wedge\Omega) = 0 \quad \Leftrightarrow \quad F_{a\bar{b}}\,\text{d}z^{\bar{b}} \wedge \mu_2^a = \mathcal{F}(\mu_2) = -\overline{\partial}_A \alpha_2 \,,$$

where $\mu_2 \in H^{(0,1)}(T^{(1,0)}X)$, $\alpha_2 = \delta_2 A^{(0,1)} \in \Lambda^{(0,1)}(\text{End}(V))$.

Kernels and the Atiyah Algebroid

It follows that μ_2 is in the kernel of Atiyah map [Atiyah 57, Anderson et al 10]

$$\mathcal{F} \;:\quad H^{(0,1)}(T^{(1,0)}X) \to H^{(0,2)}(\text{End}(V))\;.$$

We thus see that the infinitesimal moduli of a complex manifold with holomorphic bundle is

$$T\mathcal{M}_1 = H^{(0,1)}(\text{End}(V)) \oplus \text{ker}(\mathcal{F})\;.$$

Which Cohomology (Which differential)?

Can be put in terms of holomorphic structure, or $(0,1)$-differential

$$\overline{\partial}_1 = \begin{pmatrix} \overline{\partial}_A & \mathcal{F} \\ 0 & \overline{\partial} \end{pmatrix} \;:\; \Lambda^{(q,p)}\begin{pmatrix} \text{End}(V) \\ T^{(1,0)}X \end{pmatrix} \to \Lambda^{(q,p+1)}\begin{pmatrix} \text{End}(V) \\ T^{(1,0)}X \end{pmatrix}\,,$$

on the bundle $Q_1 = \text{End}(V) \oplus T^{(1,0)}X$.

Note that $\overline{\partial}_1^2 = 0$ due to the Bianchi identity $\overline{\partial}_A F = 0$.

One then finds

$$T\mathcal{M}_1 = H^{(0,1)}_{\overline{\partial}_1}(Q_1) = .. = H^{(0,1)}(\text{End}(V)) \oplus \text{ker}(\mathcal{F})\;.$$

Conditions from the Anomaly

From the second order variation of W we also find the condition (modulo "technical assumption")

$$\mathcal{H}(\mu_2,\alpha_2)_b \mathrm{d}z^b = 2\Delta_2^a \wedge i\partial_{[a}\omega_{b]\overline{c}}\mathrm{d}z^{b\overline{c}} - \frac{\alpha'}{2}\mathrm{tr}\,\alpha_2\wedge F = \overline{\partial}\tau_2^{(1,1)}\ .$$

Can think of $\tau_2^{(1,1)}$ as complexified α'-corrected Kähler deformations.

$\Rightarrow (\mu_2,\alpha_2)\in H^{(0,1)}(Q_1)$ is in the kernel of

$$\mathcal{H}\ :\quad H^{(0,1)}(Q_1)\to H^{(0,2)}(T^{*(1,0)}X)\ .$$

$\mathcal{H}$ is a map between cohomologies by the heterotic Bianchi Identity

$$\mathrm{d}H = -2i\partial\overline{\partial}\omega = \frac{\alpha'}{4}\mathrm{tr}\,F\wedge F\ .$$

Naively the massless moduli are given by

$$T\mathcal{M}_2 = H^{(0,1)}(T^{*(1,0)}X)\oplus \ker(\mathcal{H})\ ,\quad \ker(\mathcal{H})\in H^{(0,1)}(Q_1)\ ,$$

where $H^{(0,1)}(T^{*(1,0)}X)\cong H^{(1,1)}(X)$ are hermitian moduli.

The Heterotic Differential

What differential computes the massless moduli?

Let $Q_2 = T^{*(1,0)}X \oplus \text{End}(V) \oplus T^{(1,0)}X$. Can define holomorphic structure on the differential complex $\Lambda^{(q,p)}(Q_2)$

$$\overline{\partial}_2 = \begin{pmatrix} \overline{\partial} & \mathcal{H} \\ 0 & \overline{\partial}_1 \end{pmatrix} : \ \Lambda^{(q,p)}\begin{pmatrix} T^{*(1,0)}X \\ Q_1 \end{pmatrix} \ \rightarrow \ \Lambda^{(q,p+1)}\begin{pmatrix} T^{*(1,0)}X \\ Q_1 \end{pmatrix},$$

Note that the heterotic Bianchi Indentity implies that $\overline{\partial}_2^2 = 0$.

Compute first cohomology

$$T\mathcal{M} = H^{(0,1)}_{\overline{\partial}_2}(Q_2) = .. = \left(H^{(1,1)}(X)\Big/\text{Im}(\mathcal{F}_0)\right) \oplus \ker(\mathcal{H}) \ ,$$

where

$$\mathcal{F}_0 \ : \quad H^0(\text{End}(V)) \rightarrow H^{(1,1)}(X)$$

is defined for $\alpha \in H^0(\text{End}(V))$, $\ \mathcal{F}_0(\alpha) = \text{tr}\,(F\alpha)$.

Incorporates Green-Schwarz gauge transformation of B-field, and preserving the Yang-Mills condition.

Compactifications to 3d

G_2-structure compactifications

$N = 1$ Supersymmetric compactifications to three dimensions require internal geometry Y to have a G_2-structure, tangent bundle has a reduced structure group $G_2 \subset Spin(7)$.

Determined by a non-degenerate G_2 invariant three-form φ ($\psi = *\varphi$), satisfying

$$\begin{aligned} \mathrm{d}\varphi &= \tau_0 \psi + 3\tau_1 \wedge \varphi + *\tau_3 \\ \mathrm{d}\psi &= 4\tau_1 \wedge \psi + *\tau_2 \ . \end{aligned}$$

Supersymmetry requires [Papadopoulos et al 05, Lukas et al 10, ..],

$$\begin{aligned} \tau_0 &= \mathrm{dvol}_{\mathcal{M}_3} \lrcorner H \ , \quad \tau_1 = \frac{1}{2}\mathrm{d}\phi \\ \tau_2 &= 0 \ , \quad \tau_3 = -H + \frac{1}{6}\tau_0\varphi - \tau_1 \lrcorner \psi \ . \end{aligned}$$

Note, this is an *integrable* G_2-structure.

Gauge bundle: $F \wedge \psi = 0 \quad \Leftrightarrow$ Curvature is an instanton.

The Canonical G_2 Complex

For integrable G_2 structure (in particular G_2 holonomy) we have the complex [Reyes-Carrion 93, Fernandez et al 98]

$$0 \to \Lambda^0_1 \xrightarrow{\check{\mathrm{d}}} \Lambda^1_7 \xrightarrow{\check{\mathrm{d}}} \Lambda^2_7 \xrightarrow{\check{\mathrm{d}}} \Lambda^3_1 \to 0\,.$$

Here $\check{\mathrm{d}} = \pi \circ \mathrm{d}$, where π is the projection onto the appropriate representation.

This is a differential complex, i.e. $\check{\mathrm{d}}^2 = 0$, iff the G_2 structure is integrable.

This complex generalises to bundles (V, A)

$$0 \to \Lambda^0_1(V) \xrightarrow{\check{\mathrm{d}}_A} \Lambda^1_7(V) \xrightarrow{\check{\mathrm{d}}_A} \Lambda^2_7(V) \xrightarrow{\check{\mathrm{d}}_A} \Lambda^3_1(V) \to 0\,.$$

where $\check{\mathrm{d}}_A = \pi \circ \mathrm{d}_A$. This is a differential complex iff $F \wedge \psi = 0$.

These complexes are elliptic [Reyes-Carrion 93] $\Rightarrow$ the corresponding cohomologies $\check{H}^*(V)$ are finite dimensional.

The G_2 Holonomy Case

Infinitesimal Geometric Moduli

Joyce: Infinitesimal geometric moduli counted by $H^3(Y)$.

Infinitesimal moduli are given by

$$\partial_t\varphi = -\frac{1}{2}\Delta_t^m \wedge \varphi_{mnp}\mathrm{d}x^{np}\,, \quad \Delta_t^m = \Delta_{t\,n}{}^m\,\mathrm{d}x^n \in \Lambda^1(TY)\,.$$

where Δ is a symmetric matrix.

Can show that $\check{\mathrm{d}}_\nabla\Delta = 0$, where ∇ is the Levi-Civita connection for G_2 holonomy. Modulo diffeomorphisms we find

$$\Delta \in \check{H}^1(TY)\,.$$

It follows that $H^3(Y)$ embeds into $\check{H}^1(TY)$, given by “the symmertric part” of $\check{H}^1(TY)$.

The anti-symmetric part of $\check{H}^1(TY)$ can be shown to correspond to closed two-forms. We can interpret this as deformations of the heterotic B-field.

Moduli of instanton bundles

Let us consider an infinitesimal deformation of the instanton condition $\partial_t\left(F \wedge \psi\right) = 0$, which gives

$$\check{\mathrm{d}}_A \partial_t A = \check{\mathcal{F}}(\Delta_t)\ ,$$

where

$$\check{\mathcal{F}}(\Delta) = \pi_7\left(F_{mn}\mathrm{d}x^n \wedge \Delta^m\right)\ .$$

Note that $\check{\mathcal{F}}$ is a map in cohomology

$$\check{\mathcal{F}}\ :\quad \check{H}^p(TY)\quad\rightarrow\quad \check{H}^{p+1}(\mathrm{End}(V))\ ,$$

as can be checked by the Bianchi identity for F together with the instanton condition.

I.e. infinitesimal geometric moduli $\Delta \in \ker(\check{\mathcal{F}})$. Note: elements of $\check{H}^1(TY)$ corresponding to B-field deformations are naturally in the kernel of $\check{\mathcal{F}}$.

Theorem (delaOssa-Larfors-EES) The zeroth order infinitesimal heterotic moduli space on G_2 manifolds to 3d Minkowski is

$$T\mathcal{M} = \check{H}^1(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})\ .$$

In analogy with the complex case, we can define a differential

$$\check{\mathrm{d}}_1 = \begin{pmatrix} \check{\mathrm{d}}_A & \check{\mathcal{F}} \\ 0 & \check{\mathrm{d}}_\nabla \end{pmatrix} : \check{\Lambda}^p \begin{pmatrix} \mathrm{End}(V) \\ TY \end{pmatrix} \to \check{\Lambda}^{p+1} \begin{pmatrix} \mathrm{End}(V) \\ TY \end{pmatrix} ,$$

Note that $\check{\mathrm{d}}_1^2 = 0$ due to Bianchi identity for F and instanton condition. Here $\check{\Lambda}^p$ denotes projection of p-forms onto the appropriate representation.

Get short exact extension of complexes, $\mathcal{Q}_1 = \mathrm{End}(V) \oplus TY$

$$0 \to \check{\Lambda}^*(\mathrm{End}(V)) \to \check{\Lambda}^*(\mathcal{Q}_1) \to \check{\Lambda}^*(TY) \to 0 ,$$

where differential of $\check{\Lambda}^*(\mathcal{Q}_1)$ is given by $\check{\mathrm{d}}_1$.

Long exact sequence in cohomology gives

$$T\mathcal{M}_1 = \check{H}^1(\mathcal{Q}_1) = \check{H}^1(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}}) .$$

as expected.

The Heterotic G_2 System

The heterotic G_2 system naturally gives rise to a differential

$$\check{\mathcal{D}} = \begin{pmatrix} \check{\mathrm{d}}_A & \check{\mathcal{F}} \\ \check{\mathcal{F}} & \check{\mathrm{d}}_\theta \end{pmatrix} \; : \; \check{\Lambda}^p \begin{pmatrix} \mathrm{End}(V) \\ TY \end{pmatrix} \;\to\; \check{\Lambda}^{p+1} \begin{pmatrix} \mathrm{End}(V) \\ TY \end{pmatrix} ,$$

where the map $\check{\mathcal{F}} \; : \; \check{\Lambda}^p(\mathrm{End}(V)) \to \check{\Lambda}^{p+1}(TY)$ is given by

$$\check{\mathcal{F}}(\alpha)^m = \frac{\alpha'}{4}\, \pi \left[\mathrm{tr}\left(g^{mn} F_{nq} \mathrm{d}x^q \wedge \alpha \right) \right] ,$$

and π denotes the appropriate projection.

The connection d_θ has conncetion symbols

$$\theta_{mn}{}^p = \Gamma_{nm}{}^p \; ,$$

where $\Gamma_{nm}{}^p$ denote the connection symbols of the $SU(3)$-structure connection ∇ (the BPS connection, $\nabla\varphi = 0$).

NOTE: The connection θ is NOT an instanton when α'-corrections are included. However

$$R_\theta \wedge \psi = 0 + \mathcal{O}(\alpha') \; .$$

We compute the square of the differential

$$\check{\mathcal{D}}^2 = \begin{pmatrix} \check{\mathrm{d}}_A^2 + \check{\mathcal{F}}^2 & \check{\mathcal{F}}\check{\mathrm{d}}_\theta + \check{\mathrm{d}}_A\check{\mathcal{F}} \\ \check{\mathrm{d}}_\theta\check{\mathcal{F}} + \check{\mathcal{F}}\check{\mathrm{d}}_A & \check{\mathrm{d}}_\theta^2 + \check{\mathcal{F}}^2 \end{pmatrix} = 0\,.$$

- Instanton condition implies $\check{\mathrm{d}}_A^2 = 0 \;\Rightarrow\; F \in \Lambda^2_{\mathbf{14}}(\mathrm{End}(V))$.
  Then $\check{\mathcal{F}}^2 = 0$ follows from representation theory arguments.
  NOTE: Can be shown to be the only solution within the α'-expansion.
- Using $\mathrm{d}_A F = 0$ and instanton condition, we also find

$$\check{\mathcal{F}}\check{\mathrm{d}}_\theta + \check{\mathrm{d}}_A\check{\mathcal{F}} = \check{\mathrm{d}}_\theta\check{\mathcal{F}} + \check{\mathcal{F}}\check{\mathrm{d}}_A = 0\,.$$

- Heterotic Bianchi identity $\quad\Rightarrow\quad \check{\mathrm{d}}_\theta^2 + \check{\mathcal{F}}^2 = 0$.

Theorem (delaOssa-Larfors-EES) The infinitesimal moduli space of the heterotic G_2 system is

$$T\mathcal{M} = H^1_{\check{\mathcal{D}}}(\mathcal{Q}_1) \text{ “=” } H^1_{\check{d}_\theta}(TY) + \mathcal{O}(\alpha') \text{ and bundles}\,.$$

Scetch of proof: Identify infinitesimal deformations with $\check{\mathcal{D}}$-closed forms. Identify Heterotic symmetries (diffeomorphisms and gauge-transformations, B-field transformations) with $\check{\mathcal{D}}$-exact forms.

Differences with $SU(3)$-System

The heterotic G_2-system has advantages over the $SU(3)$-system:

- The system is more compact (not an extension).
- No separation of structures, $\check{\mathcal{D}}$ has everything. Opposite direction holds within α'-expansion: $\check{\mathcal{D}}^2 = 0$ implies most of the heterotic G_2-system, except that Lee form is exact (the dilaton).
- Embedding $SU(3)$-system in G_2-system puts holomorphic and hermitian structures on equal footing.
- The differential $\check{\mathcal{D}}$ makes it clear that the heterotic $SU(3)$-system can be viewed as a holomorphic Yang-Mills connection on $\mathcal{Q}$.
- No “technical assumption”.

The disadvantages are:

- The system is more compact (not an extension) $\Rightarrow$ harder to identify usual cohomologies within $H^1_{\check{\mathcal{D}}}(\mathcal{Q}_1)$.
- Harder to compute $H^1_{\check{\mathcal{D}}}(\mathcal{Q}_1)$. No clear notion of Poincare lemma or sheaf cohomology $\Rightarrow$ hard to get to by algebraic methods.
Homogeneous examples?

Higher Order Deformations of $SU(3)$-System

A Heterotic Effective Theory

Considering *generic* higher deformations of the heterotic $SU(3)$-system is in general a very hard problem. Get some complicated L_∞-algebra.

Clues from physics:

Know that superpotential is *holomorphic* $\Rightarrow$ A *finite* and *holomorphic* deformation of the heterotic $SU(3)$-system can be represented as a Q_2-valued $(0,1)$-form

$$y = (x, \alpha, \mu) \in \Omega^{(0,1)}(Q_2)\ ,$$

where $\mu \in \Omega^{(0,1)}(T^{(1,0)}X)$, $\alpha \in \Omega^{(0,1)}(\mathrm{End}(V))$ and $x \in \Omega^{(0,1)}(T^{*(1,0)}X)$.

Deforming the superpotential, one finds

$$\Delta W = \int_X \left(\langle y, \overline{\partial}_2 y\rangle + \tfrac{1}{3}\langle y, [y,y]\rangle + \mu^a \partial_a b\right) \wedge \Omega\ ,$$

where for $y_1, y_2 \ \in\ \Omega^{(0,*)}(Q_2)$, $\langle y_1, y_2\rangle = \mu_1^a x_{2\,a} + \mu_2^a x_{1\,a} + \mathrm{tr}\ (\alpha_1\alpha_2)$, $b \in \Omega^{(0,2)}(X)$ is auxillary, and

$$[\ ,\ ]\ :\ \ \Omega^{(0,p)}(Q_2) \times \Omega^{(0,q)}(Q_2) \to \Omega^{(0,p+q)}(Q_2)$$

satisfies Leibniz rule w.r.t. $\overline{\partial}_2$, and Jacobi identity modulo ∂_a-exact terms.

Maurer-Cartan Equation

From ΔW we derive the following equations of motion

$$\partial\Omega(\mu) = 0\,,\quad \Omega(\mu) = \tfrac{1}{2}\Omega_{abc}\mu^a\,\mathrm{d}z^{bc}$$

$$\overline{\partial}_2 y + \tfrac{1}{2}[y, y] = \partial_a\text{-exact}\,.$$

The last equation is the Maurer-Cartan equation for the DGLA

$$\mathcal{A} = \left(\Omega^{(0,*)}\left(Q_2/\{\partial_a\text{-exact}\}\right), \overline{\partial}_2, [\,,\,]\right)\,.$$

Note that infinitesimal moduli y satisfy

$$\overline{\partial}_2 y = \partial_a\gamma\,,$$

for some $\gamma \in \Omega^{(0,2)}(X)$. It follows that $\overline{\partial}\partial\gamma = 0 \Rightarrow \overline{\partial}\gamma = 0$.
Employ "technical assumption" $\Rightarrow\ \gamma = \overline{\partial}c$ for some $(0,1)$-field c.
This can be absorbed in $\tilde{y}$ to get

$$\overline{\partial}_2\tilde{y} = 0\,.$$

Conclusions

Conclusions:

- Heterotic geometries give nice generalisations of torsion-free geometries when bundles are included, but the moduli problem is hard.
- We have derived the infinitesimal moduli of the heterotic $SU(3)$-system.
- We have derived the infinitesimal moduli of the heterotic G_2-system, and compared with the $SU(3)$-system.
- We discussed higher order deformations of the heterotic $SU(3)$-system, and the heterotic DGLA.

Outlook, and work in progress:

- So far mostly a mathematical investigation into the structures. Interesting to look for applications (particle physics, AdS/CFT, other areas of string theory?).
- Further investigation into higher order obstructions, and the corresponding DGLAs. What are the *integrable* deformations?
- Higher order deformations of the heterotic G_2-system. What is the corresponding L_∞-algebra?
- What about non-perturbative effects, world sheet instantons, NS5-branes? Correct the Bianchi Identity

$$\mathrm{d}H + W_5 = \frac{\alpha'}{4}(\mathrm{tr}\, F^2 - \mathrm{tr}\, R^2)\ , \quad [W_5] \in H^{(2,2)}(X)\ .$$

$\Rightarrow$ Spoils holomorphic structure $\overline{\partial}_2^2 \neq 0$.

- Quantum corrections; Quantise quasi-topological action ΔW? Is there a corresponding topological world-sheet theory, or open string field theory ala Witten? In progress with A. Ashmore, R. Minasian, C. Strickland-Constable.

Thank you!

Thank you for your attention!