Lepton Flavor Violation

in effective field theories

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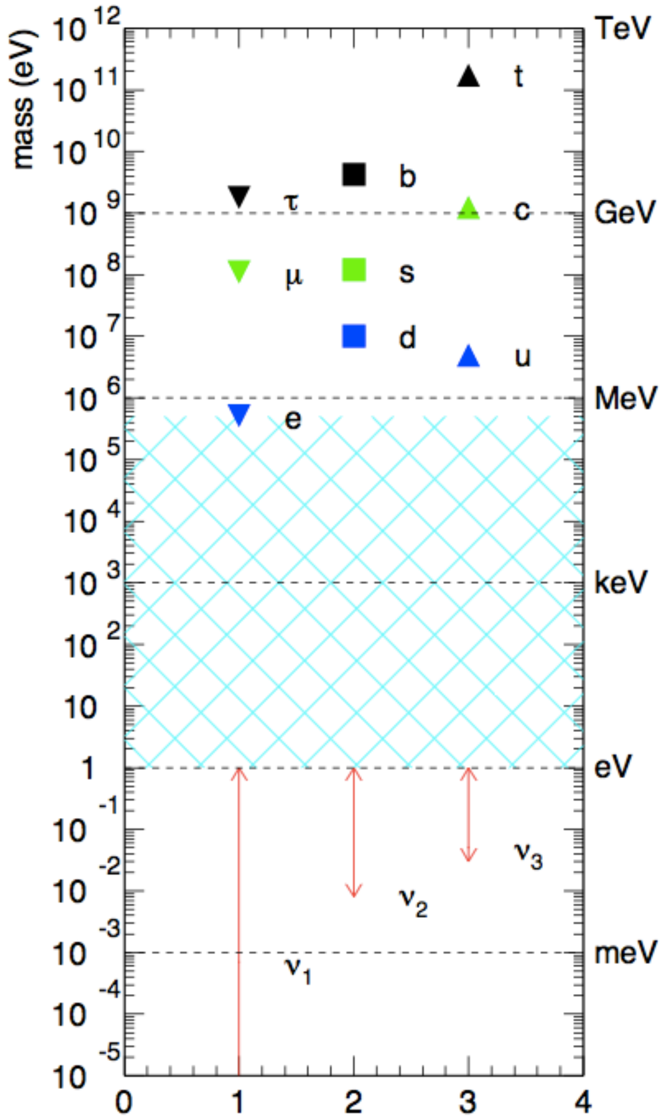
Invitation: SM matter sector, experimental data



★ Ratios of masses of quarks and leptons

- quarks

$$\frac{m_d}{m_u} \simeq 2 , \quad \frac{m_s}{m_d} \simeq 21 ,$$
$$\frac{m_t}{m_c} \simeq 267 , \quad \frac{m_c}{m_u} \simeq 431 , \quad \frac{m_t}{m_u} \simeq 1.2 \times 10^5 .$$

- leptons

$$\frac{m_\tau}{m_\mu} \simeq 17 , \quad \frac{m_\mu}{m_e} \simeq 207 .$$

★ Quark mixing (Cabibbo-Kobayashi-Maskawa) matrix parameters

$$V_{\rm CKM} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

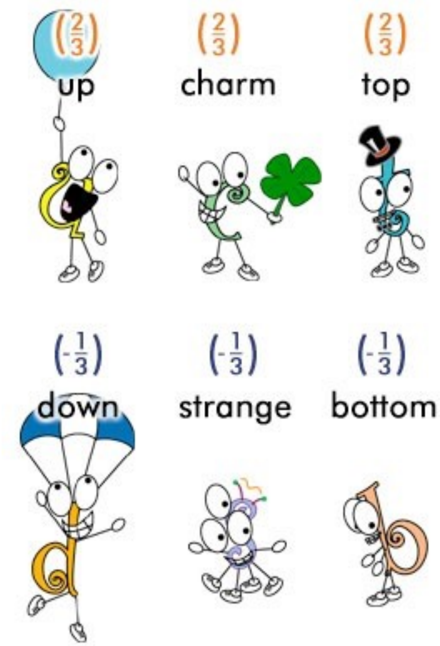
$$V_{ud} \sim 1, V_{us} \sim 0.2,\ V_{cb} \sim 0.04,\ V_{ub} \sim 0.004$$

A couple of questions...

Gauge forces in SM do not distinguish between fermions of different generations:

"Lepton universality"

- e, μ, τ have same electrical charge
- quarks have same color charge



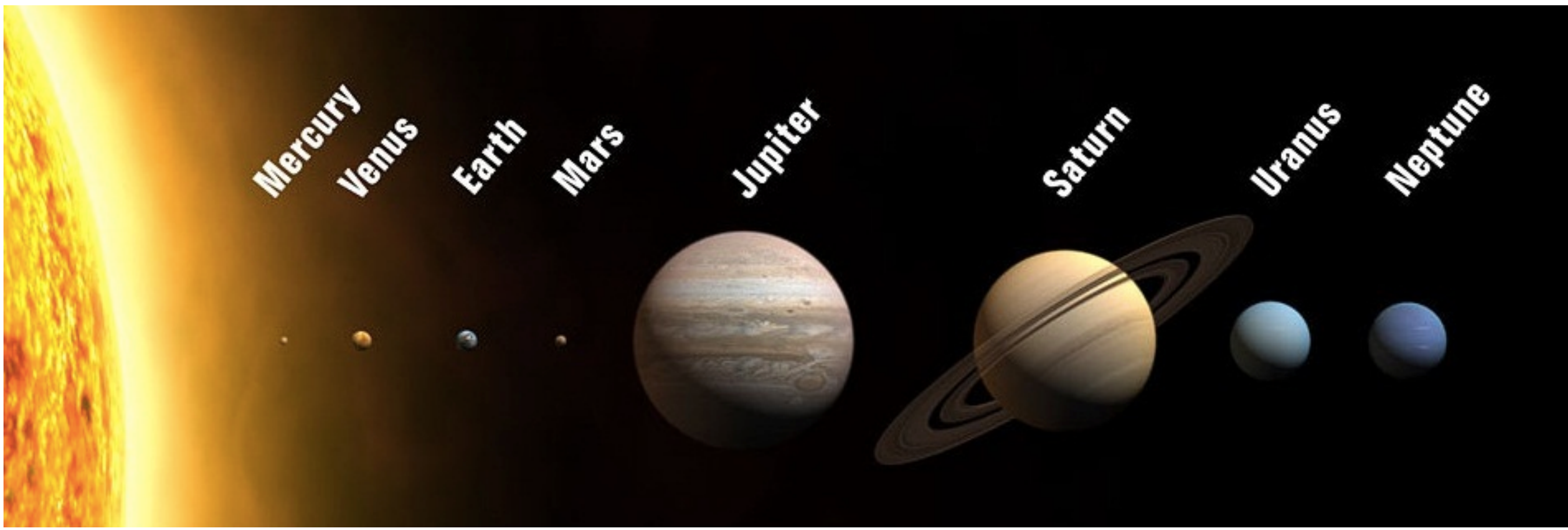
★ Why generations? Why only 3? Are there only 3?

★ Why hierarchies of masses and mixings?

★ Can there be transitions between quarks/leptons of the same charge but different generations?

The flavor puzzle

Caution: fermion mass hierarchy might not have a single reason…



Why is $M_{Jupiter} >> M_{Mercury}$?

1. Introduction: leptonic FCNC

★ Why study flavor-changing neutral currents (FCNC)?

★ No trivial FCNC vertices in the Standard Model: sensitive NP tests

★ Possible experimental studies in a lepton sector

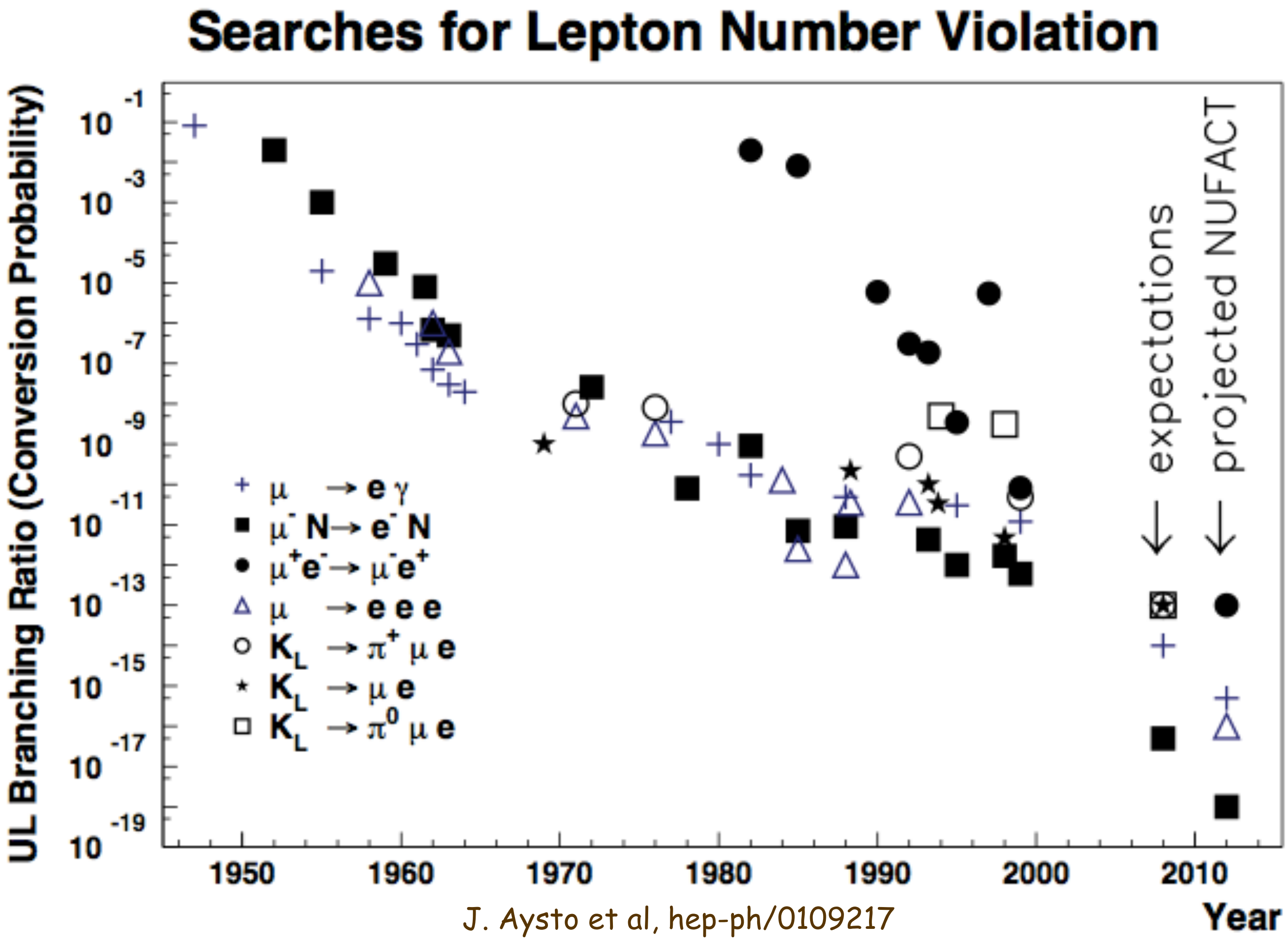
- lepton-flavor violating processes
  - $\mu \to e\gamma$, $\tau \to e\gamma$, etc.
  - $\mu \to eee$, $\tau \to \mu ee$, etc.
  - $\mu^+ e^- \to e^- \mu^+$
  - $Z^0 \to \mu e$, τe, etc.
  - $H \to \mu e$, τe, etc.
  - K^0 (B^0, D^0, ...) $\to \mu e$, τe, etc.
  - K^+ (B^+, D^+, ...) $\to \pi^+ \mu e$, $\pi^+ \tau e$, etc.
  - $\mu^- + (A, Z) \to e^- + (A, Z)$

Channel	Babar $\mathcal{L}$ (fb^{-1})	Babar $\mathcal{B}_{\mathrm{UL}}$ (10^{-8})	BELLE $\mathcal{L}$ (fb^{-1})	BELLE $\mathcal{B}_{\mathrm{UL}}$ (10^{-8})
$\tau^\pm \to e^\pm \gamma$	232	11	535	12
$\tau^\pm \to \mu^\pm \gamma$	232	6.8	535	4.5
$\tau^\pm \to \ell^\pm \ell^\mp \ell^\pm$	92	11 - 33	535	2 - 4
$\tau^\pm \to e^\pm \pi^0$	339	13	401	8.0
$\tau^\pm \to \mu^\pm \pi^0$	339	11	401	12
$\tau^\pm \to e^\pm \eta$	339	16	401	9.2
$\tau^\pm \to \mu^\pm \eta$	339	15	401	6.5
$\tau^\pm \to e^\pm \eta'$	339	24	401	16
$\tau^\pm \to \mu^\pm \eta'$	339	14	401	13

- lepton number and lepton-flavor violating processes
  - $(A, Z) \to (A, Z\pm 2) + e^\mp e^\mp$
  - $\mu^- + (A, Z) \to e^+ + (A, Z-2)$

$$\mathrm{BR}\,(K_L^0 \to \mu e) < 4.7 \times 10^{-12}$$
$$\mathrm{BR}\,(B_d^0 \to \mu e) < 1.7 \times 10^{-7} \quad \text{[Belle]}$$
$$\mathrm{BR}\,(B_s^0 \to \mu e) < 6.1 \times 10^{-6} \quad \text{[CDF]}$$

★ Highly suppressed in the Standard Model, e.g. $Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i} U_{ei} \frac{m^2_{\nu_i}}{M_W^2}\right|^2 < 10^{-54}$



J. Aysto et al, hep-ph/0109217

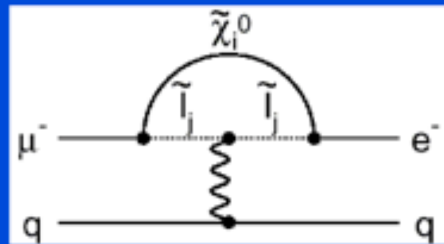
★ Let us be more systematic: consider LFV in quark processes

- only one FCNC: flavor conservation on the quark side
- use EFT to classify operators

A multitude of models…

Supersymmetry

Predictions at 10^{-15}

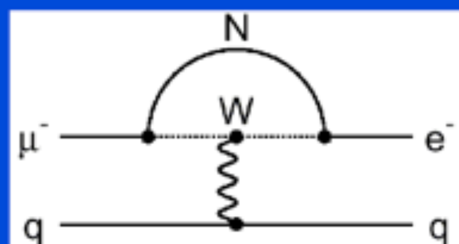


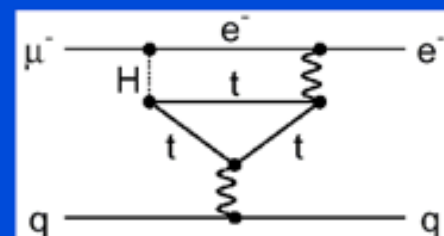
Compositeness

$\Lambda_c = 3000$ TeV

Heavy Neutrinos

$\left|U^*_{\mu N} U_{eN}\right|^2 = 8\times10^{-13}$



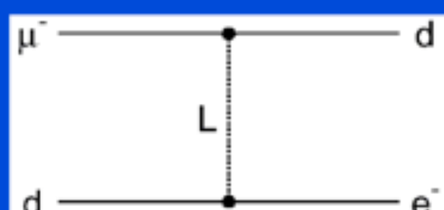


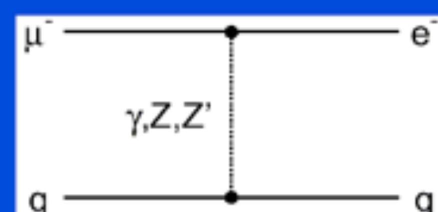
Second Higgs doublet

$g_{H_{\mu e}} = 10^{-4} \times g_{H_{\mu\mu}}$

Leptoquarks

$M_L = 3000\sqrt{\lambda_{\mu d}\lambda_{ed}}$ TeV/c^2





Heavy Z', Anomalous Z coupling

$M_{Z'} = 3000$ TeV/c^2

$B(Z \to \mu e) < 10^{-17}$

After W. Marciano

James Miller, 2006

High energy vs low energy

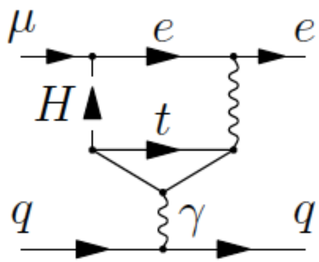
★ Leptonic FCNC could be generated by New Physics

★ E.g. FCNC Higgs decays H → μe, τe, etc.: $Y_{ij} = \frac{m_i}{v}\delta_{ij} + \frac{v^2}{\sqrt{2}\Lambda^2}\hat{\lambda}_{ij}$

Harnik, Kopp, Zupan

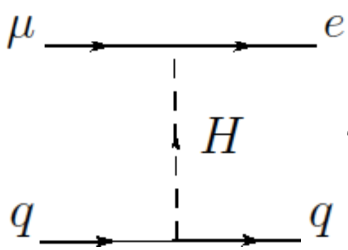
★ FCNC Higgs model & muon conversion/quarkonium decays

e.g.



$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level

★ … but note: couplings of new physics to light quarks are suppressed

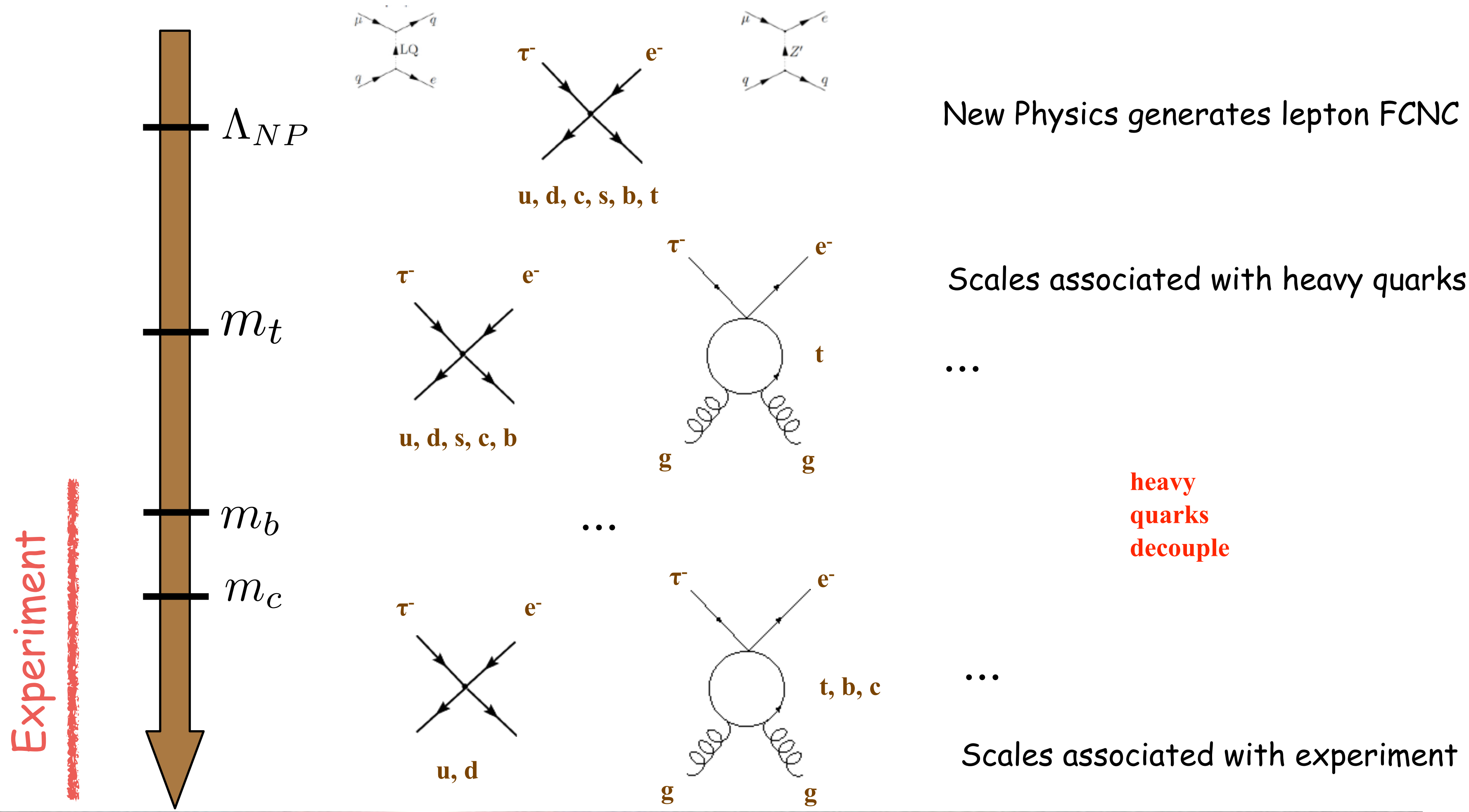
Aside: can leptons interact with gluons?

Can we correlate low energy and high energy data?
(will not discuss purely leptonic LFV interactions)

2. Effective Lagrangians for LFV transitions

★ Modern approach to flavor physics calculations: effective field theories

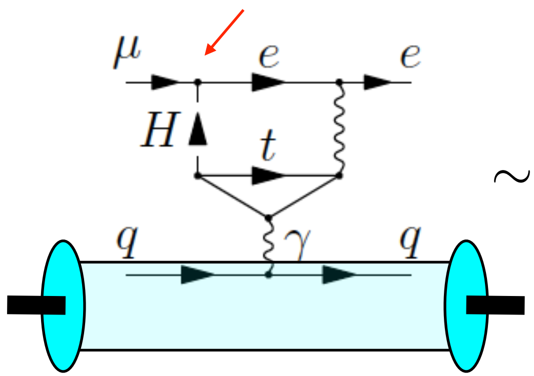
★ It is important to understand ALL relevant energy scales for the problem at hand



All operators are important

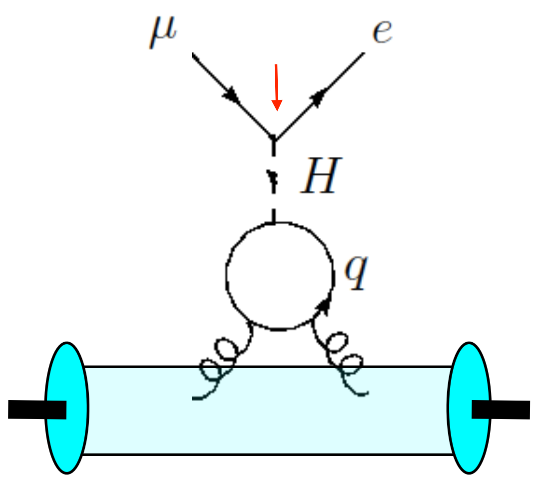
★ Contribution of heavy quarks can, in principle, be large even at low energies

★ Two-loop sensitivity to NP in muon conversion experiment...



$\sim \mathcal{O}(\alpha^2)$

★ ... becomes one-loop!



➡ gluonic couplings to hadrons are not (always) suppressed!
➡ NP couplings to heavy quarks are not well constrained and could be large

Effective Lagrangians

★ Naive power counting: largest contribution from lowest dimensional operators

★ Can write the most general LFV Lagrangian $\mathcal{L}_{LFV} = \mathcal{L}_D + \mathcal{L}_{lq} + \mathcal{L}_G + \ldots$

- dipole operators

$$\mathcal{L}_D = -\frac{m_2}{\Lambda^2}\left[\left(C_{DR}\bar{\ell}_1\sigma^{\mu\nu}P_L\ell_2 + C_{DR}\bar{\ell}_1\sigma^{\mu\nu}P_R\ell_2\right)F_{\mu\nu} + h.c.\right]$$

- four-fermion operators

$$\begin{aligned}\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2}\sum_q \Big[& \left(C_{VR}^{q\ell_1\ell_2}\;\bar{\ell}_1\gamma^\mu P_R\ell_2 + C_{VL}^{q\ell_1\ell_2}\;\bar{\ell}_1\gamma^\mu P_L\ell_2\right)\;\bar{q}\gamma_\mu q \\ & + \left(C_{AR}^{q\ell_1\ell_2}\;\bar{\ell}_1\gamma^\mu P_R\ell_2 + C_{AL}^{q\ell_1\ell_2}\;\bar{\ell}_1\gamma^\mu P_L\ell_2\right)\;\bar{q}\gamma_\mu\gamma_5 q \\ & + m_2 m_q G_F\left(C_{SR}^{q\ell_1\ell_2}\;\bar{\ell}_1 P_L\ell_2 + C_{SL}^{q\ell_1\ell_2}\;\bar{\ell}_1 P_R\ell_2\right)\;\bar{q}q \\ & + m_2 m_q G_F\left(C_{PR}^{q\ell_1\ell_2}\;\bar{\ell}_1 P_L\ell_2 + C_{PL}^{q\ell_1\ell_2}\;\bar{\ell}_1 P_R\ell_2\right)\;\bar{q}\gamma_5 q \\ & + m_2 m_q G_F\left(C_{TR}^{q\ell_1\ell_2}\;\bar{\ell}_1\sigma^{\mu\nu}P_L\ell_2 + C_{TL}^{q\ell_1\ell_2}\;\bar{\ell}_1\sigma^{\mu\nu}P_R\ell_2\right)\;\bar{q}\sigma_{\mu\nu}q \;+\; h.c.\;\Big].\end{aligned}$$

- gluonic operators

$$\begin{aligned}\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2}\frac{\beta_L}{4\alpha_s}\Big[& \left(C_{GR}\bar{\ell}_1 P_R\ell_2 + C_{GL}\bar{\ell}_1 P_L\ell_2\right)G^a_{\mu\nu}G^{a\mu\nu} \\ & + \left(C_{\tilde{G}R}\bar{\ell}_1 P_R\ell_2 + C_{\tilde{G}L}\bar{\ell}_1 P_L\ell_2\right)G^a_{\mu\nu}\widetilde{G}^{a\mu\nu} + h.c.\Big]\end{aligned}$$

There are many effective operators, so a single operator dominance hypothesis (SODH) is usually applied to get constraints on relevant Wilson coefficients.

Is it necessary?

Effective Lagrangians: designer states

★ By selecting appropriate quantum numbers of a decaying state we can probe all Wilson coefficients of LFV Lagrangian!

$$
\begin{aligned}
\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_q \Big[& \left(C_{VR}^{q\ell_1\ell_2} \, \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1\ell_2} \, \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \, \bar{q}\gamma_\mu q \\
& + \left(C_{AR}^{q\ell_1\ell_2} \, \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2} \, \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \, \bar{q}\gamma_\mu \gamma_5 q \\
& + m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2} \, \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2} \, \bar{\ell}_1 P_R \ell_2 \right) \, \bar{q} q \\
& + m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2} \, \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2} \, \bar{\ell}_1 P_R \ell_2 \right) \, \bar{q}\gamma_5 q \\
& + m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2} \, \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2} \, \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \, \bar{q}\sigma_{\mu\nu} q \; + \; h.c. \Big].
\end{aligned}
$$

- Can (partially) do away with SODH if designer initial/final states are used
- This can be done in case of restricted kinematics (e.g. 2-body decays)

Effective Lagrangians: designer states

★ By selecting appropriate quantum numbers of a decaying state we can probe all Wilson coefficients of LFV Lagrangian!

$$
\begin{aligned}
\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_q \Big[& \boxed{\left(C_{VR}^{q\ell_1\ell_2} \; \overline{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1\ell_2} \; \overline{\ell}_1 \gamma^\mu P_L \ell_2 \right) \; \overline{q} \gamma_\mu q} \\
& + \left(C_{AR}^{q\ell_1\ell_2} \; \overline{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2} \; \overline{\ell}_1 \gamma^\mu P_L \ell_2 \right) \; \overline{q} \gamma_\mu \gamma_5 q \\
& + m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2} \; \overline{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2} \; \overline{\ell}_1 P_R \ell_2 \right) \; \overline{q} q \\
& + m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2} \; \overline{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2} \; \overline{\ell}_1 P_R \ell_2 \right) \; \overline{q} \gamma_5 q \\
& + \boxed{m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2} \; \overline{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2} \; \overline{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \; \overline{q} \sigma_{\mu\nu} q} + \; h.c. \Big].
\end{aligned}
$$

also dipole operators

Vector meson decays: $\Upsilon(nS) \to \overline{\mu}\tau, \psi(nS) \to \overline{\mu}\tau, \rho \to \overline{\mu}e, \ldots$

D. Hazard and A.A.P., PRD94 (2016), 074023

Effective Lagrangians: designer states

★ By selecting appropriate quantum numbers of a decaying state we can probe all Wilson coefficients of LFV Lagrangian!

$$
\begin{aligned}
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& + \boxed{\left(C_{AR}^{q\ell_1\ell_2}\ \bar{\ell}_1\gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2}\ \bar{\ell}_1\gamma^\mu P_L \ell_2 \right)\ \bar{q}\gamma_\mu\gamma_5 q} \\
& + m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2}\ \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2}\ \bar{\ell}_1 P_R \ell_2 \right)\ \bar{q} q \\
& + \boxed{m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2}\ \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2}\ \bar{\ell}_1 P_R \ell_2 \right)\ \bar{q}\gamma_5 q} \\
& + m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2}\ \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2}\ \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right)\ \bar{q}\sigma_{\mu\nu} q \ + \ h.c. \Big].
\end{aligned}
$$

also gluonic operators

Pseudoscalar meson decays: $\eta_b \to \overline{\mu} e, \eta_c \to \overline{\mu}\tau, \eta^{(\prime)} \to \overline{\mu} e, \ldots$

D. Hazard and A.A.P., PRD94 (2016), 074023

Effective Lagrangians: designer states

★ By selecting appropriate quantum numbers of a decaying state we can probe all Wilson coefficients of LFV Lagrangian!

$$
\begin{aligned}
\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_q \Big[& \left(C_{VR}^{q\ell_1\ell_2}\, \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1\ell_2}\, \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q}\gamma_\mu q \\
& + \left(C_{AR}^{q\ell_1\ell_2}\, \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2}\, \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q}\gamma_\mu \gamma_5 q \\
& + m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_R \ell_2 \right) \bar{q} q \\
& + m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2}\, \bar{\ell}_1 P_R \ell_2 \right) \bar{q}\gamma_5 q \\
& + m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2}\, \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2}\, \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \bar{q}\sigma_{\mu\nu} q \; + \; h.c. \Big].
\end{aligned}
$$

also gluonic operators

Scalar meson decays: $\chi_{b0} \to \overline{\mu}\tau, \;\; \chi_{c0} \to \overline{\mu}\tau, \;\; \ldots$

D. Hazard and A.A.P., PRD94 (2016), 074023

3a. LFV vector quarkonia decays

★ Most LFV experimental data available V → μe, τe, etc.

$\ell_1\ell_2$	$\mu\tau$	$e\tau$	$e\mu$
$\mathcal{B}(\Upsilon(1S) \to \ell_1\ell_2)$	6.0×10^{-6}	$\cdots$	$\cdots$
$\mathcal{B}(\Upsilon(2S) \to \ell_1\ell_2)$	3.3×10^{-6}	3.2×10^{-6}	$\cdots$
$\mathcal{B}(\Upsilon(3S) \to \ell_1\ell_2)$	3.1×10^{-6}	4.2×10^{-6}	$\cdots$
$\mathcal{B}(J/\psi \to \ell_1\ell_2)$	2.0×10^{-6}	8.3×10^{-6}	1.6×10^{-7}
$\mathcal{B}(\phi \to \ell_1\ell_2)$	FPS	FPS	4.1×10^{-6}
$\mathcal{B}(\ell_2 \to \ell_1\gamma)$	4.4×10^{-8}	3.3×10^{-8}	5.7×10^{-13}

★ Decay amplitude:

$$\mathcal{A}(V \to \ell_1\bar{\ell}_2) = \bar{u}(p_1, s_1)\left[A_V^{\ell_1\ell_2}\gamma_\mu + B_V^{\ell_1\ell_2}\gamma_\mu\gamma_5 + \frac{C_V^{\ell_1\ell_2}}{m_V}(p_2-p_1)_\mu + \frac{iD_V^{\ell_1\ell_2}}{m_V}(p_2-p_1)_\mu\gamma_5\right] v(p_2, s_2)\ \epsilon^\mu(p).$$

★ Decay rate:

$$\frac{\mathcal{B}(V \to \ell_1\overline{\ell}_2)}{\mathcal{B}(V \to e^+e^-)} = \left(\frac{m_V(1-y^2)}{4\pi\alpha f_V Q_q}\right)^2 \left[(|A_V^{\ell_1\ell_2}|^2 + |B_V^{\ell_1\ell_2}|^2) + \frac{1}{2}(1-2y^2)(|C_V^{\ell_1\ell_2}|^2 + |D_V^{\ell_1\ell_2}|^2) + y\mathrm{Re}(A_V^{\ell_1\ell_2}C_V^{\ell_1\ell_2*} + iB_V^{\ell_1\ell_2}D_V^{\ell_1\ell_2*})\right].$$

Form-factors depend on vector, tensor, and dipole Wilson coefficients

LFV vector quarkonia decays

★ Most general decay rate for V → μe, τe, etc. ($V = \Upsilon(nS), \psi(nS), \rho, \phi, \ldots$):

$$\frac{\mathcal{B}(V\to \ell_1\bar{\ell}_2)}{\mathcal{B}(V\to e^+e^-)} = \left(\frac{m_V(1-y^2)}{4\pi\alpha f_V Q_q}\right)^2 [(|A_V^{\ell_1\ell_2}|^2+|B_V^{\ell_1\ell_2}|^2) + \frac{1}{2}(1-2y^2)(|C_V^{\ell_1\ell_2}|^2+|D_V^{\ell_1\ell_2}|^2) + y\mathrm{Re}(A_V^{\ell_1\ell_2}C_V^{\ell_1\ell_2*}+iB_V^{\ell_1\ell_2}D_V^{\ell_1\ell_2*})].$$

D. Hazard and A.A.P., PRD94 (2016), 074023

… and the decay rate is

$$A_V^{\ell_1\ell_2} = \frac{f_V m_V}{\Lambda^2}[\sqrt{4\pi\alpha}Q_q y^2(C_{DL}^{\ell_1\ell_2}+C_{DR}^{\ell_1\ell_2}) + \kappa_V(C_{VL}^{q\ell_1\ell_2}+C_{VR}^{q\ell_1\ell_2}) + 2y^2\kappa_V\frac{f_V^T}{f_V}G_F m_V m_q(C_{TL}^{q\ell_1\ell_2}+C_{TR}^{q\ell_1\ell_2})],$$

$$B_V^{\ell_1\ell_2} = \frac{f_V m_V}{\Lambda^2}[-\sqrt{4\pi\alpha}Q_q y^2(C_{DL}^{\ell_1\ell_2}-C_{DR}^{\ell_1\ell_2}) - \kappa_V(C_{VL}^{q\ell_1\ell_2}-C_{VR}^{q\ell_1\ell_2}) - 2y^2\kappa_V\frac{f_V^T}{f_V}G_F m_V m_q(C_{TL}^{q\ell_1\ell_2}-C_{TR}^{q\ell_1\ell_2})],$$

$$C_V^{\ell_1\ell_2} = \frac{f_V m_V}{\Lambda^2}y[\sqrt{4\pi\alpha}Q_q(C_{DL}^{\ell_1\ell_2}+C_{DR}^{\ell_1\ell_2}) + 2\kappa_V\frac{f_V^T}{f_V}G_F m_V m_q(C_{TL}^{q\ell_1\ell_2}+C_{TR}^{q\ell_1\ell_2})],$$

$$D_V^{\ell_1\ell_2} = i\frac{f_V m_V}{\Lambda^2}y[-\sqrt{4\pi\alpha}Q_q(C_{DL}^{\ell_1\ell_2}-C_{DR}^{\ell_1\ell_2}) - 2\kappa_V\frac{f_V^T}{f_V}G_F m_V m_q(C_{TL}^{q\ell_1\ell_2}-C_{TR}^{q\ell_1\ell_2})].$$

LFV vector quarkonia decays: dipoles

★ Constraints on Wilson coefficients of dipole low energy operators

Dipole Wilson coefficient [GeV^{-2}]	Leptons $\ell_1\ell_2$	Initial state $\Upsilon(1S)(b)$	$\Upsilon(2S)(b)$	$\Upsilon(3S)(b)$	$J/\psi(c)$	$\phi(s)$	$\ell_2 \to \ell_1\gamma$
$\lvert C_{DL}^{\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	2.0×10^{-4}	1.6×10^{-4}	1.4×10^{-4}	2.5×10^{-4}	FPS	2.6×10^{-10}
	$e\tau$	$\cdots$	1.6×10^{-4}	1.6×10^{-4}	5.3×10^{-4}	FPS	2.7×10^{-10}
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	1.1×10^{-3}	0.2	3.1×10^{-7}
$\lvert C_{DR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	2.0×10^{-4}	1.6×10^{-4}	1.4×10^{-4}	2.5×10^{-4}	FPS	2.6×10^{-10}
	$e\tau$	$\cdots$	1.6×10^{-4}	1.6×10^{-4}	5.3×10^{-4}	FPS	2.7×10^{-10}
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	1.1×10^{-3}	0.2	3.1×10^{-7}

D. Hazard and A.A.P.,
PRD94 (2016), 074023

★ Much tighter constraints are obtained from lepton radiative decays: drop from quarkonium decay analyses in what follows

LFV vector quarkonia decays: 4f operators

★ Constraints on Wilson coefficients of four-fermion low energy operators

Wilson coefficient [GeV^{-2}]	Leptons $\ell_1\ell_2$	Initial state (quark) $\Upsilon(1S)(b)$	$\Upsilon(2S)(b)$	$\Upsilon(3S)(b)$	$J/\psi(c)$	$\phi(s)$
$\lvert C_{VL}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	FPS
	$e\tau$	$\cdots$	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	FPS
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	1.0×10^{-5}	2×10^{-3}
$\lvert C_{VR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	FPS
	$e\tau$	$\cdots$	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	FPS
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	1.0×10^{-5}	2×10^{-3}
$\lvert C_{TL}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	FPS
	$e\tau$	$\cdots$	3.3×10^{-2}	3.2×10^{-2}	2.4	FPS
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	4.8	1×10^{4}
$\lvert C_{TR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	FPS
	$e\tau$	$\cdots$	3.3×10^{-2}	3.2×10^{-2}	2.4	FPS
	$e\mu$	$\cdots$	$\cdots$	$\cdots$	4.8	1×10^{4}

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LFV pseudoscalar/scalar quarkonia decays

★ Very scarce LFV experimental data available P/S → μe, τe, etc.

- no data for pseudoscalar heavy-flavored meson decays
- no data for any scalar meson decays

$\ell_1\ell_2$	$e\mu$
$\mathcal{B}(\eta \to \ell_1\ell_2)$	6×10^{-6}
$\mathcal{B}(\eta' \to \ell_1\ell_2)$	4.7×10^{-4}
$\mathcal{B}(\pi^0 \to \ell_1\ell_2)$	3.6×10^{-10}

$$P = \eta_b, \eta_c, \eta^{(\prime)}, \ldots$$
$$S = \chi_{b0}, \chi_{c0}, \ldots$$

★ Decay amplitudes:

$$\mathcal{A}(P \to \ell_1 \overline{\ell}_2) = \overline{u}(p_1, s_1)[E_P^{\ell_1\ell_2} + iF_P^{\ell_1\ell_2}\gamma_5]v(p_2, s_2)$$

$$\mathcal{A}(S \to \ell_1 \overline{\ell}_2) = \overline{u}(p_1, s_1)[E_S^{\ell_1\ell_2} + iF_S^{\ell_1\ell_2}\gamma_5]v(p_2, s_2)$$

★ Decay rates:

$$\mathcal{B}(P \to \ell_1 \overline{\ell}_2) = \frac{m_P}{8\pi\Gamma_P}(1-y^2)^2[|E_P^{\ell_1\ell_2}|^2 + |F_P^{\ell_1\ell_2}|^2].$$

$$\mathcal{B}(S \to \ell_1 \overline{\ell}_2) = \frac{m_S}{8\pi\Gamma_S}(1-y^2)^2[|E_S^{\ell_1\ell_2}|^2 + |F_S^{\ell_1\ell_2}|^2]$$

Form-factors depend on axial, pseudoscalar, and gluonic operator Wilson coefficients (P)
scalar and gluonic operator Wilson coefficients (S)

LFV pseudoscalar quarkonia decays

★ Most general decay rate for P → μe, τe, etc. ($P = \eta_b, \eta_c, \eta^{(\prime)}, \dots$):

$$\mathcal{B}(P \to \ell_1 \overline{\ell}_2) = \frac{m_P}{8\pi\Gamma_P}(1-y^2)^2[|E_P^{\ell_1\ell_2}|^2 + |F_P^{\ell_1\ell_2}|^2].$$

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… and the decay rate is

$$\begin{aligned} E_P^{\ell_1\ell_2} = {} & y\frac{m_P}{4\Lambda^2}[-if_P[2(C_{AL}^{q\ell_1\ell_2} + C_{AR}^{q\ell_1\ell_2}) \\ & - m_P^2 G_F(C_{PL}^{q\ell_1\ell_2} + C_{PR}^{q\ell_1\ell_2})] + 9G_F a_P(C_{\tilde{G}L}^{\ell_1\ell_2} + C_{\tilde{G}R}^{\ell_1\ell_2})], \\ F_P^{\ell_1\ell_2} = {} & -y\frac{m_P}{4\Lambda^2}[f_P[2(C_{AL}^{q\ell_1\ell_2} - C_{AR}^{q\ell_1\ell_2}) \\ & - m_P^2 G_F(C_{PL}^{q\ell_1\ell_2} - C_{PR}^{q\ell_1\ell_2})] + 9iG_F a_P(C_{\tilde{G}L}^{\ell_1\ell_2} - C_{\tilde{G}R}^{\ell_1\ell_2})]. \end{aligned}$$

Gluonic operators?

LFV scalar quarkonia decays

★ Most general decay rate for S → μe, τe, etc. ($S = \chi_{b0}, \chi_{c0}, \ldots$):

$$\mathcal{B}(S \to \ell_1 \overline{\ell}_2) = \frac{m_S}{8\pi\Gamma_S}(1-y^2)^2\left[|E_S^{\ell_1\ell_2}|^2 + |F_S^{\ell_1\ell_2}|^2\right]$$

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… and the decay rate is

$$E_S^{\ell_1\ell_2} = y\frac{m_S G_F}{4\Lambda^2}\left[2if_S m_S m_q (C_{SL}^{ql_1l_2} + C_{SR}^{ql_1l_2}) + 9a_S(C_{GL}^{ql_1l_2} + C_{GR}^{ql_1l_2})\right],$$

$$F_S^{\ell_1\ell_2} = y\frac{m_S G_F}{4\Lambda^2}\left[2f_S m_S m_q (C_{SL}^{ql_1l_2} - C_{SR}^{ql_1l_2}) - 9ia_S(C_{GL}^{ql_1l_2} - C_{GR}^{ql_1l_2})\right].$$

Gluonic operators?

LFV pseudoscalar/scalar quarkonia decays

★ Constraints on Wilson coefficients of low energy operators

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Wilson coefficient	Leptons $\ell_1\ell_2$	Initial state η_b	η_c	$\eta(u/d)$	$\eta(s)$	$\eta'(u/d)$	$\eta'(s)$
$\lvert C_{AL}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\mu$	$\cdots$	$\cdots$	3×10^{-3}	2×10^{-3}	2.1×10^{-1}	1.9×10^{-1}
$\lvert C_{AR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\mu$	$\cdots$	$\cdots$	3×10^{-3}	2×10^{-3}	2.1×10^{-1}	1.9×10^{-1}
$\lvert C_{PL}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\mu$	$\cdots$	$\cdots$	2×10^{3}	1×10^{3}	3.9×10^{4}	3.6×10^{4}
$\lvert C_{PR}^{q\ell_1\ell_2}/\Lambda^2\rvert$	$\mu\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\tau$	$\cdots$	$\cdots$	FPS	FPS	FPS	FPS
	$e\mu$	$\cdots$	$\cdots$	2×10^{3}	1×10^{3}	3.9×10^{4}	3.6×10^{4}

★ More data is needed: use radiative decays: $\boxed{\mathcal{B}(V\to\gamma\ell_1\overline{\ell}_2)=\mathcal{B}(V\to\gamma M)\mathcal{B}(M\to\ell_1\overline{\ell}_2)}$

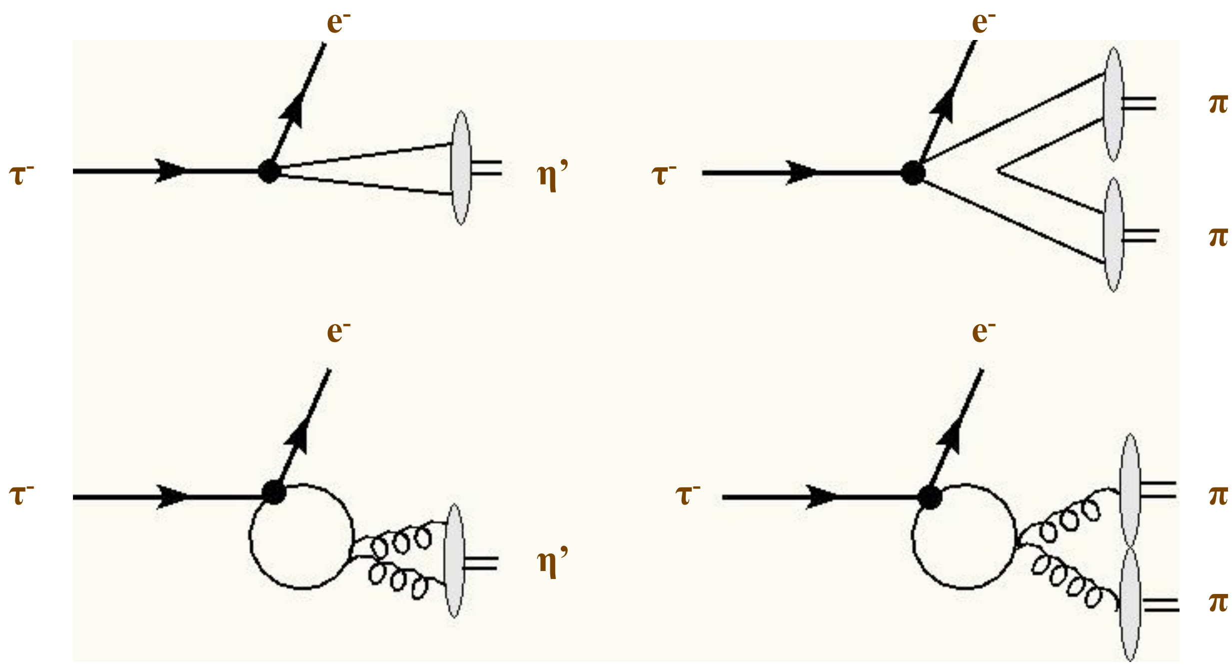
$$\mathcal{B}(\psi(2S)\to\gamma\chi_{c0}(1P))=9.99\pm0.27\%,$$
$$\mathcal{B}(\psi(3770)\to\gamma\chi_{c0}(1P))=0.73\pm0.09\%.$$
$$\mathcal{B}(J/\psi\to\gamma\eta_c)=1.7\pm0.4\%,$$
$$\mathcal{B}(\psi(2S)\to\gamma\eta_c)=0.34\pm0.05\%.$$

$$\mathcal{B}(\Upsilon(2S)\to\gamma\chi_{b0}(1P))=3.8\pm0.4\%,$$
$$\mathcal{B}(\Upsilon(3S)\to\gamma\chi_{b0}(1P))=0.27\pm0.04\%,$$
$$\mathcal{B}(\Upsilon(3S)\to\gamma\chi_{b0}(2P))=5.9\pm0.6\%.$$

3b. Probing LFV gluonic operators in tau decays

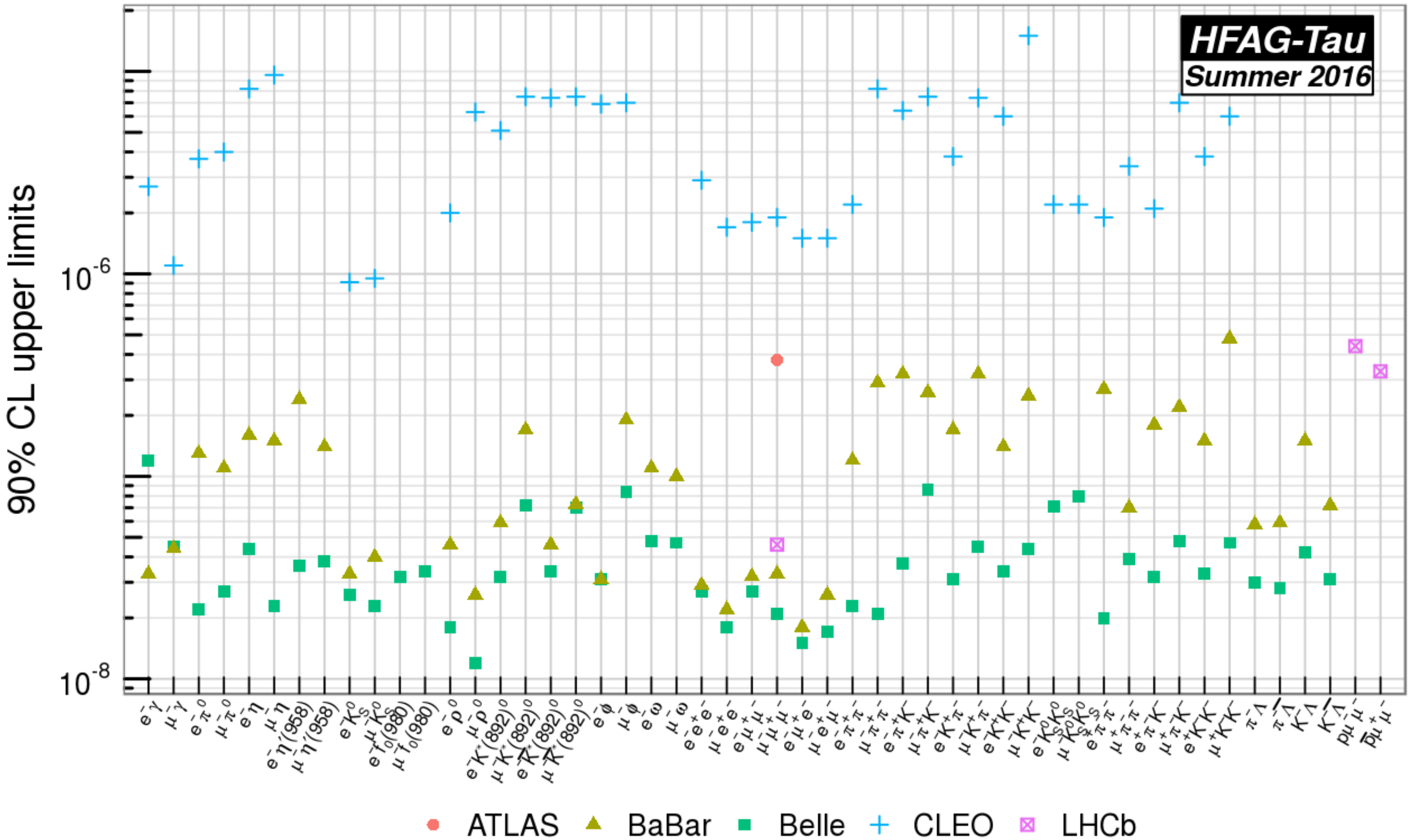
★ Let's compute FCNC tau decays (concentrate on those sensitive to gluonic operators)

AAP and D. Zhuridov
PRD89 (2014) 3, 033005



Partity-violating operators

Parity-conserving operators



What can we learn about LFV gluons operators from this data?

Hadronic physics I

★ To calculate FV tau decays we need a bit of hadronic physics

- matrix elements of parity-conserving operators

$$\langle \pi^+\pi^- | \bar{q}q | 0 \rangle = \langle K^+K^- | \bar{q}q | 0 \rangle = \delta_q^M B_0$$

$$\langle M^+M^- | \bar{q}\gamma_\mu q | 0 \rangle = \delta_q^M G_M^{(q)}(Q^2)\,(p_+ - p_-)_\mu$$

$$\langle M^+M^- | \frac{\alpha_s}{4\pi} G^{a\mu\nu} G^a_{\mu\nu} | 0 \rangle = -\frac{2}{9} q^2,$$

- ... where B_0=1.96 GeV from $m_\pi^2 = (m_u + m_d)\,B_0$

Black, Han, He, Sher

- ... and we used $\theta^\mu_\mu = -\frac{b\alpha_s}{8\pi} G^{\mu\nu a} G^a_{\mu\nu} + \sum_{q=u,d,s} m_q \bar{q} q$

Voloshin

★ Can do better on hadronic side by using data

Celis, Cirigliano, Passemar

Hadronic physics II

★ To calculate FV tau decays we need a bit of hadronic physics

- matrix elements of parity-violating operators

$$\langle M(p)|\bar{q}\gamma^\mu\gamma_5 q|0\rangle = -ib_q f_M^q p^\mu,$$
$$\langle M(p)|\bar{q}\gamma_5 q|0\rangle = -ib_q h_M^q,$$
$$\langle M(p)|\frac{\alpha_s}{4\pi} G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}|0\rangle = a_M,$$

- ... where q=u,d,s and $b_{u,d}=1/2^{1/2}$, while $b_s=1$

- ... and in the FKS scheme of eta-eta' mixing

$$a_\eta = -\frac{m_{\eta'}^2 - m_\eta^2}{2}\sin 2\phi\left(-f_q b_q \sin\phi + f_s\cos\phi\right),$$
$$a_{\eta'} = -\frac{m_{\eta'}^2 - m_\eta^2}{2}\sin 2\phi\left(f_q b_q \sin\phi + f_s\cos\phi\right),$$

Bounds: parity conserving

★ Looking at the scalar operators only

$$\frac{d\Gamma(\tau \to \ell M^+ M^-)}{dq^2} = \frac{m_\tau}{32(2\pi)^3\Lambda^4}\left[|A_{MM}|^2 + |B_{MM}|^2\right] \times \sqrt{1 - \frac{4m_M^2}{q^2}}\left(1 - \frac{q^2}{m_\tau^2}\right)^2,$$

– ... with the following coefficients

$$A_{MM} = -\frac{2c_1^{\ell\tau}}{9}q^2 + \frac{1}{2}\sum_{q=u,d,s}\left(C_1^{q\ell\tau} + C_2^{q\ell\tau}\right)\delta_q^M B_0,$$
$$B_{MM} = -\frac{2c_3^{\ell\tau}}{9}q^2 + \frac{1}{2}\sum_{q=u,d,s}\left(C_3^{q\ell\tau} + C_4^{q\ell\tau}\right)\delta_q^M B_0.$$

Coef	Bound on $\lvert c_i^{\ell\tau}\rvert/\Lambda^2$, GeV^{-3}							
	$\mathcal{B}(\tau \to \mu\ \pi^+\pi^-)$ $< 2.1 \times 10^{-8}$	$\mathcal{B}(\tau \to e\ \pi^+\pi^-)$ $< 2.3 \times 10^{-8}$	$\mathcal{B}(\tau \to \mu K^+K^-)$ $< 4.4 \times 10^{-8}$	$\mathcal{B}(\tau \to eK^+K^-)$ $< 3.3 \times 10^{-8}$	$\mathcal{B}(\tau \to \mu\eta')$ $< 1.3 \times 10^{-7}$	$\mathcal{B}(\tau \to e\eta')$ $< 1.6 \times 10^{-7}$	$\mathcal{B}(\tau \to \mu\eta)$ $< 1.3 \times 10^{-7}$	$\mathcal{B}(\tau \to e\eta)$ $< 1.6 \times 10^{-7}$
c_1	6.8×10^{-8}	6.5×10^{-8}	9.4×10^{-8}	8.2×10^{-8}	–	–	–	–
c_2	–	–	–	–	2.3×10^{-7}	2.5×10^{-7}	1.6×10^{-7}	1.5×10^{-7}
c_3	6.8×10^{-8}	6.5×10^{-8}	9.4×10^{-8}	8.2×10^{-8}	–	–	–	–
c_4	–	–	–	–	2.3×10^{-7}	2.5×10^{-7}	1.6×10^{-7}	1.5×10^{-7}

Bounds: parity violating

★ Again, looking at the scalar operators only

$$\Gamma(\tau \to \mu M) = \frac{m_\tau}{8\pi\Lambda^4}\left[|A_M|^2 + |B_M|^2\right]\left(1 - \frac{m_M^2}{m_\tau^2}\right)^2$$

– ... with the following coefficients

$$A_M = -\frac{2i}{9}c_2^{\ell\tau}a_M + \sum_{q=u,d,s}\left(C_2^{q\ell\tau} - C_1^{q\ell\tau}\right)\frac{b_q h_M^q}{4m_q} + \frac{1}{2}m_\mu \sum_{q=u,d,s}\left(C_5^{q\ell\tau} - C_6^{q\ell\tau}\right) b_q f_M^q - \frac{1}{2}m_\tau \sum_{q=u,d,s}\left(C_7^{q\ell\tau} - C_8^{q\ell\tau}\right) b_q f_M^q$$

$$B_M = -\frac{2i}{9}c_4^{\ell\tau}a_M + \sum_{q=u,d,s}\left(C_4^{q\ell\tau} - C_3^{q\ell\tau}\right)\frac{b_q h_M^q}{4m_q} - \frac{1}{2}m_\tau \sum_{q=u,d,s}\left(C_5^{q\ell\tau} - C_6^{q\ell\tau}\right) b_q f_M^q + \frac{1}{2}m_\mu \sum_{q=u,d,s}\left(C_7^{q\ell\tau} - C_8^{q\ell\tau}\right) b_q f_M^q$$

Coef	Bound on $\lvert c_i^{\ell\tau}\rvert/\Lambda^2$, GeV^{-3}							
	$\mathcal{B}(\tau\to\mu\,\pi^+\pi^-)$ $<2.1\times10^{-8}$	$\mathcal{B}(\tau\to e\,\pi^+\pi^-)$ $<2.3\times10^{-8}$	$\mathcal{B}(\tau\to\mu K^+K^-)$ $<4.4\times10^{-8}$	$\mathcal{B}(\tau\to eK^+K^-)$ $<3.3\times10^{-8}$	$\mathcal{B}(\tau\to\mu\eta')$ $<1.3\times10^{-7}$	$\mathcal{B}(\tau\to e\eta')$ $<1.6\times10^{-7}$	$\mathcal{B}(\tau\to\mu\eta)$ $<1.3\times10^{-7}$	$\mathcal{B}(\tau\to e\eta)$ $<1.6\times10^{-7}$
c_1	6.8×10^{-8}	6.5×10^{-8}	9.4×10^{-8}	8.2×10^{-8}	–	–	–	–
c_2	–	–	–	–	2.3×10^{-7}	2.5×10^{-7}	1.6×10^{-7}	1.5×10^{-7}
c_3	6.8×10^{-8}	6.5×10^{-8}	9.4×10^{-8}	8.2×10^{-8}	–	–	–	–
c_4	–	–	–	–	2.3×10^{-7}	2.5×10^{-7}	1.6×10^{-7}	1.5×10^{-7}

3c. Probing LFV gluonic operators with muon conversion

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

★ Nuclear averages are often done as an approximation. For a general operator Q

$$\langle N|Q|N\rangle = \int d^3r \left[Z \rho_p(r)\langle p|Q|p\rangle + (A-Z)\,\rho_n(r)\langle n|Q|n\rangle\right]$$

p(n) densities

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}, \qquad \int d^3\rho_{p(n)}(r) = 1$$

★ Matrix elements of light quark currents are easily computed

- since $(m_\mu - m_e) \ll m_N$ we can neglect space components of the quark current

$$\langle p|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|p\rangle = 2 + c_d$$
$$\langle n|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|n\rangle = 1 + 2c_d$$

count number of quarks

Probing LFV gluonic operators with muon conversion

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Lepton wave functions are taken as solutions of Dirac equation

- with usual substitutions $u_1(r) = r\, g(r)$ and $u_2(r) = r\, f(r)$

$$\frac{d}{dr}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_i \\ -(W - V - m_i) & \kappa/r \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

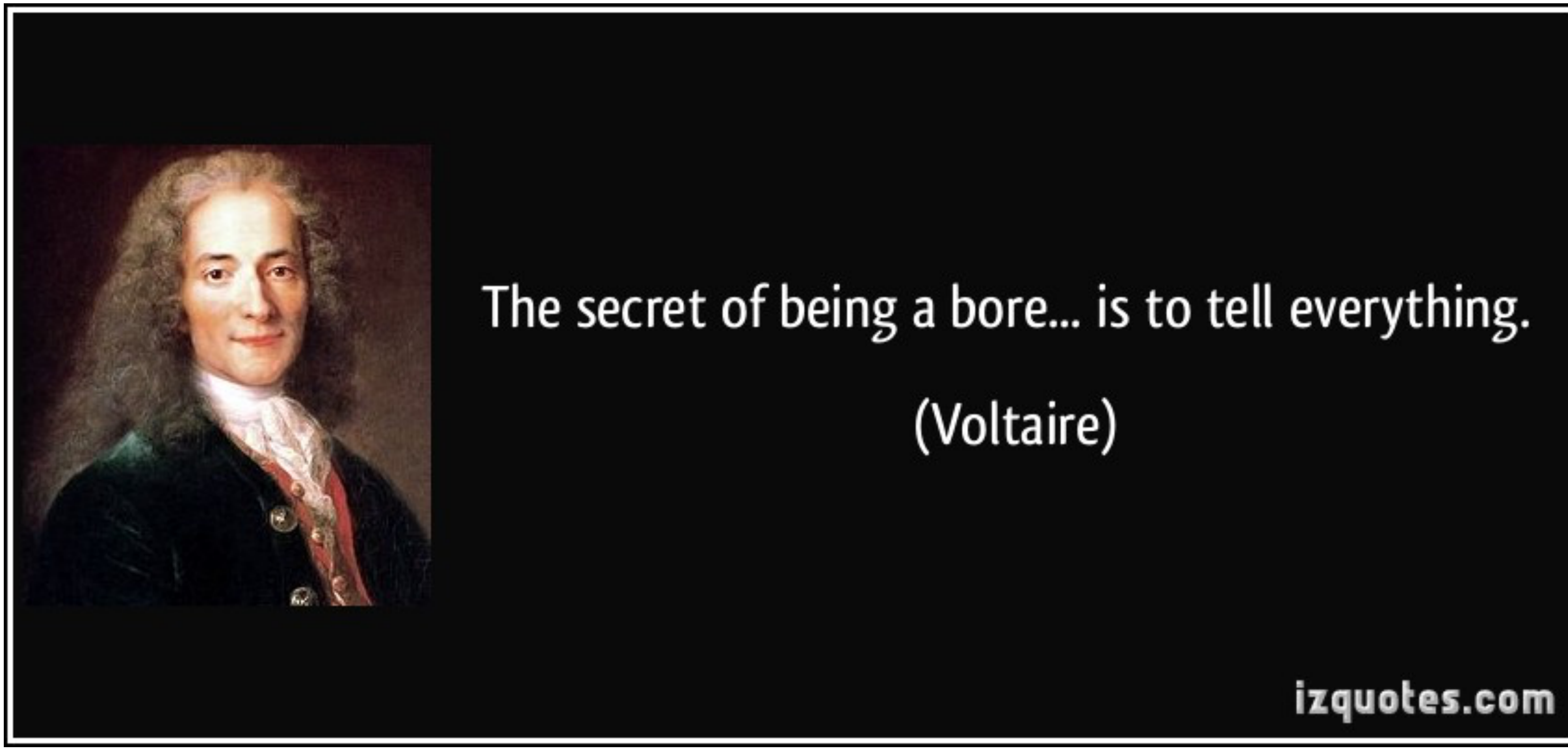
$$\psi = \psi^{\mu}_{\kappa} = \begin{pmatrix} g(r)\chi^{\mu}_{\kappa}(\theta,\phi) \\ i f(r)\chi^{\mu}_{-\kappa}(\theta,\phi) \end{pmatrix}$$

★ ... with Dirac equation in a potential $V(r) = -e\int_r^\infty E(r')dr'$

$$E(r) = \frac{Ze}{r^2}\int_0^r r'^2 \rho^{(p)}(r')dr'$$

$$\text{SINDRUM II (PSI), 2006 :} \qquad R_{\mu e} < 7\times 10^{-13}$$

$$\text{M2e goal :} \qquad R_{\mu e} < \text{a few} \times 10^{-17}$$



Le secret d'ennuyer est celui de tout dire

Probing gluonic operators with muons

★ Let's calculate the conversion amplitude

$$M^{\mu e}_{NN'} = \frac{1}{\Lambda^2 M_Q} \int d^3x \quad \left[\left(c_1 \bar{\psi}^{\mu(e)}_{\kappa,W} P_L \psi^{(\mu)}_{1s} + c_3 \bar{\psi}^{\mu(e)}_{\kappa,W} P_R \psi^{(\mu)}_{1s} \right) \langle N' | \frac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu} | N \rangle \right.$$
$$+ \quad \left. \left(c_2 \bar{\psi}^{\mu(e)}_{\kappa,W} P_L \psi^{(\mu)}_{1s} + c_4 \bar{\psi}^{\mu(e)}_{\kappa,W} P_R \psi^{(\mu)}_{1s} \right) \langle N' | \frac{\alpha_s}{4\pi} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} | N \rangle \right]$$

★ Relate nucleon and nuclear matrix elements...

$$\left\langle N \left| \frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu} \right| N \right\rangle = -\frac{9}{2} \left[Z G^{(g,p)} \rho^{(p)} + (A-Z) G^{(g,n)} \rho^{(n)} \right]$$

★ ... and calculate (relevant) parity-conserving nucleon matrix element

$$G^{(g,\mathcal{N})} = \left\langle \mathcal{N} \left| \frac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu} \right| \mathcal{N} \right\rangle = \text{-189 MeV}$$

Numerical estimates

★ Conversion probability (factoring out lightest heavy quark mass)

$$\Gamma_{\text{conv}}(\mu N \to eN) = \frac{4}{\Lambda^4}\left(|c_1|^2 + |c_3|^2\right)a_N^2$$

★ ... where we defined $a_N = G^{(g,p)}S^{(p)} + G^{(g,n)}S^{(n)}$

★ ... and also

$$S^{(p)} = \frac{1}{2\sqrt{2}}\int_0^\infty dr r^2 Z \rho^{(p)}(g_e^- g_\mu^- - f_e^- f_\mu^-),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}}\int_0^\infty dr r^2 (A-Z) \rho^{(n)}(g_e^- g_\mu^- - f_e^- f_\mu^-)$$

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}$$

Nucleus	Model	c, fm	z, fm	$S^{(p)}$	$S^{(n)}$
$^{48}_{22}$Ti	FB	–	–	0.0368	0.0435
$^{197}_{79}$Au	2pF	6.38	0.535	0.0614	0.0918

TABLE I. Nucleon densities model parameters and the overlap integrals in the unit of $m_\mu^{5/2}$ for several nuclei.

Numerical estimates

★ Conversion probability

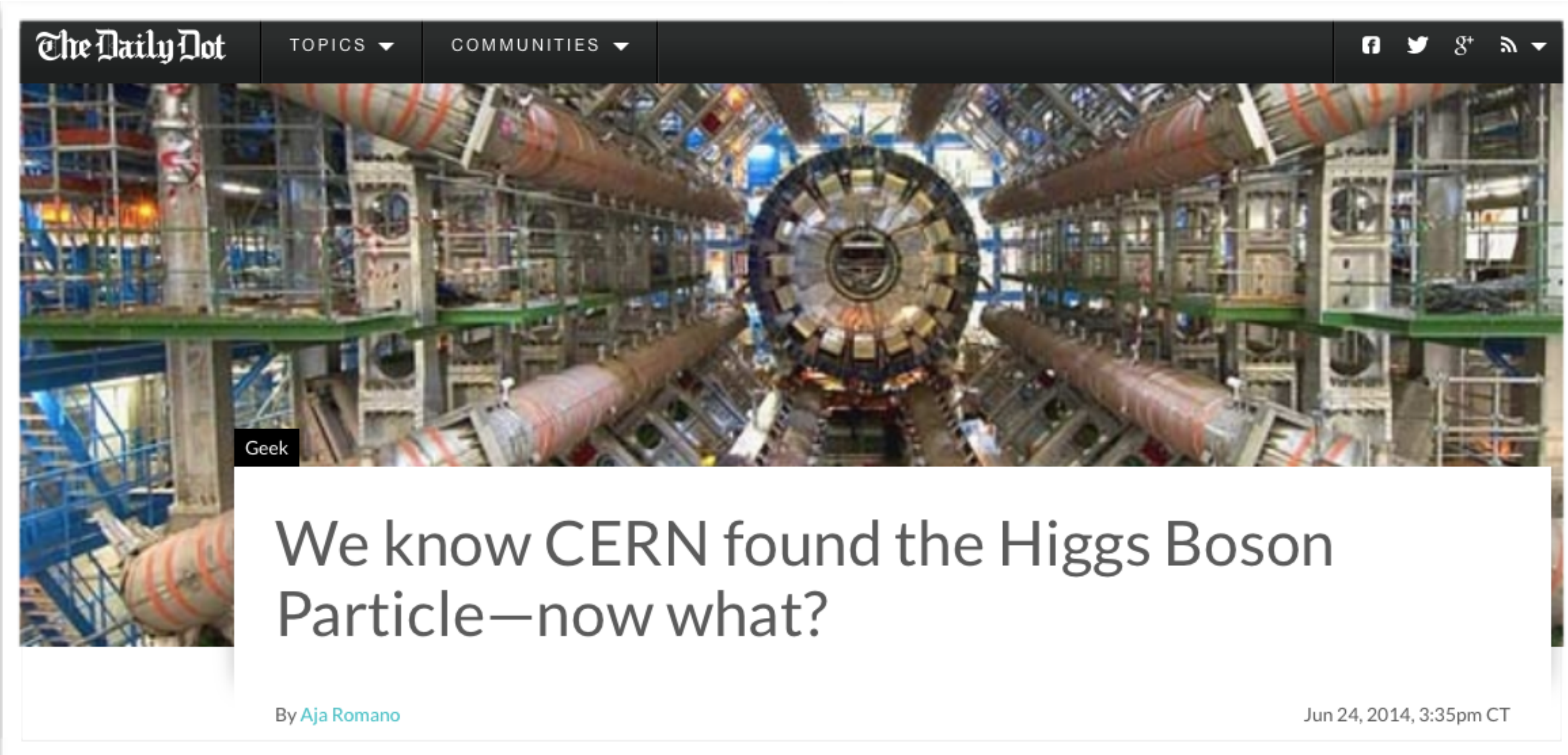
$$B^N_{\mu e} \equiv \Gamma_{\rm conv}(\mu^- N \to e^- N_{\rm g.s.})/\Gamma_{\rm capture}(\mu^- N)$$

★ ... results in constraints on scale/Wilson coefficient

Coefficient	Bound on $\lvert c_i^{e\mu}\rvert/\Lambda^2$, GeV^{-3}	
	Conversion on ${}^{48}_{22}$Ti	Conversion on ${}^{197}_{79}$Au
c_1	5.7×10^{-12}	2.6×10^{-12}
c_2	N/A	N/A
c_3	5.7×10^{-12}	2.6×10^{-12}
c_4	N/A	N/A

★ Important: can only probe parity-conserving operators!!!

Now what?


The Daily Dot
TOPICS
COMMUNITIES
Geek
We know CERN found the Higgs Boson Particle—now what?
By Aja Romano
Jun 24, 2014, 3:35pm CT

4. Matching to high scale physics

★ Consider example: leptoquark interactions

- scalar leptoquarks

$$\mathcal{L}_S = \left(\lambda_{LS_0}\bar{q}_L^c i\tau_2 \ell_L + \lambda_{RS_0}\bar{u}_R^c e_R\right) S_0^\dagger + \left(\lambda_{LS_{1/2}}\bar{u}_R \ell_L + \lambda_{RS_{1/2}}\bar{q}_L i\tau_2 e_R\right) S_{1/2}^\dagger + \text{H.c.},$$

- vector leptoquarks

$$\mathcal{L}_V = \left(\lambda_{LV_0}\bar{q}_L \gamma_\mu \ell_L + \lambda_{RV_0}\bar{d}_R \gamma_\mu e_R\right) V_0^{\mu\dagger} + \left(\lambda_{LV_{1/2}}\bar{d}_R^c \gamma_\mu \ell_L + \lambda_{RV_{1/2}}\bar{q}_L^c \gamma_\mu e_R\right) V_{1/2}^{\mu\dagger} + \text{H.c.},$$

Davidson, Bailey, Campbell

★ Matching to the general result above, get

C_i^u/Λ^2	Expression	C_i^d/Λ^2	Expression
$\frac{C_1^u}{\Lambda^2}$	$\frac{\lambda_{RS_{1/2}}^{\ell_1 u}\lambda_{LS_{1/2}}^{\ell_2 u}}{2M_{S_{1/2}}^2}$	$\frac{C_1^d}{\Lambda^2}$	$\frac{\lambda_{LV_{1/2}}^{\ell_2 b}\lambda_{RV_{1/2}}^{\ell_1 b}}{M_{V_{1/2}}^2}$
$\frac{C_2^u}{\Lambda^2}$	$\frac{\lambda_{RS_0}^{\ell_1 u}\lambda_{LS_0}^{\ell_2 u}}{2M_{S_0}^2}$	$\frac{C_2^d}{\Lambda^2}$	$\frac{\lambda_{LV_0}^{\ell_2 b}\lambda_{RV_0}^{\ell_1 b}}{M_{V_0}^2}$
$\frac{C_3^u}{\Lambda^2}$	$\frac{\lambda_{RS_0}^{\ell_2 u}\lambda_{LS_0}^{\ell_1 u}}{2M_{S_0}^2}$	$\frac{C_3^d}{\Lambda^2}$	$\frac{\lambda_{LV_0}^{\ell_1 b}\lambda_{RV_0}^{\ell_2 b}}{M_{V_0}^2}$
$\frac{C_4^u}{\Lambda^2}$	$\frac{\lambda_{RS_{1/2}}^{\ell_2 u}\lambda_{LS_{1/2}}^{\ell_1 u}}{2M_{S_{1/2}}^2}$	$\frac{C_4^d}{\Lambda^2}$	$\frac{\lambda_{LV_{1/2}}^{\ell_1 b}\lambda_{RV_{1/2}}^{\ell_2 b}}{M_{V_{1/2}}^2}$

Leptoquarks as an example

★ Leptoquark interaction parameters for tau-mu transitions

$$\frac{|\lambda^{\mu t}_{RS_0}\lambda^{\tau t}_{LS_0}|}{M^2_{S_0}} = \frac{|\lambda^{\mu t}_{RS_{1/2}}\lambda^{\tau t}_{LS_{1/2}}|}{M^2_{S_{1/2}}} < 2.3 \times 10^{-4}\ \mathrm{GeV}^{-2},$$

$$\frac{|\lambda^{\tau b}_{LV_0}\lambda^{\mu b}_{RV_0}|}{M^2_{V_0}} = \frac{|\lambda^{\tau b}_{LV_{1/2}}\lambda^{\mu b}_{RV_{1/2}}|}{M^2_{V_{1/2}}} < 4.4 \times 10^{-6}\ \mathrm{GeV}^{-2}$$

★ ... and the same for tau-e

$$\frac{|\lambda^{et}_{RS_0}\lambda^{\tau t}_{LS_0}|}{M^2_{S_0}} = \frac{|\lambda^{et}_{RS_{1/2}}\lambda^{\tau t}_{LS_{1/2}}|}{M^2_{S_{1/2}}} < 2.2 \times 10^{-4}\ \mathrm{GeV}^{-2}$$

$$\frac{|\lambda^{\tau b}_{LV_0}\lambda^{eb}_{RV_0}|}{M^2_{V_0}} = \frac{|\lambda^{\tau b}_{LV_{1/2}}\lambda^{eb}_{RV_{1/2}}|}{M^2_{V_{1/2}}} < 4.2 \times 10^{-6}\ \mathrm{GeV}^{-2}$$

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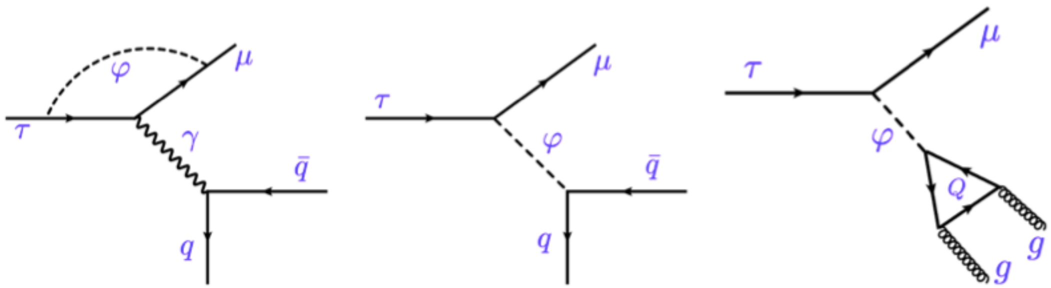
FCNC Higgs as an example

Celis, Cirigliano, Passemar

★ FCNC Higgs gives another example



★ FCNC Higgs gives another example

Process	$(\mathrm{BR} \times 10^8)$ 90% C.L.	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	<4.4 [86]	<0.016	Dipole
$\tau \to \mu\mu\mu$	<2.1 [87]	<0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	<2.1 [88]	<0.13	Scalar, gluon, dipole
$\tau \to \mu\rho$	<1.2 [89]	<0.13	Scalar, gluon, dipole
$\tau \to \mu\pi^0\pi^0$	$<1.4 \times 10^3$ [90]	<6.3	Scalar, gluon

Process	$(\mathrm{BR} \times 10^8)$ 90% CL	$\sqrt{\lvert Y^h_{e\tau}\rvert^2 + \lvert Y^h_{\tau e}\rvert^2}$	Operator(s)
$\tau \to e\gamma$	<3.3 [86]	<0.014	Dipole
$\tau \to eee$	<2.7 [87]	<0.12	Dipole
$\tau \to e\pi^+\pi^-$	<2.3 [88]	<0.14	Scalar, gluon, dipole
$\tau \to e\rho$	<1.8 [89]	<0.16	Scalar, gluon, dipole
$\tau \to e\pi^0\pi^0$	$<6.5 \times 10^2$ [90]	<4.3	Scalar, gluon

Celis, Cirigliano, Passemar

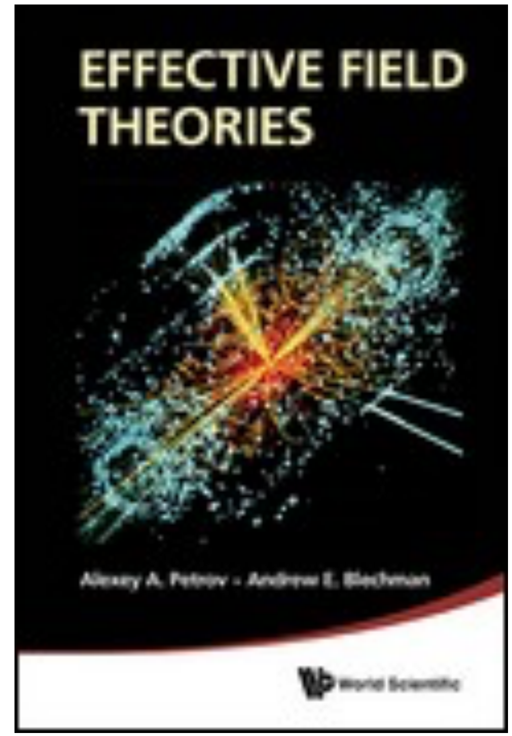
5. Conclusions

- Flavor-changing neutral current transitions provide great opportunities for studies of lepton flavor in the SM and BSM
  - charge lepton transitions offer practically SM-background-free playground
  - large contributions from New Physics are possible, but not seen
  - EFT approach can be useful in studies of quarkonium/tau FCNC decays
  - ... as current methods rarely go beyond dim-6 operators
  - ... and thus do not constrain NP-heavy fermion couplings very well
- Need more data from Belle-II (or LHCb) on LFV quarkonia and tau decays!
  - there is NO DATA for LFV radiative decays, e.g. $\psi(nS) \to \gamma\overline{\mu}e, \gamma\overline{\mu}\tau, \ldots$
- More data from ATLAS/CMS/(LHCb?) on pp $\to$ $\tau\mu$ + X
  - possible effects from gg $\to$ $\tau\mu$ due to large gluon luminosity of LHC

Bhattacharya, Morgan, AAP

- Maybe flavor physics will be the only place to see glimpses of New Physics
- ...but then again, maybe not.


EXTRA!


EFFECTIVE FIELD THEORIES
Alexey A. Petrov · Andrew E. Blechman
World Scientific

Thank you for your attention!

JOURNAL OF APPLIED BEHAVIOR ANALYSIS 1974, 7, 497 NUMBER 3 (FALL 1974)

THE UNSUCCESSFUL SELF-TREATMENT OF A CASE OF "WRITER'S BLOCK"[1]

DENNIS UPPER

VETERANS ADMINISTRATION HOSPITAL, BROCKTON, MASSACHUSETTS

REFERENCES

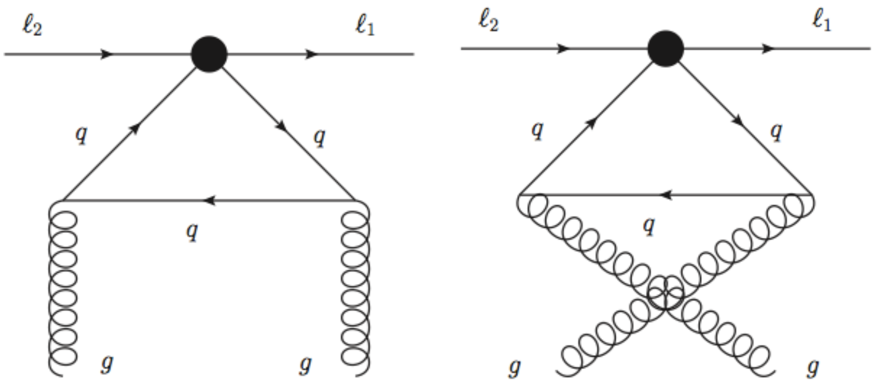
[1]Portions of this paper were not presented at the 81st Annual American Psychological Association Convention, Montreal, Canada, August 30, 1973. Reprints may be obtained from Dennis Upper, Behavior Therapy Unit, Veterans Administration Hospital, Brockton, Massachusetts 02401.

Received 25 October 1973.
(Published without revision.)

Hopefully, I did better than him...

Effective Lagrangians: gluonic operators

★ Coefficients of gluonic operators depend on the number of active flavors



$$
\begin{aligned}
\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2}\frac{\beta_L}{4\alpha_s}\Big[\Big(C_{GR}\overline{\ell}_1 P_R \ell_2 + C_{GL}\overline{\ell}_1 P_L \ell_2\Big) G^a_{\mu\nu} G^{a\mu\nu} \\
+ \Big(C_{\tilde{G}R}\overline{\ell}_1 P_R \ell_2 + C_{\tilde{G}L}\overline{\ell}_1 P_L \ell_2\Big) G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} + h.c.\Big]
\end{aligned}
$$

★ we can calculate their contribution to meson or tau decay rates!
★ also relevant for muon conversion experiments
★ c_i probe couplings of heavy quarks to New Physics

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Effective Lagrangians: gluonic operators

★ ... get an effective Lagrangian

$$\mathcal{L}^{(7)}_{\ell_1\ell_2} = \frac{1}{\Lambda^2}\sum_{i=1}^{4} c_i^{\ell_1\ell_2} O_i^{\ell_1\ell_2} + \mathrm{H.c.},$$

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...where we defined operators

$$O_1^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu},$$

$$O_2^{\ell_1\ell_2} = \bar{\ell}_{1R}\ell_{2L}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu},$$

$$O_3^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu},$$

$$O_4^{\ell_1\ell_2} = \bar{\ell}_{1L}\ell_{2R}\,\frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu},$$

...and Wilson coefficients

$$c_1^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}\left(C_1^{q\ell_1\ell_2} + C_2^{q\ell_1\ell_2}\right),$$

$$c_2^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}\left(C_1^{q\ell_1\ell_2} - C_2^{q\ell_1\ell_2}\right),$$

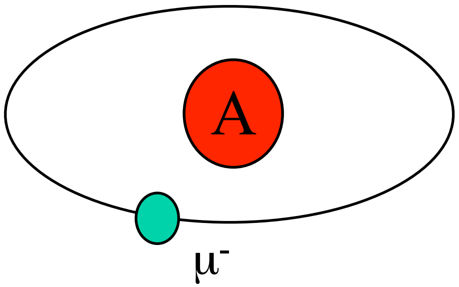
$$c_3^{\ell_1\ell_2} = -\frac{2}{9}\sum_{q=c,b,t}\frac{I_1(m_q)}{m_q}\left(C_3^{q\ell_1\ell_2} + C_4^{q\ell_1\ell_2}\right),$$

$$c_4^{\ell_1\ell_2} = \frac{2\mathrm{i}}{9}\sum_{q=c,b,t}\frac{I_2(m_q)}{m_q}\left(C_3^{q\ell_1\ell_2} - C_4^{q\ell_1\ell_2}\right),$$

$$I_1 = \frac{1}{3}, \qquad I_2 = \frac{1}{2}.$$

3c. Probing LFV gluonic operators with muon conversion

★ Basic idea for the muon conversion experiment



★ take low energy muons (~ 30 MeV) to be stopped in a target A(Z,A-Z): muons cascade to atomic 1s state

★ Binding energy and orbit radius for muonic hydrogen-like state

$$E_b = -\frac{Z^2 m e^4}{8n^2} \sim \frac{Z^2 m}{n^2}$$

$$r = \frac{n^2}{Z \pi m e^2} \sim \frac{n^2}{Zm}$$

muonic atom is 200x stronger bound
radius is 200x smaller

★ Radial wave function for hydrogen-like system: $R_{nl} \sim r^\ell Z^{3/2}$

overlap probability: $p \sim r^{2\ell} Z^3$

large overlap for an s-wave and high-Z nucleus

Measure $R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$ to probe NP

Experimental ideas

★ Examples of nuclei suitable for muon conversion experiments

Nucleus	$R_{\mu e}(Z)$ / $R_{\mu e}(Al)$	Bound lifetime	Atomic Bind. Energy(1s)	Conversion Electron Energy	Prob decay >700 ns
Al(13,27)	1.0	.88 μs	0.47 MeV	104.97 MeV	0.45
Ti(22,~48)	1.7	.328 μs	1.36 MeV	104.18 MeV	0.16
Au(79,~197)	~0.8-1.5	.0726 μs	10.08 MeV	95.56 MeV	negligible

J. Miller, 2006

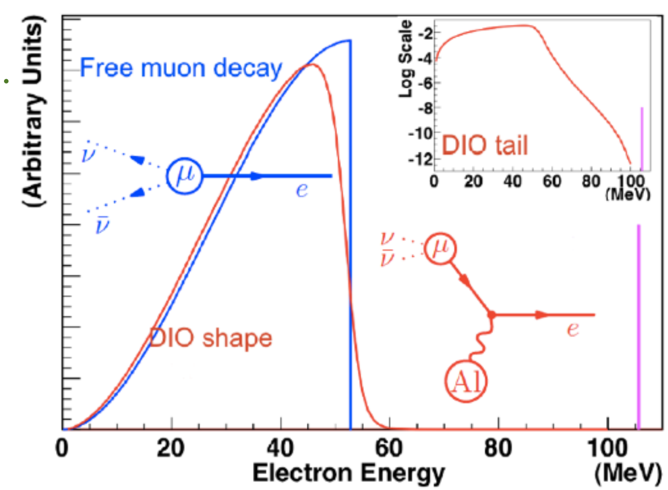
★ The experiment is tricky

✓ Muon conversion gives monoenergetic electrons...
✓ ... yet, there are other sources of electrons as well!

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu & \text{- decay (40\%)} \\ \mu^- + Al \to X + \nu_\mu & \text{- capture (60\%)} \\ \mu^- + Al \to e^- + Al & \text{- conversion} \end{cases}$$

SINDRUM II (PSI), 2006 : $R_{\mu e} < 7 \times 10^{-13}$

M2e goal : $R_{\mu e} <$ a few $\times 10^{-17}$



Czarnecki, Marciano, Tormo