Quantum breaks in supernova neutrino physics, and in some other systems

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Agenda

A. Put one new thing on the table

(as if the table were not overcrowded already)

B. Look at ways in which what we have learned from

theory of ν's in SN can apply to other physics

Rules

1. "Flavor" may be used to distinguish other species beyond ν ' s

2. We deal only with 2 -> 2 reactions in which particle momenta do not change,

$$\vec{p} + \vec{q} \rightarrow \vec{p} + \vec{q}$$

AND in which a neutrino (or a whatever) sees the same environment (densities and angular distributions) for the duration of the calculation

Momenta are like "sites for spins" in infinite range Ising model analogy.

3. Flavor assignments evolve in time under interactions that involve flavor coordinates only.

(for SN neutrino in ν-surface region this is good physics over distances of a few hundred meters or so)

4. Calculations will be in a periodic box of volume, *Vol.* containing N neutrinos (or whatevers)

Also : the time (or distance) duration of the calculation should not be greater than the dephasing time that comes from

ν mass: m

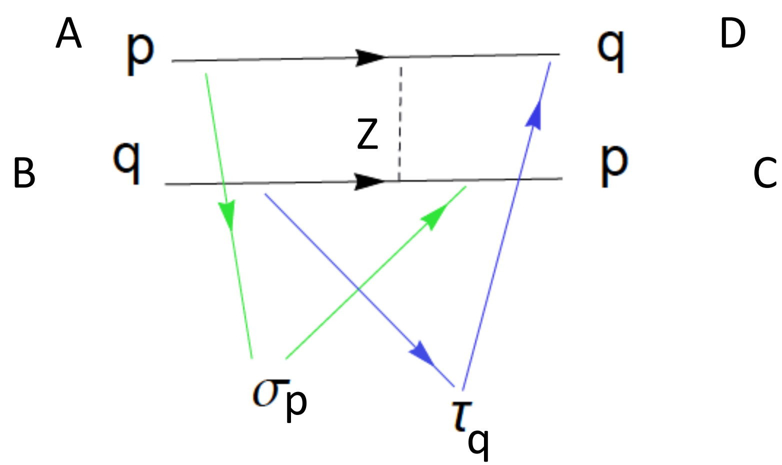
$$t_{dephase} \approx 2\, p/m^2$$

(gets you 1 km. for the neutrino in SN neutrino-sphere region)

Totally forward interaction between nearly massless particles with different flavors:

$$H_{eff} = g\,(\mathrm{Vol.})^{-1} \sum_{\mathbf{p},\mathbf{q}} [\, \mathrm{a}_{\mathbf{p}}\, \mathrm{b}_{\mathbf{q}}\, \mathrm{c}^{\dagger}_{\mathbf{p}}\, \mathrm{d}^{\dagger}_{\mathbf{q}} + \mathrm{H.C}\,](1 - \cos\theta_{\mathbf{p},\mathbf{q}})$$

a_p , b_q , c_p , d_q annihilate single particles of flavors A,B,C,D.



These flavors could, e.g , be 2 of the 3 neutrino flavors.

Or they could stand for

1. Photon-ness and polarization
2. Graviton-ness
3. Light-scalar-particle-ness

Stripped-down exercise to show essence of instabilities:

Head-on beam-on-beam

Collective "spins"

$$\vec{\sigma} = \sum_{j}^{N} \vec{\sigma}_j \qquad \vec{\tau} = \sum_{j}^{N} \vec{\tau}_j$$

$$H_S = gV^{-1}\left[\sigma^{(+)}\tau^{(-)} + \sigma^{(-)}\tau^{(+)} + \frac{1}{2}\lambda\sigma^{(3)}\tau^{(3)}\right]$$

Initial values:

$\sigma^{(3)} = N$

$\tau^{(3)} = -N$

$\sigma^{(+)} = \tau^{(+)} = 0$

$$i\dot{\sigma}^{(+)} = gV^{-1}[\sigma^{(3)}\tau^{(+)} + \lambda\sigma^{(+)}\tau^{(3)}]$$

$$i\dot{\tau}^{(+)} = gV^{-1}[\tau^{(3)}\sigma^{(+)} + \lambda\sigma^{(3)}\tau^{(+)}]$$

$$i\dot{\sigma}^{(3)} = gV^{-1}[\tau^{(+)}\sigma^{(-)} - \tau^{(-)}\sigma^{(+)}]$$

For Z exchange in ν-ν : λ=1

For ϒ +ϒ -> graviton+ graviton : λ =0

Instabilities

n=N/Vol= number density

$$i\dot{\sigma}^{(+)} = gn[\tau^{(+)} - \lambda\sigma^{(+)}]$$

$$i\dot{\tau}^{(+)} = gn[-\sigma^{(+)} + \lambda\tau^{(+)}]$$

$|\lambda| < 1$ -> growing mode

$|\lambda| > 1$ -> oscillating mode

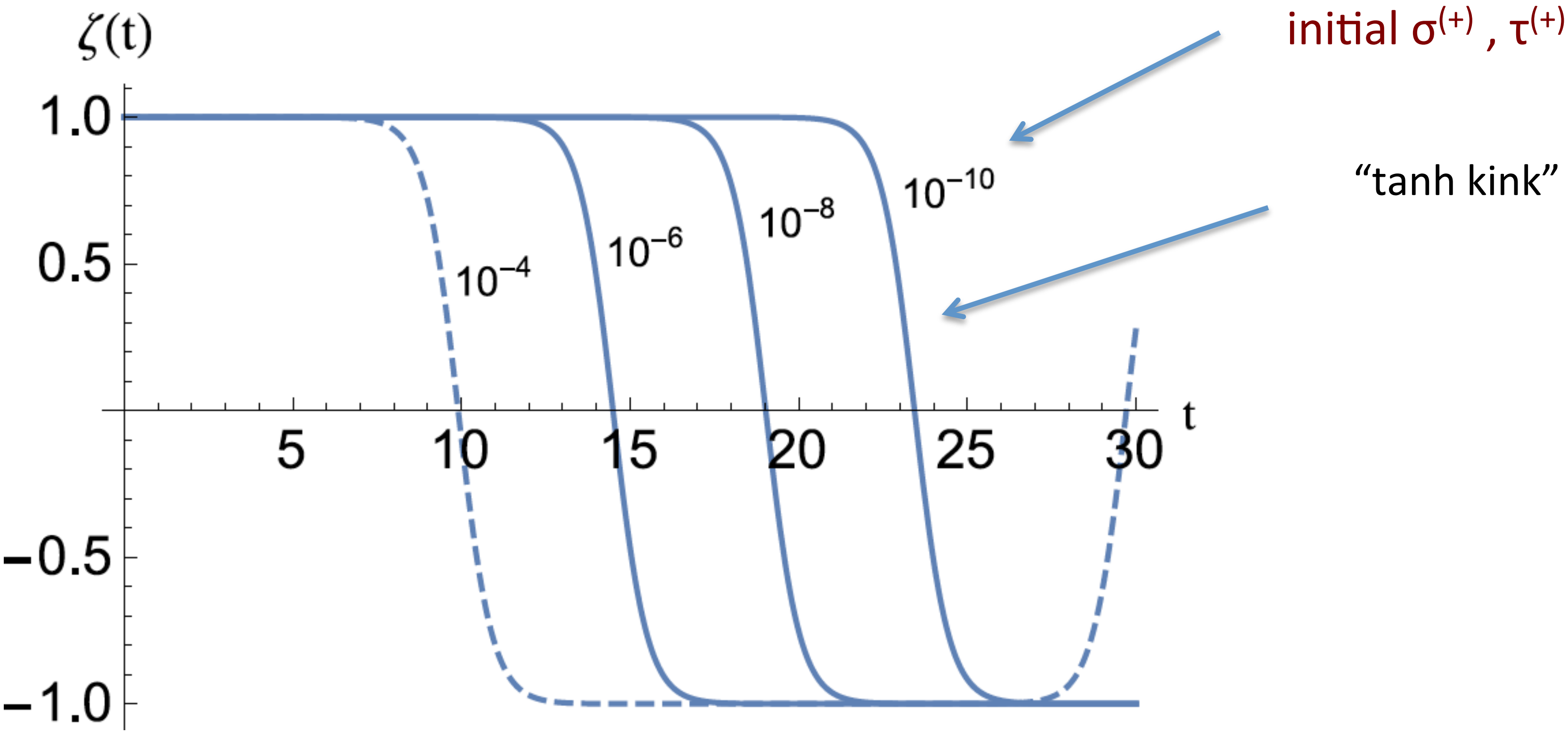
Note: For ν-ν case we can use the above for well-collimated beams.

For an isotropic velocity distribution, e.g. , we have a new variable for each angle.

Distribution in energies is irrelevant because the Z exchange force is independent of energies.

FLAVOR SWAPS

$\lambda = 0$ case



$$\zeta = \langle a_p^\dagger a_p \rangle - \langle c_p^\dagger c_p \rangle = \langle \sigma_3 \rangle$$

Seeds are supplied by ν oscillation terms

NOTE: equal spacing "break time" ~ $(g\ n)^{-1}$ log [seed]

$g=G_F$

(not "quantum!")

BY CONTRAST:

Standard model two-beam problem, where $\lambda=1$ (marginally stable).

Tiny seeds $\longrightarrow$ tiny flavor oscillations for 2 beams

But

More complex angular distributions (discretized in m angles) , $\sigma^{(+)} \rightarrow \sigma_i^{(+)}$
can give complex eigenvalue of 2m dimensional linear response matrix

If we take flavor dependent initial angular distributions……

Then there can be **fast** flavor exchanges in the vicinity of the ν − surface.

(serious calculations take $\bar{\nu}$ into account)

Recent Lit. :

S. Chakraborty, R. S. Hansen, I. Izaguirre and G. Raffelt, JCAP 1603, no. 03, 042 (2016) ;
arXiv:1602.00698

B. Dasgupta, A. Mirizzi and M. Sen JCAP 02 (2017) 019 ; arXiv:1609.00528.

RFS, Phys. Rev. Lett. 116, 081101 (2016); arXiv:1509.03323

Those swaps are not any form of MSW transformation.

They do **not** involve a changing background environment as seen by a ν

nor

a position dependent angular distribution for the ν cloud.

Something fairly new:

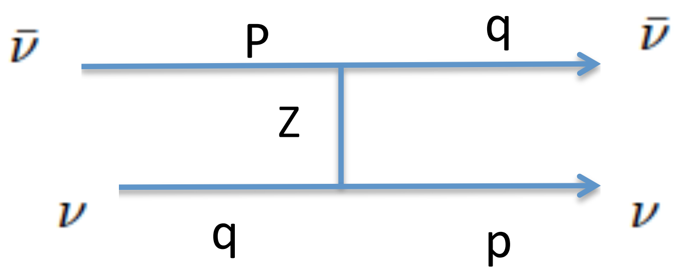
$$\bar{\nu} + \nu \rightarrow \bar{\nu} + \nu$$

Single (0rdinary) flavor, for simplicity

λ =0

UNSTABLE to spectrum swap

for two clashing beam case
and also for isotropic distributions



S. Chakraborty, R. S. Hansen, I. Izaguirre and G. Raffelt
j.nucl.physb.2016.02.012, arXiv:1602.02766

I. Izaguirre, G. Raffelt, I. Tamborra, Phys. Rev. Lett. 118, 021101 (2017), arXiv:1610.01612

M. Wu and I. Tamborra, arXiv:1701.06580

Because of $\lambda=0$, the instability appears to be quite pervasive

-- not demanding specific
flavor dependent angular distributions.

But If the interactions are to be just Z exchange and ordinary ν-mass terms…..

Then there is no seed, and nothing happens????

A mass term would have to mix $\bar{\nu}$ and ν !

Otherwise---

TO THE RESCUE…..

“Quantum Break©” © Microsoft Corp.

A. Vardi , J. R. Anglin, Phys. Rev. Lett. 86, 568 (2001), arXiv: physics/0007054
(Applications to Bose condensates in atomic physics)

BEYOND MEAN FIELD THEORY

QUANTUM BREAK TIME

Back to the demo equations.

$$i\dot{\sigma}^{(+)} = gV^{-1}[\sigma^{(3)}\tau^{(+)} + \lambda\sigma^{(+)}\tau^{(3)}]$$

$$i\dot{\tau}^{(+)} = gV^{-1}[\tau^{(3)}\sigma^{(+)} + \lambda\sigma^{(3)}\tau^{(+)}]$$

$$i\dot{\sigma}^{(3)} = gV^{-1}[\tau^{(+)}\sigma^{(-)} - \tau^{(-)}\sigma^{(+)}]$$

plus 3 more

In the beginning σ and τ were bilinears in the operators a,b,c,d and their adjoints.

Expectations of these operators in the evolving system -> the flavor density matrix. BUT

$<\sigma^{(+)}\ \tau^{(3)}> \neq\ <\sigma^{(+)}><\tau^{(3)}>$ for later time even if we have it in the initial condition.

The implicit assumption before,

$<\sigma^{(+)}\ \tau^{(3)}> =\ <\sigma^{(+)}><\tau^{(3)}>$ was "Mean Field Theory"

The equations WERE for 6 functions, rather than for operators.

NOW without seeds or MFT we need the full 2 N component wave function.

For the ("marginally stable") case $\lambda = 1$ this problem was exactly solved in:

A. Friedland and C. Lunardini, JHEP {\bf 310}, 43 (2003) ; arXiv:hep-ph/0307140

In the limit of large N with fixed #density the time required for macroscopic changes is ~ $N^{1/2}$ $(g\ n)^{-1}$

Taking $N=10^{40}$, ...uh ..... we can't wait that long,

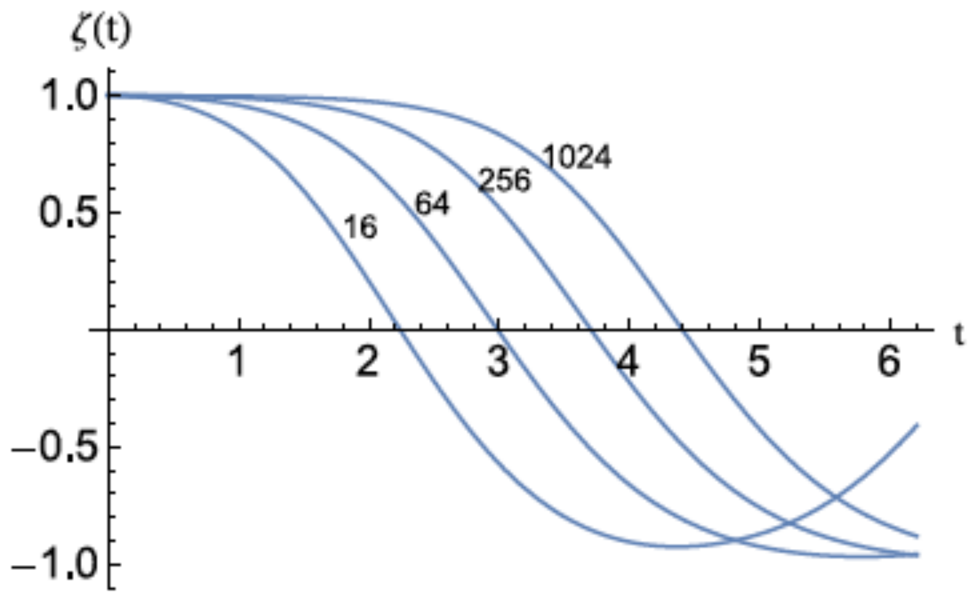
For the unstable (by MFT criteria, $\lambda=0$) case the outcome is very different:

Calculate wave-function $\psi(t)$ for $\lambda=0$. (6 N nonlinear, coupled 1st order ODE's). Lap-top will do up to N=1000. Plot

$$\zeta = < \sigma_3(t) >$$

λ=0 unstable but seedless

QUANTUM BREAKS for N= 16 to 1024



(RFS arXiv 1702.03013)

Time is in units $(n\,G_F)^{-1}$

BREAK TIME ~ $(n\,G_F)^{-1}$ Log[N]

Can we tolerate the Log[N] in a SN application? YES

A general statement ??

A system that:

1. has an N-component mean field description

   AND

2. is exactly at an unstable equilibrium point.

   moves rapidly away from that point AT (not "IN") :

   a QUANTUM BREAK TIME ~ log[N]

Can we trust eyeballing those curves for values N_{max}= 1000
to show that the leading behavior is exactly ~ log[N] ???

“Bogolyubov procedure” -- perhaps gives an improvement of MFT:

e.g. $< \tau^{(+)} \tau^{(-)} \sigma^{(3)} > \cong < \tau^{(+)} \tau^{(-)} > < \sigma^{(3)} >$ etc., etc.

from RFS: PRL 93, 13360 (2004) ; arxiv hep-ph/0402127 --

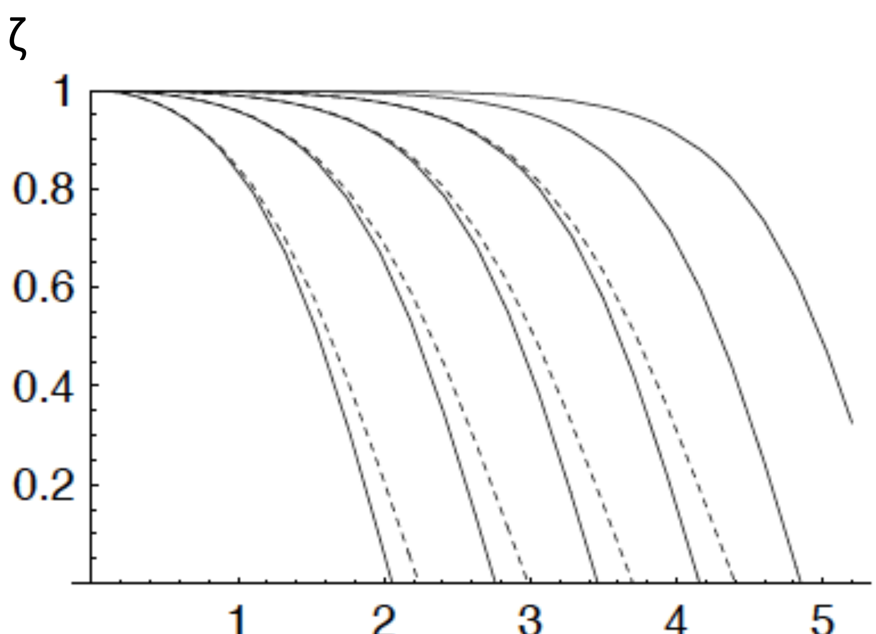
re: polarization exchange in ϒ+ϒ ->ϒ+ϒ (in vacuum)

$$\bar{w} = \langle \sigma^{(3)} \rangle$$

$$\frac{d^2}{dt^2}\bar{w} = -2(ng)^2\left[\bar{w}(1-\bar{w}^2) - \frac{4\bar{w}^2}{N}\right]$$

asymptotic Log[N] can now be proven for solutions

with initial conditions, $\bar{w}[0] = 1$, $\bar{w}'[0] = 0$.



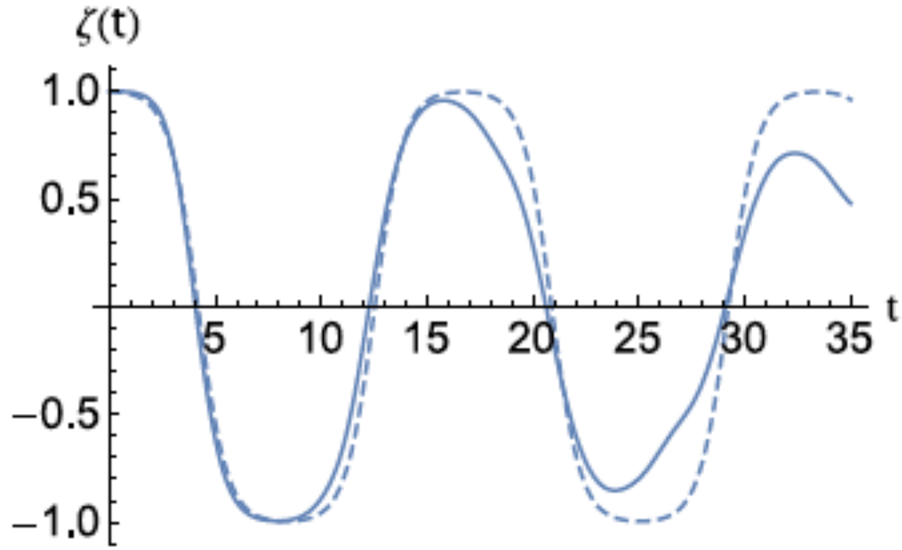
for N= 16 , 64 , 256 , 512 …. L to R

Solid= “Bogolyubov”

Dashed=exact

Helps a little, maybe

N=256 seeded MFT, compared to exact, with adjustment of time scale

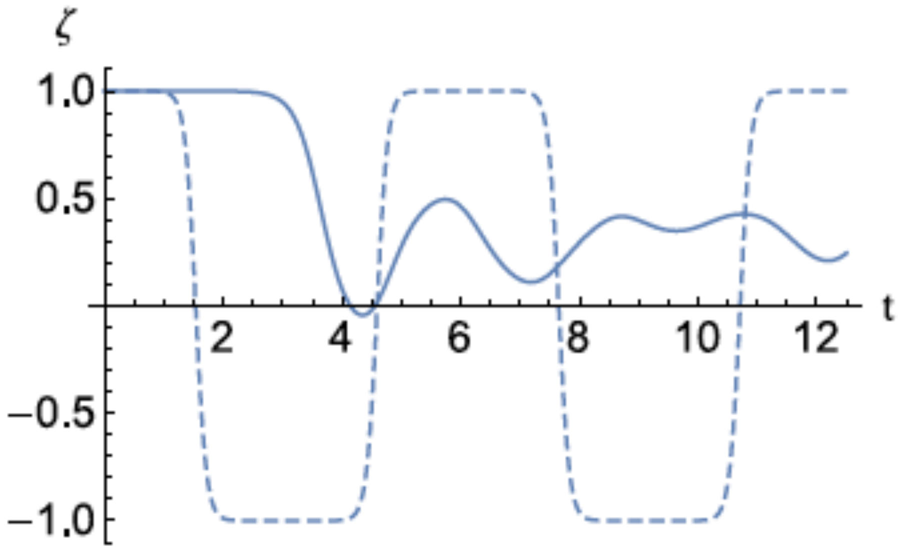


Dashed = seeded

What happens when the initial clouds are ISOTROPIC ?

In MFT we can take variables σ_i , τ_i i=1,…100 for q_i directions

uniformly distributed on sphere.



solid = isotropic

dashed= beams

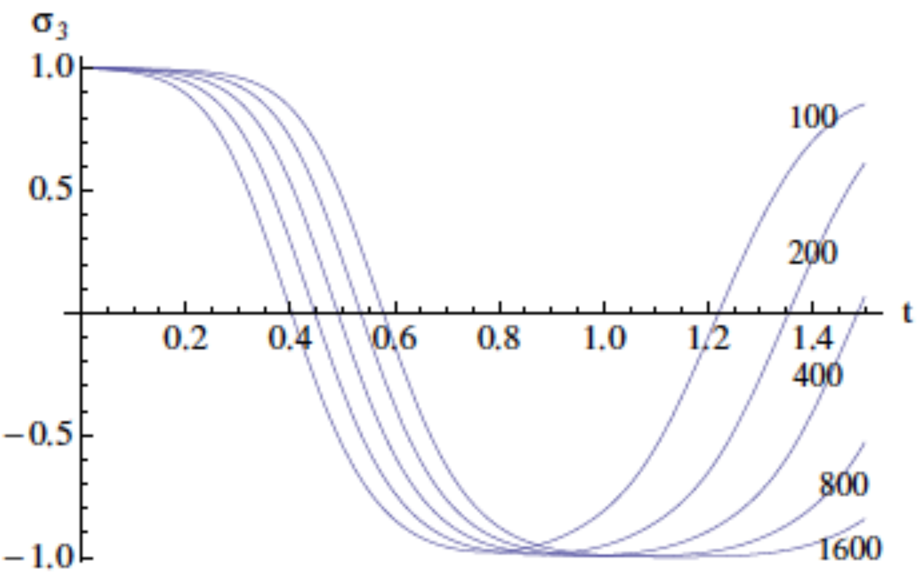
(Same # of ν 's in Vol.)

For an unseeded isotropic case :

Looking for the quantum break by solving for the N body wave function:

much too big for my laptop.

Head-on circularly polarized ϒ's exchanging spins—in an atomic gas

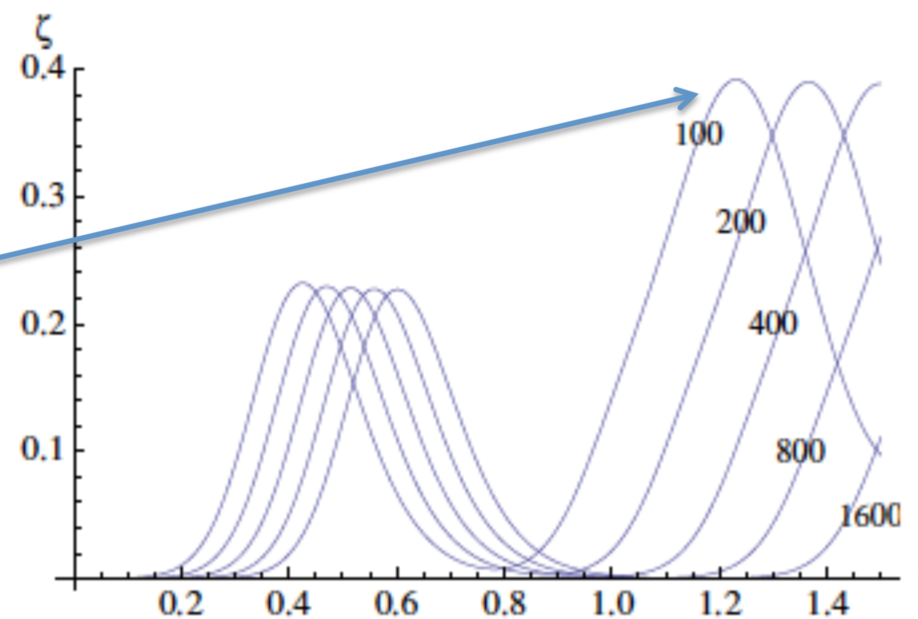


$<\sigma_3>$ = average circular polarization in one beam

$\zeta = - < \sigma_3 \quad \tau_3 > + < \sigma_3 > < \tau_3 >$

approaching Schrodinger cat-ness!

$\zeta = 0$ in MFT



GRAVITON + GRAVITON --> ϒ + ϒ

(helicities 2+2 ⟶ 1+1)

AMPLITUDE $M = -16\pi G|\mathbf{q}||\mathbf{k}|(\mathbf{1} - \mathbf{cos}\theta_{\mathbf{q},\mathbf{k}})$

N. E. J. Bjerrum-Bohr, B. R. Holstein, L. Planté, and P. Vanhove
Phys. Rev. D **91**, 064008 (2015)

$$H_{eff} = g\,(\mathrm{Vol.})^{-1} \sum_{\mathbf{p},\mathbf{q}} [\, \mathrm{a}_{\mathbf{p}}\, \mathrm{b}_{\mathbf{q}}\, \mathrm{c}^{\dagger}_{\mathbf{p}}\, \mathrm{d}^{\dagger}_{\mathbf{q}} + \mathrm{H.C}\ ](1 - \cos\theta_{\mathbf{p},\mathbf{q}})$$

a and b annihilate gravitons ; c and d annihilate photons

$g = 4\,\pi\, G$

G = gravitational const.= $6.7 \times 10^{-45}\ \mathrm{MeV}^{-2}$

begin with $\cos\theta = -1$ again

Could graviton number densities really ever be large enough to make this happen?

LIGO’s big event: ~250 Hz gravitons, from region (??) 100 km in radius has (for 10^{-3} sec) a high density of gravitons n. In our estimate (leaving out the log)

$n\,G = (100\text{ km})^{-1}$??

We fall short by the log of factor 100, it is true. But if we had been counting on scattering to mix up the species, then we would be short by a factor of 10^{85} .

Something enters here that did not enter in the earlier stuff

The classical motions of these giant objects generate coherent states of graviton --- with $\langle a_p \rangle \neq 0$.

If the objects have a trapped magnetic field and are then moved around they should generate EM radiation in some sense coherent with grav. Waves.

Then in a single-mode idealization we could have some tiny initial N_γ ---a seed--

and we can show that log[N] in the Break Time result is replaced by:

$\log[N/N_\gamma]$

Time scale then contains no $\hbar$ dependence

when N_γ is expressed as E^2 Vol. $/\omega$.

Homework problem: find the photon-graviton instability from classical GR .

CONCLUSIONS

1. A number of questions have been raised.

2. Some might be relevant to supernova;

(for example, "quantum breaks" ….

to complete the " ν , $\bar{\nu}$ " instability story)

3. It would be fun to see transfer of results and technology from our subject into other applications in physics.