Viability of the Asymptotic Safety scenario beyond renormalizability ?

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The fundamental problem:

Give a meaning to ("define", "renormalize", "take the continuum limit of", ...) a functional integral over all metrics on a space time $\mathcal{M}$:

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S[\hat{g}_{\mu\nu}]}$$

S: diff $(\mathcal{M})$ - invariant bare action,

e.g. S_{EH} + counter terms

$$\mathcal{D}\hat{g}_{\mu\nu} \equiv \prod_{x\in\mathcal{M}} \prod_{\mu,\nu} dg_{\mu\nu}(x)$$

$\uparrow$ requires regularization (UV Cutoff)

The strategy:

MR, 1996

Define and compute the functional integral indirectly by means of the associated Effective Average Action (EAA):

$\Gamma_k[g_{\mu\nu}, \cdots]$, one-parameter family of action functionals, $0 \leq k < \infty$.

The problem, reformulated:

Construct fully extended integral curves ("RG trajectories") $k \longmapsto \Gamma_k[\cdot]$, $0 \leq k < \infty$ of an infinite dimensional flow $(\mathcal{T}, \beta)$.

$\mathcal{T}$: "theory space" $\ni A[g_{\mu\nu}, \cdots]$ specified by field contents and symmetries

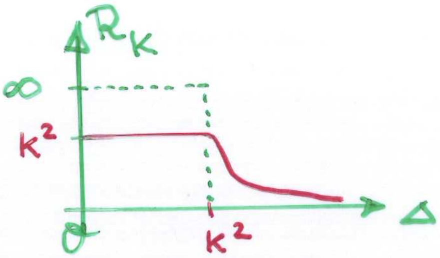
β : vector field on $\mathcal{T}$ defined by the functional renormalization group equation (FRGE) satisfied by the EAA:

$$k\,\partial_k \Gamma_k = \beta(\Gamma_k)$$

The Effective Average Action

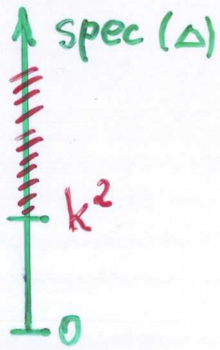
$$e^{W_k[J]} :=$$

$$\int \mathcal{D}\hat{\phi}\, e^{-S[\hat{\phi}]} \cdot e^{\int dx\, J\hat{\phi}} \cdot e^{-\frac{1}{2}\int \hat{\phi}\, R_k(\Delta)\, \hat{\phi}}$$



suppresses low eigenvalue ("low momentum", "large wave length") eigen-modes of Δ:

IR cutoff at (mass) scale $k \in [0, \infty)$



$$\Gamma_k[\phi] := \Big(\text{Legendre transf. of } W_k[J]\Big) - \frac{1}{2}\int \phi\, R_k(\Delta)\, \phi$$

Interpolating property:

$$\Gamma \xleftarrow{k \to 0} \Gamma_k \xrightarrow{k \to \infty} \sim S$$

ordinary eff. action   bare action

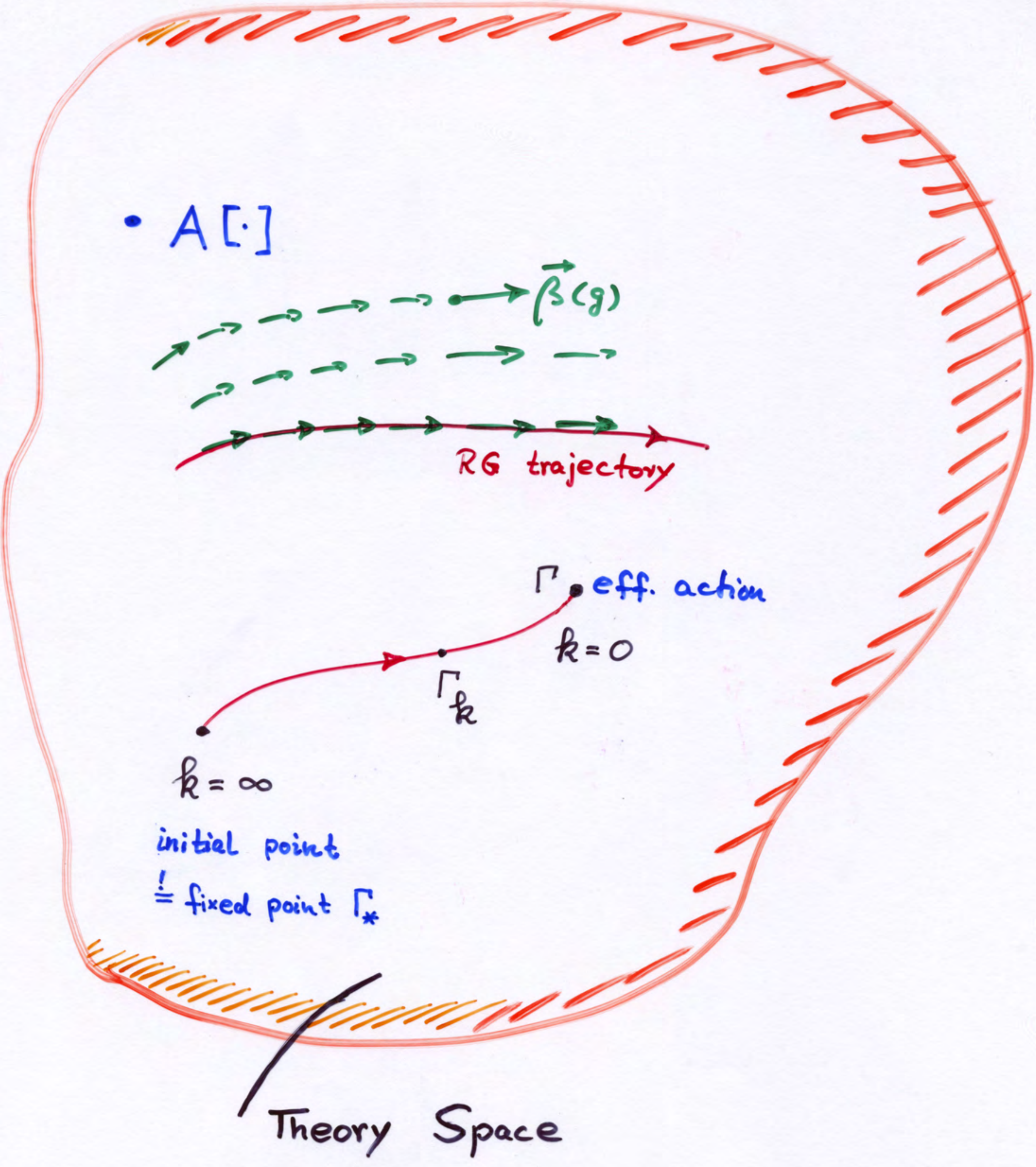
Functional Renormalization Group Equation (FRGE):

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k \right]$$

The Asymptotic Safety idea:

- Take the infinite-cutoff limit of an UV-regularized quantum theory of gravity at a non-trivial RG fixed point with a finite dimensional UV-critical hypersurface, assuming it exists.
- The resulting continuum theory is predictive and well behaved at arbitrarily short distances.

(S. Weinberg, 1979, 2009)



$A[\cdot]$
$\vec{\beta}(g)$
RG trajectory
Γ eff. action
$k=0$
Γ_k
$k=\infty$
initial point
$\stackrel{!}{=}$ fixed point Γ_*
Theory Space

The Einstein-Hilbert Truncation

MR, 1996

Ansatz:

$$\Gamma_k = -\frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left(R - 2\Lambda_k \right)$$

+classical gauge fixing and ghost terms

Running coupling constants:

Newton constant G_k , dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k , dimensionless: $\lambda(k) = k^{-2} \Lambda_k$

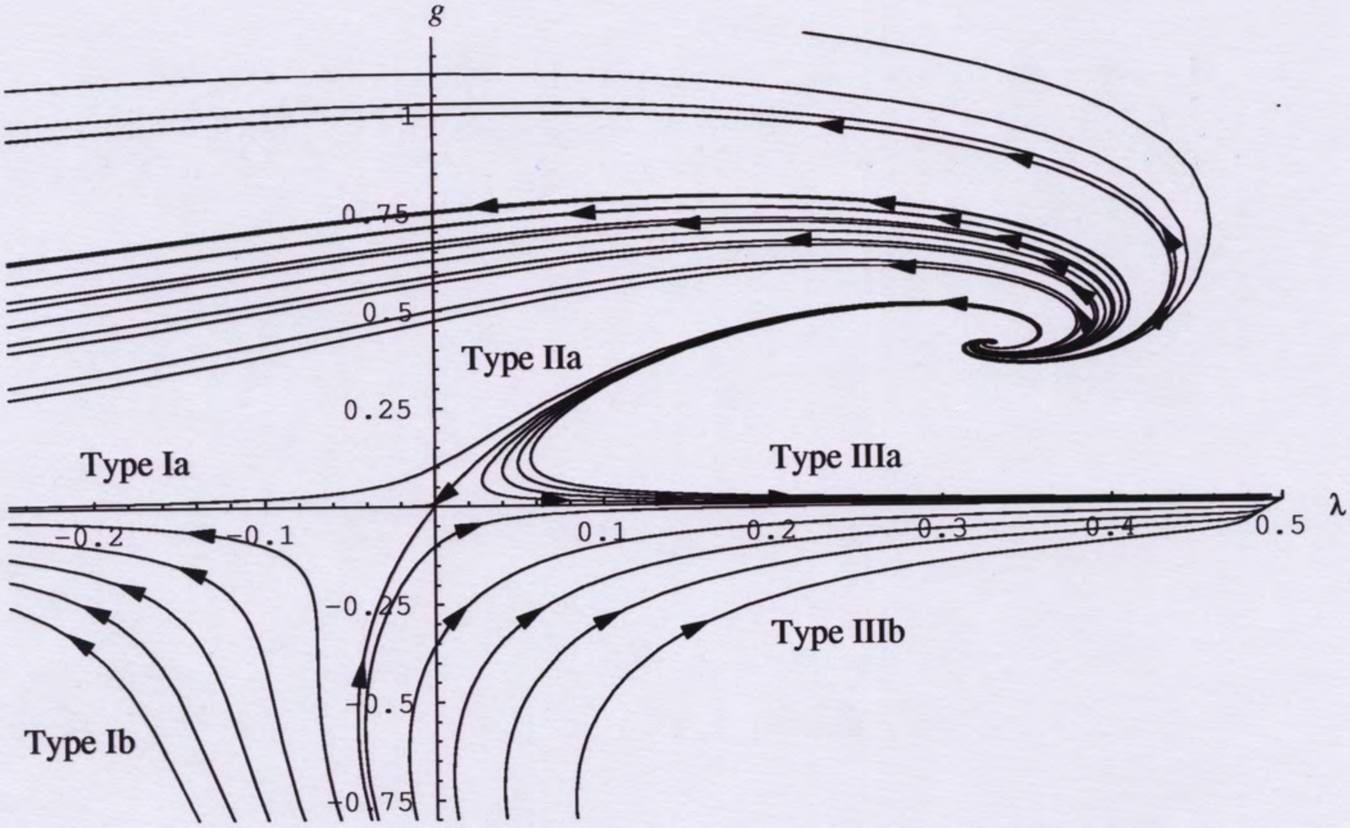
Insert ansatz into FRGE, "project out" monomials retained:

$$\mathrm{Tr}\left[\cdots\right] = (\cdots) \int \sqrt{g} + (\cdots) \int \sqrt{g} R + \cdots \Rightarrow$$

$$\boxed{\begin{aligned} k\,\partial_k\, g(k) &= \beta_g(g,\lambda) \\ k\,\partial_k\, \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

Einstein – Hilbert Truncation :

RG Flow on the $g-\lambda$ plane



M.R., F. Saueressig, hep-th/0110054

Beyond nonperturbative renormalizability

$$\{\text{nonperturbative renormalizability}\}$$

$$\cap \quad \{\text{Background Independence}\}$$

$$\cap \quad \{\text{Hilbert space positivity}\}$$

$$\neq \quad \emptyset \quad ?$$

Does the non-Gaussian fixed point of Quantum Einstein Gravity define a conformal field theory?

How is QEG in $2+\varepsilon$ dimensions, $\varepsilon \to 0$, related to the strictly 2D approaches to quantum gravity?

(Liouvilly theory, matrix models, stat. mech. simulations, etc.)

1. The two-dimensional limit of QEG taken at the level of action functionals

2. The NGFP as a conformal field theory

3. Unitarity vs. Stability vs. Locality

4. Outlook

Lit.: A. Nink and MR, JHEP 02 (2016) 167

and arXiv: 1512. 06805

Effective Einstein equation re-interpreted

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \text{(or similar)}$$

matter-like field on classical spacetime with metric $\bar{g}_{\mu\nu}$

<u>Generic EAA:</u> $$\Gamma_k[\Phi, \bar{\Phi}] \equiv \Gamma_k[\varphi; \bar{\Phi}]$$

$$\varphi^i \equiv \Phi^i - \bar{\Phi}^i \quad = \langle \hat{\varphi}^i \rangle$$

$$\frac{1}{\sqrt{\bar{g}}} \frac{\delta \Gamma_k[\varphi; \bar{\Phi}]}{\delta \varphi^i} + \left(\mathcal{R}_k\right)_{ij} \varphi^j = J_i$$

<u>Self-consistent backgrounds</u> from tadpole condition:

$$\frac{\delta}{\delta \varphi^i} \Gamma_k[\varphi; \bar{\Phi}] \Big|_{\varphi=0,\ \bar{\Phi} = \bar{\Phi}^{sc}_k} = 0$$

Or:

$$\frac{\delta}{\delta \Phi^i} \Gamma_k[\Phi, \bar{\Phi}] \Big|_{\Phi = \bar{\Phi} = \bar{\Phi}^{sc}_k} = 0$$

Example:

$$\Gamma_k = \Gamma_k^{grav}[g,\bar{g}] + \Gamma_k^{M}[g,A,\bar{g}] + S_{gf} + S_{gh}$$

Tadpole eqs. (@ vanishing ghosts):

$$0 = \frac{\delta}{\delta g_{\mu\nu}} \left\{ \Gamma_k^{grav}[g,\bar{g}] + \Gamma_k^{M}[g,A,\bar{g}] \right\} \Bigg|_{\substack{g=\bar{g}=\bar{g}_k^{sc} \\ A=\bar{A}=\bar{A}_k^{sc}}}$$

$$0 = \frac{\delta}{\delta A} \;\cdots\cdots$$

Stress tensors for A and $h_{\mu\nu}$:

$$T^{M}[\bar{g},A]^{\mu\nu} := \frac{2}{\sqrt{\bar{g}}} \frac{\delta}{\delta g_{\mu\nu}} \Gamma_k^{M}[g,A,\bar{g}] \Bigg|_{g=\bar{g}}$$

$$T^{grav}[\bar{g}]^{\mu\nu} := \frac{2}{\sqrt{\bar{g}}} \frac{\delta}{\delta g_{\mu\nu}} \Gamma_k^{grav}[g,\bar{g}] \Bigg|_{g=\bar{g}}$$

$$= \frac{2}{\sqrt{\bar{g}}} \frac{\delta}{\delta h_{\mu\nu}} \Gamma_k^{grav}[h;\bar{g}] \Bigg|_{h=0}$$

Effective Einstein eq. has zero LHS:

$$0 = T^{grav}_{\mu\nu}[\bar{g}_k^{sc}] + T^{M}_{\mu\nu}[\bar{g}_k^{sc}, \bar{A}_k^{sc}]$$

$d = 2+\varepsilon$:

QEG in the Einstein - Hilbert truncation

$$\Gamma_k^{grav}[g,\bar{g}] \equiv \Gamma_k^{grav}[g] = -\frac{1}{16\pi G_k}\int d^{2+\varepsilon}x\,\left(R - 2\Lambda_k\right)$$

$$g_k = k^{\varepsilon} G_k \quad , \quad \lambda_k = k^{-2}\Lambda_k$$

$$\partial_t g_k = \varepsilon g_k - b\, g_k^2 + O(g^3)$$

NGFP:

$$g_* = \frac{\varepsilon}{b}$$

$$\lambda_* \sim \varepsilon$$

RG trajectories near (g_*, λ_*) :

$$(g_k, \lambda_k) =: \varepsilon\,(\breve{g}_k, \breve{\lambda}_k)$$

$$(G_k, \Lambda_k) =: \varepsilon\,\underbrace{(\breve{G}_k, \breve{\Lambda}_k)}_{\text{finite for } \varepsilon \to 0}$$

Stress tensor of $h_{\mu\nu}$

$$T^{\text{grav}}_{\mu\nu}[\bar{g}] = \frac{1}{8\pi G_k}\left(\bar{G}_{\mu\nu} + \Lambda_k \bar{g}_{\mu\nu}\right)$$

Trace $\bar{g}^{\mu\nu} T^{\text{grav}}_{\mu\nu}[\bar{g}] \equiv \Theta_k[\bar{g}]$:

$$\Theta_k[\bar{g}] = \frac{1}{16\pi \underbrace{G_k}_{=\varepsilon \breve{G}_k}} \left[-\underbrace{(d-2)}_{=\varepsilon}\bar{R} + 2d\underbrace{\Lambda_k}_{=\varepsilon\breve{\Lambda}_k}\right]$$

has an unambiguous limit $\varepsilon \to 0$:

$$\Theta_k[\bar{g}] = \frac{1}{16\pi \breve{G}_k}\left[-\bar{R} + 4\breve{\Lambda}_k\right] + O(\varepsilon)$$

$$= \underbrace{\frac{1}{16\pi \breve{g}_k}\left[-\bar{R} + 4\breve{\lambda}_k k^2\right]}_{\text{"lives" in exactly 2 dim.!}} + O(\varepsilon)$$

For the theory defined by the constant NGFP trajectory:

$$\Theta_k^{\text{NGFP}}[\bar{g}] = \frac{b}{16\pi}\left[-\bar{R} + 4\lambda_* k^2\right]$$

$$\Theta_{k=0}^{\text{NGFP}}[\bar{g}] = -\frac{b}{16\pi}\bar{R}$$

Intrinsically 2D description of the limiting theory: $\Gamma_k^{\text{grav, 2D}}[g]$

Take limit $\varepsilon \to 0$ at the level of actions.

For spacetimes of arbitrary topological type:

Euler number

moduli

$$\boxed{\begin{aligned} \frac{1}{\varepsilon}\int d^{2+\varepsilon}x\,\sqrt{g}\,R &= \frac{4\pi}{\varepsilon}\,\chi + C(\{\tau\}) \\ &\quad - \frac{1}{4}\int d^2x\,\sqrt{g}\,R\,\Box^{-1}R + O(\varepsilon) \end{aligned}}$$

Polyakov (induced gravity) action: non-local !

$$\Gamma_k^{\text{grav, 2D}}[g] = \frac{1}{64\pi\,\breve{G}_k}\int d^2x\,\sqrt{g}\,R\,\Box^{-1}R + \frac{\breve{\Lambda}_k}{8\pi\,\breve{G}_k}\int d^2x\,\sqrt{g} \quad + \text{topolog.}$$

Proof of a simple special case:

trivial spacetime topology allowing $g_{\mu\nu} = e^{2\sigma} \hat{g}_{\mu\nu}$ globally, with $\hat{R} \equiv R(\hat{g}) = 0$.

$$R = e^{-2\sigma} \left[\hat{R} - (d-1)(d-2) \hat{D}_\mu \sigma \hat{D}^\mu \sigma - 2(d-1) \hat{\Box} \sigma \right]$$

$$\frac{1}{\varepsilon} \int d^d x \sqrt{g}\, R = \frac{1}{\varepsilon} \int d^d x \sqrt{\hat{g}}\, e^{\overbrace{(d-2)}^{\varepsilon}\sigma} \Big[\underbrace{\hat{R}}_{=0} + (d-1)\underbrace{(d-2)}_{=\varepsilon} \big(\hat{D}_\mu \sigma\big)^2 \Big]$$

$$= \int d^2 x \sqrt{\hat{g}}\, \hat{D}_\mu \sigma \hat{D}^\mu \sigma \qquad + O(\varepsilon)$$

$$= \int d^2 x \underbrace{\sigma}_{= -\frac{1}{2} \Box^{-1} R} \quad \underbrace{\sqrt{\hat{g}}\,(-\hat{\Box})\sigma}_{= \frac{1}{2} \sqrt{g}\, R} \qquad + O(\varepsilon)$$

$$= -\frac{1}{4} \int d^2 x \sqrt{g}\, R\, \Box^{-1} R \quad + \dots$$

In $d=2$ for $\hat{R} = 0$:

$$\sqrt{g}\, R = -2 \sqrt{\hat{g}}\, \hat{\Box} \sigma$$

$$R = -2\, \Box \sigma$$

$$\rightsquigarrow \quad \sigma = -\frac{1}{2} \Box^{-1} R$$

Nonlocal EAA in exactly 2 dimensions

$$\boxed{\Gamma_k^{\mathrm{grav},2D}[g] = \left(\frac{3}{2}\frac{1}{\breve{g}_k}\right)\frac{1}{96\pi}\int d^2x\sqrt{g}\left(R\,\Box^{-1}R + 8\breve{\lambda}_k k^2\right)}$$

implies the following stress tensor:

$$T^{\mathrm{grav}}_{\mu\nu}[g] = \left(\frac{3}{2}\frac{1}{\breve{g}_k}\right)\frac{1}{96\pi}\Big[g_{\mu\nu} D_\gamma(\Box^{-1}R) D^\gamma(\Box^{-1}R)$$

$$-2 D_\mu(\Box^{-1}R) D_\nu(\Box^{-1}R) + 4 D_\mu D_\nu(\Box^{-1}R)$$

$$-4 g_{\mu\nu} R + 8\breve{\lambda}_k k^2 g_{\mu\nu}\Big]$$

reproduces $\quad \Theta_k[g] = \left(\frac{3}{2}\frac{1}{\breve{g}_k}\right)\frac{1}{24\pi}\left[-R + 4\breve{\lambda}_k k^2\right]$ ✓

Effective field eq. for pure gravity:

$$\boxed{0 = T^{\mathrm{grav}}_{\mu\nu}[\bar{g}^{\mathrm{sc}}_k]} \quad \Longrightarrow \quad \Theta_k[\bar{g}^{\mathrm{sc}}_k] = 0$$

$$\Longleftrightarrow \quad \boxed{R(\bar{g}^{\mathrm{sc}}_k) = 4\breve{\lambda}_k k^2}$$

Higher correlators $\langle T^{grav}_{\mu\nu}(x_1)\, T^{grav}_{\alpha\beta}(x_2) \cdots \rangle$

generated by multiple differentiation of $\Gamma_k^{grav,\,2D}$

<u>Example:</u> Trace 2pt. function at $k=0$ for $g_{\mu\nu} = \delta_{\mu\nu}$

$$\boxed{\langle \Theta_0(x)\, \Theta_0(y) \rangle = -\frac{1}{12\pi}\, c^{NGFP}_{grav}\, \partial^\mu \partial_\mu \delta(x-y)}$$

<u>Sign of the Schwinger term</u> is crucial:

Smeared operator $\Theta_0[f] := \int d^2x\, f(x)\, \Theta_0(x)$ satisfies

$$\underbrace{\langle \Theta_0[f]^2 \rangle}_{\text{formally positive}} = \frac{1}{12\pi}\, c^{NGFP}_{grav} \underbrace{\int d^2x\, (\partial_\mu f)\, \delta^{\mu\nu} (\partial_\nu f)}_{\text{positive}}$$

$\Longrightarrow$ It is impossible to define $\Theta_0[f]^2$ as a positive comp. operator unless $c^{NGFP}_{grav} > 0$!

The "NGFP theory":

$$k \mapsto (g_*, \lambda_*) \quad \forall k \in [0,\infty)$$

$$\Gamma_k^{\mathrm{grav,\,2D,\,NGFP}}[g] = \left(\tfrac{3}{2}b\right)\frac{1}{96\pi}\int d^2x\sqrt{g}\left(R\,\Box^{-1}R + 8\breve{\lambda}_* k^2\right)$$

Standard EA:

$$\boxed{\Gamma^{\mathrm{grav,\,2D,\,NGFP}}[g] = \left(\tfrac{3}{2}b\right)\frac{1}{96\pi}\int d^2x\sqrt{g}\,R\,\Box^{-1}R}$$

has vanishing cosmological constant;

self-consistent backgrounds are flat: $R(\bar{g}_k^{\mathrm{sc}}) = 0$

Reminder:

Induced gravity by a generic CFT

Consider a 2D theory of matter fields X, couple it to gravity by diff. inv. action $S[X,g]$, and compute

$$e^{-S_{ind}[g]} = \int \mathcal{D}X \, e^{-S[X,g]}$$

Then the (conserved, symmetric) stress tensor

$$T[g]^{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S_{ind}[g]}{\delta g_{\mu\nu}}$$

has a trace of the form

$$T[g]^{\mu}{}_{\mu} = -c \frac{1}{24\pi} R \qquad + \text{const}$$

corresponding to

$$S_{ind}[g] = +c \frac{1}{96\pi} \int d^2x \sqrt{g} \, R \, \square^{-1} R \qquad + \dots$$

iff the original theory is a CFT and has central charge c.

NB: The modes of the (traceless, non-conserved) tensor $T'_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\gamma}{}_{\gamma}$ satisfy Virasoro algebra with central extension $\sim c$.

Unitary vs. non-unitary CFT's

A conformal field theory can be unitary only if its central charge is positive, $c > 0$.

unitary theories: Virasoro algebra represented on a true Hilbert space, no negative norm states

non-unitary models: no quantum mechanical probability interpretation !
$\longrightarrow$ stat. mech.

Asymptotic Safety: conformality and unitarity

(a) According to the NGFP theory, the "dynamics" of the metric fluctuations is governed by a <u>conformal</u> field theory.

(b) The central charge of this theory is

$$\boxed{c_{\text{grav}}^{\text{NGFP}} = \frac{3}{2}\, b = \frac{3}{2}\,\frac{1}{\breve{g}_*}}$$

(c) In pure gravity (and gravity coupled to not too many matter fields) the 'grav' sector is described by a <u>unitary</u> CFT

– thus complying with the requirement of Hilbert space positivity.

$$c_{\text{grav}}^{\text{NGFP}} = \begin{cases} 19 - N & \text{Linear param. } g = \bar{g} + h \\ 25 - N & \text{exponential p. } g = \bar{g}\, e^{h} \end{cases}$$

The stability - unitarity antagonism

$$\Gamma_{k=0}^{\text{grav, 2D, NGFP}}[g] = \frac{C_{\text{grav}}^{\text{NGFP}}}{96\pi} \underbrace{\int d^2x \sqrt{g}\, R\, \Box^{-1} R}_{\text{negative definite fctl. of } g_{\mu\nu}}$$

$$\left.\begin{array}{l} C_{\text{grav}}^{\text{NGFP}} < 0 \\[2em] C_{\text{grav}}^{\text{NGFP}} > 0 \end{array}\right\} \Longleftrightarrow \text{eff. action is} \left\{\begin{array}{l} \text{positive def.} \\[2em] \text{negative def.} \end{array}\right.$$

$\downarrow$

instability ?

no !

Liouville representation of the NGFP theory

fix topological class, moduli ;

conformal gauge: $g_{\mu\nu} = e^{2\phi}\, \hat{g}_{\mu\nu}$

$$\boxed{\Gamma_k^{\text{grav, 2D, NGFP}}[e^{2\phi}\hat{g}] = \frac{c_{\text{grav}}^{\text{NGFP}}}{96\pi}\int \hat{R}\,\hat{\Box}^{-1}\hat{R} + \Gamma_k^{L}[\phi;\hat{g}]}$$

due to the reference metric

Liouville action

$$\Gamma_k^{L}[\phi;\hat{g}] = \frac{c_{\text{grav}}^{\text{NGFP}}}{96\pi}\int d^2x\,\sqrt{\hat{g}}\,\Big\{-\tfrac{1}{2}\hat{D}_\mu\phi\,\hat{D}^\mu\phi - \tfrac{1}{2}\hat{R}\phi + \breve{\lambda}_* k^2 e^{2\phi}\Big\}$$

Kinetic term is negative iff central charge is positive !

"Wrong" sign analogous to 4D conformal factor "problem" :

physically correct (attractivity of gravity),

no real instability after imposing physical state conditions;

c.f. Gauss law: $\vec{\nabla}^2 A_0(t,\vec{x}) = (\psi^+\psi)(t,\vec{x})$

$\rightsquigarrow$ ϕ, A_0 fully constrained fields, determined by matter

Central charge of the Liouville - CFT per se

Γ_k^L implies eom $\boxed{\hat{\Box}\phi + (2\check{\lambda}_* k^2)\, e^{2\phi} = \frac{1}{2}\hat{R}}$

and stress tensor $T_k^L[\phi;\hat{g}]^{\mu\nu} \equiv \frac{2}{\sqrt{\hat{g}}} \frac{\delta \Gamma_k^L}{\delta \hat{g}_{\mu\nu}}$

with the trace

$$\Theta_k^L[\phi;\hat{g}] \equiv \hat{g}_{\mu\nu} T_k^L[\phi;\hat{g}]^{\mu\nu} = \frac{c_{\text{grav}}^{\text{NGFP}}}{12\pi}\left[\hat{\Box}\phi + (2\check{\lambda}_* k^2) e^{2\phi}\right]$$

$$\underset{\text{on shell}}{=} \frac{c_{\text{grav}}^{\text{NGFP}}}{24\pi}\hat{R}$$

$$=: -\frac{c^L}{24\pi}\hat{R}$$

$\Rightarrow$ Γ_k^L describes CFT, with central charge c^L:

$$\boxed{c^L =: -c_{\text{grav}}^{\text{NGFP}}}$$

$\Rightarrow$ $\begin{Bmatrix} c^L < 0 \\ c^L > 0 \end{Bmatrix}$ $\Longleftrightarrow$ pure Liouville CFT $\begin{Bmatrix} \text{not unitary} \\ \text{unitary} \end{Bmatrix}$

$\Longleftrightarrow$ ϕ-mode $\begin{Bmatrix} \text{unstable} \\ \text{stable} \end{Bmatrix}$

$\Longleftrightarrow$ $c_{\text{grav}}^{\text{NGFP}}$ $\begin{Bmatrix} \text{positive} \\ \text{negative} \end{Bmatrix}$

$\Longleftrightarrow$ full 'grav' - CFT $\begin{Bmatrix} \text{unitary} \\ \text{not unitary} \end{Bmatrix}$

Pure a.s. gravity has $c^L < 0$!

Combined Liouville + background system

Trace of the <u>total</u> stress tensor

from $\Gamma_k^{\text{grav, 2D, NGFP}}[e^{2\phi}\hat{g}]$:

$$\frac{2\hat{g}_{\mu\nu}}{\sqrt{\hat{g}}}\frac{\delta}{\delta\hat{g}_{\mu\nu}}\left(\frac{c_{\text{grav}}^{\text{NGFP}}}{96\pi}\int\sqrt{g}\,\hat{R}\,\hat{\Box}^{-1}\hat{R}\right) + \Theta_k^{L}[\phi;\hat{g}]$$

$$= \frac{c_{\text{grav}}^{\text{NGFP}}}{24\pi}\left[-R(\hat{g}) + \underbrace{2\hat{\Box}\phi + (4\check{\lambda}_* k^2)\,e^{2\phi}}_{\overset{\text{o.s.}}{=}\,+R(\hat{g})}\right]$$

explains the relative minus sign

$$= \frac{c_{\text{grav}}^{\text{NGFP}}}{24\pi}\left[-\underbrace{\left(R(\hat{g}) - 2\hat{\Box}\phi\right)e^{-2\phi}}_{=R(e^{2\phi}\hat{g})\,=\,R(g)} + 4\check{\lambda}_* k^2\right]e^{2\phi}$$

$$= e^{2\phi}\cdot\Theta_k[g]$$

Same off-shell result as above!

Vanishes on-shell, i.e. for $g = \bar{g}_k^{sc}$.

Locality

True degree of physical (non-) locality is in general only weakly related to the naive appearance of the action.

($\longrightarrow$ correlators of observables)

apparent (non-) locality $\longleftrightarrow$ param. of field dof's

$\longleftrightarrow$ apparent (in-) stability

Polyakov action $\int R \,\Box^{-1} R$ becomes local

(a) in certain gauges (conformal, light-cone, ...)

(b) by introducing additional fields in a covariant way;

example: Feigin - Fuks theory

$$e^{-\frac{25-N}{96\pi}\int R\,\Box^{-1} R}$$

!

$$= \int \mathcal{D}B \; e^{-\frac{24-N}{96\pi}\int d^2x \sqrt{g}\,\left\{ D_\mu B D^\mu B + 2RB \right\}}$$

The reconstructed functional integral

There exists a UV-regularized functional integral with the bare ("classical") action

$$S^{grav}[g] = \frac{25-N}{96\pi} I[g] \quad , \qquad I[g] \equiv \int d^2x \sqrt{g}\, R \Box^{-1} R \,.$$

Quantization à la Polyakov:

$$\boxed{Z = \int_{moduli} d\tau \int \mathcal{D}_{e^{2\phi}\hat{g}} \phi \; Z_{gh}[e^{2\phi}\hat{g}] \; Z_{matter}[e^{2\phi}\hat{g}] \; Y_{grav}[e^{2\phi}\hat{g}]}$$

$$Y_{grav}[g] = e^{-S^{grav}[e^{2\phi}\hat{g}]} \qquad = Y_{grav}[\hat{g}]\, e^{+\frac{(25-N)}{12\pi}\Delta I}$$

$$Z_{matter}[g] = e^{-\frac{N}{96\pi} I[e^{2\phi}\hat{g}]} \quad = Z_{matter}[\hat{g}]\, e^{+\frac{N}{12\pi}\Delta I}$$

$$Z_{gh}[g] = Z_{gh}[\hat{g}]\, e^{+\frac{(-26)}{12\pi}\Delta I}$$

$$\mathcal{D}_{e^{2\phi}\hat{g}}\phi = \mathcal{D}_{\hat{g}}\phi \quad e^{+\frac{1}{12\pi}\Delta I}$$

$$\Delta I[\phi;\hat{g}] \equiv \frac{1}{2}\int d^2x \sqrt{\hat{g}} \left\{ \hat{D}_\mu\phi \hat{D}^\mu\phi + \hat{R}\phi \right\}$$

$$Z = \int d\tau \, Z_{gh}[\hat{g}] \, Z_{matter}[\hat{g}] \, Y_{grav}[\hat{g}] \int \mathcal{D}_{\hat{g}} \phi$$

Total anomaly budget:

$$\boxed{c_{tot} \equiv \underbrace{(25-N)}_{\text{NGFP}} + \underbrace{N}_{\text{matter}} + \underbrace{1}_{\text{Jacobian}} + \underbrace{(-26)}_{\text{bc-ghosts}} = 0}$$

In asym. safe quantum gravity, c_{tot} vanishes <u>always</u>, i.e. for <u>any</u> matter system, unlike in string theory.

$$N \xrightarrow[\text{safety}]{\text{asymptotic}} N_{eff} = N + (25-N) = 25$$

Cancellation of explicit matter contribution to the fctl. int. and the implicit matter dependence of the bare $\equiv$ NGFP gravity action.

$\Longrightarrow$ <u>Quenching of KPZ scaling:</u>

$N_{eff} = 25 \rightsquigarrow \phi$ decouples from gravitationally dressed observables of matter CFT:

no quantum corrections to flat space - scaling dimensions

A second universality class ?

$c_{grav}^{NGFP} = 25$ $\quad g = \bar{g} e^{h}$ $\quad$ weakly coupled Liouville – DDK – KPZ th.

$c_{grav}^{NGFP} = 19$ $\quad g = \bar{g} + h$ $\quad$?

Perhaps more than a cooincidence:

The Virasoro algebra allows for unitary truncations in the "critical dimensions of non-critical string theory" advocated by Gervais:

$$D_{crit} = 0 \; , \; 7 \; , \; 13 \; , \; 19 \; , \; 25$$

$$\hat{=} \quad c_{grav} = 25 \; , \; \underline{19} \; , \; 13 \; , \; 7 \; , \; 0$$

Summary and Outlook

What we found:

In the limit $d \to 2$, the $h_{\mu\nu}$-sector of BRST gauge fixed QEG, in the Einstein-Hilbert trunc., is described by a conformal and unitary quantum field theory. Its states inhabit a true Hilbert space, $\mathcal{H}_{grav}$.

After imposing the physical state conditions on the total space (computing the BRST cohomology), no physical states are left though:

$$\underbrace{\mathcal{H}_{grav}}_{\text{pos.!}} \oplus \mathcal{H}_{ghost} \longrightarrow \emptyset$$

What we must show ultimately:

In $d = 4$, the total space does contain physical states, and they form a Hilbert space with a positive definite inner product.