Renormalization of SU(2) Yang–Mills theory with Flow Equations

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Main features

The renormalization theory based on Flow Equations (Wilson, Wegner, Polchinski) without using Feynman graphs applied to Perturbative Yang–Mills theory: Reuter, Wetterich; Becchi; Bonini, D'Attanasio, Marchesini (BAM); Morris; Fröb and Hollands...

The main features of the present work

- Distributions in four dimmensional Euclidean space.
- Bounds on the 1PI functions in momentum space.
- Convergence of 1PI functions in the UV limit.
- Slavnov–Taylor identities in the UV limit.
- Renormalization conditions are imposed at physical points.

Notations

Lie algebra in the adjoint representation $su(2) \to GL(3, \mathbb{R})$

$$(t_c)^{ab} = i\epsilon_{abc}, \qquad [t_a, t_b] = i\epsilon_{abc} t_c, \qquad a, b, c \in \{1, 2, 3\} .$$

Λ, Λ_0 are IR and UV cutoffs, $0 < \Lambda \leqslant \Lambda_0$.
M is a fixed mass parameter such that $0 < M \leqslant \Lambda_0$.

$$\delta(\sum_{i=0}^{n-1} p_i)\Gamma^{\vec{\phi}} = \prod_{i=0}^{n-1} \frac{\delta}{\delta \phi_i} \Gamma \Big|_{\vec{\phi}=0}, \qquad p_0 = -\sum_{i=1}^{n-1} p_i,$$

$$|\vec{p}|^2 = \sum_{i=0}^{n-1} p_i^2, \qquad \Gamma^{\vec{\phi};w} = \prod_{i=1}^{n-1} \left(\frac{\partial}{\partial p_i}\right)^{w_i} \Gamma^{\vec{\phi}},$$

$$K = \left(j_\mu^a, \quad b^a, \quad \bar{\eta}^a, \quad \eta^a\right), \qquad \Phi = \left(A_\mu^a, \quad B^a, \quad c^a, \quad \bar{c}^a\right),$$

$$\langle f, g \rangle = \int d^4x \, f(x) g(x) , \qquad K \cdot \Phi = \sum \langle \Phi_i, K_i \rangle,$$

$$\dot{\Gamma}^{\Lambda\Lambda_0} = \partial_\Lambda \Gamma^{\Lambda\Lambda_0},$$

Terminology

We define the η-function by

$$\eta(\vec{p}) = \min_{S\in\wp_{n-1}\setminus\{\emptyset\}} (|\sum_{i\in S} p_i|, M)\,.$$

where $\wp_{n-1}$ denotes the power set of $[n-1]$. A momentum configuration $\vec{p}$ is said nonexceptional iff $\eta(\vec{p}) \neq 0$.

- $[\Gamma^{\vec{\phi}}] < 0$ irrelevant,
- $[\Gamma^{\vec{\phi}}] \geqslant 0$ relevant,
- $[\Gamma^{\vec{\phi}}] = 0$ marginal,
- $[\Gamma^{\vec{\phi}}] > 0$ strictly relevant,

where $[...]$ stands for the mass dimension.

Faddeev–Popov quantization with Lorenz gauge fixing

Semiclassical Lagrangian density

$$
\begin{aligned}
\tilde{\mathcal{L}}_0^{tot} =& \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2 - \partial_\mu \bar{c}^a (D_\mu c)^a, \\
D_\mu c =& \partial_\mu c - ig[A_\mu, c], \\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu],
\end{aligned}
$$

$\xi > 0$ is the Feynman parameter.

$F_{\mu\nu}$, c, A_μ are elements of the su(2) algebra, e.g. $A_\mu = t^a A^a_\mu$.

The counterterms

All counterterms respecting the global symmetries and having ghost number zero

$$
\begin{aligned}
\mathcal{L}_{ct}^{\Lambda_0\Lambda_0} &= r^{\bar{c}c\bar{c}c}\bar{c}^b c^b \bar{c}^a c^a + r_1^{\bar{c}cAA}\bar{c}^b c^b A_\mu^a A_\mu^a + r_2^{\bar{c}cAA}\bar{c}^a c^b A_\mu^a A_\mu^b \\
&+ r_1^{A^4} A_\mu^b A_\nu^b A_\mu^a A_\nu^a + r_2^{A^3} A_\nu^b A_\nu^b A_\mu^a A_\mu^a + 2\epsilon_{abc} r^{A^3}(\partial_\mu A_\nu^a) A_\mu^b A_\nu^c \\
&- r_1^{A\bar{c}c}\epsilon_{abd}(\partial_\mu \bar{c}^a) A_\mu^b c^d - r_2^{A\bar{c}c}\epsilon_{abd}\bar{c}^a A_\mu^b \partial_\mu c^d + \Sigma^{\bar{c}c}\bar{c}^a \partial^2 c^a \\
&- \frac{1}{2}\Sigma_T^{AA} A_\mu^a(\partial^2\delta_{\mu\nu} - \partial_\mu\partial_\nu) A_\nu^a + \frac{1}{2\xi}\Sigma_L^{AA}(\partial_\mu A_\mu^a)^2 .
\end{aligned}
$$

There are eleven such counterterms which depend on Λ_0.
At tree level $\mathcal{L}_{ct}^{\Lambda_0\Lambda_0} = 0$.

The complex measure

Let $d\nu_{\Lambda\Lambda_0}$ be a Gaussian measure with the characteristic function

$$\chi^{\Lambda\Lambda_0}(j,b,\bar\eta,\eta) = e^{\frac{1}{\hbar}\langle\bar\eta,S^{\Lambda\Lambda_0}\eta\rangle - \frac{1}{2\hbar}\langle j,C^{\Lambda\Lambda_0}j\rangle - \frac{1}{2\hbar\xi}\langle b,b\rangle}.$$

To obtain AGE we need an auxiliary field B.

$$\boxed{d\mu_{\Lambda\Lambda_0}(A,B,c,\bar c) = d\nu_{\Lambda\Lambda_0}(A,B-i\frac{1}{\xi}\partial A,c,\bar c).}$$

$$C^{\Lambda\Lambda_0}_{\mu\nu} = \frac{1}{p^2}(\delta_{\mu\nu} + (\xi-1)\frac{p_\mu p_\nu}{p^2})\,\sigma_{\Lambda\Lambda_0}(p^2)\,,$$

$$S^{\Lambda\Lambda_0}(p) = \frac{1}{p^2}\,\sigma_{\Lambda\Lambda_0}(p^2)\,,\quad \sigma_{\Lambda\Lambda_0}(p^2) = e^{-\frac{p^4}{\Lambda_0^4}} - e^{-\frac{p^4}{\Lambda^4}}\,.$$

Generating functionals

Partition function of Yang–Mills theory

$$Z^{\Lambda\Lambda_0}(K) = \int d\mu_{\Lambda\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}L^{\Lambda_0\Lambda_0}}\; e^{\frac{1}{\hbar}K\cdot\Phi}.$$

$$\mathcal{L}^{\Lambda_0\Lambda_0} = \mathcal{L}_0^{\Lambda_0\Lambda_0} + \mathcal{L}_{ct}^{\Lambda_0\Lambda_0}\,,$$

$$\mathcal{L}_0^{\Lambda_0\Lambda_0} = g\epsilon_{abc}\partial_\mu A_\nu^a A_\mu^b A_\nu^c + \frac{g^2}{4}\epsilon_{cab}\epsilon_{cds}A_\mu^a A_\nu^b A_\mu^d A_\nu^s$$
$$- g\epsilon_{abc}\partial_\mu \bar{c}^a A_\mu^b c^c\,.$$

The tree level interaction $\mathcal{L}_0^{\Lambda_0\Lambda_0}$ does not depend on the B field.
Generating functional of the Connected Schwinger (CS) functions with regulator $\sigma_{\Lambda\Lambda_0}$

$$W^{\Lambda\Lambda_0} = \hbar\log Z^{\Lambda\Lambda_0}.$$

Generating functional of the Connected Amputated Schwinger (CAS) functions

$$L^{\Lambda\Lambda_0}(\Phi) = -\hbar\log\int d\mu_{\Lambda\Lambda_0}(\Phi')\, e^{-\frac{1}{\hbar}L^{\Lambda_0\Lambda_0}(\Phi'+\Phi)}$$

The BRST symmetry

The full tree level Lagrangian density in the limit $\Lambda \to 0$, $\Lambda_0 \to \infty$

$$\mathcal{L}_0^{tot} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{\xi}{2} B^2 - iB \partial_\mu A_\mu - \partial_\mu \bar{c} D_\mu c \, .$$

is invariant under the infinitesimal BRST transformation

$$\delta^{BRS} A = \epsilon \, Dc, \qquad \delta^{BRS} c = \epsilon \, \frac{1}{2} ig \{c, c\},$$

$$\delta^{BRS} \bar{c} = \epsilon \, iB, \qquad \delta^{BRS} B = 0,$$

where ϵ is a Grassmann parameter, and $\{c, c\}^d = i\epsilon_{abd} c^a c^b$. Defining the classical operator s

$$\delta^{BRS} \Phi = \epsilon \, s \, \Phi, \qquad s^2 = 0.$$

BRST invariance is broken by the regulators.

The flow equation for L (Wilson, Wegner, Polchinski...)

For any polynomial $P(\Phi)$ we have

$$\frac{d}{d\Lambda}\int d\mu_{\Lambda\Lambda_0}(\Phi)P(\Phi) = \hbar\int d\mu_{\Lambda\Lambda_0}(\Phi)\langle\frac{\delta}{\delta\Phi},\hat{\mathbf{1}}\dot{\mathbf{C}}\frac{\delta}{\delta\Phi}\rangle P(\Phi).$$

Using this equation one obtains the FE

$$\boxed{\dot{L}^{\Lambda\Lambda_0} = \frac{\hbar}{2}\langle\frac{\delta}{\delta\Phi},\hat{\mathbf{1}}\dot{\mathbf{C}}\frac{\delta}{\delta\Phi}\rangle L^{\Lambda\Lambda_0} - \frac{1}{2}\langle\frac{\delta L^{\Lambda\Lambda_0}}{\delta\Phi},\hat{\mathbf{1}}\dot{\mathbf{C}}\frac{\delta L^{\Lambda\Lambda_0}}{\delta\Phi}\rangle\ .}$$

$\mathbf{C}^{\Lambda\Lambda_0}$ is a 7x7 matrix,

$$\mathbf{C}^{\Lambda\Lambda_0} := \begin{pmatrix} C_{\mu\nu}^{\Lambda\Lambda_0}, & S^{\Lambda\Lambda_0}p_\mu, & 0, & 0 \\ -S^{\Lambda\Lambda_0}p_\nu, & \frac{1}{\xi}(1-\sigma_{\Lambda\Lambda_0}), & 0, & 0 \\ 0, & 0, & 0, & -S^{\Lambda\Lambda_0} \\ 0, & 0, & S^{\Lambda\Lambda_0}, & 0 \end{pmatrix}$$

The effective action

Let $K^{\Lambda\Lambda_0}(\underline{\Phi})$ be a solution of the system of equations

$$\underline{A} - \frac{\delta W^{\Lambda\Lambda_0}}{\delta j}\Big|_{K^{\Lambda\Lambda_0}(\underline{\Phi})} = 0, \qquad \underline{B} - \frac{\delta W^{\Lambda\Lambda_0}}{\delta b}\Big|_{K^{\Lambda\Lambda_0}(\underline{\Phi})} = 0,$$

$$\underline{c} - \frac{\delta W^{\Lambda\Lambda_0}}{\delta \bar{\eta}}\Big|_{K^{\Lambda\Lambda_0}(\underline{\Phi})} = 0, \qquad \underline{\bar{c}} + \frac{\delta W^{\Lambda\Lambda_0}}{\delta \eta}\Big|_{K^{\Lambda\Lambda_0}(\underline{\Phi})} = 0.$$

The effective action is

$$\Gamma^{\Lambda\Lambda_0}(\underline{\Phi}) = K^{\Lambda\Lambda_0}(\underline{\Phi}) \cdot \underline{\Phi} - W^{\Lambda\Lambda_0}(K^{\Lambda\Lambda_0}(\underline{\Phi}))\,.$$

Reduced effective action

$$\boxed{\mathsf{\Gamma}^{\Lambda\Lambda_0}(\underline{\Phi}) = \Gamma^{\Lambda\Lambda_0}(\underline{\Phi}) - \frac{1}{2}\langle \underline{\Phi}, \mathbf{C}^{-1}_{\Lambda\Lambda_0}\underline{\Phi}\rangle.}$$

The flow equation for Γ (Wetterich, BAM, Morris...)

$$W^{\Lambda\Lambda_0}(K) = \frac{1}{2}\langle K, \mathbf{C}^{\Lambda\Lambda_0} K\rangle - L^{\Lambda\Lambda_0}(\hat{\mathbf{1}}_{\bar{c}}\mathbf{C}^{\Lambda\Lambda_0} K), \tag{1}$$

$$\underline{\Phi} = \left(\Phi - \mathbf{C}^{\Lambda\Lambda_0} \frac{\delta L^{\Lambda\Lambda_0}}{\delta\Phi}\right)\Big|_{\Phi(\underline{\Phi})}, \tag{2}$$

$$\Gamma^{\Lambda\Lambda_0}(\underline{\Phi}) = \left(L^{\Lambda\Lambda_0}(\Phi) - \frac{1}{2}\langle(\underline{\Phi} - \Phi), \mathbf{C}^{-1}_{\Lambda\Lambda_0}(\underline{\Phi} - \Phi)\rangle\right)\Big|_{\Phi(\underline{\Phi})}. \tag{3}$$

Applying ∂_Λ to (3) and using the FE for L

$$\dot{\Gamma}^{\Lambda\Lambda_0}(\underline{\Phi}) = \frac{\hbar}{2}\langle\frac{\delta}{\delta\Phi}, \hat{\mathbf{1}}\dot{\mathbf{C}}^{\Lambda\Lambda_0}\frac{\delta}{\delta\Phi}\rangle L^{\Lambda\Lambda_0}\Big|_{\Phi(\underline{\Phi})}.$$

Finally using $\int d^4z\, W^{\Lambda\Lambda_0}_{xz}\hat{\mathbf{1}}_{\bar{c}}\Gamma^{\Lambda\Lambda_0}_{zy}\,\hat{\mathbf{1}}_c = \delta_{xy}$ the FE gets the form

$$\boxed{\dot{\Gamma} = \frac{\hbar}{2}\langle\dot{\mathbf{C}}\,\delta_\phi\delta_{\bar{\phi}}\Gamma\sum_{m=0}^{\infty}\left(-\hat{\mathbf{1}}\mathbf{C}\,\delta_{\bar{\phi}}\delta_\phi\Gamma\right)^m\rangle\,.}$$

A solution for the B field

Vanishing renormalization conditions for all relevant terms with at least one B field

$$\Gamma^{B\vec{\phi};0\Lambda_0;w}_{l;\vec{\varkappa}}(\vec{q}) = 0\,,$$

where $\vec{\varkappa} = (\gamma, \omega, \ldots)$ are sources of any local operator insertions with no dependence on the B-fields.

$$\Gamma^{B\vec{\phi};\Lambda\Lambda_0;w}_{l;\vec{\varkappa}}(\vec{q}) = 0\,.$$

$\Gamma^{B\vec{\phi};\Lambda_0\Lambda_0;w}_{l;\vec{\varkappa}} = 0 \implies$ no counterterms with the field B. Dependence on the B-fields known explicitly:

$$\Gamma^{\Lambda\Lambda_0}(\underline{A}, \underline{B}, \underline{c}, \underline{\bar{c}}) = \frac{1}{2\xi}\langle(\xi\underline{B} - i\partial\underline{A})^2\rangle + \tilde{\Gamma}^{\Lambda\Lambda_0}(\underline{A}, \underline{\bar{c}}, \underline{c})\,.$$

Violated Slavnov–Taylor Identities (VSTI)

Introduce the Lagrangian density

$$\mathcal{L}_{vst}^{\Lambda_0\Lambda_0} = \mathcal{L}^{\Lambda_0\Lambda_0} + \gamma\psi^{\Lambda_0} + \omega\Omega^{\Lambda_0},$$

where γ, ω are external sources, $R_i^{\Lambda_0} = 1 + O(\hbar)$,

$$\psi^{\Lambda_0} = R_1^{\Lambda_0}\partial c - igR_2^{\Lambda_0}[A, c], \qquad \Omega^{\Lambda_0} = \frac{1}{2i}gR_3^{\Lambda_0}\{c, c\}\,.$$

We will study the change of variables $\Phi \mapsto \Phi + \delta_\epsilon\Phi$

$$Z^{\Lambda\Lambda_0}(K) = \int d\mu_{\Lambda\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}L_{vst}^{\Lambda_0\Lambda_0}}\, e^{\frac{1}{\hbar}K\cdot\Phi}.$$

$$\delta_\epsilon A = \epsilon\,\sigma_{0\Lambda_0}\psi^{\Lambda_0}, \qquad \delta_\epsilon c = -\epsilon\,\sigma_{0\Lambda_0}\Omega^{\Lambda_0}, \qquad \delta_\epsilon \bar{c} = \epsilon\,\sigma_{0\Lambda_0} iB\,.$$

BRST invariance is broken

$$
\begin{aligned}
Q_\rho^{\Lambda_0} =& \frac{\delta L^{\Lambda_0\Lambda_0}}{\delta A_\mu}\sigma_{0\Lambda_0}\psi_\mu^{\Lambda_0} - \frac{\delta L^{\Lambda_0\Lambda_0}}{\delta c}\sigma_{0\Lambda_0}\Omega^{\Lambda_0} - \frac{1}{\xi}\frac{\delta L^{\Lambda_0\Lambda_0}}{\delta \bar{c}}\sigma_{0\Lambda_0}\partial A \\
& + AC^{-1}\psi^{\Lambda_0} - \bar{c}S^{-1}\Omega^{\Lambda_0} + \frac{1}{\xi}\partial A S^{-1}c\,, \\
Q_\beta^{\Lambda_0} =& \sigma_{0\Lambda_0}\Big(\frac{\delta L^{\Lambda_0\Lambda_0}}{\delta \bar{c}} - \partial\psi^{\Lambda_0}\Big) - S^{-1}c\,.
\end{aligned}
$$

$$
\boxed{\Big(\langle\hbar\frac{\delta}{\delta\rho(x)}\rangle + i\frac{1}{\xi}\langle b, \hbar\frac{\delta}{\delta\beta}\rangle\Big) Z_{aux}^{0\Lambda_0}\Big|_{\rho,\beta=0} = \mathcal{S} Z_{aux}^{0\Lambda_0}\Big|_{\rho,\beta=0}}
$$

$$
\begin{aligned}
\mathcal{S} =& \langle j, \sigma_{0\Lambda_0}\hbar\frac{\delta}{\delta\gamma}\rangle + \langle\bar{\eta}, \sigma_{0\Lambda_0}\hbar\frac{\delta}{\delta\omega}\rangle - i\langle\sigma_{0\Lambda_0}\hbar\frac{\delta}{\delta b}, \eta\rangle, \\
\mathcal{L}_{aux}^{\Lambda_0\Lambda_0} =& \mathcal{L}_{vst}^{\Lambda_0\Lambda_0} + \beta Q_\beta^{\Lambda_0} + \rho Q_\rho^{\Lambda_0} + \rho\gamma_\mu^a Q_{\rho\gamma_\mu^a}^{\Lambda_0} + \rho\omega^a Q_{\rho\omega^a}^{\Lambda_0}\,.
\end{aligned}
$$

$$\boxed{\hbar \int d^4x \left(W_{\rho(x)} + \frac{i}{\xi} b_x W_{\beta(x)} \right) = \mathcal{S} W^{0\Lambda_0}}$$

$$W^{0\Lambda_0}_{\rho} = \frac{\delta W^{0\Lambda_0}_{aux}}{\delta\rho}\Big|_{\substack{\rho=0\\ \beta=0}} \qquad W^{0\Lambda_0}_{\beta} = \frac{\delta W^{0\Lambda_0}_{aux}}{\delta\beta}\Big|_{\substack{\rho=0\\ \beta=0}}$$

The Legendre transform gives the BRST anomalies Γ_1 and Γ_β

$$\Gamma^{0\Lambda_0}_{\chi} = \frac{\delta \Gamma^{0\Lambda_0}_{aux}}{\delta\chi}\Big|_{\substack{\rho=0\\ \beta=0}} \qquad \chi \in \{\rho, \beta\}, \qquad \Gamma^{0\Lambda_0}_{1} = \int d^4x\, \Gamma^{0\Lambda_0}_{\rho(x)}$$

The operator $\mathcal{S}$ in a generalized form

$$\Gamma_1^{0\Lambda_0} + \langle (i\underline{B} + \frac{1}{\xi}\partial\underline{A}), \Gamma_\beta^{0\Lambda_0}\rangle = \frac{1}{2}\mathcal{S}\underline{\Gamma}^{0\Lambda_0},$$

where we have introduced an auxiliary functional $\underline{\Gamma}^{0\Lambda_0}$

$$\underline{\Gamma}^{0\Lambda_0} = i\langle \underline{B}, \bar{\omega}\rangle + \underline{\tilde{\Gamma}}^{0\Lambda_0}, \qquad \underline{\tilde{\Gamma}}^{0\Lambda_0} = \tilde{\Gamma}^{0\Lambda_0} + \frac{1}{2\xi}\langle \underline{A}, \partial\partial\,\underline{A}\rangle\,,$$

and $\mathcal{S} = \mathcal{S}_{\tilde{c}} + \mathcal{S}_A - \mathcal{S}_c$, with

$$\mathcal{S}_\phi = \langle \frac{\delta\underline{\Gamma}^{0\Lambda_0}}{\delta\underline{\phi}}, \sigma_{0\Lambda_0}\frac{\delta}{\delta\phi^\star}\rangle + \langle \frac{\delta\underline{\Gamma}^{0\Lambda_0}}{\delta\phi^\star}, \sigma_{0\Lambda_0}\frac{\delta}{\delta\underline{\phi}}\rangle,$$

$$(\underline{\phi}, \phi^\star) \in \{(\underline{A}, \gamma), (\underline{c}, \omega), (\underline{\tilde{c}}, \bar{\omega})\}, \quad \frac{\delta}{\delta\underline{\tilde{c}}} = \frac{\delta}{\delta\underline{\bar{c}}} - \partial\frac{\delta}{\delta\gamma}\,.$$

Nilpotency of S

We rewrite equation VSTI in the following form

$$\langle i\underline{B}, \tilde{\Gamma}^{0\Lambda_0}_{\beta}\rangle = \frac{1}{2}\mathcal{S}_{\tilde{c}}\underline{\Gamma}^{0\Lambda_0} = \langle i\underline{B}, \sigma_{0\Lambda_0}\frac{\delta}{\delta\tilde{c}}\tilde{\underline{\Gamma}}^{0\Lambda_0}\rangle,$$
$$\tilde{\mathrm{F}}^{0\Lambda_0}_{1} = \frac{1}{2}\tilde{S}\tilde{\underline{\Gamma}}^{0\Lambda_0},$$

where

$$\tilde{S} = \mathcal{S}_A - \mathcal{S}_c, \qquad \tilde{\mathrm{F}}^{0\Lambda_0}_{1} = \tilde{\Gamma}^{0\Lambda_0}_{1} + \frac{1}{\xi}\langle \partial A, \tilde{\Gamma}^{0\Lambda_0}_{\beta}\rangle\,.$$

Important properties of $\mathcal{S}_\phi$: $\forall \phi, \phi' \in \{A, c, \tilde{c}\}$

$$(\mathcal{S}_\phi\mathcal{S}_{\phi'} + \mathcal{S}_{\phi'}\mathcal{S}_\phi)\underline{\Gamma}^{0\Lambda_0} = 0.$$

It follows that $\tilde{S}^2\tilde{\underline{\Gamma}}^{0\Lambda_0} = 0$, $\tilde{S}\mathcal{S}_{\tilde{c}} = -\mathcal{S}_{\tilde{c}}\tilde{S}$ and consequently

$$\tilde{S}\tilde{\mathrm{F}}^{0\Lambda_0}_{1} = 0, \qquad \tilde{S}\tilde{\Gamma}_{\beta} + \sigma_{0\Lambda_0}\left(\frac{\delta}{\delta\underline{c}} - \partial\frac{\delta}{\delta\gamma}\right)\tilde{\mathrm{F}}^{0\Lambda_0}_{1} = 0.$$

Finally setting $\gamma, \omega = 0$ we get the violated antighost equation (VAGE) and the violated Slavnov–Taylor identities (VSTI)

$$\tilde{\Gamma}_{\beta}^{0\Lambda_0} = \sigma_{0\Lambda_0}\Big(\frac{\delta \tilde{\underline{\Gamma}}^{0\Lambda_0}}{\delta \underline{\bar{c}}} - \partial \tilde{\underline{\Gamma}}_{\gamma}^{0\Lambda_0}\Big) \qquad (VAGE),$$

$$\tilde{\mathrm{F}}_{1}^{0\Lambda_0} = \langle \frac{\delta \tilde{\underline{\Gamma}}^{0\Lambda_0}}{\delta \underline{A}}, \sigma_{0\Lambda_0} \tilde{\underline{\Gamma}}_{\gamma}^{0\Lambda_0} \rangle - \langle \frac{\delta \tilde{\underline{\Gamma}}^{0\Lambda_0}}{\delta \underline{c}}, \sigma_{0\Lambda_0} \tilde{\underline{\Gamma}}_{\omega}^{0\Lambda_0} \rangle \qquad (VSTI).$$

The goal is to show that $\tilde{\Gamma}_{\beta}^{0\infty} = 0$, $\tilde{\mathrm{F}}_{1}^{0\infty} = 0$ which imply $\tilde{\Gamma}_{1}^{0\infty} = 0$.

Another form of the VSTI

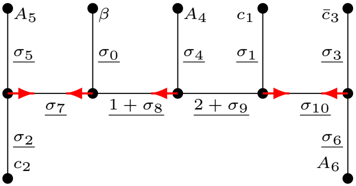
$$\tilde{\Gamma}_{1}^{0\Lambda_0} = \langle \frac{\delta \tilde{\Gamma}^{0\Lambda_0}}{\delta \underline{A}}, \sigma_{0\Lambda_0} \tilde{\Gamma}_{\gamma}^{0\Lambda_0} \rangle - \langle \frac{\delta \tilde{\Gamma}^{0\Lambda_0}}{\delta \underline{c}}, \sigma_{0\Lambda_0} \tilde{\Gamma}_{\omega}^{0\Lambda_0} \rangle - \frac{1}{\xi} \langle \partial \underline{A}, \sigma_{0\Lambda_0} \frac{\delta \tilde{\Gamma}^{0\Lambda_0}}{\delta \underline{\bar{c}}} \rangle \, .$$

Renormalization conditions

- $\Gamma^{0\Lambda_0;\vec{\phi};w}_{\vec{\varkappa}}(0) = 0$, for all strictly relevant terms.
- $\Gamma^{M\Lambda_0;c\bar{c}c\bar{c}}(0) = 0$, $\Gamma^{M\Lambda_0;c\bar{c}A^2}(0) = 0$, $\partial_A \Gamma^{M\Lambda_0;c\bar{c}A}(0) = 0$, We show that the counterterms $r^{\bar{c}c\bar{c}c}$, $r_1^{\bar{c}cA^2}$, $r_2^{\bar{c}cA^2}$, $r_2^{A\bar{c}c}$ vanish.
- The renormalization constants r^{A^3} , Σ_T^{AA} , $\Sigma^{\bar{c}c}$ are free.
- The remaining renormalization constants must satisfy 7 additional relations in order to make the marginal terms $\Gamma_1^{\vec{\phi};w}$, $\Gamma_\beta^{\vec{\phi};w}$ at the renormalization point comply with the bounds **3** and **4** below.
- We prove the existence of a solution for this system of relations that does not depend on the UV cutoff.

Trees $\mathcal{T}_{\vec{\varphi}}$ and $\mathcal{T}_{1\vec{\varphi}}$

- Vertices of valence one and three, $V = V_1 \cup V_3$.
- E_φ is the set of edges incident to $V_\varphi \subset V_1$.
- Edges may have "*" labels. $E_* = \{e \in E : \text{labeled by } *\}$.
  $\mathcal{T}_{1\vec{\varphi}}$ are $\mathcal{T}_{\vec{\varphi}}$ are trees with one or zero "*"-edges.
- An edge e carries momentum p_e. Momentum conservation at the vertices.
- To any edge are associated $\rho(e) \in \{0, 1, 2\}$ and the number of momentum derivatives $\sigma(e)$.
- An edge has the θ-weight, $\theta(e) = \rho(e) + \sigma(e)$.



$$\chi : V_\bullet \to E \backslash E_1$$

$$\rho(e) = 2 - |\chi^{-1}(e)|$$

The main elements of the bounds

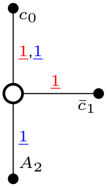
For a tree τ we sum over the family of θ-weights $\Theta^w_\tau = \{\theta_j(e)\}_j$.

$$\Pi^{\Lambda}_{\tau,\theta}(\vec{p}) = \prod_{e\in E} \frac{1}{(\Lambda + |p_e|)^{\theta(e)}} \,,$$

$$Q^{\Lambda;w}_{\tau}(\vec{p}) = \frac{\prod\limits_{e\in E_*} (\Lambda + |p_e|)}{\prod\limits_{e\in E_\varkappa} (\Lambda + |p_e|)} \begin{cases} \inf\limits_{i\in\mathbb{I}} \sum\limits_{\Theta^{w'(i)}_\tau} \Pi^{\Lambda}_{\tau,\theta}(\vec{p}), & |V_1| = 3\,, \\ \sum\limits_{\Theta^{w}_\tau} \Pi^{\Lambda}_{\tau,\theta}(\vec{p}), & otherwise\,. \end{cases}$$

$w'(i)$ is obtained from w by diminishing w_i by one unit, and, for nonvanishing w, $\mathbb{I} = \{i : w_i > 0\}$.

$$Q^{\Lambda;(0,1,1)}_{c\bar{c}A} \in \inf \begin{cases} \frac{1}{\Lambda+|p_{\bar{c}}|} + \frac{1}{\Lambda+|p_c|} & w'(2) = (0, 1, 0) \\ \frac{1}{\Lambda+|p_A|} + \frac{1}{\Lambda+|p_c|} & w'(1) = (0, 0, 1) \end{cases}$$



Bounds on vertex functions

Let $d = 4 - 2n_{\varkappa} - \mathrm{N} - \|w\|$.

1.a) $d \geqslant 0$ or $\mathrm{N} + n_{\varkappa} = 2$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}(\vec{p})| \leqslant (\Lambda + |\vec{p}|)^d P_r^{\Lambda\Lambda}(\vec{p})\,,$$

1.b) $d < 0$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}(\vec{p})| \leqslant \sum_{\tau \in \mathcal{T}_{\vec{\varkappa}\vec{\phi}}} Q_{\tau}^{\Lambda;w}(\vec{p})\; P_r^{\Lambda\Lambda}(\vec{p})\,,$$

$$P_r^{\Lambda\Lambda'}(\vec{p}) = \mathcal{P}_r^{(0)}\left(\log_+ \frac{\max(|\vec{p}|, M)}{\Lambda + \eta(\vec{p})}\right) + \mathcal{P}_r^{(1)}\left(\log_+ \frac{\Lambda'}{M}\right).$$

$\mathcal{P}_r$ denotes polynomials with nonnegative coefficients and degree r. r is linear in the loop number l.

Convergence

Let $d = 4 - 2n_{\varkappa} - \mathrm{N} - \|w\|$.

2.a) $d \geqslant 0$ or $\mathrm{N} + n_{\varkappa} = 2$

$$|\partial_{\Lambda_0} \Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}(\vec{p})| \leqslant \frac{\Lambda + M + |\vec{p}|}{\Lambda_0^2} (\Lambda + |p|)^d P_r^{\Lambda\Lambda_0}(\vec{p}) \,,$$

2.b) $d < 0$

$$|\partial_{\Lambda_0} \Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}(\vec{p})| \leqslant \frac{\Lambda + M + |\vec{p}|}{\Lambda_0^2} \sum_{\tau \in \mathcal{T}_{\vec{\varkappa}\vec{\phi}}} Q_\tau^{\Lambda;w}(\vec{p})\, P_r^{\Lambda\Lambda_0}(\vec{p}) \,.$$

Cauchy criterion

$$|\Gamma^{\Lambda\Lambda_0';\vec{\phi};w}_{\vec{\varkappa};l} - \Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}| \leqslant \int_{\Lambda_0}^{\Lambda_0'} d\lambda_0 \, |\partial_{\lambda_0} \Gamma^{\Lambda\lambda_0;\vec{\phi};w}_{\vec{\varkappa};l}| \,.$$

The restoration of AGE

Let $d = 3 - 2n_{\varkappa} - \mathrm{N} - \|w\|$.

3.a) $d \geqslant 0$ or $\mathrm{N} + n_{\varkappa} = 1$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\beta\vec{\varkappa};l}(\vec{p})| \leqslant \frac{M + |\vec{p}| + \Lambda}{\Lambda_0}(\Lambda + |\vec{p}|)^d P^{\Lambda\Lambda_0}_{r_\beta\, s_\beta}(\vec{p})\,,$$

3.b) $d < 0$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{\beta\vec{\varkappa};l}(\vec{p})| \leqslant \frac{M + |\vec{p}| + \Lambda}{\Lambda_0} \sum_{\tau \in \mathcal{T}_{\beta\vec{\varkappa}\vec{\phi}}} Q^{\Lambda;w}_{\tau}(\vec{p})\, P^{\Lambda\Lambda_0}_{r_\beta\, s_\beta}(\vec{p})\,,$$

$$P^{\Lambda\Lambda_0}_{r\,s}(\vec{p}) = \left(1 + \left(\frac{|\vec{p}|}{\Lambda_0}\right)^4\right)\mathcal{P}^{(2)}_s\left(\frac{|\vec{p}|}{\Lambda + M}\right) P^{\Lambda\Lambda_0}_r(\vec{p})\,.$$

The restoration of STI

Let $d = 5 - 2n_{\varkappa} - \mathrm{N} - \|w\|$.

4.a) $d > 0$ or $\mathrm{N} + n_{\varkappa} = 2$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{1\vec{\varkappa};l}(\vec{p})| \leqslant \frac{M + |\vec{p}| + \Lambda}{\Lambda_0}(\Lambda + |\vec{p}|)^d P^{\Lambda\Lambda_0}_{r_1\, s_1}(\vec{p})\,,$$

4.b) $d \leqslant 0$

$$|\Gamma^{\Lambda\Lambda_0;\vec{\phi};w}_{1\vec{\varkappa};l}(\vec{p})| \leqslant \frac{M + |\vec{p}| + \Lambda}{\Lambda_0} \sum_{\tau \in \mathcal{T}_{1\vec{\varkappa}\vec{\phi}}} Q^{\Lambda;w}_{\tau}(\vec{p})\, P^{\Lambda\Lambda_0}_{r_1\, s_1}(\vec{p})\,.$$

An overview of the proof

We prove the theorems one by one in this order 1-4.

We have to verify that the bounds hold at tree level.

For each of them we proceed by induction in the loop order l.

The irrelevant terms are constructed by integrating the FE from Λ_0 down to Λ.

$$\Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w} = \Gamma_l^{\Lambda_0\Lambda_0;\vec{\phi};w} + \int_{\Lambda_0}^{\Lambda} d\lambda\, \dot{\Gamma}_l^{\lambda\Lambda_0;\vec{\phi};w}, \quad \Gamma_l^{\Lambda_0\Lambda_0;\vec{\phi};w} = 0$$

$$\dot{\Gamma}^{\vec{\phi}} = \frac{\color{red}{\hbar}}{2} \sum_{\vec{\Phi}=(\vec{\phi}_1,\ldots)} (-)^{\pi_a} \langle \dot{C}\mathcal{F}^{A\vec{\Phi}A} + \dot{S}(\mathcal{F}^{\bar{c}\vec{\Phi}c} - \mathcal{F}^{c\vec{\Phi}\bar{c}})\rangle$$

$$\mathcal{F}^{\zeta_1\vec{\Phi}\bar{\zeta}_m} = \Gamma^{\zeta_1\vec{\phi}_1\bar{\zeta}_1} \prod_{j=2}^{m} \mathbf{C}_{\zeta_j\bar{\zeta}_{j-1}} \Gamma^{\zeta_j\vec{\phi}_j\bar{\zeta}_j}, \quad \Gamma_{l=0}^{\zeta\bar{\zeta}} = 0$$

For the relevant terms we integrate the FE from $\Lambda = 0$ up to arbitrary Λ at the renormalization point $\vec{q}$

$$|\Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w}(\vec{q}) - \Gamma_l^{0\Lambda_0;\vec{\phi};w}(\vec{q})| \leqslant \int_0^{\Lambda} d\lambda \, |\dot{\Gamma}_l^{\lambda\Lambda_0;\vec{\phi};w}(\vec{q})|$$

We interpolate this to arbitrary momenta $\vec{p}$ integrating over the corresponding irrelevant terms;

$$|\Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w}(\vec{p}) - \Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w}(\vec{q})| \leqslant \int_{\vec{q}}^{\vec{p}} |\partial\Gamma_l^{\Lambda\Lambda_0;\vec{\phi};w}|$$

Renormalization conditions for $\Gamma_1^{\vec{\phi};w}$ and $\Gamma_\beta^{\vec{\phi};w}$ are given by the rhs of VSTI and VAGE respectively.

Thank you!

https://arxiv.org/abs/1704.06799