New AdS/CFT pairs from non-Abelian T-duality

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1. Introduction & motivation: NATD in AdS/CFT

Non-Abelian T-duality (NATD) has proved to be very useful as a solution generating technique in AdS/CFT

Its realization in the CFT remains however quite unknown

Interestingly, some examples suggest that, contrary to its Abelian counterpart, NATD may change the CFT:

NATD of $AdS_5 \times S^5$: Gaiotto & Maldacena geometry (dual to N=2 SCFTs (Gaiotto theories))

NATD of $AdS_5 \times T^{1,1}$: Bah, Beem, Bobev, Wecht geometry (dual to N=1 SCFTs (Sicilian quivers))

Indeed, contrary to its Abelian counterpart, NATD has not been proven to be a symmetry of string theory

Applying NATD to an AdS/CFT pair, a new AdS background is generated which may have associated a different CFT dual, which, moreover, may only exist in the strong coupling regime

This will be the focus of this talk

Based on:
- Y.L., Carlos Núñez, Salomón Zacarías, 1703.00417
- Y.L., Carlos Núñez, 1603.04440

Outline:

1. Introduction and motivation: NATD in AdS/CFT

2. Basics of NATD: i) NATD vs Abelian T-duality
   ii) NATD as a solution generating technique

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   3.1. Short review about Gaiotto-Maldacena geometries

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2. Basics of NATD: i) NATD vs Abelian T-duality

Using the string sigma-model Rocek and Verlinde proved that Abelian T-duality is a symmetry to all orders in g_s and α'

(Buscher'88; Rocek, Verlinde'92)

The extension to arbitrary wordsheets determines the global properties of the dual variable:

$$\theta \in [0, 2\pi] \quad \xrightarrow{\text{T}} \quad \tilde{\theta} \in [0, 2\pi]$$

In the non-Abelian case neither proof works

Variables living in a group manifold are substituted by variables living in its Lie algebra

$$g \in SU(2) \quad \xrightarrow{\text{NAT}} \quad \chi \in \mathbb{R}^3$$

In the absence of global information the new variables remain non-compact

ii) NATD as a solution generating technique

Need to know how the RR fields transform

Sfetsos and Thompson (2010) extended Hassan's derivation in the Abelian case:
Implement the relative twist between left and right movers in the bispinor formed by the RR fields

3. The ST $AdS_5 \times S^2$ background

$$
\begin{tikzcd}
AdS_5 \times S^5 \arrow[r, leftrightarrow, "\text{NATD}"] \arrow[d, leftrightarrow] & \text{N=2} \quad AdS_5 \times S^2 \in \text{GM} \arrow[d, leftrightarrow] \\
\text{N=4 SYM} & \text{N=2 SCFT}
\end{tikzcd}
$$

(Sfetsos, Thompson'10)

- Gaiotto-Maldacena geometries encode the information about the dual CFT
- Useful example to study the CFT realization of NATD

- Take the $AdS_5 \times S^5$ background

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2 + \cos^2\alpha ds^2(S^3)\Big)$$

$$F_5 = 8L^4 \sin\alpha \cos^3\alpha d\alpha \wedge d\beta \wedge \mathrm{Vol}(S^3)$$ + Hodge dual

- Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2\Big) + \frac{d\rho^2}{L^2\cos^2\alpha} + \frac{L^2\cos^2\alpha\rho^2}{\rho^2 + L^4\cos^4\alpha} ds^2(S^2)$$

$$B_2 = \frac{\rho^3}{\rho^2 + L^4\cos^4\alpha}\mathrm{Vol}(S^2)\,, \qquad e^{-2\phi} = L^2\cos^2\alpha(L^4\cos^4\alpha + \rho^2)$$

$$F_2 = L^4 \sin\alpha\cos^3\alpha d\alpha \wedge d\beta, \qquad F_4 = B_2 \wedge F_2$$

- New Gaiotto-Maldacena geometry
- What about ρ ?
  - Background perfectly smooth for all $\rho \in \mathbb{R}^+$
  - No global properties inferred from the NATD
  - How do we interpret the running of ρ to infinity in the CFT?
- Singular at $\alpha = \pi/2$ where the original S^3 shrinks (due to the presence of NS5-branes)

This is the tip of a cone with S^2 boundary $\rightarrow$

Large gauge transformations $B_2 \rightarrow B_2 - n\pi \mathrm{Vol}(S^2)$
for $\rho \in [(n-1)\pi, n\pi]$

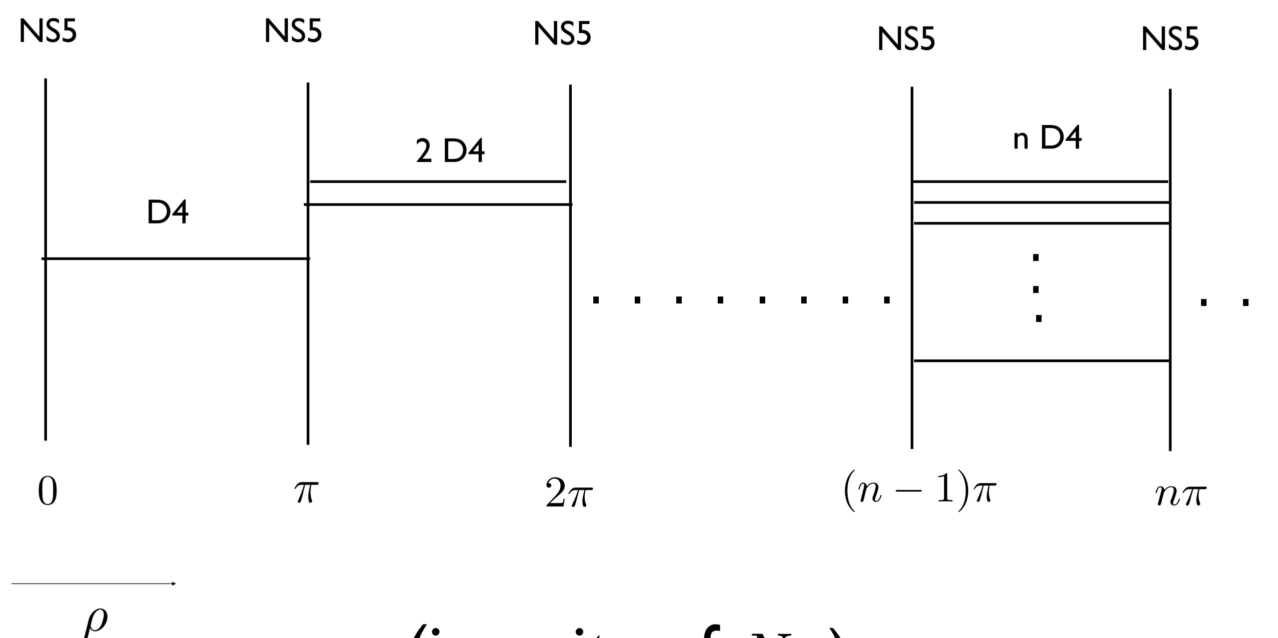
This modifies the Page charges such that $N_4 = nN_6$ in each $[(n-1)\pi, n\pi]$ interval

We have also N_5 charge, such that every time we cross a π interval one unit of NS5 charge is created

This is compatible with a D4/NS5 brane set-up:

D4: $\mathbb{R}^{1,3}, \rho$

NS5: $\mathbb{R}^{1,3}, \alpha, \beta$



(in units of N_6)

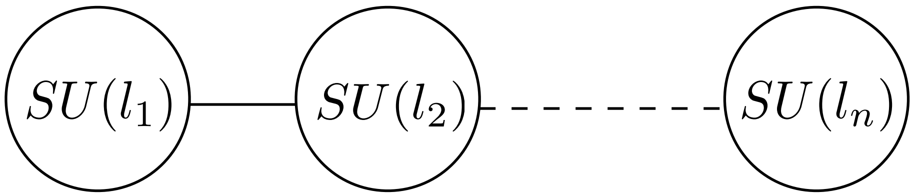
These D4/NS5 brane set-ups realize 4d $\mathcal{N}=2$ field theories with gauge groups connected by bifundamentals (Witten'97)

Having the D4 finite extension in the ρ direction, the field theory living in them is 4d at low energies, with effective gauge coupling:

$$\frac{1}{g_4^2} \sim \rho_{n+1} - \rho_n$$

For l_n D4-branes in $[\rho_n, \rho_{n+1}]$ the gauge group is $SU(l_n)$ and there are (l_{n-1}, l_n) and (l_n, l_{n+1}) hypermultiplets.

The field theory is then described by a quiver



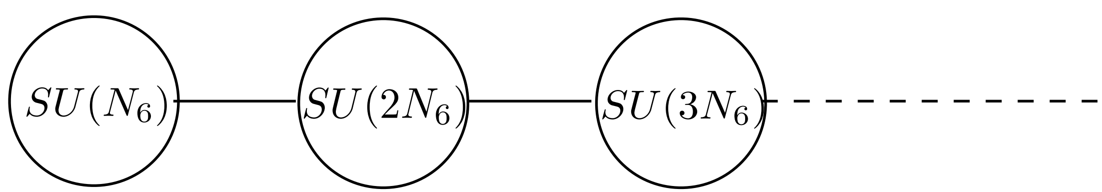
The bifundamentals contribute to the $SU(l_n)$ beta function as $l_{n-1}+l_{n+1}$ flavors.

The beta function thus vanishes at each interval if

$$2l_n = l_{n+1} + l_{n-1}$$

This condition is satisfied by our brane configuration, which has $l_n = nN_6$

It corresponds to an infinite linear quiver:



This is in agreement with Gaiotto-Maldacena

3.1 Short review of GM geometries

Generic backgrounds dual to 4d N=2 SCFTs.

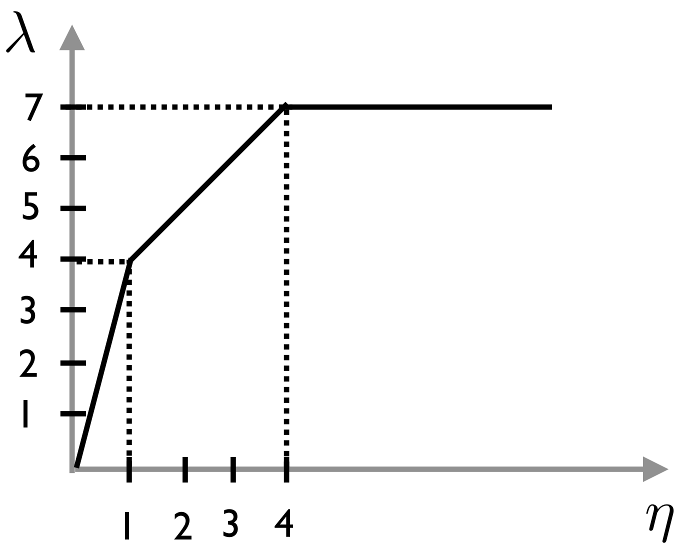
Described in terms of a function $V(\sigma,\eta)$ solving a Laplace eq. with a given charge density $\lambda(\eta)$ at $\sigma=0$

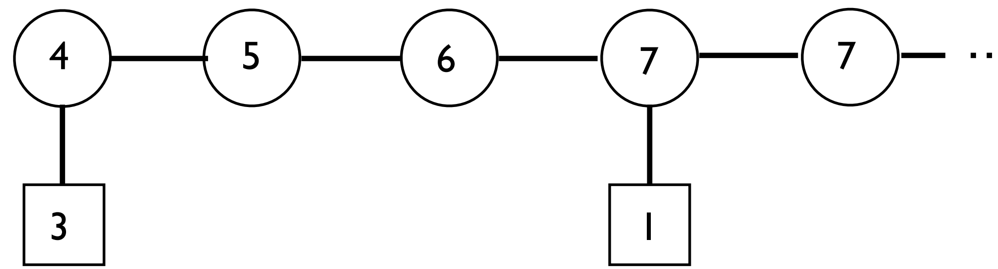
$$\partial_\sigma[\sigma\partial_\sigma V]+\sigma\partial_\eta^2 V=0, \qquad \lambda(\eta)=\sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0}$$

Regularity and quantization of charges impose strong constraints on the allowed form of $\lambda(\eta)$, which encodes the information of the dual CFT:

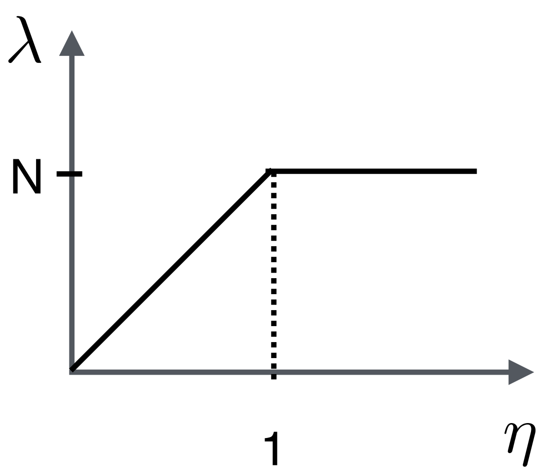
- A $SU(n_i)$ gauge group is associated to each integer value of $\eta=\eta_i$, with n_i given by $\lambda(\eta_i)=n_i$
- A kink in the line profile corresponds to extra k_i fundamentals attached to the gauge group at the node n_i

For example:

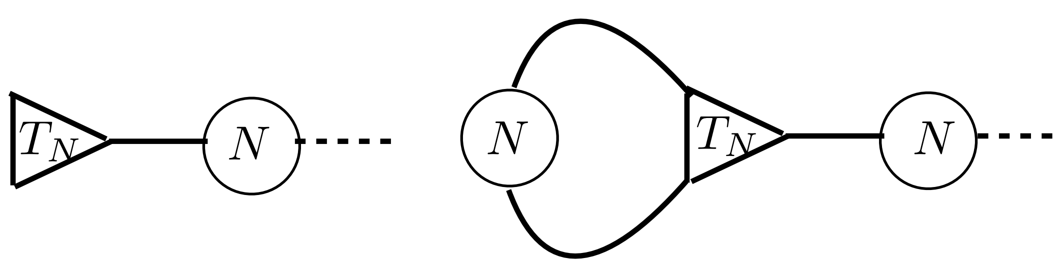




The Maldacena-Nunez solution:



$$\lambda_{MN}(\eta) = \frac{N}{2}(|\eta+1| - |\eta-1|)$$



Interesting for our work:

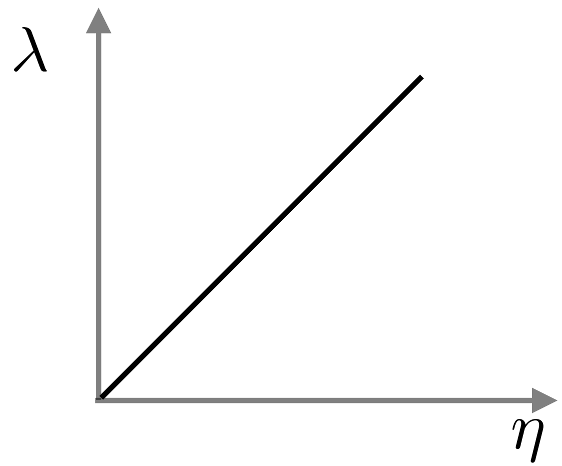
Following Reid-Edwards and Stefanski'10 (see also Aharony, Berdichevsky, Berkooz'12) , the MN solution can be taken as a building block for N=2 IIA solutions: Any allowed profile of the line charge density can be viewed as a sum of suitably re-scaled and shifted λ_{MN} profiles

We can use this to *complete* the NATD solution

3.2. The NATD as a GM geometry

GM geometry with $\lambda(\eta) = \eta\,,\ \eta \sim \rho\,,\ \sigma = \sin\alpha$

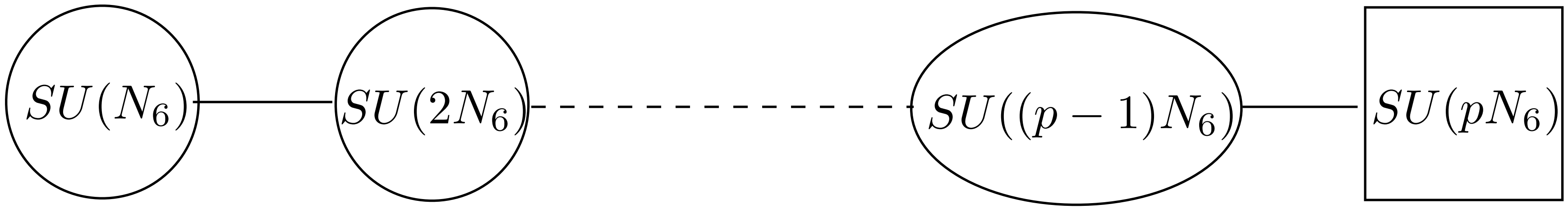
$\lambda(\eta) = \eta \Rightarrow$ Infinite linear quiver, consistent with the brane set-up:



Next, we will complete the quiver and, using holography, *complete* the geometry (both for large ρ and at the singularity)

Example in which the field theory *informs* the geometry

A natural way to complete the quiver is by adding fundamentals:



This completion reproduces correctly the value of the holographic central charge:

From the geometry:

$$c_{NATD} \sim V_{int} \sim \int_0^{\eta_*} f(\eta) d\eta = \frac{N_6^2 N_5^3}{12}$$

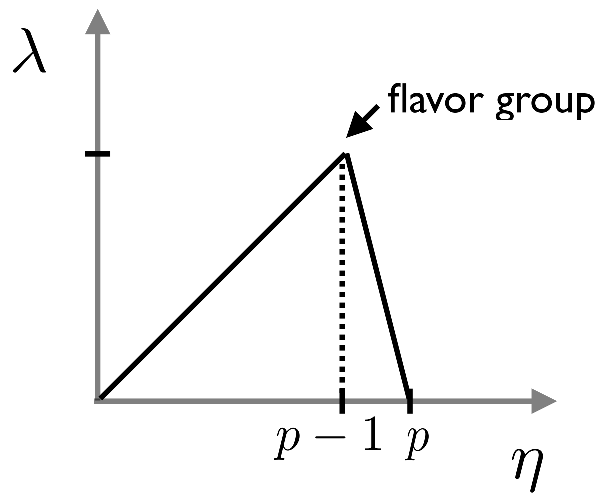
(Klebanov, Kutasov, Murugan'08)

In the field theory we can use: $c = \frac{1}{12}(2n_v + n_h)$ (Shapere, Tachikawa'08)

This gives

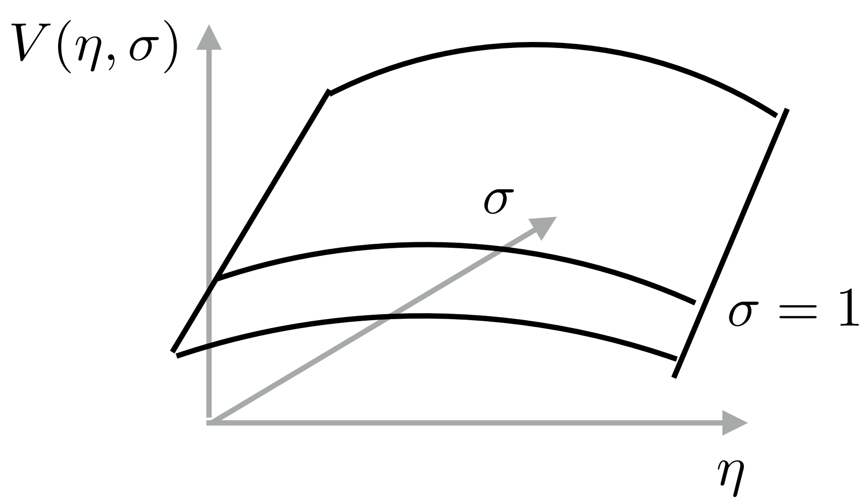
$$c = \frac{N_6^2 p^3}{12}\left[1 - \frac{1}{p} - \frac{2}{p^2 N_6^2} + \frac{2}{N_6^2 p^3}\right] \approx \frac{N_6^2 p^3}{12}$$

In the geometry, the completed quiver corresponds to



$$\frac{\lambda(\eta)}{N_6} = \begin{cases} \eta & 0 \leq \eta \leq p-1 \\ (1-p)\eta + (p^2-p) & (p-1) \leq \eta \leq p \end{cases}$$

This charge density can be obtained as a superposition of MN solutions:



This superposition completes the NATD solution, and removes the singularity

The singularity can be interpreted as a result of cutting the space at $\sigma = 1$

Can we find other examples where the NATD solution belongs to a classification with known field theory dual, to check these ideas?

We showed in Y.L., Macpherson, Montero, Núñez, 1609.09061 that the same idea works in a certain N=4 AdS4 NATD solution

What happens if we dualize on the AdS subspace?

4. The $R_t \times S^2 \times S^5$ example

$$\begin{tikzcd} AdS_5 \times S^5 \arrow[r, leftrightarrow, "\text{NATD}"] \arrow[d, leftrightarrow] & \mathbb{R}_t \times S^2 \times S^5 \text{ geometry in LM} \arrow[d, leftrightarrow] \\ \text{N=4 SYM} & \text{BMN vacuum} \end{tikzcd}$$

- Lin-Maldacena geometries encode the information about the dual CFT
- (Another) useful example to study the CFT realization of NATD

- Take the $AdS_5 \times S^5$ background

$$ds^2 = L^2\Big(-\cosh^2 r dt^2 + dr^2 + \sinh^2 r ds^2(S^3) + ds^2(S^5)\Big)$$

- Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = L^2\Big(-\cosh^2 r dt^2 + dr^2 + \frac{d\rho^2}{L^4 \sinh^2 r} + \frac{\rho^2 \sinh^2 r}{\rho^2 + L^4 \sinh^4 r} ds^2(S^2) + ds^2(S^5)\Big)$$

$$B_2 = \frac{\rho^3}{\rho^2 + L^4 \sinh^4 r}\mathrm{Vol}(S^2)\,, \qquad e^{-2\phi} = L^2 \sinh^2 r(L^4 \sinh^4 r + \rho^2)$$

$$F_2 = L^4 \sinh^3 r \cosh r dt \wedge dr\,, \qquad F_4 = B_2 \wedge F_2$$

- New LLM geometry with $SU(2|4)$ supergroup
- What about ρ ?
- Singular at $r=0$ where the original S^3 shrinks (due to the presence of NS5-branes)

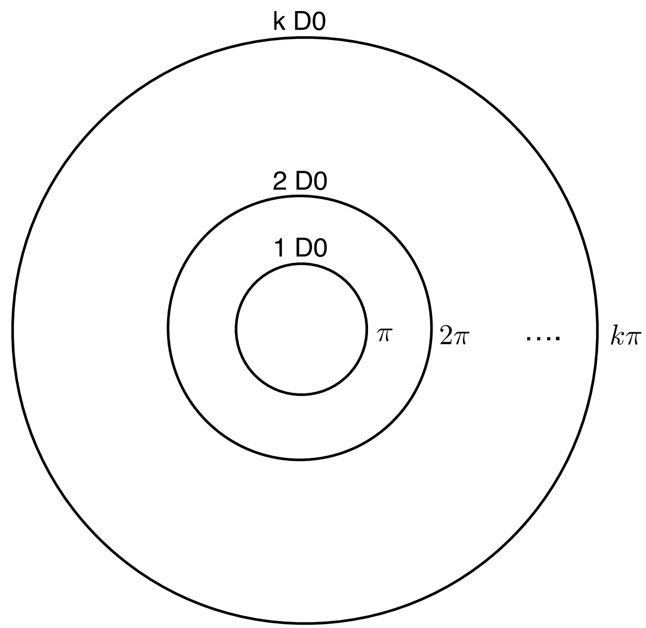
This is the tip of a cone with S^2 boundary $\rightarrow$

Large gauge transformations $B_2 \rightarrow B_2 - n\pi \mathrm{Vol}(S^2)$ for $\rho \in [(n-1)\pi, n\pi]$

This modifies the Page charges such that $N_0 = nN_2$ in each $[(n-1)\pi, n\pi]$ interval.

Total number of D0-branes: $N = \sum_{n=1}^{\infty} nN_2(n)\,, \quad N_2(n) \sim n$

This is consistent with a brane set-up of concentric spherical D2-branes with n D0-charge and radius $n\pi$



The partition of the total number of D0-branes is exactly of the same form of the partitions that define BMN vacua

This is in agreement with Lin-Maldacena

4.1. Short review of Lin-Maldacena geometries

All gravity solutions with $SU(2|4)$ supergroup were classified by LLM.

The bosonic symmetries, $R_t \times SO(3) \times SO(6)$, act geometrically $\to S^2, S^5$

In IIA they are described in terms of a function $V(\sigma, \eta)$ satisfying a Laplace equation $\partial_\sigma[\sigma \partial_\sigma V] + \sigma \partial_\eta^2 V = 0$

The difference with the GM geometries is in the boundary conditions. These were discussed by Lin and Maldacena

Consider in particular the LM geometry dual to a BMN vacuum:

The BMN Matrix Model:

U(N) QM model obtained by reducing N=4 SYM on $\mathbb{R} \times S^3$ on the S^3

It has $R_t \times SO(3) \times SO(6)$ global symmetries

Its vacua are in one to one correspondence with SU(2) reps. of dim N:

$$N = \sum_n n N(n)$$

Each choice of partition gives a different vacuum

Each vacuum is dual to a different LM geometry

(Lin, Maldacena'05)

The potential consists on a *background potential,* common to all vacua, plus a, vacuum specific, $\phi(\sigma, \eta)$ contribution

$\phi(\sigma, \eta)$ solves the Laplace eq. with specific boundary conds:

These consist on an infinite conducting plane at $\eta = 0$ plus a number of conducting disks at $\eta_i \sim N_5^i$ with charges $Q_i \sim N_2^i$ where N_5^i, N_2^i are the NS5 and D2 brane charges associated to the background fluxes

For a given electrostatic configuration N satisfies

$$N = \sum_i \Big(\sum_{j<i} N_5^j \Big) N_2^i$$

which is what we found for the NATD: $N = \sum_{n=1}^{\infty} n N_2(n)$

4.2 The NATD as a LM geometry

We seem to be describing the BMN vacuum with $N(n) \sim n$

But, why is the NATD solution singular?, and,

how can we fix the ever-growing dimension of the irreps?

Finding explicit solutions to the full electrostatic problem is very hard.

Instead of a discrete distribution of plates, one can consider a continuous distribution of point charges sitting at $\sigma = 0$

This gives an exact solution to the Laplace equation which however only satisfies the boundary conditions partially

$\rightarrow$ Singular solution (Bak, Siwach, Yee'05)

Shieh, van Anders, Van Raamsdonk'07; Donos, Simon'10: "Coarse-grained" LM geometry

The solution to such electrostatic problem is:

$$\phi(\sigma,\eta)=\int_0^\infty dz\frac{\lambda(z)}{\sqrt{\sigma^2+(z-\eta)^2}}$$

with $\lambda(z)\sim z$. This gives exactly the potential of the NATD solution. The NATD arises as the coarse-grained geometry associated to the $N(n)\sim n$ BMN vacuum

On the other hand, regularizing as (Donos, Simon'10)

$$\int_0^\infty dz\frac{z}{\sqrt{\sigma^2+(z-\eta)^2}}=\int_0^L dz\frac{z}{\sqrt{\sigma^2+(z-\eta)^2}}+L+\eta\log 2L$$

we obtain a well-behaved solution with the right D0-brane asymptotics, and a well-behaved dual CFT

As in other examples, the NATD solution arises as a result of zooming-in around the small η region

Interestingly, in this example, this zooming-in can be made more precise:

It can be seeing that the NATD solution arises as the result of taking the Penrose limit on the superstar solution in $AdS_7 \times S^4$, describing the back-reaction of giant gravitons in this geometry (Alishahiha, Yavartanoo'05)

This makes very precise the idea that the NATD solution focuses on a patch of a more generic manifold

5. Conclusions

We have focused on NATD on $AdS_5 \times S^5$:

- On S^5 : GM geometry dual to an infinite quiver, that we have completed, and thereof the geometry, resolving the singularity and defining the background globally

- On AdS_5 : Coarse-grained Lin-Maldacena geometry, dual to a BMN vacuum. Vacuum and geometry defined by cutting the space at a finite distance. Singularity resolved

Both examples show that NATD changes the dual CFT

In both cases, the NATD solution focuses on a patch of a globally well-defined manifold.

This is very concrete in the second example $\leftrightarrow$ Penrose limit

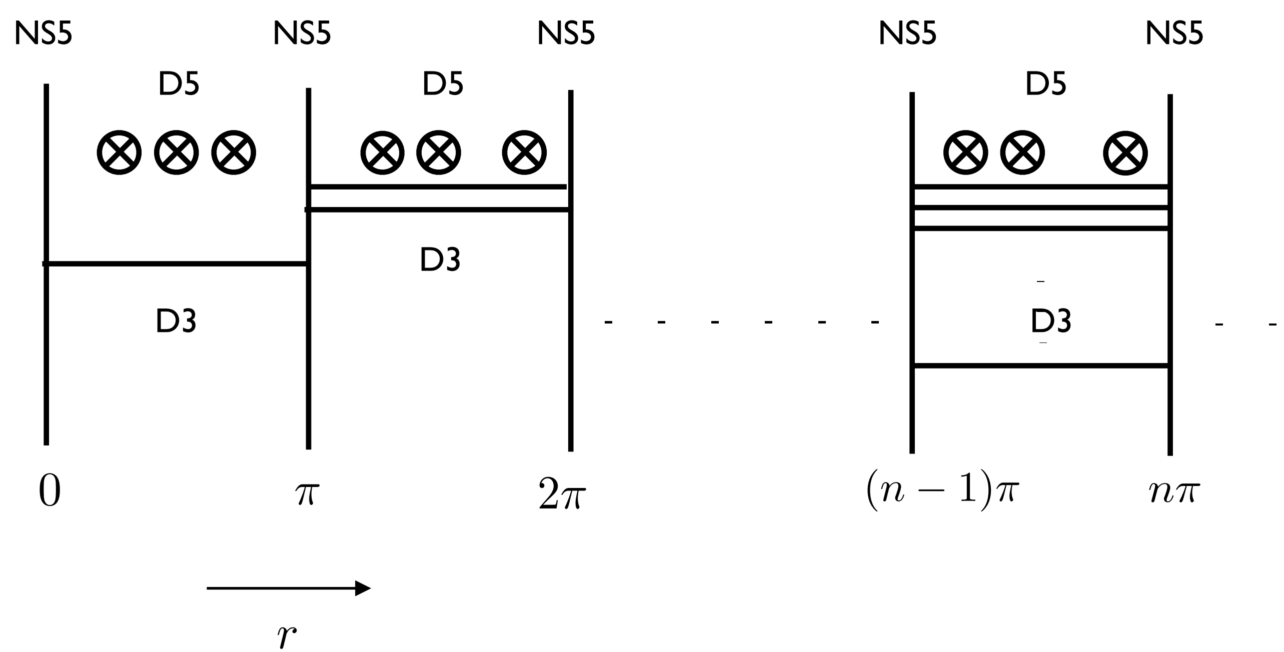
THANKS!

4. The $AdS_4 \times S^2 \times S^2$ example

Non-Abelian T-duality on a reduction to IIA of $AdS_4 \times S^7/\mathbb{Z}_k$

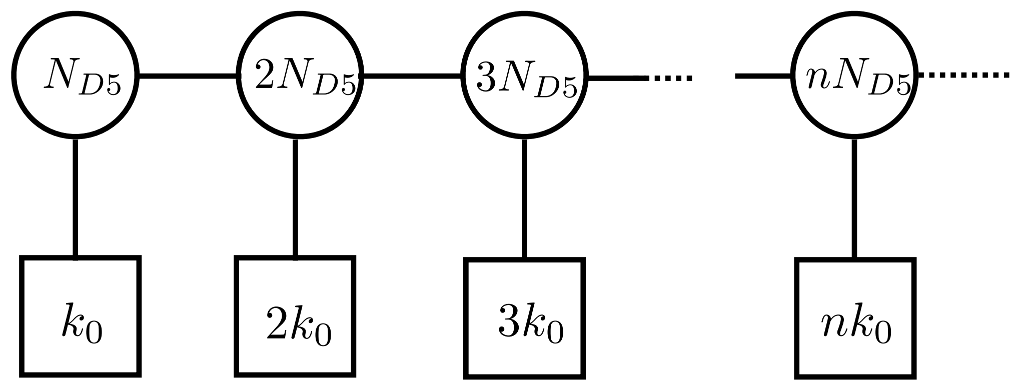
$\rightarrow$ IIB $AdS_4 \times S^2 \times S^2$ background, N=4 SUSY, in the classification of D'Hoker, Estes and Gutperle'07

Analysis of charges: (D3,NS5,D5) brane set-up:



Gaiotto and Witten'08: 3d N=4 $T^{\hat{\rho}}_{\rho}(N)$ theories

$T_\rho^{\hat\rho}(N)$ field theories flow to CFTs in the infrared if the partitions satisfy certain conditions, that are satisfied by our brane set-up



The holographic duals of these CFTs are known (Assel, Bachas, Estes and Gomis'11)

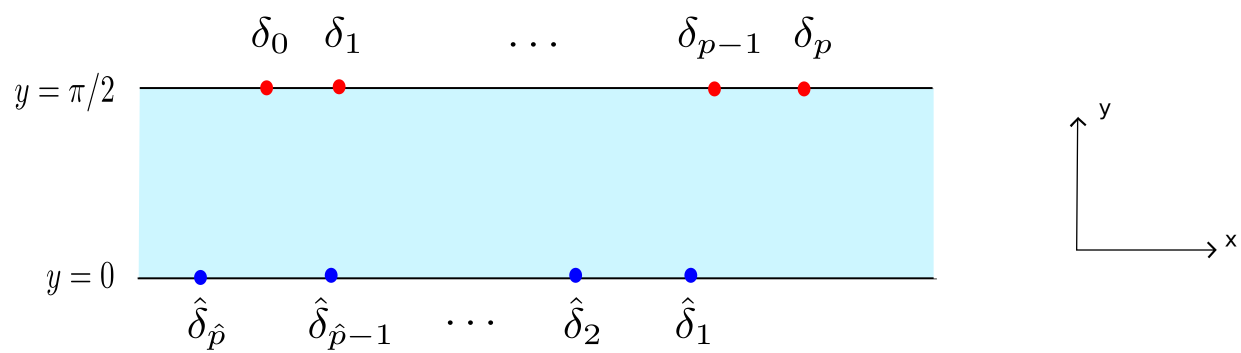
They belong to the general class of $AdS_4 \times S^2 \times S^2$ geometries in D'Hoker, Estes and Gutperle'07

These are fibrations of $AdS_4 \times S^2 \times S^2$ over a Riemann surface that can be completely determined from two harmonic functions $h_1(z,\bar z), h_2(z,\bar z)$

Assel, Bachas, Estes and Gomis'11 showed how to determine these functions from the (D3, NS5, D5) brane set-ups associated to $T_\rho^{\hat\rho}(N)$ theories:

$$h_1 = -\frac{1}{4}\sum_{a=1}^{p} N_5^a \log\tanh\left(\frac{i\frac{\pi}{2}+\delta_a - z}{2}\right) + cc$$

$$h_2 = -\frac{1}{4}\sum_{b=1}^{\hat p} \hat N_5^b \log\tanh\left(\frac{z-\hat\delta_b}{2}\right) + cc$$



The positions of the D5 and NS5 branes are determined, in turn, from the linking numbers of the configuration:

$$\hat\delta_b - \delta_a = \log\tan\left(\frac{\pi}{2}\frac{l_a\hat l_b}{N}\right)$$

The h_1, h_2 functions computed from our *completed* brane set-up agree with those associated to the non-Abelian T-dual geometry in the region $x, y \sim 0$, far from the location of the branes

The non-Abelian T-dual arises as a result of zooming-in in a particular region of the *completed* solution

This completion smoothes out the singularities and defines the geometry globally

The free energy of the completed solution satisfies the bound $N^2 \log N$ found by Assel, Estes, Yamazaki'12

5. Conclusions

- NATD geometries dual to infinite linear quivers

  $\rightarrow$ Different CFTs after NATD

  D3 $\rightarrow$ (D4,NS5) (D2,D6) $\rightarrow$ (D3,NS5,D5)

- Quivers completed, and thereof the geometries, to define the CFTs

- NATD as a zooming-in in a patch of the completed geometry

  Penrose limit of superstar solution (in progress)

- General pattern?

  $AdS_6 \times S^4$: (D4,D8) system $\rightarrow$ (D5,NS5,D7)

  $AdS_6 \times S^2$ IIB solutions recently classified by D'Hoker, Gutperle, Karch and Uhlemann'16