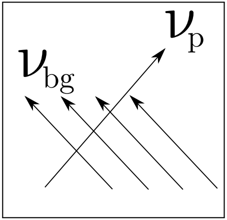
Neutrino-neutrino refraction with linear equations



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based on work in collaboration with

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MITP SNOBS2017, Mainz
October 11

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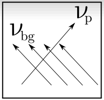


Linear equations for free streaming neutrinos

Observation

Inherently, the oscillations of neutrinos in a SN is a linear problem.

Idea

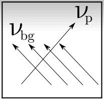
How much of the neutrino-neutrino refration can we describe using linear equations.

Ultimate goal

General conclusions about the behaviour of neutrino oscillations in presence of neutrino-neutrino refraction.

Methods

- Solve equations from first principle, analytic and numeric. (this talk)
- Describe complicated systems using effective potentials.



General equations

Probe neutrino in arbitraty neutrino and matter background.
Flavour state described by ψ.

$$i\partial_t \psi = H^{(p)} \psi$$

$$H^{(p)} = \frac{1}{2} \begin{pmatrix} -c_{2\theta}\omega_p + V_e + V_\nu & s_{2\theta}\omega_p + 2\bar{V}_\nu e^{i\phi_B} \\ s_{2\theta}\omega_p + 2\bar{V}_\nu e^{-i\phi_B} & c_{2\theta}\omega_p - V_e - V_\nu \end{pmatrix}, \qquad (1)$$

$$V_\nu = \int d\mathbf{k}\, V_\nu^0(\mathbf{k}) \left[\psi_e(\mathbf{k})\psi_e^*(\mathbf{k}) - \psi_\tau(\mathbf{k})\psi_\tau^*(\mathbf{k})\right],$$

$$\bar{V}_\nu e^{i\phi_B} = \int d\mathbf{k}\, V_\nu^0(\mathbf{k}) \psi_e(\mathbf{k})\psi_\tau^*(\mathbf{k})$$

$$V_\nu^0(\mathbf{k}) = \sqrt{2} G_F n(\mathbf{k}) \left(1 - \mathbf{v}_{bg} \cdot \mathbf{v}_p\right)$$

General equations - rotated

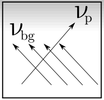
Rewrite the off-diagonal as $V' e^{i\phi'} = s_{2\theta}\omega_p + 2\bar{V}_\nu e^{i\phi_B}$. Apply the transformation $U = \mathrm{diag}\left(e^{i\phi'/2}, e^{-i\phi'/2}\right)$.

$$H^{(p)} = \frac{1}{2}\begin{pmatrix} V^r & V' \\ V' & -V^r \end{pmatrix},$$

where

$$V' = \sqrt{4\bar{V}_\nu^2 + 4 s_{2\theta}\omega_p \cos\phi_B \bar{V}_\nu + s_{2\theta}^2\omega_p^2},$$

$$V^r = V_e + V_\nu + \dot{\phi}' - c_{2\theta}\omega_p.$$



Conditions for a large conversion

Our Ansatz is that every case where a large conversion happens can be described in terms of at least one of these frameworks:

1. **Resonance**

   Vanishing diagonal:

   $$V_e + V_\nu + \dot{\phi}' - c_{2\theta}\omega_p = 0.$$

2. **Adiabatic conversion**

   Fast oscillations in V^r and V' can be removed by going to a rotating frame. This can result in a Hamiltonian describing adiabatic evolution.

3. **Parametric enhancement**

   Present if the period of oscillation equals the period of change of mixing angle.

Neutrino and matter background

Oscillation of neutrino background affected by matter background:

$$P_{e\tau} = \sin^2 2\theta_m \sin^2 \frac{1}{2}\phi_m \approx \left(\frac{\omega_{bg}}{V_e}\right)^2 s_{2\theta}^2 \sin^2 \frac{1}{2}\phi_m$$

for $\omega_{bg}, V_\nu \ll V_e$ and
$\phi_m = tA_\beta\omega_m = tA_\beta\sqrt{(V_e - c_{2\theta}\omega_{bg})^2 + s_{2\theta}^2\omega_{bg}^2}$. The resulting potentials are

$$V_\nu = \int d\omega_{bg}\, V_\nu^0 (1 - 2P_{e\tau}),$$

$$\bar{V}_\nu = \int d\omega_{bg}\, V_\nu^0 \sqrt{P_{e\tau}(1 - P_{e\tau})}, \qquad \cos\phi_B = \frac{-\cos 2\theta_m \sin \frac{1}{2}\phi_m}{\sqrt{1 - P_{e\tau}}}$$

Rotating frame for single ω_{bg}

$$H^{(p)} = \frac{1}{2}\begin{pmatrix} V^r & V' \\ V' & -V^r \end{pmatrix},$$

where

$$V' \approx s_{2\theta}\omega_p + 2V_\nu^0 s_{2\theta}\sin^2\frac{1}{2}\phi_m$$

$$V^r \approx V_e + V_\nu^0 - c_{2\theta}\omega_p + \frac{\omega_m A_\beta\left(\frac{\omega_m}{V_\nu^0}\cos\phi_m + \cos\phi_m - 1\right)}{\left(1 - \cos\phi_m + \frac{\omega_m}{V_\nu^0}\right)^2 + \sin^2\phi_m}.$$

Analytic approximation

$$H = \frac{1}{2}\begin{pmatrix} d + \dot{\phi}' & g + f\cos\alpha t \\ g + f\cos\alpha t & -d - \dot{\phi}' \end{pmatrix}$$

Rotate away g and average the diagonal

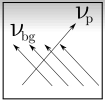
$$H \approx \frac{1}{2\sqrt{g^2/d^2+1}}\begin{pmatrix} d & f\cos\alpha - \frac{g}{d}\dot{\phi}' \\ f\cos\alpha - \frac{g}{d}\dot{\phi}' & -d \end{pmatrix},$$

$\dot{\phi}' \approx V_\nu^0 A_\beta \cos\alpha t$.

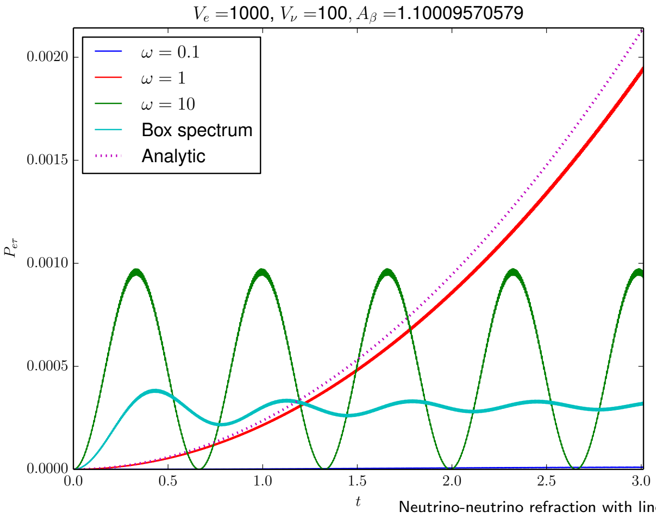
Resulting transition probability:

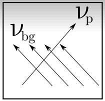
$$P_{e\tau} = \frac{\left(g(d-\alpha) - \frac{1}{2}d(f - \frac{g}{d}V_\nu^0 A_\beta)\right)^2}{\gamma^2(d^2+g^2)}\sin^2\frac{1}{2}\gamma t$$

$$\gamma = \sqrt{(d-\alpha)^2 + \left(f - \frac{g}{d}V_\nu^0 A_\beta\right)^2}$$

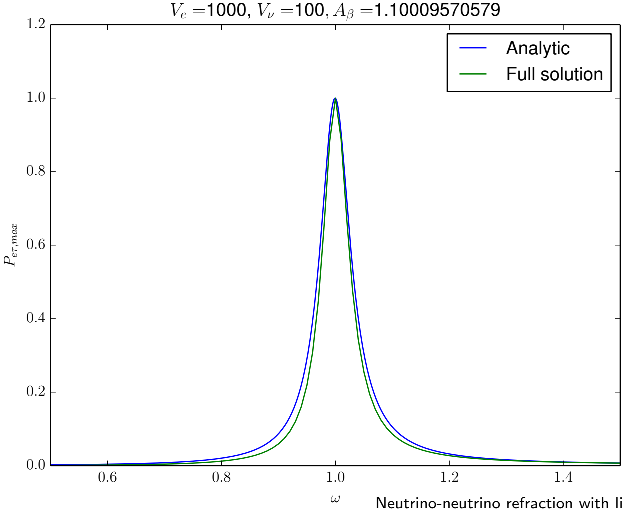


Energy spectrum




ν_p
ν_{bg}

Resonance


V_e =1000, V_ν =100, A_β =1.10009570579
Analytic
Full solution
$P_{e\tau,max}$
ω
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.6
0.8
1.0
1.2
1.4

Conclusions

- Large conversion is found for a neutrino propagating in a matter and neutrino potential.
- The conversion can be interpreted as a parametric resonance.
- The parametric resonance condition is only satisfied for a narrow range of energies, and a broader spectrum weakens the efficiency of the conversion.