

TENSOR NETWORK STATES FOR 1+1D LATTICE GAUGE THEORIES

Mari-Carmen Bañuls

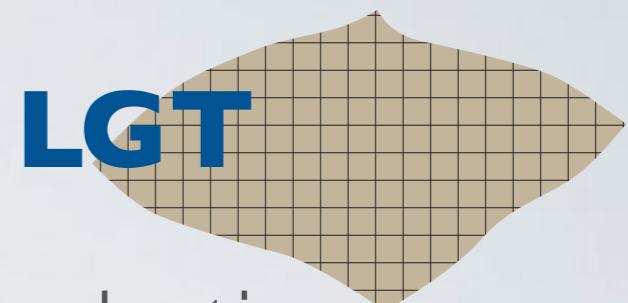
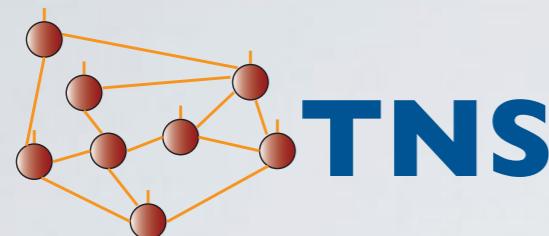
with K. Cichy (Frankfurt), K. Jansen (DESY), H. Saito,
J.I. Cirac, S. Kühn (MPQ)



Max-Planck-Institut
für Quantenoptik
(Garching b. München)

Mainz MITP 20.9.2017

WHY?



Non-perturbative way of
solving QFT (QCD)

Mostly path-integral
formalism & MC

4D lattice

spectrum

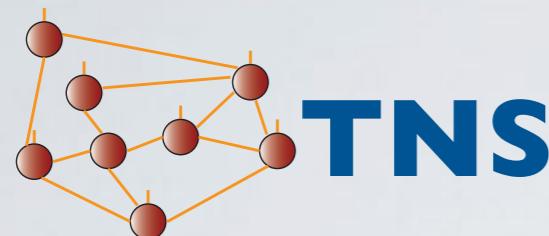
finite T

large 3+1 dim lattices

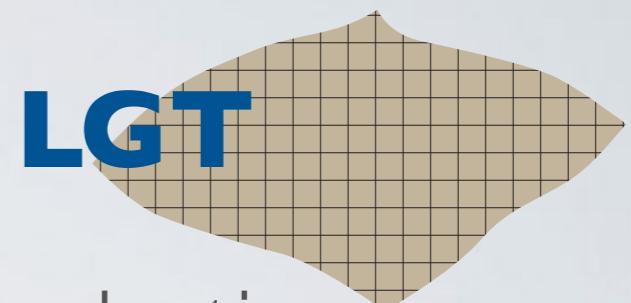
chemical potential

time evolution

WHY?



Non-perturbative for
Hamiltonian systems



Non-perturbative way of
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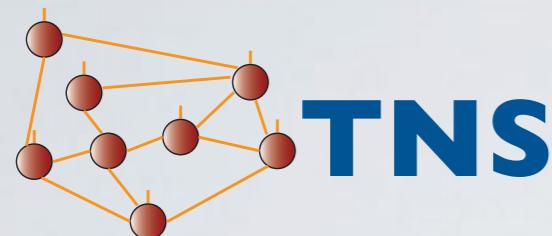
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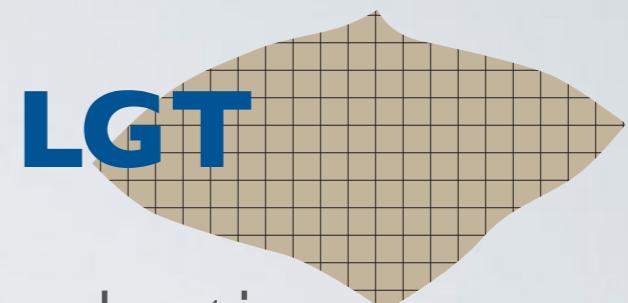
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Non-perturbative for
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Extremely successful for
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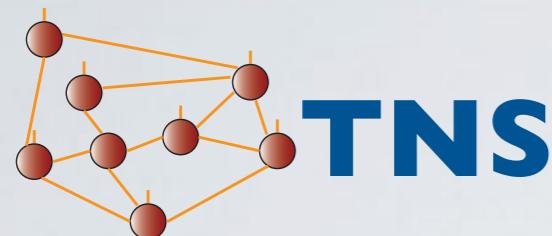
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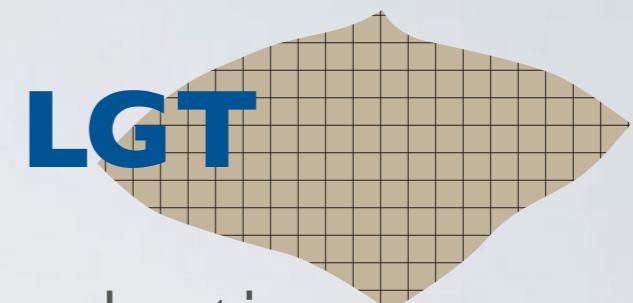
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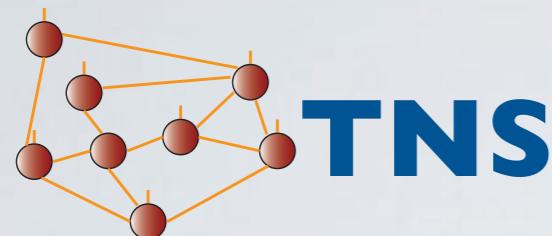
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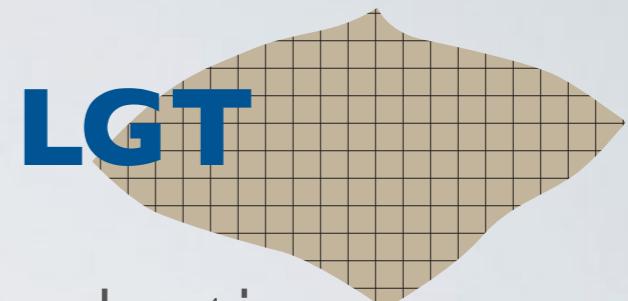
Promising improvements
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ground states

low-lying excitations

thermal states

time evolution



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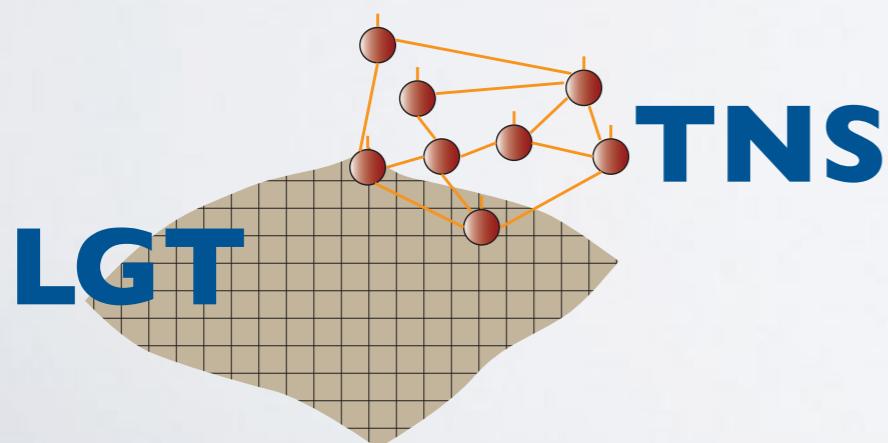
time evolution

In this talk...

Using TNS/MPS for LGT

Overview of recent results:

from Abelian to non-Abelian



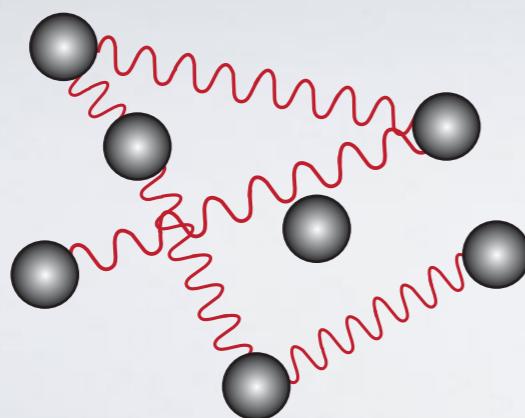
WHAT ARE TNS?

- TNS = Tensor Network States

A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle$$

N

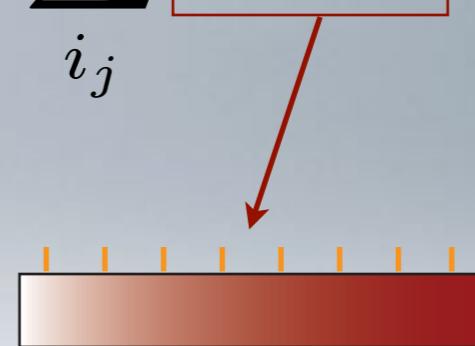


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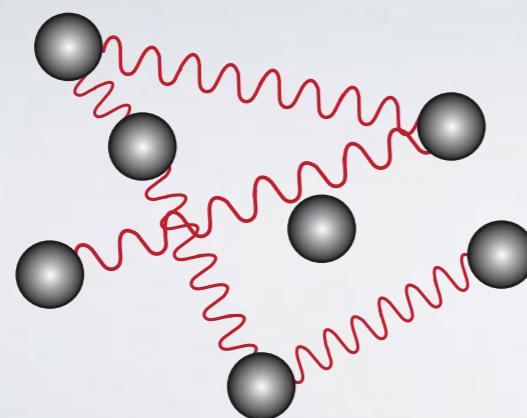
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N-legged
tensor

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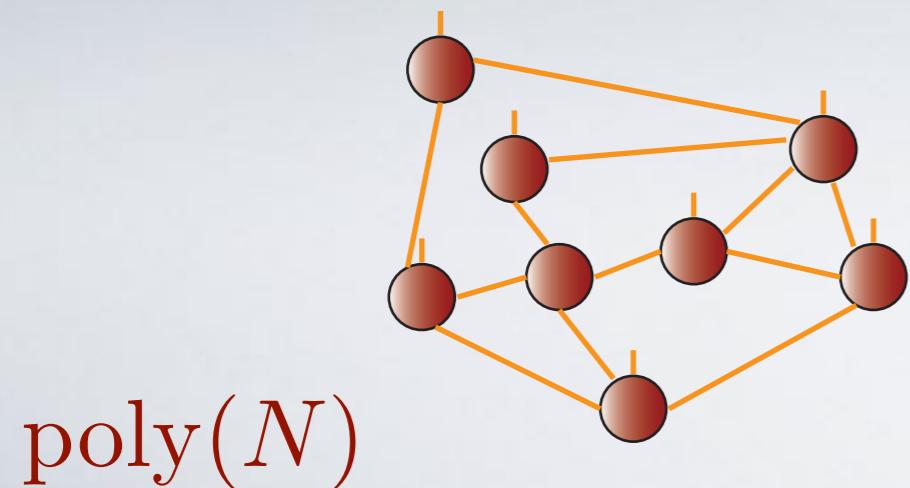


d^N

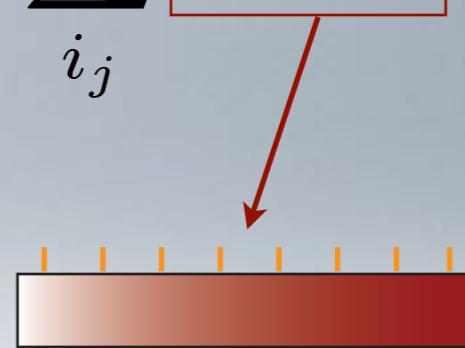
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ATNS has only a polynomial number of parameters

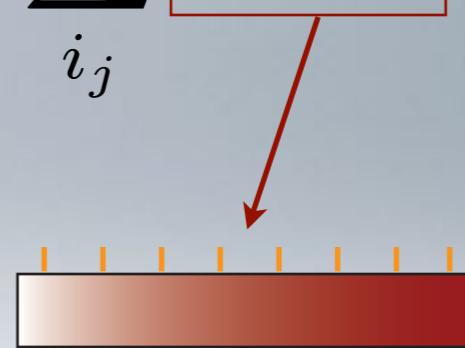
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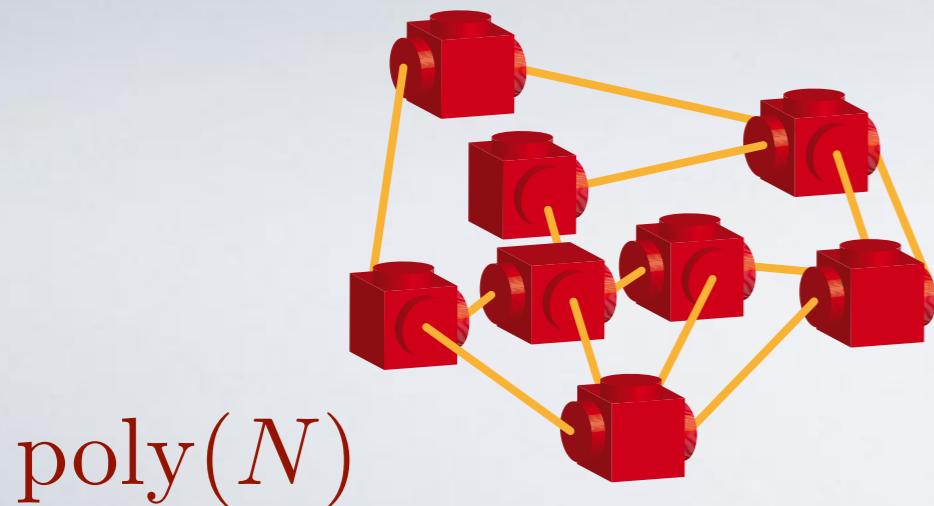
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$\text{poly}(N)$

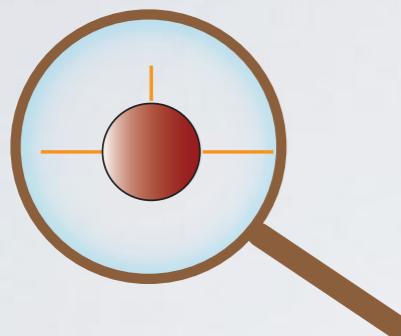
USING TNS FOR QMB

a formal approach

numerical algorithms

USING TNS FOR QMB

a formal approach



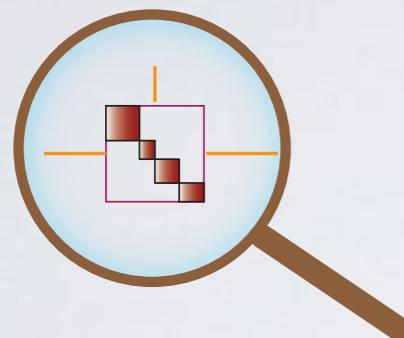
classifying tensors
constructing states

Chen et al PRB 2011
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Wahl et al PRL 2013; Yang et al PRL 2015
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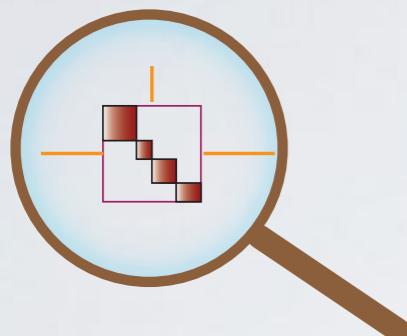
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great descriptive power: phases,
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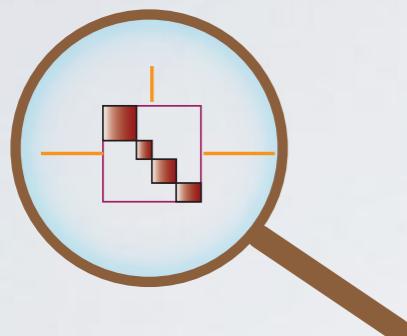
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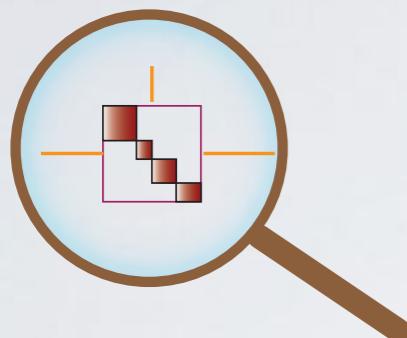
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no sign problem

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tensor networks describe
partition functions (observables)

need to contract a TN
TRG approaches

Nishino, JPSJ 1995

Levin & Wen PRL 2008

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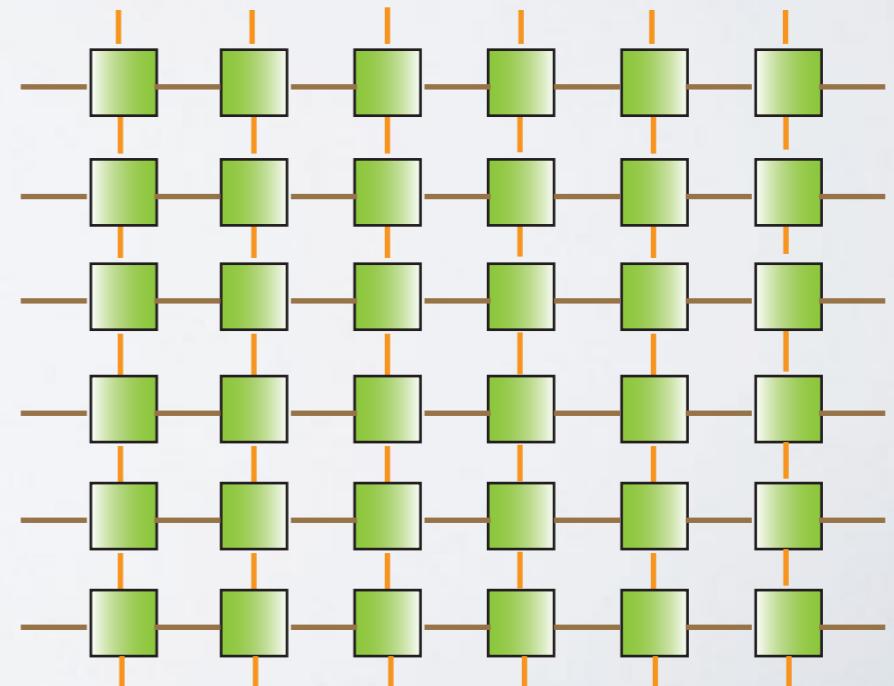
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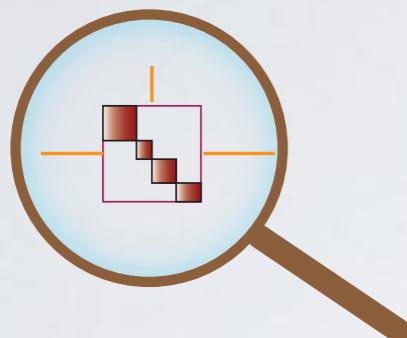
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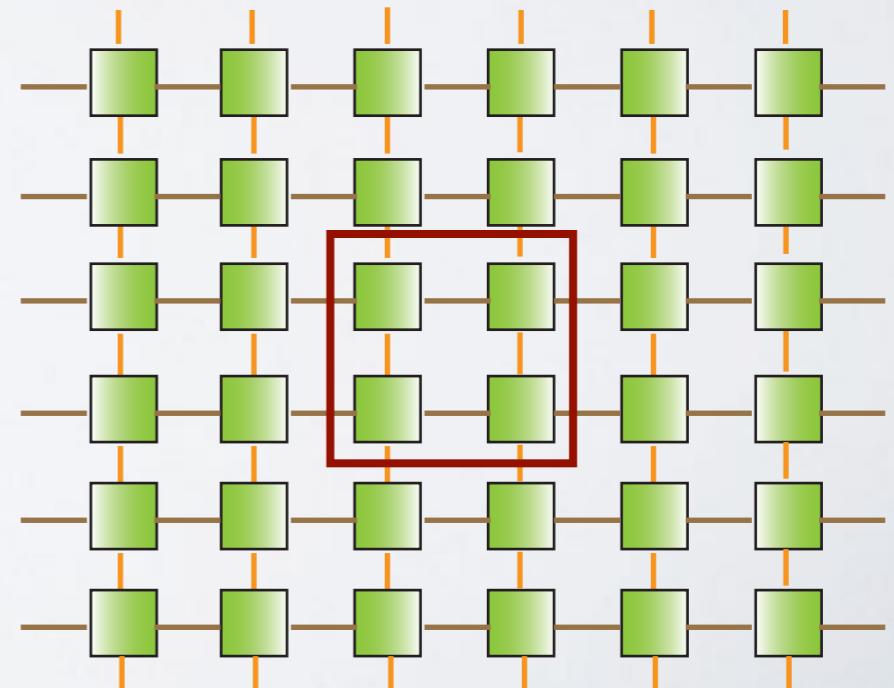
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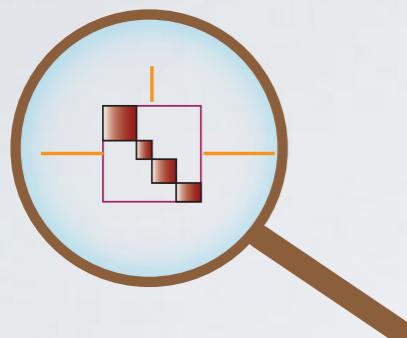
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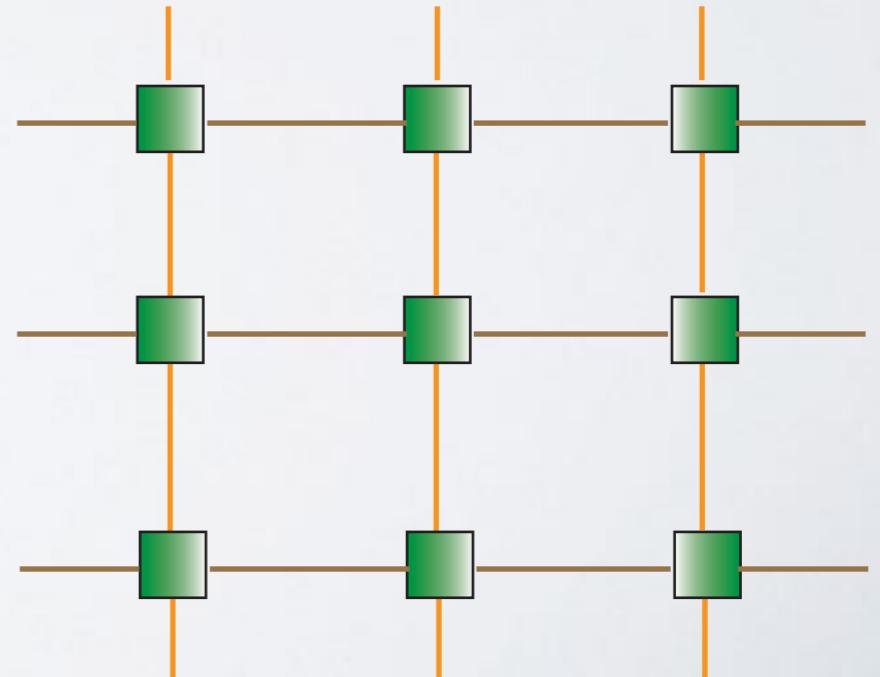
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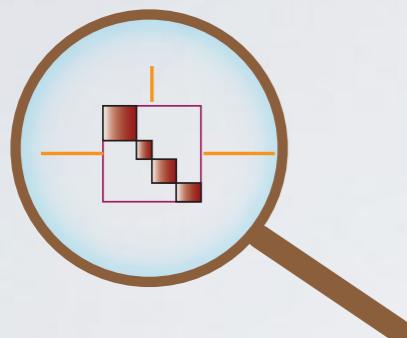
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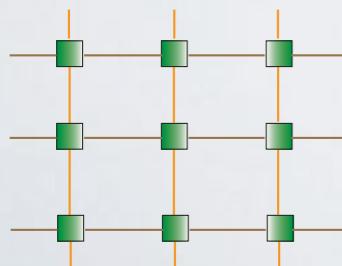
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TNS as ansatz for the state

efficient algorithms for GS, low
excited states, thermal, dynamics

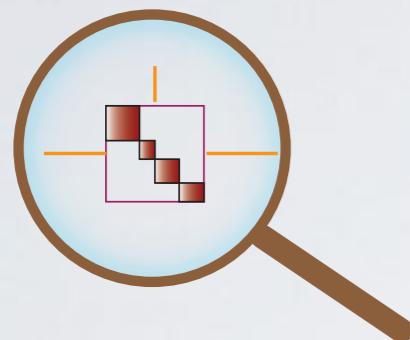
White PRL 1992; Schollwöck RMP 2011

Vidal PRL 2003; Verstraete et al PRL 2004

Verstraete et al Adv Phys 2008; Orús Ann Phys 2014

USING TNS FOR LGT

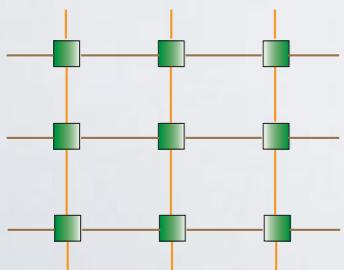
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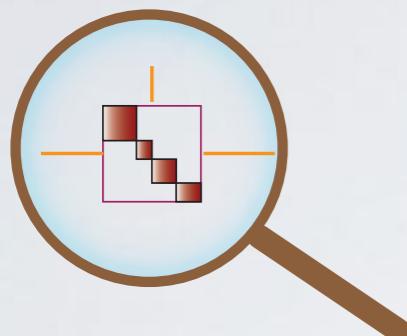


numerical algorithms
↓
TNS as ansatz for the state



USING TNS FOR LGT

a formal approach



gauging the symmetry
explicitly invariant states

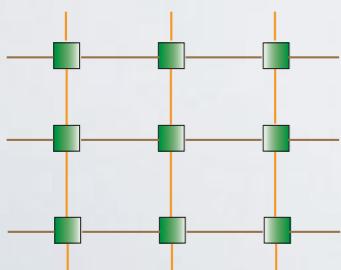
general prescriptions, $U(1)$, $SU(2)$

Tagliacozzo et al PRX 2014
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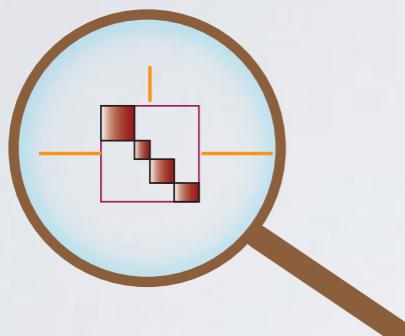


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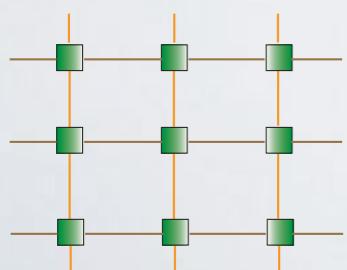
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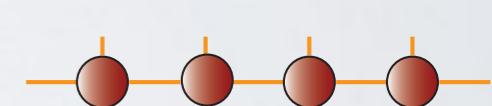


TRG approaches to classical
and quantum models

Liu et al PRD 2013

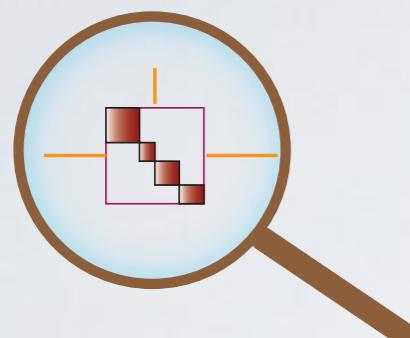
Shimizu, Kuramashi, PRD 2014

Kawauchi, Takeda 2015



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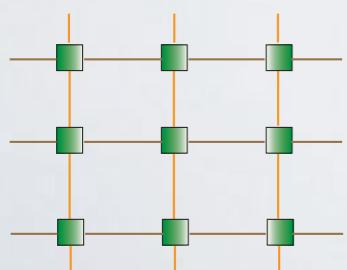
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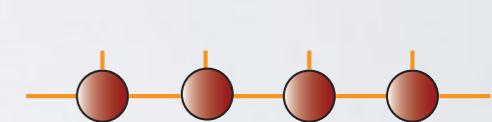
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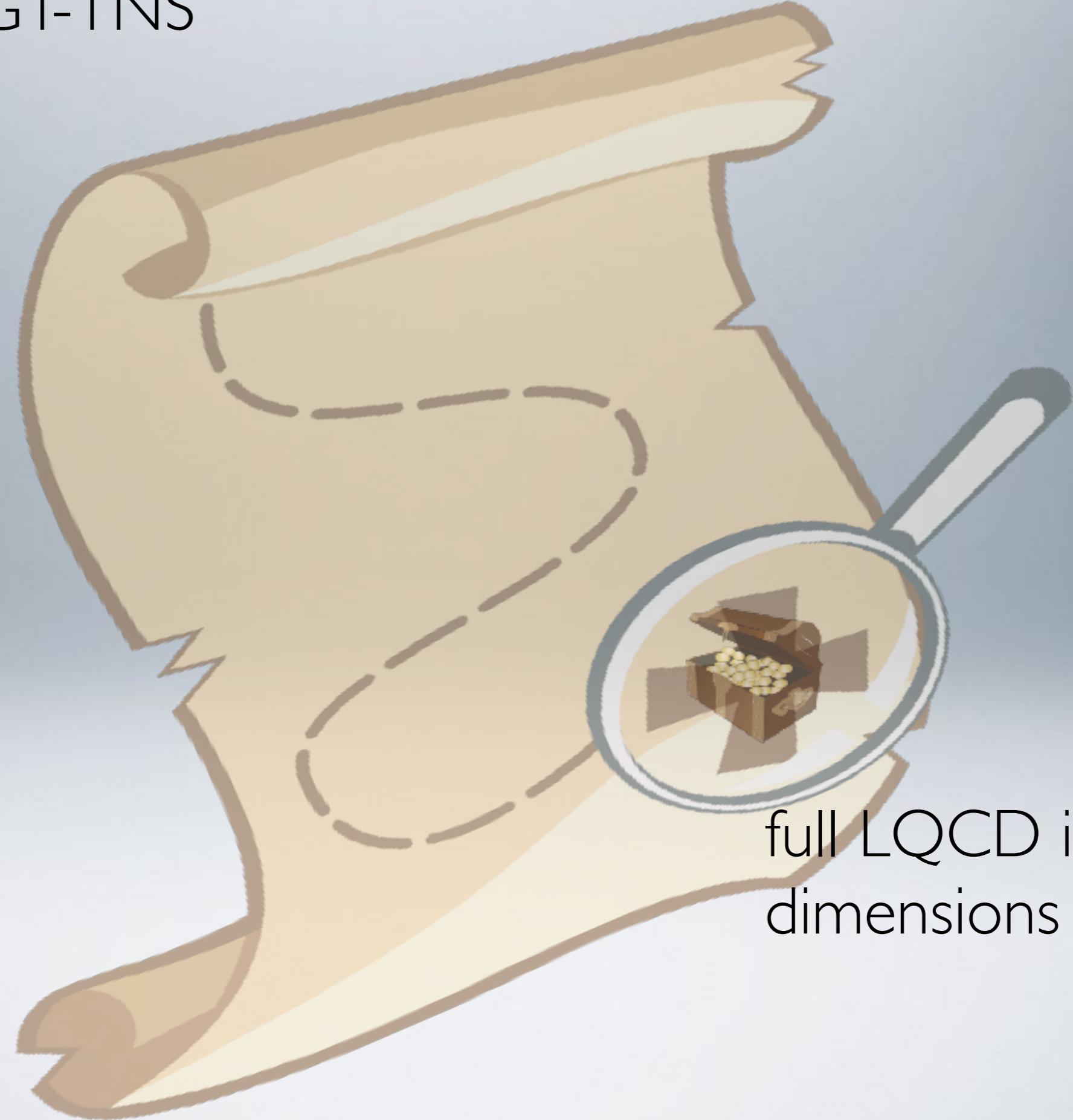
next...



a possible LGT-TNS
roadmap...



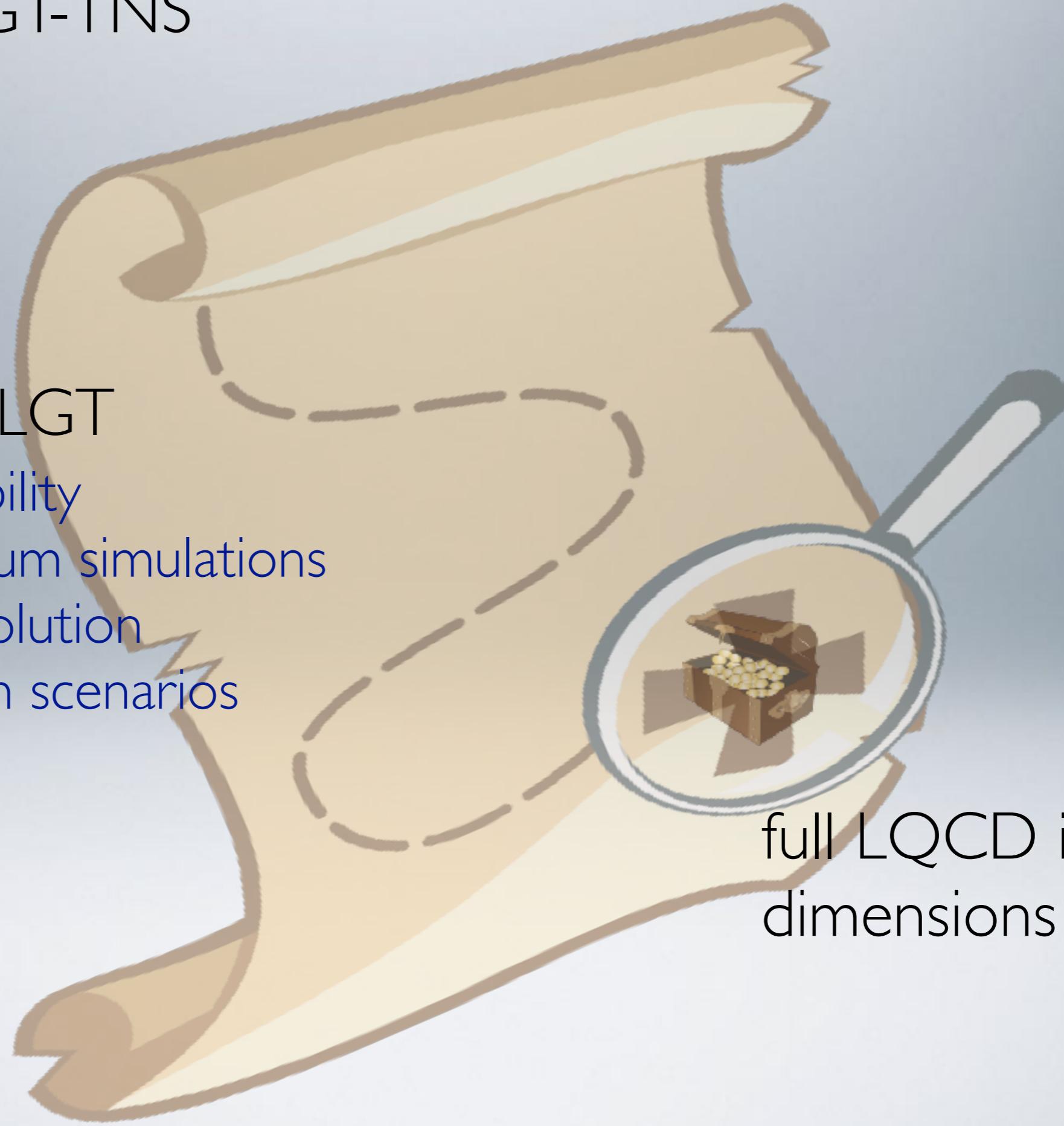
a wishful LGT-TNS
roadmap...



full LQCD in 3+1
dimensions

a wishful LGT-TNS roadmap...

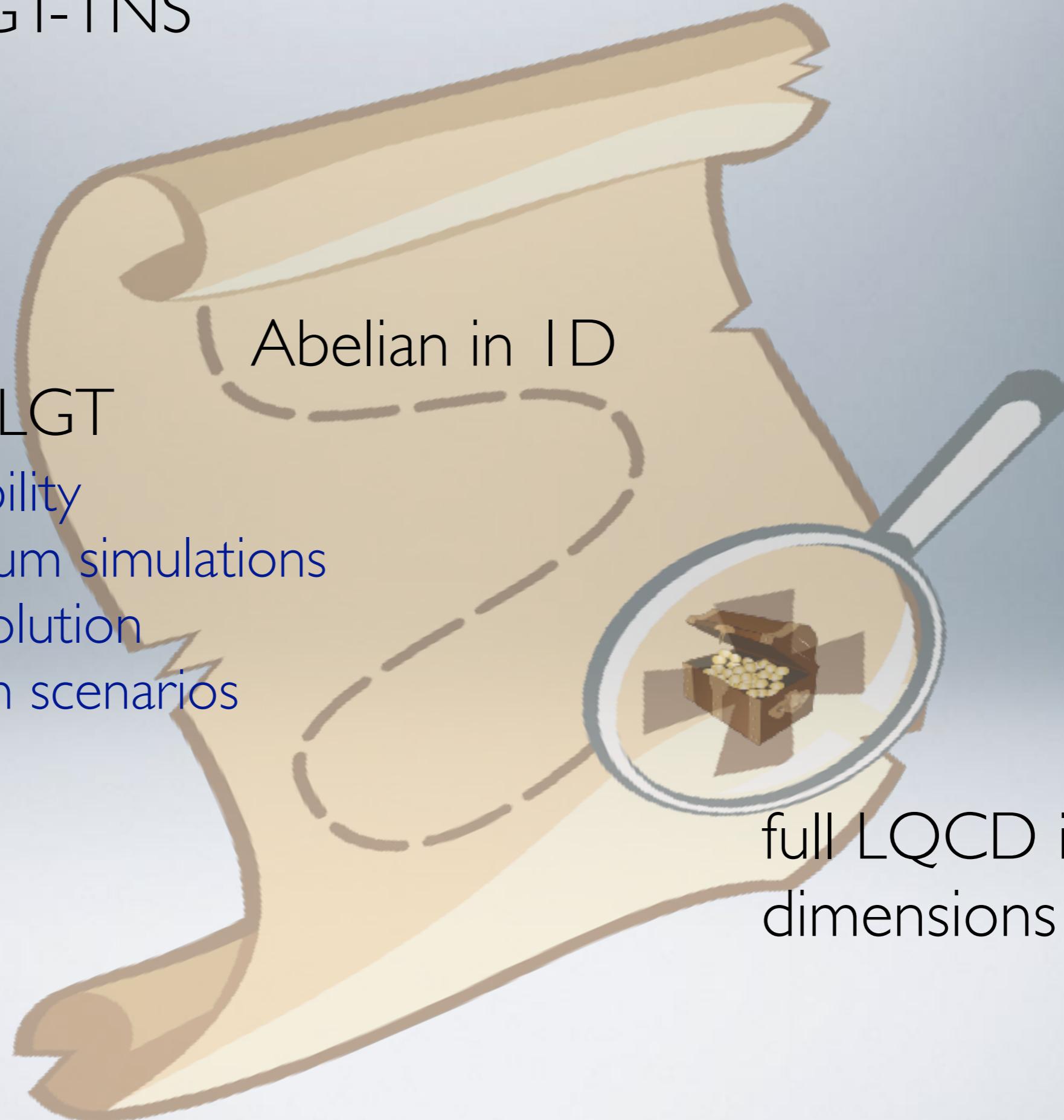
I+1D LGT
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precise equilibrium simulations
time evolution
sign problem scenarios



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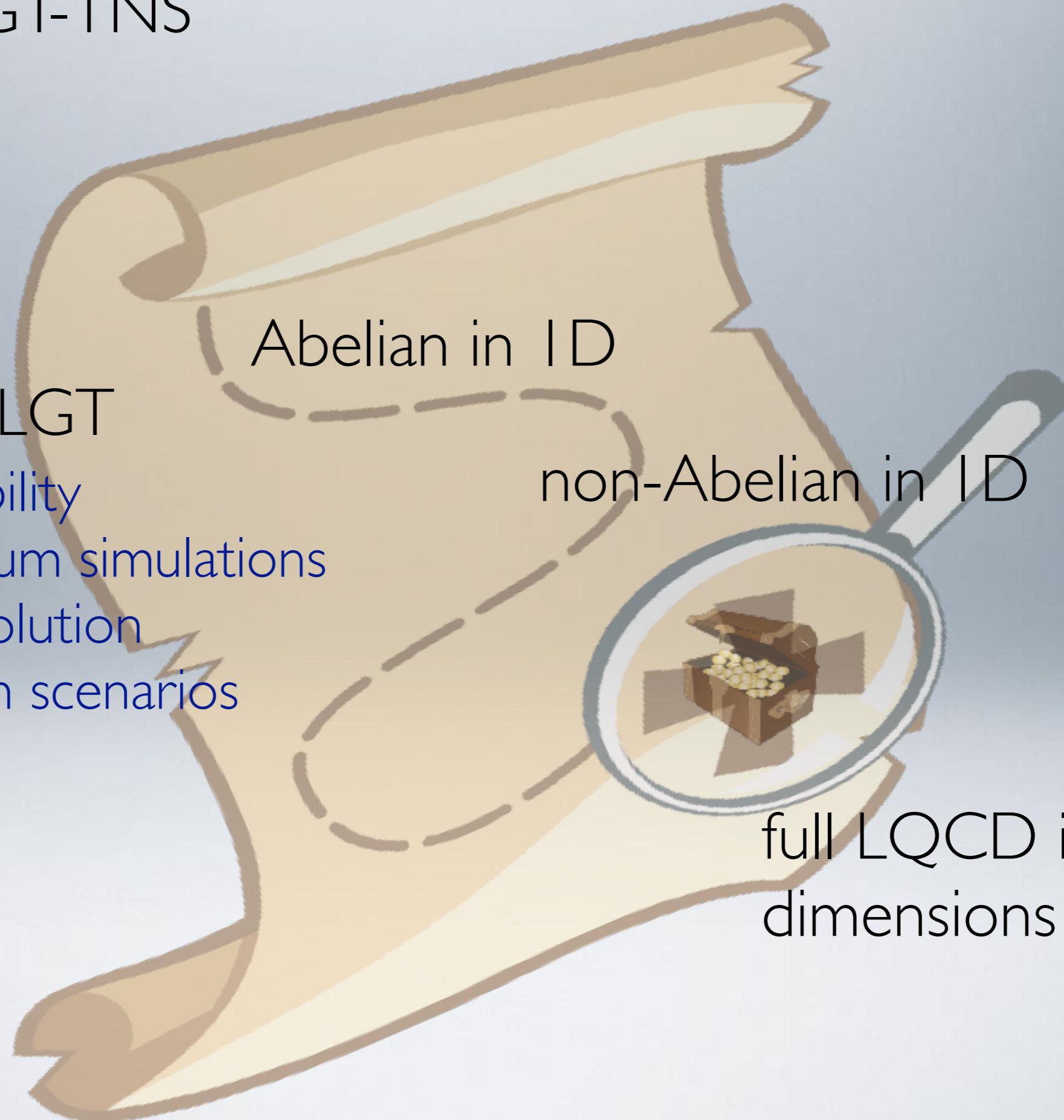
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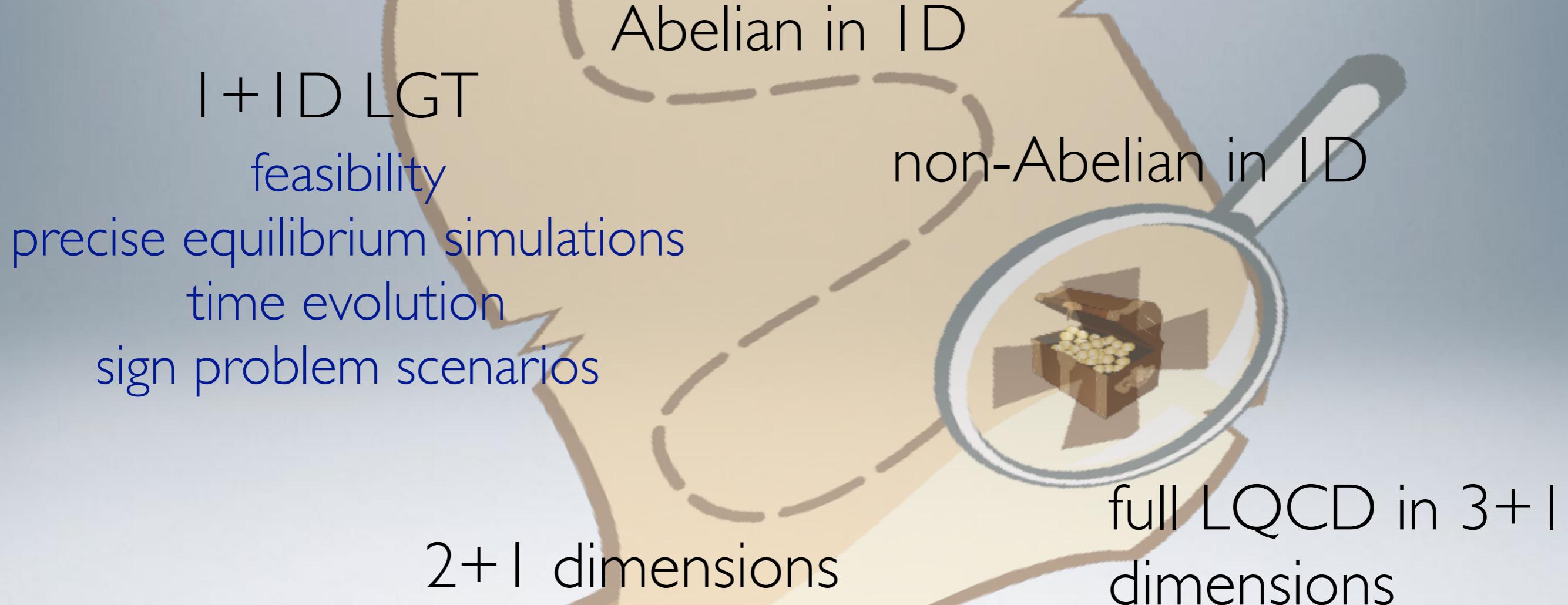


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TNS FOR LGT

early
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TNS FOR LGT

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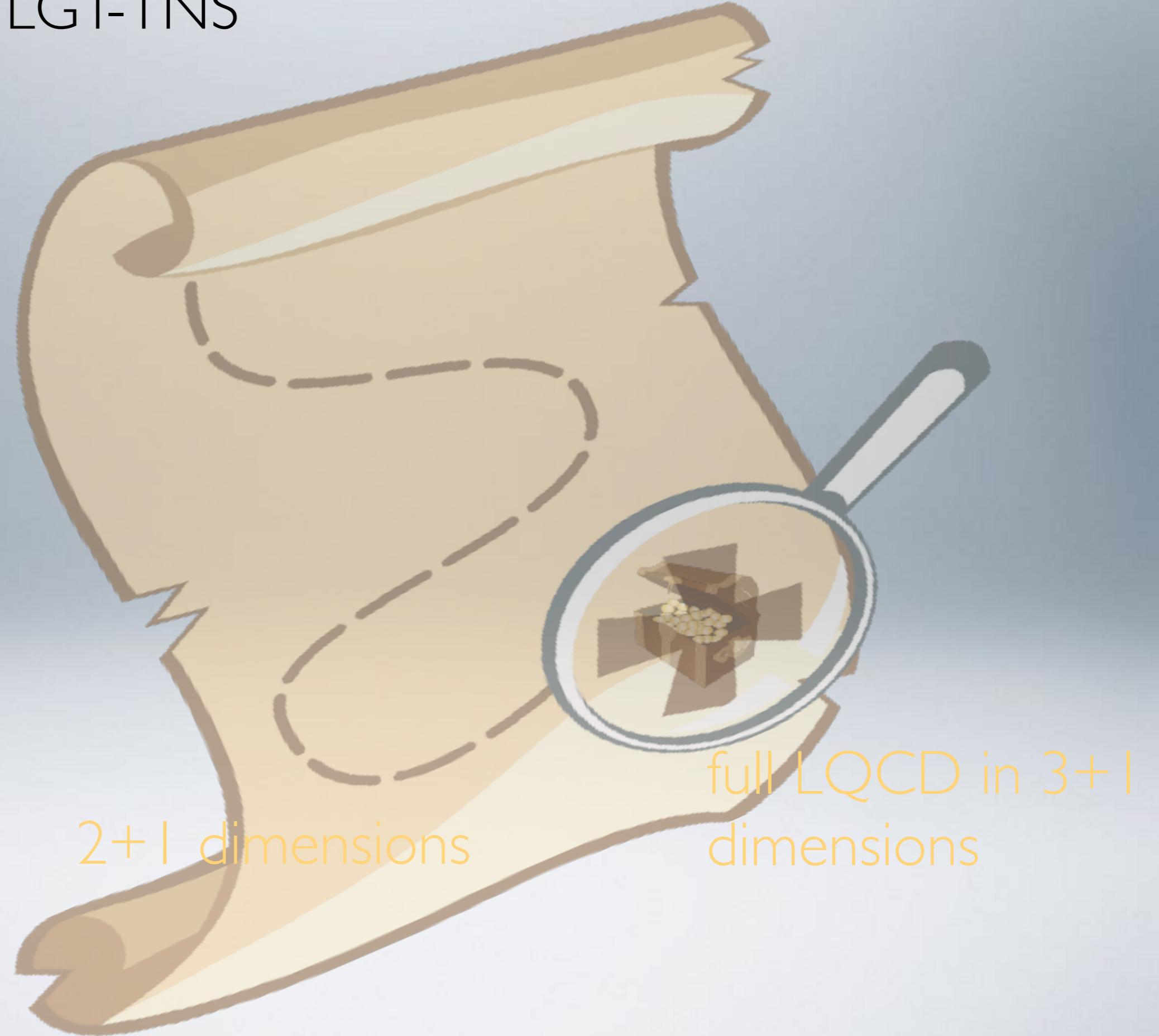
MPS for critical QFT

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TNS for classical gauge models

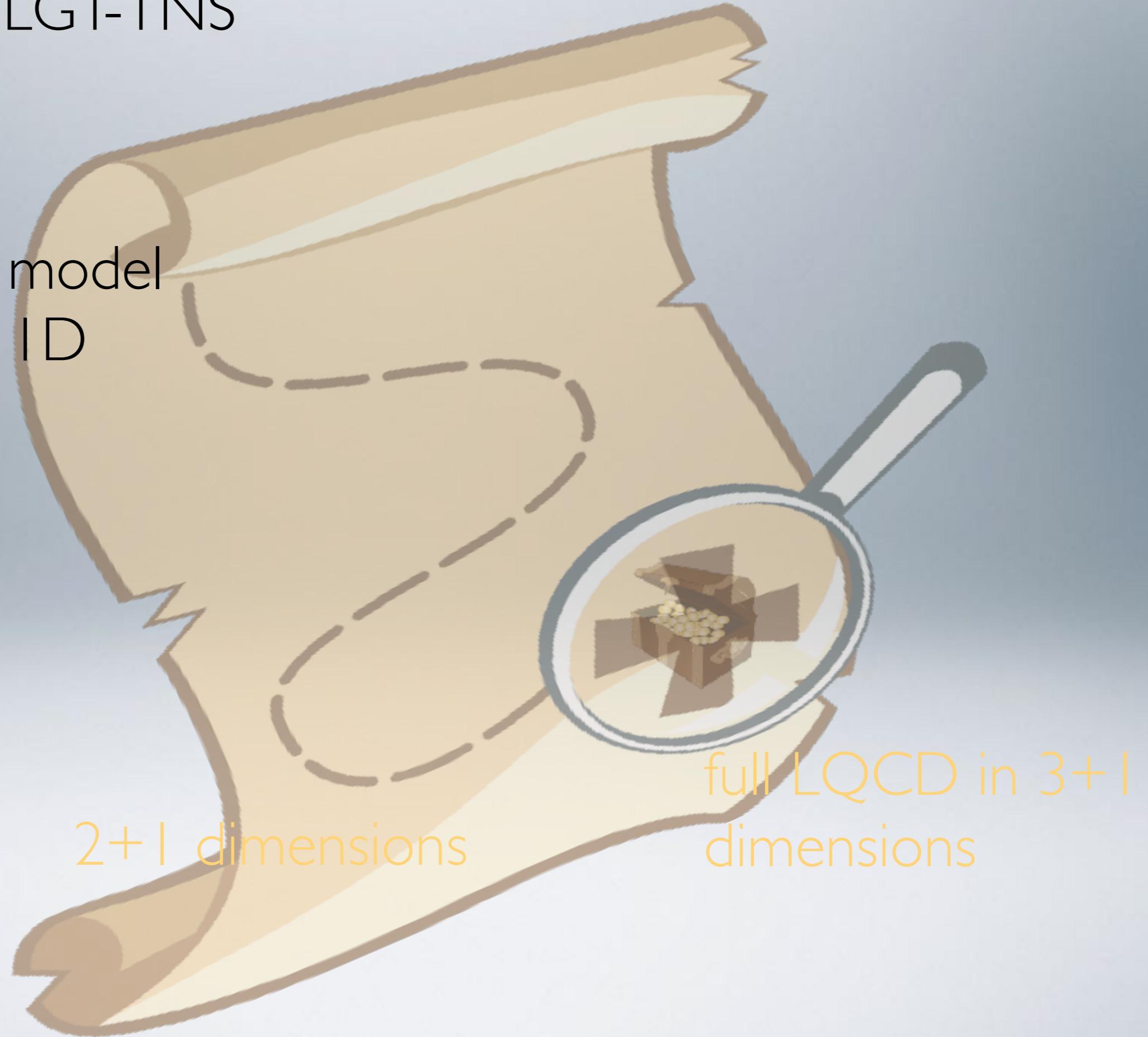
Meurice et al. 2013

an ongoing LGT-TNS
roadmap...



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Schwinger model
 $U(1)$ in 1D



an ongoing LGT-TNS roadmap...

Schwinger model
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precise equilibrium
simulations,
feasibility of QSim

MCB et al JHEP 11(2013)158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
S. Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review Dalmonte, Montangero, Cont. Phys. 2016

finite density

S. Kühn et al, PRL 118 (2017) 071601;

2+1 dimensions

full LQCD in 3+1
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Non-Abelian in 1D
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see also Silvi et al., Quantum 2017
S. Kühn et al. arXiv:1707.06434



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other models in
1+1 dimensions
in progress



full LQCD in 3+1
dimensions

an ongoing LGT-TNS roadmap...

Schwinger model
 $U(1)$ in 1D
precise equilibrium
simulations,
feasibility of QSim

MCB et al JHEP 11 (2013) 158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
S. Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review Dalmonte, Montangero, Cont. Phys. 2016

2+1 dimensions

R. Orús' talk

finite density

S. Kühn et al, PRL 118 (2017) 071601;

Non-Abelian in 1D
string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130;
see also Silvi et al., Quantum 2017
S. Kühn et al. arXiv:1707.06434

other models in
1+1 dimensions
in progress



full LQCD in 3+1
dimensions

SOLVING LGT WITH TNS

GENERAL STRATEGY

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Hamiltonian formulation

acting on a Hilbert space

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Finite dimensional degrees of freedom

gauge dof require attention

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Common ingredients for quantum simulation

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Common ingredients for quantum simulation

Schwinger model example

SCHWINGER MODEL

continuum

$$H = \int dx [-i\bar{\Psi}\gamma^1\partial_1\Psi + g\bar{\Psi}\gamma^1A_1\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2]$$

plus constraint: Gauss' Law

$$\partial_1 E = g\bar{\Psi}\gamma^0\Psi$$

SCHWINGER MODEL

discretized

$$H = -\frac{i}{2a} \sum_n (\phi_n^\dagger e^{i\theta_n} \phi_{n+1} - \text{h.c.}) + m \sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2} \sum_n L_n^2$$

plus constraint: Gauss' Law

$$L_n - L_{n-1} = \phi_n^\dagger \phi_n - \frac{1}{2} [1 - (-1)^n]$$

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MPS representation

basis $| \dots s_e \ell s_o \ell s_e \ell s_o \dots \rangle$

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SCHWINGER MODEL

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are local

can be implemented with explicitly
gauge invariant tensors,
truncating values of electric flux

Buyens et al., PRL 2014;
Silvi et al, NJP 2014;
See also Buyens et al., PRD 2017

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$|\ell_0 \dots s_e s_o s_e s_o \dots\rangle$ non-local
terms

COMPUTING CONTINUUM QUANTITIES

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Scan parameters

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m/g

mass gaps and GS energy density
in the continuum $x \rightarrow \infty$

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$x \in [5, 600]$

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$N \propto x$ (up to ~ 850)

COMPUTING CONTINUUM QUANTITIES

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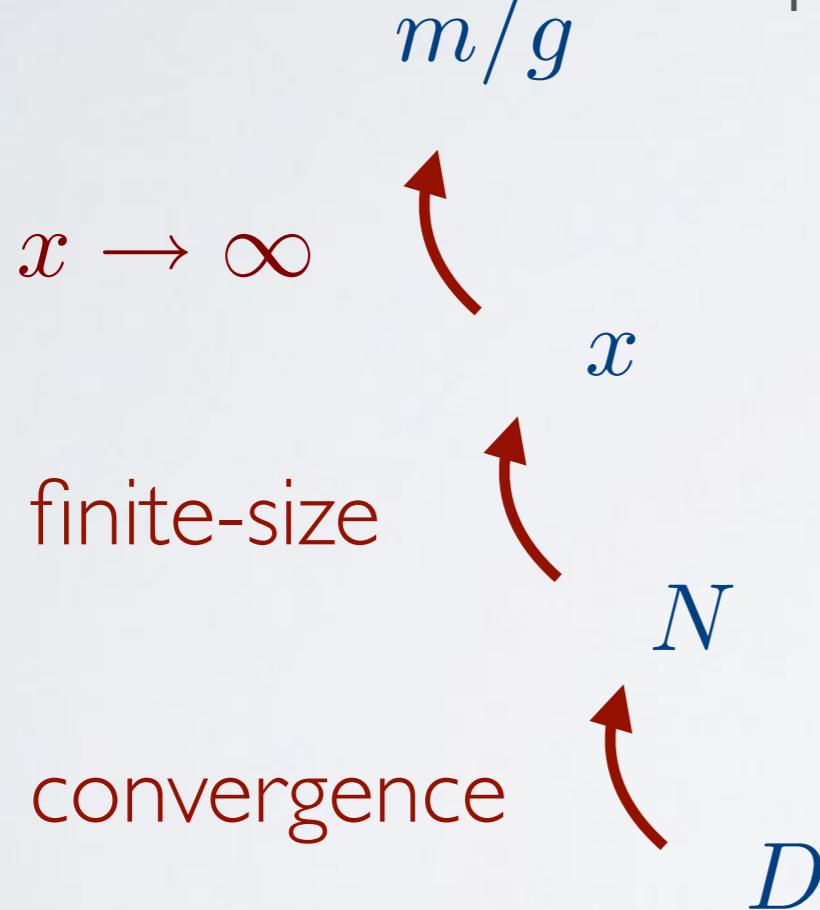
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D

$D \in [20, 120]$

COMPUTING CONTINUUM QUANTITIES

Scan parameters

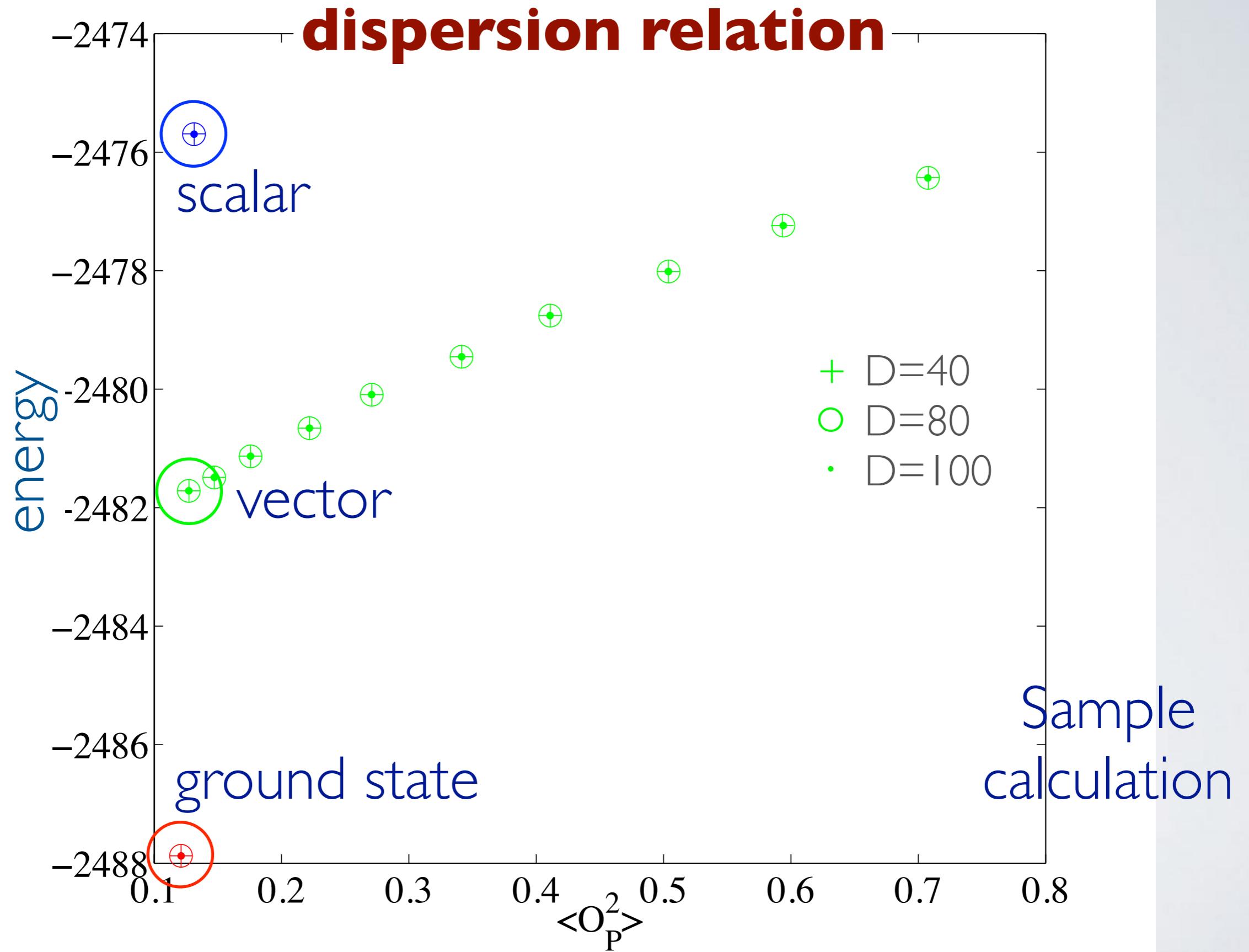


mass gaps and GS energy density
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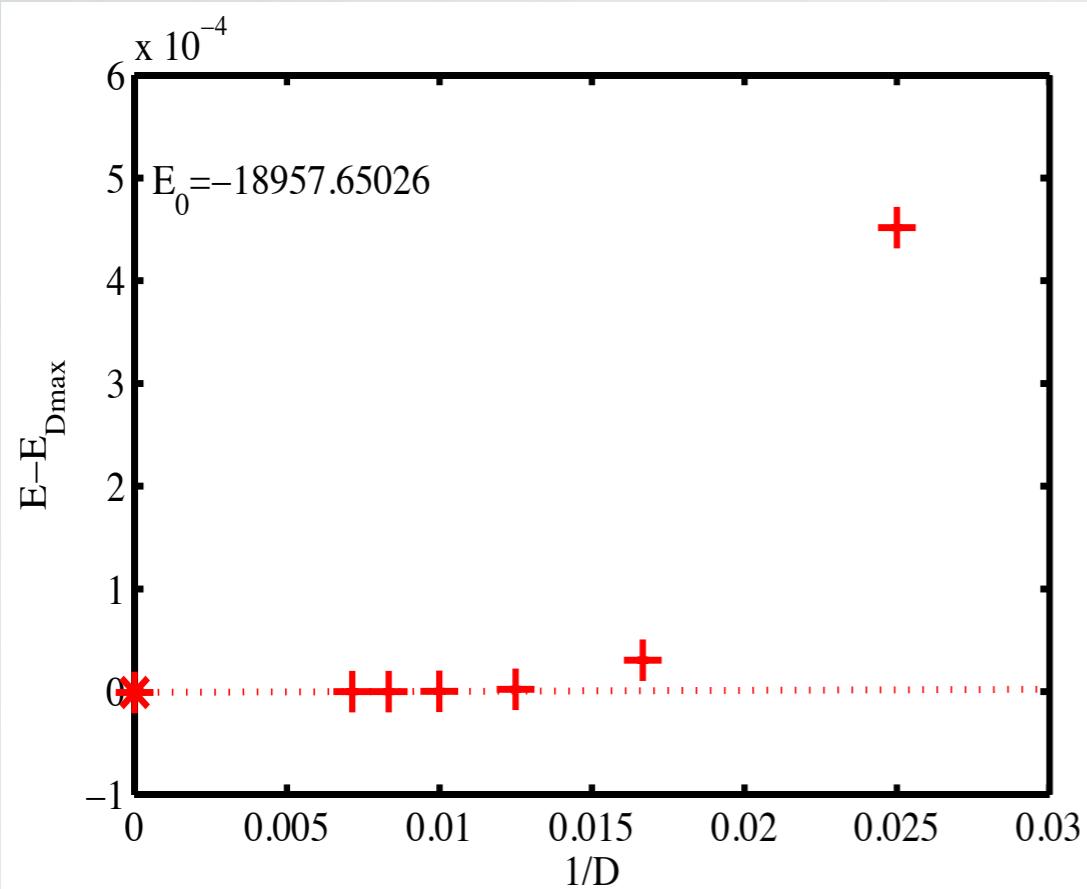
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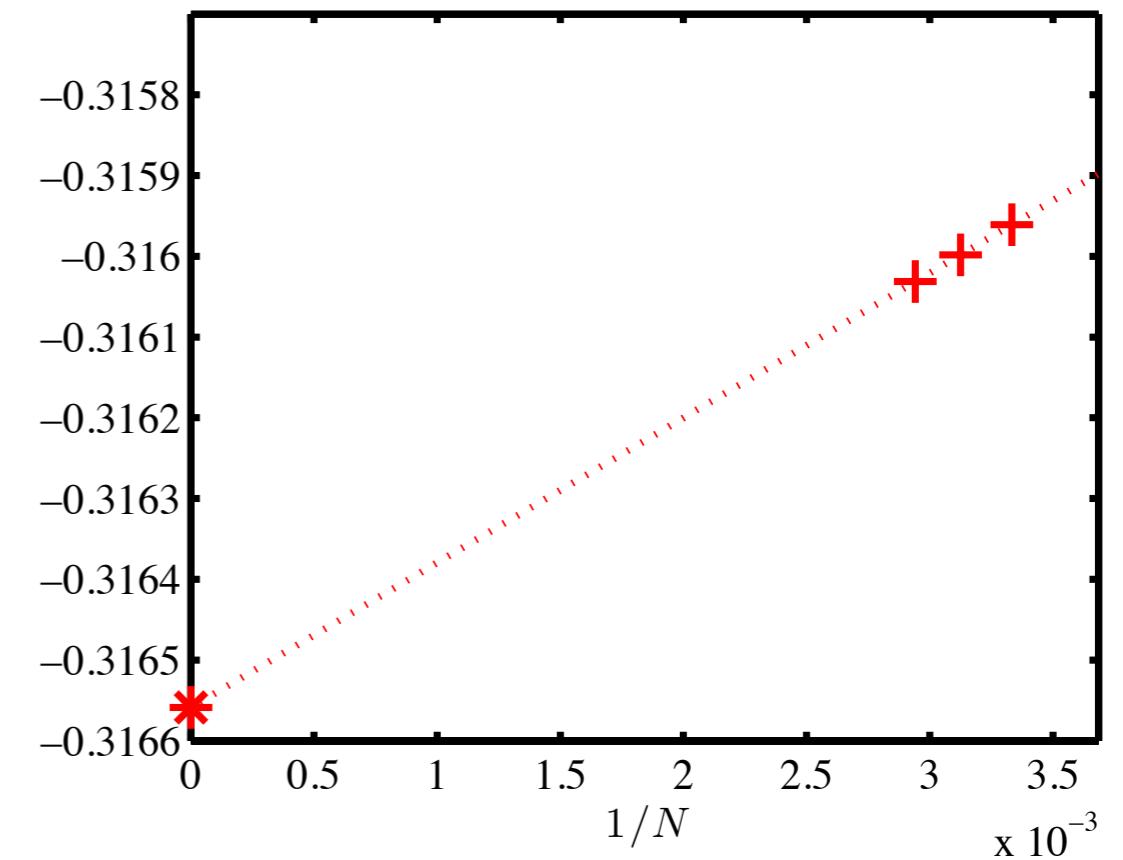
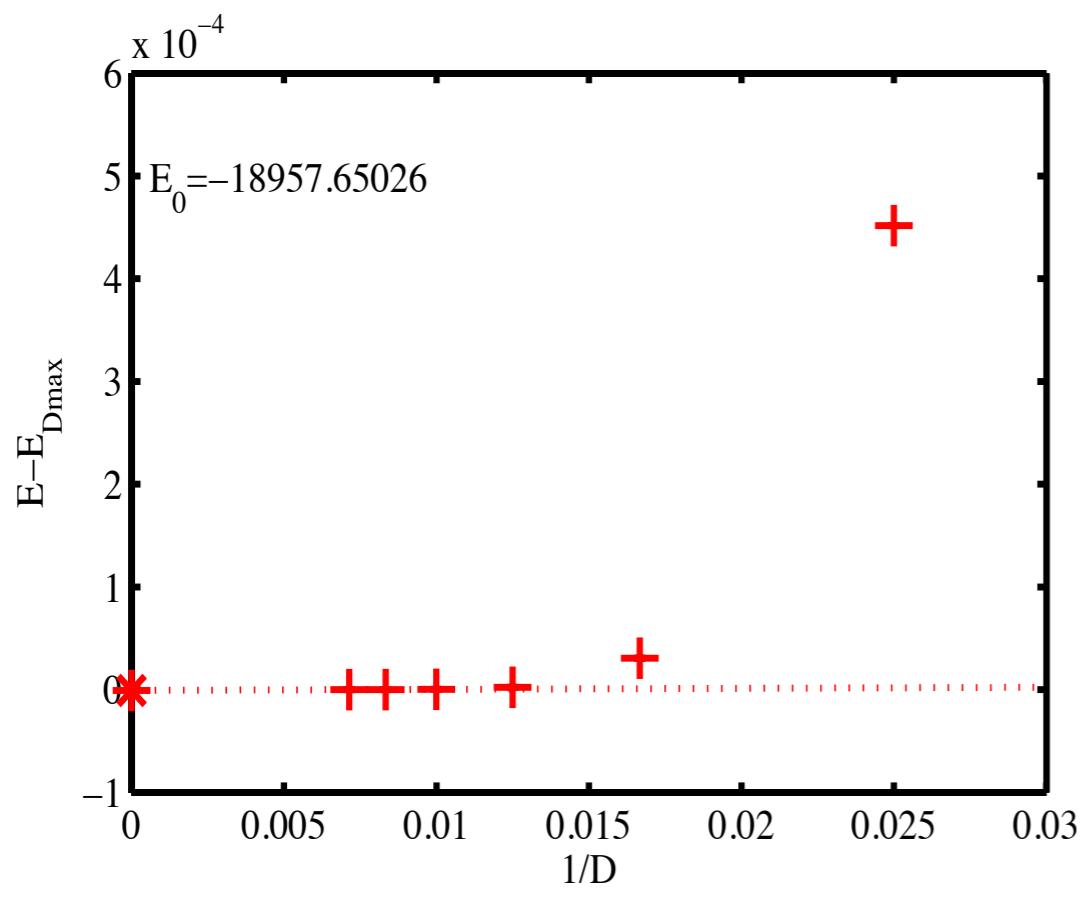
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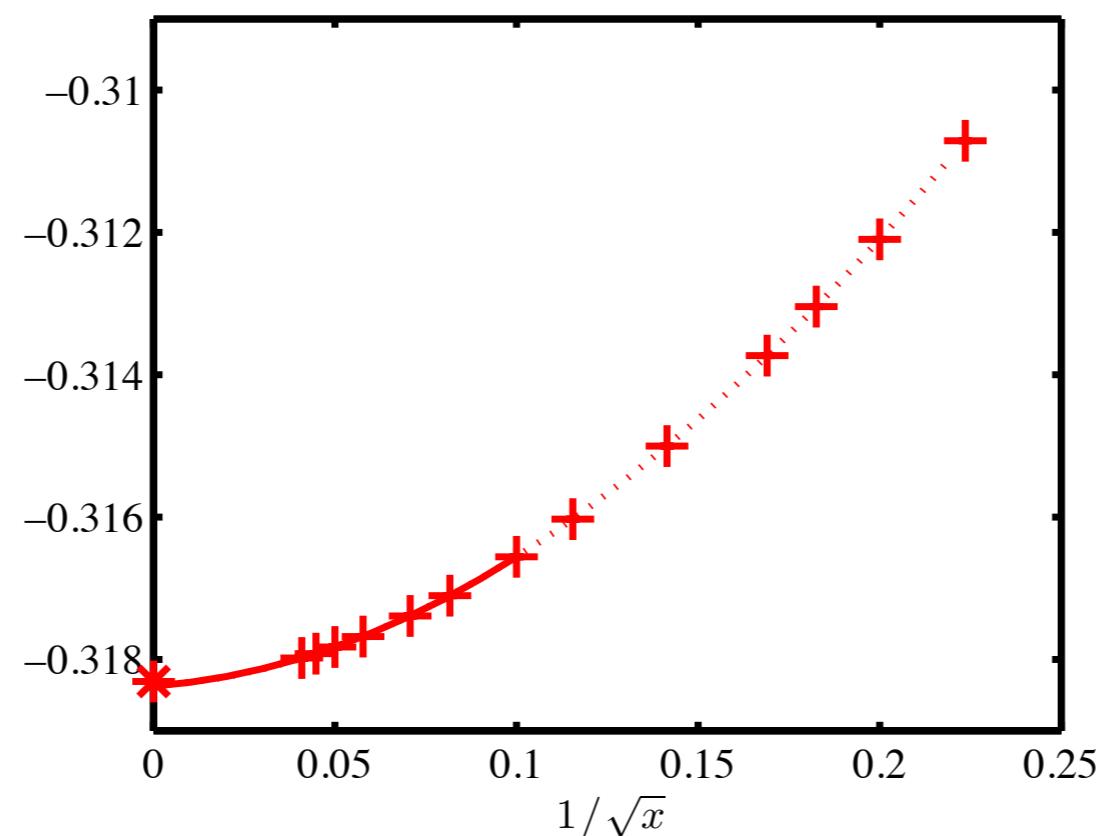
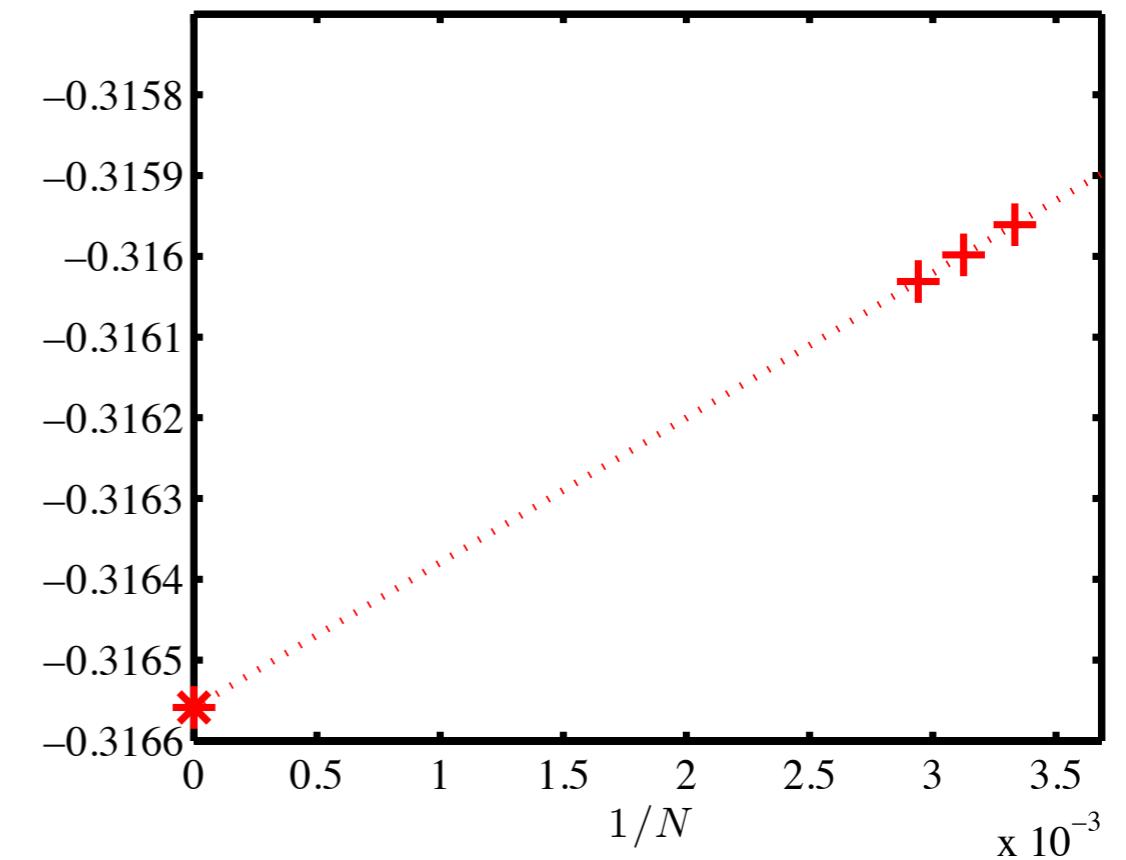
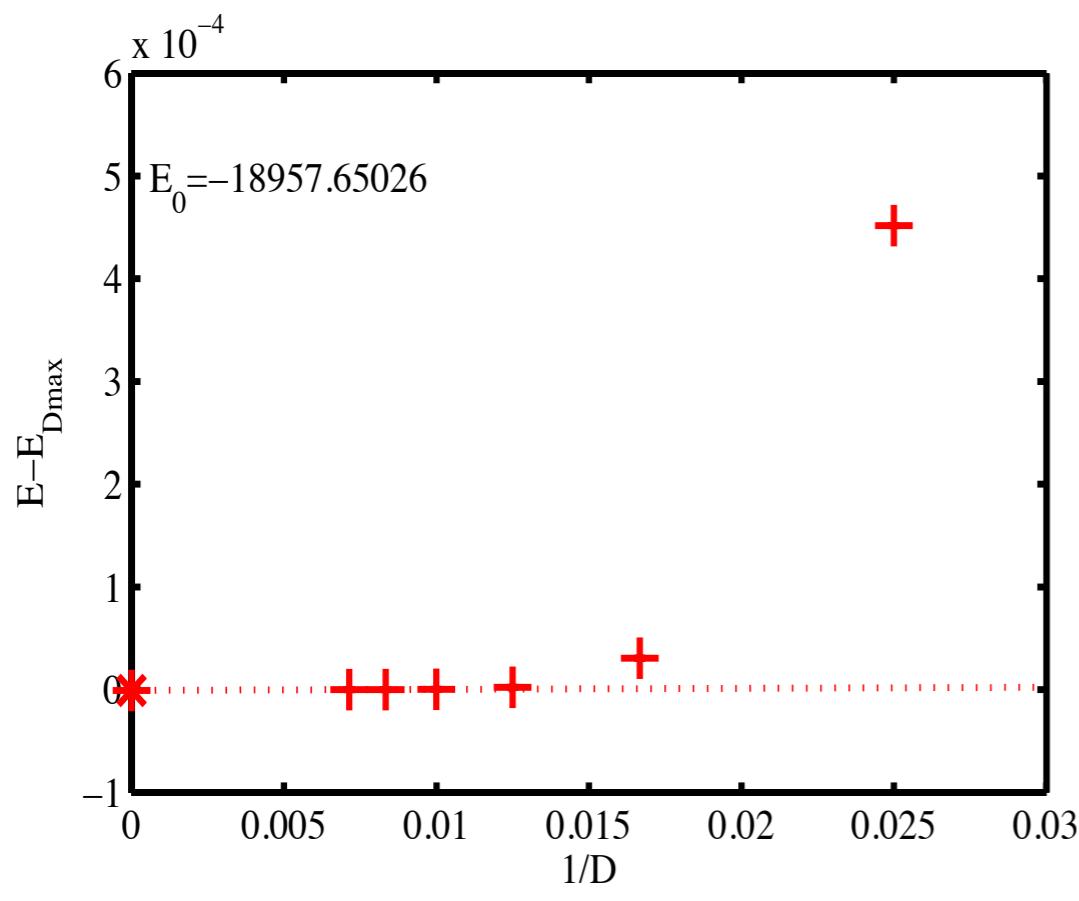


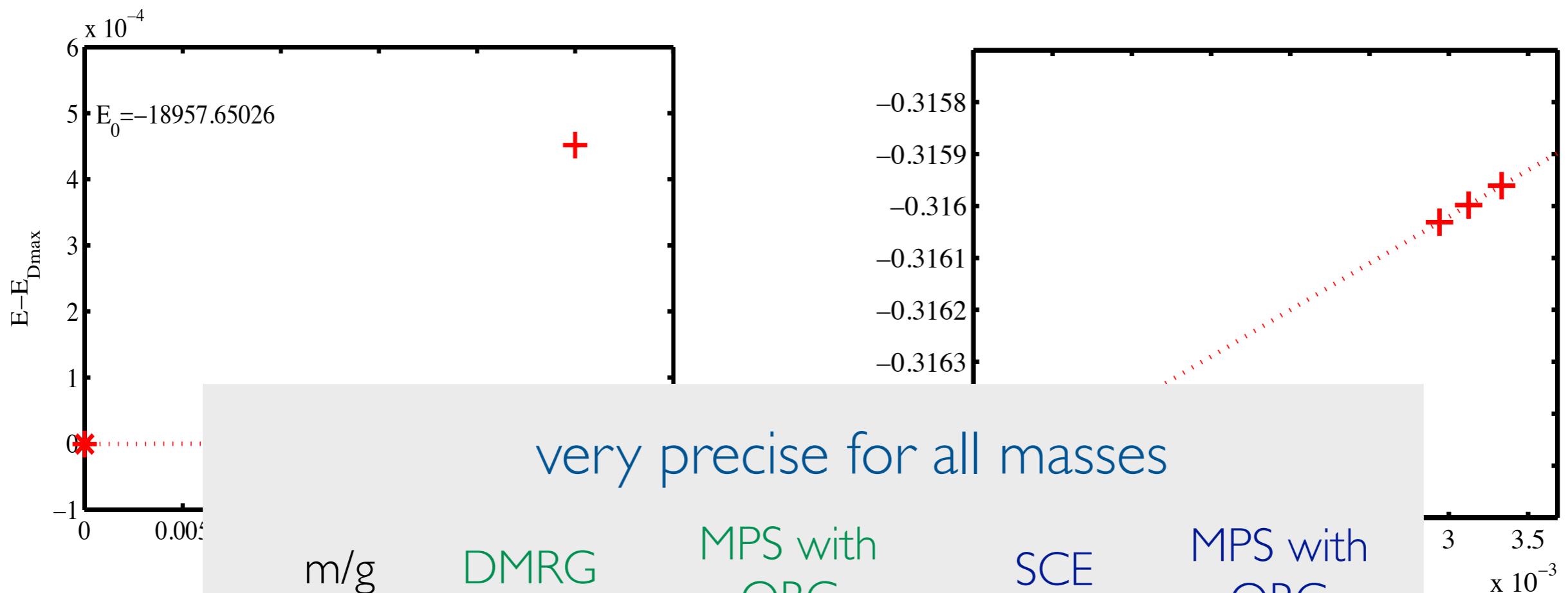
I

truncation error

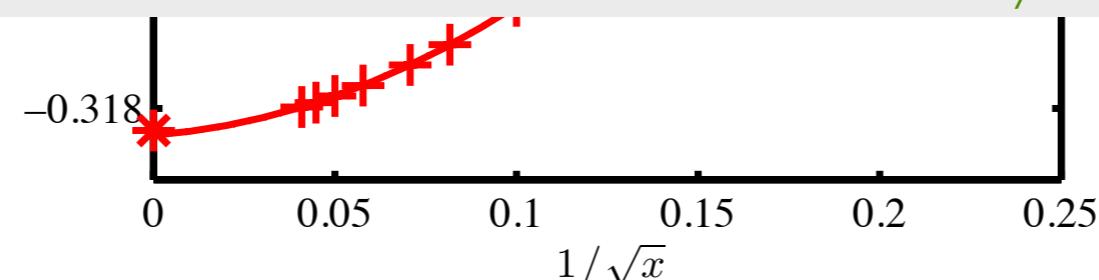
 $m/g = 0 \quad x = 100$ $N = 300$ 







see also Buyens et al., PRL 2014



TESTBENCH: SCHWINGER MODEL

Relevant states can be described as MPS

TN allow reliable continuum limit

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Mass spectrum

Chiral condensate (order parameter of chiral symmetry breaking)

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Thermal equilibrium states well approximated by MPO

Temperature dependence of chiral condensate

MCB, Cichy, Cirac, Jansen, Saito, PRD 2015, PRD 2016
also Buyens et al., PRD 2016

BEYOND SCHWINGER

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Scenario suffering from sign-problem

chemical potential

S. Kuehn et al, PRL 118 (2017) 071601

several flavours needed

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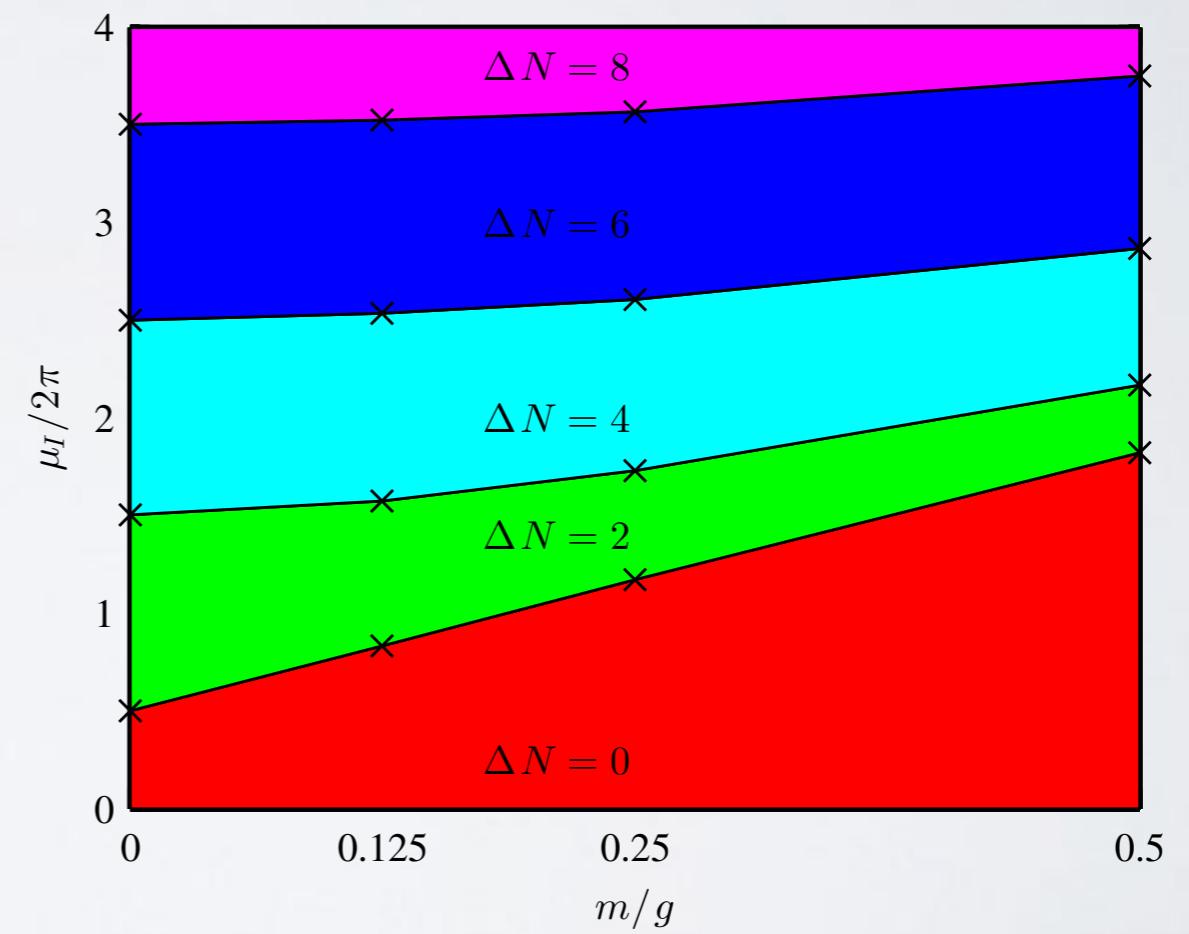
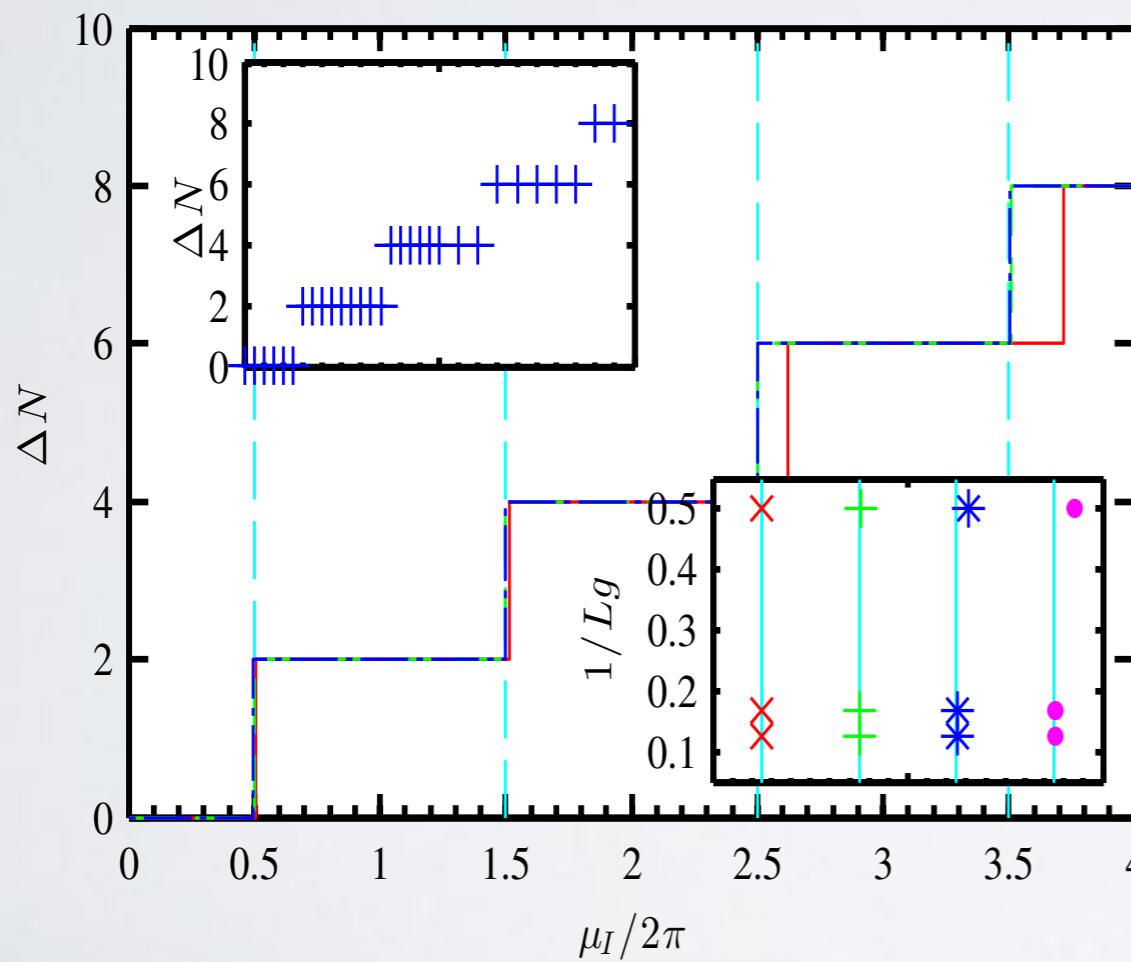
Non-Abelian model

SU(2) natural next step

FINITE DENSITY WITH MPS

Several fermion flavors, chemical potentials

ground state density changes (first order PT)



TESTBENCH: SCHWINGER MODEL

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Phys. Rev. D 93, 094512 (2016)

Multiflavour Schwinger model

also Buyens et al., PRD 94, 085018 (2016)

Phase diagram at finite density: no sign problem

S. Kühn et al., PRL 118, 071601 (2017)

NON-ABELIAN

SU(2) MODEL

SU(2) matrices

$$H = \frac{1}{2a} \sum_{n,b,c} \left(\phi_n^b {}^\dagger U_n^{bc} \phi_{n+1}^c + \text{h.c.} \right) + m \sum_{n,b} (-1)^n \phi_n^b {}^\dagger \phi_n^b + \frac{ag^2}{2} \sum_n \mathbf{J}_n^2$$

Kogut-Susskind '75

plus non-Abelian Gauss' Law $G_n^\nu |\Psi_{\text{phys}}\rangle = 0$

$$G_n^\nu = L_n^\nu - R_{n-1}^\nu - Q_n^\nu,$$

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SU(2) MODEL

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Truncating the gauge dof

SU(2) MODEL

Truncating the gauge dof
gauge invariant truncation
also for quantum simulations!

Zohar, Burrello PRD 2015
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Simplest case: link variables with dimension 5
plus two fermions per site

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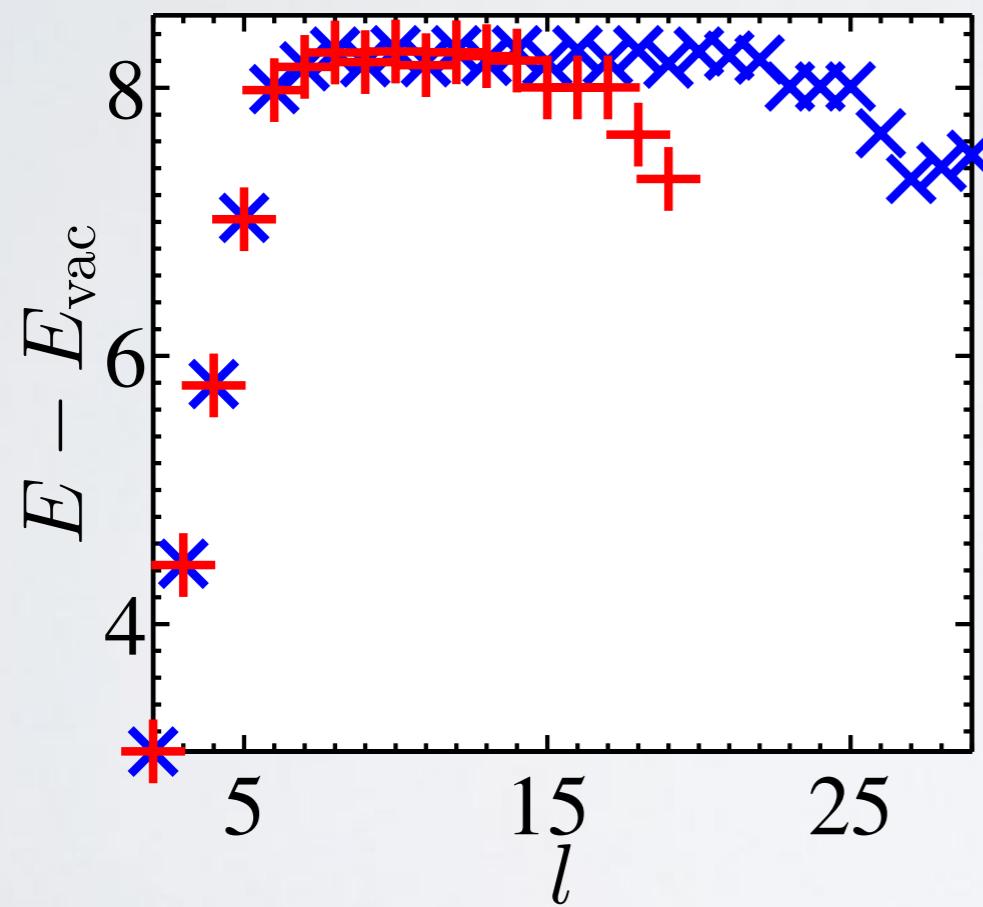
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Simulated **statical and dynamical** properties

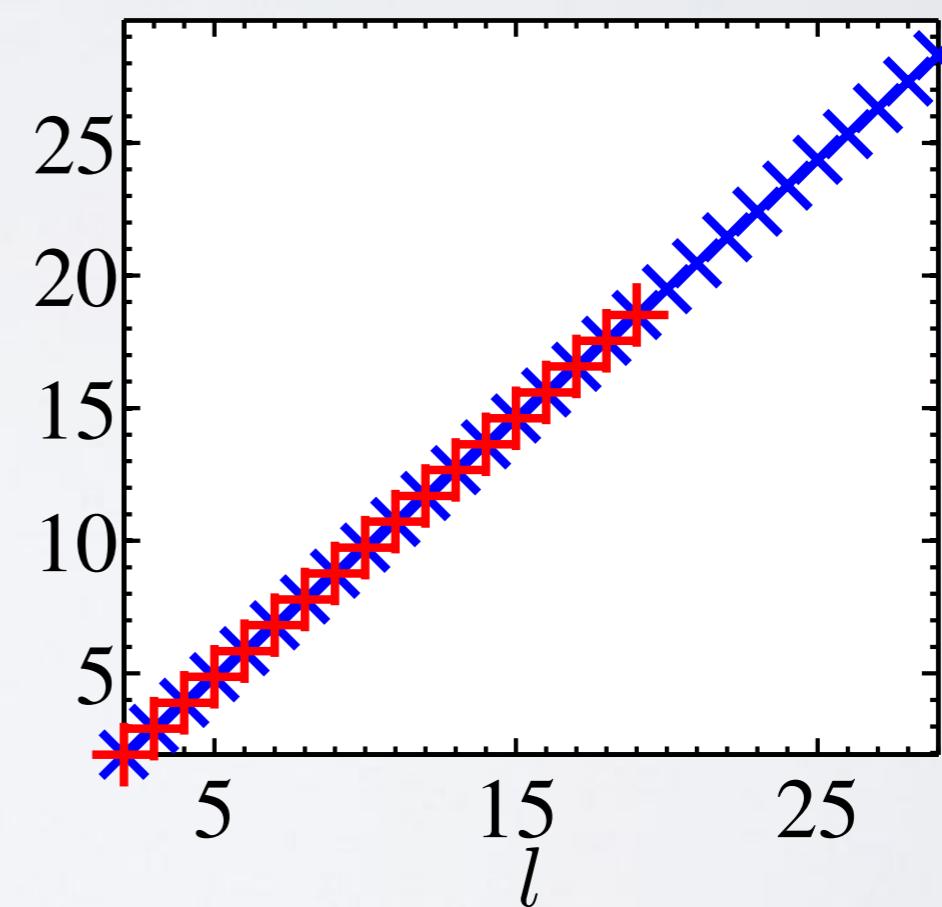
SU(2) STRING BREAKING

Ground state energy with external charges

$m = 3$



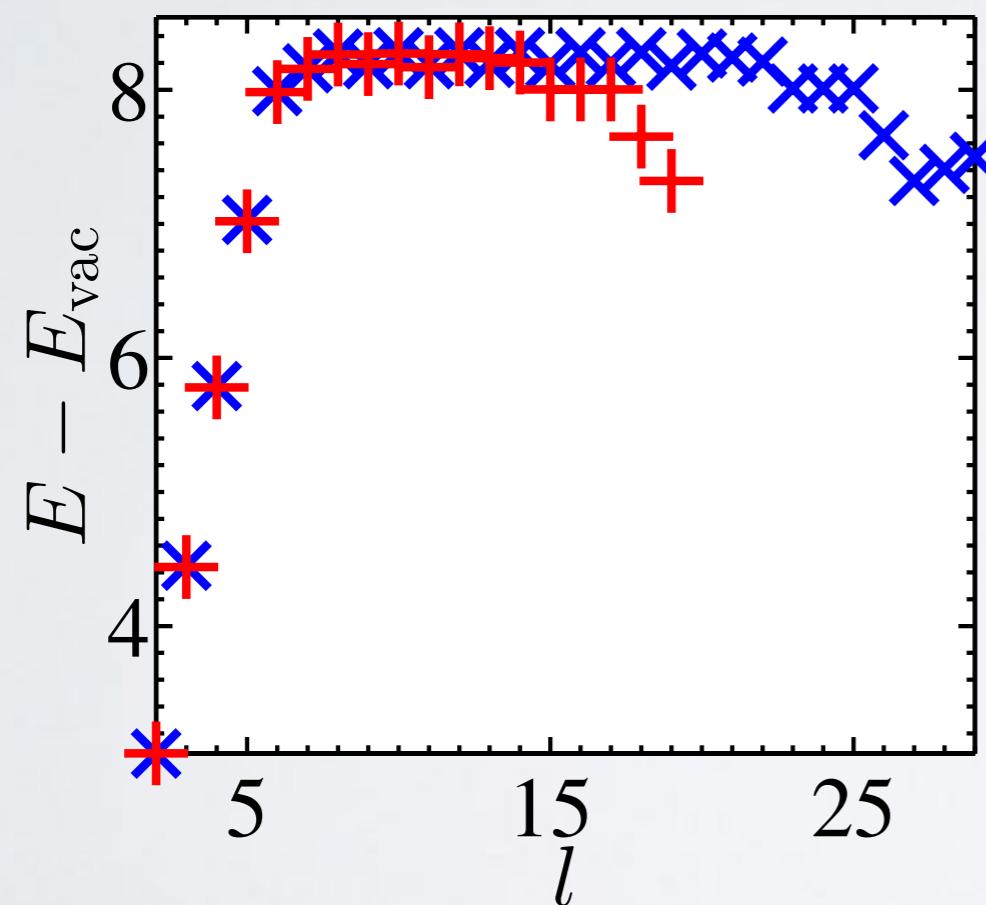
$m = 10$



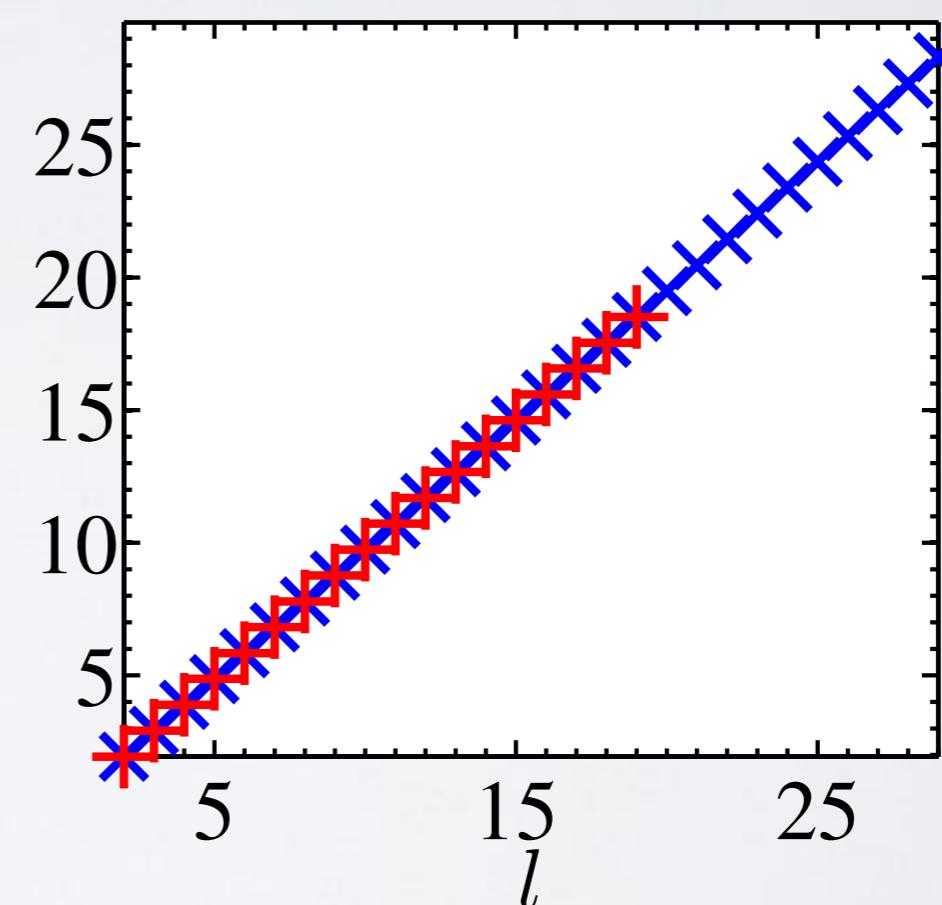
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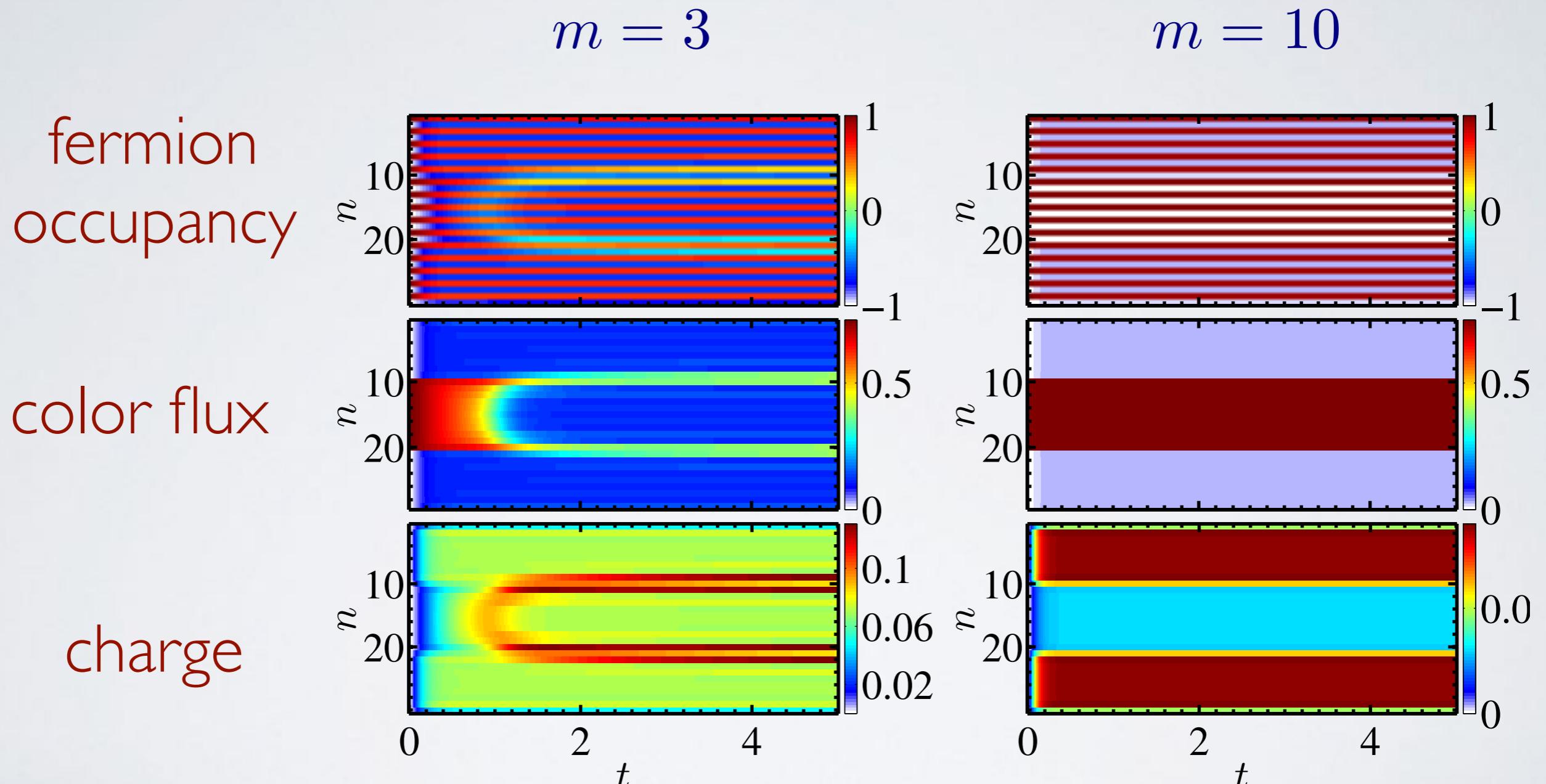


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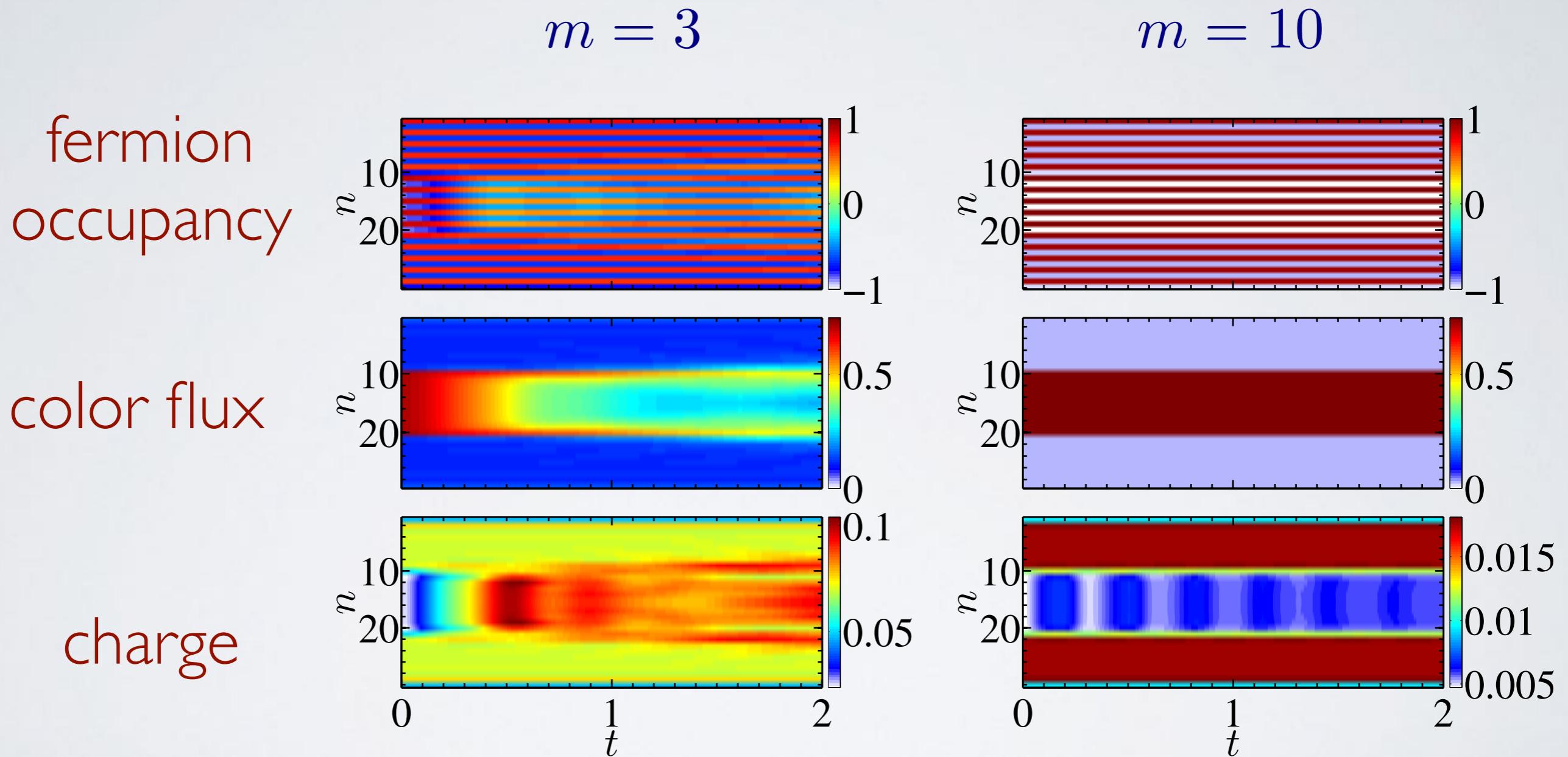
Proposed observables to detect string

SU(2) STRING BREAKING



external charges,
imaginary time

SU(2) STRING BREAKING



external charges,
real time

SU(2) MODEL

SU(2) MODEL

Without truncating the gauge dof
Physical subspace

SU(2) MODEL

Without truncating the gauge dof
Physical subspace

Physical states: color singlets

Hamer '82

$$| \dots n j \ n j \dots \rangle$$

representing gauge invariant combination

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$+ \begin{array}{c} \text{Diagram: } \text{O} \xrightarrow{\left| \frac{1}{2} -- \right\rangle} \text{O} \\ \text{O: green dot} \end{array} + \begin{array}{c} \text{Diagram: } \text{O} \xrightarrow{\left| \frac{1}{2} -+ \right\rangle} \text{O} \\ \text{O: red dot} \end{array}$ $+ \begin{array}{c} \text{Diagram: } \text{O} \xrightarrow{\left| \frac{1}{2} +- \right\rangle} \text{O} \\ \text{O: red dot} \end{array} + \begin{array}{c} \text{Diagram: } \text{O} \xrightarrow{\left| \frac{1}{2} ++ \right\rangle} \text{O} \\ \text{O: red dot} \end{array}$

representing gauge invariant combination

$$| 0 \ 1 \ \frac{1}{2} \ 1 \ 0 \rangle$$

SU(2) MODEL

Without truncating the gauge dof
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transitions only between
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representing gauge invariant combination

transitions only between such states



$$| \dots j_1 \ 1 \ j_2 \ 1 \ j_1 \dots \rangle$$



$$| \dots j_1 \ 2 \ j_1 \ 0 \ j_1 \dots \rangle$$

$$(-1)^{j_2-j_1-1/2} \sqrt{\frac{2j_2+1}{2j_1+1}}$$

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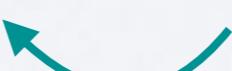
Physical states: color singlets

$$| \dots n \textcolor{red}{j} \textcolor{blue}{n} j \dots \rangle$$

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Physical states: color singlets

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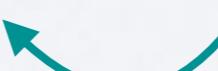
Gauss law

SU(2) MODEL

Without truncating the gauge dof
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Physical states: color singlets

$$| \dots n j \ n j \dots \rangle$$


Gauss law $j, j \pm 1/2$

SU(2) MODEL

Without truncating the gauge dof
Physical subspace

Physical states: color singlets

$$| \dots n j \ n j \dots \rangle$$

encode

Gauss law $j, j \pm 1/2$

SU(2) MODEL

Without truncating the gauge dof
Physical subspace

Physical states: color singlets

$$|\dots n j \ n j \dots \rangle$$

encode $\{|0\rangle, |1+\rangle, |1-\rangle, |2\rangle\}$

Gauss law $j, j \pm 1/2$



SU(2) MODEL

Without truncating the gauge dof
Physical subspace

Physical states: color singlets

$$| \dots \tilde{n} \tilde{n} \dots \rangle$$



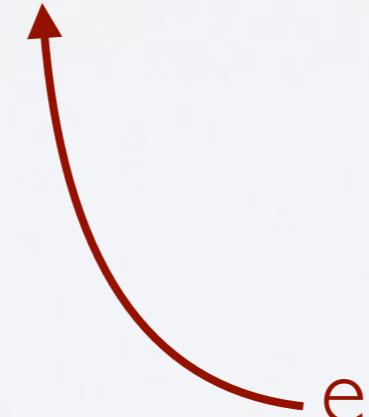
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OBC: color electric flux j
can be recovered from
fermion content

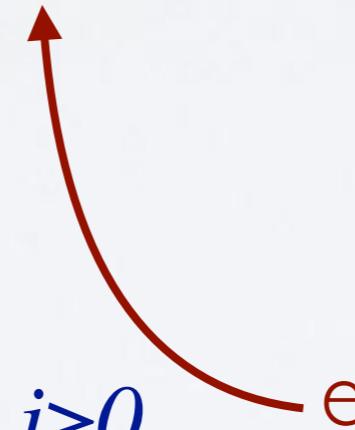
SU(2) MODEL

Without truncating the gauge dof
Physical subspace

Physical states: color singlets

$$| \dots \tilde{n} \tilde{n} \dots \rangle$$

condition such that all $j \geq 0$



encode $\{|0\rangle, |1+\rangle, |1-\rangle, |2\rangle\}$

OBC: color electric flux j
can be recovered from
fermion content

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Without truncating the gauge dof
Physical subspace

Physical states: color singlets

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OBC: color electric flux j
can be recovered from
fermion content

condition such that all $j \geq 0$

dimension of physical space

$$C_k = (2k)!/(k+1)!k!$$

$$4^N \left(1 - \sum_{k=1}^N \frac{C_k}{4^k} \right)$$

SU(2) MODEL

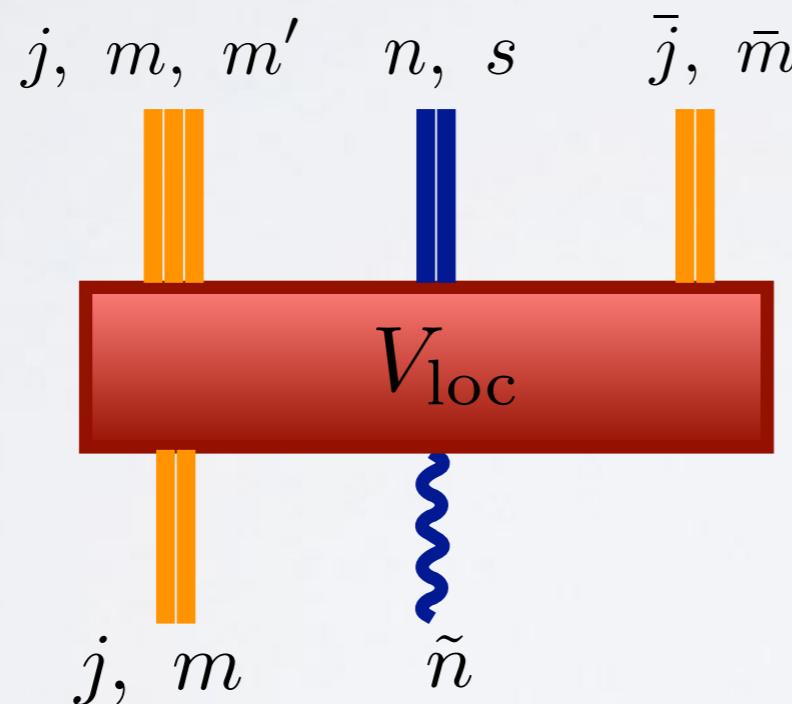
Isometric relation between reduced and full basis

$$|\dots \tilde{n} \tilde{n} \dots\rangle \longrightarrow |\dots n^1 n^2 j \ell \ell' n^1 n^2 \dots\rangle$$

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constructed from local
pieces

$$\sum_j \sum_{\tilde{n}} \sum_{m, m' = -j}^j \sum_{s = -|q|}^{|q|} \frac{C(jm', qs; j + q, \bar{m})}{\sqrt{2(j + q) - 1}} |jm'm'; ns; j + q, m' + s\rangle \langle jm; \tilde{n}|$$

SU(2) MODEL

Isometric relation between reduced and full basis

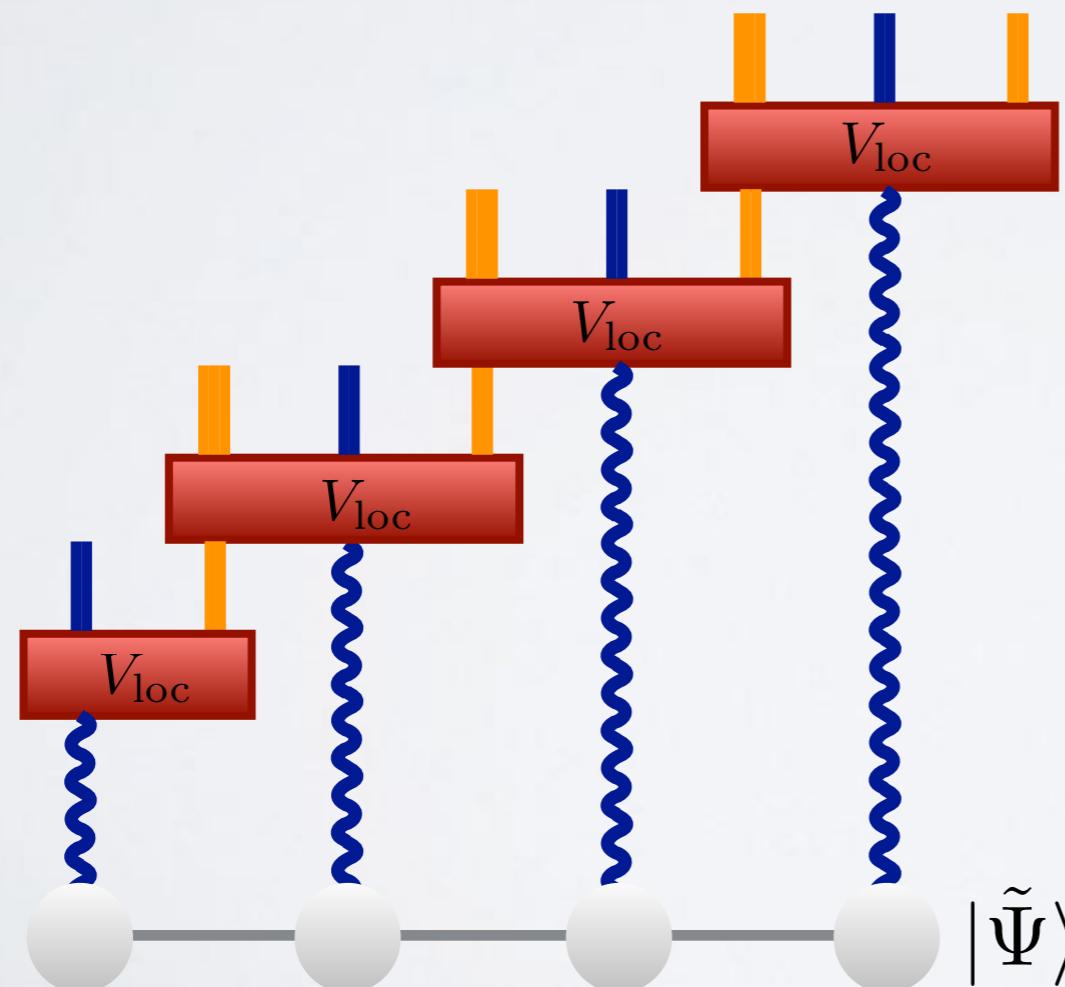
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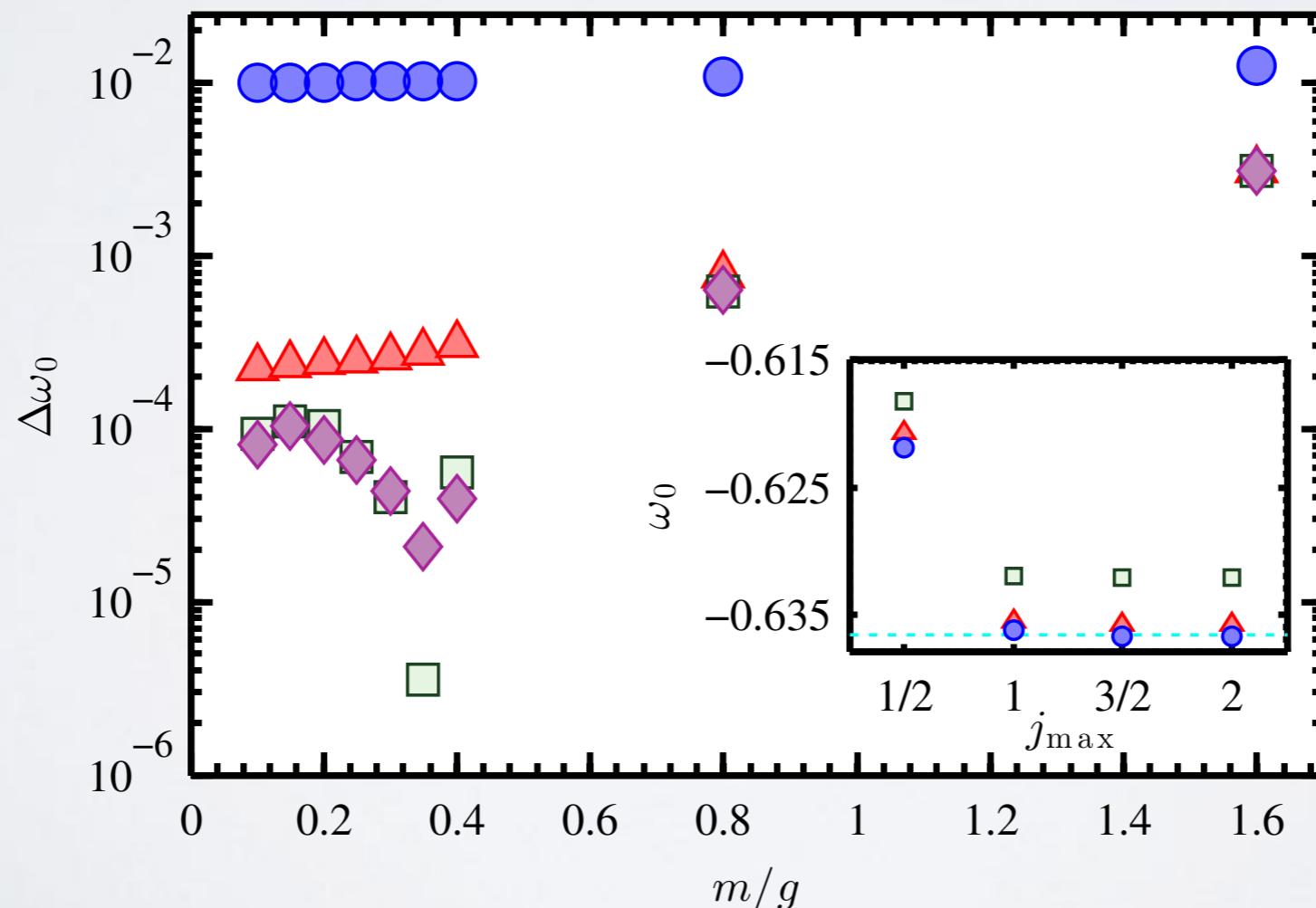
efficient calculation of spectral properties and entropies

SU(2) MODEL

We can study the effect of the truncation in j
e.g. ground state energy compared to full model

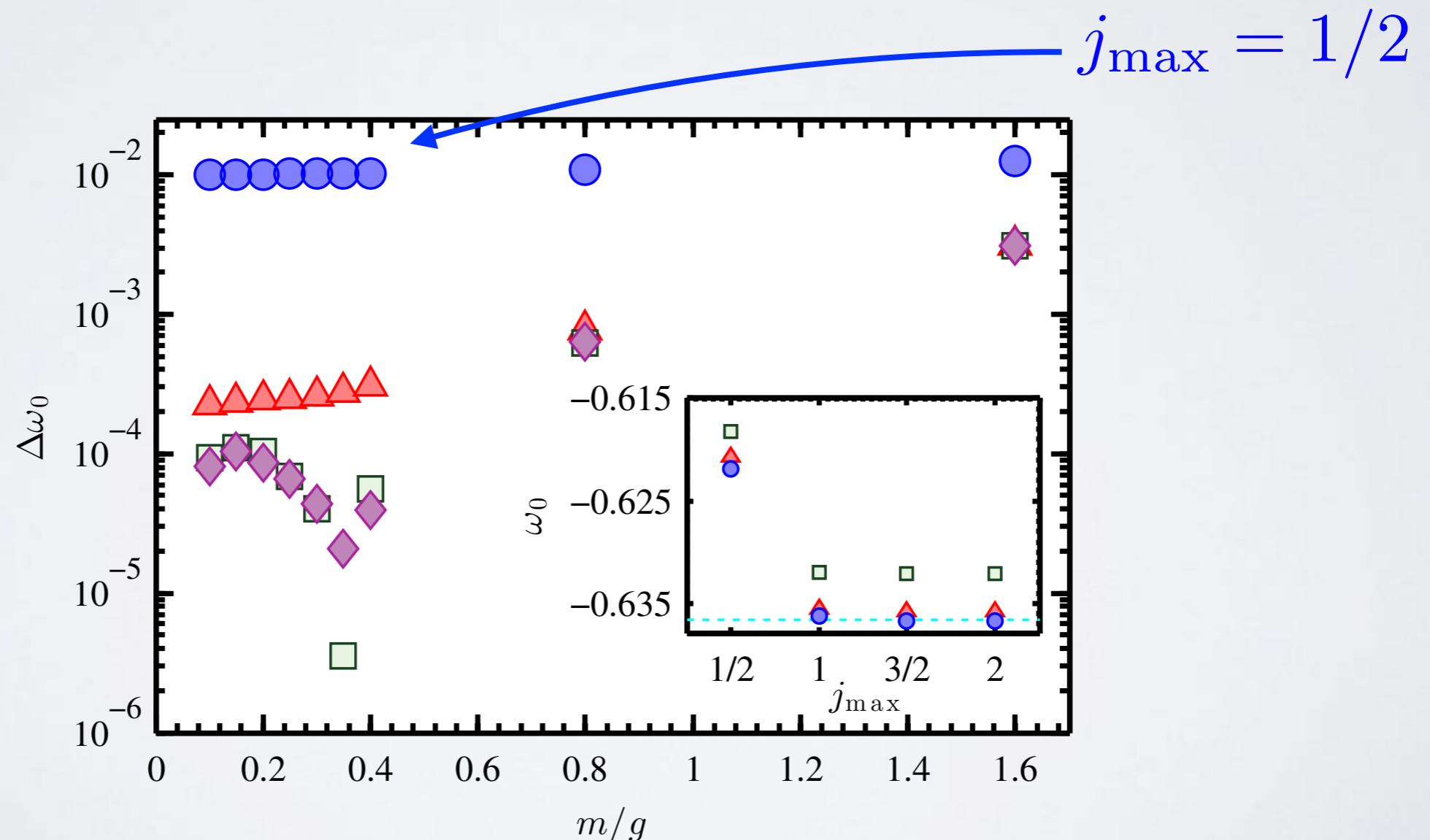
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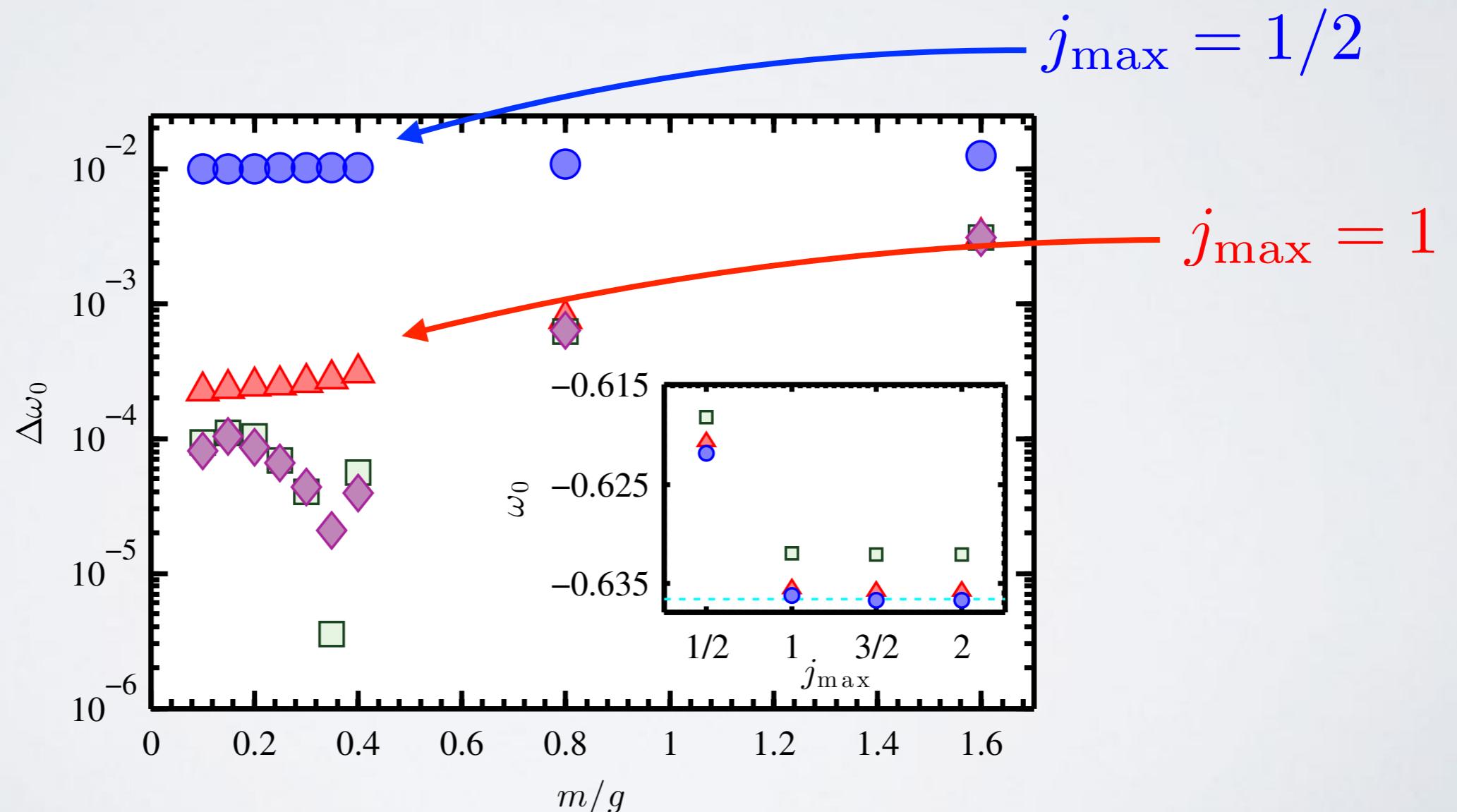
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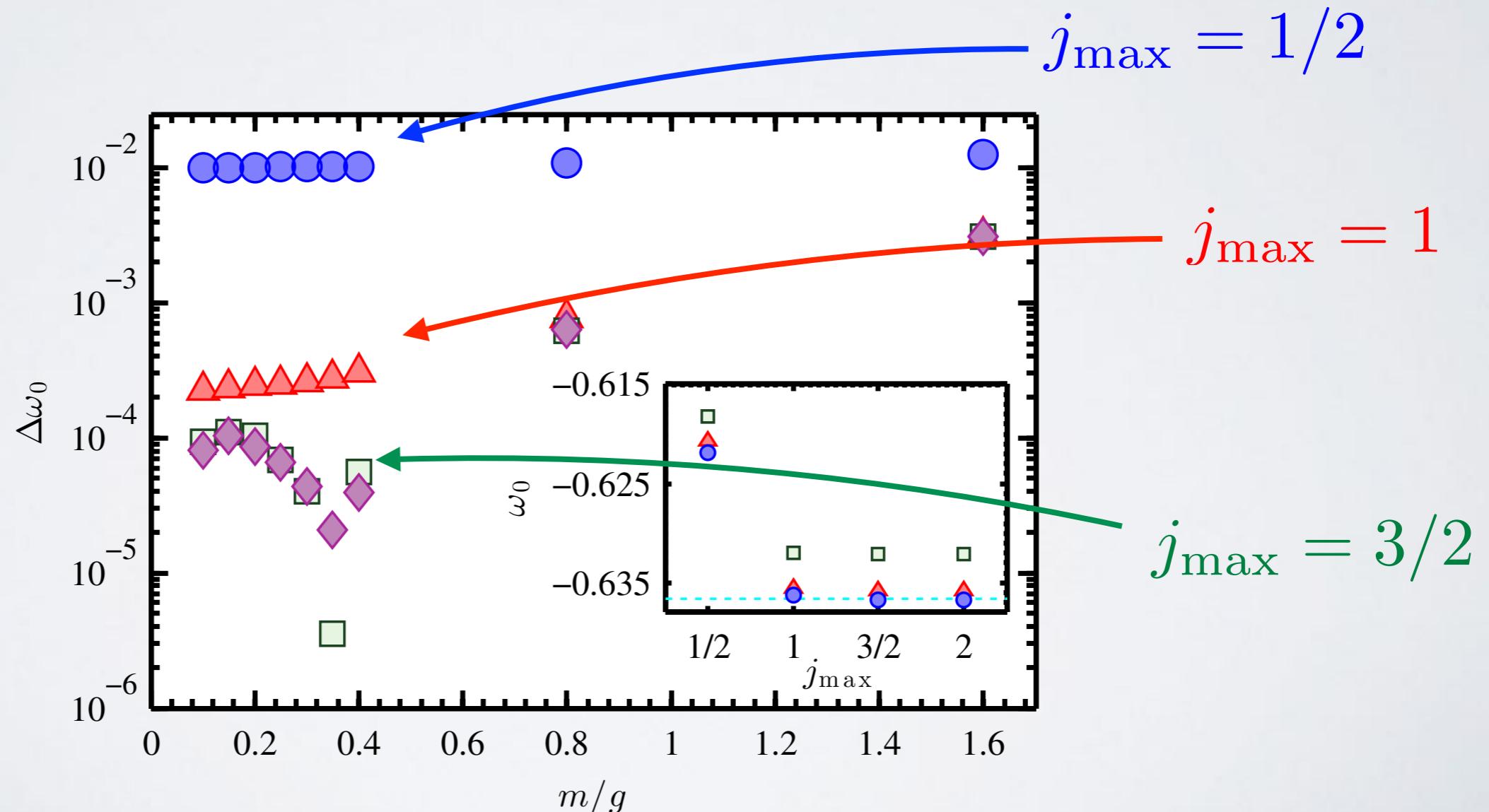
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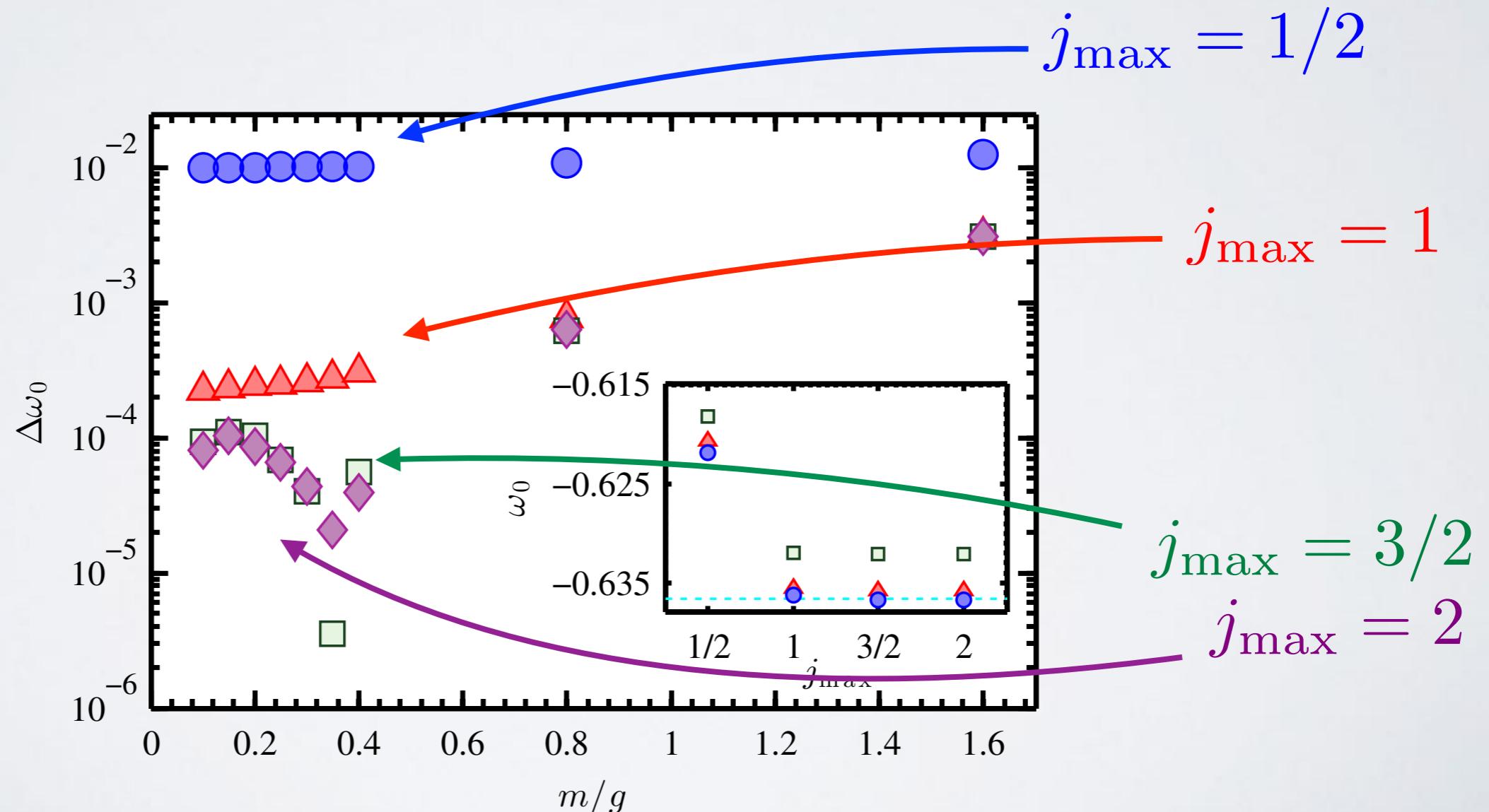
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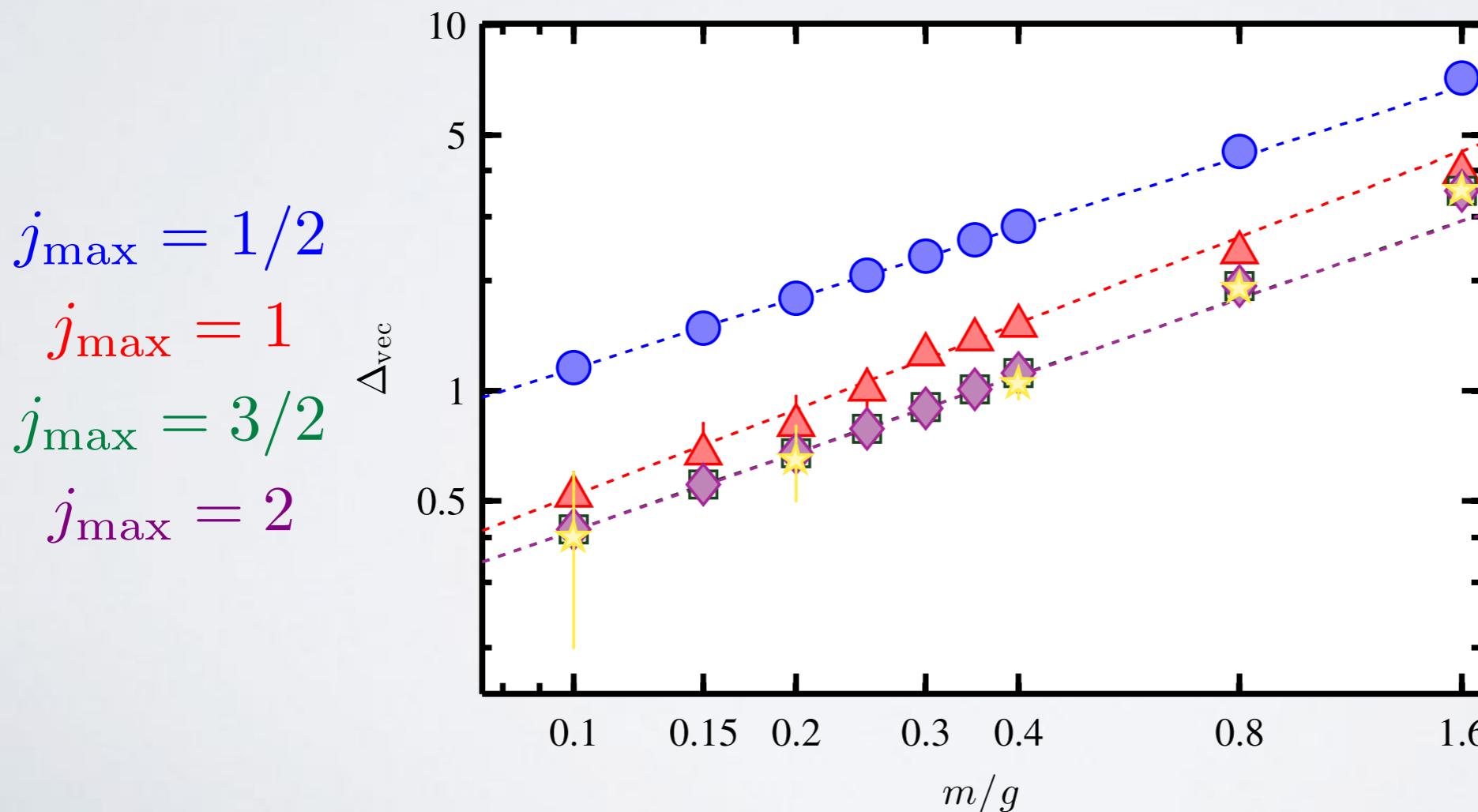
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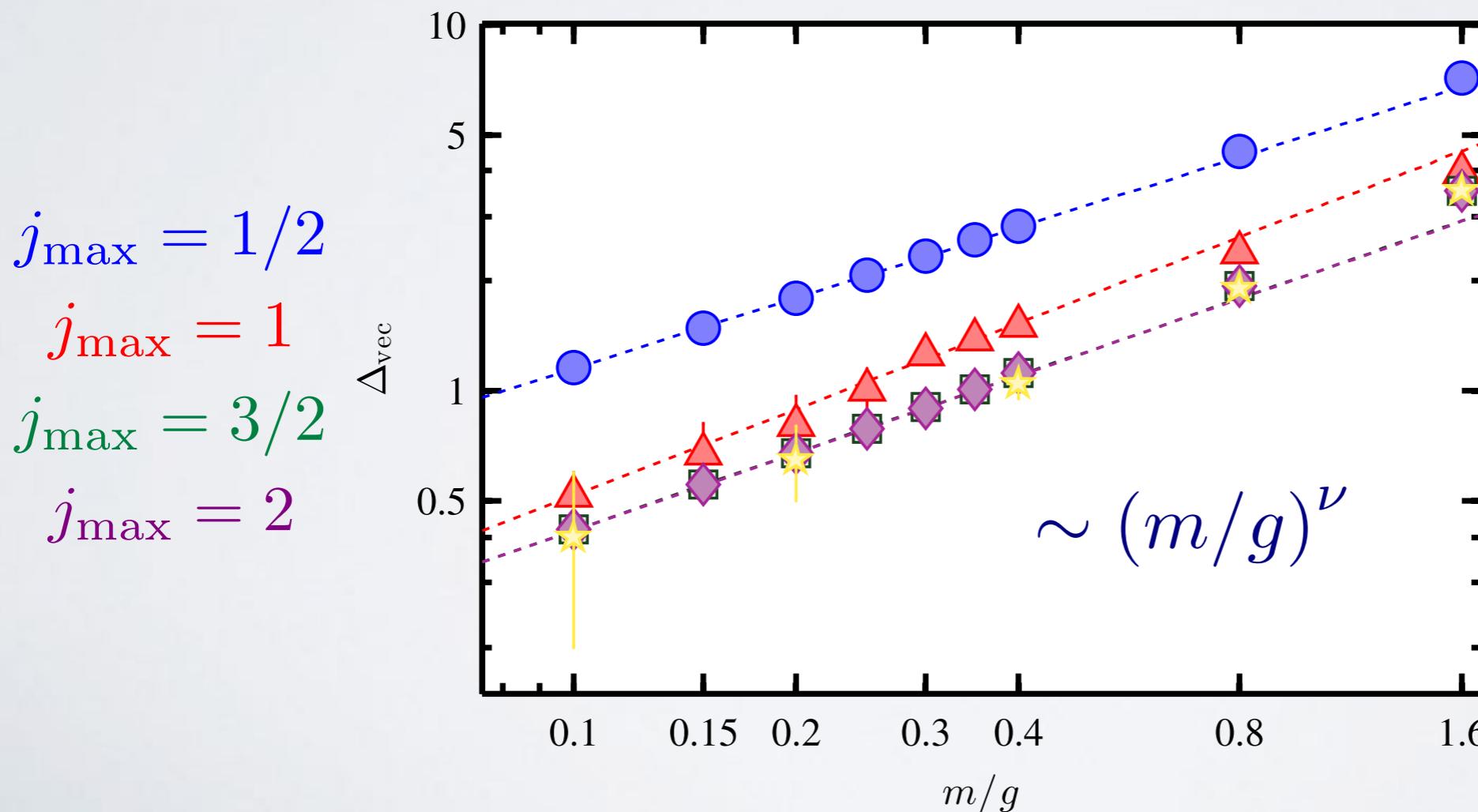
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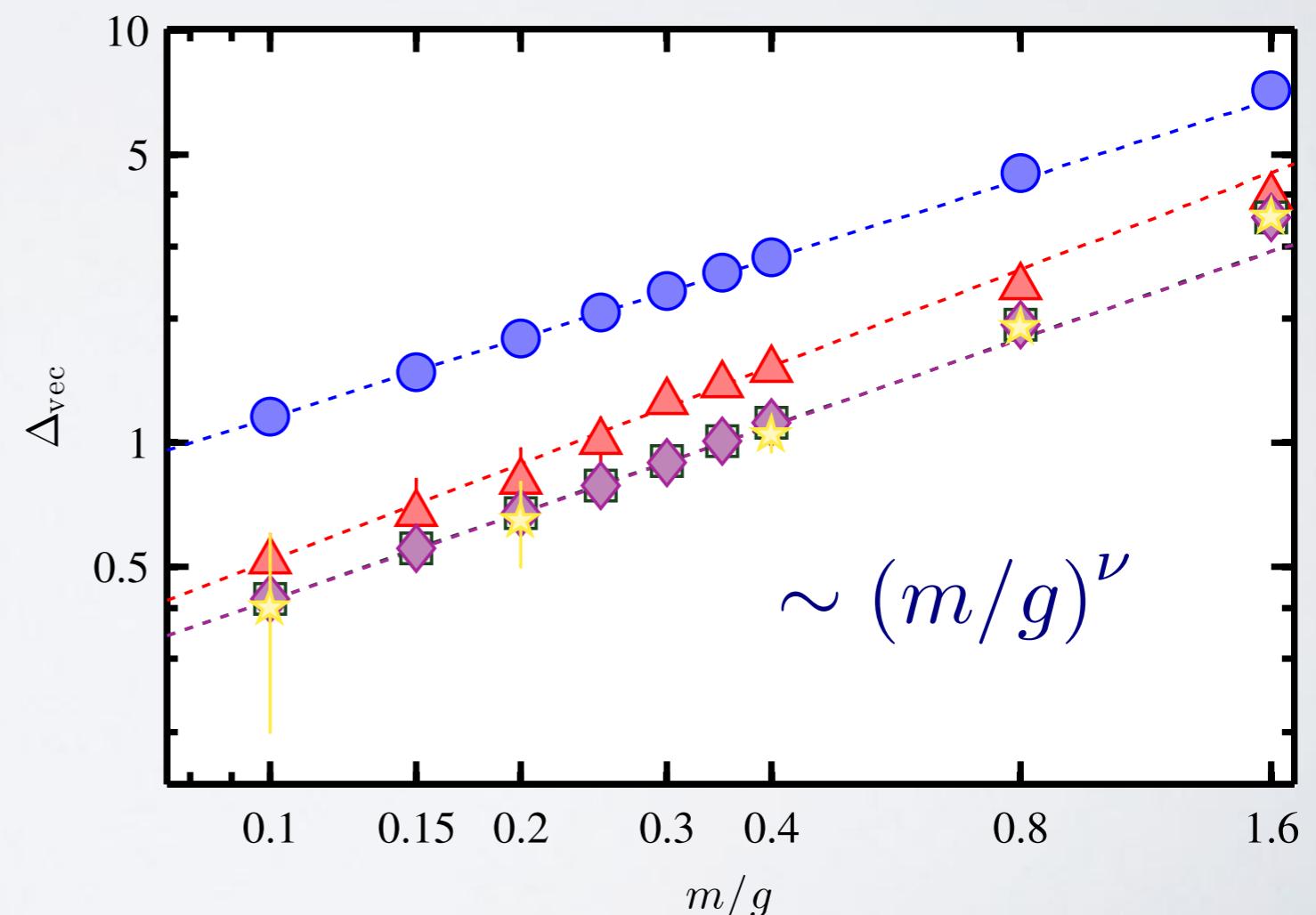


SU(2) MODEL

We can study the effect of the truncation in j
e.g. vector meson mass gap

critical exponent

$j_{\max} = 1/2$	~ 0.639
$j_{\max} = 1$	$0.781(93)(75)$
$j_{\max} = 3/2$	$0.700(29)(11)$
$j_{\max} = 2$	$0.700(29)(11)$



SU(2) MODEL

We can compute entropies efficiently

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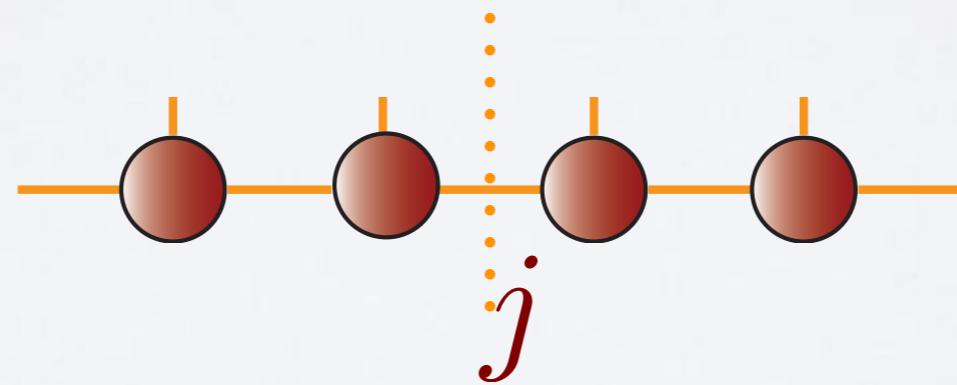
gauge constraints not purely local \Rightarrow not all entropy physical

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$$S(\rho) = - \sum_j p_j \log_2(p_j) + \sum_j p_j S(\rho_j)$$



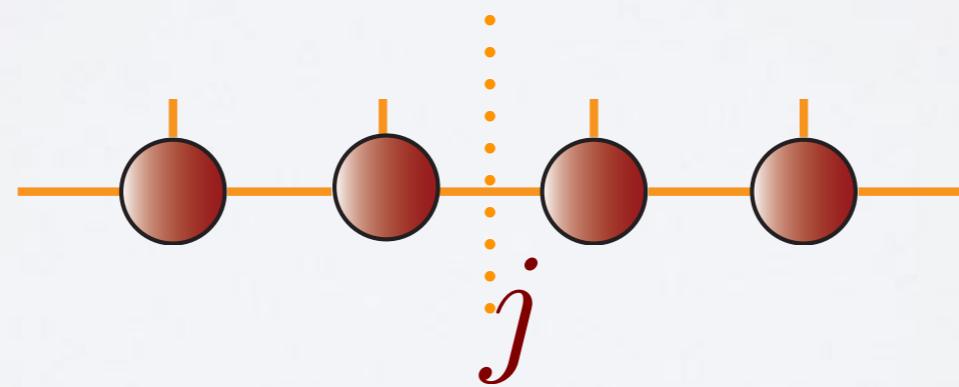
Gosh et al JHEP 2015
Soni, Trivedi JHEP 2016
van Acleyen et al PRL 2016

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Gosh et al JHEP 2015
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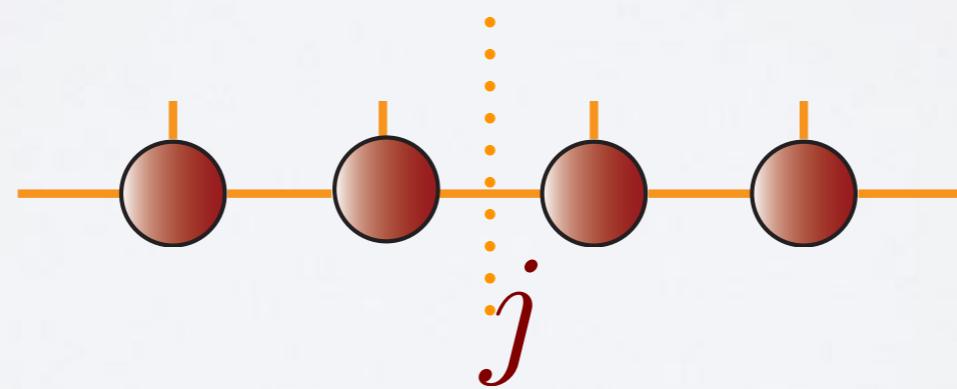
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S_{class}



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SU(2) MODEL

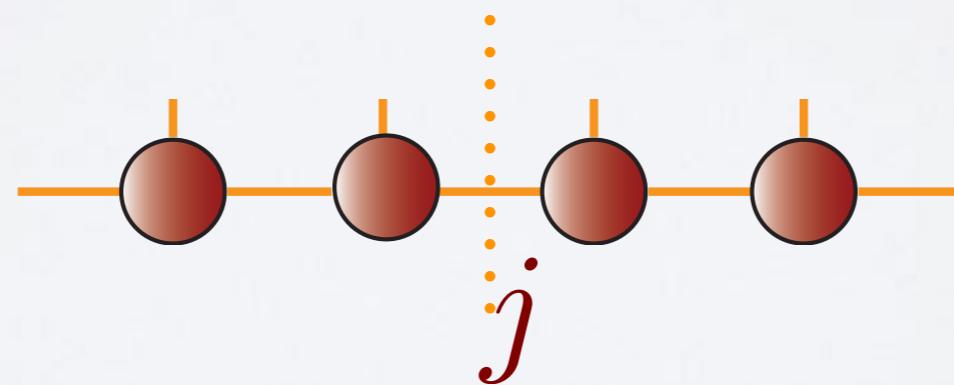
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S_{class}

S_{repr}



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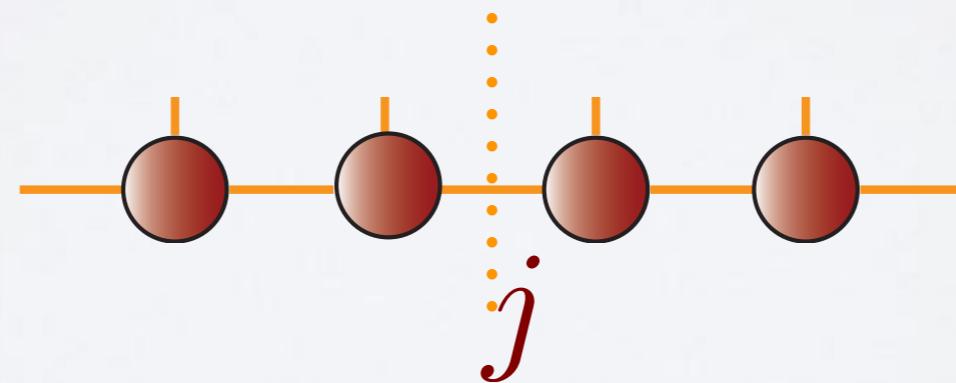
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S_{class} S_{repr} S_{dist}



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S_{class} S_{repr} S_{dist}

divergence continuum limit

$$S \propto \frac{c}{6} \log_2 \frac{\xi}{a}$$

Calabrese, Cardy JStatMech 2004

spin theory gapped

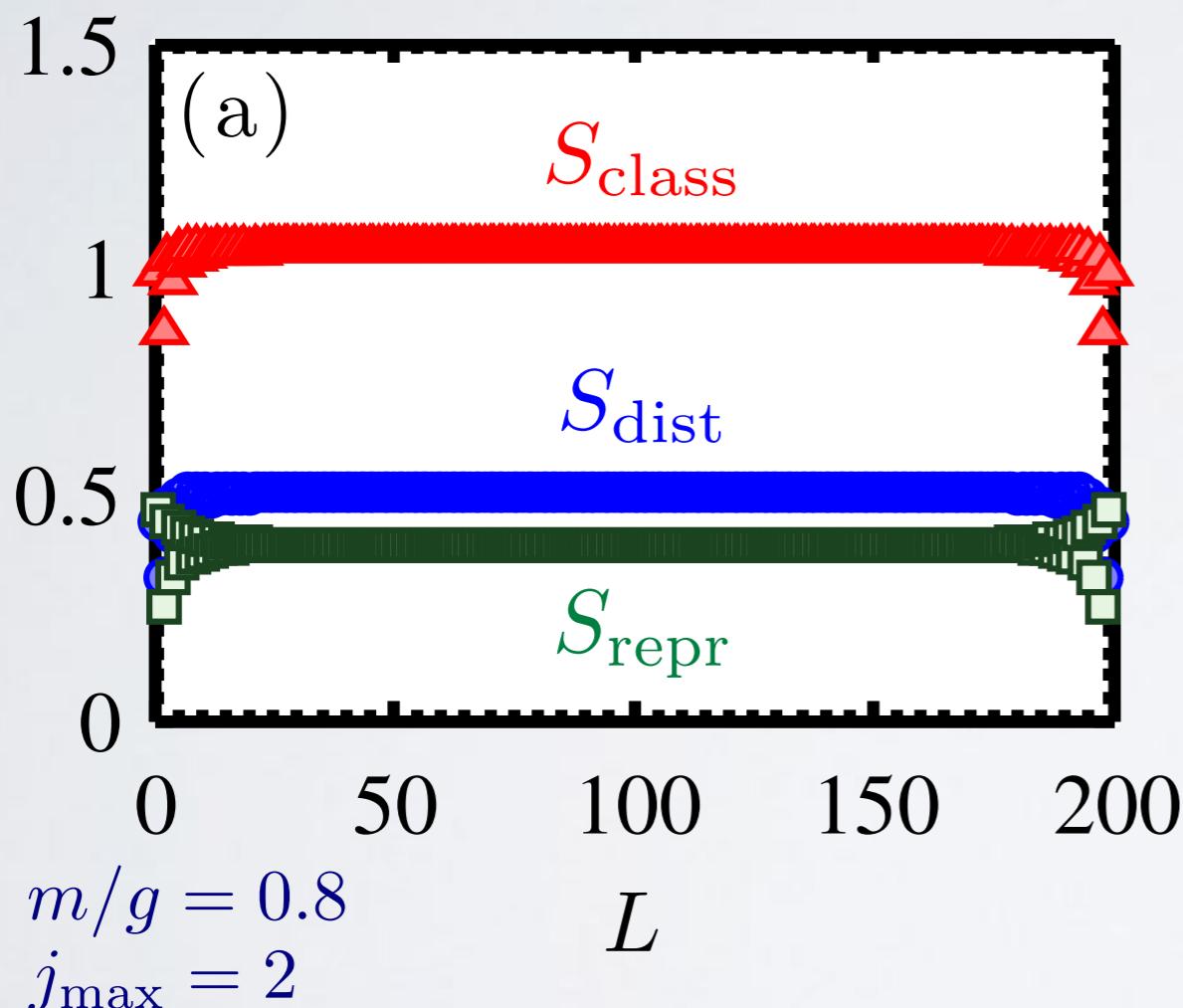
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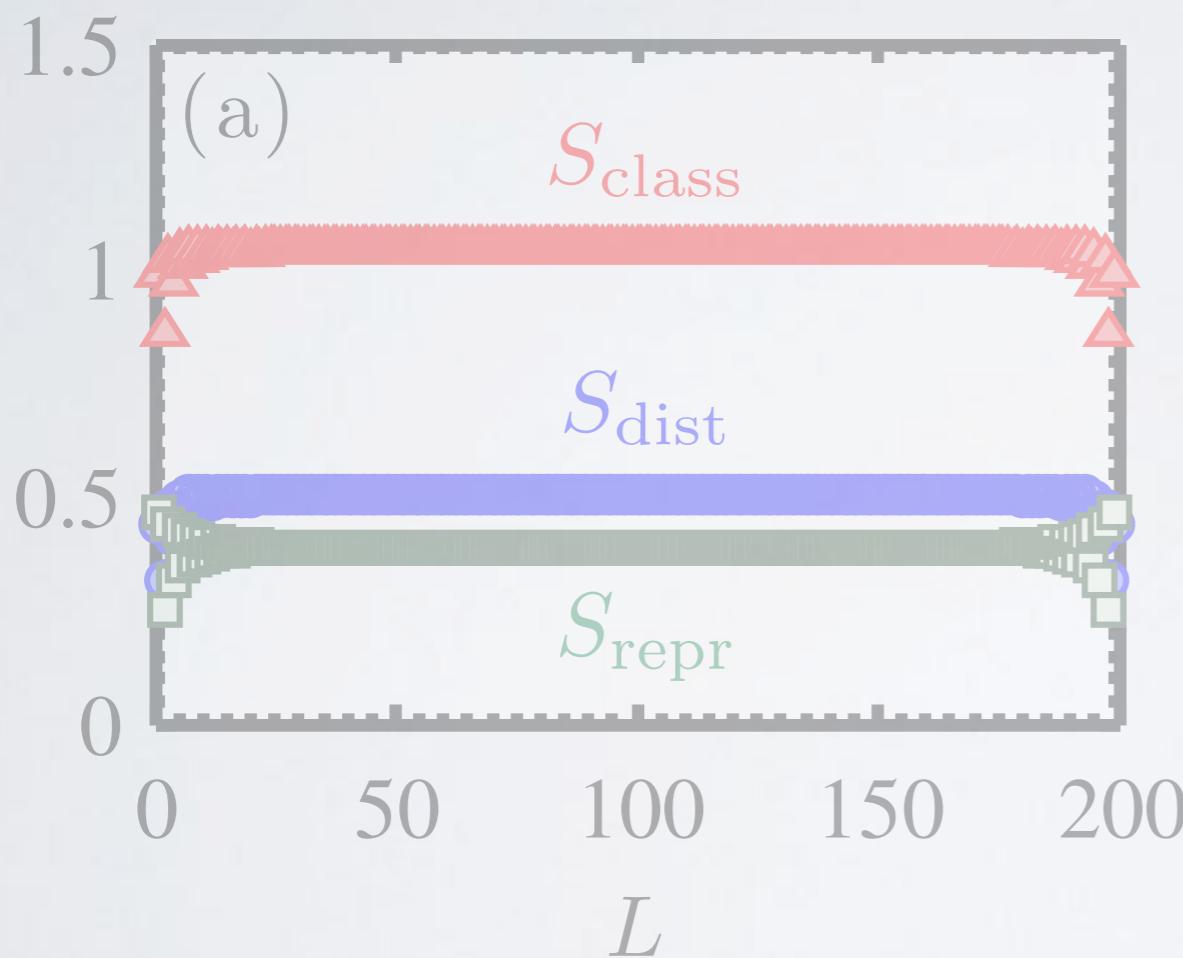
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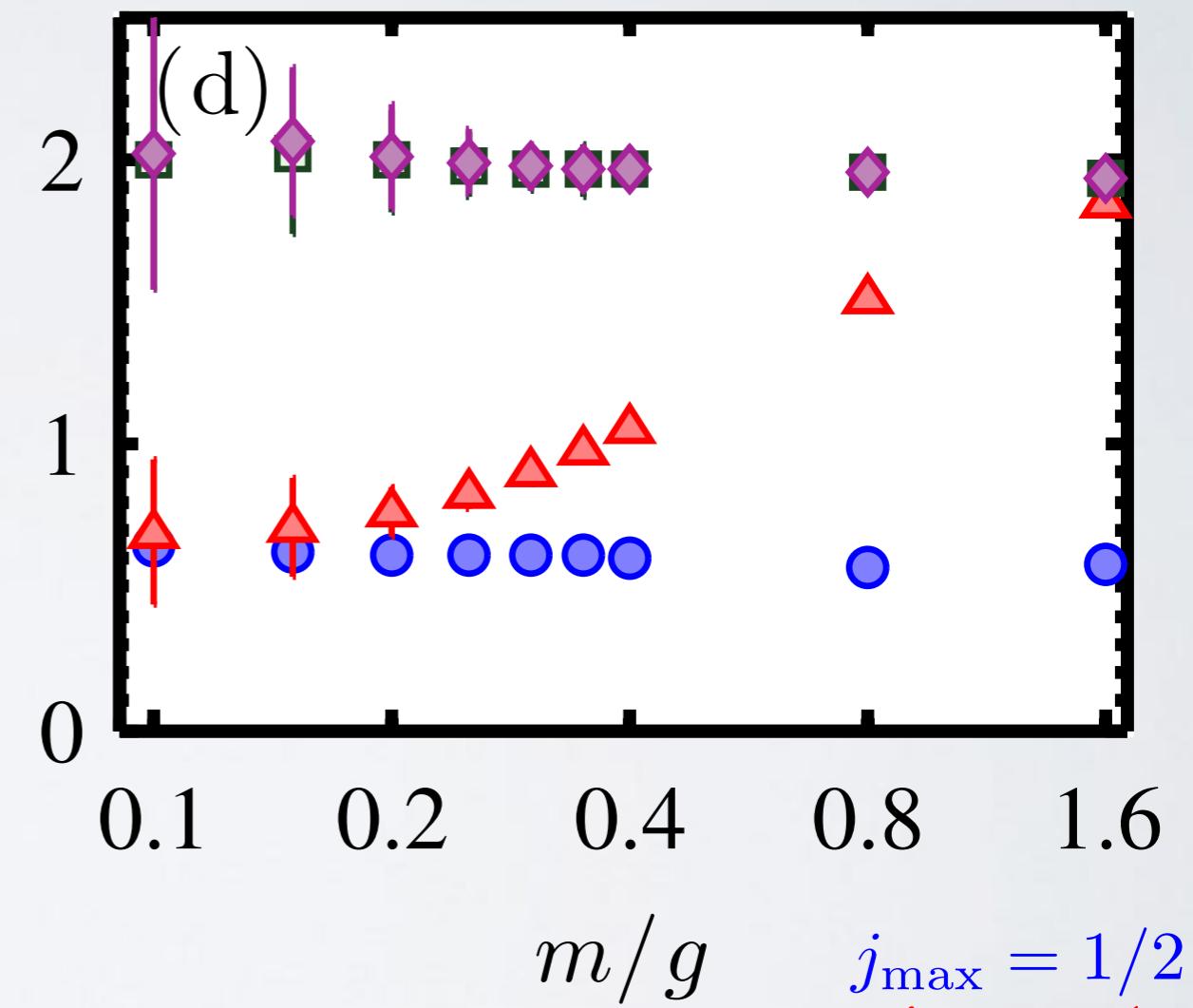
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extracting the central charge



$$j_{\max} = 1/2$$
$$j_{\max} = 1$$
$$j_{\max} = 3/2$$
$$j_{\max} = 2$$

our ongoing LGT-TNS
roadmap...

Schwinger model
 $U(1)$ in 1D
precise equilibrium
simulations,
feasibility of QSim

S. Kühn et al., PRA 90, 042305 (2014)

2+1 dimensions

JHEP 11(2013)158
PRD 92, 034519 (2015)

finite density

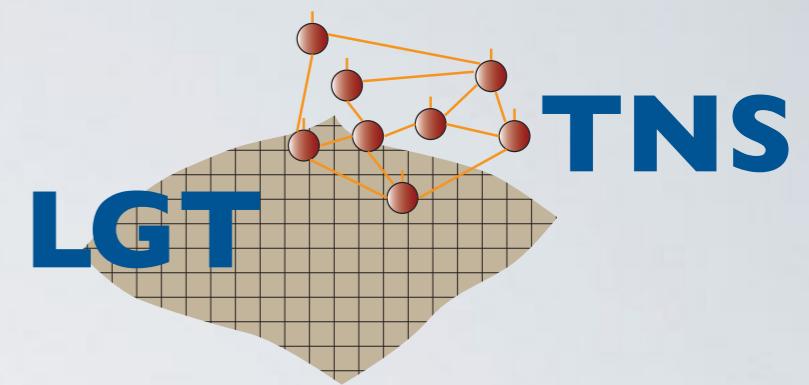
S. Kuehn et al, PRL 118 (2017) 071601

Non-Abelian in 1D
string breaking dynamics
S. Kühn et al., JHEP 07 (2015) 130;
arXiv:1707.06434

other models in
1+1 dimensions
in progress

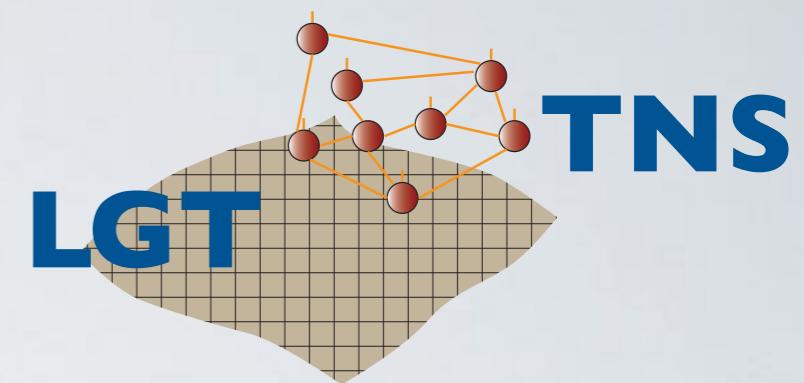


full LQCD in 3+1
dimensions



To conclude...

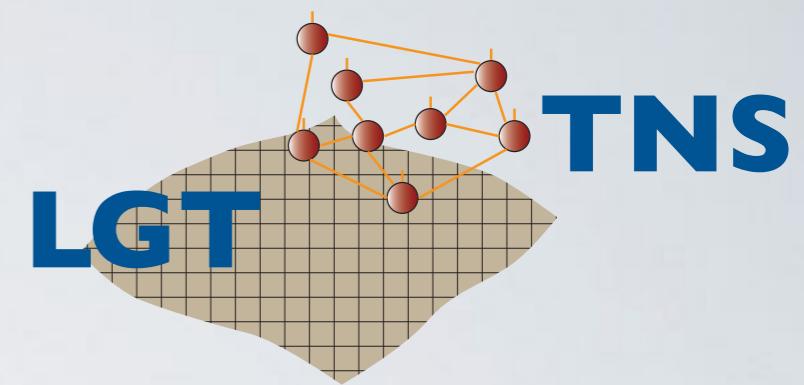
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1+1D tested feasibility for LQFT



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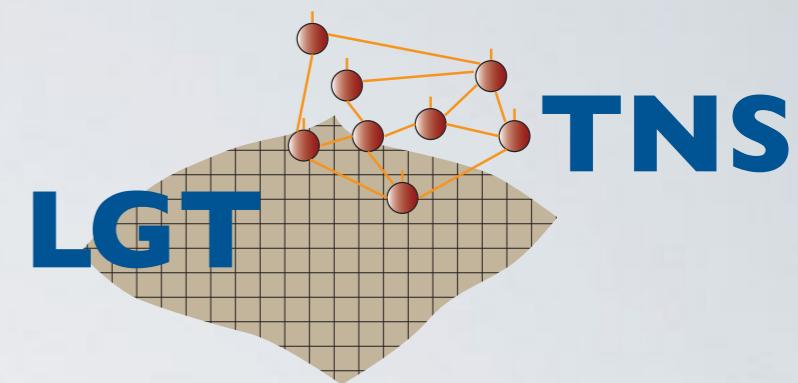
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spectrum, thermal equilibrium, finite density,
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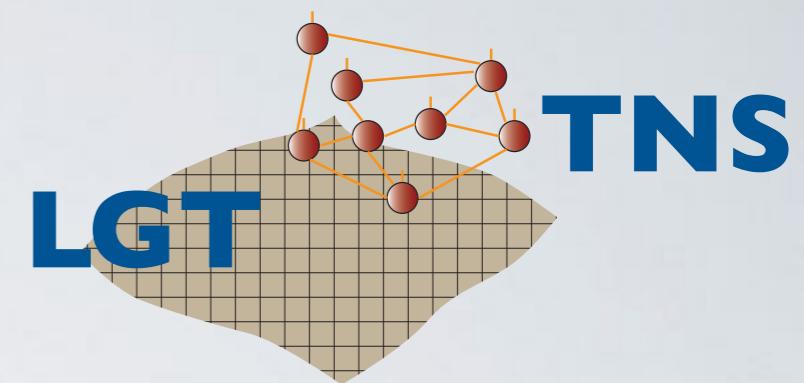
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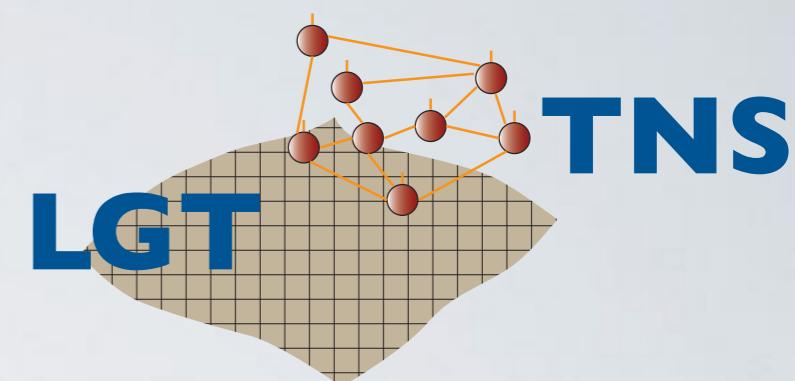
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THANKS



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