Fast flavor conversions - a classical picture !

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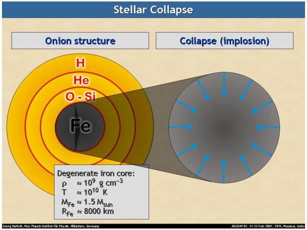
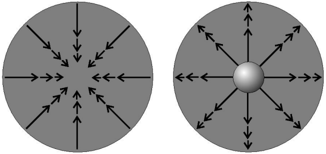
In collaboration with Basudeb Dasgupta.

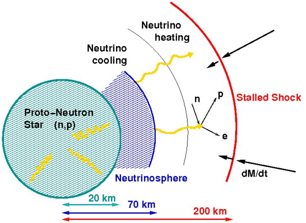
Supernova explosion



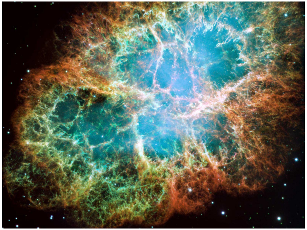
Explosion of a massive $6 - 8\ M_{\odot}$ star



Collapse of degenerate core. Bounce and Shock.



Stalled shock and accretion



Explosion!

Talk by Dighe, HRI (2016).

Flavor Oscillations in dense media: Collective effects

- Neutrino oscillation usually involves two kinds of terms:
  1. Evolution due to vacuum oscillation term ($\omega \equiv \Delta m^2/2E$).
  2. MSW potential term (matter effects $\equiv \lambda$)

- High neutrino density causes an MSW-like potential ($\equiv \mu$). Leads to self-interactions. $\mu \gg \omega$.

  Pantaleone (1992)

- Causes collective oscillations $\rightarrow$ large flavor conversions even for tiniest mixing angle.

- Bipolar oscillations $\propto \sqrt{\omega\mu}$. Intuitive understanding through pendulum analogy.

  Hannestad et. al. (2007)

- Fast oscillations $\propto \mu$. Even for $\omega = 0$.

  Sawyer (2016), Raffelt et. al. (2016), Dasgupta, Mirizzi and MS (2016).

Polarization vector picture

- Neutrino flavor density matrix $\varrho_{ex} = \langle \nu_e | \nu_x \rangle$.
- Define $\mathbf{P} = \left(\text{Re}\left(\varrho_{ex}\right), \text{Im}\left(\varrho^*_{ex}\right), \varrho_{ee} - \varrho_{xx}\right)$. z-component of $\mathbf{P}$ encodes flavor information. $\mathbf{P}(0) = \overline{\mathbf{P}}(0) = (0, 0, 1)$.
- EOMs

$$\begin{aligned} \dot{\mathbf{P}}_{\mathbf{p}} &= \left(\omega_{\mathbf{p}}\mathbf{B} + \lambda\mathbf{L} + H^{\nu\nu}_{\mathbf{p}}\right) \times \mathbf{P}_{\mathbf{p}}, \\ \dot{\overline{\mathbf{P}}}_{\mathbf{p}} &= \left(-\omega_{\mathbf{p}}\mathbf{B} + \lambda\mathbf{L} + H^{\nu\nu}_{\mathbf{p}}\right) \times \overline{\mathbf{P}}_{\mathbf{p}}, \end{aligned}$$

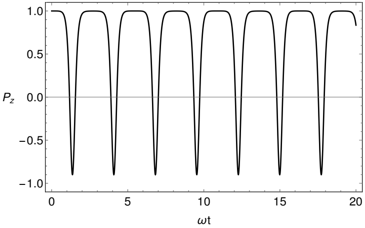
where

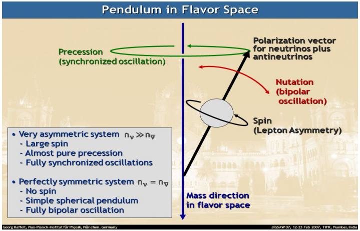
$$\begin{aligned} \mathbf{B} &= (\sin 2\vartheta_0, 0, -\cos 2\vartheta_0) \text{ for a vacuum mixing angle } \vartheta_0, \\ \mathbf{L} &= (0, 0, 1), \\ H^{\nu\nu}_{\mathbf{p}} &= \mu \int \frac{d^3q}{(2\pi)^3} (1 - \vec{v_{\mathbf{p}}}.\vec{v_{\mathbf{q}}})(\mathbf{P}_{\mathbf{q}} - \overline{\mathbf{P}}_{\mathbf{q}}). \end{aligned}$$

Bipolar Oscillations

- Coherent $\nu_e \bar{\nu}_e \leftrightarrow \nu_x \bar{\nu}_x$ oscillations.
- Can be mapped to a pendulum in flavor space.
- Depending on mass hierarchy, pendulum can be inverted (unstabe) or stable.

Hannestad et. al. (2007)



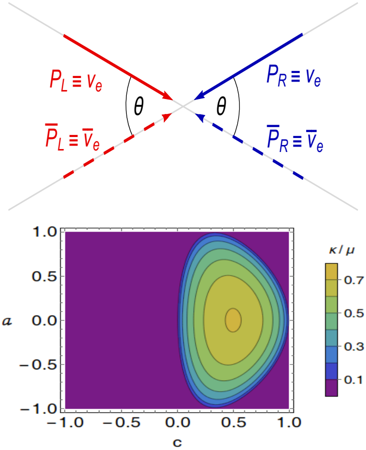


Talk by Raffelt, JIGSAW, 2007.

Fast Oscillations

- Simplest system which shows fast conversions, in absence of any spatial inhomogeneities.
- Only exists in the left-right symmetry breaking solution.
- Conversions obtained for $c \equiv \cos\theta > 0$.

  Raffelt et. al. (2016)
- Is a classical analogy possible?



Eqn. of motion

Redefine the basis:

$$
\begin{aligned}
\mathbf{Q} &\equiv (\mathbf{P}_L + \mathbf{P}_R) + (\overline{\mathbf{P}}_L + \overline{\mathbf{P}}_R) - \tfrac{2\omega}{\mu(3-c)}\mathbf{B}\,, \\
\mathbf{D} &\equiv (\mathbf{P}_L + \mathbf{P}_R) - (\overline{\mathbf{P}}_L + \overline{\mathbf{P}}_R)\,, \\
\mathbf{X} &\equiv (\mathbf{P}_L - \mathbf{P}_R) + (\overline{\mathbf{P}}_L - \overline{\mathbf{P}}_R)\,, \\
\mathbf{Y} &\equiv (\mathbf{P}_L - \mathbf{P}_R) - (\overline{\mathbf{P}}_L - \overline{\mathbf{P}}_R)\,.
\end{aligned}
$$

The EoMs:

$$
\begin{aligned}
\dot{\mathbf{Q}} &= \frac{\mu}{2}(3-c)\,\mathbf{D}\times\mathbf{Q} + \frac{\mu}{2}(1+c)\,\mathbf{X}\times\mathbf{Y}\,, \\
\dot{\mathbf{D}} &= \omega\,\mathbf{B}\times\mathbf{Q}\,, \\
\dot{\mathbf{X}} &= \left[\omega\left(\frac{3+c}{3-c}\right)\mathbf{B} + \mu\,c\,\mathbf{Q}\right]\times\mathbf{Y} + \mu\,\mathbf{D}\times\mathbf{X}\,, \\
\dot{\mathbf{Y}} &= \left[\omega\left(\frac{2}{3-c}\right)\mathbf{B} - \frac{\mu}{2}(1-c)\mathbf{Q}\right]\times\mathbf{X} + \frac{\mu}{2}(3+c)\,\mathbf{D}\times\mathbf{Y}\,.
\end{aligned}
$$

Exactly and approximately conserved quantities

- Clearly, $|\mathbf{P}|$ is conserved. So is flavor lepton number, given by $\mathbf{B} \cdot \mathbf{D}$. *Exactly* conserved.
- In the fast conversion limit $\omega/\mu \to 0$, extra conserved quantities, involving $\mathbf{Q}$, $\mathbf{D}$, $\mathbf{X}$ and $\mathbf{Y}$.
- Reduces the 12-variable problem to a 3-variable problem involving $\mathbf{Q}$.
- Leads to a closed form EoM for $\mathbf{Q}$.

Fast conversion: The quartic potential

- The closed equation for $\mathbf{Q}$:

$$\ddot{\mathbf{Q}} = -\mu^2 \, c \, (1-c) \Big[|\mathbf{Q}_0|^2 - \mathbf{Q} \cdot \mathbf{Q} \Big] \mathbf{Q} \, ,$$

  where $|\mathbf{Q}_0|$ is the length of $\mathbf{Q}$ at time $t = 0$.
- The corresponding Lagrangian

$$\mathcal{L}_{\mathbf{Q}} = \frac{1}{2} |\dot{\mathbf{Q}}|^2 - \, \mu^2 \, c \, (1-c) \Big[|\mathbf{Q}_0|^2 - \frac{\mathbf{Q} \cdot \mathbf{Q}}{2} \Big] \frac{\mathbf{Q} \cdot \mathbf{Q}}{2} \, .$$

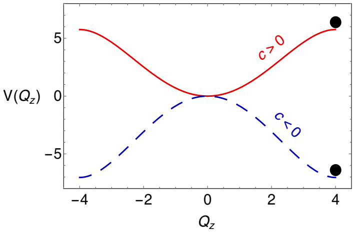
- $Q_x, Q_y \approx 0$, motion primarily governed by Q_z. Clearly, motion in a *quartic* potential.

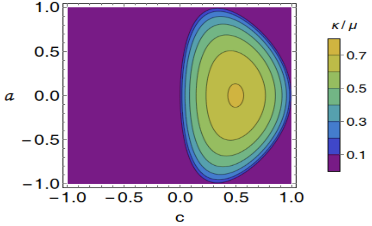
Fast oscillation as motion in the quartic potential

- The analytic potential

$$V(Q_z) = \mu^2 c\,(1-c)\left[|\mathbf{Q}_0|^2 - \frac{Q_z^2}{2}\right]\frac{Q_z^2}{2}$$

- Unstable for $c > 0$.
- LSA gives non-zero instability growth rate κ only for $c \equiv \cos\theta > 0$.



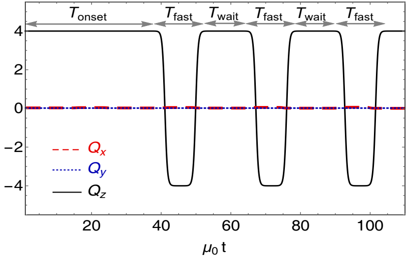


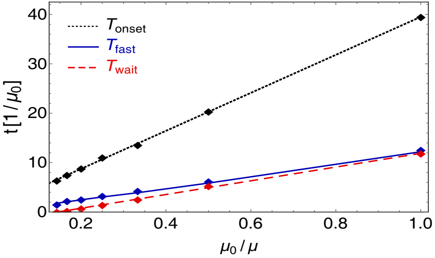
Time-period of the fast oscillations

- Three timescales in the problem. All vary as $1/\mu$.
- Use energy conservation to get

$$T_{\rm fast} = 2\int_{Q_z^{\rm max}}^{Q_z^{\rm min}} \frac{dQ_z}{\sqrt{2(E - V(Q_z))}}\,.$$

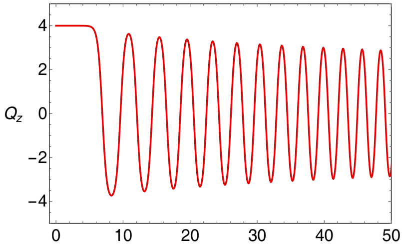
- $T_{\rm onset}$ and $T_{\rm wait}$ depend logarithmically on subleading $\mathcal{O}(\omega/\mu)$ terms.

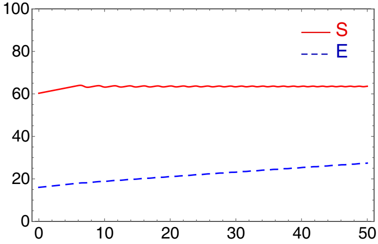




Varying neutrino density: Adiabatic Invariance

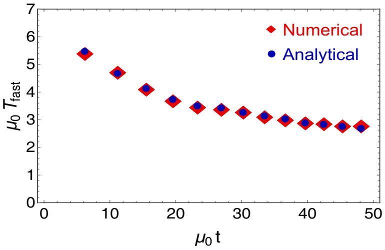
- Can extend argument for a varying neutrino density, for e.g., $\mu(t) = \mu_0(1 + t/100)$.
- Use adiabatic invariance $\rightarrow$ action remains constant, while energy and time period changes.

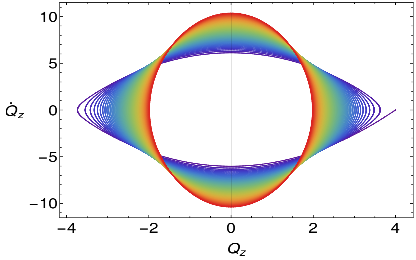




Varying neutrino density: Adiabatic Invariance

- Can calculate T_{fast} for varying μ using adiabatic invariance. Excellent agreement with numerics.
- Details of motion captured in phase-plot.





Conclusions

- Simplest homogeneous system showing fast conversion $\equiv$ particle rolling down a quartic potential. A classical analogue indeed exists!
- Potential well if $\cos\theta > 0$. Potential barrier if $\cos\theta < 0$. Explains angular dependence of fast oscillations in such systems.
- Explain the time period of fast oscillation analytically, both for constant and varying μ.
- Subleading terms important for onset and waiting period.
- Above analysis for zero neutrino-antineutrino asymmetry. Can extend to non-zero asymmetry. Analogous to the presence of external electric and magnetic fields.

Thank You!