Weak signals of quantum spacetime from electromagnetic interactions of neutral dark matter

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joint work with S. Doplicher, K. Fredenhagen N. Pinamonti
Phys. Rev. D 95 (2017)

Foundational and Structural Aspects of Gauge Theories

Mainz Institute of Theoretical Physics, May 29 - June 2, 2017

Outline

Introduction

Generally accepted view: continuous description of spacetime should break down at distance of the order of the Planck length $\lambda_P = \sqrt{\hbar G/c^3} \simeq 1.6 10^{-33}$ cm

- analysis of [Doplicher, Fredenhagen, Roberts '95]: QM + GR $\Rightarrow$ accuracies Δq^μ on spacetime coordinates of an event in Minkowski satisfy Spacetime Uncertainty Relations (STUR), implemented by commutation relations between the q^μ's
- Minkowski spacetime replaced by a Quantum (noncommutative) Spacetime $\mathcal{E}$ (C*-algebra generated by q^μ's)
- QFT on QST has remarkable properties [Bahns, Doplicher, Fredenhagen, Piacitelli '01,'03,'04,...]
- it can also serve as a (partial) substitute of inflation [Doplicher, M., Pinamonti '13]

This talk:

Look for physical effects that can be explained only by QST

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Operational limitations to localizability of events

QM: need energy $E \simeq 1/L$ to prepare a quantum state localized in a small region of size L
GR: large energy E creates a trapped surface (event horizon) of Schwarzschild radius $r \simeq E$ around localization region

$$\Downarrow$$

localization has operational meaning only if $L \geq r$ i.e. $L \gtrsim 1 = \lambda_P$ (in natural units)

Principle of gravitational stability against localization

The gravitational field generated by the concentration of energy required by the Heisenberg Uncertainty Principle to localize an event in spacetime should not be so strong to hide the event itself to any distant observer

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Spacetime Uncertainty Relations

If only one coordinate is localized with high precision, TS will not form: transferred energy density goes to zero

[DFR] analysis:

- quantum state localized in region supp f of sizes Δq^μ, $\mu = 0, \dots, 3$

$$\omega_f(A) = \langle e^{i\phi(f)}\Omega, A e^{i\phi(f)}\Omega \rangle$$

  energy $E \simeq 1/\min_\mu\{\Delta q^\mu\} \implies$ energy density ρ
- solution of linearized Einstein equations with source ρ given by retarded potential
- condition of non formation of TS: $g_{00} > 0$

Spacetime Uncertainty Relations (STURs)

$$\Delta q^0 \sum_{j=1}^{3} \Delta q^j \geq \lambda_P^2, \qquad \sum_{i<j=1}^{3} \Delta q^i \Delta q^j \geq \lambda_P^2$$

Necessary conditions imposed by the principle of gravitational stability

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Quantum Spacetime

STURs can be realized by assuming that Δq^μ's are standard deviations of quantum operators q^μ satisfying suitable commutation relations, as for Heisenberg uncertainty relations

Quantum Conditions

$$[q^\mu, q^\nu] = i\lambda_P^2 Q^{\mu\nu}, \quad [q^\rho, Q^{\mu\nu}] = 0,$$

$$Q_{\mu\nu}Q^{\mu\nu} = 0, \quad \left(\frac{1}{4}Q^{\mu\nu}(*Q)_{\mu\nu}\right)^2 = 1$$

- Noncommutative C*-algebra $\mathcal{E}$ of Quantum Spacetime (QST) generated by q^μ's replaces algebra of functions on Minkowski
- It is equipped with action of the Poincaré group $q^\mu \to \Lambda^\mu_\nu q^\nu + a^\nu$
- $\mathcal{E}$ has nontrivial center $Z(\mathcal{E})$ = functions on a manifold $\Sigma \simeq T\mathbb{S}^2 \times \mathbb{Z}_2$ and $\mathcal{E} \simeq C_0(\Sigma, \mathcal{K})$, $\mathcal{K}$ = compact operators

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Optimal localization on QST

In an irreducible representation q^μ is a Lorentz transform of Schroedinger's (x_1, x_2, p_1, p_2)

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There exists states of optimal localization ω on $\mathcal{E}$, minimizing

$$\sum_\mu (\Delta q^\mu)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta p_1)^2 + (\Delta p_2)^2$$

given by translates of the harmonic oscillator ground states
They are the best approximation of points on QST

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Free quantum fields on QST

ϕ free (scalar) field on Minkowski can be defined on QST through
Weyl-von Neumann-Moyal quantizazion

$$\phi(q) = \int d^4k \, \check{\phi}(k) \otimes e^{ikq}$$

(formal) element of $\mathfrak{F} \otimes \mathcal{E}$, $\mathfrak{F}$ field algebra

- it satisfies Klein-Gordon equation (derivatives on $\mathcal{E}$ defined by $\partial_\mu \phi(q) := \frac{\partial}{\partial x^\mu} \phi(q + x\mathbb{1})$)
- ω_x, ω_y optimally localized states around $x, y \implies$ $[\mathrm{id} \otimes \omega_x(\phi(q)), \mathrm{id} \otimes \omega_y(\phi(q))]$ falls off as a Gaussian of width λ_P for large spacelike $x - y$

Locality is lost at distances small w.r.t. λ_P, but recovered as $\lambda_P \to 0$

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(Perturbative) interacting fields on QST

Several (inequivalent) possibilities of defining perturbative interacting fields

- Hamiltonian approach (interaction picture) with interaction Lagrangian defined by $:\phi(q)^n:$ [DFR]
- Yang-Feldman equation and quasi-planar Wick products [Bahns, Doplicher, Fredenhagen, Piacitelli '02 & '04]
- Hamiltonian approach with interaction defined by quantum Wick product $:\phi^n(q):_Q$, which yields UV-finite (IR-cutoff) theory to all orders [Bahns, Doplicher, Fredenhagen, Piacitelli '03]

$:\phi^n(q):_Q$ defined by generalizing point-splitting to QST:
e.g., for $n = 2$

$$:\phi^2:(x) := \lim_{y \to x} \phi(x)\phi(y) - \langle \Omega, \phi(x)\phi(y)\Omega \rangle$$

limit $y \to x$ has to be performed in a way compatible with the STURs

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Quantum Wick product

- Introduce quantum coordinates of independent events

$$q_1^\mu := q^\mu \otimes \mathbb{1}, \qquad q_2^\mu := \mathbb{1} \otimes q^\mu$$

tensor product of Z-moduli $\implies [q_1^\mu, q_1^\nu] = i\lambda_P^2 Q^{\mu\nu} = [q_2^\mu, q_2^\nu]$

- introduce center of mass and relative coordinates

$$\bar{q}^\mu := \frac{1}{2}(q_1^\mu + q_2^\mu), \qquad \xi^\mu := \frac{1}{\lambda_P}(q_1^\mu - q_2^\mu)$$

identification of commutators $\implies [\bar{q}^\mu, \xi^\nu] = 0$

- evaluating optimally localized state on ξ^μ yields a map $E^{(2)} : \mathcal{E} \otimes_Z \mathcal{E} \to \mathcal{E} \simeq C^*(\bar{q}^\mu)$

Quantum Wick product

$$\begin{aligned} :\phi^2(\bar{q}):_Q &:= E^{(2)}(:\phi(q_1)\phi(q_2):) \\ &= \int d^4k_1 d^4k_2 :\check{\phi}(k_1)\check{\phi}(k_2): e^{-\frac{\lambda_P^2}{4}|k_1-k_2|^2} e^{i(k_1+k_2)\bar{q}} \end{aligned}$$

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U(1) gauge theory on quantum spacetime 1/3

- gauge group of U(1) gauge theory on commutative spacetime: $\mathcal{U}(C_b(\mathbb{R}^4))$
- on QST replaced by $\mathcal{G} := \mathcal{U}(M(\mathcal{E}))$
- it has a nontrivial action on a real scalar field $\varphi(q)$:

$$\varphi(q) \to U\varphi(q)U^*, \quad U \in \mathcal{G}$$

- physicist's recipe to write down an invariant Lagrangian: replace ∂_μ with a covariant derivative D_μ, i.e., derivation on $\mathcal{E}$ transforming under $\mathcal{G}$ as

$$D_\mu\varphi(q) \to UD_\mu\varphi(q)U^*$$

U(1) gauge theory on quantum spacetime 2/3

Solution:

$$D_\mu\varphi(q) = \partial_\mu\varphi(q) - ie[A_\mu(q), \varphi(q)]$$

where A_μ is a field transforming under $\mathcal{G}$ as

$$A_\mu(q) \to UA_\mu(q)U^* + \frac{i}{e}U\partial_\mu U^*$$

- writing $U = e^{i\Lambda}$ and going to commutatitve spacetime the trasformation law of A_μ becomes

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\Lambda(x)$$

  $\Rightarrow A_\mu(q)$ is identified with the electromagnetic potential on QST
- coupling constant e is the electron charge, since A_μ has to interact also with electron field ψ transforming as $\psi \to U\psi$
- Problem: how to accommodate quark fields? Discussed by [Schupp, Wess, ... $\simeq$ '00] using (perturbative) Seiberg-Witten map

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U(1) gauge theory on quantum spacetime 3/3

Gauge invariant lagrangian for φ:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2} D_\mu\varphi(q) D^\mu\varphi(q) - \frac{1}{2} m^2 \varphi(q)^2 \\ &= \frac{1}{2}\partial_\mu\varphi(q)\partial^\mu\varphi(q) - \frac{1}{2} m^2\varphi(q)^2 \\ &\underbrace{-\frac{ie}{2}\{[A_\mu(q),\varphi(q)],\partial^\mu\varphi(q)\} - \frac{e^2}{2}[A_\mu(q),\varphi(q)][A^\mu(q),\varphi(q)]}_{\mathcal{L}_I}\end{aligned}$$

commutators $\Rightarrow \mathcal{L}_I = 0$ on commutative spacetime, effective coupling $e\lambda_P^2$

For $A(q) = (0, \frac{1}{2}\boldsymbol{B}\wedge\boldsymbol{q})$ (uniform external magnetic field) and neglecting A^2, $\mathcal{L}_I$ is the energy of interaction with a magnetic moment density

$$M_j = \frac{e\lambda_P^2}{2}\left[\frac{1}{2}(\{\partial_l\varphi,\partial^l\varphi\}\delta_{jk} - \{\partial_j\varphi,\partial^k\varphi\})m_k - \varepsilon_{jkh}\{\partial_0\varphi,\partial^h\varphi\}e_k\right]$$

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Possibly observable consequences? 1/3

- lightest stable dark matter particle should be neutral, but on QST it could actually emit (very weak) em radiation with no background
- in order to obtain observable signal one would need very large magnetic moment
- one could conceive a compact “star” of dark matter in rapid rotation around a very massive companion, akin to binary black hole (gravitational waves)

Possibly observable consequences? 2/3

Order of magnitude estimate of this effect obtained as follows:

- magnetic moment of a φ particle with sharp momentum $\boldsymbol{k}$ in a frame where $\boldsymbol{m} = \boldsymbol{e}$

$$\mu_{\boldsymbol{k}} = e\lambda_P^2 \left\{ 2\boldsymbol{k} \wedge \boldsymbol{e} + \frac{|\boldsymbol{k}|^2}{\omega_m(\boldsymbol{k})} \left[\boldsymbol{e} - \frac{\boldsymbol{k}\cdot\boldsymbol{e}}{|\boldsymbol{k}|^2}\boldsymbol{k} \right] \right\}$$

- magnetic moment of a mass M star of classical particles with such magnetic moment in orbit of radius R and angular frequency ω

$$\boldsymbol{M}_S(t) = e\lambda_P^2 M \left[R\omega\cos(\omega t)\boldsymbol{M}_1 + R\omega\sin(\omega t)\boldsymbol{M}_2 + O(R^2\omega^2) \right]$$

$\boldsymbol{M}_i$ O(1) fixed vectors

- classical electromagnetic power radiated by such a variable moment

$$\frac{dE}{dt} \simeq e^2\lambda_P^4 M^2 R^2 \omega^6 = e^2 \left(\frac{\tau_P}{T}\right)^6 \left(\frac{R}{\lambda_P}\right)^2 M^2$$

Possibly observable consequences? 3/3

- Planck time $\tau_P \simeq 10^{-44}s$
- $T \simeq 10^{-2}s$, $R \simeq 10^3 km$, $M \simeq 10^{56} GeV$ (same order of the GW150914 binary black hole parameters)

results in a fraction of energy radiated per unit time

$$\frac{1}{E}\frac{dE}{dt} \simeq 10^{-89}s^{-1}$$

really very small... (for GW150914 it was $\simeq 10^{-2}$)
But obtained in the unrealistic approximation of Minkowski spacetime
Question: could the effect be enhanced by taking into account the highly curved background typical of such situation?
This is suggested by the results of [Doplicher, M., Pinamonti '13]

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Localizability in a spherically symmetric spacetime

Aim: produce a rigorous version of DFR argument on curved spacetime

Strategy:

1. consider a (scalar massless) free quantum field ϕ on a background $(M, g_{\mu\nu})$ in a (Hadamard) state such that
$$\Box\phi = 0, \qquad G_{\mu\nu} = 8\pi\omega(T_{\mu\nu})$$
2. prepare a localized state: for $f \in C_c^\infty(M)$
$$\omega_f(A) := \frac{\omega(\phi(f)A\phi(f))}{\omega(\phi(f)\phi(f))}, \qquad A \in \mathcal{A}$$
3. evaluate change to expectation value of $T_{\mu\nu}$ after localization
4. estimate backreaction on metric and formation of TS by Raychauduri equation (no linearization of gravity)
5. impose principle of gravitational stability

Step 4 (and 5) only under assumption of spherical symmetry of background metric

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Spherical symmetry

To evaluate backreaction, we should solve

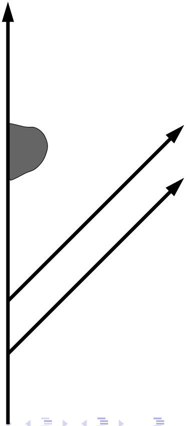
$$G_{\mu\nu} = 8\pi\, \omega_f(T_{\mu\nu})$$

It is **very** difficult. Assume spherical symmetry

- Spacetime is $I \times \mathbb{R}_+ \times \mathbb{S}^2$, retarded coordinates:
- spanned by future null geodesic emanated from the center of the sphere
  - u proper time on the worldline γ of center
  - s retarded distance: affine parameter along the null geodesics with $s(0) = 0$ and $\dot{s}(0) = 1$
- The general spherically symmetric metric is

$$ds^2 := -A(u,s)du^2 - 2ds\,du + r(u,s)^2 d\Omega^2$$

- Fix u, the family of null geodesics forms a cone $\mathcal{C}_u$

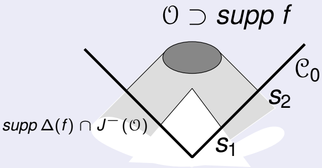


Backreaction and trapped surfaces

Theorem ([Doplicher, M., Pinamonti '13])

For a large class of spherically symmetric $(M, g_{\mu\nu})$ and ω (including cosmological ones), and for $f \in C_c^\infty(M)$ as in figure with

$$s_1 < s_2 < \frac{3}{2}s_1, \quad (s_2)^2 < \bar{s}^2 , \quad \bar{s}^2 := \frac{1}{6\,C}$$



the future of $\mathcal{C}_0$ contains a trapped surface.

For a flat Friedmann-Robertson-Walker spacetime with metric

$$ds^2 = -dt^2 + a(t)^2[dr^2 + r^2 d\Omega^2]$$

the limitation becomes $r \gtrsim \frac{\lambda_P}{a(t)} \Rightarrow$ effective Planck length diverges near the singularity, as argued by [Doplicher, '01]

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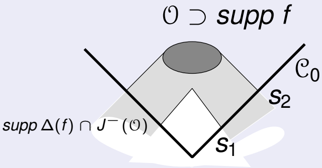
Theorem ([Doplicher, M., Pinamonti '13])

For a large class of spherically symmetric $(M, g_{\mu\nu})$ and ω (including cosmological ones), and for $f \in C_c^\infty(M)$ as in figure with

$$s_1 < s_2 < \frac{3}{2}s_1, \quad (s_2)^2 < \bar{s}^2 \,, \quad \bar{s}^2 := \frac{1}{6\,C}$$

the future of $\mathcal{C}_0$ contains a trapped surface.



For a flat Friedmann-Robertson-Walker spacetime with metric

$$ds^2 = -dt^2 + a(t)^2[dr^2 + r^2 d\Omega^2]$$

the limitation becomes $r \gtrsim \frac{\lambda_P}{a(t)} \Rightarrow$ effective Planck length diverges near the singularity, as argued by [Doplicher, '01]

Outline

Oppenheimer-Snyder solution

Consider a compact dark matter object rotating as before around a collapsing star of dust of mass M_0 and initial radius $R_0 \geq 2GM_0$

- outside the collapsing star, the metric is Schwarzschild, not FRW
- inside the collapsing star, the metric is closed FRW

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + \sin^2\chi d\Omega^2)$$

with $a(t)$ given by the Oppenheimer-Snyder solution

$$a(t) = \frac{1}{2}\sqrt{\frac{R_0^3}{2GM_0}}(1 + \cos\eta)$$

$$t = \frac{1}{2}\sqrt{\frac{R_0^3}{2GM_0}}(\eta + \sin\eta)$$

$\eta \in [0, \pi]$ is the conformal time, $\eta_{\text{coll}} = \arccos(\frac{4GM_0}{R_0} - 1)$ at collapse ($R(t_{\text{coll}}) = 2GM_0$)

- the two solutions are matched continuously

Radiation from dark matter around a collapsing object

We take the results of [Doplicher, M., Pinamonti '13] and the Oppenheimer-Snyder solution as a justification of the ansatz

$$\frac{dE}{dt} = e^2 \left(\frac{\lambda_P a(0)}{a(t)} \right)^4 M^2 R^2 \omega^6$$

for the em power radiated by our dark matter object on QST. As $M_0 \to 0$ the old formula is recovered

On the other hand em energy is emitted at the cost of kinetic and potential energy of the object. Considering only the orbital kinetic energy

$$E \simeq \frac{1}{2} M R^2 [\omega_0^2 - \omega_{\text{coll}}^2]$$

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ω_{coll} can be computed. Two regimes:

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  but the result could be sensibly altered taking into account the red-shift of the radiation emitted near the horizon and anyway for realistic matter collpase stops much before becasue of pressure

- if M_0 finite then

$$E \simeq e^2\lambda_P^4 M^2 R^2 \omega_0^6 \left(\frac{R_0^3}{2GM_0}\right)^{1/2} F(\eta_{\text{coll}})$$

  and for $M_0 \simeq 10^{56}GeV$, $R_0 \simeq 10^{-1}km \Rightarrow E \simeq 10^{-40}GeV$

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Conclusions

Summary:

- On QST a U(1) gauge theory becomes nonabelian, and neutral matter can develop a magnetic moment
- Compact rotating object of neutral matter with a variable magnetic moment emit em radiation of order $O(e^2 \lambda_P^4)$
- Order of magnitude esitmates, taking also into account the divergence of the effective λ_P near singularities, indicate that the effect is by far too small to be presently detected

Remarks:

- Difficult to accommodate non-integer electric charges in this framework. Investigate actions of $\mathcal{G}$ on a $\mathcal{E}$-module
- The effect should be compared with graviton mediated dark matter-photon interaction and with Hawking radiation
- Could be interesting to analyze self-gravitating Bose-Einstein condensates of dark matter

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