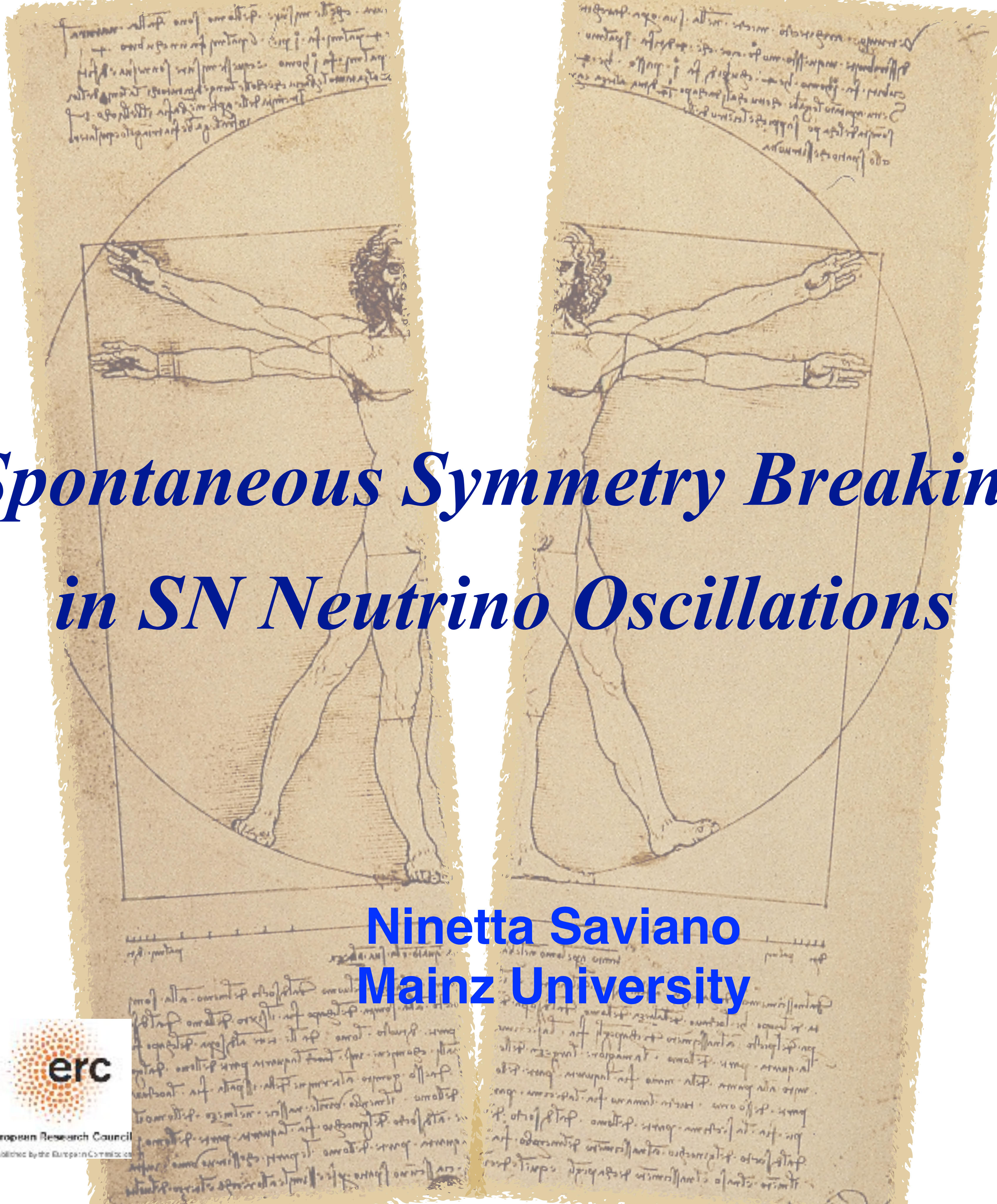
SN workshop, MITP, Mainz



Spontaneous Symmetry Breaking in SN Neutrino Oscillations

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EQUATIONS OF MOTION FOR A DENSE NEUTRINO GAS

$$\partial_t \varrho_{\mathbf{p},\mathbf{x}} + \mathbf{v}_\mathbf{p} \cdot \nabla_\mathbf{x} \varrho_{\mathbf{p},\mathbf{x}} + \dot{\mathbf{p}} \cdot \nabla_\mathbf{p} \varrho_{\mathbf{p},\mathbf{x}} = -i[\Omega_{\mathbf{p},\mathbf{x}}, \varrho_{\mathbf{p},\mathbf{x}}]$$

Liouville operator Hamiltonian

$\partial_t \varrho_{\mathbf{p},\mathbf{x}} \longrightarrow$ Explicit time evolution

$\mathbf{v}_\mathbf{p} \cdot \nabla_\mathbf{x} \varrho_{\mathbf{p},\mathbf{x}} \longrightarrow$ Drift term due to space inhomogeneities

$\dot{\mathbf{p}} \cdot \nabla_\mathbf{p} \varrho_{\mathbf{p},\mathbf{x}} \longrightarrow$ Force term acting on neutrinos (negligible)

$$\Omega_{p,x} = \Omega_{vac} + \Omega_{matt} + \Omega_{\nu\nu}$$

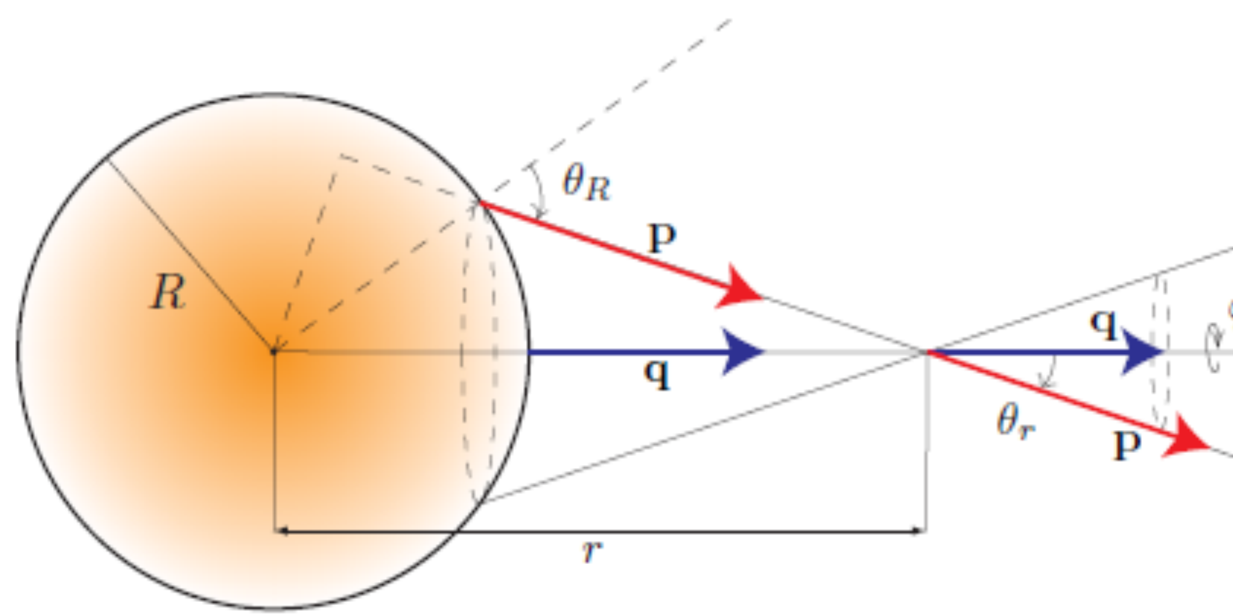
7-dimensional problem never solved in its complete form. Symmetries have been used to reduce the complexity of the problem.

TIME HOMOGENEITY

$$\partial_t \rho_{p,x} + v_p \cdot \nabla_x \rho_{p,x} = -i[\Omega_{p,x}, \rho_{p,x}]$$

Stationary space evolution (SN neutrinos)

Numerical approach typically based on the so called "Bulb Model"



Further simplification: ***pure radial dynamics*** —> $\mathbf{v_p} \cdot \nabla_\mathbf{x} \to v_r\, d/dr$

Many numerical investigations were possible within this model

ATTEMPT BEYOND THE "BULB MODEL"

Validity of bulb model recently questioned removing some of the symmetries…

→ new instabilities can be triggered in the flavor evolutions

MAA instability

~~Axial~~ symmetry in ν propag. → breaking of the spherical symmetry after the onset of oscillations. Matter effects can suppress this effects

Raffelt, Sarikas & Seixas, 1305.7140; Duan, 1309.7377;
Chackraborty & Mirizzi 1308.5255;
Chackraborty, Mirizzi, Saviano and Seixas, 1402.1767…

Breaking of space-time symmetries by self-intercating ν

Translational ~~symmetry~~ in time

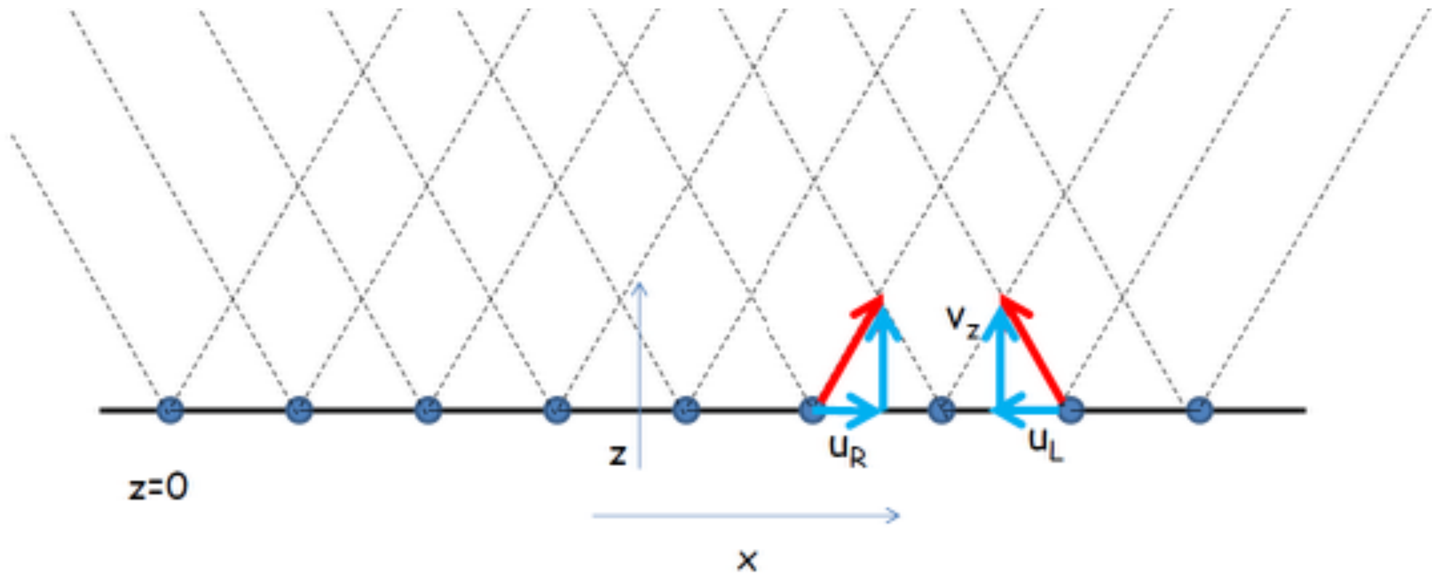
Mangano, Mirizzi, Saviano, 1403.1892
Dasgupta & Mirizzi, 1509.03171…

Translational ~~symmetry~~ in space

Duan & Shalgar, 1412.7097; Mirizzi, Mangano, Saviano, 1503.03485,
Mirizzi, 1506.06805; Chakraborty, Hansen, Izaguirre , Raffelt, 1507.07569
….

SPACE INHOMOGENEITIES: EoM FOR THE 2D MODEL

ν evolving in the plane (x,z) emitted from an infinite plane at z=0, in only two directions (L and R). Excess of ν_e over $\overline{\nu_e}$ (=α)



$$\hat{\mathbf{v}}_\zeta = (\mathsf{u}_\zeta, 0, v_z) \quad (\zeta = L, R)$$
$$0 < v_z < 1$$

For the L mode: (analogous for the R mode: L <—> R symmetry)

$$\begin{aligned} \hat{\mathbf{v}}_L \cdot \nabla_{\mathbf{x}} \mathsf{P}_L(x,z) &= [+\omega \mathsf{B} + \mu \mathsf{D}_R(x,z)] \times \mathsf{P}_L(x,z) \\ \hat{\mathbf{v}}_L \cdot \nabla_{\mathbf{x}} \overline{\mathsf{P}}_L(x,z) &= [-\omega \mathsf{B} + \mu \mathsf{D}_R(x,z)] \times \overline{\mathsf{P}}_L(x,z) \end{aligned}$$

$\rho_p = \frac{1}{2}(1 + P \cdot \sigma)$ Two-flavor polarization vectors $\qquad \mathsf{D}_{L,R} = \mathsf{P}_{L,R} - \overline{\mathsf{P}}_{L,R}$

$\omega = \frac{\Delta m^2}{2E}$ Vacuum oscillation frequency

$B \cdot \hat{e}_3 = -\cos\theta$ Mass eigenstate direction in flavor space

$\mu = \sqrt{2} G_F [F^0_{\bar{\nu}_e} - F^0_{\bar{\nu}_x}](1 - \hat{\mathbf{v}}_L \cdot \hat{\mathbf{v}}_R)$ ν−ν potential (F⁰ flux at the boundary)

SOLVING THE PROBLEM IN FOURIER SPACE

The partial differential equations can be transformed into a tower of ordinary differential equations for the Fourier modes coupled through the interaction term

$$\mathsf{P}_{L(R),k}(z) = \int_{-\infty}^{+\infty} dx \;\; \mathsf{P}_{L(R)}(x,z) e^{-ikx}$$

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We assume a monochromatic perturbation (with wave-number $k_0 = 2\pi/\lambda_0$) in the translational symmetry along x at z=0 :

$$P^3_{L,R}(x,0) = \langle P^3_{L,R}(x,0) \rangle + \epsilon \cos(k_0 x)$$

unperturbed value

Perturbation ($\epsilon \ll 1$)

$$P^1_{L,R}(x,0) = P^2_{L,R}(x,0) = 0$$

pure flavour state

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$$P^3_{L,R}(x,0) = \langle P^3_{L,R}(x,0)\rangle + \epsilon\cos(k_0 x) \qquad P^1_{L,R}(x,0) = P^2_{L,R}(x,0) = 0$$

unperturbed value — Perturbation ($\epsilon \ll 1$) — pure flavour state

Non-linear interaction:

$$v_z \frac{d}{dz}\mathsf{P}_{L,n}(z) \ = \ -iu_L k_n \mathsf{P}_{L,n} + \omega \mathsf{B}\times\mathsf{P}_{L,n} \ + \ \mu \sum_{j=-\infty}^{+\infty} \mathsf{D}_{R,n-j}\times\mathsf{P}_{L,j}$$

$$k_n = nk_0$$

Solution in real space by inverse Fourier transform $P(x,z) = \int_{-\infty}^{+\infty} dk P_k(z)e^{ikx}$

2D FLAVOR EVOLUTION IN THE PLANE

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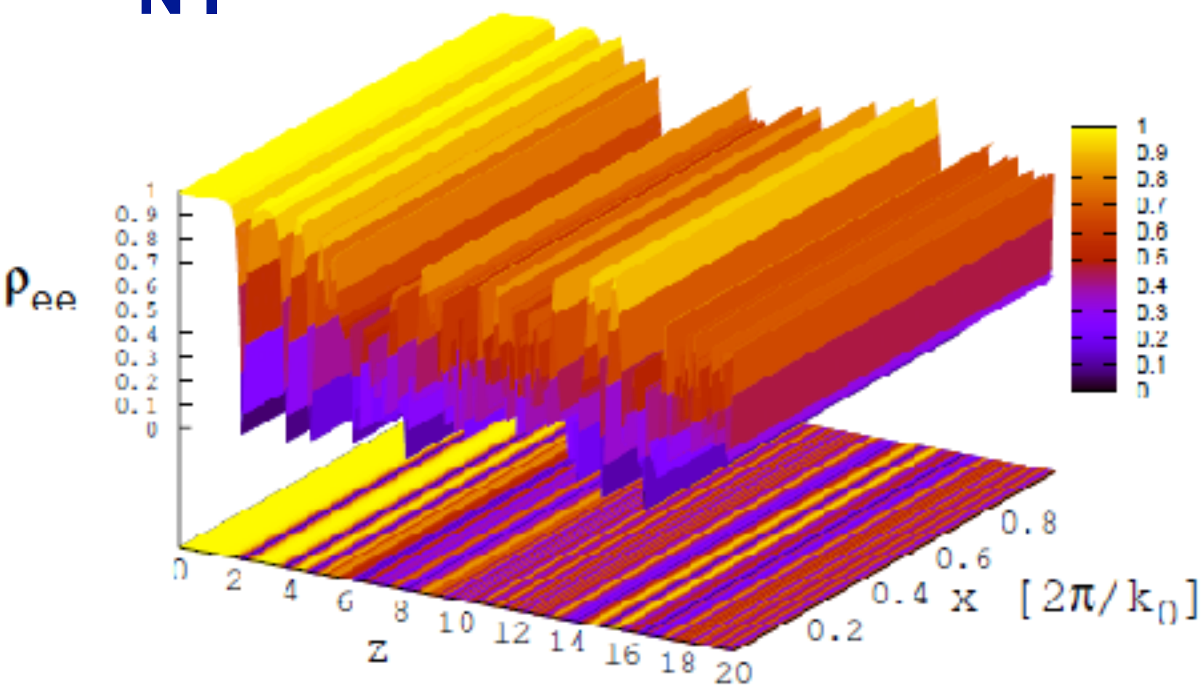


AXIAL ⟺	L-R
SPHERICAL ⟺	TRANSLATIONAL

~~L-R~~

TRANSLATIONAL

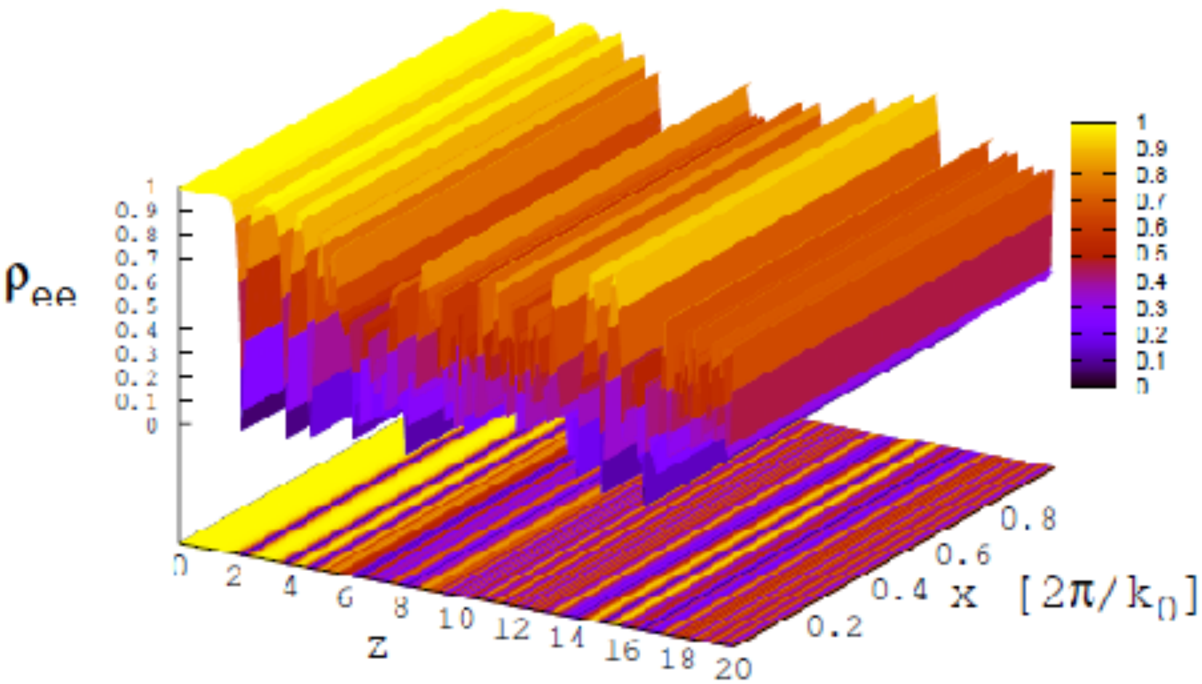
Evolution uniform in the x direction.

Coherent behavior along x direction.

2D FLAVOR EVOLUTION IN THE PLANE

AXIAL $\Longleftrightarrow$ L-R

SPHERICAL $\Longleftrightarrow$ TRANSLATIONAL



~~L-R~~

TRANSLATIONAL

Evolution uniform in the x direction.

Coherent behavior along x direction.

$\mu = 10, \; \omega = 1, \alpha = 0.3$

$k_0 = 0.2\sqrt{2\omega\mu}$

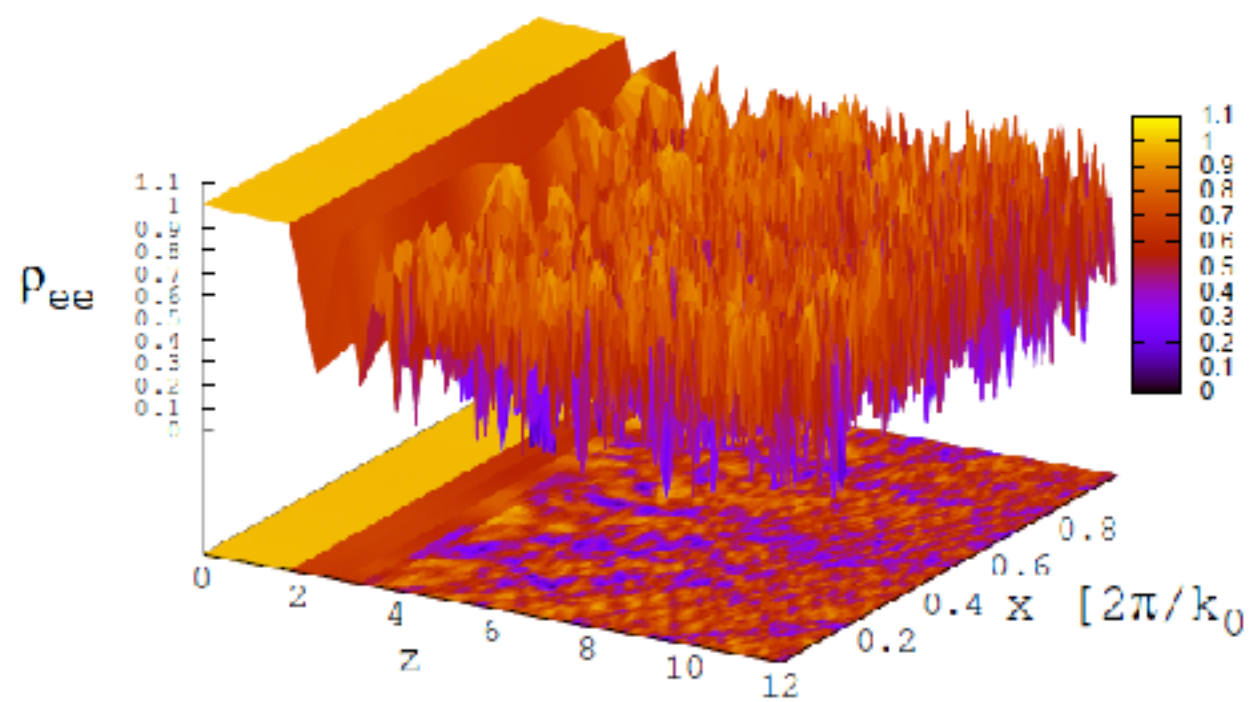
~~L-R~~

~~TRANSLATIONAL~~ $(\epsilon = 0.01)$

Large variations in the x direction at smaller and smaller scales.

Planes of common phase broken.

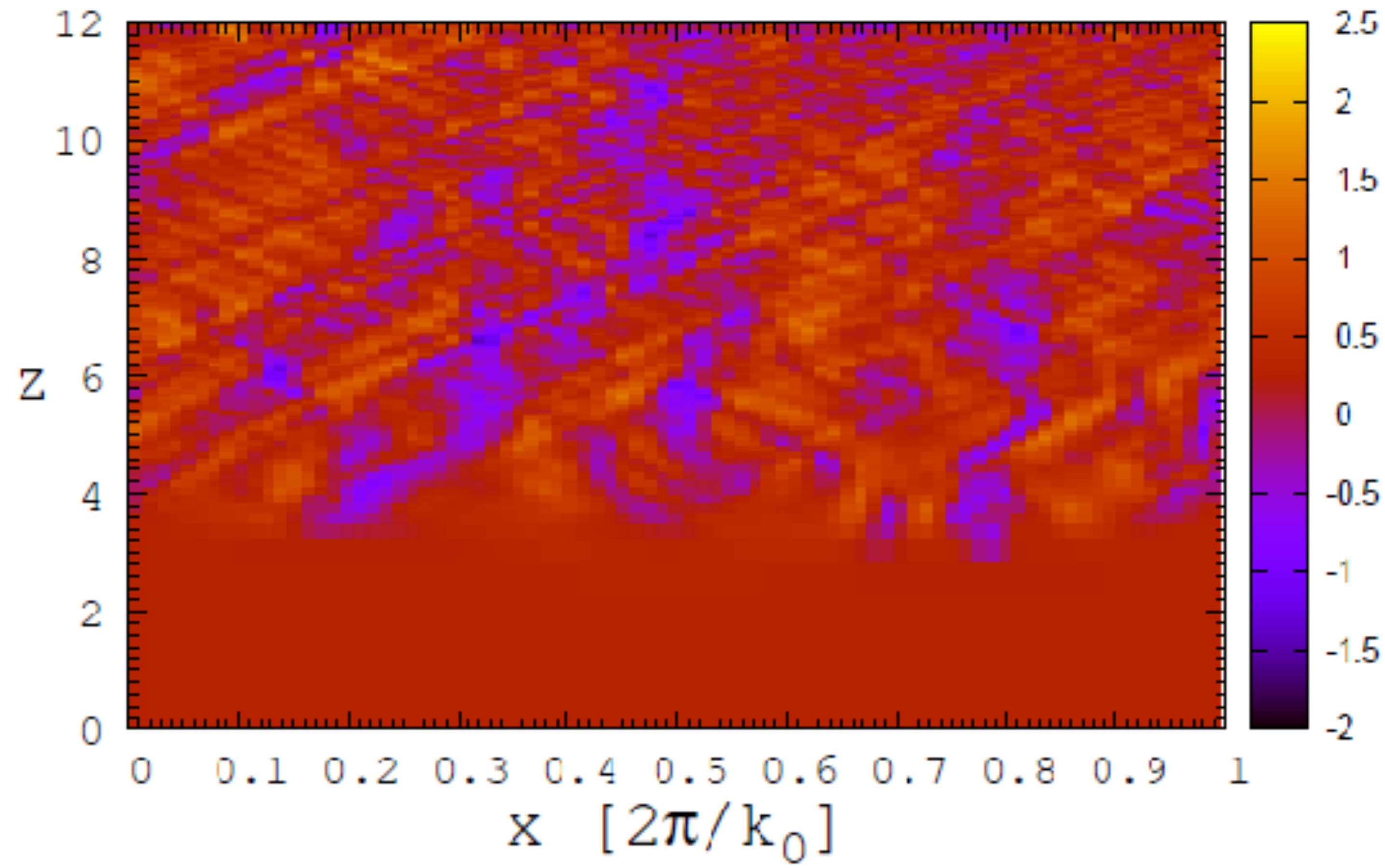
Coherent behavior of oscillation lost.



LEPTON NUMBER

Large space variations of the initial ν-ν asymmetry $\boldsymbol{\alpha}$

Map of L_0



~~TRANSLATIONAL~~ → L_0 shows a non-trivial domain structure with different net lepton number flux

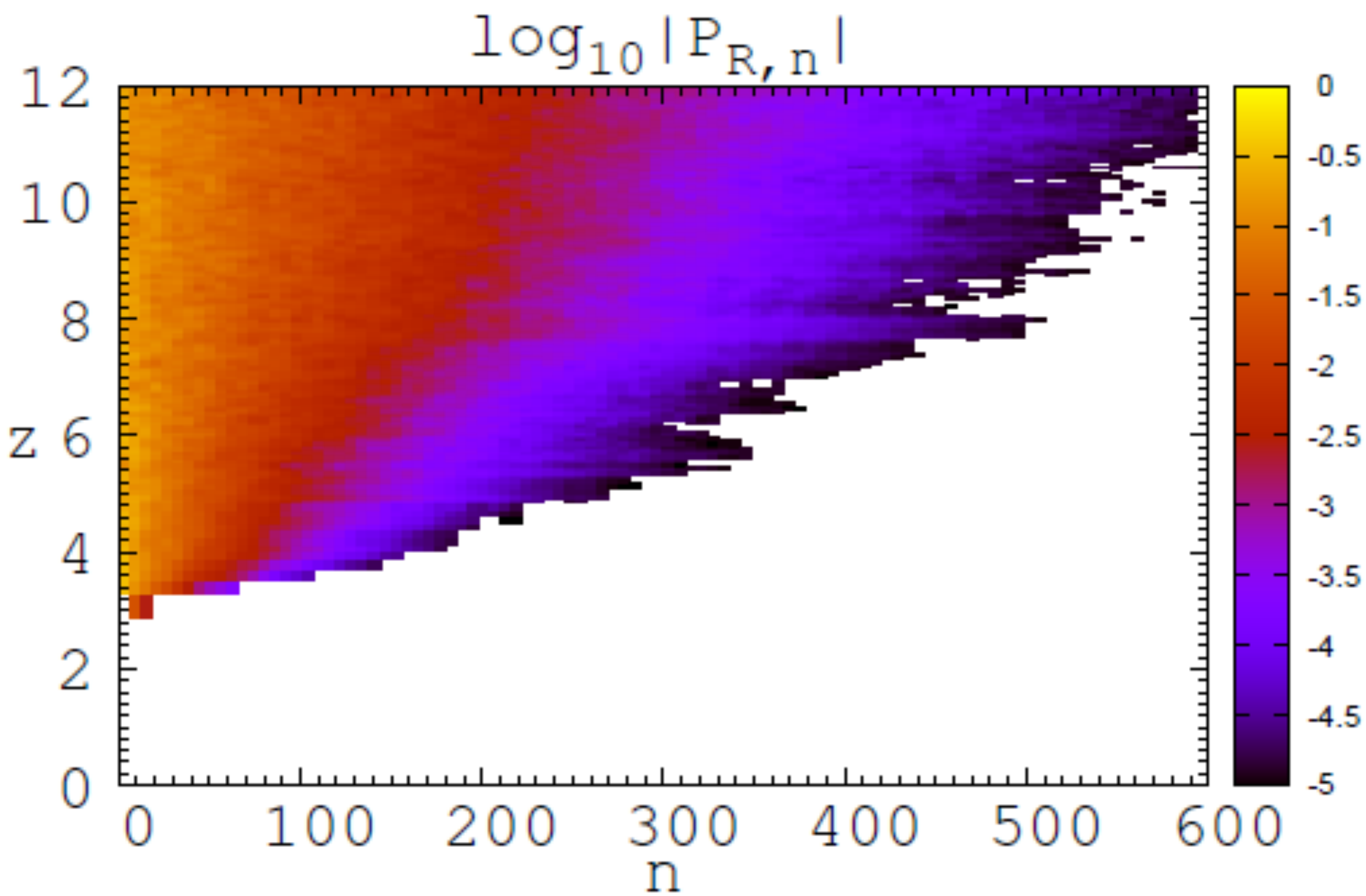
Lepton current $L^{\mu} = (L_0, L)$

$$\mathrm{L}_0 = \mathrm{D}_L \cdot \mathrm{B} + \mathrm{D}_R \cdot \mathrm{B} \ ,$$
$$\mathbf{L} = \hat{\mathrm{v}}_L(\mathrm{D}_L \cdot \mathrm{B}) + \hat{\mathrm{v}}_R(\mathrm{D}_R \cdot \mathrm{B})$$

→ Continuity equation

$$\partial_t \mathrm{L}_0 + \nabla_{\mathbf{x}} \cdot \mathbf{L} = \nabla_{\mathbf{x}} \cdot \mathbf{L} = 0$$

GROWTH OF FOURIER MODES



Evolution of different Fourier modes in the plane n-z

~~TRANSLATIONAL~~ → Growth of $n > 0$ modes in Fourier space. Cascade process. Flavor wave diffuses to higher harmonics (smaller scales)

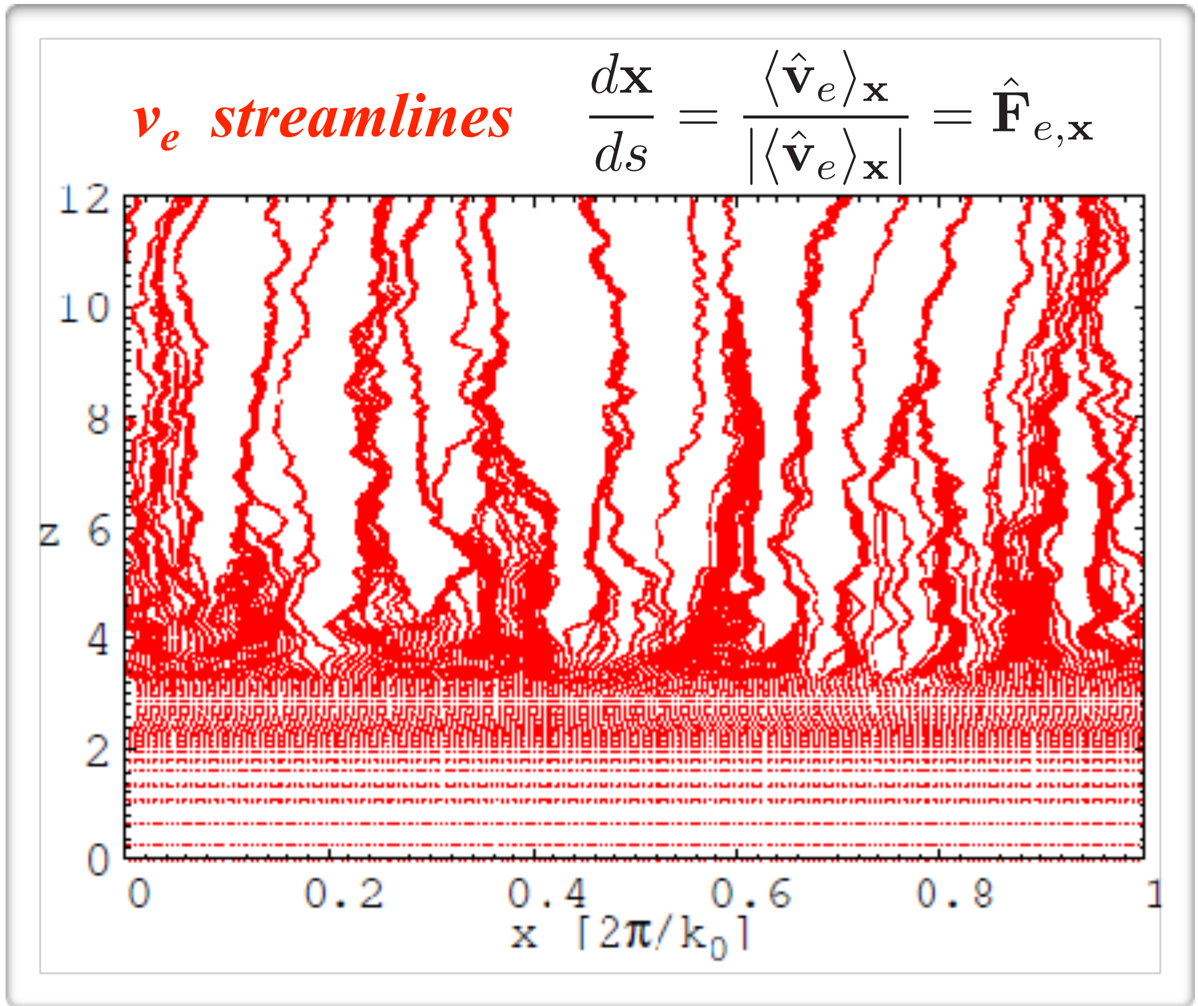
ANALOGY WITH A TURBULENT FLUID

ν_e average velocity: $\langle \hat{\mathbf{v}}_e \rangle_{\mathbf{x}} = \dfrac{\varrho_{ee,L}\hat{\mathbf{v}}_L + \varrho_{ee,R}\hat{\mathbf{v}}_R}{\varrho_{ee,L} + \varrho_{ee,R}}$

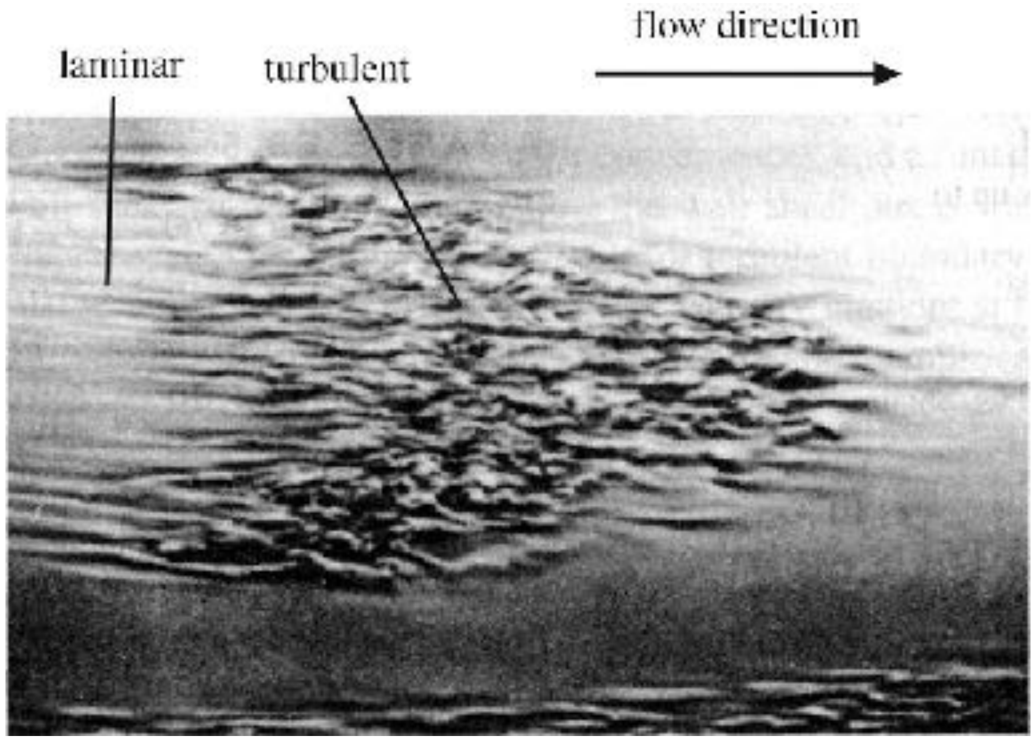
Symmetry unbroken: $\langle \hat{\mathbf{v}}_e \rangle_{\mathbf{x}} \simeq v_z$

TRANSLATIONAL: $\langle \hat{\mathbf{v}}_e \rangle_{\mathbf{x}}$ starts to acquire a transverse component in the x direction

Streamlines irregular, no longer || z and large variations



fluid streamlines



Analogy between the behaviour of ν gas in this model and the behaviour of a streaming fluid:

transition btw the coherent → incoherent behaviour of the ν oscillations

&

transition btw the laminar → turbulent behaviour of a fluid. (Non-linear Navier-Stokes equations)

OPEN ISSUES

How to get from toy models to real SNe?

(Neutrino momentum distribution not limited to "outward" direction; Important "halo" flux even at large distance; Large 3 D effects (in toy model symmetric boundary is assumed)

Which is the outcome of self-induced flavor conversions?

(Probably spectral swaps and splits are possible only in bulb model; Is flavor decoherence generic in the presence of inhomogeneities?)

Which is the experimental strategy to test this possible outcome?

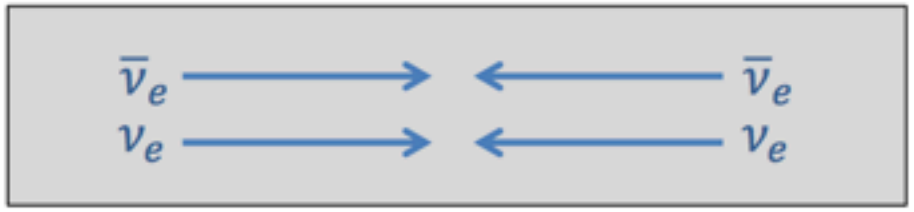
(Till now most of pheno studies focussed on signatures of swaps and splits.
Still lacking studies on how to test the flavor equilibrium, i.e. equal nu flavor fluxes)

Which is the impact on SN physics?

(Are possible self-induced flavor conversions below the shock-wave?, If yes, does it help the shock revival? How to test in schematic way in SN simulations?)

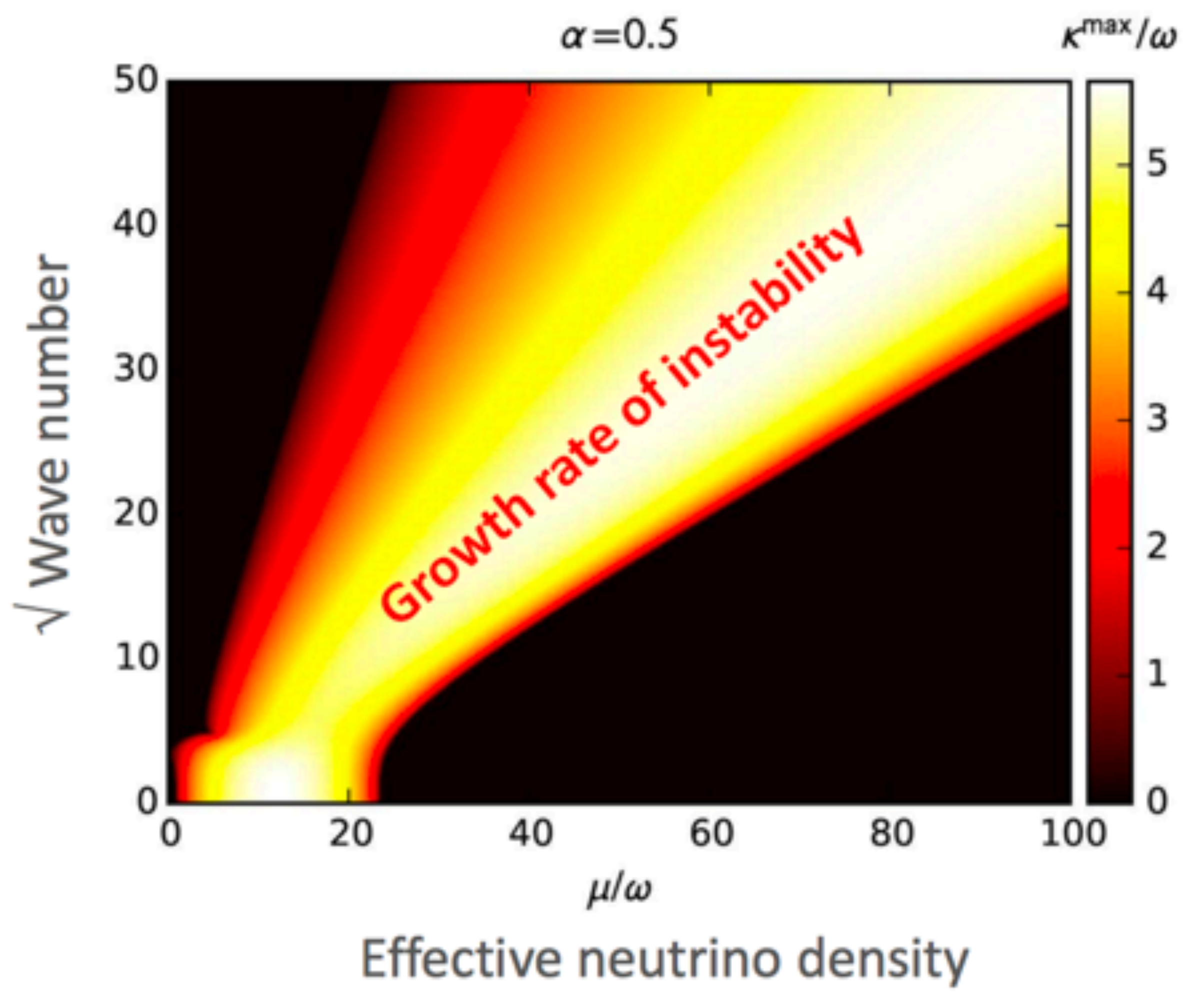
Thank You

Spatial Symmetry Breaking (SSB)



Linearized stability analysis for colliding-beam model

Duan & Shalgar, arXiv:1412.7097

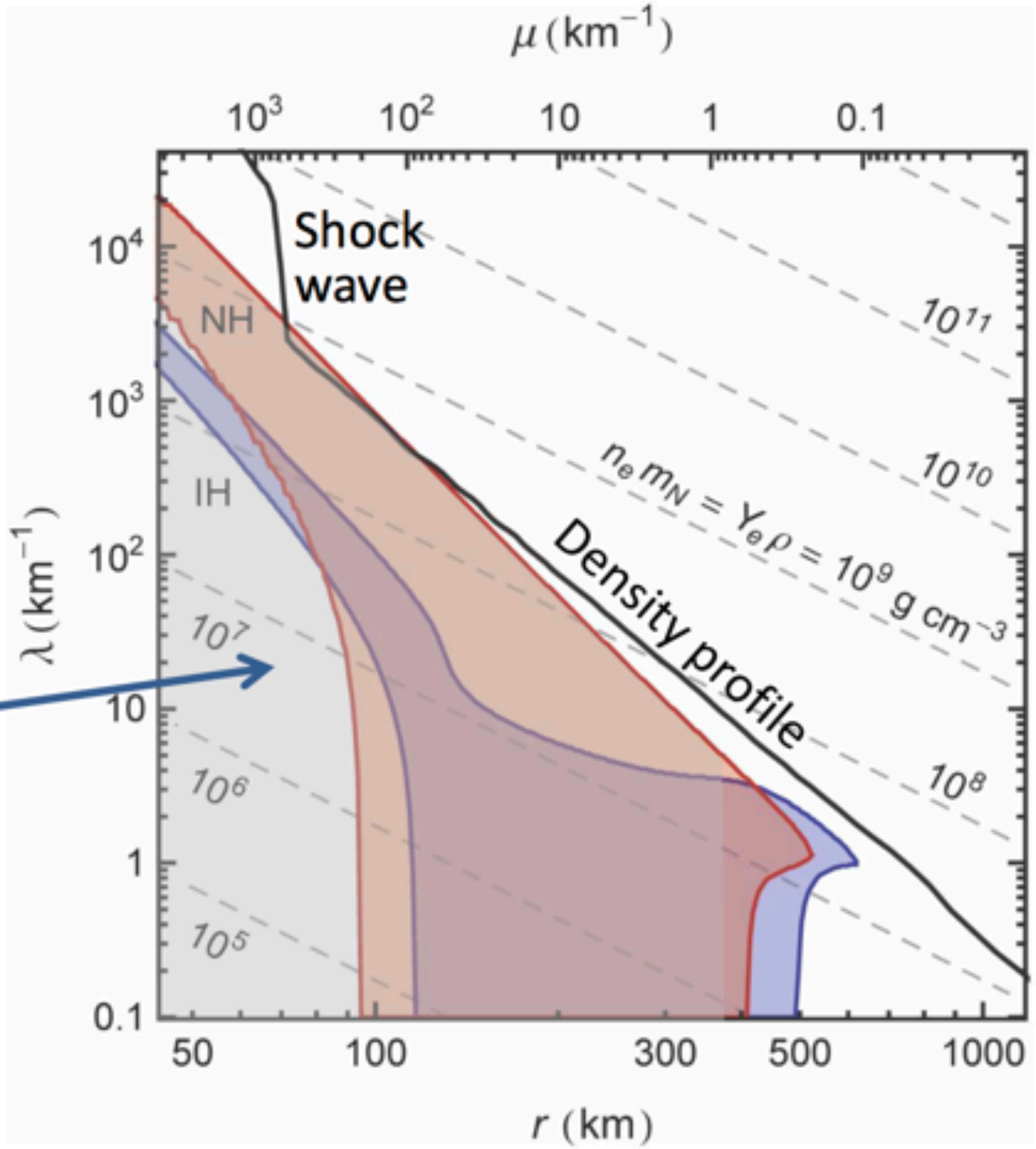


- Instability footprint shifted to larger neutrino density μ for larger wave number k
- For any neutrino density, unstable for some k-range
- No flavor-stable conditions exist for homogeneous neutrino gas (no "sleeping top" regime)

From Georg's presentation

Small-Scale Instabilities

- Small-scale modes “fill in” the stability footprint for large neutrino density
- Largest-scale mode is “most dangerous” to cross SN density profile



Chakraborty, Hansen, Izaguirre & Raffelt,

From Georg’s presentation

MULTI-ANGLE CASE

With respect to the model considered before we assume $\alpha = 1, \ldots N$ neutrino emission modes labeled in terms of the velocities $\hat{\mathbf{v}}_\alpha = (v_{x,\alpha}, 0, v_{z,\alpha})$, whose components are $v_{x,\alpha} = \cos\vartheta_\alpha$ and $v_{z,\alpha} = \sin\vartheta_\alpha$

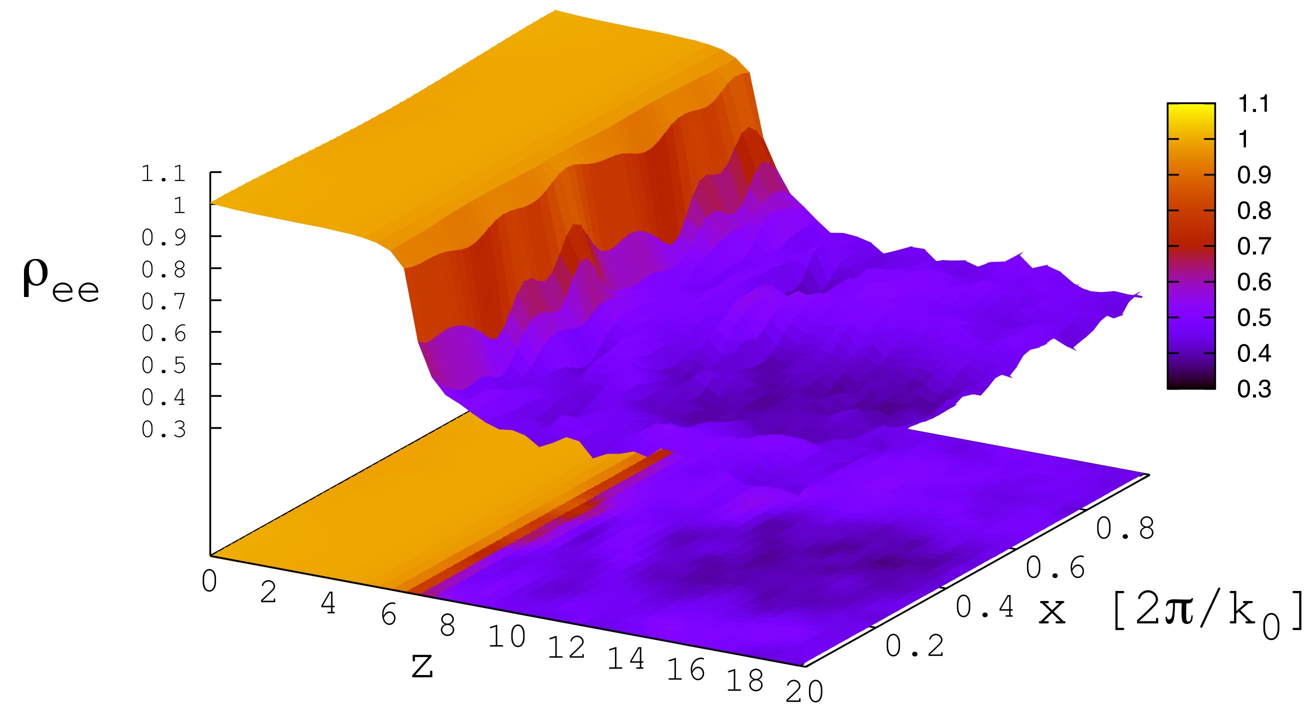
Modification with respect to the single-angle case:

Flavor equilibrium with $\varrho_{ee}(x,z) \simeq 0.5$ across the plane.

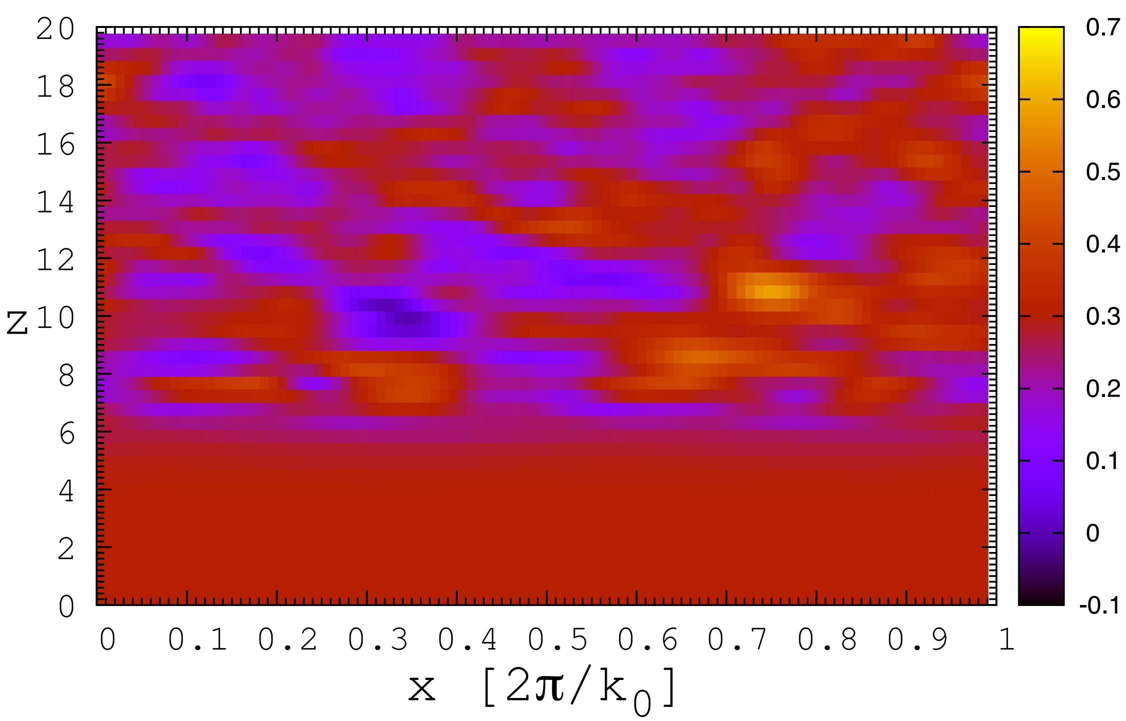
The flavor variations along the x direction are smoothed

with respect to the single-angle case being at most ~ 20%

Multi-angle decoherence



Spatial variation of L_0 reduced



Number of excited Fourier modes reduced

reason: when the decoherence quickly starts the length of |P0| is significantly shortened, reducing the growth of the higher order harmonics