Trapping Centers at the Superfluid-Mott-Insulator Criticality: Transition between Charge-quantized States

Boris Svistunov

University of Massachusetts, Amherst

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Teaching an old dog new tricks…

The old dog:

O(2)-symmetric Wilson-Fisher conformal field theory in (2+1) simulated/emulated by Worm algorithm, or ultracold atoms

The new trick:

Novel mechanism of charge fractionalization:
Halon, a polaron with half-integer charge

Collaborators:

Kun Chen

UMass Amherst,
USTC Hefei
(Now at Rutgers)

Yuan Huang

UMass Amherst,
USTC Hefei

Youjin Deng

USTC Hefei

+ Fruitful discussions with Nikolay Prokof’ev

Polaron/impurity in a quantum-critical environment is a well-known fundamental problem…

M. Punk and S. Sachdev, Phys. Rev. A 87, 033618 (2013)

S. Sachdev, C. Buragohain, and M. Vojta, Science, 286, 2479 (1999)

S. Sachdev, M. Troyer, and M. Vojta, Phys. Rev. Lett. 86, 2617 (2001)

M. Vojta, C. Buragohain, and S. Sachdev, Physical Review B, 61, 15152 (2000)

S. Sachdev and M. Vojta, Phys. Rev. B68, 064419 (2003)

Let us look at the U(1) problem from the charge-quantization perspective…

More generally: the problem of trapped quanta

Elementary excitations—quasiparticles and/or centers—can carry quanta of conserved quantities, such as:

(i) energy (always, by definition),

(ii) momentum/quasi-momentum (in translation-invariant cases),

(iii) projection of angular momentum (e.g., Kelvons),

(iv) projection of spin,

(v) the number of *genuine* (as opposed to *quasi*) particles (e.g., quasiparticles in Landau theory of Fermi liquid; and also: vacancies, interstitials, impuritons, particle/hole excitations in Mott insulators).

(vi) (topology-driven) **fractional quantization** (e.g., quantum Hall, spinons in 1D)

A rule of thumb:

The number of *genuine* particles is a *bad* quantum number whenever corresponding U(1) symmetry is broken—even if only in the topological sense, like in 1D superfluids; and is a *good* quantum number otherwise, apart from topology-driven cases, and also static impurity (i.e., a center) in a normal Fermi liquid.

The particle charge is a good quantum number:

quasiparticles in Landau Fermi liquid theory, and also for Fermi polarons/molecules in the normal Fermi sea;

vacancies, interstitials, impuritons; particle/hole excitations in Mott insulators

… good but trivial (zero):

excitons

phonons in a solid

The particle charge is a bad quantum number:

phonons in a superfluid

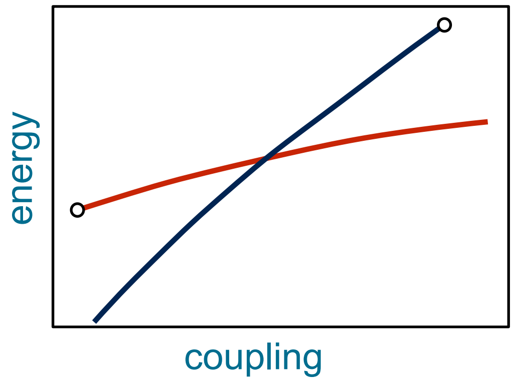
Bogoliubov quasiparticles in a superconductor

static impurity (center) in a normal Fermi liquid

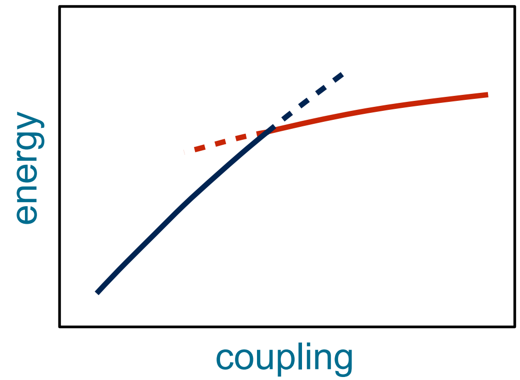
Types of transitions between two charge-quantized states of an impurity (or a trapping center)

in the absence of broken U(1) symmetry

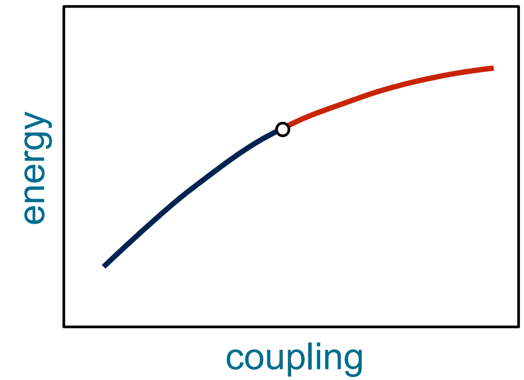
coexistence



asymptotic metastability



critical endpoint



Takes place when there is an insulating gap.

The two edges of the insulating gap define the two endpoints.

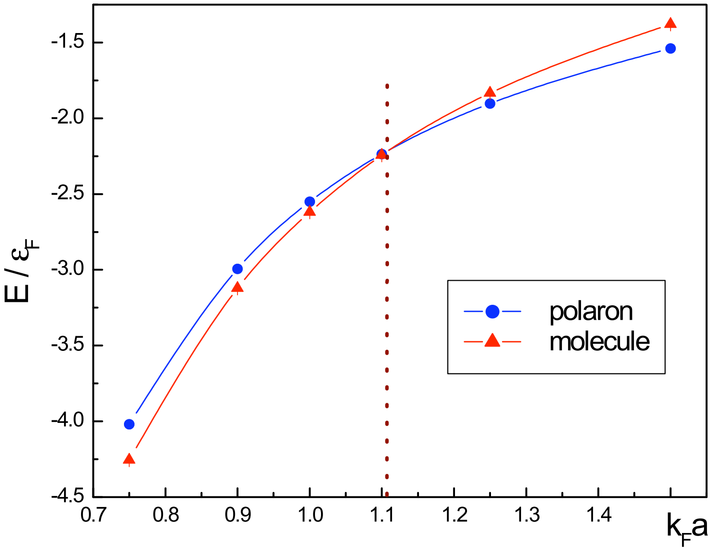
On the approach to the endpoint, the upper branch is a weakly bound state of the lower-branch impurity/center and a quasiparticle compensating the charge.

The upper branch is decaying, but metastable on approach to the transition point: decay width vanishes faster than the energy difference.

?

Example of asymptotic metastability: Resonant Fermi polaron

(Experimental realization with ultracold atoms: Feshbach resonance)



Prokof'ev and Svistunov, Phys. Rev. B **77**, 020408 (2008)

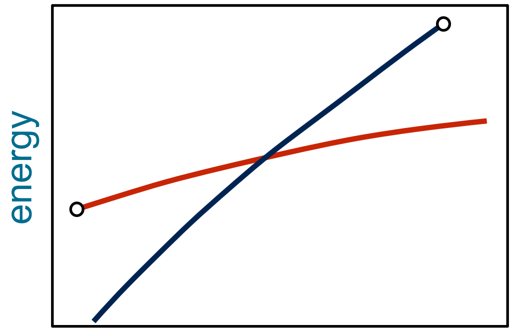
Example of coexistence: an impurity/center in the Mott insulator

Experimental realization with ultracold atoms: optical traps

$$H = -\sum_{\langle i,j\rangle} a_i^+ a_j + U\sum_i n_i\left(n_i - 1\right) + Vn_{i=0}$$

at an integer filling factor

coexistence



On the approach to the endpoint, the upper branch is a weakly bound state of the lower-branch impurity/center and a particle/hole excitation compensating the charge.

Parabolic dispersion is crucial for this type of end-point scenario: There are no weakly bound states for linear dispersion.

Q: What happens right at the superfluid-to-Mott-insulator criticality?

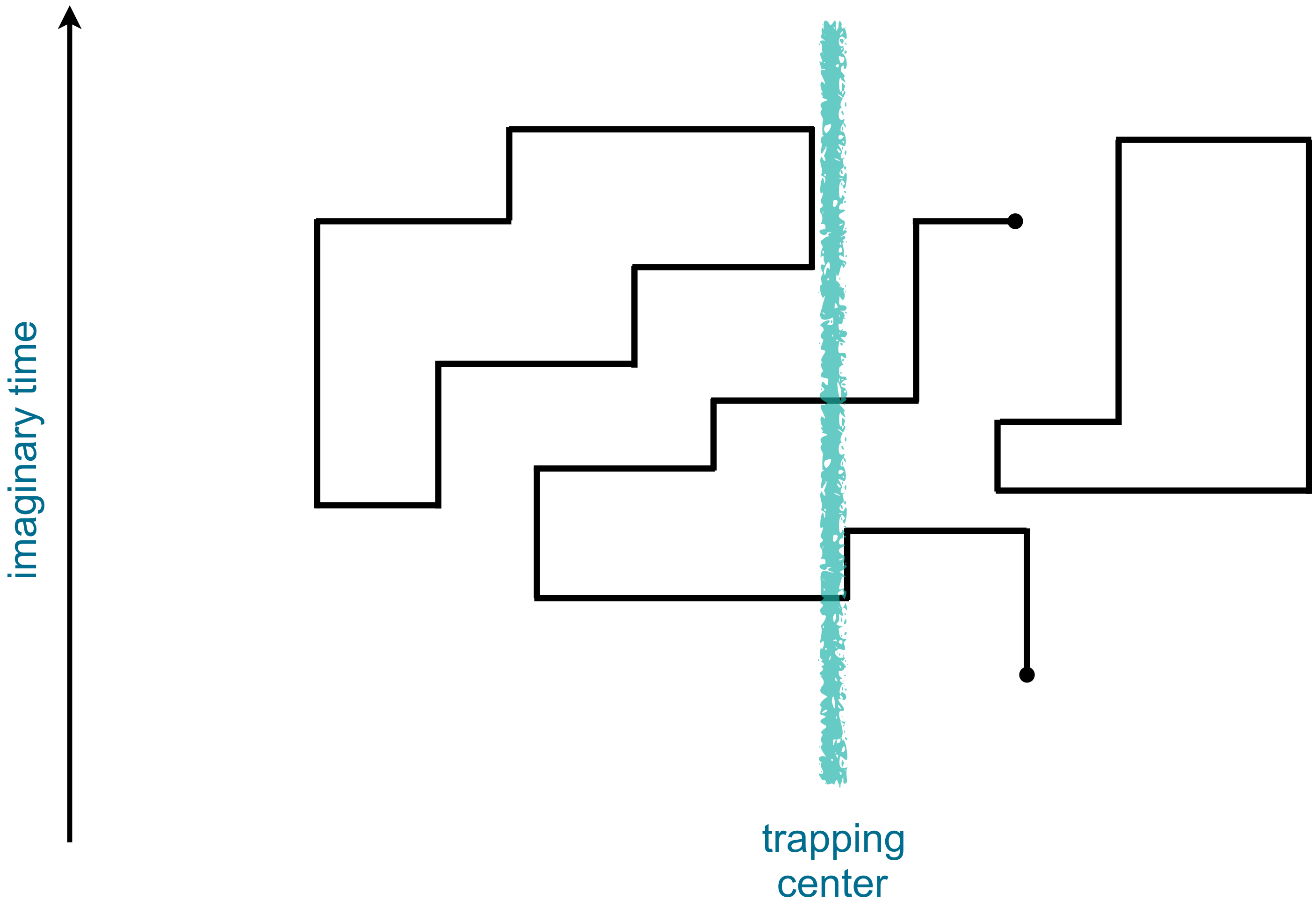
(when the system emulates O(2)-symmetric Wilson-Fisher conformal field theory)

$$H = -\sum_{\langle i,j \rangle} a_i^+ a_j + U_c \sum_i n_i \left(n_i - 1 \right) + V n_{i=0}$$ at an integer filling factor, in 2D

charge = $\int \delta n(r)\, d^2 r$

Path integral simulations by Worm algorithm

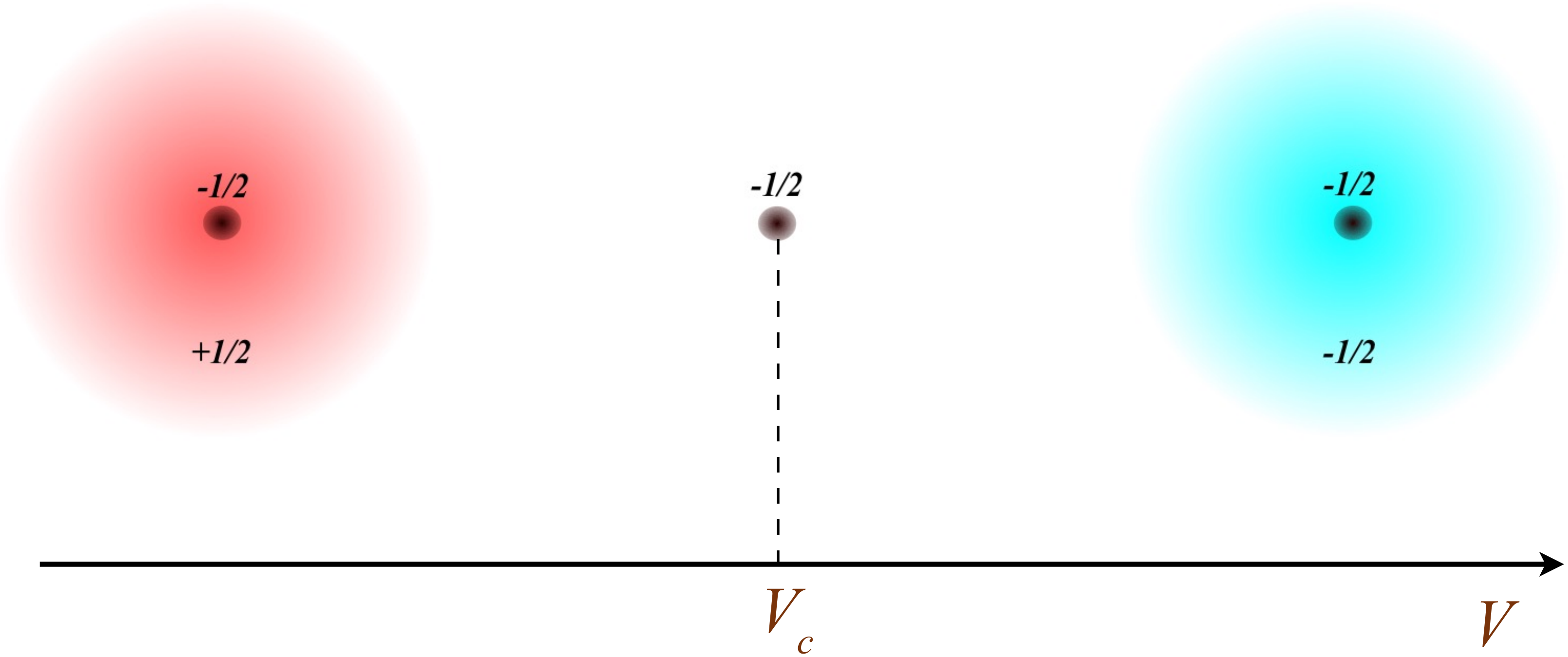
(2+1)-dimensional path-integral representation of the Hubbard model and/or 3D J-current model



Criticality: The Halon

size of the halo: $r_0 \sim |V - V_c|^{-\tilde{\nu}}$, $\tilde{\nu} = 2.33(5)$

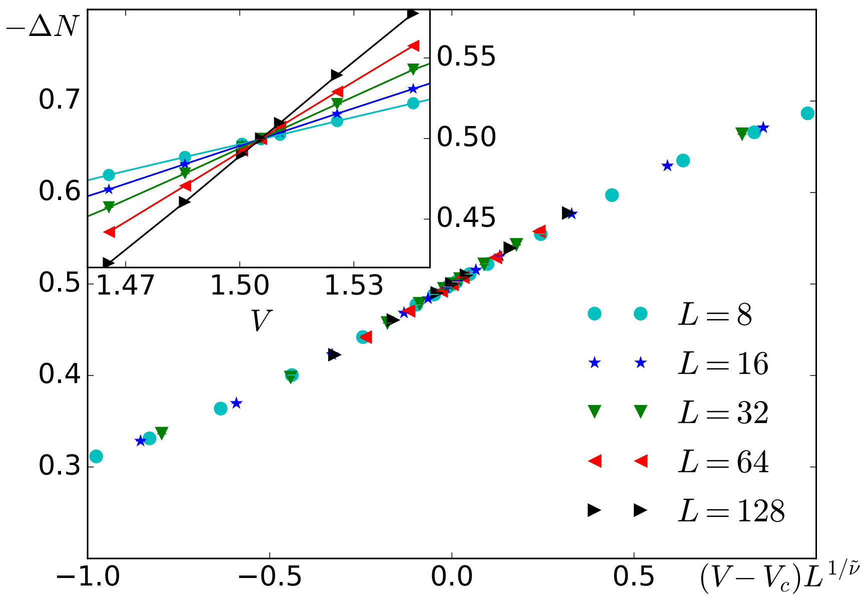
The halo charge $\pm 1/2$ is guaranteed by emergent particle-hole symmetry.



strength of the repulsive center

Extracting the value of $\tilde{\nu}$

In the *grand canonical* ensemble, calculate the change in the total number of particles as a response to the center strength at different system sizes and see which exponent collapses the data.



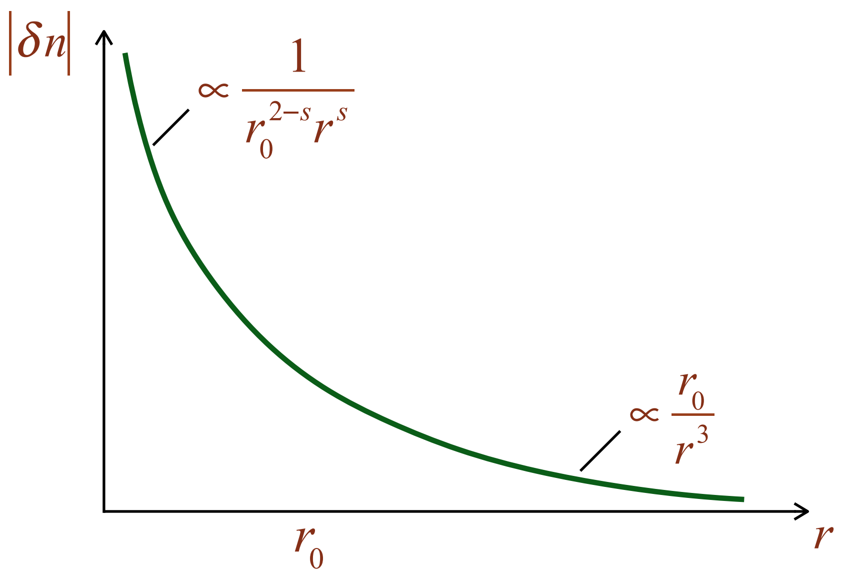
The relevant range of parameters is when the size of the halo is of the order of the system size.

Structure of the halo

$$\delta n(r) = \pm r_0^{-2} f_{\text{halo}}\left(r/r_0\right) \qquad (r \geq r_{\text{uv}})$$ (universal scaling ansatz)

$$f_{\text{halo}}(x) \propto 1/x^3, \qquad x \gg 1$$ (linear-response tail)

$$f_{\text{halo}}(x) \propto 1/x^s, \qquad \boxed{s = 1 + 1/\tilde{\nu},} \qquad x \ll 1$$ (singular core)



The amplitude of the linear-response tail diverges on the approach to the critical point, whereas the amplitude of the singular core vanishes.

Deriving relation $s = 1 + 1/\tilde{\nu}$

$$\left(r_0 \sim |V - V_c|^{-\tilde{\nu}} \right)$$

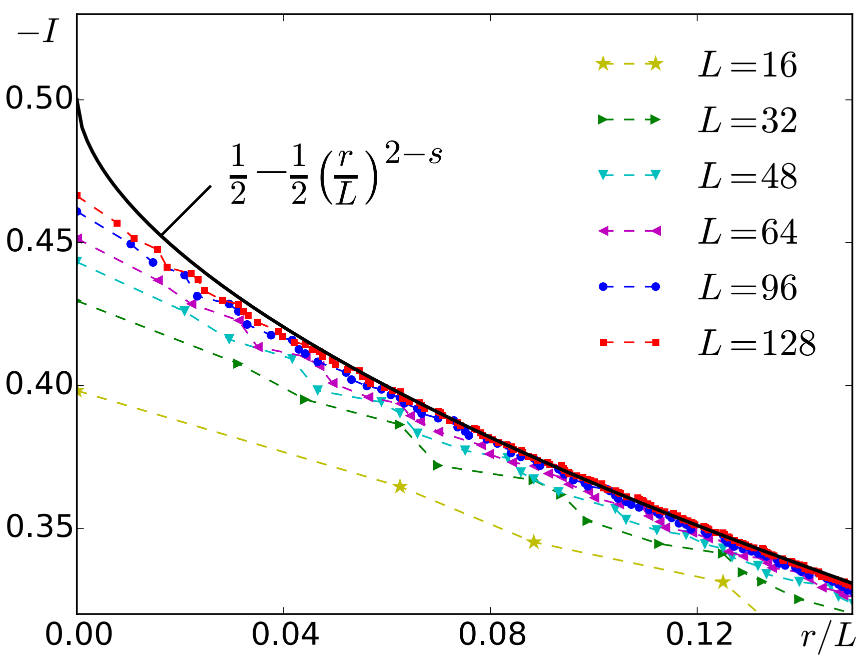
$$\left.\begin{array}{l}
H_{\mathrm{ctr}} = V\hat{n}_0 \quad \Rightarrow \quad n_0 \equiv \langle \hat{n}_0 \rangle \equiv \left\langle \dfrac{\partial H}{\partial V} \right\rangle = \dfrac{dE}{dV} \\
n_0 = \mathrm{reg.\,part} + \delta n(r \sim r_{\mathrm{uv}}) \quad \Rightarrow \quad n_0 \propto \mathrm{reg.\,part} + \dfrac{1}{r_0^{2-s}} \\
\dfrac{dE}{dV} = \mathrm{reg.\,part} + \dfrac{dE_{\mathrm{halo}}}{dV}
\end{array}\right\} \Rightarrow \quad \frac{1}{r_0^{2-s}} \propto \frac{dE_{\mathrm{halo}}}{dV}$$

$$E_{\mathrm{halo}} \propto 1/r_0 \quad \Rightarrow \quad \frac{dE_{\mathrm{halo}}}{dV} = \frac{dE_{\mathrm{halo}}}{dr_0}\frac{dr_0}{dV} \sim \frac{1}{r_0^2}\frac{dr_0}{dV} \sim \frac{1}{r_0^{1-1/\tilde{\nu}}}$$

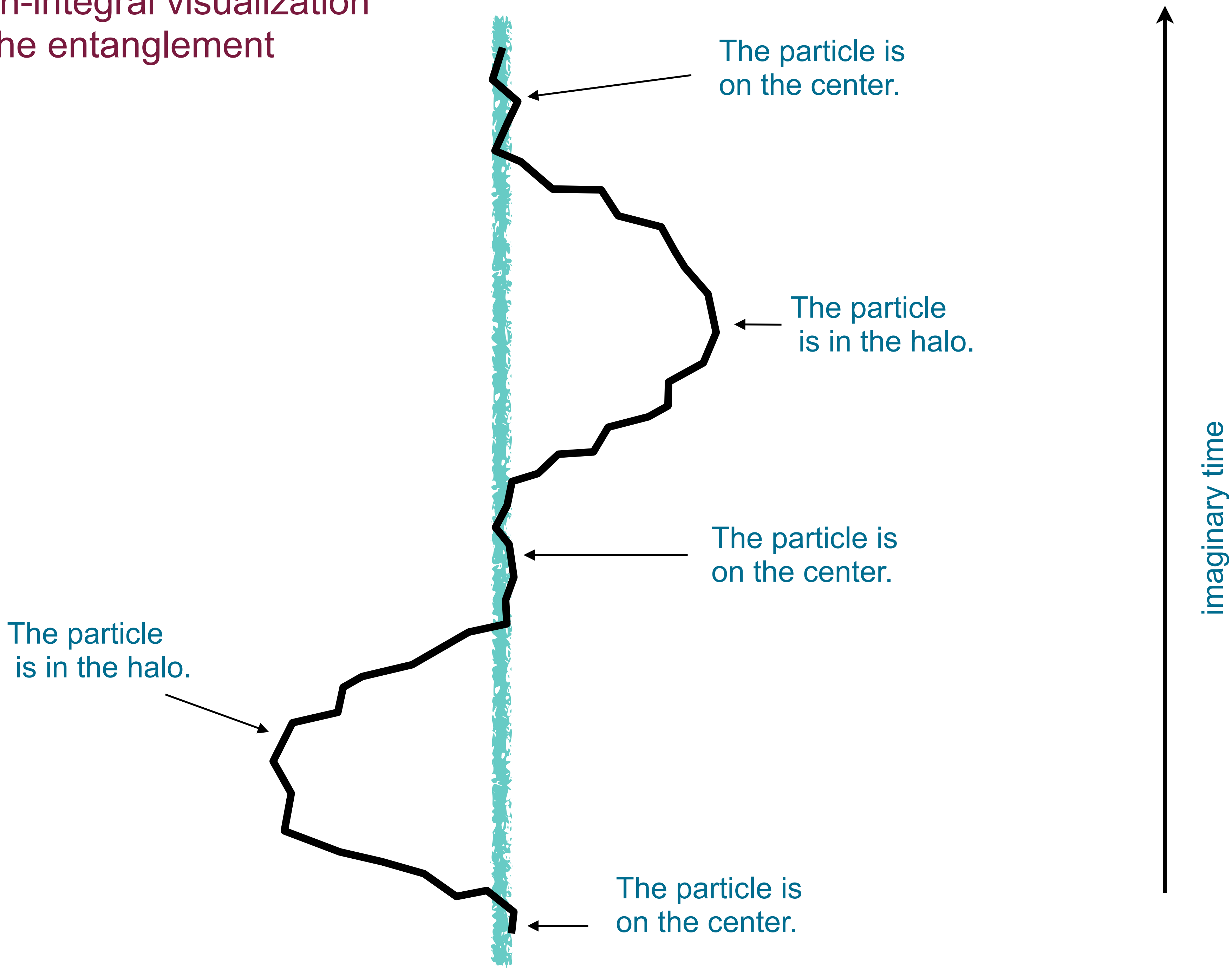
Consistency with numerics

In the *canonical* ensemble, and right at $V=V_c$, calculate the integral $I(r)$ and compare to the finite-size scaling ansatz.

$$I(r) = \int_{r'<r} \delta n(r')\, d^2r', \qquad I(r) = -\frac{1}{2} + \text{const}\left(\frac{r}{L}\right)^{2-s}$$



Path-integral visualization of the entanglement



Minimalistic model: Spin-1/2 impurity in the O(2)-critical bosonic environment

$$H_{\text{int}} = \gamma(\hat{\psi}\hat{S}_{+} + \hat{\psi}^{\dagger}\hat{S}_{-}) + h_z\hat{S}_z$$

$Q = \hat{S}_z + \int d^d r\, \hat{\psi}^{\dagger}\hat{\psi}$ the charge (takes on half-integer values)

$h_z\hat{S}_z$ creates the halon

$h_x\hat{S}_x$ yet another relevant perturbation

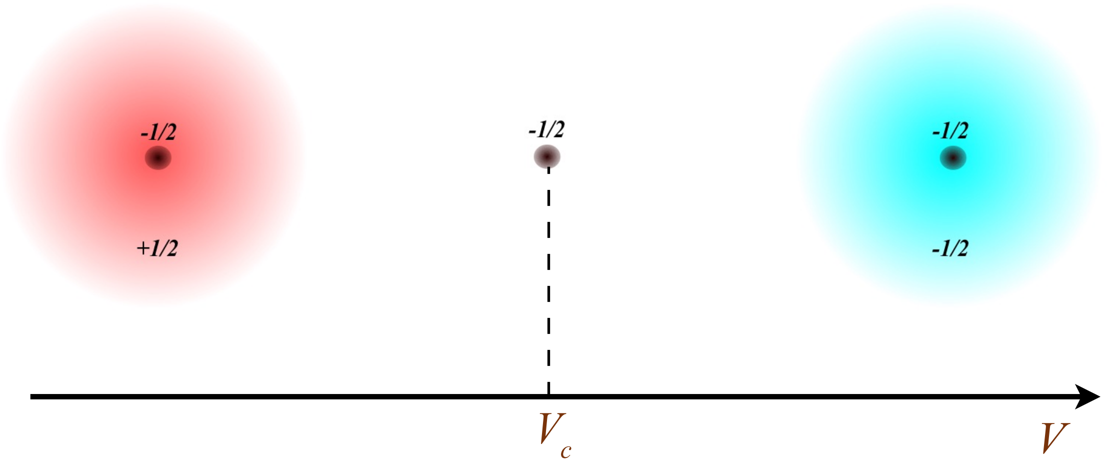
correlation length/time: $\xi_\alpha \propto |h_\alpha|^{-\nu_\alpha}$ $(\alpha = z, x)$

	Monte Carlo	RG of Ref. [1]
$\nu_z \equiv \tilde{\nu}$	2.33(5)	2.66
ν_x	1.13(2)	1.08

[1] Seth Whitsitt and Subir Sachdev, arXiv: 1709.04919

[2] Kun Chen, Yuan Huang, Youjin Deng, and BS, 2017 (to appear soon)

To summarize: the halon, a new quasiparticle with unusual properties



1. The charge of the center is half-integer.

2. The charge of the halo is plus/minus one half.

3. One particle (or hole) gets entangled between the center and the halo.