Tensor network simulation of QED on infinite lattices: learning from (1 + 1)d, and prospects for (2 + 1)d

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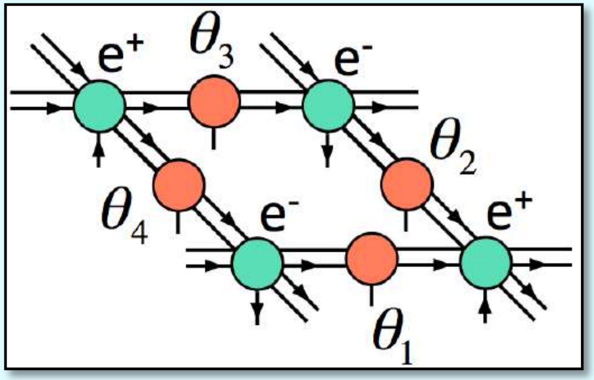
K. Zapp, RO, Phys. Rev. D 95, 114508 (2017)

Goal of this talk

1) Show TN results with infinite-DMRG for (1+1)d QED

2) Show that we have all the necessary ingredients for a meaningful simulation of (2+1)d QED with infinite-PEPS



Fermions + U(1) gauge+ plaquette interactions +
+ improved optimization + contraction schemes + ...

Outline

1) TN Basics

2) (1+1)d QED with iDMRG

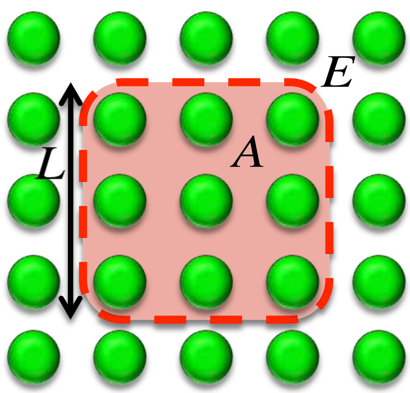
3) (2+1)d QED: roadmap for iPEPS

Entanglement obeys area-law

Entanglement → key resource in quantum information

teleportation, quantum algorithms, quantum error correction, quantum cryptography…

2d system



$$\rho_A = \mathrm{tr}_E\left(|\Psi\rangle\langle\Psi|\right)$$ Reduced density matrix of subsystem A

$$S(A) = -\mathrm{tr}\left(\rho_A \log \rho_A\right)$$ Entanglement entropy (von Neumann entropy)

For many ground states → $S(A) \sim \alpha L - S_\gamma$ „topological entropy“

$(L > \xi)$

In d dimensions

Generic state $S(A) \sim L^d$ (volume)

Ground states of (most) local Hamiltonians $S(A) \sim L^{d-1}$ (area)

Srednicki, Plenio, Eisert, Dreißig, Cramer, Wolf…

Locality of interactions ⇔ area-law ⇔ tensor network states

Hilbert space is a convenient illusion

Hilbert space of a N-body many-body system

"Exploration" time ~ $O(10^{10^{23}})$ **sec.**

Compare to…

Age of the universe ~ $O(10^{17})$ **sec.**

Set of area-law states

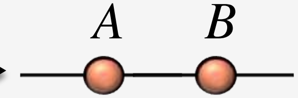
Set of product states (mean field)

We need a language to target the relevant corner of quantum states directly

Tensor Networks

e.g. RO, Annals of Physics ***349*** *(2014) 117–158*

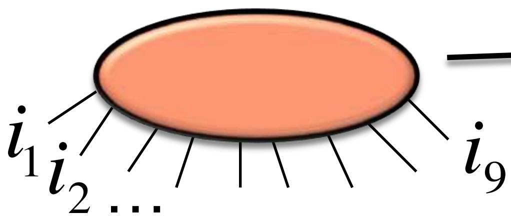
$$A \cdot B \rightarrow$$



$$|\Psi\rangle = \sum_{i's} \boxed{\Psi_{i_1 i_2 \ldots i_N}} |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

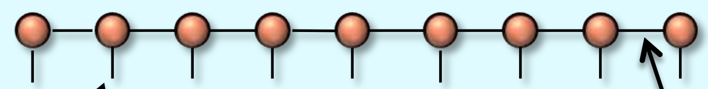
p-level systems

$$\Psi_{i_1 i_2 i_3 i_4 i_5 i_6 i_7 i_8 i_9}$$



1d

Matrix Product States (MPS)



DMRG, PWFRG, TEBD…

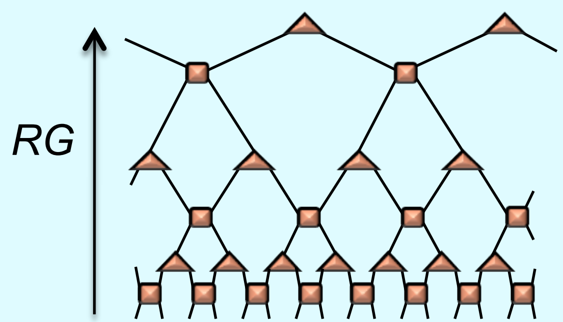
physical 1…p

bond 1..D (entanglement)

2d, 3d...

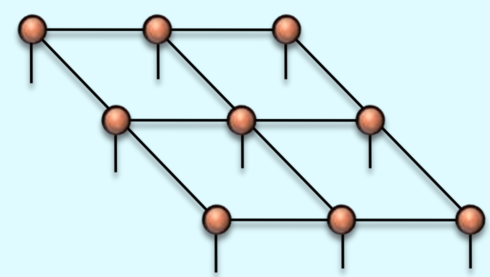
Scale-invariant

Multiscale Entanglement Renormalization Ansatz (MERA)



AdS/CFT, Entanglement Renormalization

Projected Entangled Pair States (PEPS), Tensor Product States (TPS)

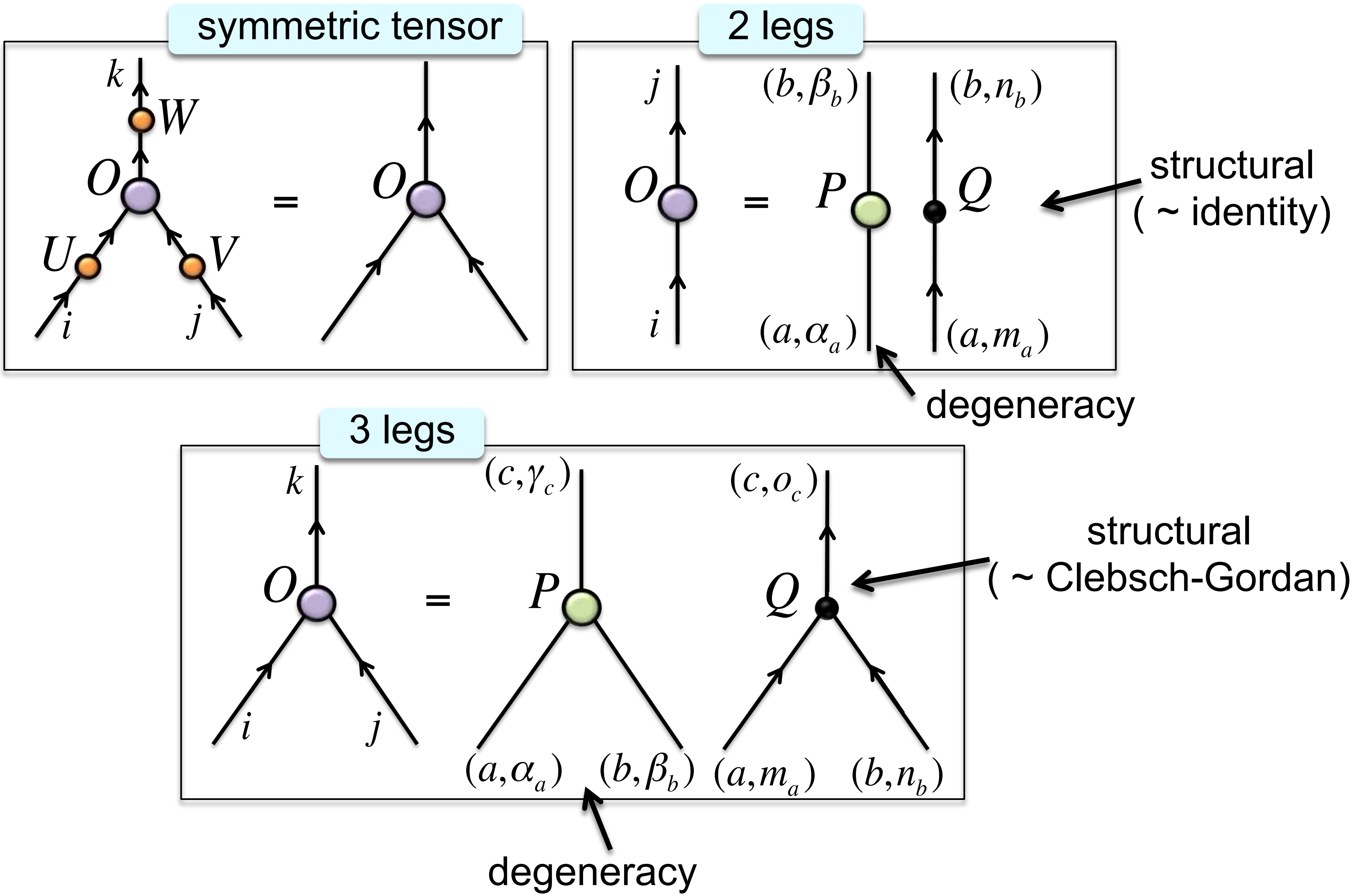


Tensor Product Variational Approach, PEPS & iPEPS algorithms, Tensor-Entanglement Renormalization, TRG/SRG/HOTRG/HOSRG…

Efficient O(poly(N)), satisfy area-law, low-energy eigenstates of local Hamiltonians

Symmetric tensors and Schur's lemma

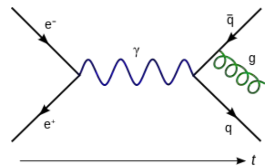
e.g., S. Singh, R. N. C. Pfeifer, G. Vidal, PRA 82, 050301 (2010)



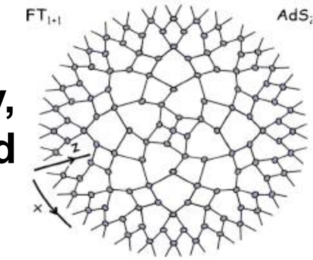
Structural part depends only on the group properties (intertwiners)

Explosion in recent years

HEP and lattice gauge theories



Quantum gravity, string theory and AdS/CFT

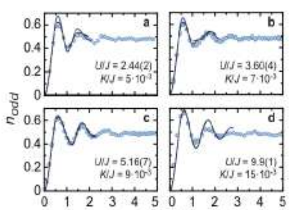


Quantum information and computation

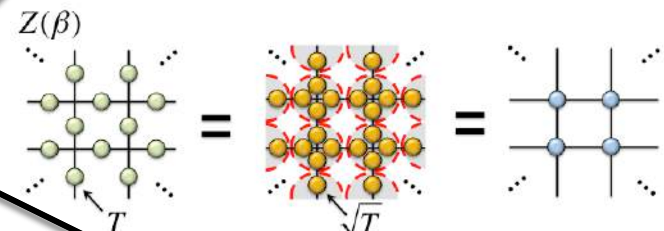
Strongly correlated systems

Entanglement and Tensor Networks

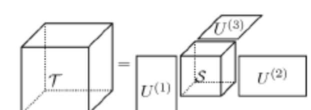
Quantum simulations



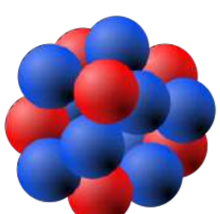
Classical statistical mechanics



Numerical tensor calculus

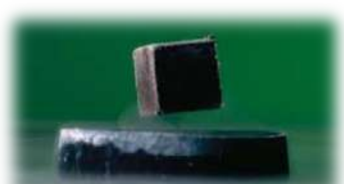


Nuclear physics

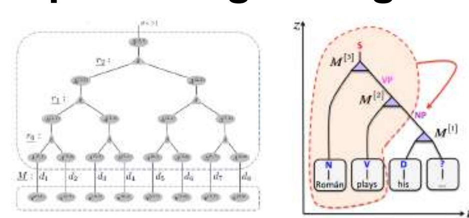


Quantum chemistry

Materials science



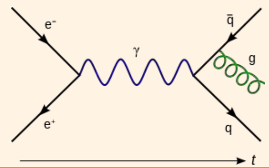
AI, Deep learning & Linguistics



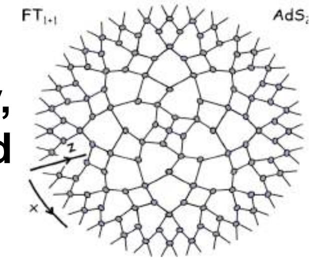
Explosion in recent years

Today's talk

HEP and lattice gauge theories



Quantum gravity, string theory and AdS/CFT

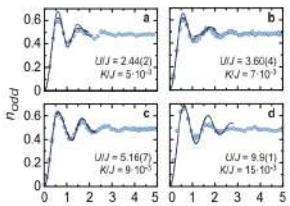


Quantum information and computation

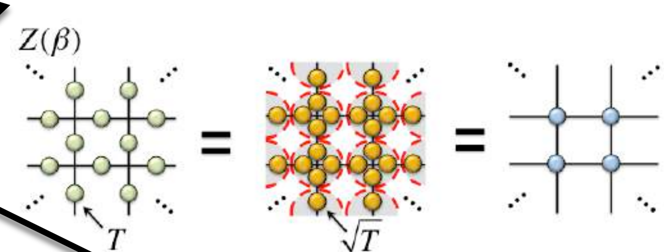
Strongly correlated systems

Entanglement and Tensor Networks

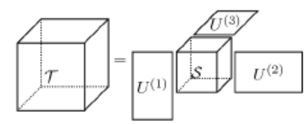
Quantum simulations



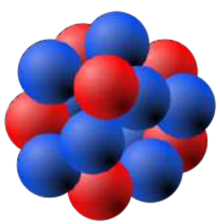
Classical statistical mechanics



Numerical tensor calculus

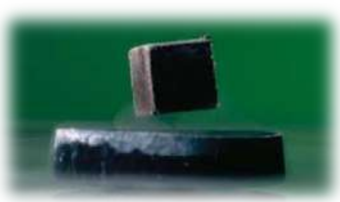


Nuclear physics

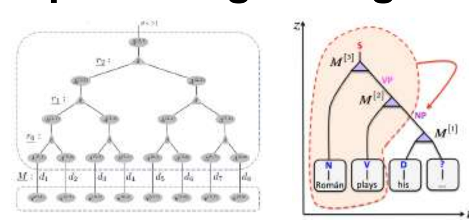


Quantum chemistry

Materials science



AI, Deep learning & Linguistics



Outline

1) TN Basics

2) (1+1)d QED with iDMRG

3) (2+1)d QED: roadmap for iPEPS

Massive Schwinger model

1) Lagrangian density

$$\mathcal{L} = \bar{\psi}\left(i\partial_\mu \gamma^\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - g\bar{\psi}A_\mu \gamma^\mu \psi$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\vec{\nabla}\vec{E} = \rho$$ Gauß' law

1-flavour fermions, U(1) gauge field, coupling g

2) Lattice Hamiltonian, Kogut-Susskind staggered formulation

$$H = -\frac{i}{2a}\sum_n \left(\phi_n^+ e^{i\theta_n}\phi_{n+1} - h.c.\right) + m\sum_n (-1)^n \phi_n^+\phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

$$\left[\theta_n, L_m\right] = i\delta_{nm}$$

$$L_n - L_{n-1} = \phi_n^+\phi_n - \frac{1}{2}\left(1-(-1)^n\right)$$ Gauß' law

- Fermionic and bosonic variables
- Local interactions
- Invariant under translations

Massive Schwinger model

3) After Jordan-Wigner transformation

$$H = \frac{g}{2\sqrt{x}}\left(\sum_n L_n^2 + \frac{\mu}{2}\sum_n (-1)^n\left(\sigma_n^z + (-1)^n\right) + x\sum_n\left(\sigma_n^+ e^{i\theta_n}\sigma_{n+1}^-\right) + h.c.\right)$$

$$x \equiv 1/(g^2a^2) \qquad \mu \equiv 2m\sqrt{x}/g \qquad L_n - L_{n-1} = \frac{1}{2}\left(\sigma_n^z + (-1)^n\right)$$ Gauß' law

Good formulation for infinite-MPS (TDVP, iDMRG...)

B. Buyens, K. Van Acoleyen, J. Haegeman, F. Verstraete, PoS(LATTICE2014)308.

4) Integrating out Gauss' law (only in 1+1 dimensions)

$$H = x\sum_n\left(\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right) + \frac{\mu}{2}\sum_n\left(1 + (-1)^n\sigma_n^z\right) + \sum_n\left(l + \frac{1}{2}\sum_{k=0}^{n}\left((-1)^k + \sigma_k^z\right)\right)^2$$

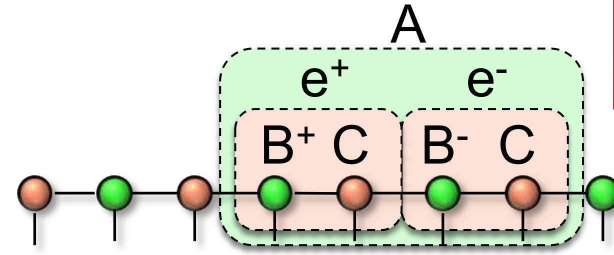
- Non-local interactions

Good formulation for finite-MPS (DMRG)

M. C. Bañuls, K. Cichy, J. I. Cirac, K. Jansen, H. Saito, PoS(LATTICE2013)332.

Gauge-invariant iDMRG

Gauge-invariant iDMRG simulations



fermion →
$$(B_n)^{s_n}_{(q,\alpha_q)(r,\beta_r)} = (b_{n,q})^{s_n}_{\alpha_q,\beta_r}\,\delta_{q+(s_n+(-1)^n)/2,r},$$

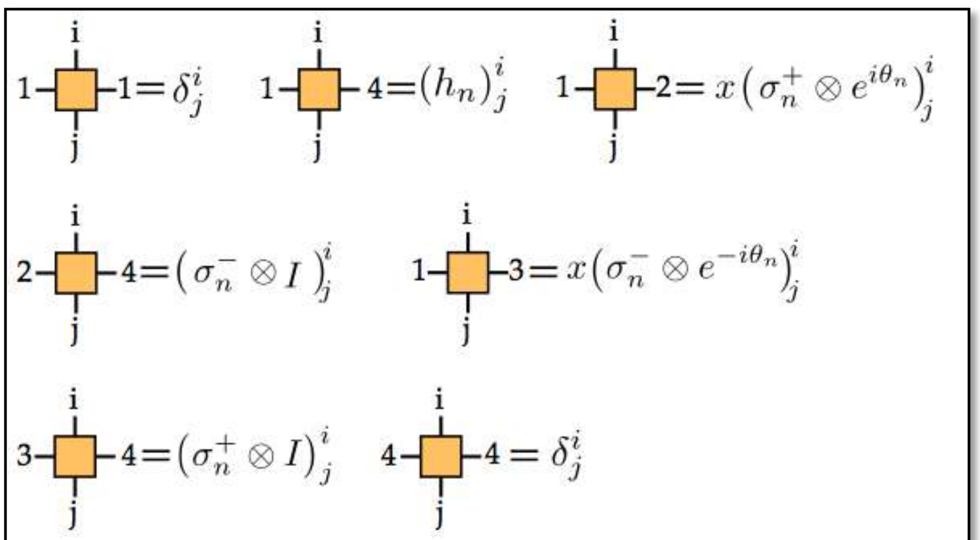
gauge boson →
$$(C_n)^{p_n}_{(q,\alpha_q)(r,\beta_r)} = (c_n)^{p_n}_{\alpha_q,\beta_r}\,\delta_{q,p_n}\,\delta_{r,p_n}.$$

B. Buyens, K. Van Acoleyen, J. Haegeman, F. Verstraete, PoS(LATTICE2014)308.

degeneracy structural

q,r indices for U(1) gauge symmetry sector (structural)

Greek degeneracy indices

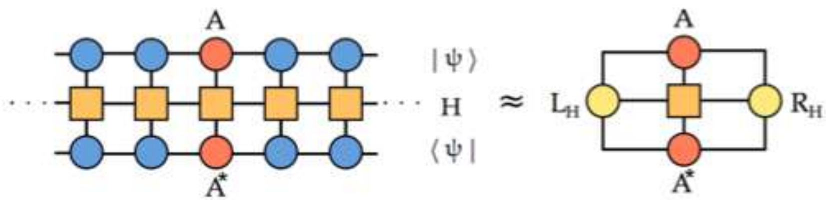


H = translation invariant MPO, bond dimension = 4

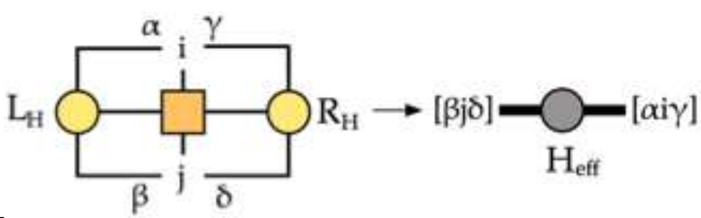
Parameter	Description
χ_c	Bond dimension of charge index
χ_d	Bond dimension of degeneracy index
p_{max}	Gauge boson truncation
N	Number of added sites
x	Inverse coupling
m/g	Dimensionless fermion mass

1-site iDMRG crash-course *(technical)*

1) Get L_H and R_H environment tensors



2) Define effective Hamiltonian

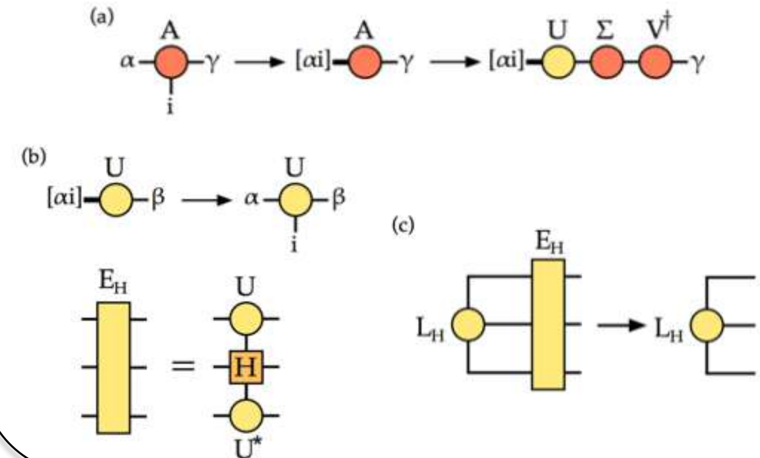


3) Solve eigenvalue problem

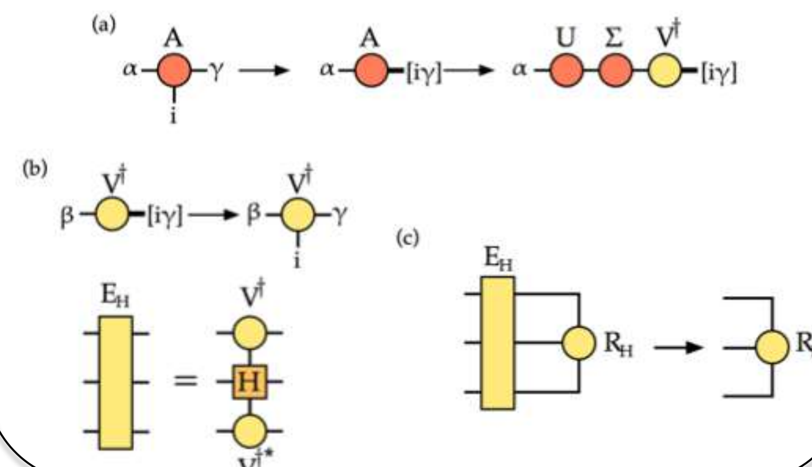
$$\mathcal{H}_{eff}\vec{A} = \lambda\vec{A}$$

Mixed canonical form, and
2-site iDMRG also possible

4) Odd step: absorb to the left



5) Even step: absorb to the right



Chiral condensate

$$\Sigma = \langle \overline{\psi}\psi \rangle \qquad \Sigma(x) = \frac{\sqrt{x}}{N}\sum_n (-1)^n \left\langle \frac{1+\sigma_n^z}{2} \right\rangle \qquad \Sigma_{sub} = \Sigma(x) - \Sigma_{free}(x)$$

m/g=0.25

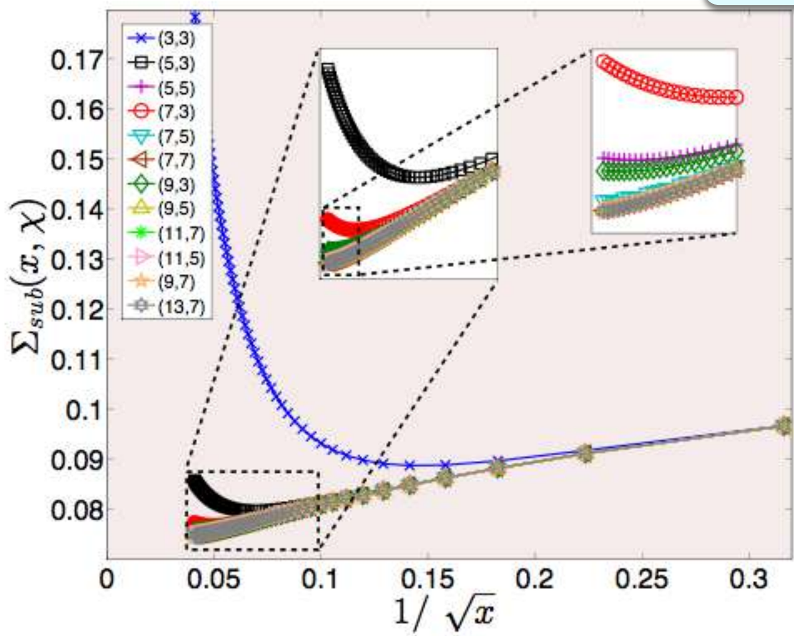


FIG. 8: [Color online] Subtracted chiral condensate for $m/g = 0.25$ as a function of $1/\sqrt{x}$ for different bond dimensions (χ_c, χ_d), and physical gauge-boson dimension 5. The insets show two consecutive zooms in the region with small lattice constant $a \sim 1/\sqrt{x}$.

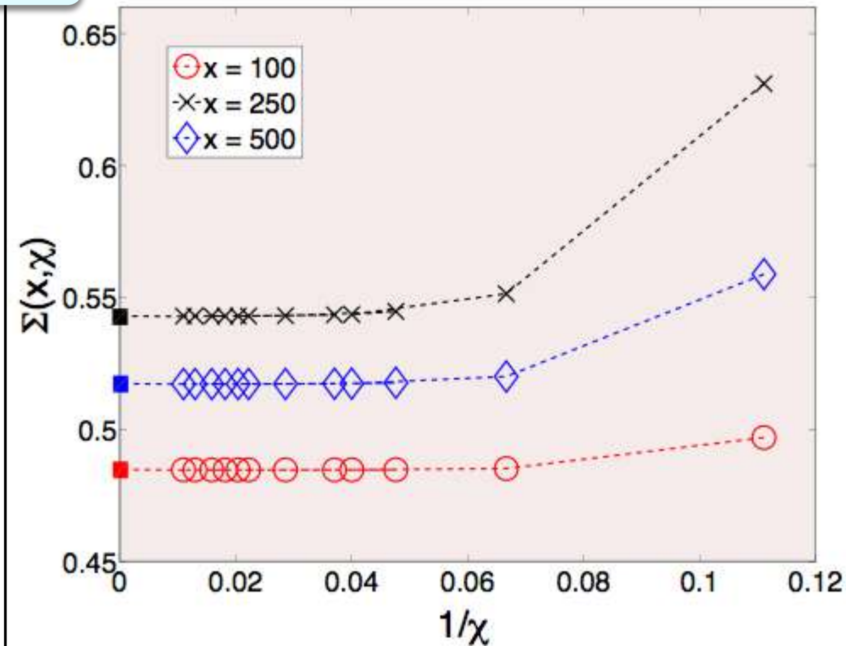


FIG. 9: [Color online] Extrapolation of the computed chiral condensate in $1/\chi$, for $x = 100, 250, 500$, at $m/g = 0.25$.

Chiral condensate

$$\Sigma = \langle \bar{\psi}\psi \rangle \qquad \Sigma(x) = \frac{\sqrt{x}}{N}\sum_n (-1)^n \left\langle \frac{1+\sigma_n^z}{2} \right\rangle \qquad \Sigma_{sub} = \Sigma(x) - \Sigma_{free}(x)$$

m/g=0.25

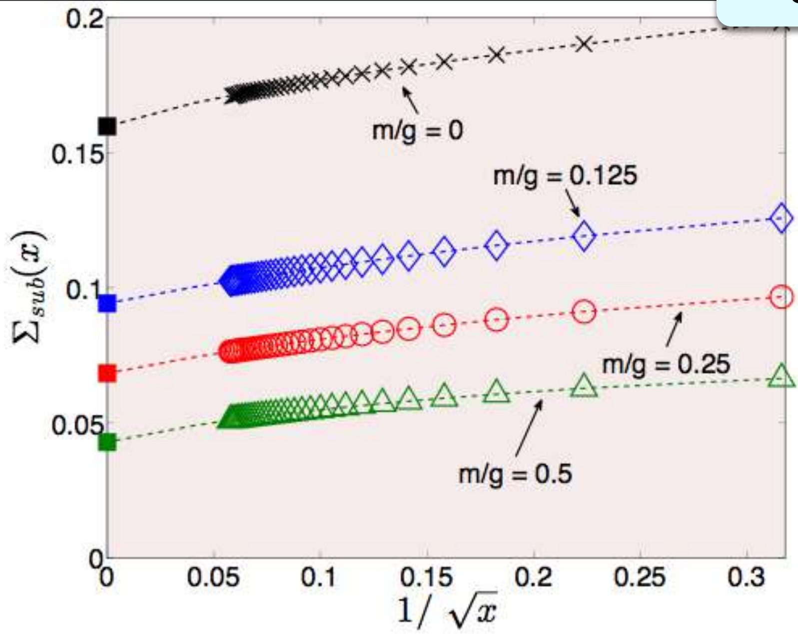


FIG. 10: [Color online] Continuum extrapolation of the subtracted chiral condensate for $m/g = 0, 0.125, 0.25, 0.5$ attained from $x \in [10, 300]$. Dashed lines correspond to the fit, and squares at $1/\sqrt{x} = 0$ to the extrapolated value in the continuum.

		DMRG	iTDVP	
m/g	One-site iDMRG	Ref.[15]	Ref.[8]	exact
0	0.15900	0.15993	0.15992	0.15992
0.125	0.09425	0.09202	0.09201	-
0.25	0.06838	0.06666	0.06664	-
0.5	0.04293	0.04238	0.04234	-
0.75	-	-	0.03062	-
1	-	-	0.02385	-
2	-	-	0.01246	-

TABLE II: Comparison: subtracted chiral condensate in the continuum. The extrapolation is in the regime $x \in [10, 300]$.

Similar results with 2-site iDMRG

A similar approach should be possible in (2+1)d

Outline

1) TN Basics

2) (1+1)d QED with iDMRG

3) (2+1)d QED: roadmap for iPEPS

Lattice (2+1)d QED with PEPS

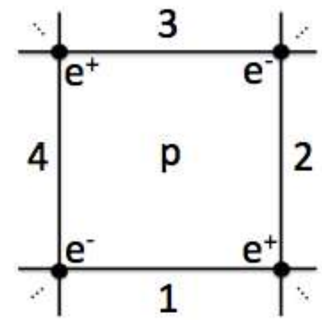
Why interesting? „True“ fermions, chiral symmetry breaking, confinement, higher dimensions,...

Staggered fermions in (2+1)d

Compact-QED

$$H = -\frac{i}{2a}\sum_{\langle i,j\rangle}\left(\phi_i^+ e^{i\theta_{ij}}\phi_j - h.c.\right) + m\sum_i (-1)^{s(i)}\phi_i^+\phi_i$$

$$+\frac{ag^2}{2}\sum_{\langle i,j\rangle} L_{ij}^2 - \frac{1}{ag^2}\sum_p \cos\left(\theta_1+\theta_2-\theta_3-\theta_4\right)$$



electric energy   magnetic energy

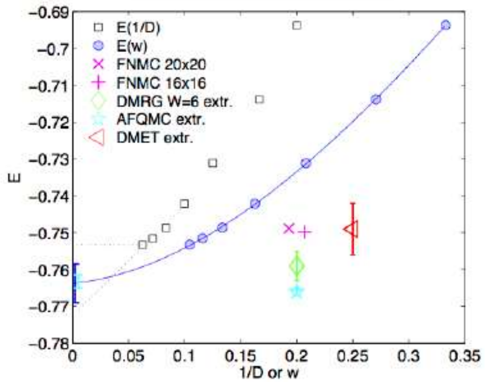
$$L_{ij} - L_{i(j+1)} = \phi_i^+\phi_i - \frac{1}{2}\left(1-(-1)^{s(i)}\right)$$ Gauß‘ law

- Jordan-Wigner no longer useful
- Plaquette interaction
- Integrating out Gauß‘ law no longer useful
- Truncation of gauge-boson Hilbert space (quantum link model)
- Gauge U(1), global fermionic parity Z_2

Lattice (2+1)d QED with PEPS

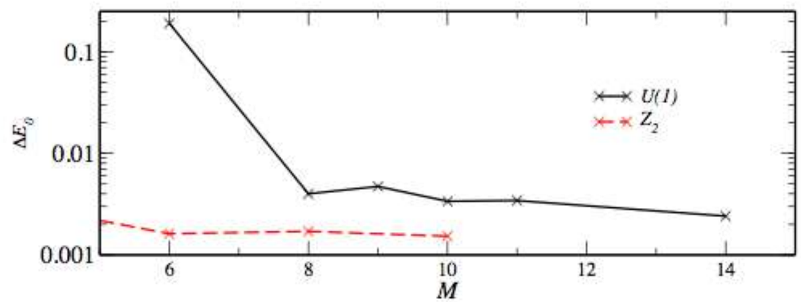
Ok, so what do we need for this simulation?

Fermions



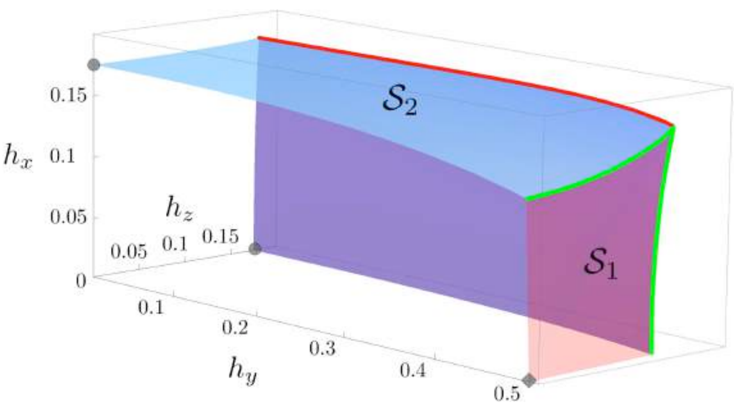
*e.g., P. Corboz, PRB **93** 045116 (2016)*

U(1) symmetry



*e.g., B. Bauer, P. Corboz, RO, M. Troyer, PRB **83** 125106 (2011)*

Plaquette interactions



e.g., S. Dusuel, M. Kamfor, RO, K. P. Schmidt, J. Vidal, PRL 106, 107203 (2011)

Efficient accurate schemes

Simple update, full update, fast full update, CTMs, TRG, TERG, boundary-MPS, TDVP, variational, imag.-time evolution, large unit cells, finite-D scaling...

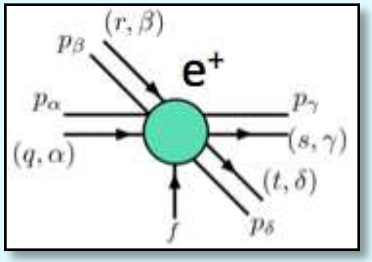
We have all the ingredients to do this simulation (a priori)

An option:

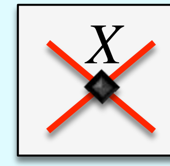
Lattice (2+1)d QED with PEPS

fermion (site)

Symmetric tensors

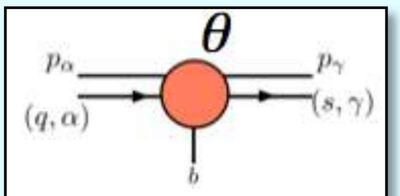


$$b^{f}_{\alpha_q,\beta_r,\gamma_s,\delta_t}\delta_{mod(p_\alpha+p_\beta+p_\gamma+p_\delta+f,2),0}\delta_{(q+r\pm f),(s+t)},$$



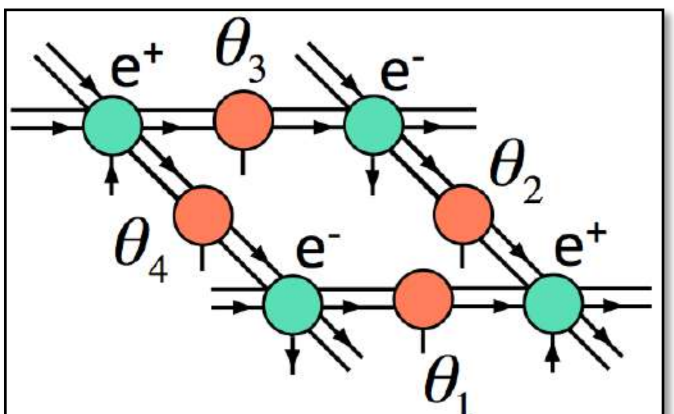
free param.   fermionic Z_2 gauge U(1)

gauge boson (link)



$$c^{b}_{\alpha_q,\gamma_s}\delta_{mod(p_\alpha+p_\gamma,2),0}\delta_{q,b}\delta_{s,b},$$

fermionic and bosonic indices
U(1) indices oriented



(2+1)d variational ansatz

- Global fermionic Z_2
- Gauge U(1)

*See also E. Zohar, M. Burrello, T. B. Wahl, J. I. Cirac, Ann. Phys. 385-439 (2015); E. Zohar, M. Burrello, NJP **18**, 043008 (2016); E. Zohar, T. B. Wahl, M. Burrello, J. I. Cirac, Ann. Phys. 84-137 (2016)*

Lattice (2+1)d QED with PEPS

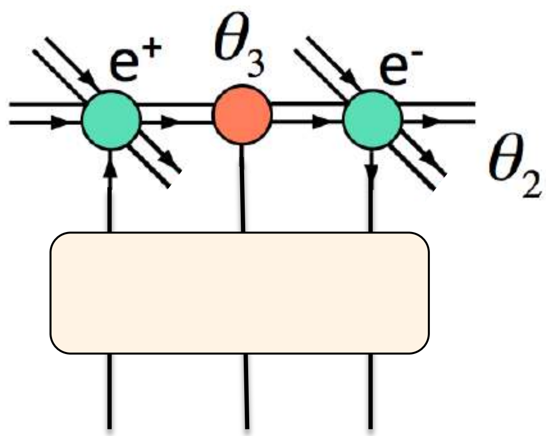
Imaginary-time evolution

$$H = \boxed{-\frac{i}{2a}\sum_{\langle i,j\rangle}\left(\phi_i^+ e^{i\theta_{ij}}\phi_j - h.c.\right)} + m\sum_i (-1)^{s(i)}\phi_i^+\phi_i$$
$$+\frac{ag^2}{2}\sum_{\langle i,j\rangle} L_{ij}^2 - \frac{1}{ag^2}\sum_p \cos\left(\theta_1+\theta_2-\theta_3-\theta_4\right)$$

$e^{-Ht}|\Psi\rangle$

Trotterize, etc

Fermion-gauge boson interaction: 3-body gate



Lattice (2+1)d QED with PEPS

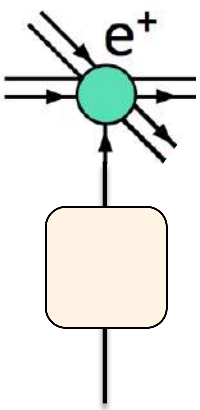
Imaginary-time evolution

$$H = -\frac{i}{2a}\sum_{\langle i,j\rangle}\left(\phi_i^{+}e^{i\theta_{ij}}\phi_j - h.c.\right) + \boxed{m\sum_i(-1)^{s(i)}\phi_i^{+}\phi_i}$$
$$+\frac{ag^2}{2}\sum_{\langle i,j\rangle}L_{ij}^2 - \frac{1}{ag^2}\sum_p \cos(\theta_1+\theta_2-\theta_3-\theta_4)$$

$e^{-Ht}|\Psi\rangle$

Trotterize, etc

Fermionic mass term:
1-body gate



Lattice (2+1)d QED with PEPS

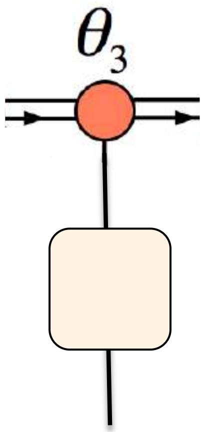
$$H = -\frac{i}{2a}\sum_{\langle i,j\rangle}\left(\phi_i^+ e^{i\theta_{ij}}\phi_j - h.c.\right) + m\sum_i (-1)^{s(i)}\phi_i^+\phi_i$$
$$+\boxed{\frac{ag^2}{2}\sum_{\langle i,j\rangle}L_{ij}^2} - \frac{1}{ag^2}\sum_p \cos\left(\theta_1+\theta_2-\theta_3-\theta_4\right)$$

Imaginary-time evolution

$e^{-Ht}|\Psi\rangle$

Trotterize, etc

Electric field energy:
1-body gate



Lattice (2+1)d QED with PEPS

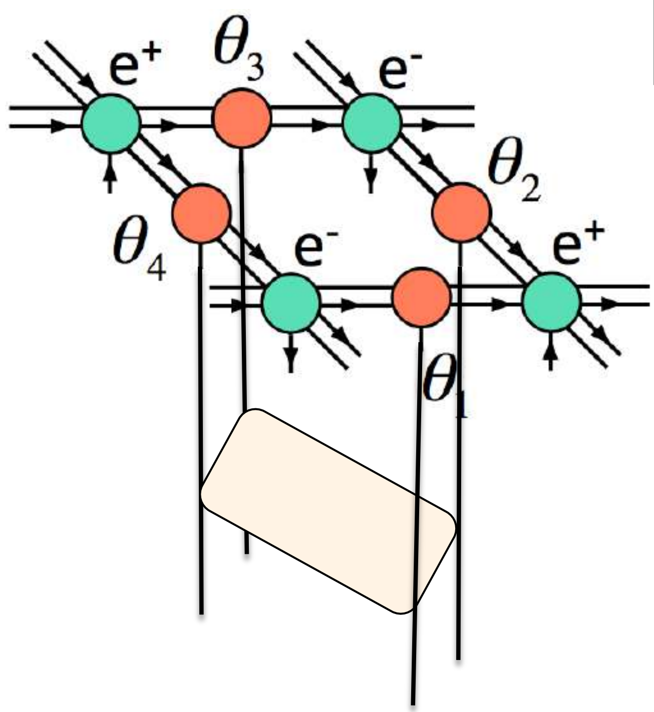
Imaginary-time evolution

$$H = -\frac{i}{2a}\sum_{\langle i,j\rangle}\left(\phi_i^+ e^{i\theta_{ij}}\phi_j - h.c.\right) + m\sum_i (-1)^{s(i)}\phi_i^+\phi_i + \frac{ag^2}{2}\sum_{\langle i,j\rangle} L_{ij}^2 \boxed{-\frac{1}{ag^2}\sum_p \cos(\theta_1+\theta_2-\theta_3-\theta_4)}$$

$e^{-Ht}|\Psi\rangle$

Trotterize, etc

Magnetic field energy: plaquette gate



Discussion

Lattice (2+1)d QED with PEPS

- Compact or non-compact?

Both should be possible, change in plaquette gates (pure gauge term)

- Chiral condensate and number of flavours?

More fermions, larger bond dimensions. Should be possible but costly

- Monopole density and number of flavours?

Same as above

- Finite-temperature BKT confinement-deconfinement transition?

Should be possible with iPEPOs and related approaches

- Which approach is better? Simple update, full update, variational...

Depends on the regime. One needs to try.

Tensor networks in Mainz

JG|U

The team

(Feb. 2016)

Funding

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Computational Science Mainz

MATERIALS IN MAINZ SCIENCE

JG|U

SPICE
SPIN PHENOMENA
INTERDISCIPLINARY CENTER

LGT / TN „chat“ ☺

Collaborators