Conformal symmetry and Supergravity

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mostly based on a paper in preparation with
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Conformal symmetry

- largest extension of spacetime group.
- needs massless fields and no dimensionful parameters
- useful for describing actions in a more symmetrical way (certainly for supergravity theories)
- Actions looks more beautiful, and exhibit their structure in a conformal setting
- Aim: describe supergravity field equations and currents
- + their properties for coupling conformal matter : rigid conformal symmetry coupled to Poincaré supergravity
- Non-linear form of Ferrara-Zumino equations (1977).
- Extension of classical results on improved currents (Callan-Coleman-Jackiw, 1969)

Plan

1. Conformal symmetry and currents (Callan-Coleman-Jackiw, 1969)
2. Superconformal algebra and multiplets

   Weyl multiplet ;

   matter multiplets;

   action for pure supergravity and for matter couplings
3. Supercurrents

   pure supergravity;

   supercurrent multiplet;

   bosonic part
4. Conclusion

conformal actions

- Rigid scale invariance : Weyl weight of fields under dilations add up to weight of Lagrangian, which should be 4 (compensating d^4x). (derivatives have weight 1)
- Conformal symmetry:
  includes also 'special conformal transformations'
  is not obvious for scale-invariant actions.
  E.g. for scalars the dilatation vector $\delta\phi^i = k_D^i \lambda_D = w\phi^i \lambda_D$
  should be a 'closed homothetic Killing vector'. $\nabla_i k_{\rm D}^j = w\delta_i^j$
- This is satisfied for a Kähler manifold if
  metric homogeneous of degree 0 in z and $\bar{z}$:
  = Kähler potential homogeneous of degree 1 in z and $\bar{z}$.
- In that case: there is also a U(1) Killing vector (indicated by T)
  $k_{\rm T}^i = k_{\rm D}^j J_j{}^i$ or with $\{\phi^i\} = \{z^\alpha, \bar{z}^{\bar{\alpha}}\}$: $k_T^\alpha = iz^\alpha;\ k_T^{\bar{\alpha}} = -i\bar{z}^{\bar{\alpha}}$.
  In conformal supersymmetry: this is the R-symmetry.

Currents

- In general: for every rigid symmetry A, there is a conserved current : $\partial_\mu J^\mu{}_A \approx 0$

$\approx$: upon use of eqns of motion (eom)

- For translations $\epsilon^A \leftarrow \xi^\mu$: $J^\mu{}_A \leftarrow T^\mu{}_\nu$: $\partial_\mu T^\mu{}_\nu \approx 0$

  Due to Lorentz rotation invariance, one can define also a symmetric energy-momentum tensor: $T^{\mu\nu} = T^{\nu\mu}$

- When there is conformal symmetry, Callan-Coleman – Jackiw proved that there is a traceless 'improved'current

  $\Theta^{\mu\nu} = T^{\mu\nu} +$ terms determined by the special conformal transformations

Simplest example:

$$
\begin{aligned}
\mathcal{L} &= -\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{4}\lambda\phi^4 \\
T_{\mu\nu} &= \partial_\mu\phi\partial_\nu\phi + \eta_{\mu\nu}\mathcal{L}\,, \\
\Theta^{\mu\nu} &= T^{\mu\nu} + \tfrac{1}{6}\left(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu\right)\phi^2
\end{aligned}
$$

$$\partial_\mu\Theta^{\mu\nu} \approx 0\,, \qquad \Theta^\mu{}_\mu \approx 0$$

C.G. Callan, S. Coleman and R. Jackiw, 1969

Currents from gauge coupling

- First order local action, e.g.

$$S = \int \mathrm{d}^4x \left[-\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\mathrm{rigid}} + A^\mu J_\mu + \dots \right]$$

$$\frac{\delta S}{\delta A^\mu} = \frac{1}{g^2}\partial^\nu F_{\nu\mu} + J_\mu \approx 0 \qquad \partial^\mu\partial^\nu F_{\nu\mu} = 0 \;\rightarrow\; \partial^\mu J_\mu \approx 0$$

- Gravity: pure grav. $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \approx 0$

matter coupled

$$S = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{g}\left[\kappa^{-2}R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \lambda\phi^4\right]$$

$$\frac{2}{\sqrt{g}}\frac{\delta S}{\delta g^{\mu\nu}} = \kappa^{-2}G_{\mu\nu} - T_{\mu\nu} \approx 0 \qquad \nabla^\mu G_{\mu\nu} = 0 \;\rightarrow\; \nabla^\mu T_{\mu\nu} \approx 0$$

Two remarks:

1. Conservation of current follows from properties of pure gauge part
2. We coupled conformal matter to gravity, but did not obtain the improved current

Conformal currents from gauge coupling

- Couple matter to gravity in a conformal way

$$S = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{g}\,\Big[\underbrace{\kappa^{-2}R}_{\text{Not conformal gravity}} - \underbrace{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \tfrac{1}{6}\phi^2 R - \lambda\phi^4}_{\text{Local Conformal matter}}\Big]$$

$$\frac{2}{\sqrt{g}}\frac{\delta S}{\delta g^{\mu\nu}} = \left(\kappa^{-2} - \tfrac{1}{6}\phi^2\right) G_{\mu\nu} - \Theta_{\mu\nu} \approx 0$$

$$\kappa^{-2}G_{\mu\nu} \approx \Theta^c_{\mu\nu}\,, \qquad \Theta^c_{\mu\nu} = \Theta_{\mu\nu} + \tfrac{1}{6}\phi^2 G_{\mu\nu}$$

with new ϕ eom

$$\nabla^\mu\Theta^c_{\mu\nu} \approx 0\,, \qquad \Theta^c{}_\mu{}^\mu \approx 0\,, \qquad G_\mu{}^\mu = -R \approx 0$$

- Remember energy-momentum tensors

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi + \eta_{\mu\nu}\mathcal{L}$$ rigid Poincaré or Poincaré gravity

$$\Theta^{\mu\nu} = T^{\mu\nu} + \tfrac{1}{6}\left(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu\right)\phi^2$$ rigid conformal

$$\Theta^c_{\mu\nu} = \Theta_{\mu\nu} + \tfrac{1}{6}\phi^2 G_{\mu\nu}$$ local conformal

Contains gravity part

Matter coupled-gravity from conformal

- Add a ‘compensating field’ ϕ_0
- ϕ_0 and ϕ have Weyl weight $w = 1$

$$S = \frac{1}{2}\int \mathrm{d}^4\sqrt{g}\left[-\phi_0\Box^C\phi_0 + \phi\Box^C\phi + \lambda\phi^4\right]$$
$$= \frac{1}{2}\int \mathrm{d}^4\sqrt{g}\left[\partial_\mu\phi_0\partial^\mu\phi_0 - \partial_\mu\phi\partial^\mu\phi + \frac{1}{6}(\phi_0^2 - \phi^2)R + \lambda\phi^4\right]$$

- gauge-fix the conformal symmetry

1. $\phi_0 = \kappa^{-1}\sqrt{6}$

$$S = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{g}\left[\kappa^{-2}R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{6}\phi^2 R - \lambda\phi^4\right]$$
$$\kappa^{-2}G_{\mu\nu} \approx \Theta^c_{\mu\nu}\,, \qquad \Theta^c{}_\mu{}^\mu \approx 0$$

2. Einstein frame : $\phi_0^2 = \phi^2 + 6\kappa^{-2}$

$$S = \frac{1}{2}\int \mathrm{d}^4\sqrt{g}\left[\kappa^{-2}R - \frac{\partial_\mu\phi\partial^\mu\phi}{1 + \frac{1}{6}\kappa^2\phi^2} + \lambda\phi^4\right]$$

$$\kappa^{-2}G_{\mu\nu} \approx T_{\mu\nu}$$

$T_{\mu\nu}$ not traceless

2. Superconformal algebra and multiplets

- In general $\begin{pmatrix} \text{conformal algebra} & Q, S \\ Q, S & R\text{-symmetry} \end{pmatrix}$

- according to dilatational weight:

$$\begin{array}{rcl} 1 & : & P_\mu \\ \frac{1}{2} & : & Q \\ 0 & : & D,\ M_{ab},\ \mathrm{U}(1) \\ -\frac{1}{2} & : & S \\ -1 & : & K_\mu \end{array}$$

- Gauged by 'Weyl multiplet'

$$\boxed{e_\mu{}^a, b_\mu, A_\mu, \psi_\mu}$$

Gauge fields of P^a, D, $U(1)$, Q

Other gauge fields are composites as e.g. $\omega_\mu{}^{ab}\,(e, b, \psi)$

Superconformal matter multiplets

- use 'superfields' of rigid susy, but assign (Weyl,Chiral) weights to all fields:

$$\delta\Phi = \left(w\lambda_D + i\, c\, \lambda_{U(1)}\right)\Phi$$

- There are restrictions.
  - E.g. "real superfield": must be c=0
- Multiplet identified by its 'first' component
  - is superconformal primary:
    in particular: first component invariant under S-susy
    leads to conditions :
    E.g. Chiral superfield: must have $w = c$.
  - transformations of other fields determined by the algebra

Example chiral multiplet

$$\delta Z = [w\lambda_{\rm D} + w\,{\rm i}\,\lambda_T]\,Z + \frac{1}{\sqrt{2}}\bar\epsilon P_L\chi\,,$$

$$\delta P_L\chi = \left[\left(w+\tfrac{1}{2}\right)\lambda_{\rm D} + \left(w-\tfrac{3}{2}\right){\rm i}\lambda_T\right]P_L\chi + \frac{1}{\sqrt{2}}P_L\left(\not{D}Z + F\right)\epsilon + \sqrt{2}wZP_L\eta$$

$$\delta F = [(w+1)\,\lambda_{\rm D} + (w-3)\,{\rm i}\lambda_T]\,F + \frac{1}{\sqrt{2}}\bar\epsilon\,P_R\,\not{D}\chi + \sqrt{2}(1-w)\bar\eta P_L\chi$$

If $w = 3$, the action formula is possible: analogue as integrating over chiral superspace

$$[Z]_F = \int {\rm d}^4x\, e\left[F + \frac{1}{\sqrt{2}}\bar\psi_\mu\gamma^\mu P_L\chi + \frac{1}{2}Z\bar\psi_\mu\gamma^{\mu\nu}P_R\psi_\nu\right] + {\rm h.c.}$$

Analogue of covariant derivative D_α : $\mathcal{D}_\alpha Z = \frac{1}{\sqrt{2}}P_L\chi$ if $w = 0$

Analogue of covariant $\overline{D}^2 = \overline{D}^{\dot\alpha}\overline{D}_{\dot\alpha}$ $T\,Z^* = F^*$ if $w = 1$

Action formula real multiplet: $[N]_D = \frac{1}{2}[\,T(N)]_F$ similar to integration over full superspace

Using these few rules (restrictions), one can take over rigid superspace results and upgrade the theory to supergravity

Actions for pure supergravity

‘Weyl multiplet’: $\{e_\mu{}^a, \cancel{b_\mu}, A_\mu, \psi_\mu\}$

We use a ‘compensating’ chiral multiplet $\{\cancel{X^0}, \cancel{P_L\Omega^0}, F^0\}$ of Weyl weight 1.

$$[-X^0\bar{X}^0]_D = \int \mathrm{d}^4x e \Big[g^{\mu\nu}\partial_\nu X^0\partial_\mu\bar{X}^0 + \tfrac{1}{2}\bar{\Omega}^0\not{D}\Omega^0 + F^0\bar{F}^0 + X^0\bar{X}^0\left(\tfrac{1}{6}R - \tfrac{1}{6}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho + A^aA_a\right) + \ldots\Big]$$

K-gauge: $b_\mu = 0|_\boxdot$, D and $U(1)$-gauge: $X^0\big|_\boxdot = \frac{\sqrt{3}}{\kappa}$, S-gauge: $\Omega^0\big|_\boxdot = 0$

$$[-X^0\bar{X}^0]_D\big|_\boxdot = \int \mathrm{d}^4x\, e\left[\frac{1}{2\kappa^2}\left(R - \bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho + 6A^aA_a\right) + F^0\bar{F}^0\right]$$

super-Poincaré action

Sugra matter couplings and conformal case

should be (2,0) should be (3,3)

$$S = \left[N(X^I, \bar{X}^{\bar{I}})\right]_D + \left[\mathcal{W}(X^I)\right]_F$$
$$= \left[3X^0\bar{X}^{\bar{0}}(-1 + \Phi_{\mathrm{M}}(S^i, \bar{S}^{\bar{\imath}})\right]_D + \left[(X^0)^3 W(S^i)\right]_F$$

We split $\{X^I\} = \left\{X^0, X^i = X^0 S^i\right\}$

- If S^i have conformal couplings
  $=\ \Phi_M$ homog. order 1 in S^i and in $\bar{S}^i$; and W order 3.
  'conformal case': is local coupling of rigid superconformal
- Difference from conformal case:

$$\Delta K \equiv S^i \Phi_{\mathrm{M}i} - \Phi_{\mathrm{M}}\,, \qquad \Delta W \equiv W - \frac{1}{3} S^i W_i$$

3. Field equations and improved supercurrents

- Pure supergravity → super-Bianchi identity
- Matter currents
- Reduce to bosonic case to see the difference

Field equations of the pure supergravity multiplet

$$S_{\rm pureSG} = \left[-X^0\bar X^0\right]_D = \left[X^0 T(\bar X^0)\right]_F$$

Weyl multiplet hidden in notation (like covariance)

- Field equation for compensating multiplet

$$\mathcal{R} = T(\bar X^0)/X^0 \approx 0 : \qquad T(\bar X^0) = \{\bar F^0,\ \not{\mathcal{D}} P_R\Omega^0,\ \Box^C\bar X^0\}$$

Chiral multiplet of (w,c)=(1,1)

Contains $R\bar X^0$

Contains γ −trace of gravitino field strength

- Field equation of Weyl multiplet

$$\begin{aligned}\mathcal{E}_a &\equiv 4e^{-1}e_a^\mu \frac{\delta[-X^0\bar X^0]_D}{\delta A^\mu} \\ &= 4\mathrm{i}X^0\mathcal{D}_a\bar X^{\bar 0} - 4\mathrm{i}\bar X^{\bar 0}\mathcal{D}_a X^0 + 2\mathrm{i}\overline{\Omega}^0 P_L\gamma_a\Omega^{\bar 0} \\ \mathcal{E}_a|_{\boxdot} &= -8\kappa^{-2}A_a\end{aligned}$$

Ferrara-Zumino, 1977 gave linearized conservation equations

$$\bar D^{\dot\alpha}E_{\alpha\dot\alpha} = D_\alpha\mathcal{R}\,, \qquad E_{\alpha\dot\alpha} = \tfrac14(\gamma^a)_{\alpha\dot\alpha}E_a$$

Consistent equation with (Weyl,chiral) weights

$$\overline{\mathcal{D}}^{\dot\alpha}\mathcal{E}_{\alpha\dot\alpha} = (X^0)^3\mathcal{D}_\alpha\left(\frac{\mathcal{R}}{X^0}\right)$$

Generalized Bianchi identity. We verified this !

Matter coupled field equations

- Field equation for compensating multiplet

$$\mathcal{R} \approx 2X^0 \Delta W - T\left(\frac{\bar{X}^{\bar{0}} \Delta K}{X^0}\right)$$

conformal case: $\mathcal{R} \approx 0$

- Field equation of Weyl multiplet

$$\mathcal{C}_a = e^{-1} e_a^\mu \frac{\delta S}{\delta A^\mu} = -\tfrac{3}{4}\left(\mathcal{E}_a + J_a\right) \approx 0$$

$$J_a \equiv -4 e^{-1} e_a^\mu \frac{\delta\left[X^0 \bar{X}^0 \Phi_{\mathrm{M}}(S, \bar{S})\right]_D}{\delta A^\mu}$$

Bianchi: $\overline{\mathcal{D}}^{\dot{\alpha}} \mathcal{E}_{\alpha\dot{\alpha}} = (X^0)^3 \mathcal{D}_\alpha \left(\frac{\mathcal{R}}{X^0}\right)$

$\rightarrow$ conservation equation

$$\begin{aligned} \overline{\mathcal{D}}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} &\approx -(X^0)^3 \mathcal{D}_\alpha \left(\frac{\mathcal{R}}{X^0}\right) \\ &\approx -(X^0)^3 \mathcal{D}_\alpha \left(2\Delta W - (X^0)^{-2} T\left(\bar{X}^0 \Delta K\right)\right) \end{aligned}$$

conformal case: $\overline{\mathcal{D}}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} \approx 0$

How does this modify the Callen-Coleman-Jackiw improved currents?

Supercurrent multiplet

$$\begin{aligned} J_a &= -\Phi_{\mathrm{M}}\mathcal{E}_a \\ &\quad + \left[2\mathrm{i}X^0\Phi_{\mathrm{M}\,i}\bar{\chi}^i\gamma_a\Omega^0 + \text{h.c.}\right] \\ &\quad + 2\mathrm{i}X^0\bar{X}^{\bar{0}}\left[2(\Phi_{\mathrm{M}\,i}\mathcal{D}_aS^i - \Phi_{\mathrm{M}\,\bar{\imath}}\mathcal{D}_a\bar{S}^{\bar{\imath}}) - \Phi_{\mathrm{M}\,i\bar{\jmath}}\bar{\chi}^i\gamma_a\chi^{\bar{\jmath}}\right] \end{aligned}$$

$$\mathcal{E}_a = 4\mathrm{i}X^0\mathcal{D}_a\bar{X}^{\bar{0}} - 4\mathrm{i}\bar{X}^{\bar{0}}\mathcal{D}_aX^0 + 2\mathrm{i}\overline{\Omega}^0P_L\gamma_a\Omega^{\bar{0}}$$

Poincaré gauge fixing ?

$$\text{Einstein frame}\quad X^0\bar{X}^{\bar{0}}(1-\Phi_{\mathrm{M}})\Big|_{\boxdot} = \kappa^{-2}$$
$$0 = 3\bar{X}^{\bar{0}}\left[(-1+\Phi_{\mathrm{M}})\Omega^0 + \Phi_{\mathrm{M}\,i}\chi^i\right]\Big|_{\boxdot}$$

X^0 and Ω^0 functions of matter, leads to Kähler manifold .

$$\mathcal{K} = -3\log(1-\kappa^2\Phi_{\mathrm{M}}))$$

No ‘conformal matter’ part recognizable.

Einstein tensor and currents mixed.

Supercurrent multiplet in conformal case

$$J_a = -\Phi_{\rm M}\mathcal{E}_a + \cancel{\left[2{\rm i}X^0\Phi_{{\rm M}\,i}\bar\chi^i\gamma_a\Omega^0 + {\rm h.c.}\right]} + 2{\rm i}X^0\bar X^{\bar 0}\left[2(\Phi_{{\rm M}\,i}\mathcal{D}_a S^i - \Phi_{{\rm M}\,\bar\imath}\mathcal{D}_a\bar S^{\bar\imath}) - \Phi_{{\rm M}\,i\bar\jmath}\bar\chi^i\gamma_a\chi^{\bar\jmath}\right]$$

$$\mathcal{E}_a = 4{\rm i}X^0\mathcal{D}_a\bar X^{\bar 0} - 4{\rm i}\bar X^{\bar 0}\mathcal{D}_a X^0 + 2{\rm i}\overline{\Omega}^0 P_L\gamma_a\Omega^{\bar 0}$$

Poincaré gauge fixing ?

$$\text{Conformal frame :}\quad X^0\big|_{\boxdot} = \kappa^{-1}\,,\qquad \Omega^0\big|_{\boxdot} = 0$$

$$\mathcal{E}_a|_{\boxdot} = -8\kappa^{-2}A_a = \kappa^{-2}E_a\,,$$

$$J_a|_{\boxdot} = 8\kappa^{-2}\Phi_M A_a + 2{\rm i}\kappa^{-2}\left[2(\Phi_{{\rm M}\,i}\mathcal{D}_a S^i - \Phi_{{\rm M}\,\bar\imath}\mathcal{D}_a\bar S^{\bar\imath}) - \Phi_{{\rm M}\,i\bar\jmath}\bar\chi^i\gamma_a\chi^{\bar\jmath}\right].$$

$$\kappa^{-2}E_a \approx -J_a \qquad\qquad \mathcal{D}^{\dot\alpha}E_{\alpha\dot\alpha} \propto \mathcal{R} \approx 0\,,\quad \mathcal{D}^{\dot\alpha}J_{\alpha\dot\alpha} \approx 0$$

Compare with gravity + matter (Callan-Coleman-Jackiw)

$$\kappa^{-2}G_{\mu\nu} \approx \Theta^c_{\mu\nu} \qquad\qquad \Theta^c_{\mu\nu} = \tfrac{1}{6}\phi^2 G_{\mu\nu} + \Theta_{\mu\nu}(\phi)$$

Conformal frame in components (for conformal case)

$$S = \int \mathrm{d}^4 x\, 3\sqrt{g}\kappa^{-2}\left[\tfrac{1}{6}R - \Phi_{Mi\bar{\jmath}} D_\mu S^i D^\mu \bar{S}^{\bar{\jmath}}\right]$$

$$D_\mu S^i = (\partial_\mu - \mathrm{i} A_\mu) S^i\,, \qquad D_\mu \bar{S}^{\bar{\imath}} = (\partial_\mu + \mathrm{i} A_\mu) S^{\bar{\imath}}$$

$$S = \int \mathrm{d}^4 x\, 3\sqrt{g}\kappa^{-2}\Big[(1-\Phi_M)(\tfrac{1}{6}R + A_\mu^2) - \Phi_{Mi\bar{\jmath}}\partial_\mu S^i \partial^\mu \bar{S}^{\bar{\jmath}} \\ - \mathrm{i} A_\mu (\Phi_{Mi}\partial_\mu S^i - \Phi_{M\bar{\imath}}\partial^\mu \bar{S}^{\bar{\imath}})\Big]$$

- When A_μ would be eliminated: not anymore Kähler, no conformal coupling
  (would be Kähler in Einstein frame)
- Keep A_μ : Kähler potential is Φ_M.

Bosonic part and improvements (for conformal case)

$$S=\int \mathrm{d}^4x\, 3\sqrt{g}\kappa^{-2}\Big[(1-\Phi_M)(\tfrac{1}{6}R+A_\mu^2)-\Phi_{Mi\bar{\jmath}}\partial_\mu S^i\partial^\mu\bar{S}^{\bar{\jmath}}$$
$$-\mathrm{i}A_\mu(\Phi_{Mi}\partial_\mu S^i-\Phi_{M\bar{\imath}}\partial^\mu\bar{S}^{\bar{\imath}})\Big]$$

- Graviton eom: $\mathcal{G}_{\mu\nu}\equiv G_{\mu\nu}+6A_\mu A_\nu-3g_{\mu\nu}A^\rho A_\rho\approx\Theta^c_{\mu\nu}$

$$\Theta^c_{\mu\nu}=3\Phi_{Mi\bar{\jmath}}\left(2\partial_{(\mu}S^i\partial_{\nu)}\bar{S}^{\bar{\jmath}}-g_{\mu\nu}\partial^\lambda S^i\partial_\lambda\bar{S}^{\bar{\jmath}}\right)$$
$$-(\nabla_\mu\partial_\nu-g_{\mu\nu}\Box)\Phi_M+\mathcal{G}_{\mu\nu}\Phi_M$$

$$\Theta^{c\,\lambda}_{\lambda}\approx 0$$

$$\mathcal{R}\approx 0\ \rightarrow\ \left\{F^0\approx 0,\ \tfrac{1}{6}R+A_\mu A^\mu\approx 0,\ \nabla^\mu A_\mu\approx 0\right\}$$

Conservation equation

$$\nabla^\mu\Theta^c_{\mu\nu}\approx\nabla^\mu(6A_\mu A_\nu-3g_{\mu\nu}A^\rho A_\rho)$$
$$\approx 6A^\mu(\partial_\mu A_\nu-\partial_\nu A_\mu)$$

Trace equation

$$\Theta^{c\,\mu}_{\mu}\approx\mathcal{G}_\mu{}^\mu=-R-6A_\mu A^\mu\approx 0$$

4. Conclusions and final remarks

- We found a supersymmetric generalization of the Callan-Coleman-Jackiw improved currents for couplings of conformal matter to supergravity
  - Using a conformal frame rather than Einstein frame
  - Keeping the R-symmetry gauge field
  - Includes a Kähler manifold with other Kähler potential
  - Improved currents modified by R-symmetry
- Importance for :
  - higher curvature invariants to classify counterterms
  - Cosmology: Starobinsky model has conformal structure
  - nonlinear realizations
  - Supergravity backgrounds for localization techniques