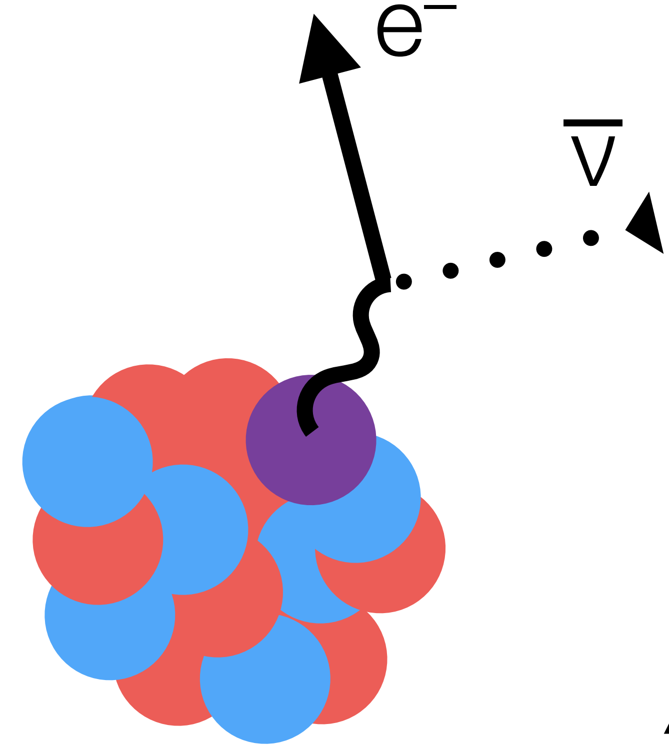
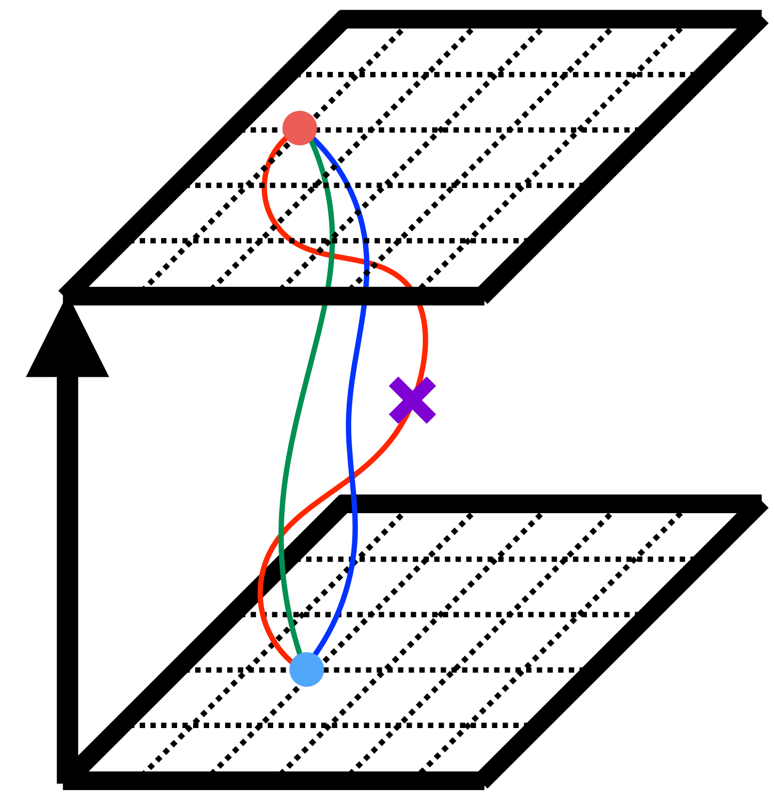
The Nucleon Axial Charge from Lattice QCD





Evan Berkowitz
Institut für Kernphysik
Institute for Advanced Simulation
Forschungszentrum Jülich

15 May 2017
Low Energy Probes of New Physics
Mainz Institute for Theoretical Physics

1701.07559
1704.01114

JÜLICH
FORSCHUNGSZENTRUM

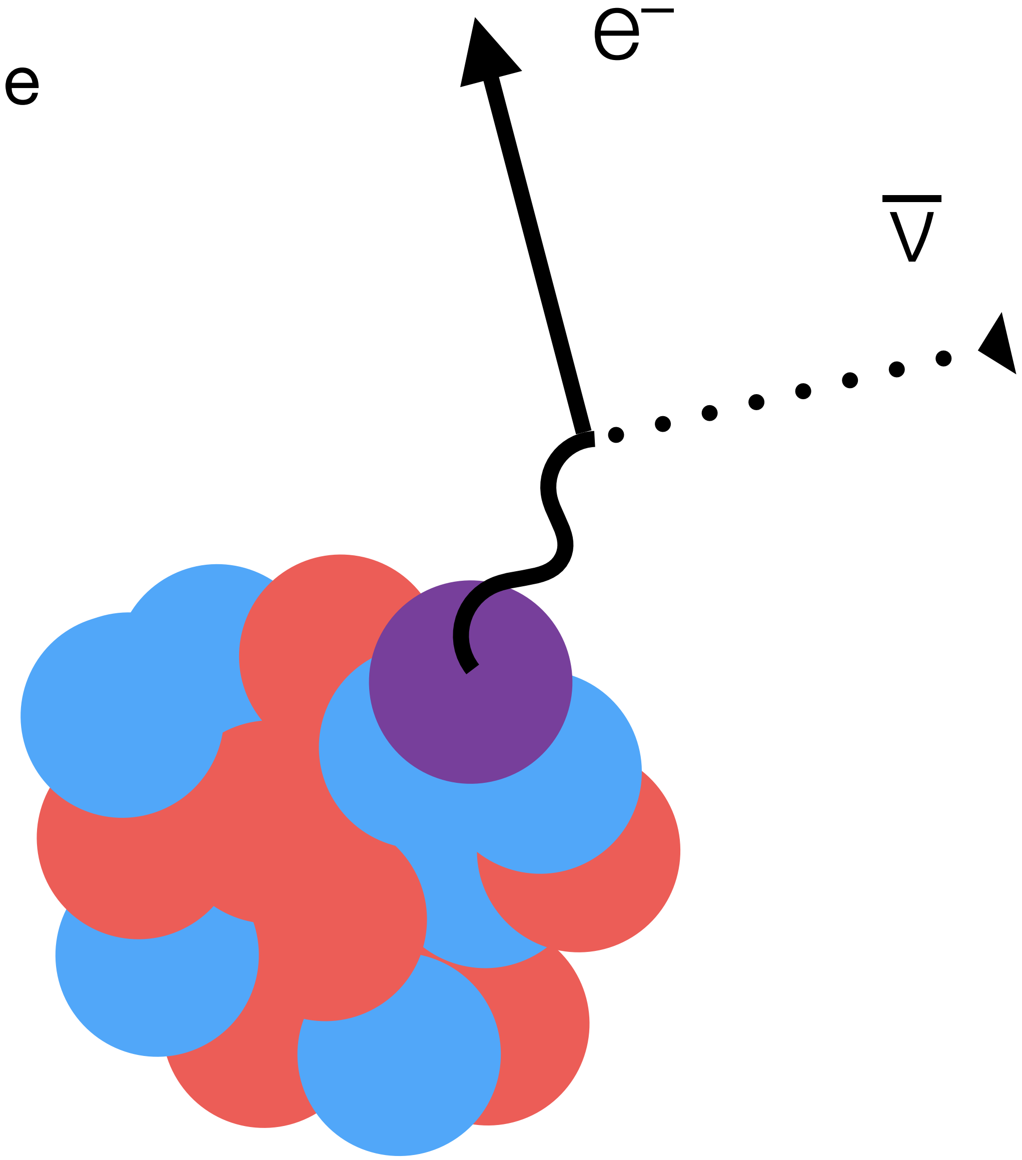
CalLat

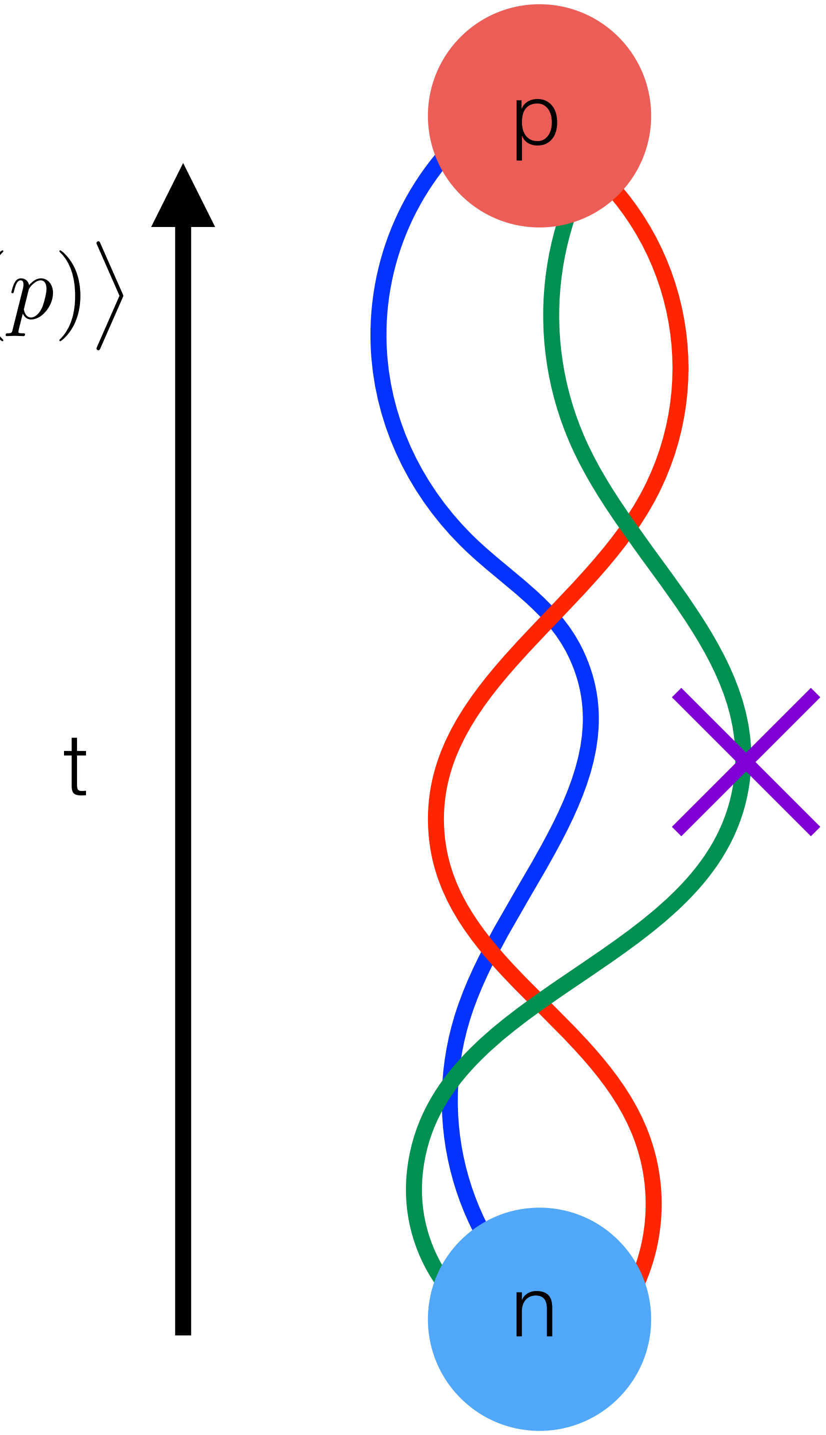
Berkeley LBL	David Brantley, Henry Monge Camacho, Chia Cheng (Jason) Chang, Ken McElvain, André Walker-Loud
RBRC	Enrico Rinaldi
FZJ	EB
JLab	Bálint Jóo
Liverpool Plymouth	Nicolas Garron
LLNL	Pavlos Vranas
NERSC	Thorsten Kurth
UNC	Amy Nicholson
nVidia	Kate Clark
Glasgow	Chris Bouchard
Rutgers	Chris Monahan
William & Mary	Kostas Orginos

The Nucleon Axial Charge

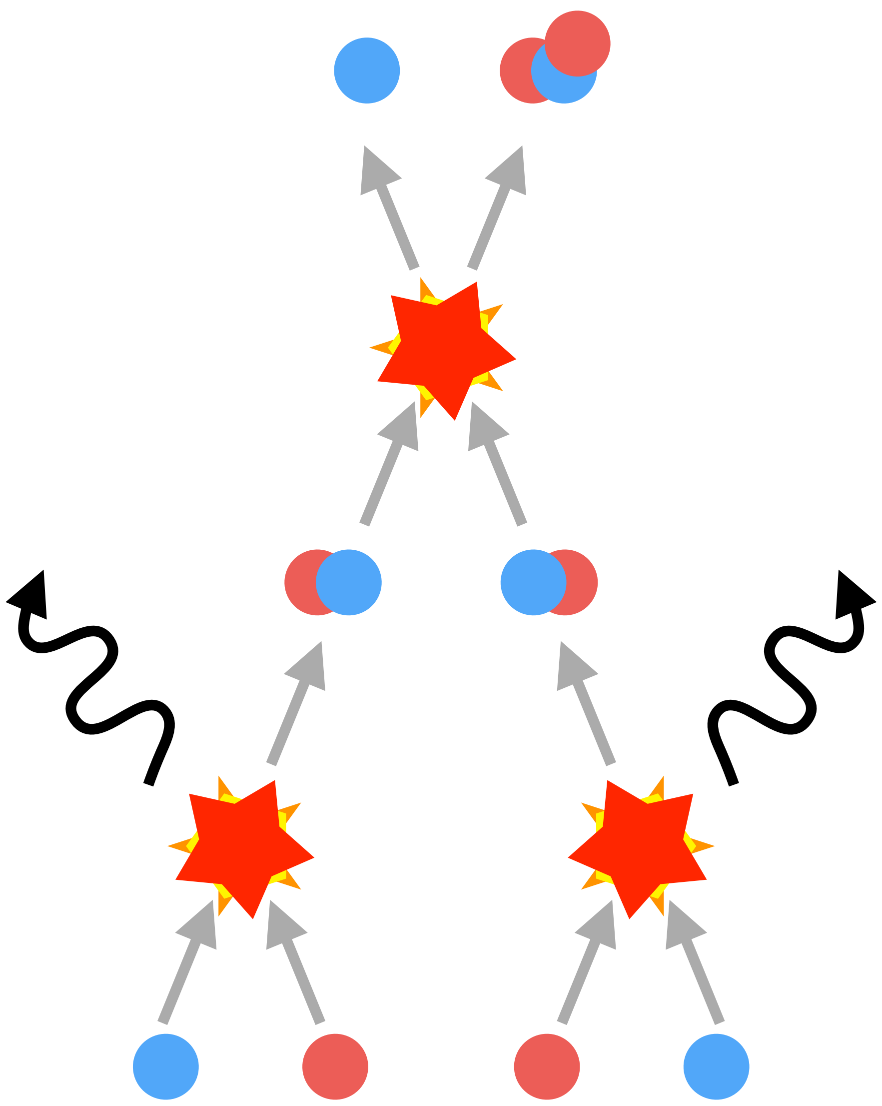
$$\langle N(p) \mid A_\mu^a \mid N(p) \rangle = \langle N(p) \mid \bar{\psi}\gamma_\mu\gamma_5\tau^a\psi \mid N(p) \rangle$$
$$= g_A \; \bar{n}(p)\gamma_\mu\gamma_5\tau^a n(p)$$

- Free neutron lifetime
- Nuclear force
- Nuclear β decay



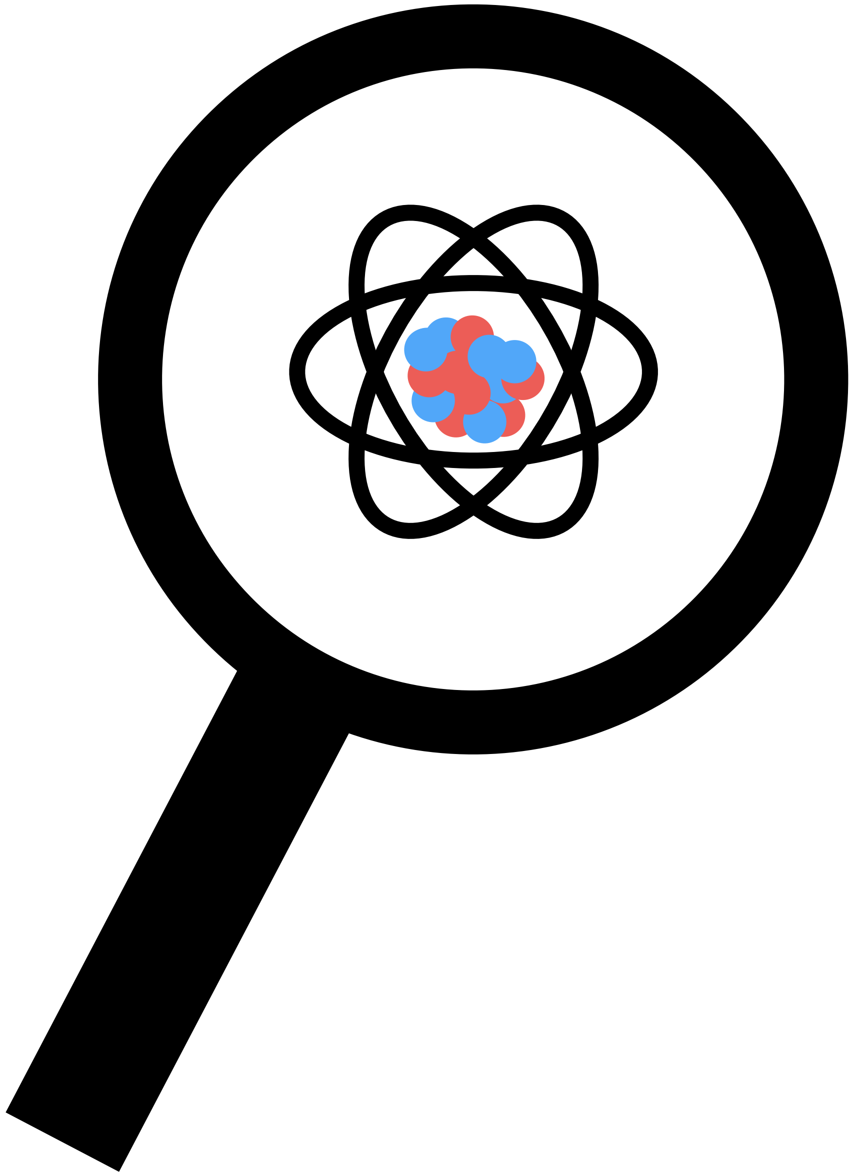


Applications



Big Bang Nucleosynthesis

Astrophysics



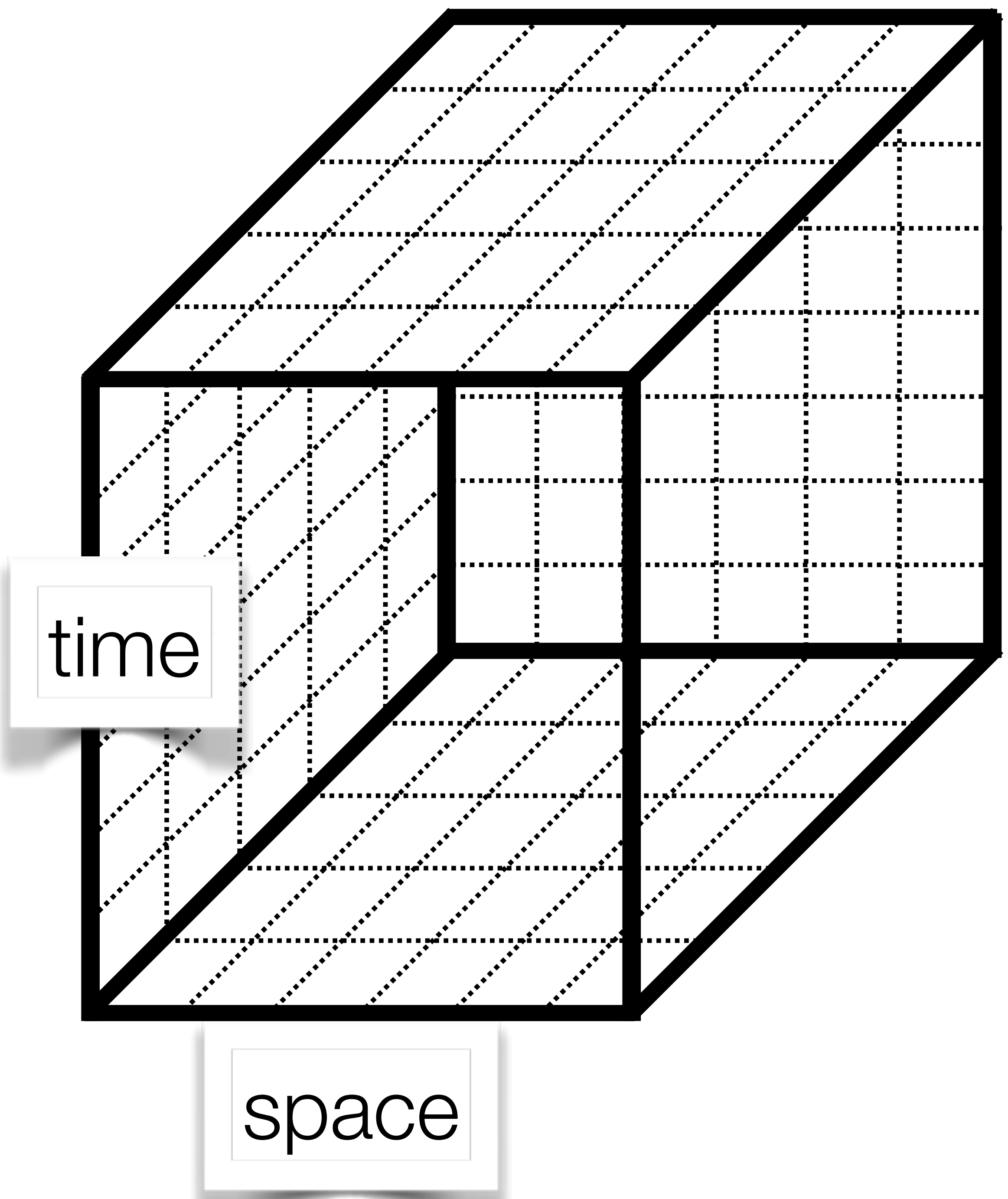
New Physics Searches

Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \mathcal{D}U \; \mathcal{O}(t)\mathcal{O}^\dagger(0) e^{-S[\bar{\psi},\psi,U]}$$

$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det\left(\not{D} + M\right) e^{-S[U]} \; \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

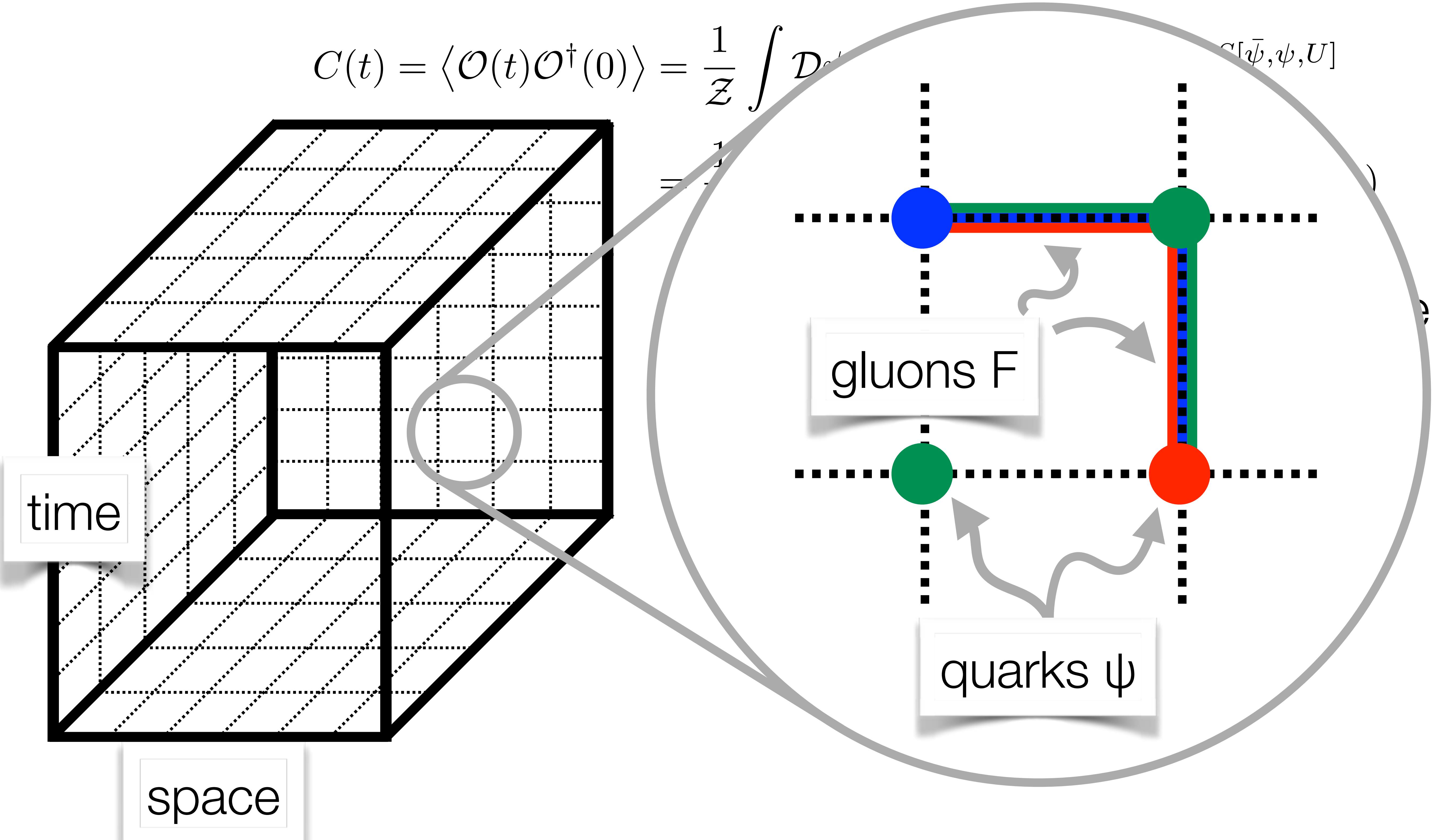


lattice
finite volume

Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}$$



Introduction to LQCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

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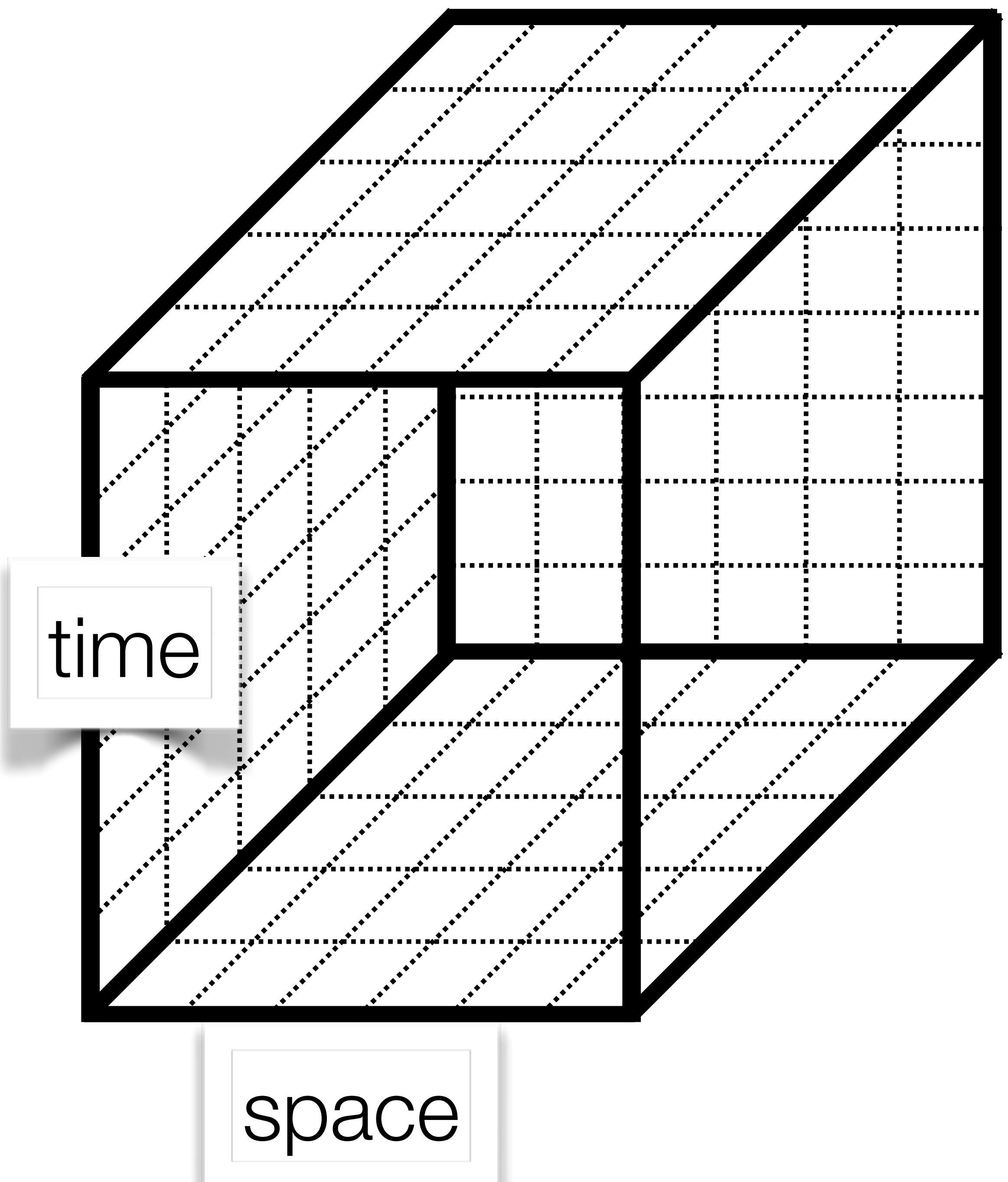
$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \underbrace{\det\left(\not{D} + M\right) e^{-S[U]}}_{\text{Probability}}\ \mathcal{O}(t)\mathcal{O}^\dagger(0)$$



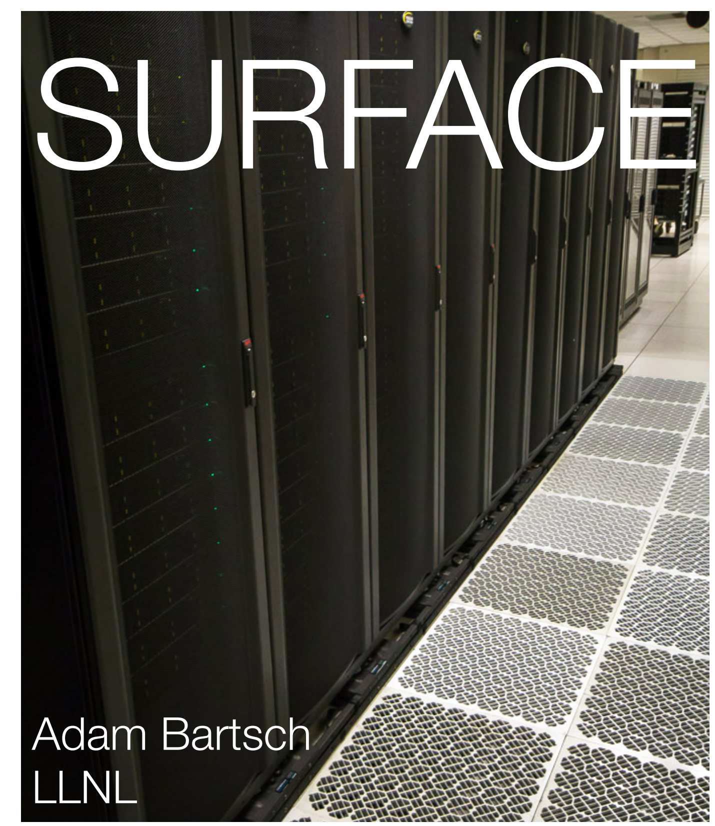
$$\{U_1, U_2, U_3, \ldots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i]\ + O\left(\frac{1}{\sqrt{N}}\right)$$

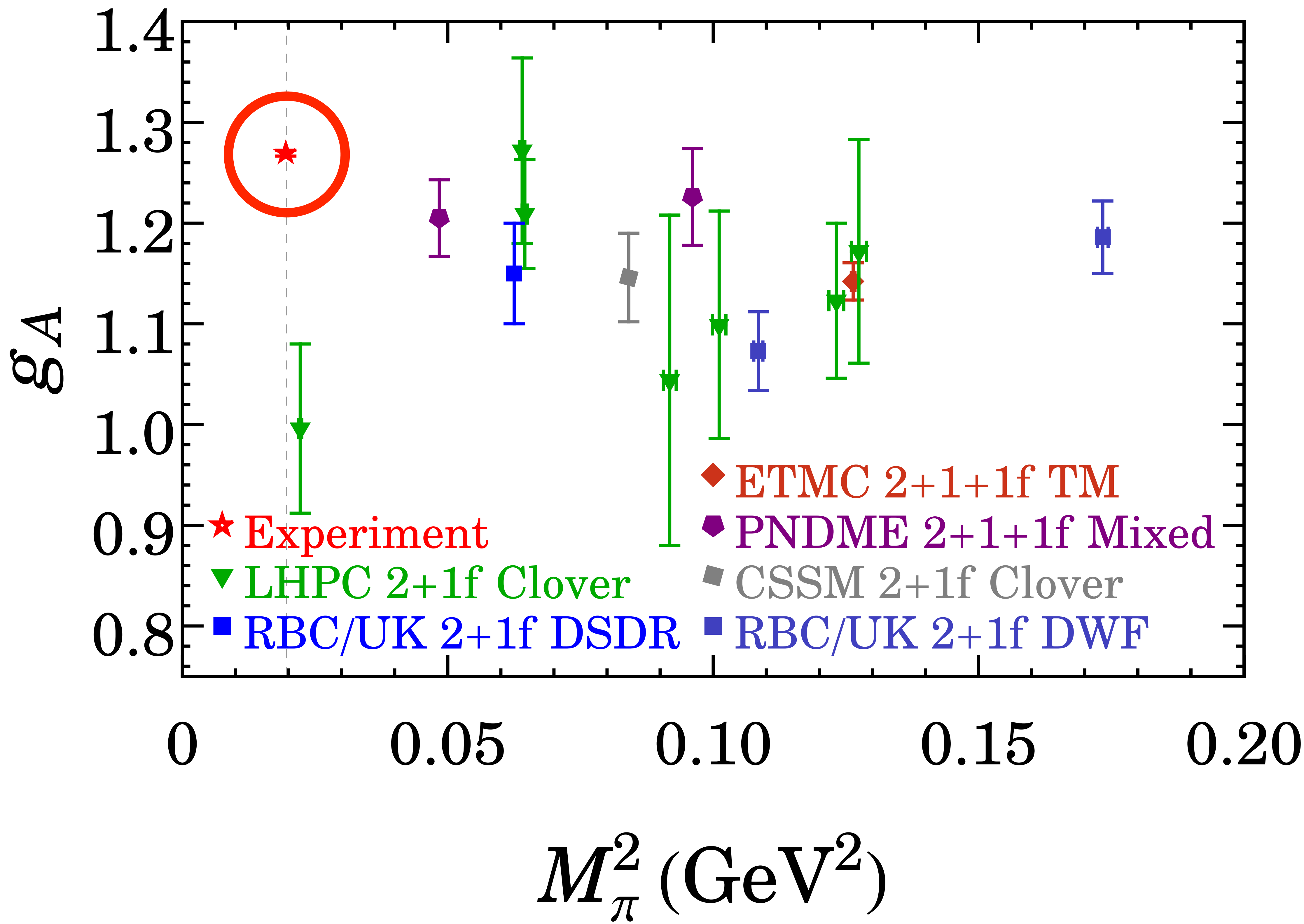

TITAN
OLCF


Edison
NERSC

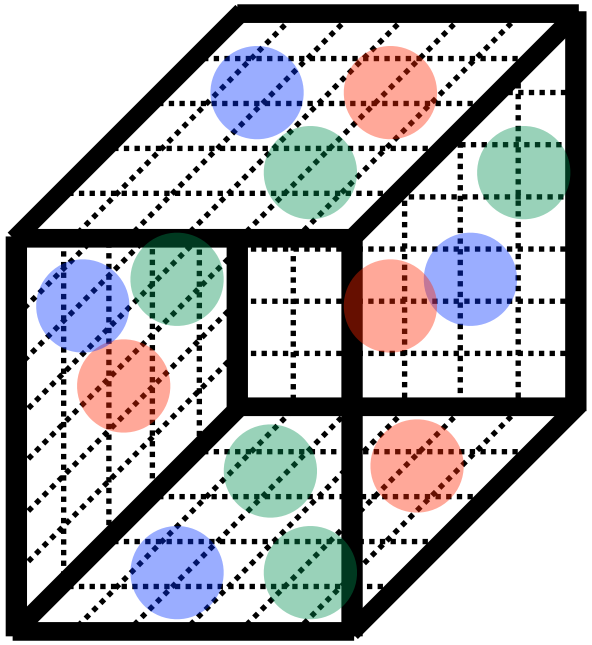

SURFACE
Adam Bartsch
LLNL

A long-outstanding problem for LQCD

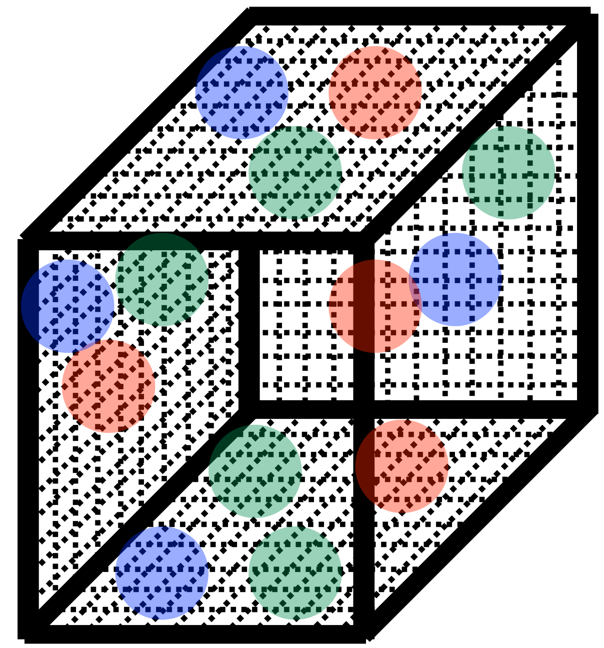
Bhattacharya, Cohen, Gupta, Joseph, Lin, Yoon PRD 89 (2014) arXiv:1306.5435



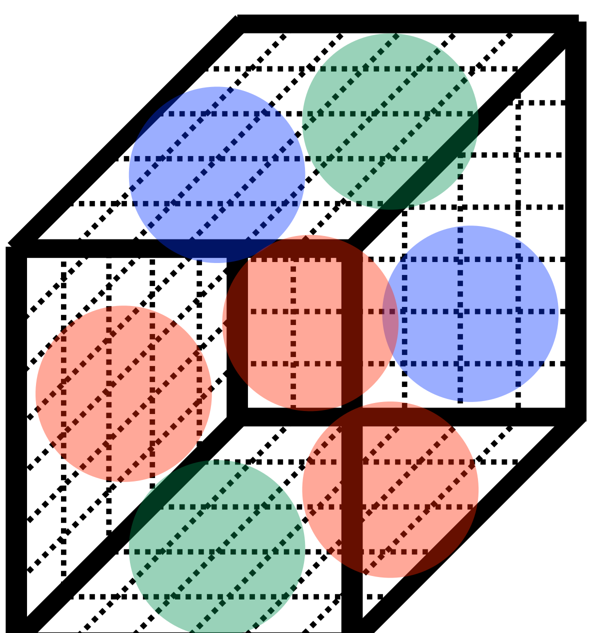
LQCD Systematics



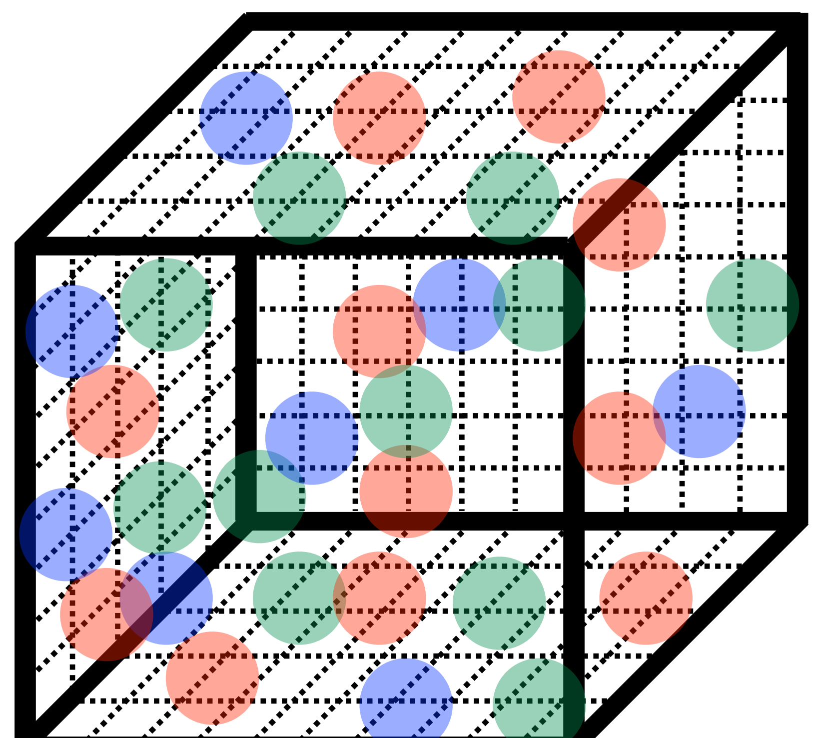
any calculation



continuum limit



physical quark masses



infinite volume limit

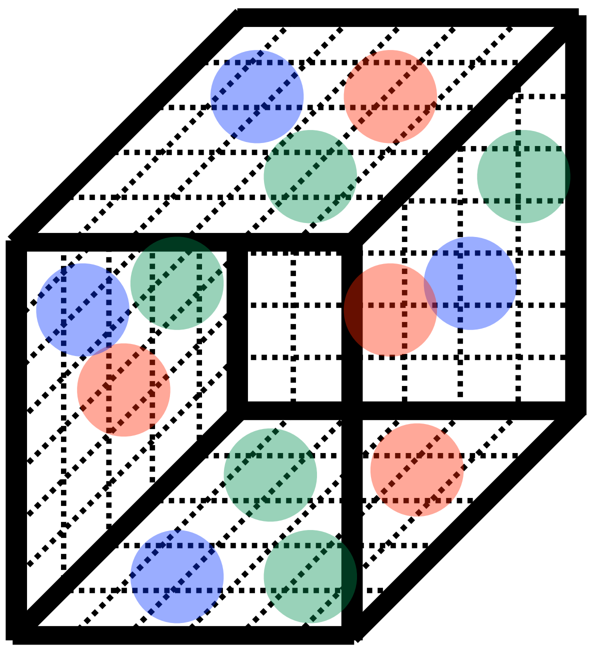
MILC Ensembles

MILC Collaboration Phys. Rev. D87 (2013) 054505

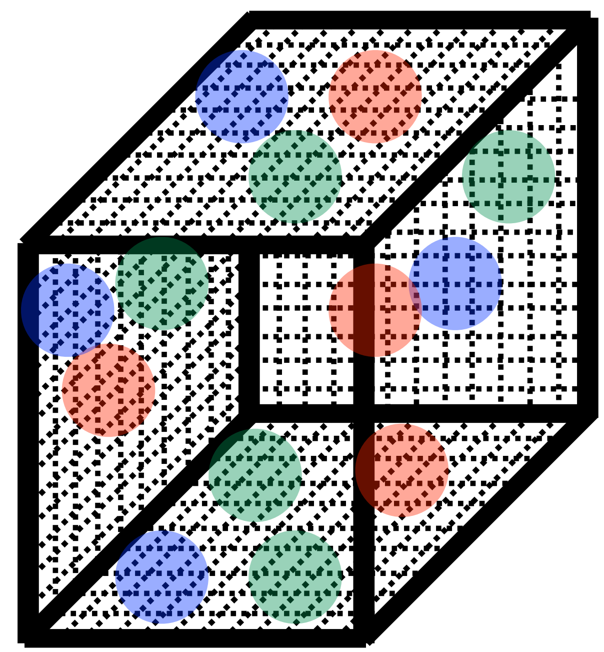
abbr. name	ensemble	N_{cfg}	N_{srcs}	volume	$\sim a$ [fm]	$\sim m_{\pi,5}$ [MeV]	$\sim m_{\pi,5}L$
a15m310	l1648f211b580m013m065m838a	1960	24	$16^3 \times 48$	0.15	307	3.78
a12m310	l2464f211b600m0102m0509m635a	1053	4	$24^3 \times 64$	0.12	305	4.54
a09m310	l3296f211b630m0074m037m440e	784	8	$32^3 \times 96$	0.09	313	4.50
a15m220	l2448f211b580m0064m0640m828a	1000	12	$24^3 \times 48$	0.15	215	3.99
a12m220S	l2464f211b600m00507m0507m628a	1000	4	$24^3 \times 64$	0.12	218	3.22
a12m220	l3264f211b600m00507m0507m628a	1000	4	$32^3 \times 64$	0.12	217	4.29
a12m220L	l4064f211b600m00507m0507m628a	1000	4	$40^3 \times 64$	0.12	217	5.36
a15m130	l3248f211b580m00235m0647m831a	1000	5	$32^3 \times 48$	0.15	131	3.30

- Anyone is free to use them
- Large statistics available
- Capable of controlling all systematic uncertainties
- We use domain wall valence on the HISQ sea, $\mathcal{O}(a^2)$ errors [1701.07559].

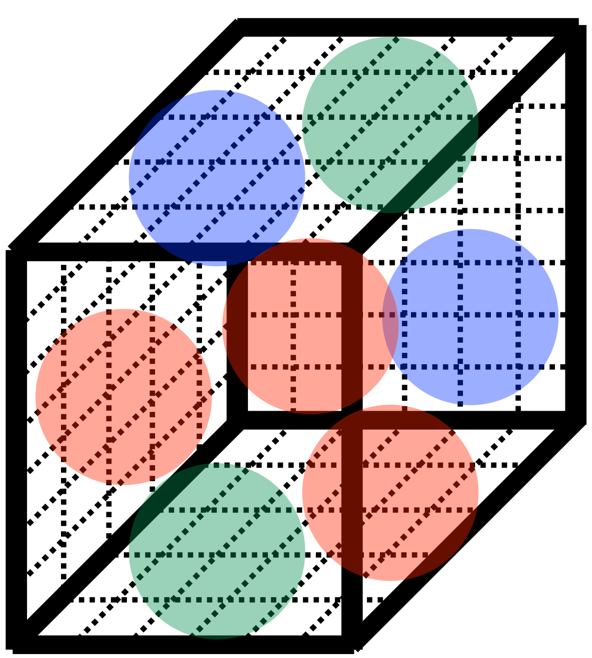
LQCD Systematics



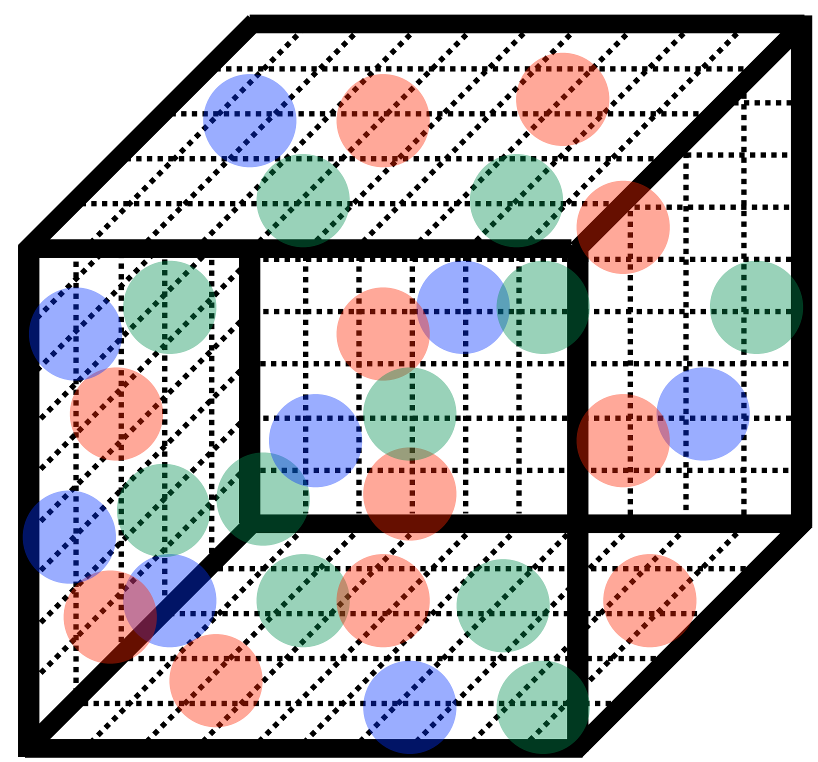
any calculation



continuum limit

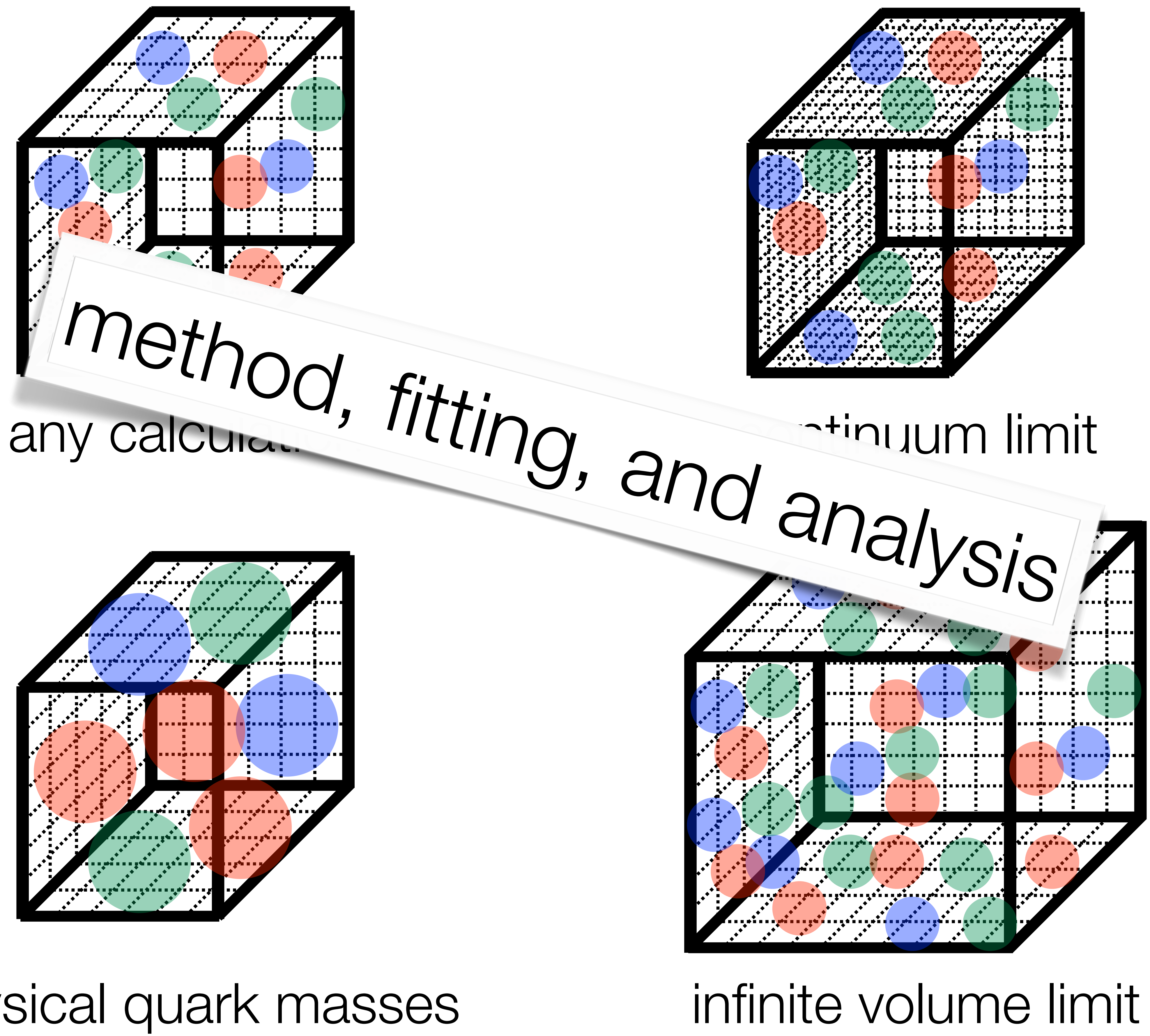


physical quark masses

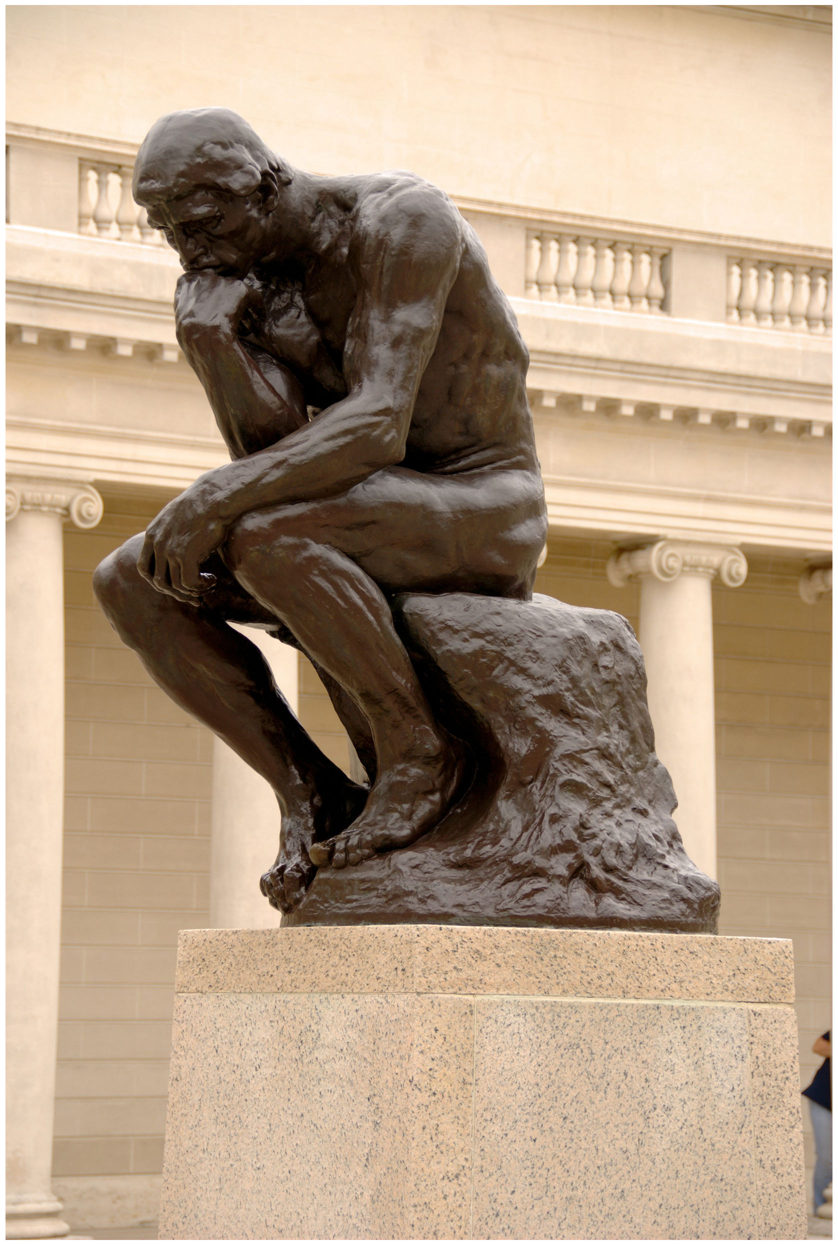


infinite volume limit

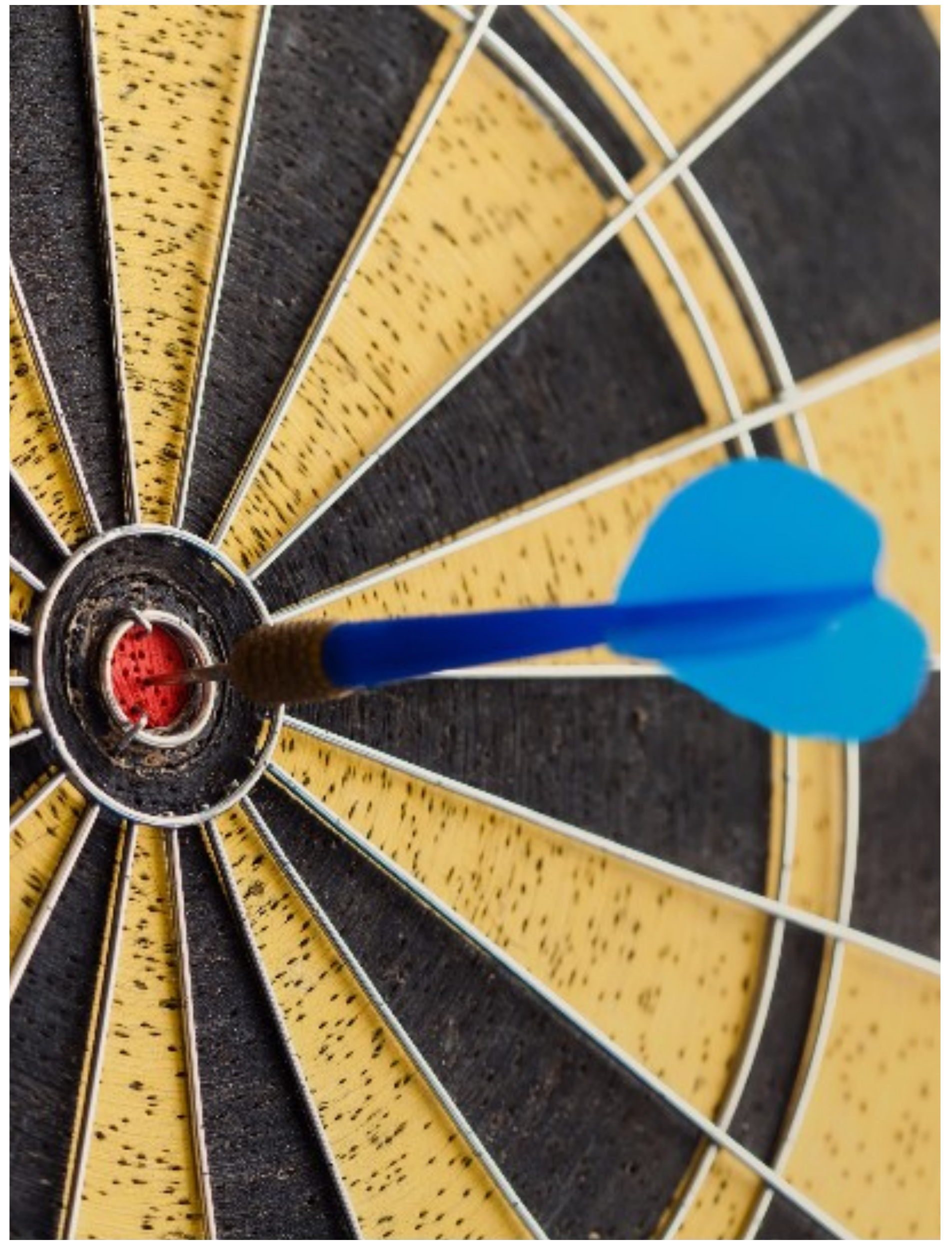
LQCD Systematics



New Methods



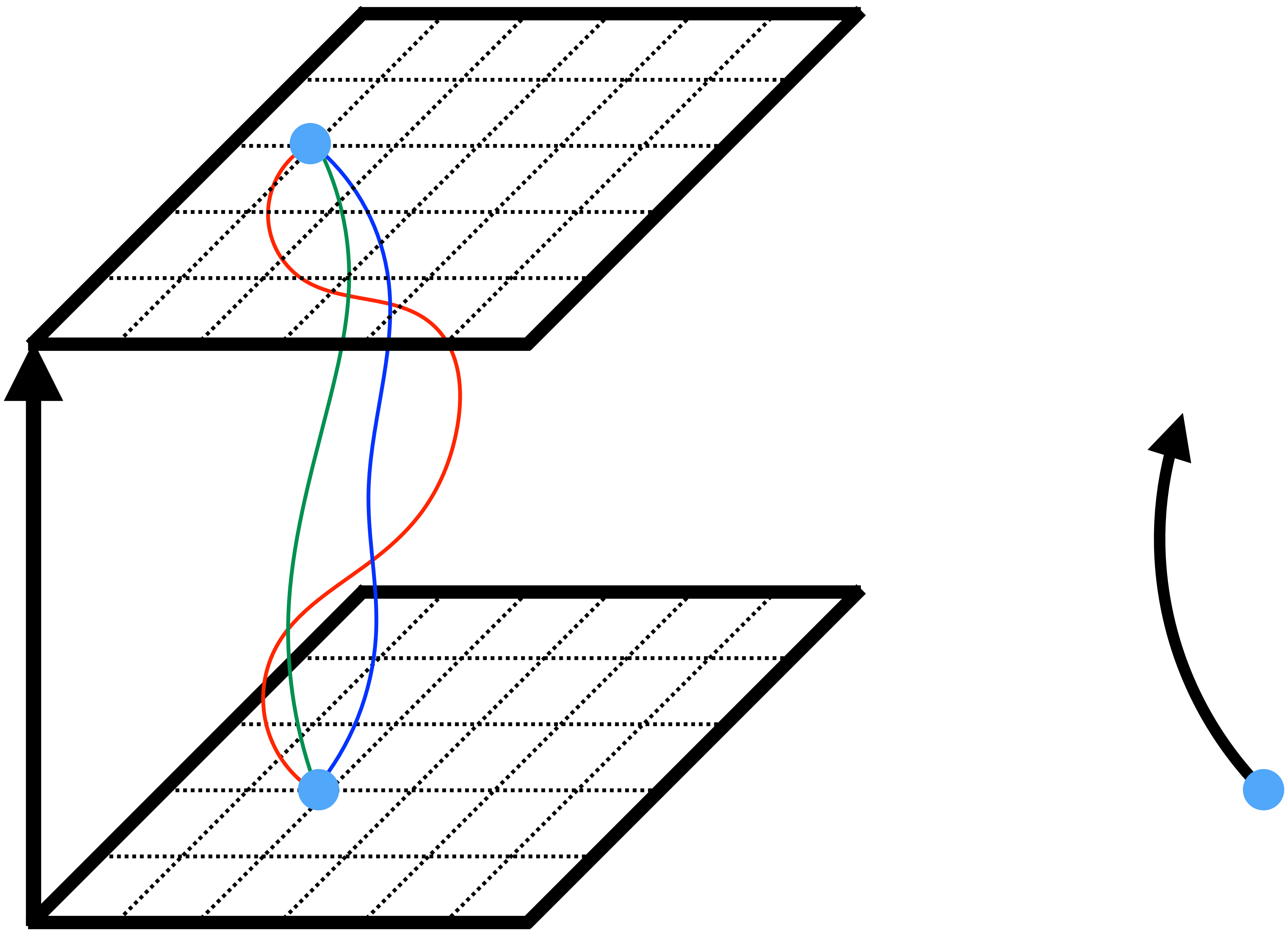
New Analytic Tools



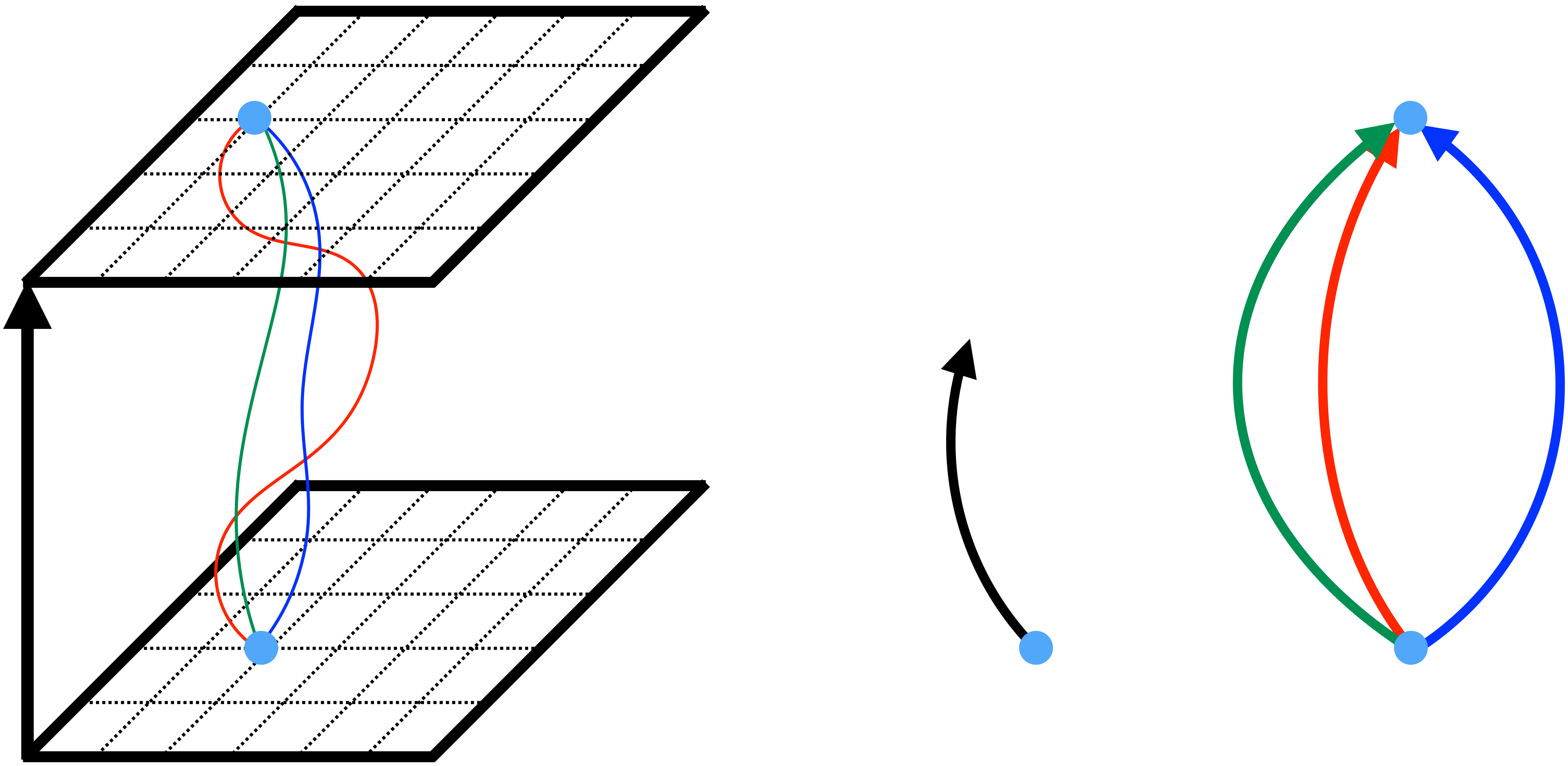
Improved Systematics

Computationally Affordable

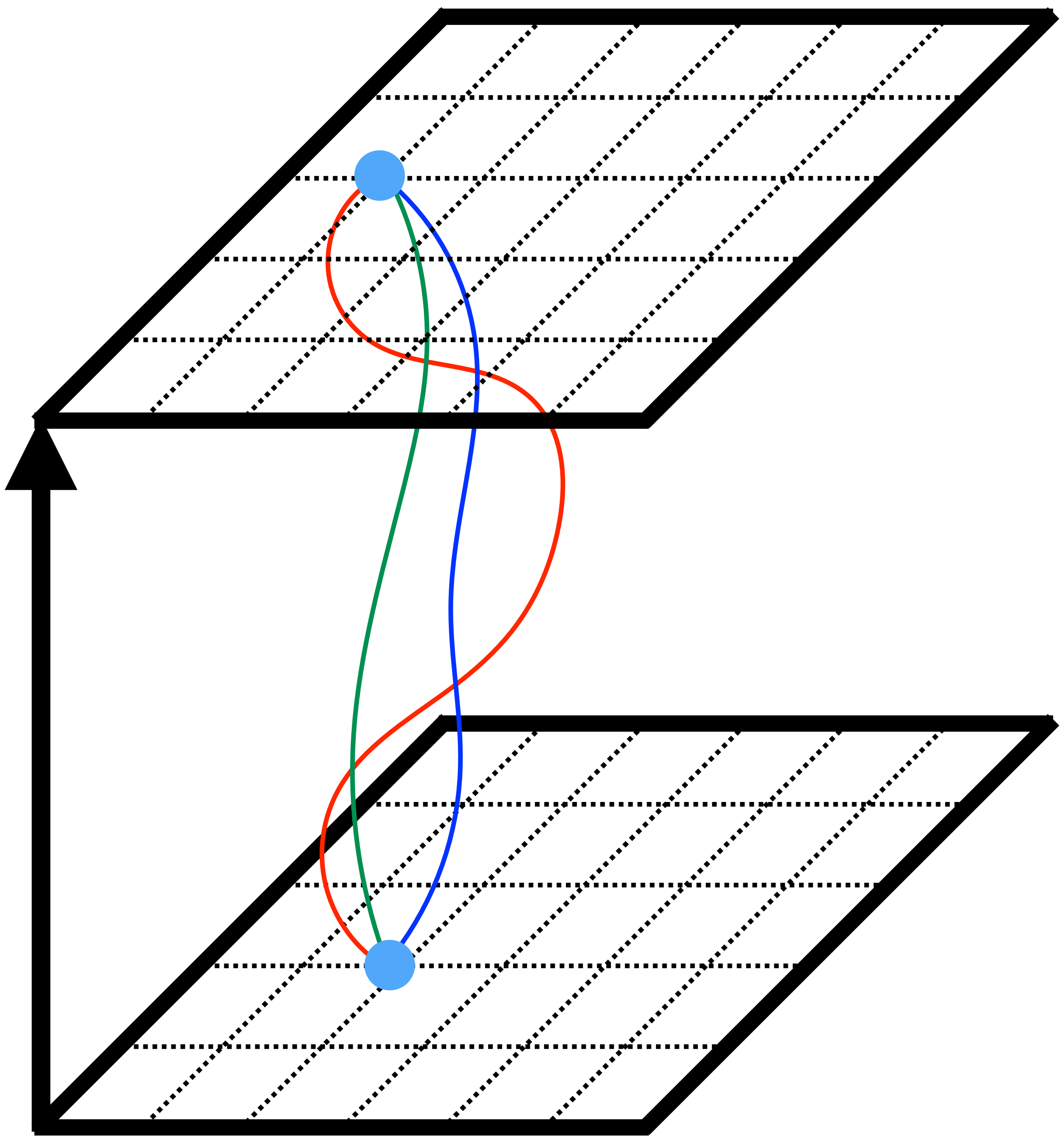
2-Point Standard Method



2-Point Standard Method



Effective Mass



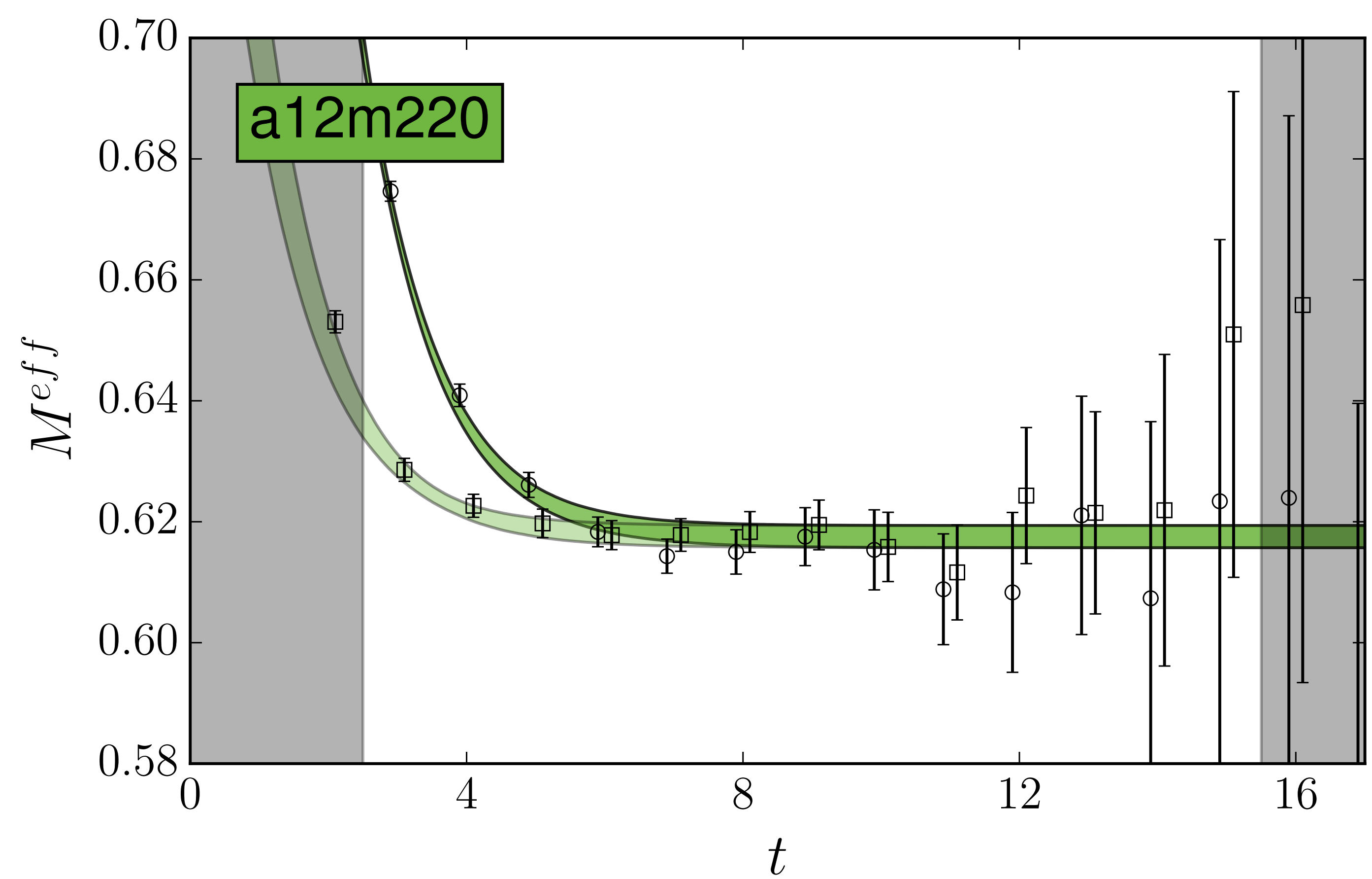
$$C(t) = \langle \Omega | \mathcal{O}(t) \mathcal{O}^\dagger(0) | \Omega \rangle$$

$$= \sum_n \langle \Omega | e^{\hat{H}t} \mathcal{O}(0) e^{-\hat{H}t} \frac{|n\rangle\langle n|}{2E_n} \mathcal{O}^\dagger(0) | \Omega \rangle$$

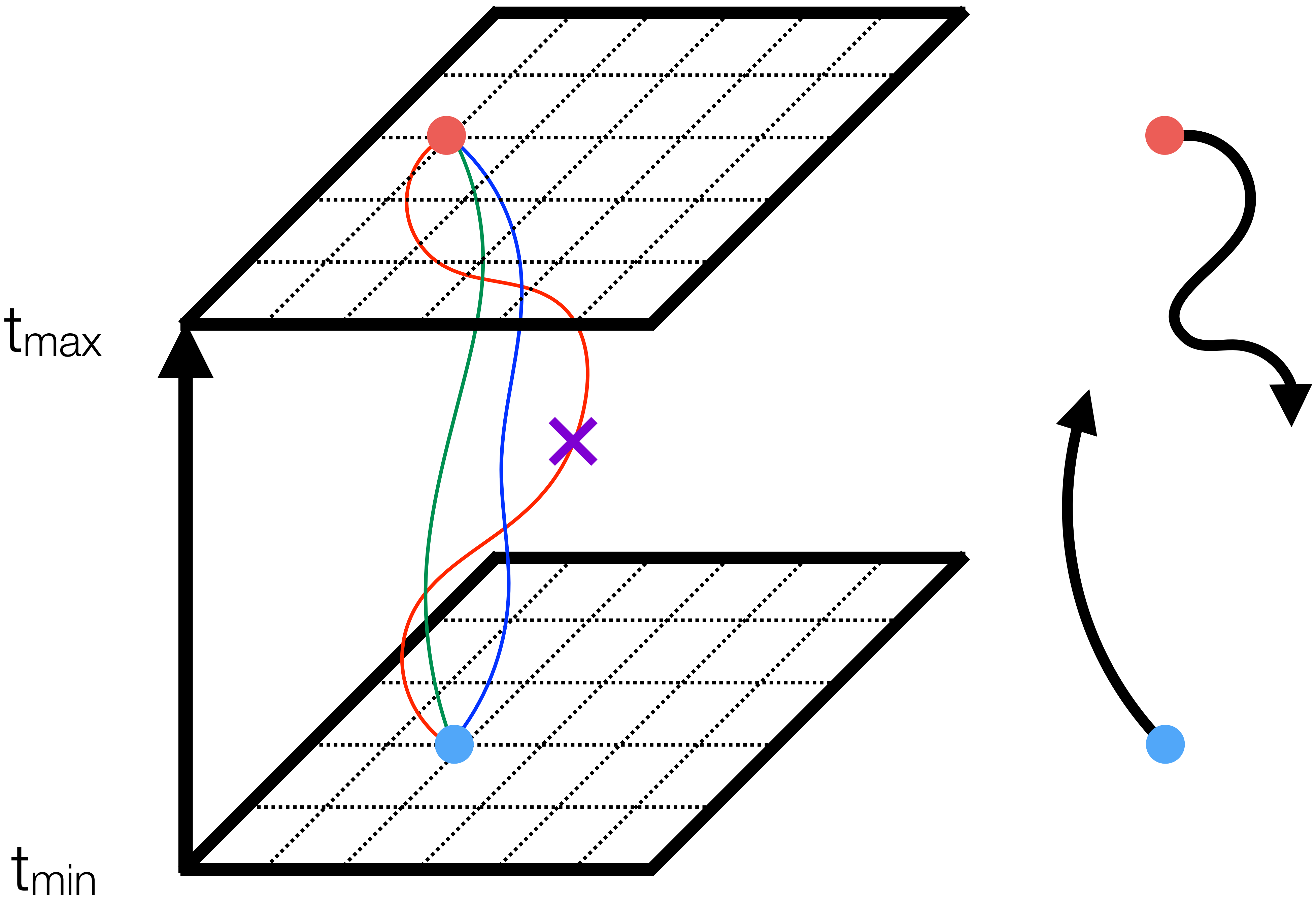
$$= \sum_n Z_n Z_n^\dagger \frac{e^{-E_n t}}{2E_n}$$

$$M^{eff}(t) = -\partial_t \ln\left(C(t)\right)$$

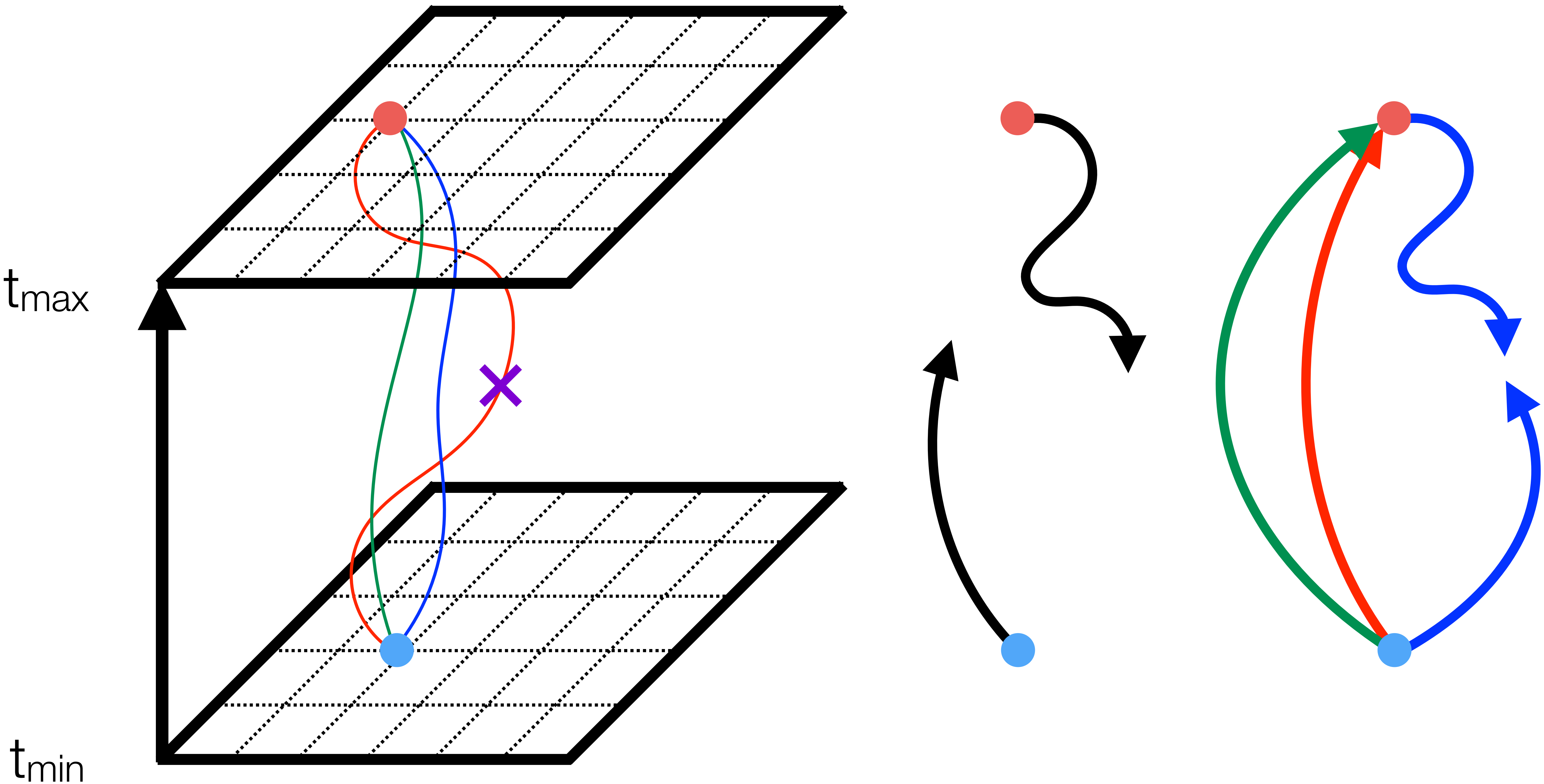
$$\lim_{t\to\infty} M^{eff}(t) = E_0$$



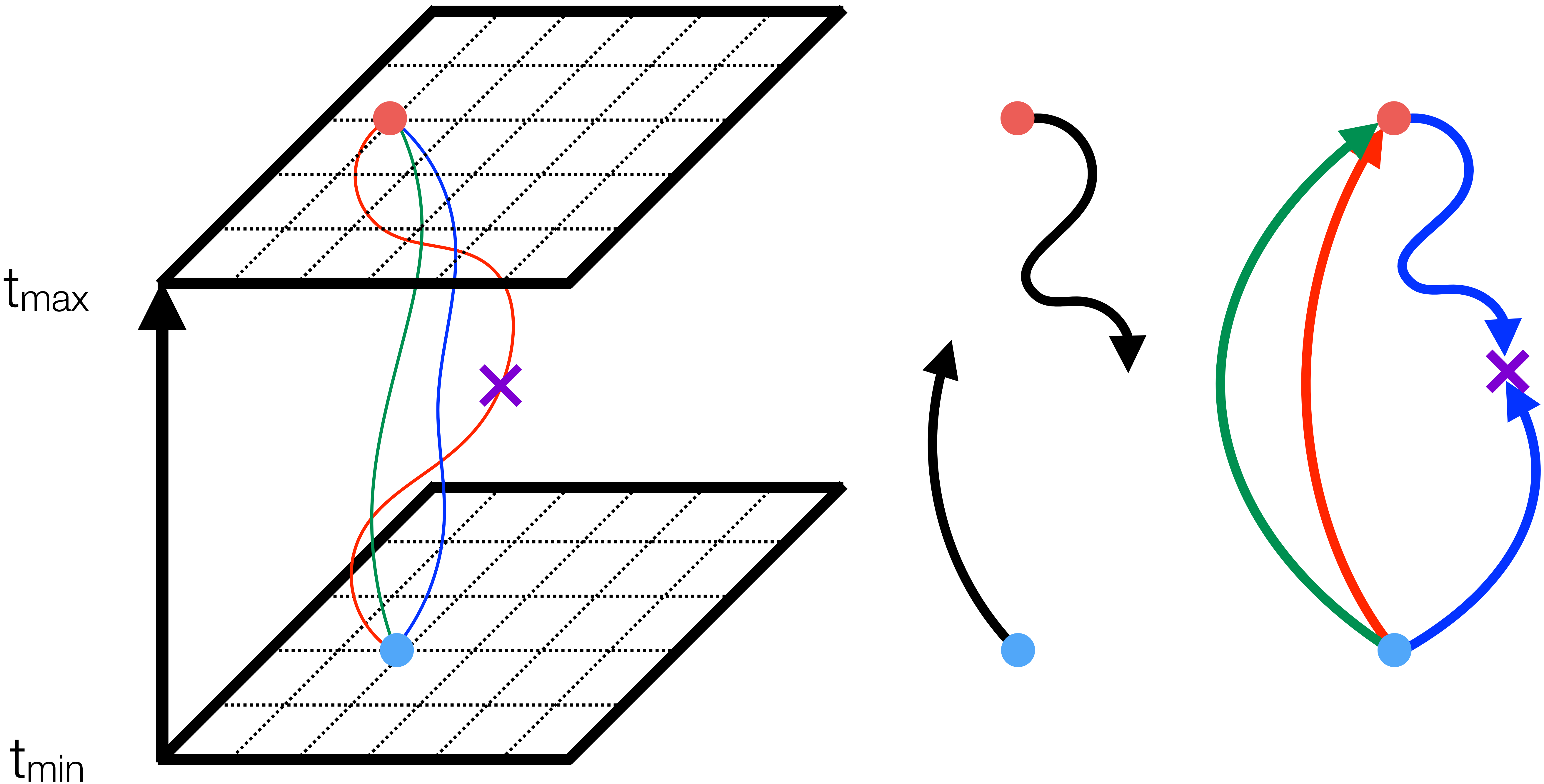
3-Point Standard Method



3-Point Standard Method

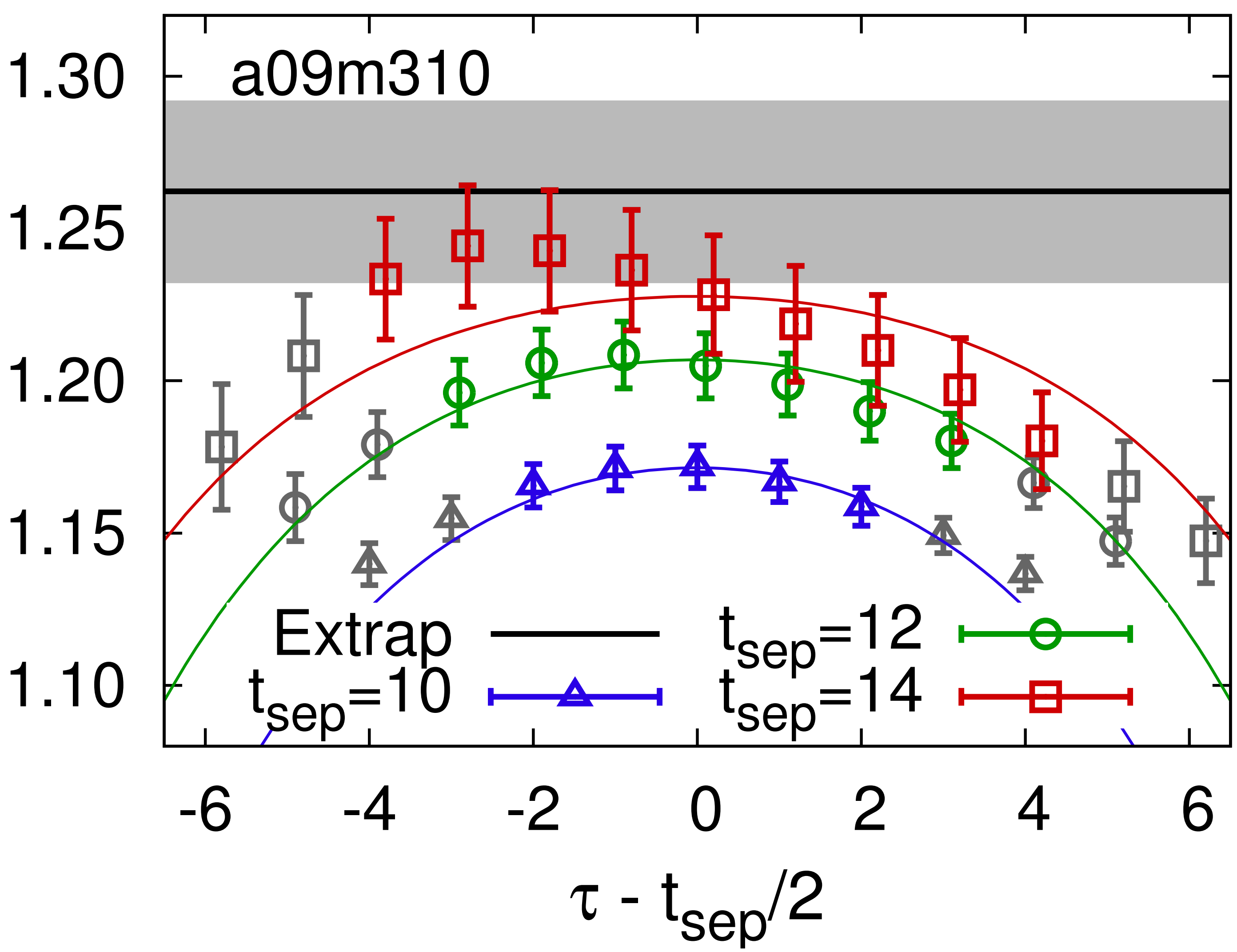


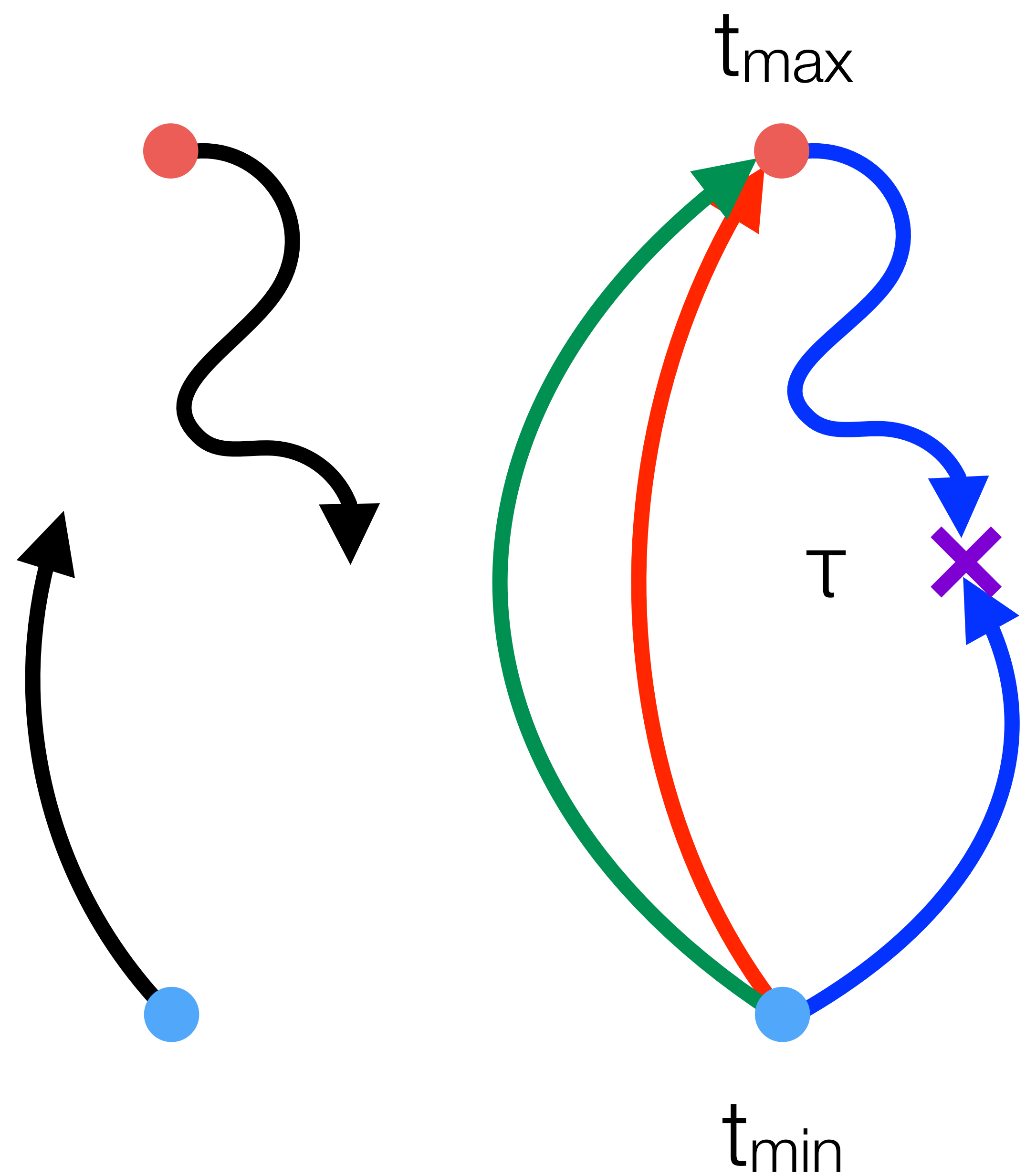
3-Point Standard Method



Standard Method

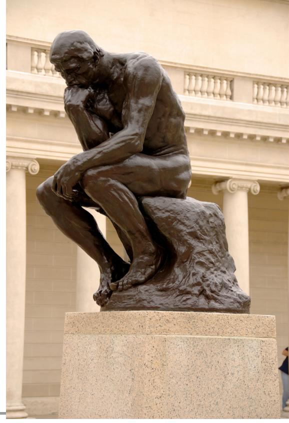
PNDME Phys. Rev. D94 (2016) arXiv:1606.07049

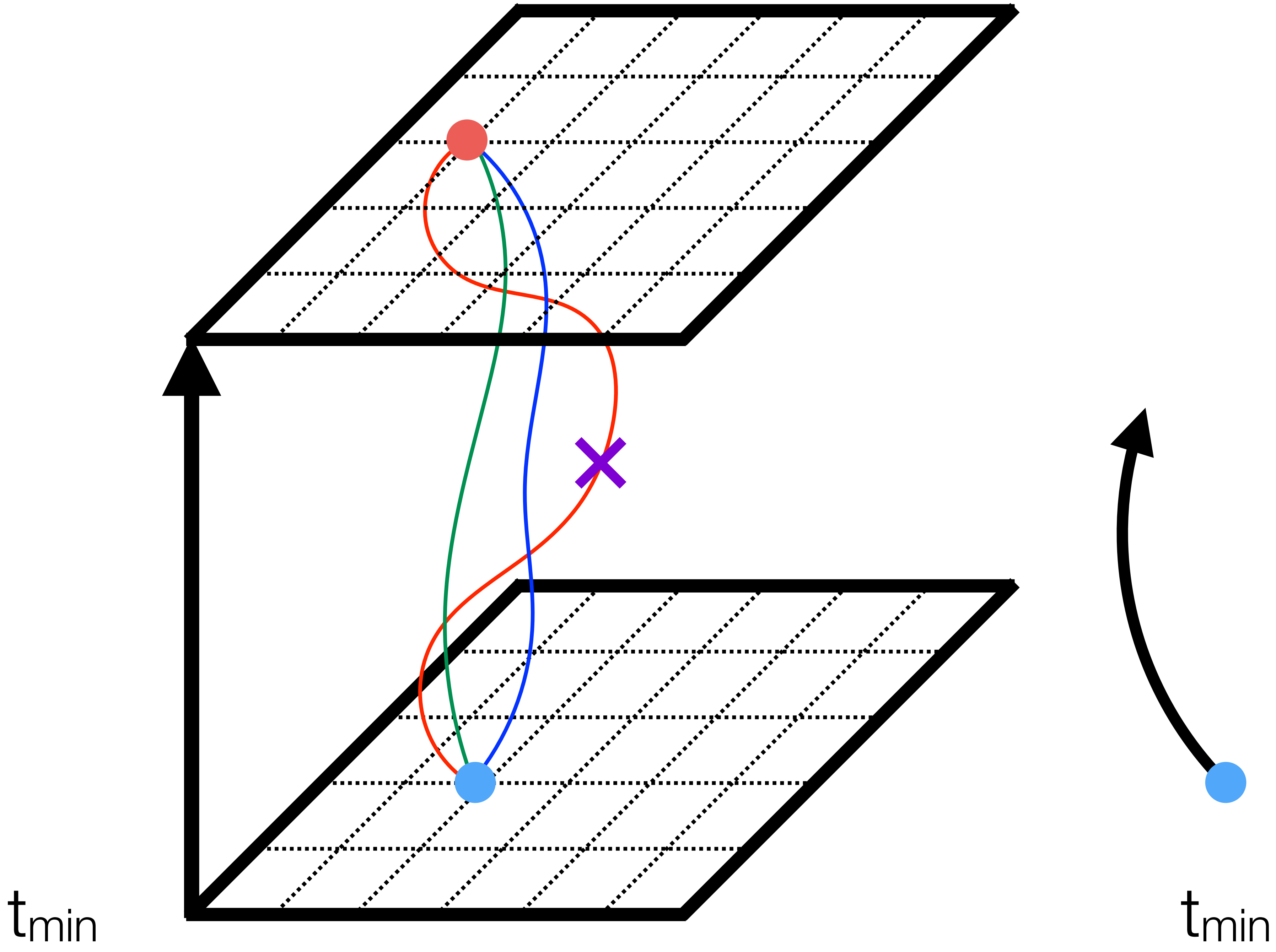




Feynman-Hellman Method

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963





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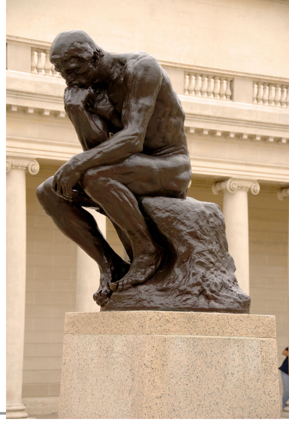
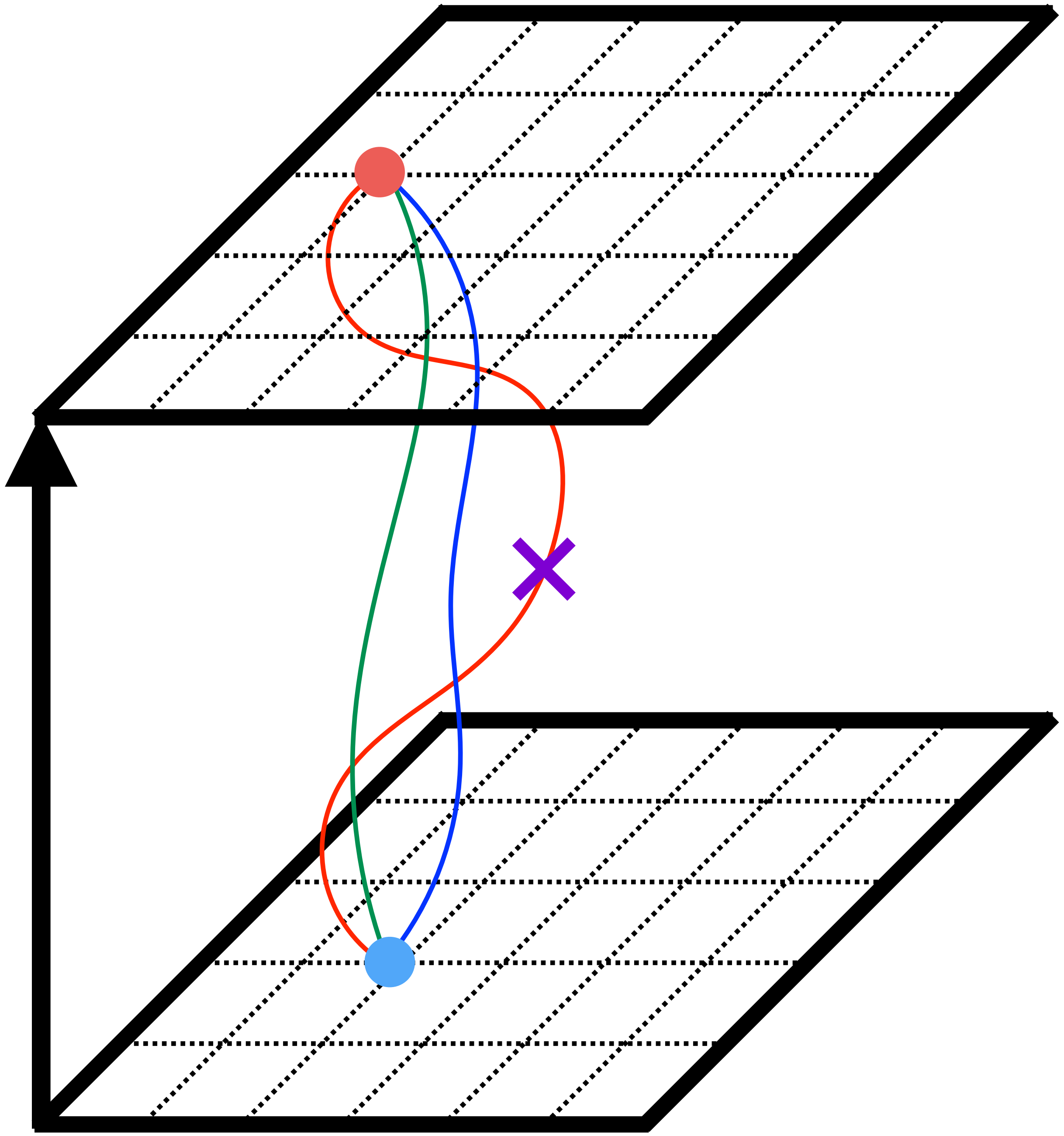
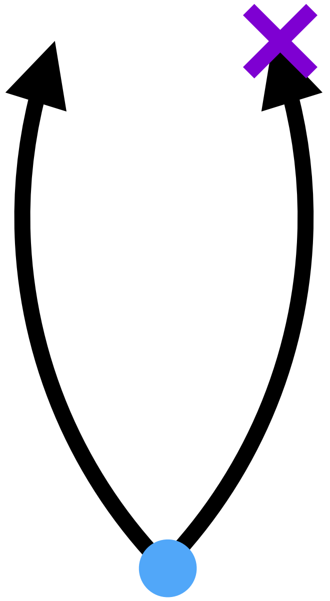




t_{min}



t_{min}

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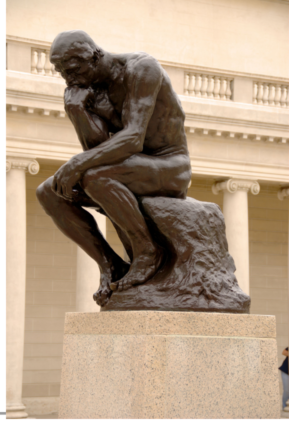

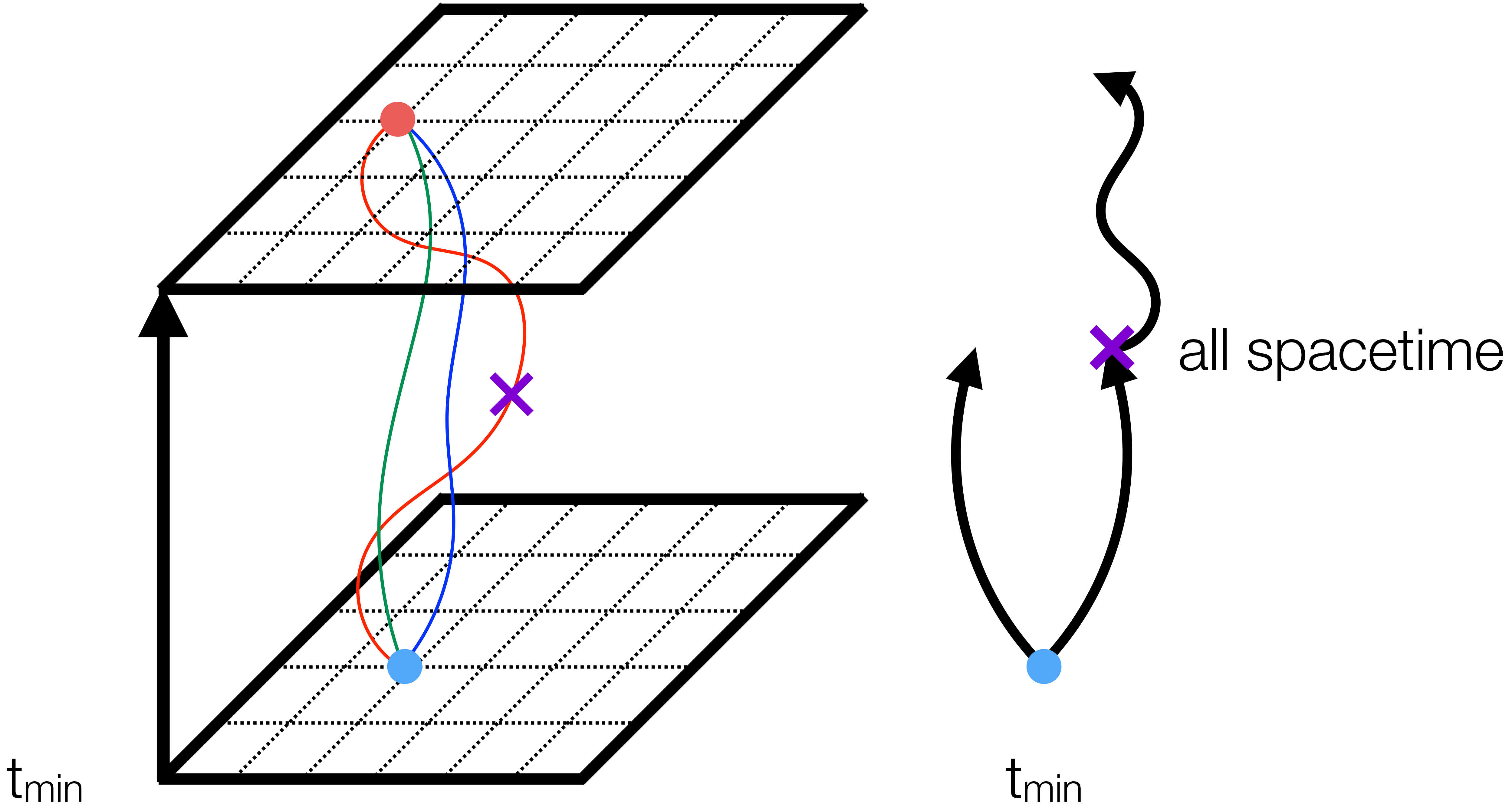

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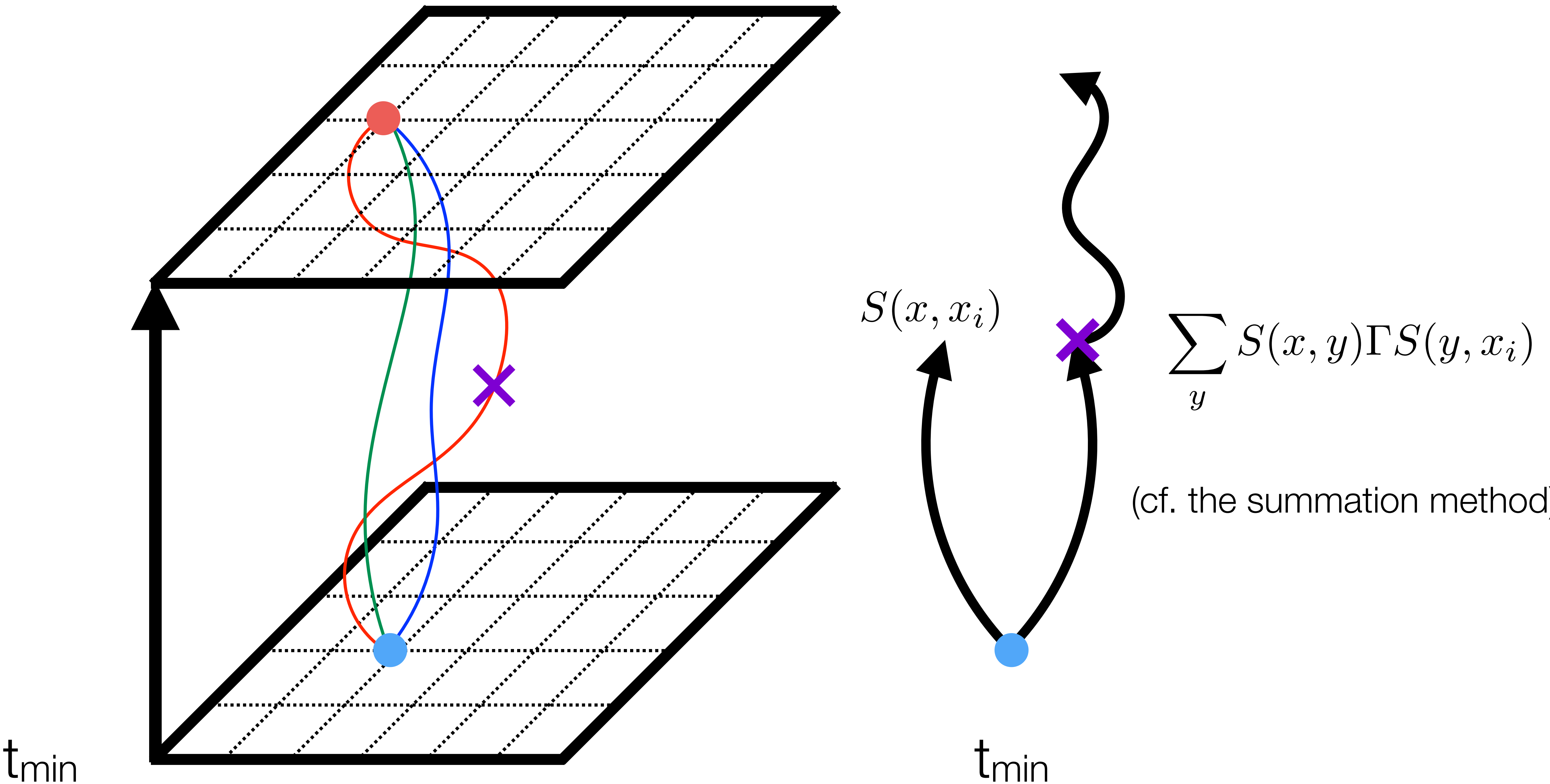

Feynman-Hellman Method

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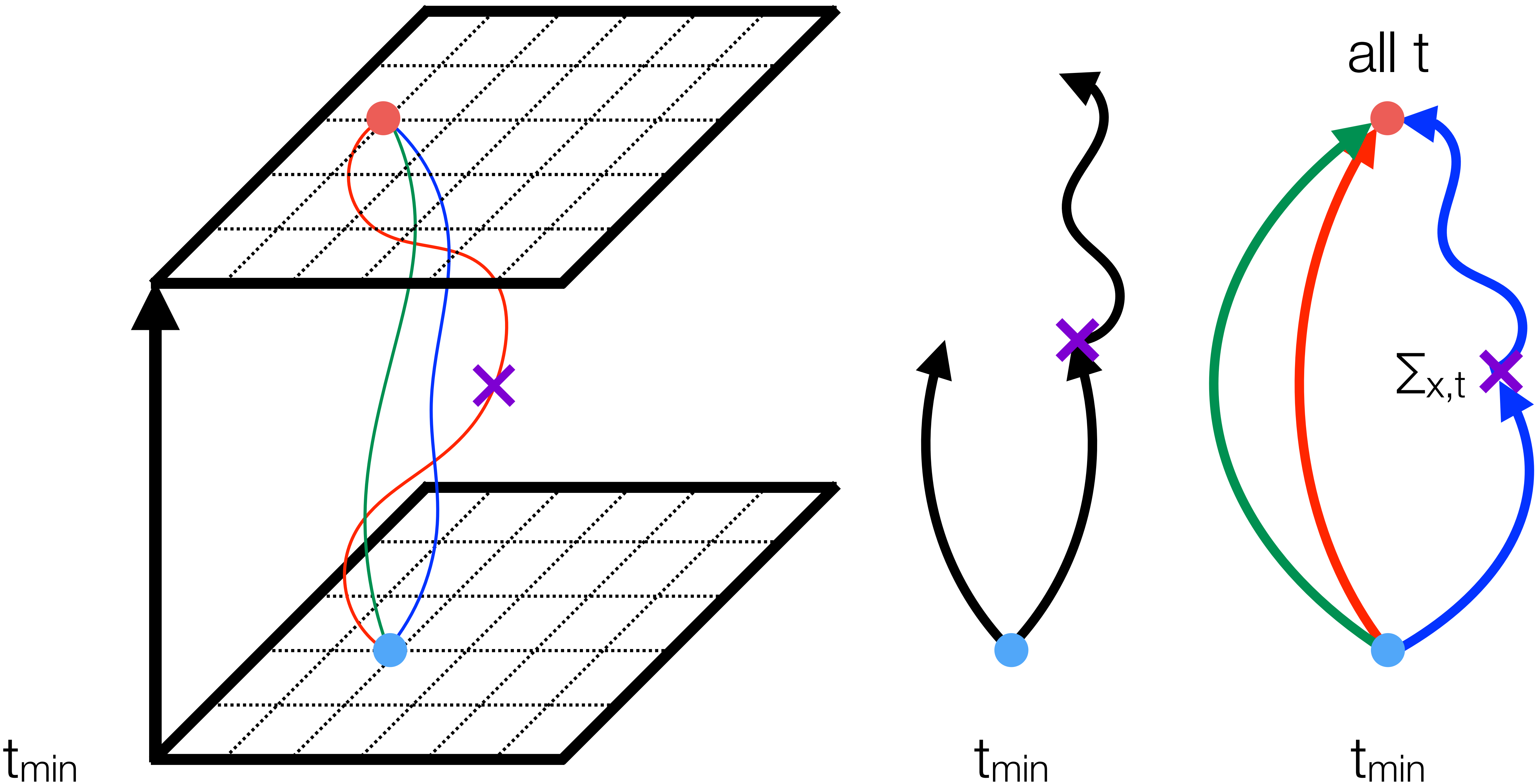
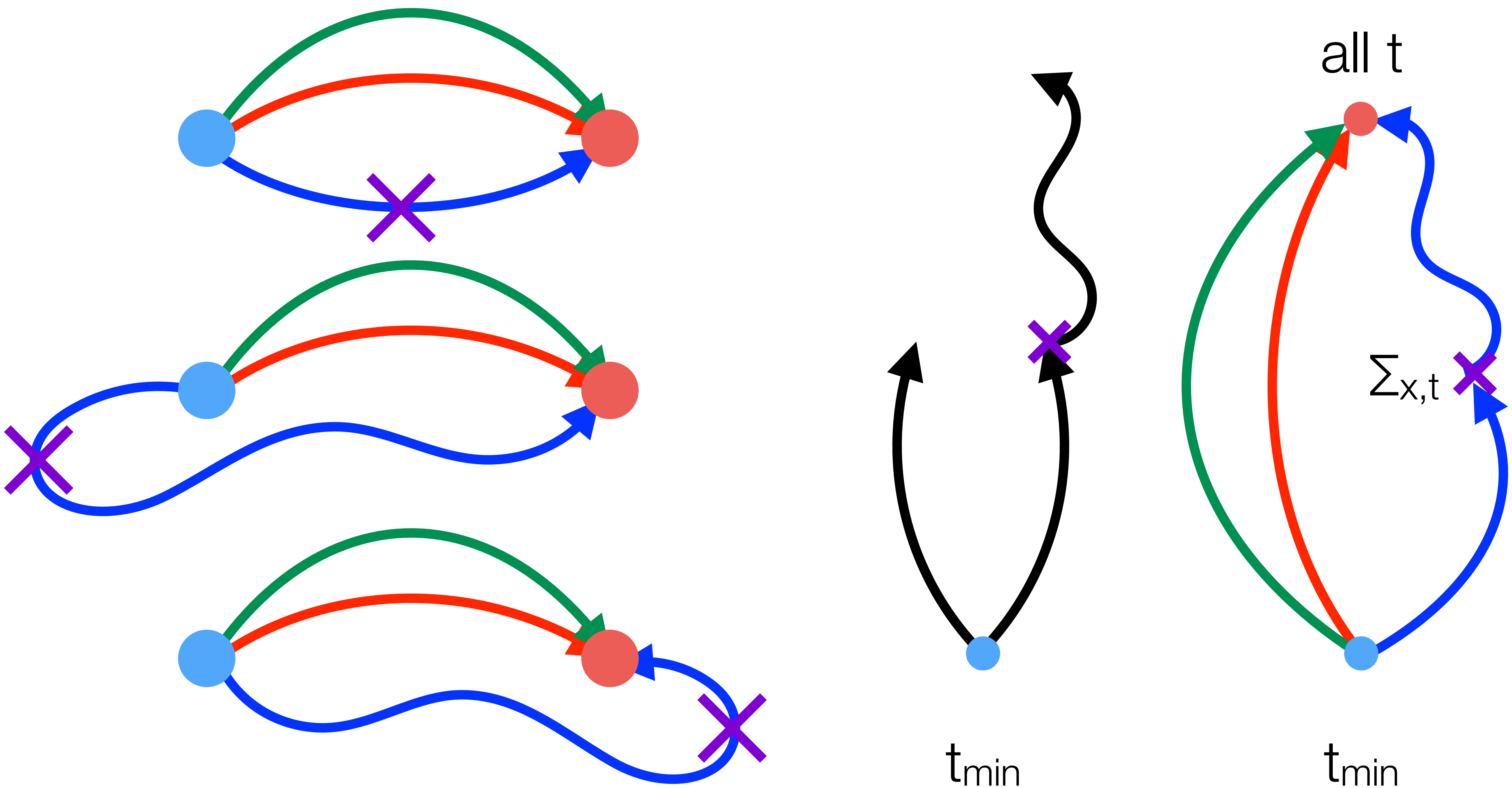


Improved systematics

Bouchard, Chang, Kurth, Orginos, and Walker-Loud arXiv:1612.06963



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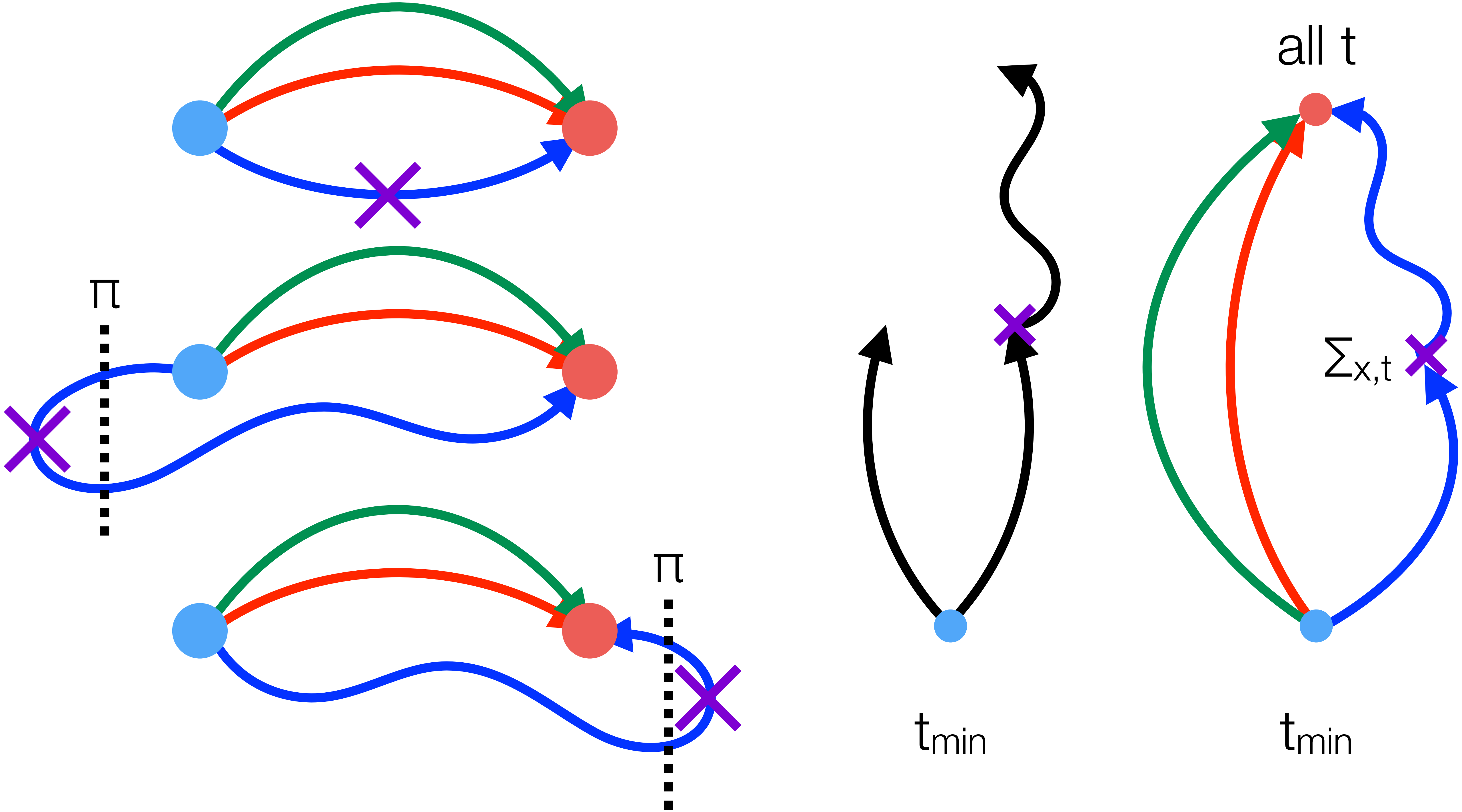

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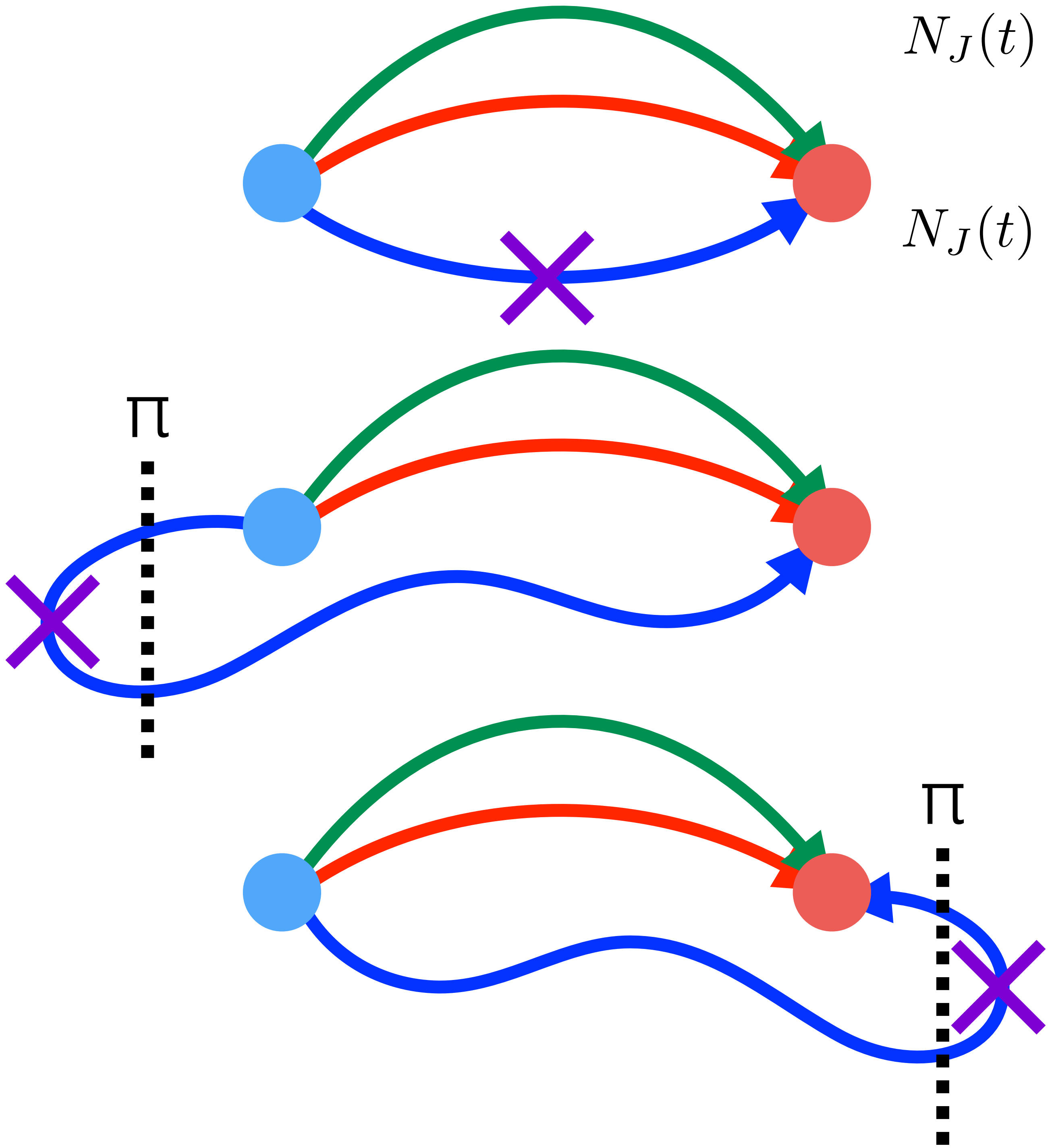


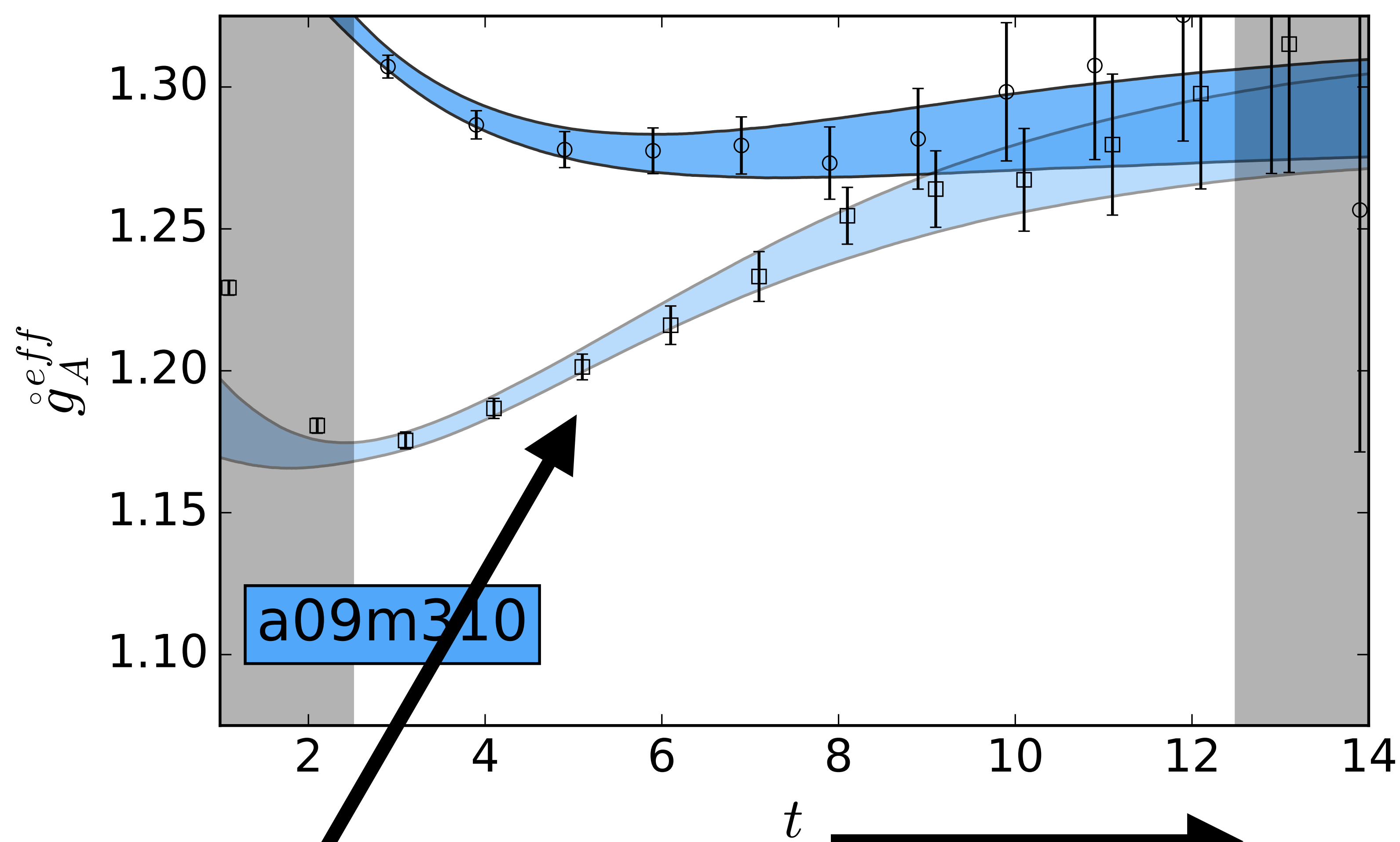
$$N_J(t) = \sum_{t'} \langle \Omega | T\{O(t) J(t') O^\dagger(0)\} | \Omega \rangle$$

$$N_J(t) = \sum_n \left[(t-1) z_n g^J_{nn} z^\dagger_n + d^J_n\right] e^{-E_n t}$$

$$+ \sum_{\substack{n \\ m \neq n}} z_n g^J_{nm} z^\dagger_m \frac{e^{-E_n t + \frac{\Delta_{nm}}{2}} - e^{-E_m t + \frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{nm}}{2}} - e^{\frac{\Delta_{mn}}{2}}}$$

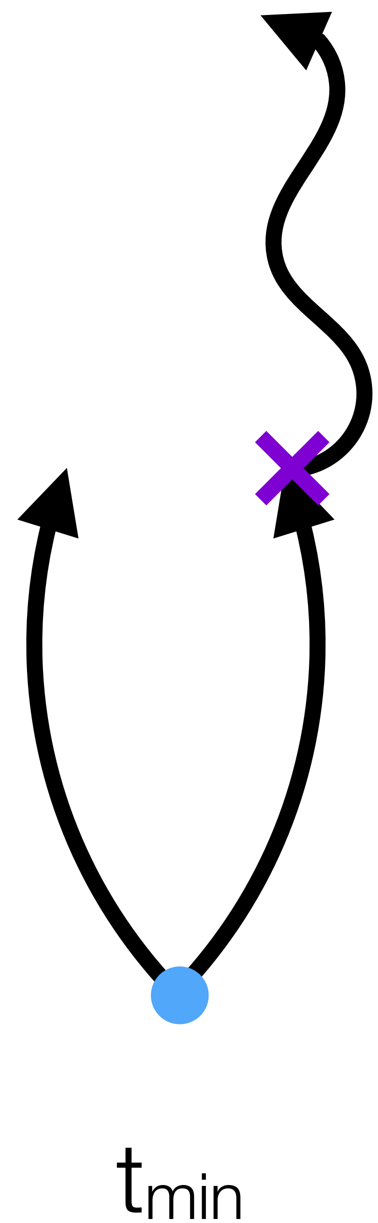
Example Effective Matrix Element

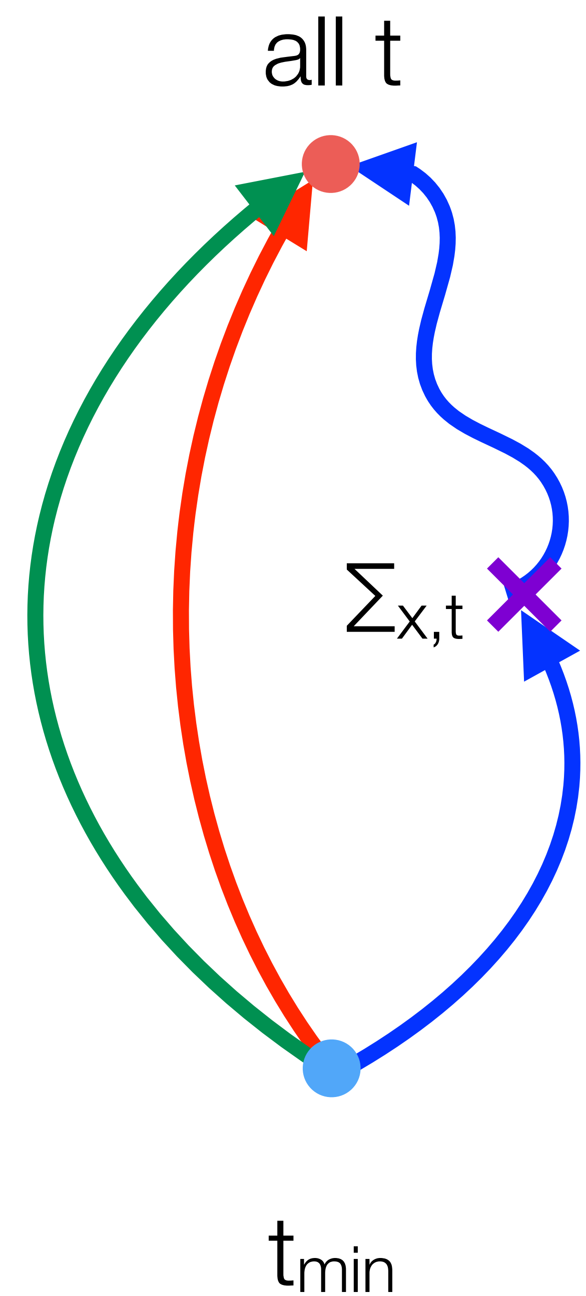
arXiv:1704.01114

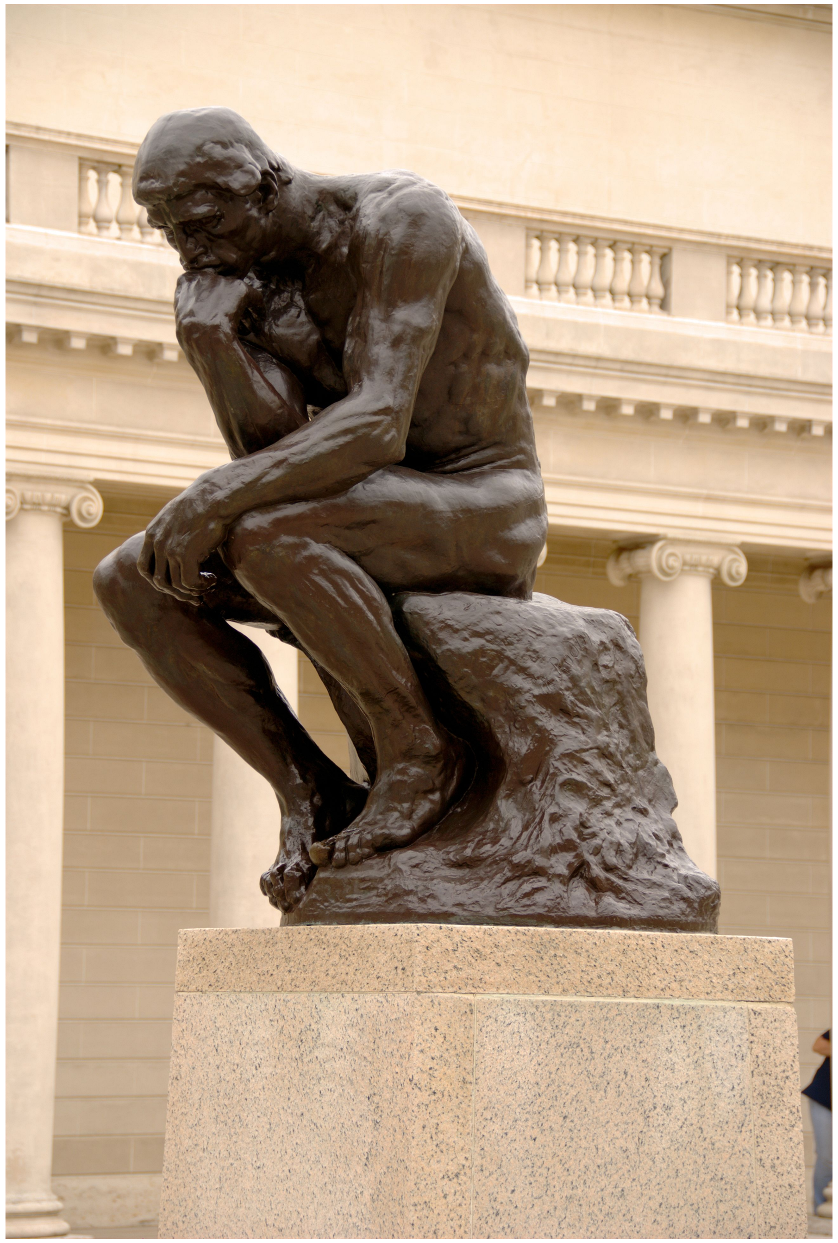


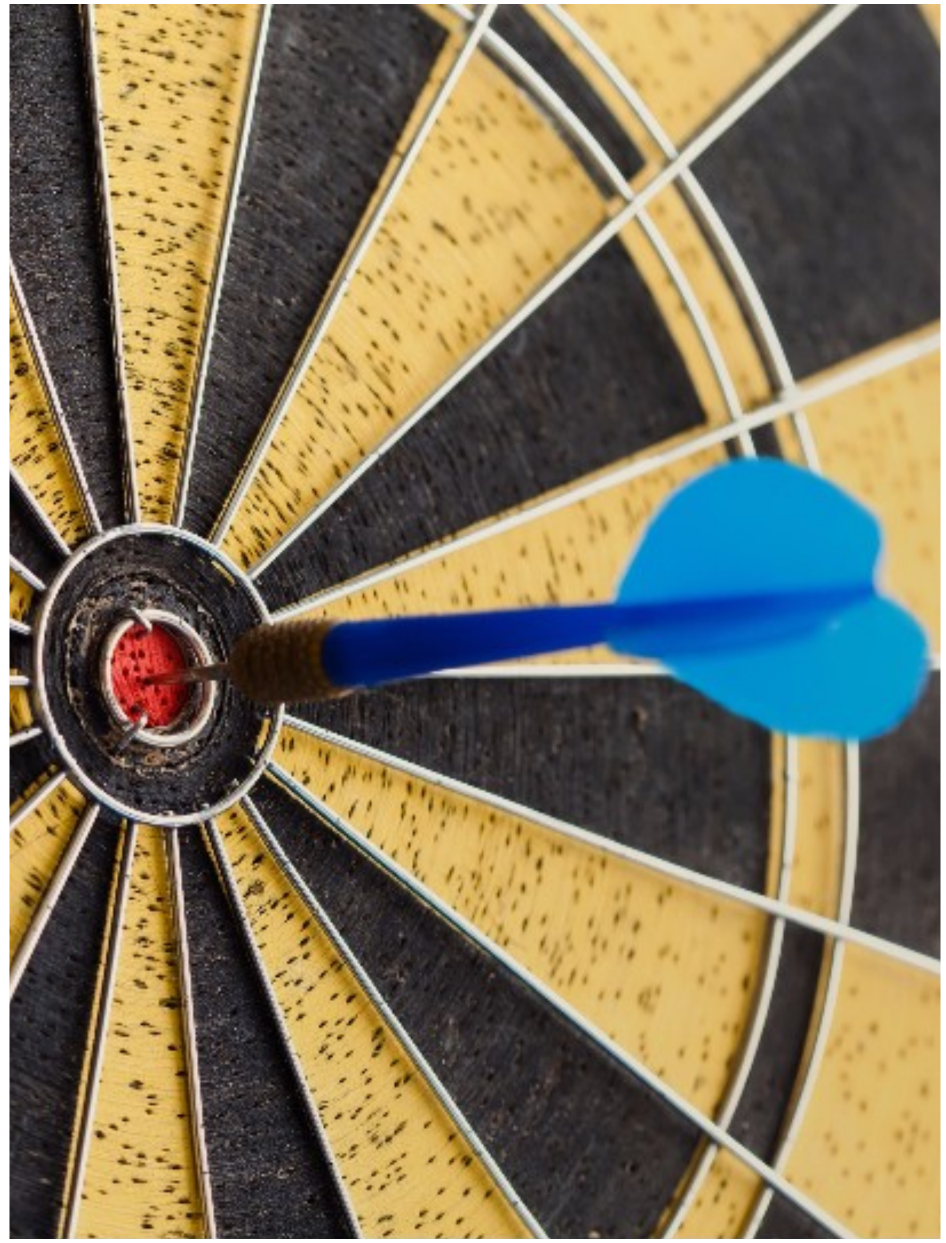
known functional form

asymptotes in just one time variable









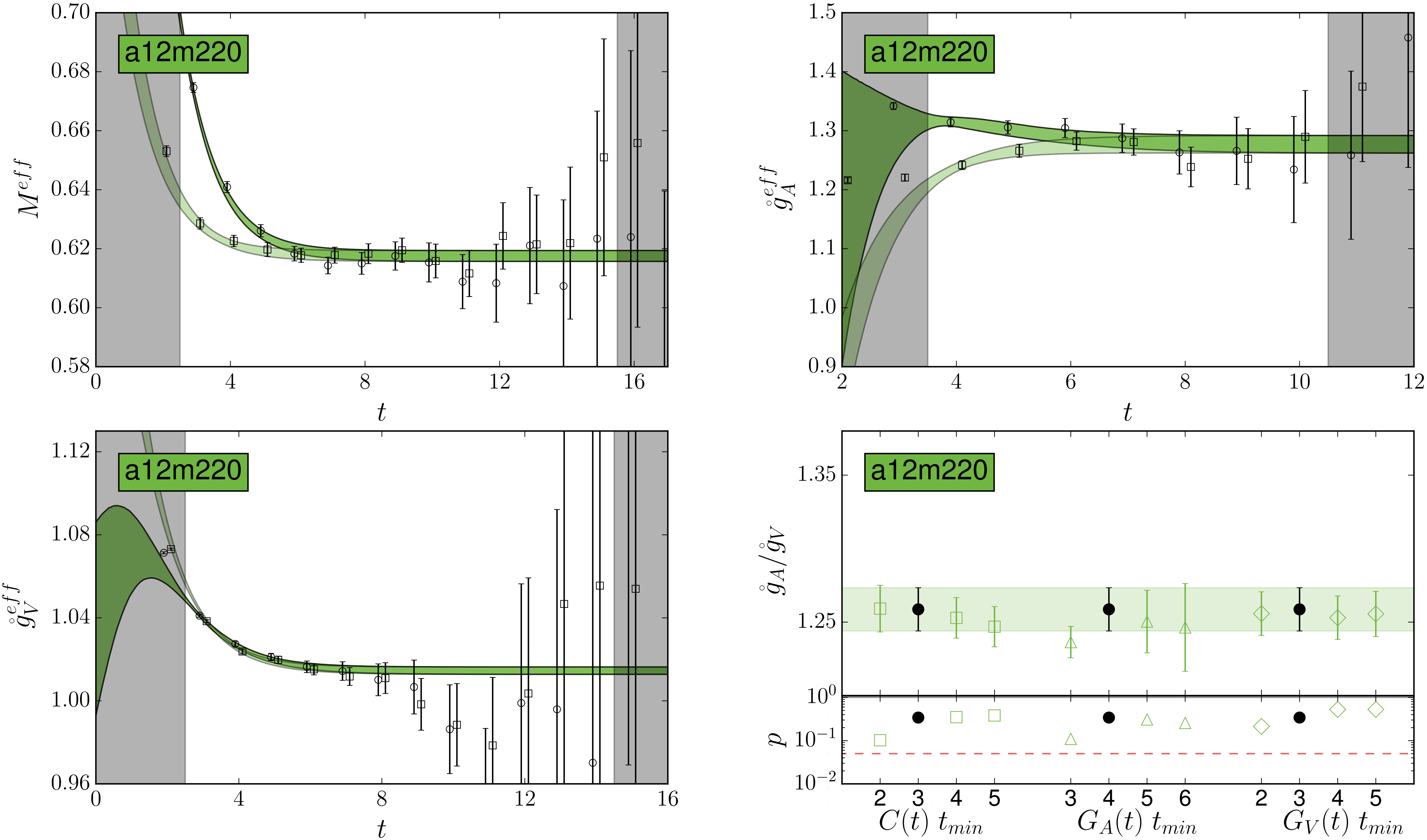
- Not QCD Specific
- Any fermion bilinear matrix element

- 3-point → 2-point function: easier fits
- Known spectral decomposition

- Stochastic enhancement
- 3/2 the cost of one temporal separation

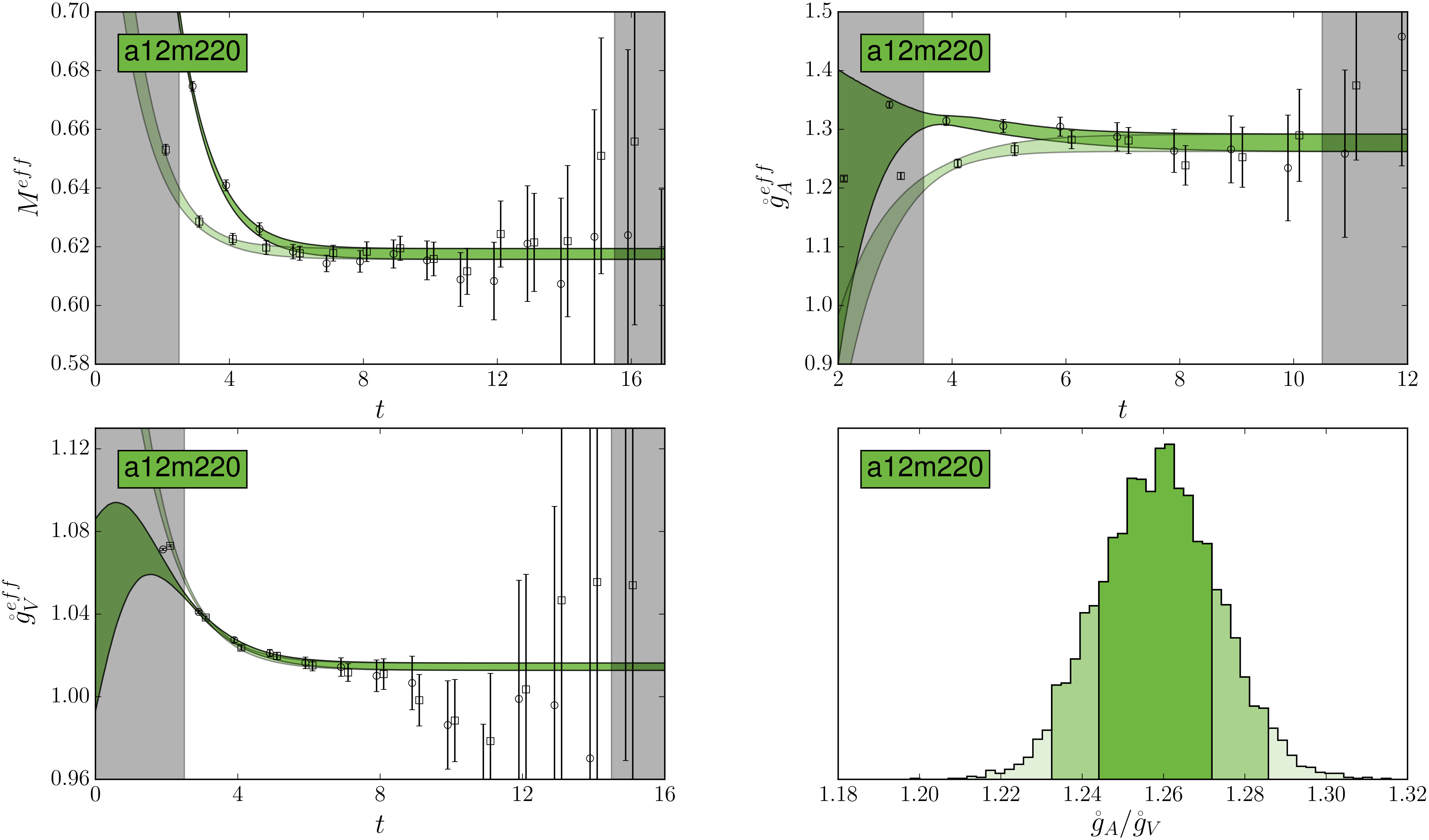
Systematics for an example point

arXiv:1704.01114



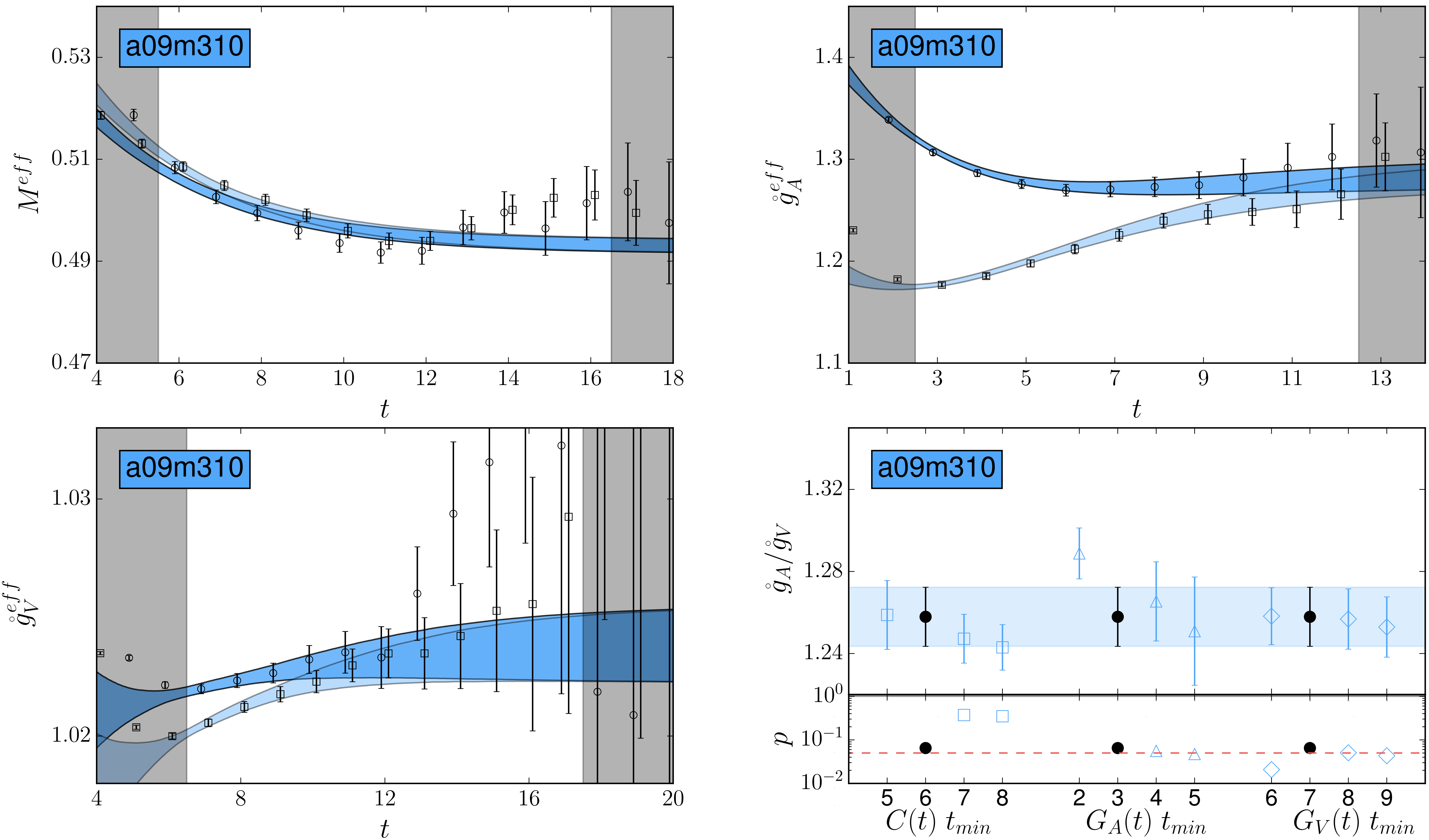
Systematics for an example point

arXiv:1704.01114



Another example point

arXiv:1704.01114



Models

arXiv:1704.01114

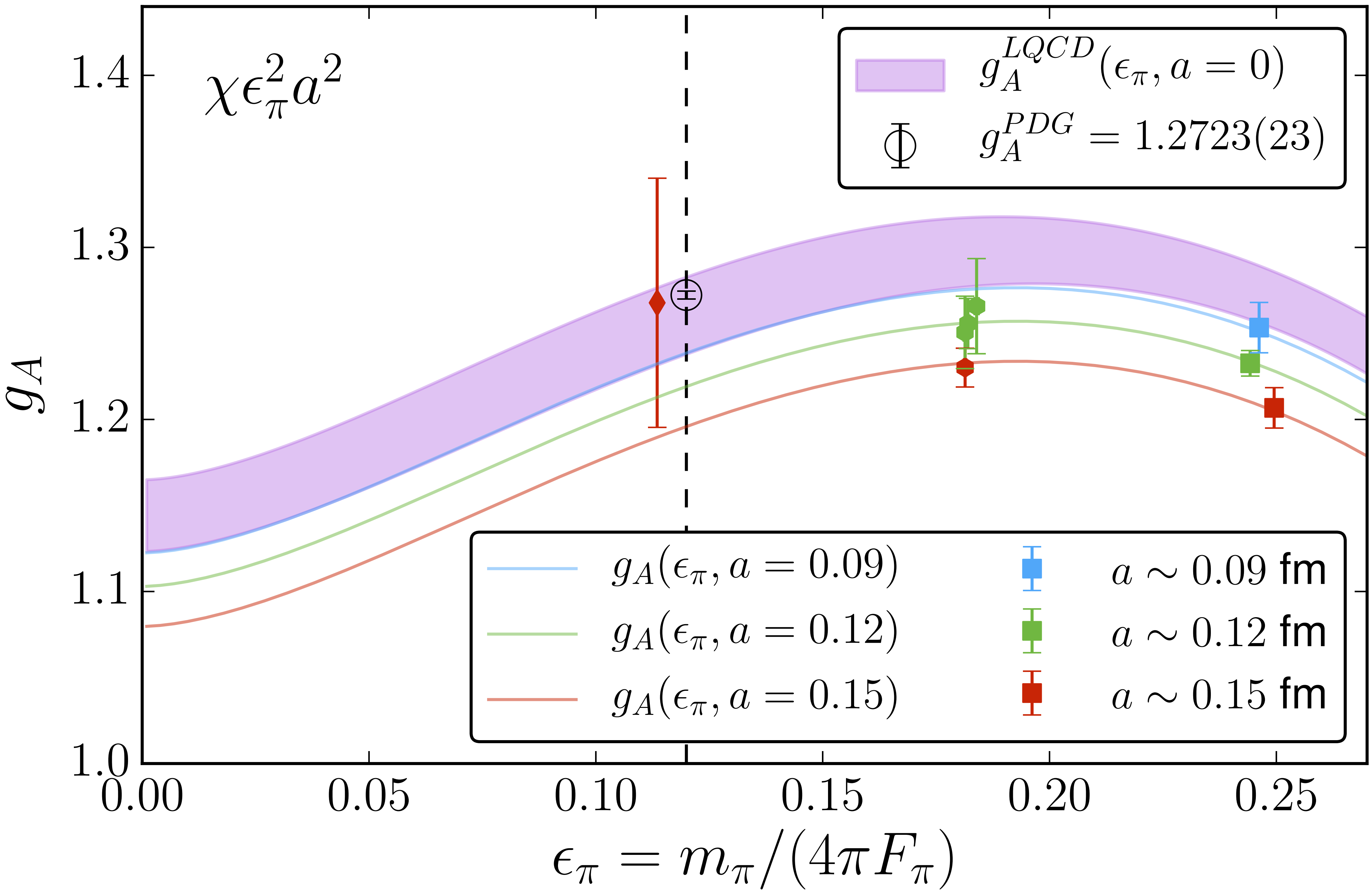
$$\epsilon_\pi = \frac{m_\pi}{4\pi f_\pi} \qquad \delta_a = c_{2a}\frac{a^2}{w_0^2}$$

Physics	Finite Volume	m_π dependence	lattice spacing dependence
Taylor Expansion	independent	ϵ_π^0	a^0
Chiral Perturbation Theory	coefficients reappear	ϵ_π^2	a^2 $\alpha_S a^2$

$$\begin{aligned}\delta_L &\equiv g_A(L) - g_A(\infty)\\ &= \frac{8}{3}\epsilon_\pi^2\left[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)\right]\end{aligned}$$

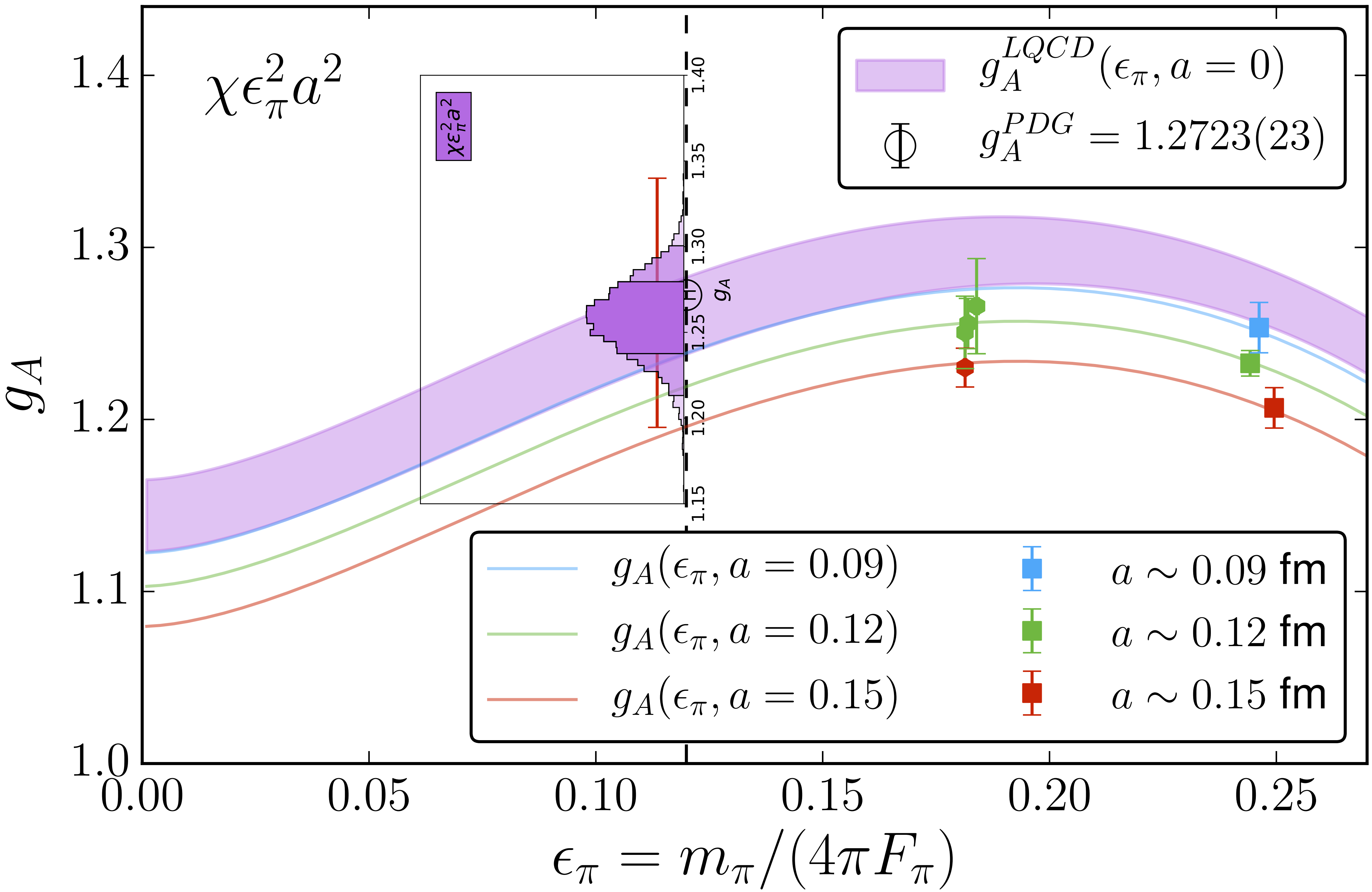
Chiral Extrapolation

arXiv:1704.01114



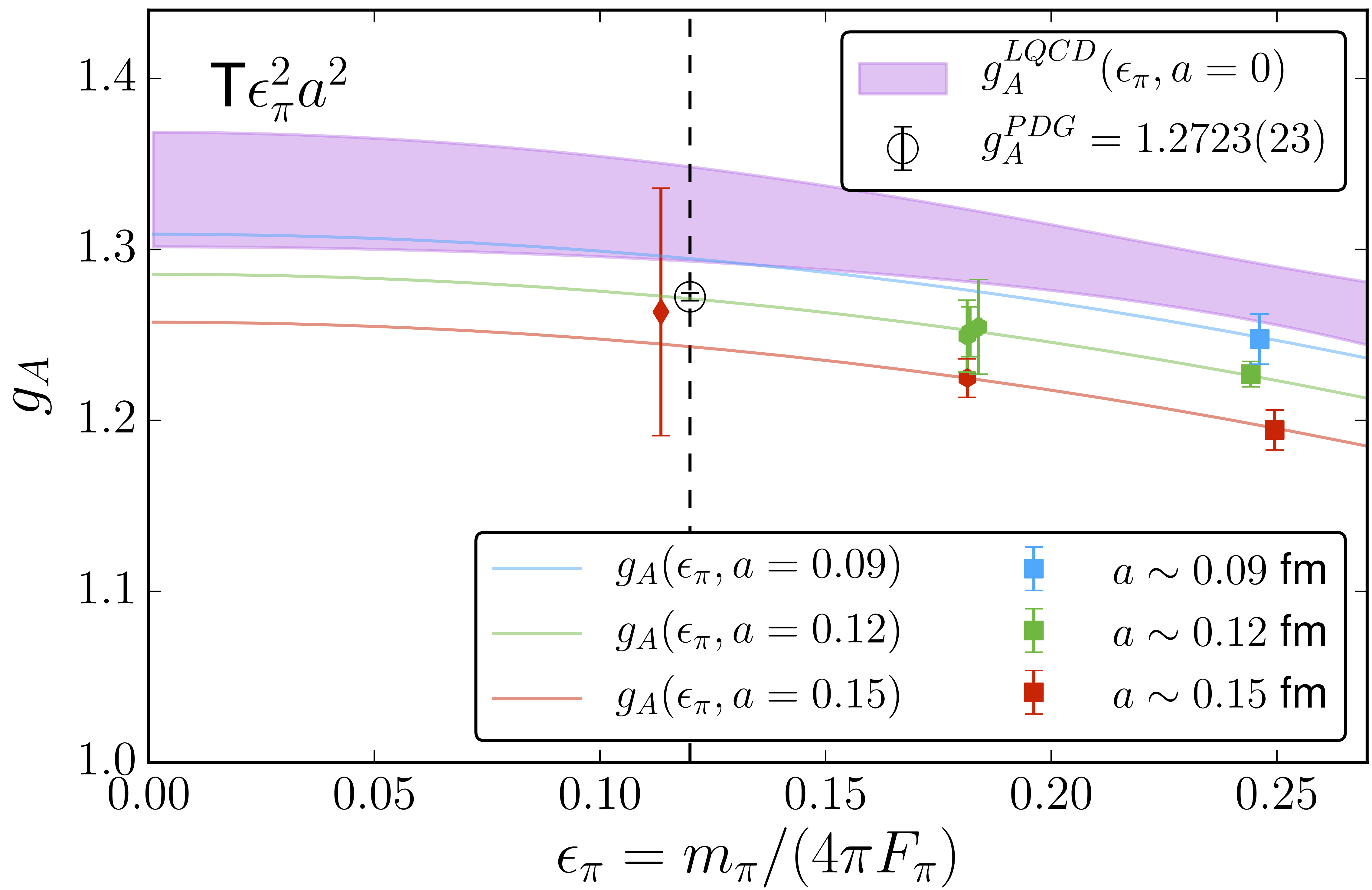
Chiral Extrapolation

arXiv:1704.01114



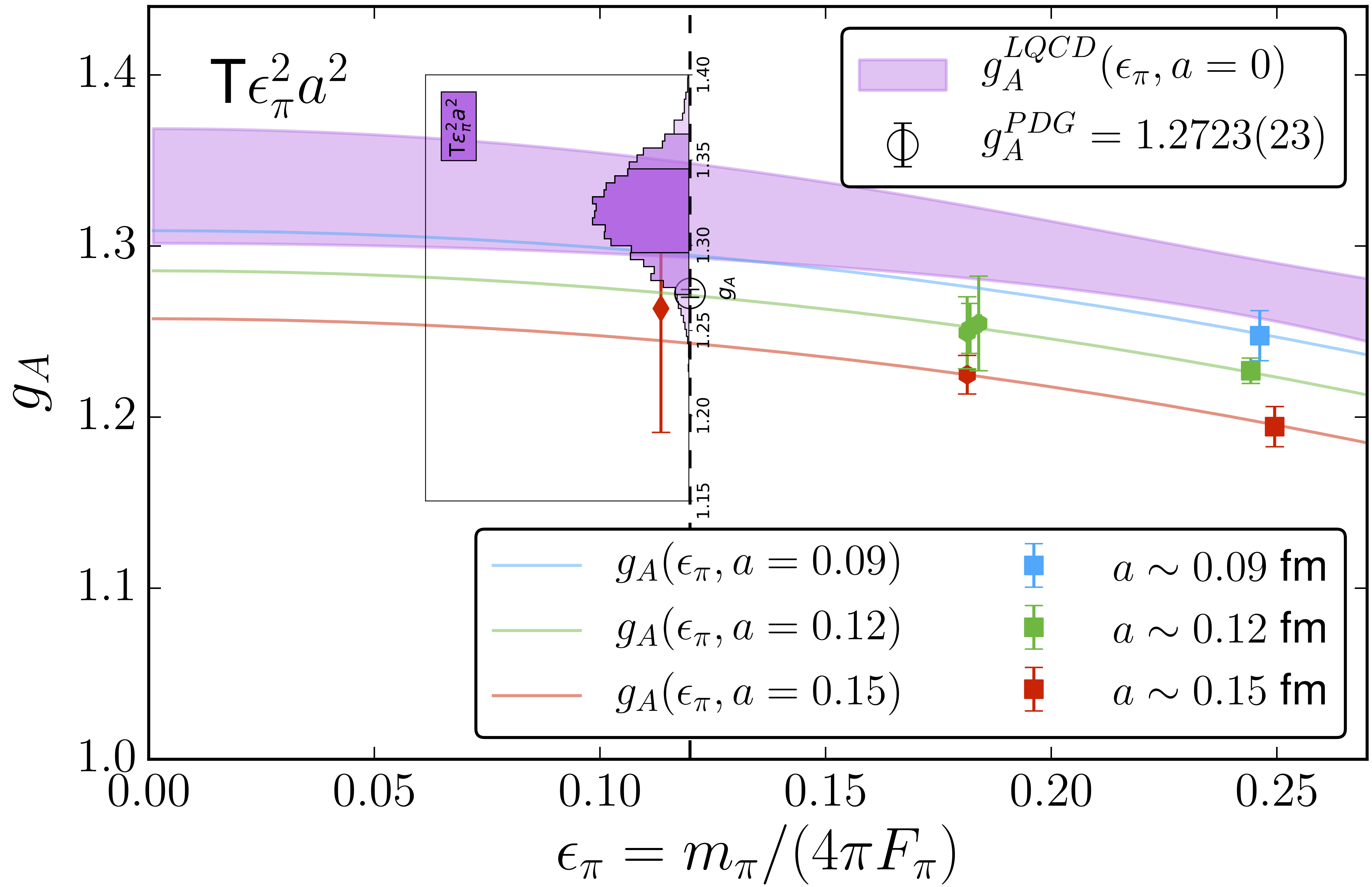
Taylor Series Extrapolation

arXiv:1704.01114



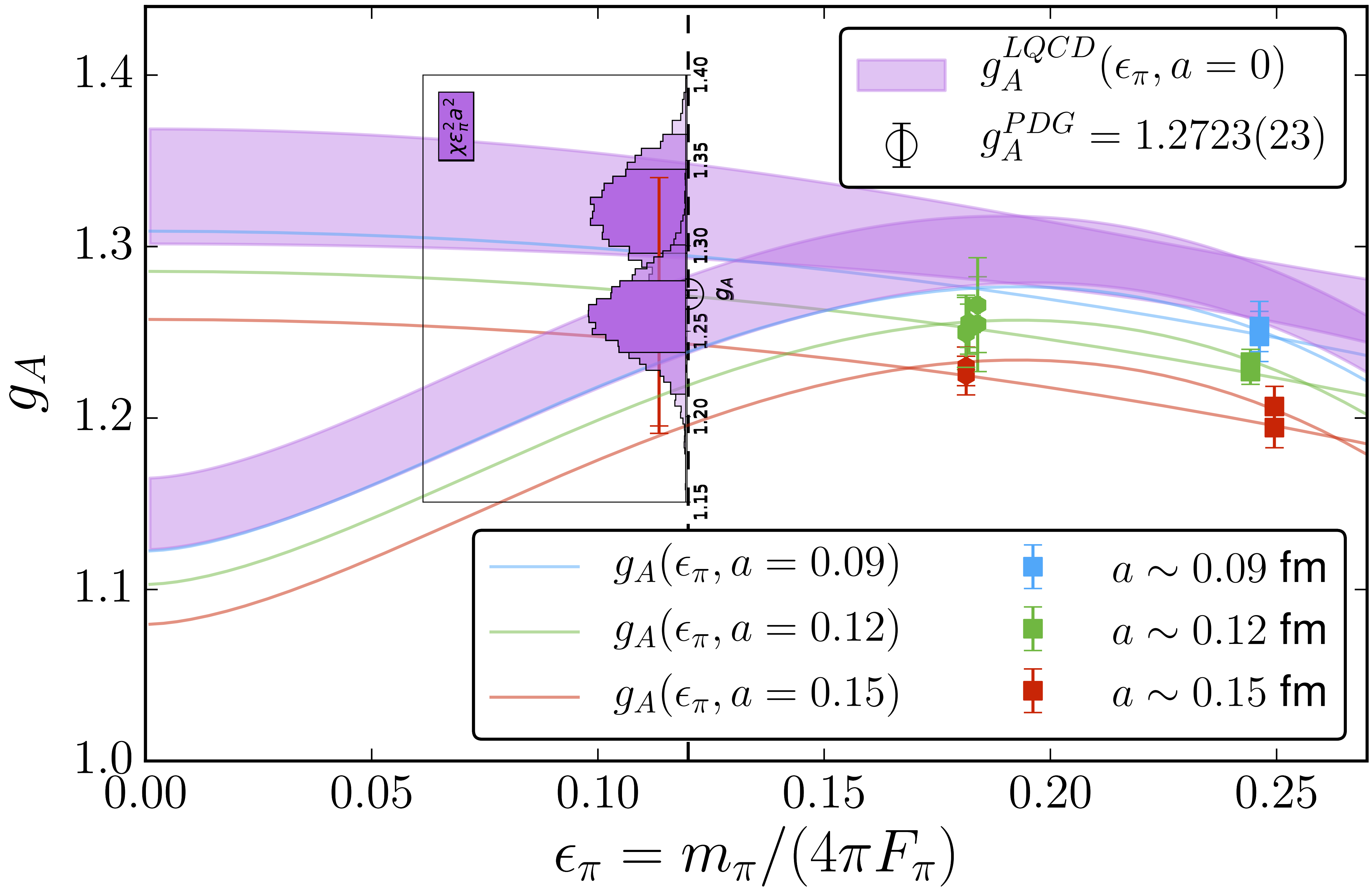
Taylor Series Extrapolation

arXiv:1704.01114



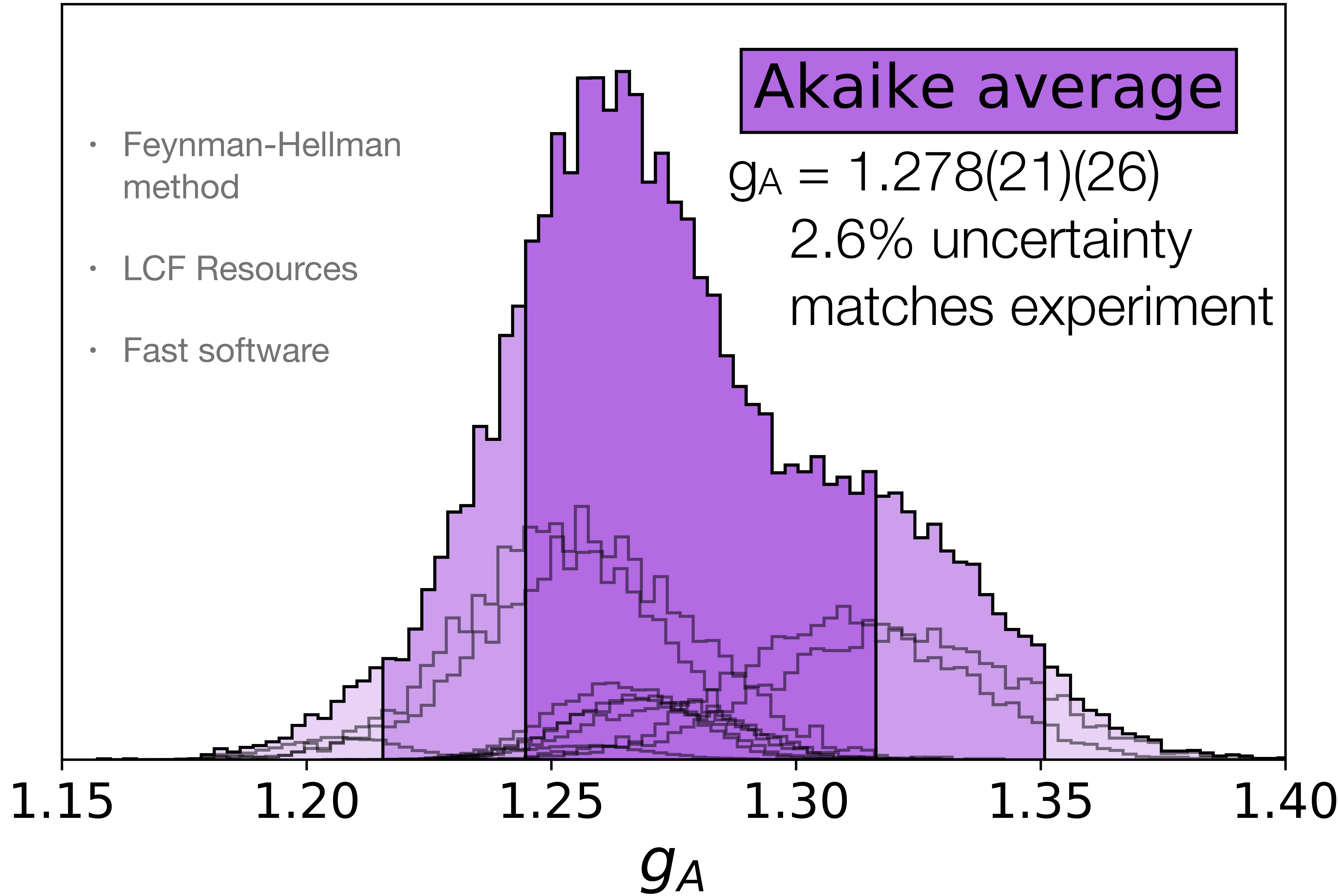
Model Comparison

arXiv:1704.01114



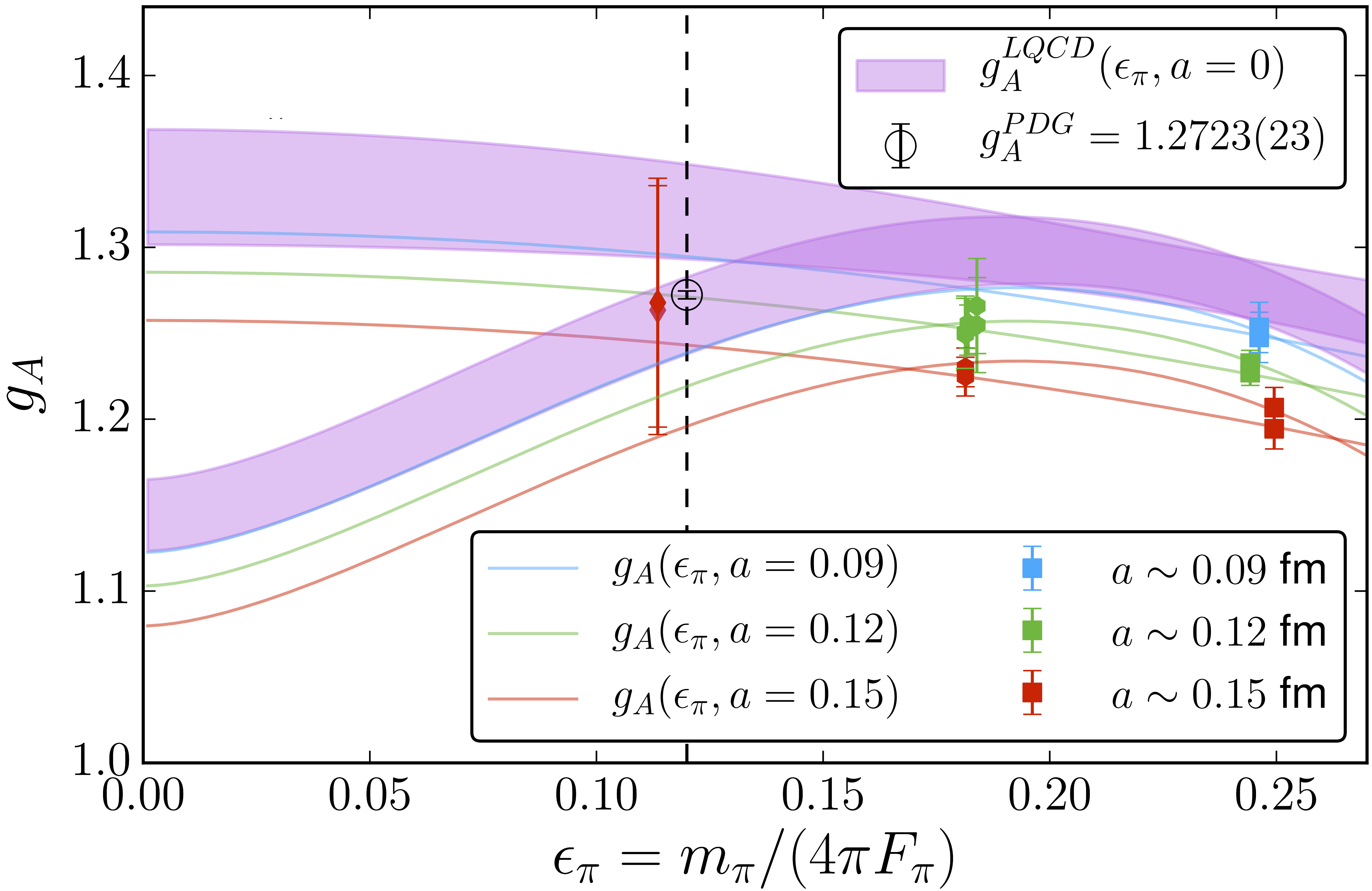
arXiv:1704.01114

- Feynman-Hellman method
- LCF Resources
- Fast software



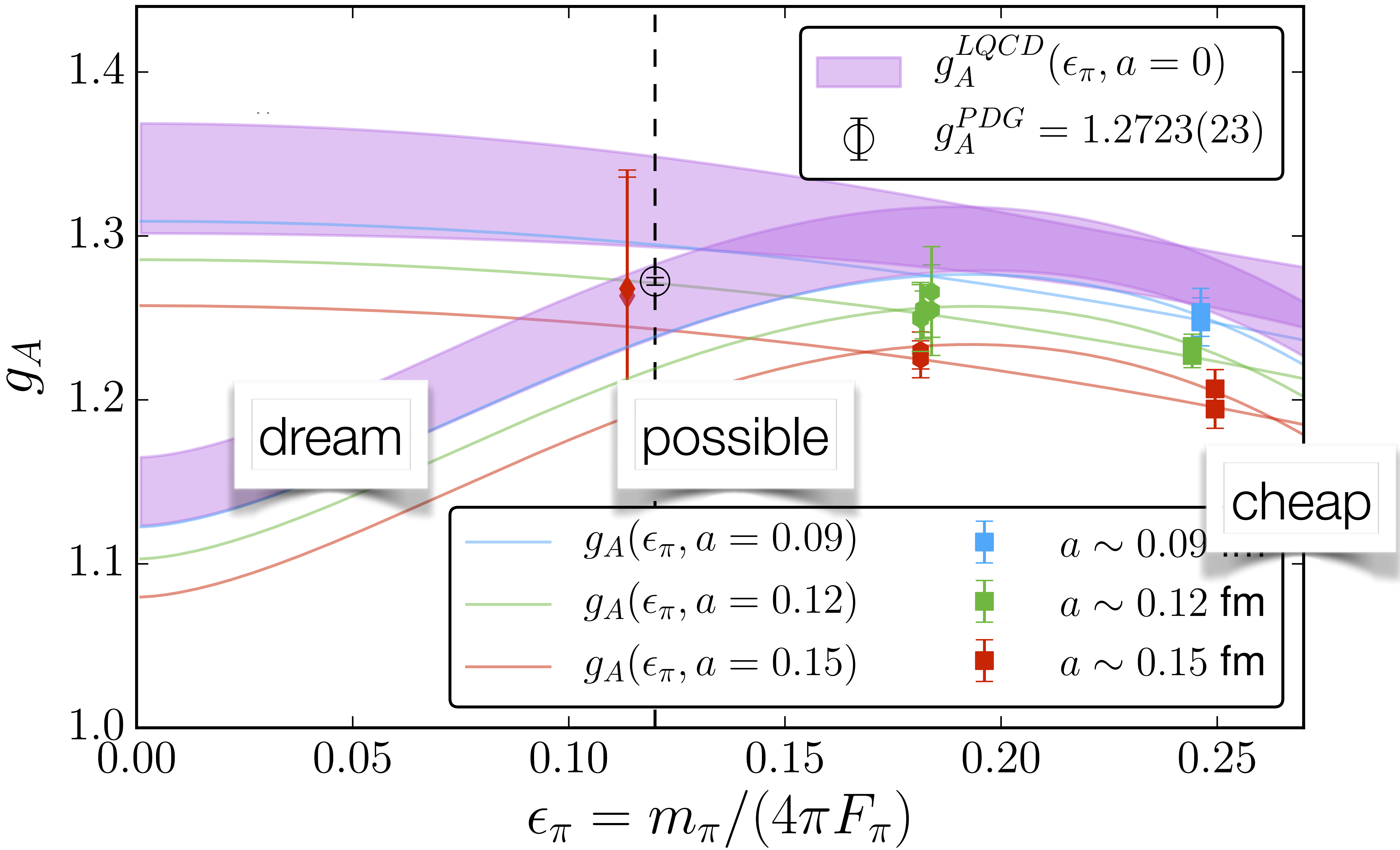
Towards 1% uncertainty

arXiv:1704.01114



Towards 1% uncertainty

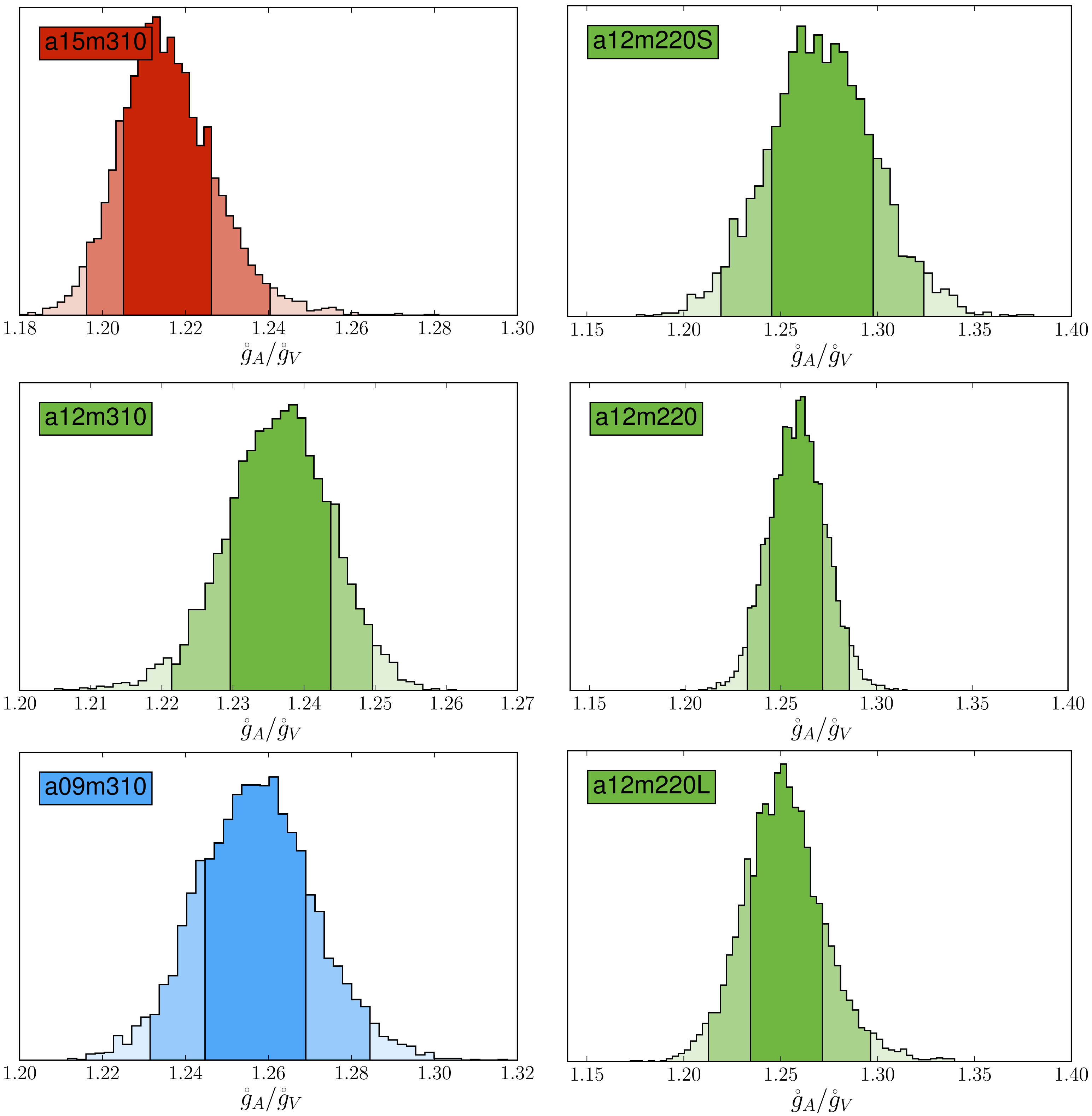
arXiv:1704.01114

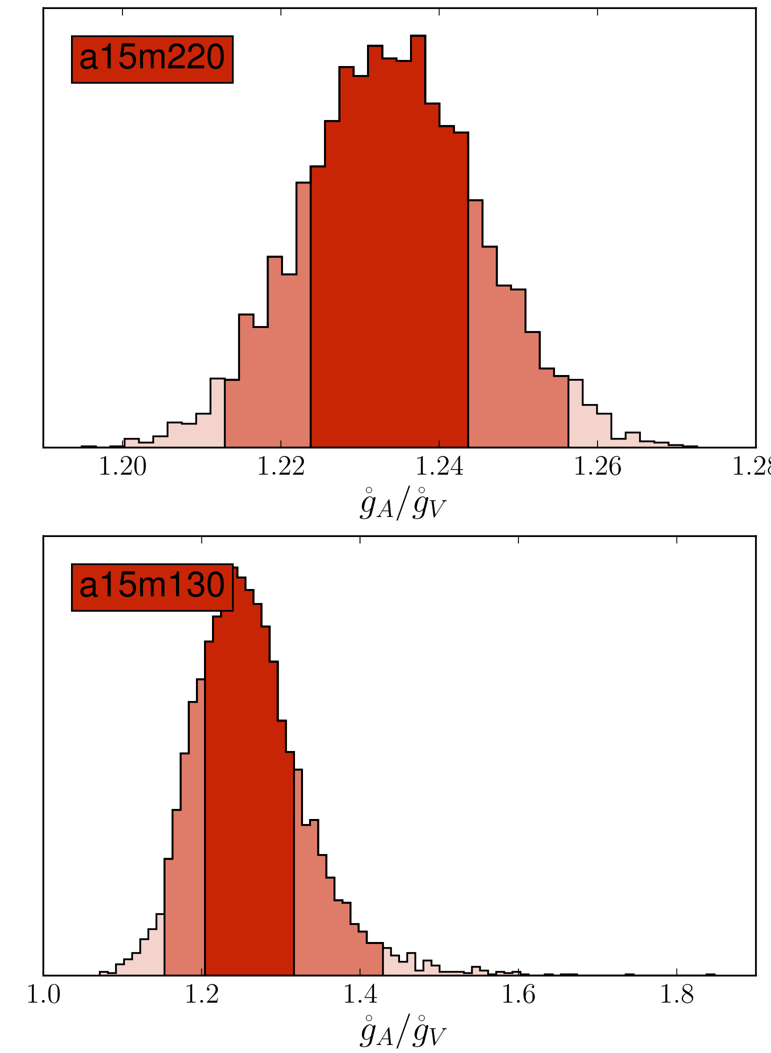


Backup Slides

All Ensembles

arXiv:1704.01114

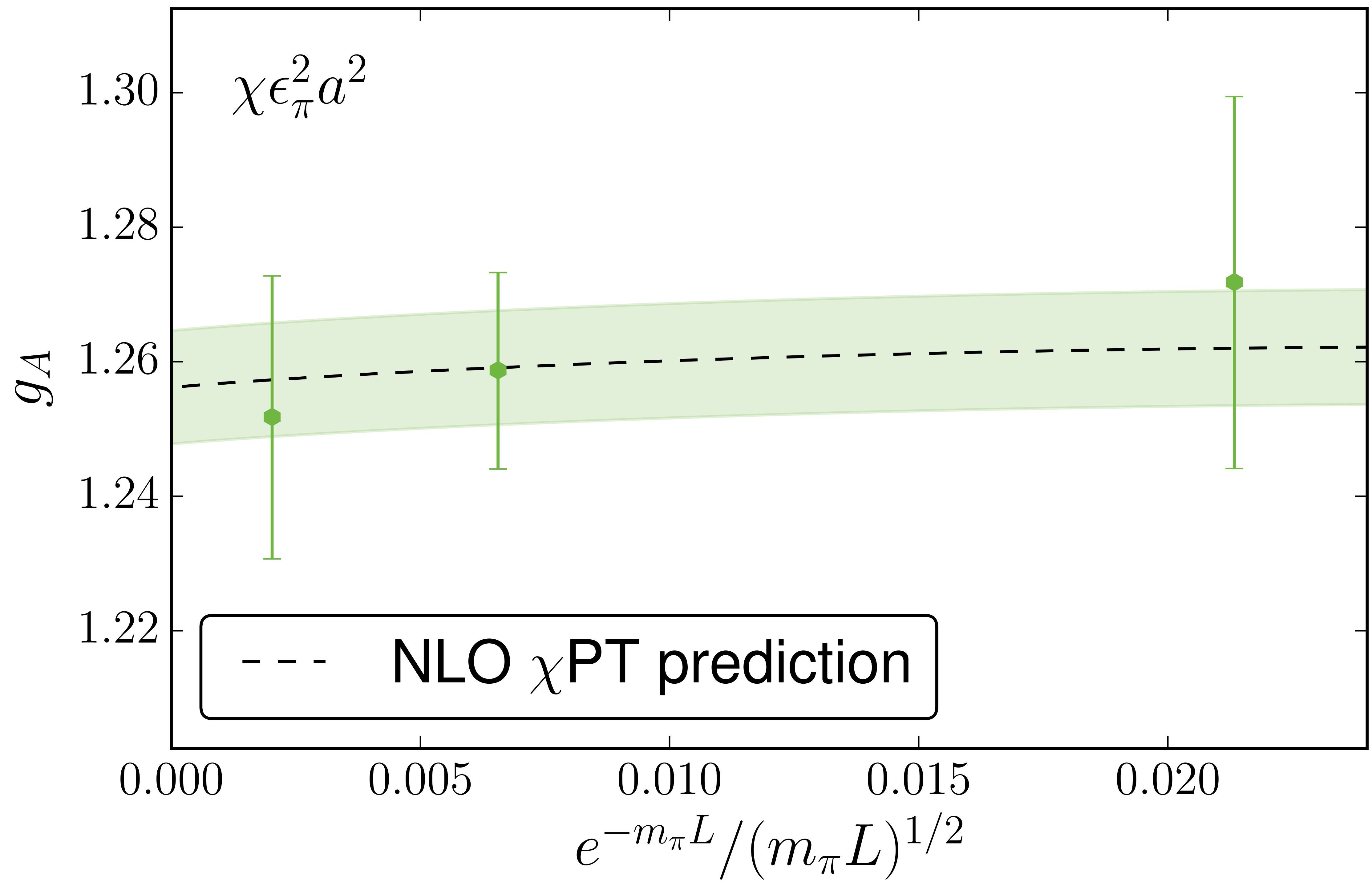




Good stability
under bootstrap

Infinite Volume Extrapolation: a12m220

arXiv:1704.01114



Model

$$g_A = c_0 + \delta_a + \delta_L \ ,$$
$$g_A = c_0 + \alpha_S \delta_a + \delta_L \ ,$$
$$g_A = c_0 + c_2 \epsilon_\pi^2 + \delta_L \ ,$$
$$g_A = c_0 + c_2 \epsilon_\pi^2 + \delta_a + \delta_L \ ,$$
$$g_A = c_0 + c_2 \epsilon_\pi^2 + \alpha_S \delta_a + \delta_L \ ,$$
$$g_A = g_0 + \delta_a + \delta_L \ ,$$
$$g_A = g_0 + \alpha_S \delta_a + \delta_L \ ,$$
$$g_A = g_0 - (g_0 + 2g_0^3)\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + \delta_L \ ,$$
$$g_A = g_0 - (g_0 + 2g_0^3)\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + \delta_a + \delta_L \ ,$$
$$g_A = g_0 - (g_0 + 2g_0^3)\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + \alpha_S \delta_a + \delta_L \ ,$$

Finite Volume

$$\begin{aligned}\delta_L &\equiv g_A(L) - g_A(\infty) \\ &= \frac{8}{3}\epsilon_\pi^2 \left[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L) \right]\end{aligned}$$

Lattice Spacing

$$\delta_a = c_{2a} \frac{a^2}{w_0^2}$$

Pion mass

$$\epsilon_\pi = \frac{m_\pi}{4\pi f_\pi}$$

Model Comparison

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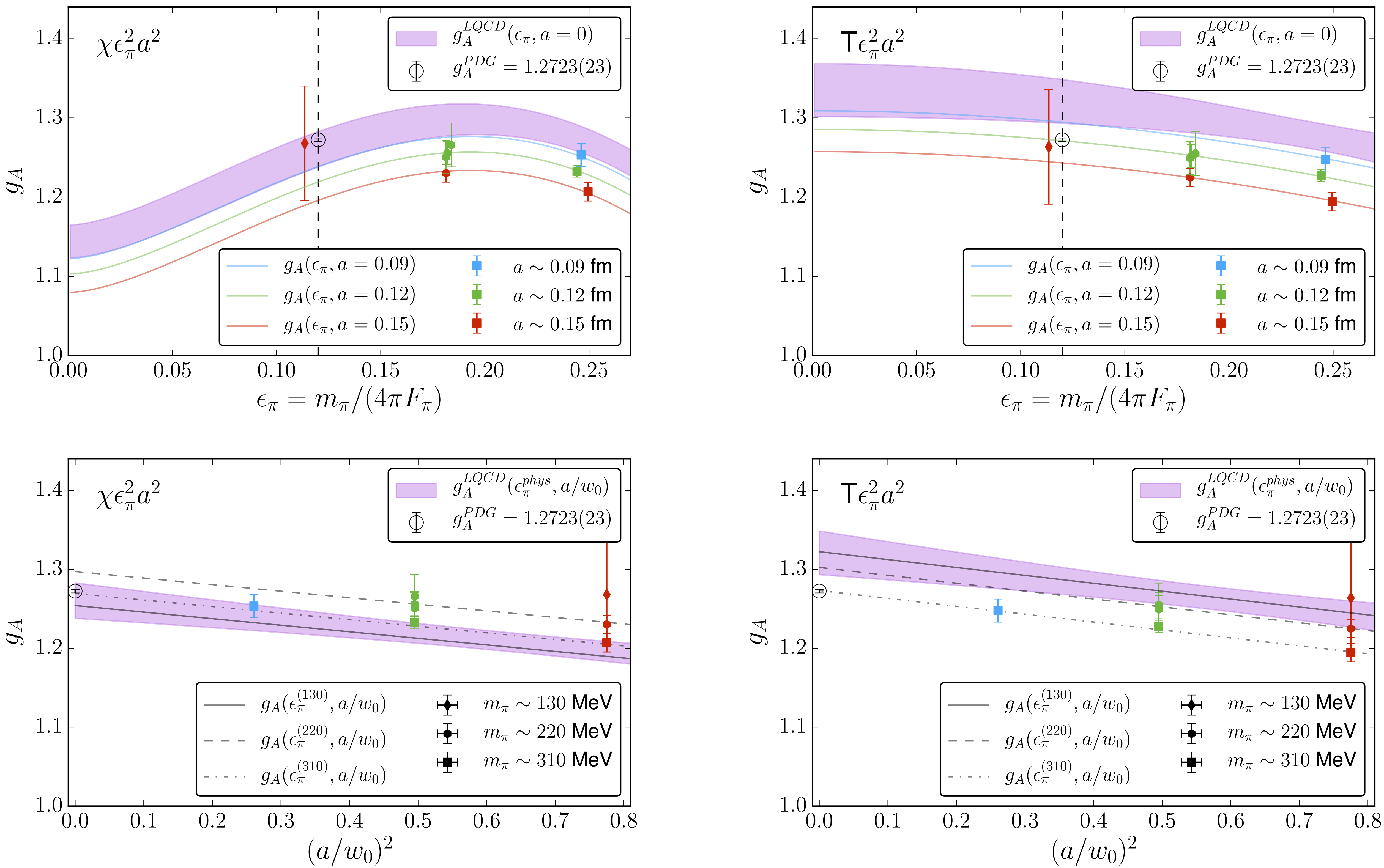
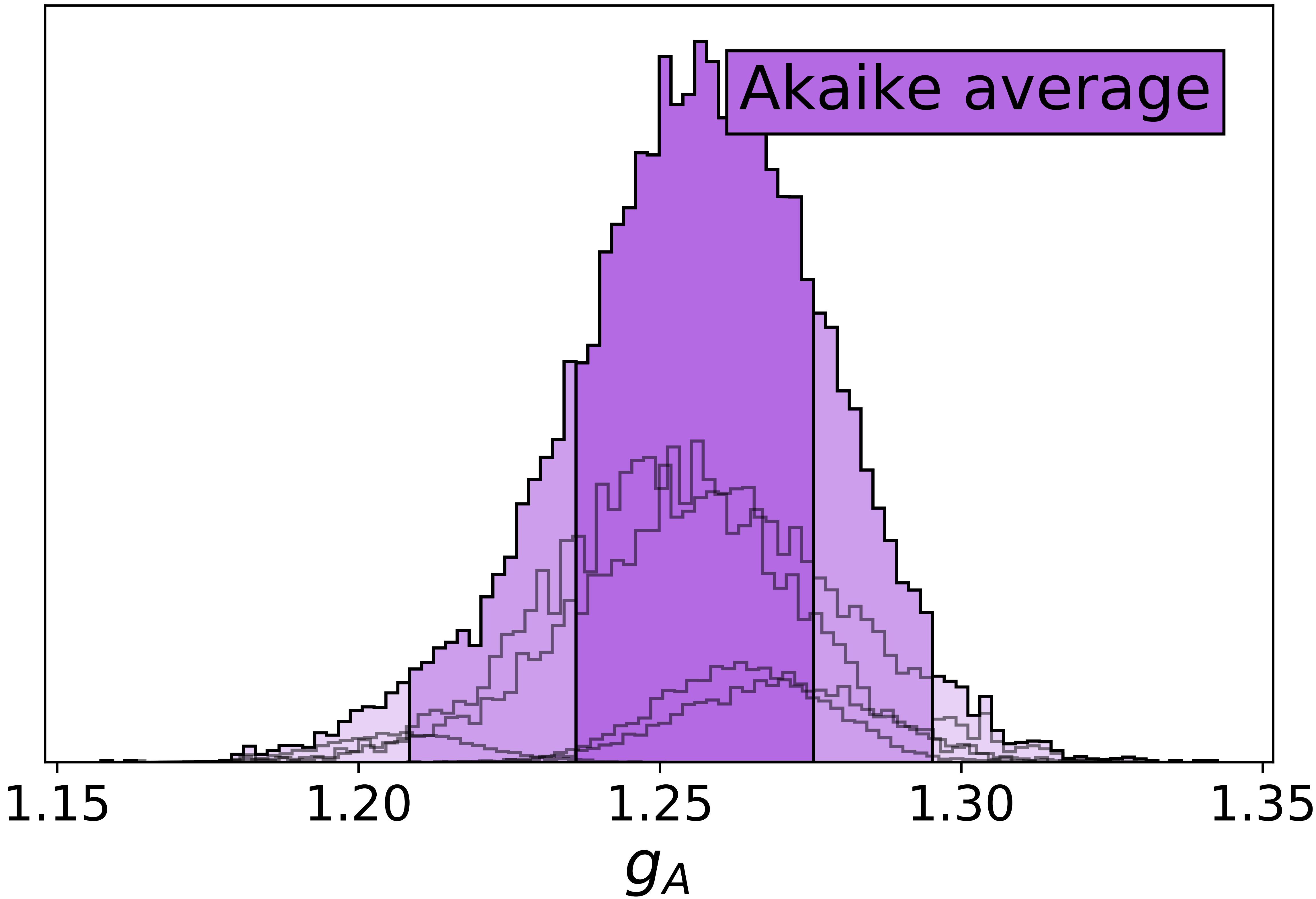
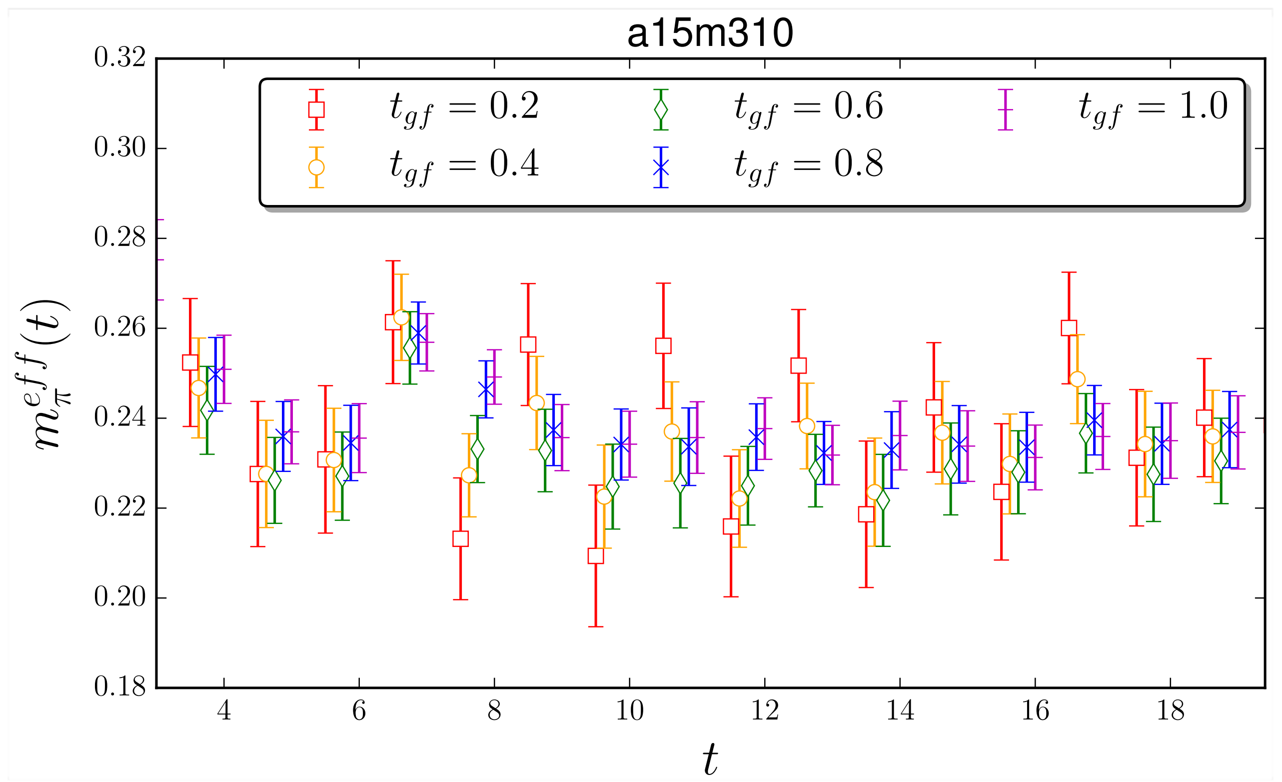


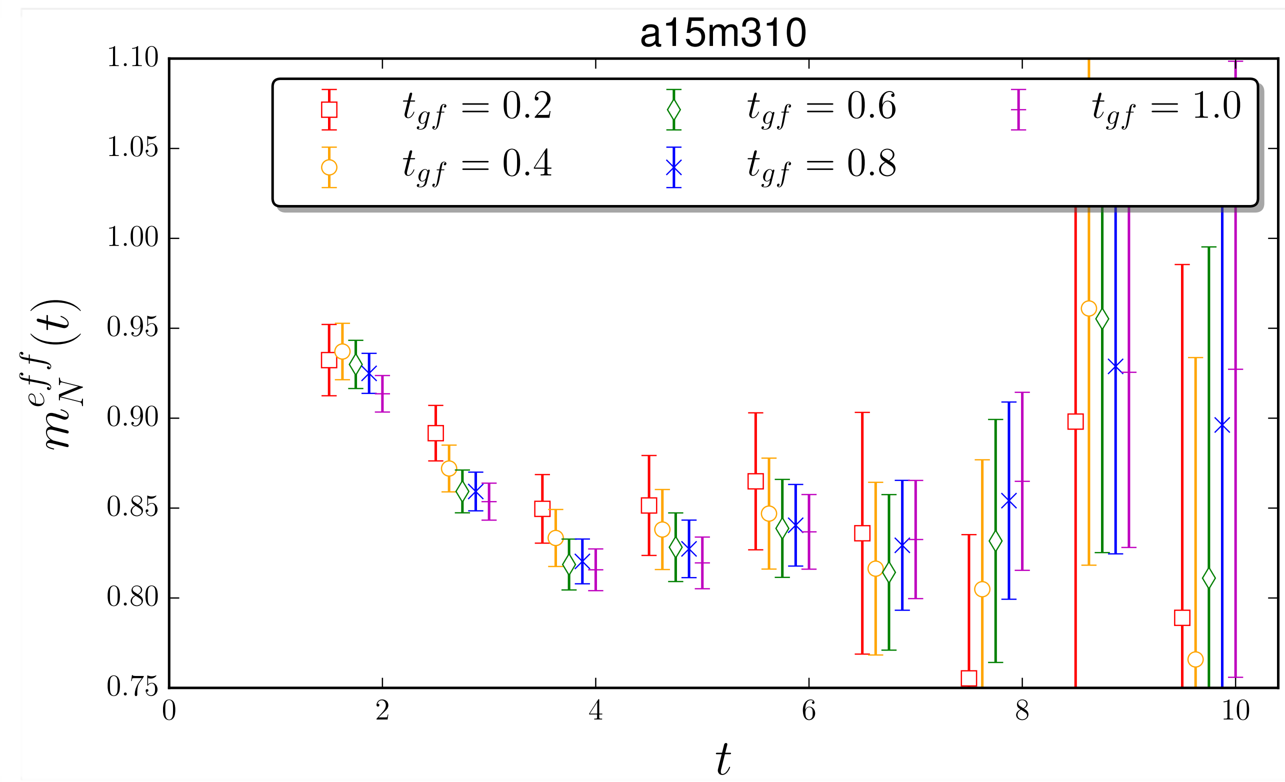
χPT only: gA=1.257(20)(09)   [1.7%]



Smearing Study

arXiv:1701.07559





Autocorrelations

arXiv:1704.01114