Foundational and structural aspects of gauge theories

In modern theoretical physics gauge symmetry represents one of the main pillars in several successful frameworks. The standard model of elementary particles is probably the most famous example and especially in the realm of quantum physics, many models implementing the gauge principle have been formulated and thoroughly studied.

Yet, it is widely accepted that our present understanding of the structural aspects of these theories and their role in a foundational and axiomatic framework for quantum field theory is not as developed as our current knowledge of their phenomenological consequences. A topical example is the quest for the construction of a good class of observables in a quantum theory of gravity in view of diffeomorphism invariance. In general, we would like to better understand in which sense the rich structure of gauge theories with its fruitful relations to mathematics, also well-known at the classical level, has a counterpart in the quantized theory.

In the past, mainly the path integral approach to quantization was used for establishing such connections. Arguments based on this approach are suggestive but typically not conclusive in view of the poorly understood mathematics behind path integrals. At the same time, approaches in axiomatic field theory are often very far from the models of interest for physics. Moreover, they are restricted by the lack of a physical principle that determines the mathematical structures of the so-called unphysical degrees of freedom. For these reasons this topical workshop was a timely event which represents an important occasion to review recent achievements and to start a productive discussion on open issues and on the directions of future research. The format of the topical workshop was the standard one: Renowned scientists gave seminal lectures, summarizing the state of the art while a substantial share of the slots was reserved for contributed talks. Younger scientists were encouraged to participate actively to the event by presenting novel results and ideas. With this format the short term impact of the meeting improved the visibility of this research field in the international scientific landscape. In the long term, one foresees the establishment of new collaborations between research groups who often only tend to have marginal contact.

The algebraic approach to quantum field theory (AQFT) is a well-established branch of theoretical physics which emphasizes the role of observables and their interplay with the notions of locality and causality. From a physical point of view, this approach has the net advantage of also being naturally applicable when the underlying background is curved.
Particularly in the past few years, the formulation of all free field theories on arbitrary globally hyperbolic spacetimes has been thoroughly analyzed and understood. The construction of the full algebra of observables has come together with the precise characterization of the quantum states which are physically admissible and with their explicit construction in a large class of interesting scenarios. More importantly, this analysis has been crucial to the definition of a Wick product of fields and to the construction of an extended algebra of observables. Starting from these tools, perturbative interactions have been rigorously formulated and the renormalization group has been extensively investigated bringing quantum field theory on arbitrary curved backgrounds almost on par with its counterpart on Minkowski spacetime.

Despite all these successes, several questions remain open and represented the main issues of the topical workshop. A notable example is that of gauge symmetry. While global gauge groups were already thoroughly studied decades ago, local gauge invariance has been neglected, being considered an insurmountable obstacle. Yet, within the past few years, several papers, reverting this attitude, have been published. At first glance they are based on different approaches: One focused on a novel functional analytic approach towards interacting and gauge theories, which emphasizes the role of functional analysis. Another one advocated the necessity of a constructive procedure, mostly based on the underlying algebra of operators, while yet another one also analyzed gauge theories from a more geometrical perspective, mainly using techniques from algebraic topology.

First of all, it emerged clearly that, combining the principle of local gauge invariance together with the algebraic framework, leads to several open issues concerning both the choice of a good algebra of gauge invariant observables and the implementation of all structural properties desired. Most notably, it appears that the principle of general local covariance, a cornerstone of the modern axiomatic approach towards the quantization of field theories on a generic curved background, cannot be implemented to its fullest extent. The reasons for such problem are manifold, but the main one can be ascribed to our current lack of a full understanding of the interplay between the topology of the underlying structures and the quantization scheme employed.

Despite these obstacles, it has been realized how the algebraic approach is well-suited to combine the infinite dimensional geometric and analytic structures proper of quantum field theories with well-established approaches towards gauge invariance such as the BRST-BV formalism.

On the one hand, one of the main goals of such an investigation was to achieve a better understanding at a quantum level of the rich structure of gauge theories with its useful relations to mathematics which are best known at the classical level. Usually the translation in a quantum framework of classical structures and properties of gauge theories is established via the path integral approach to quantization. Arguments based
on this approach are suggestive but typically not conclusive in view of the poorly understood mathematics of path integrals, especially in a e setting. The advantages of the algebraic approach in offering a mathematically rigorous description are often blunted by the difficulties in building a direct connection to models of physical interest. In this respect, one of the key goals of the topical workshop was to create a close contact with other research groups.

On the other hand, the BRST- BV formalism, combined with the tools of infinite dimensional analysis, has been applied to the quantization of general relativity. It turns out that the construction of a good class of observables for a quantum theory of gravity is problematic, especially in view of the underlying invariance under the action of the diffeomorphism group. Despite a better understanding of the problem, a complete solution is still not within our grasp and a comparison with different approaches from other areas of theoretical physics is desirable, if not essential.

These are a few examples of different communities, who do not have close contacts on a regular basis but share a common aim, namely the investigation of structural and foundational aspects of gauge theories. The topical workshop offered the ideal environment for our endeavor to build a bridge between these different communities. It also strengthened the exchange between the leading experts and younger scientists.