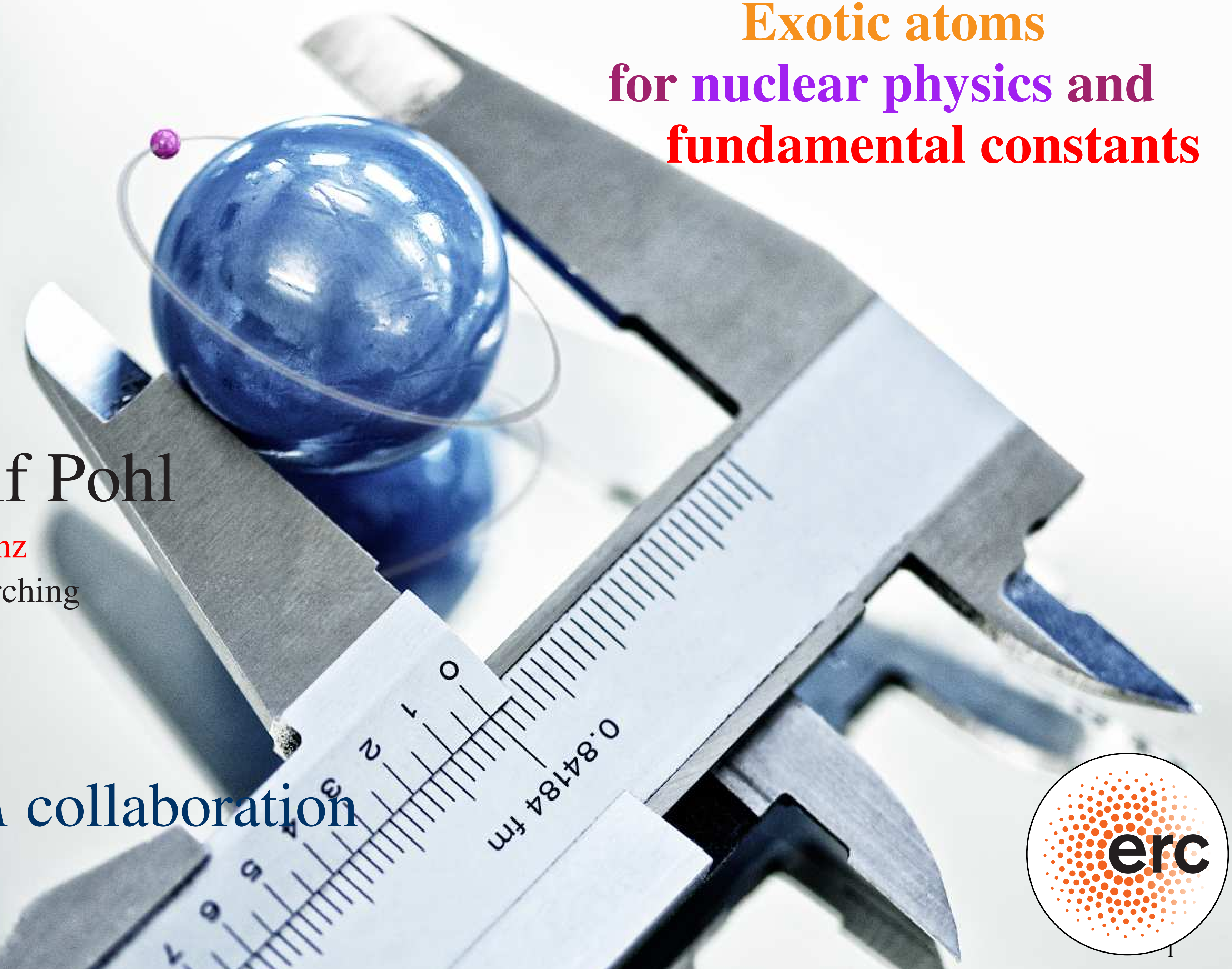
Laser spectroscopy of muonic atoms and ions

Exotic atoms
for nuclear physics and
fundamental constants



Randolf Pohl

JGU, Mainz
MPQ, Garching

for the
CREMA collaboration

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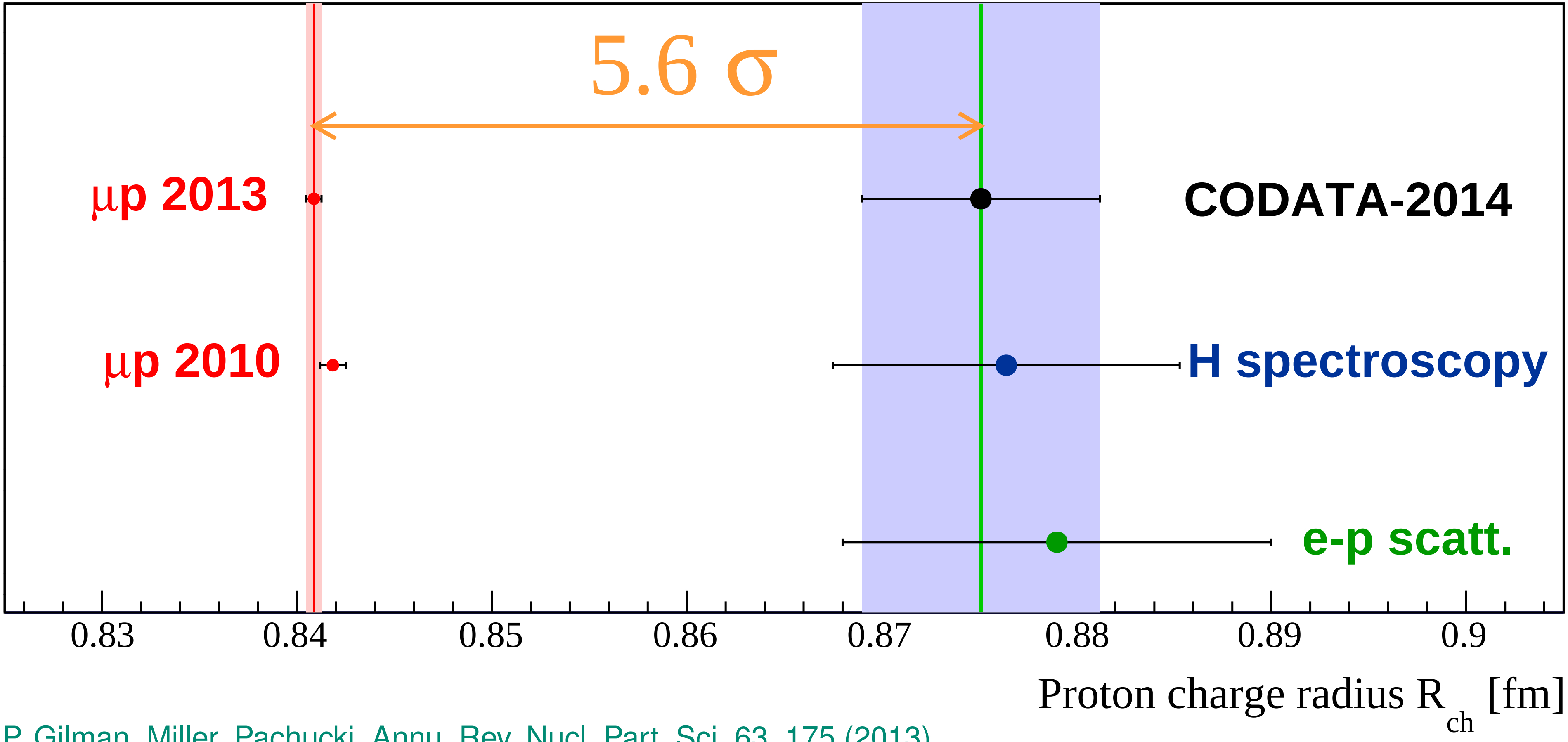
The proton radius puzzle

The proton rms charge radius measured with

electrons: 0.8751 ± 0.0061 fm

muons: 0.8409 ± 0.0004 fm



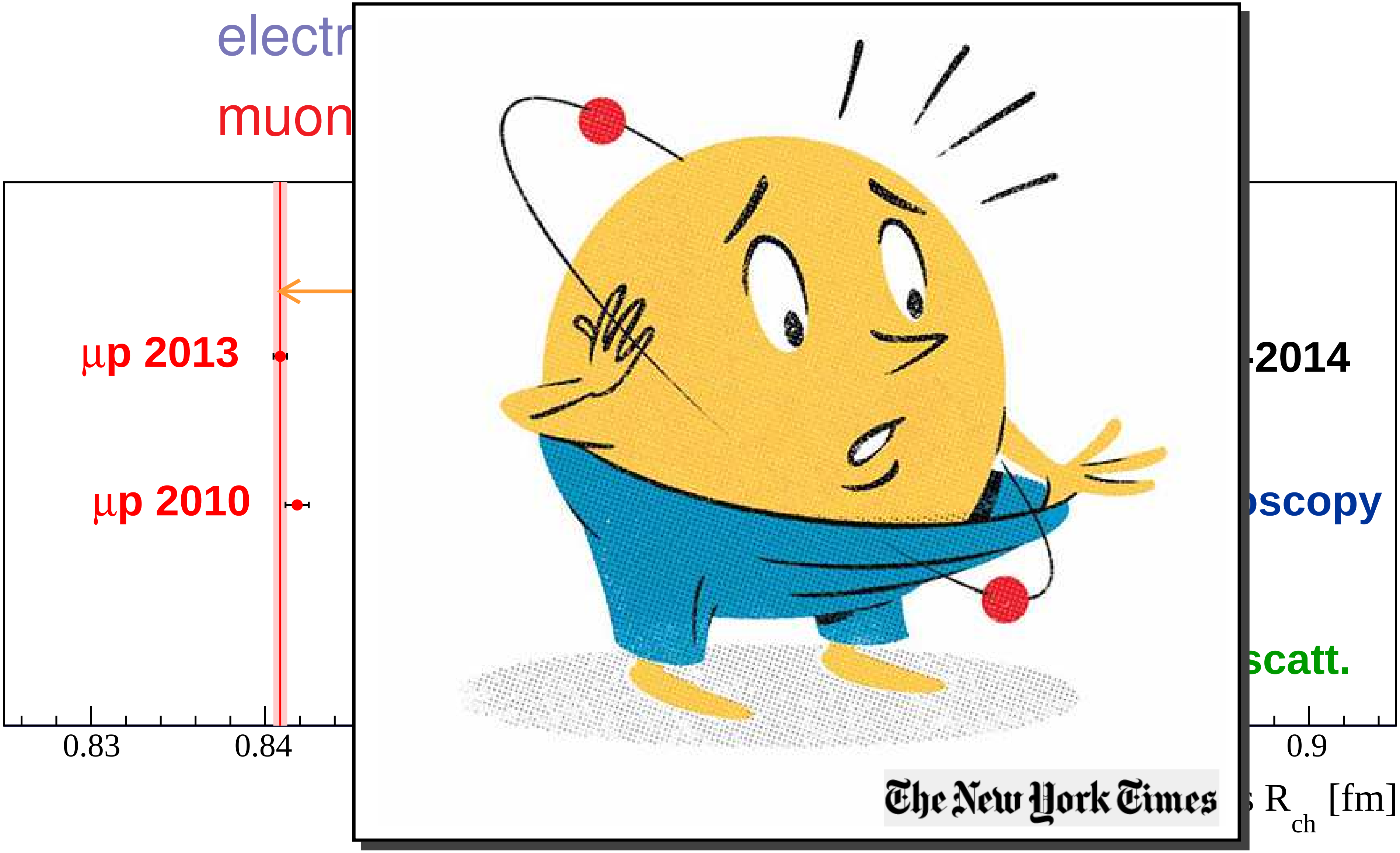
RP, Gilman, Miller, Pachucki, Annu. Rev. Nucl. Part. Sci. 63, 175 (2013).

The proton radius puzzle

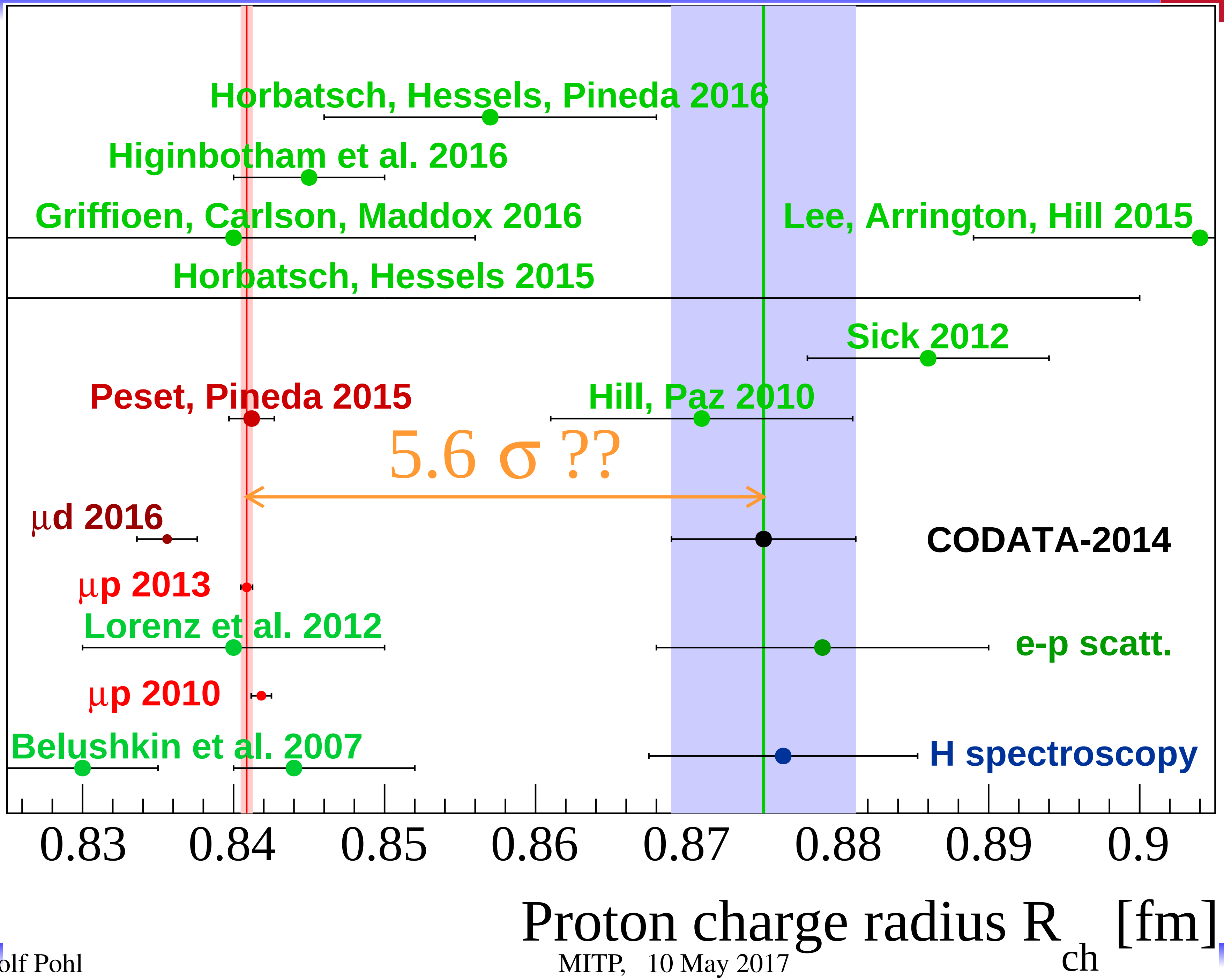
The proton rms charge radius measured with

electr

muon



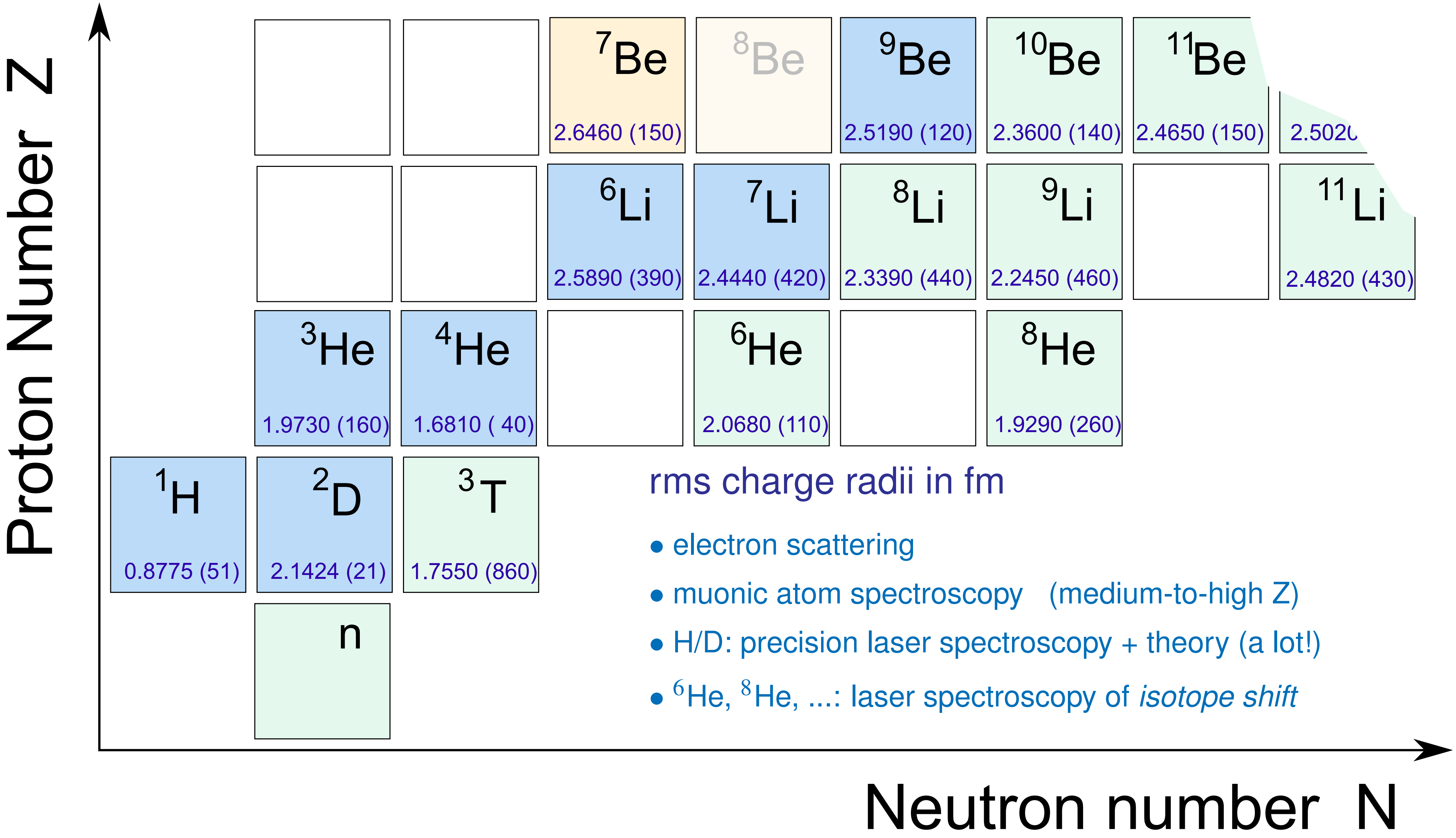
The proton radius puzzle???



Outline

- Introduction:
  How large are the proton, deuteron, helion, alpha...?
  Atomic vs. nuclear physics
- Muonic hydrogen:
  Size does matter!
- Laser spectroscopy of muonic atoms/ions
- New measurements:
  - Muonic deuterium → Another discrepancy to CODATA!
  - Muonic helium-3 and -4 ions
  - Regular hydrogen → New Rydberg constant!
- Future:
  - HFS in muonic hydrogen and helium-3
  - X-ray spectroscopy of muonic radium etc.
  - Lamb shift in muonic Li, Be, ...
  - 1S-2S in regular tritium (triton radius)
  - ...

Charge radii of light nuclei



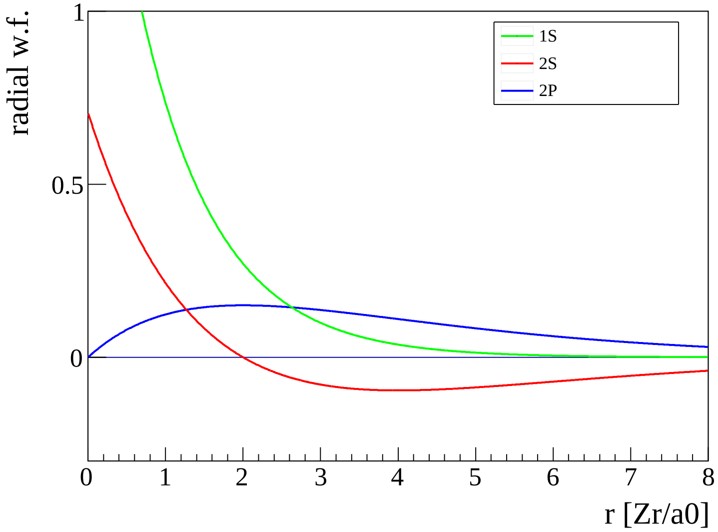
			^{7}Be 2.6460 (150)	^{8}Be	^{9}Be 2.5190 (120)	^{10}Be 2.3600 (140)	^{11}Be 2.4650 (150)	2.502
			^{6}Li 2.5890 (390)	^{7}Li 2.4440 (420)	^{8}Li 2.3390 (440)	^{9}Li 2.2450 (460)		^{11}Li 2.4820 (430)
	^{3}He 1.9730 (160)	^{4}He 1.6810 (40)		^{6}He 2.0680 (110)		^{8}He 1.9290 (260)		
^{1}H 0.8775 (51)	^{2}D 2.1424 (21)	^{3}T 1.7550 (860)						
	n							

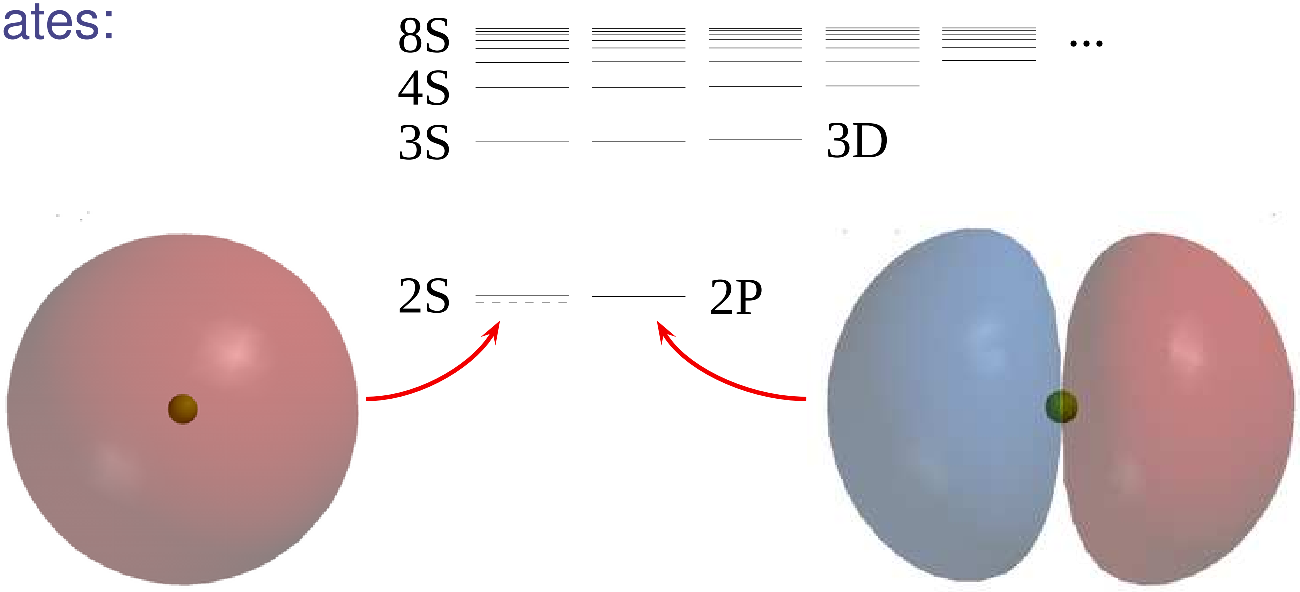
rms charge radii in fm

- electron scattering
- muonic atom spectroscopy (medium-to-high Z)
- H/D: precision laser spectroscopy + theory (a lot!)
- ^{6}He, ^{8}He, ...: laser spectroscopy of *isotope shift*

Atomic physics

Wave functions of S and P states:





S states: max. at r=0

Electron sometimes inside the proton.

S states are shifted.

Shift ist proportional to the

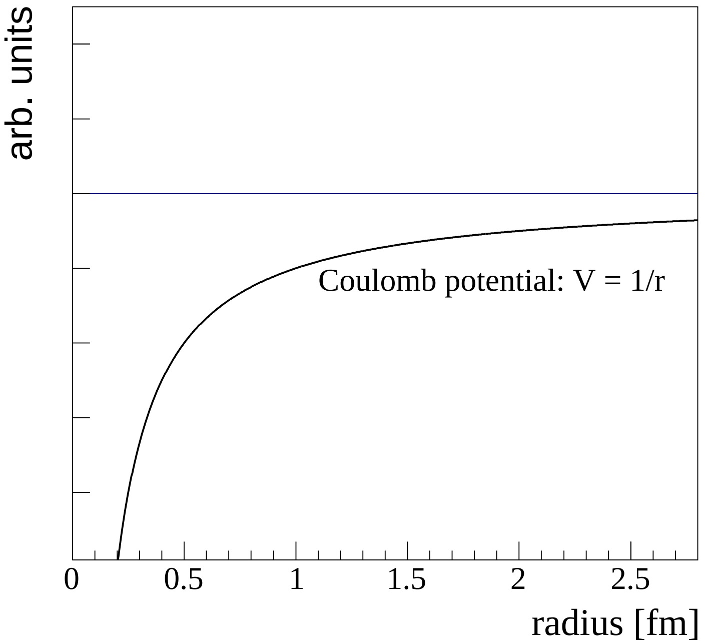
size of the proton

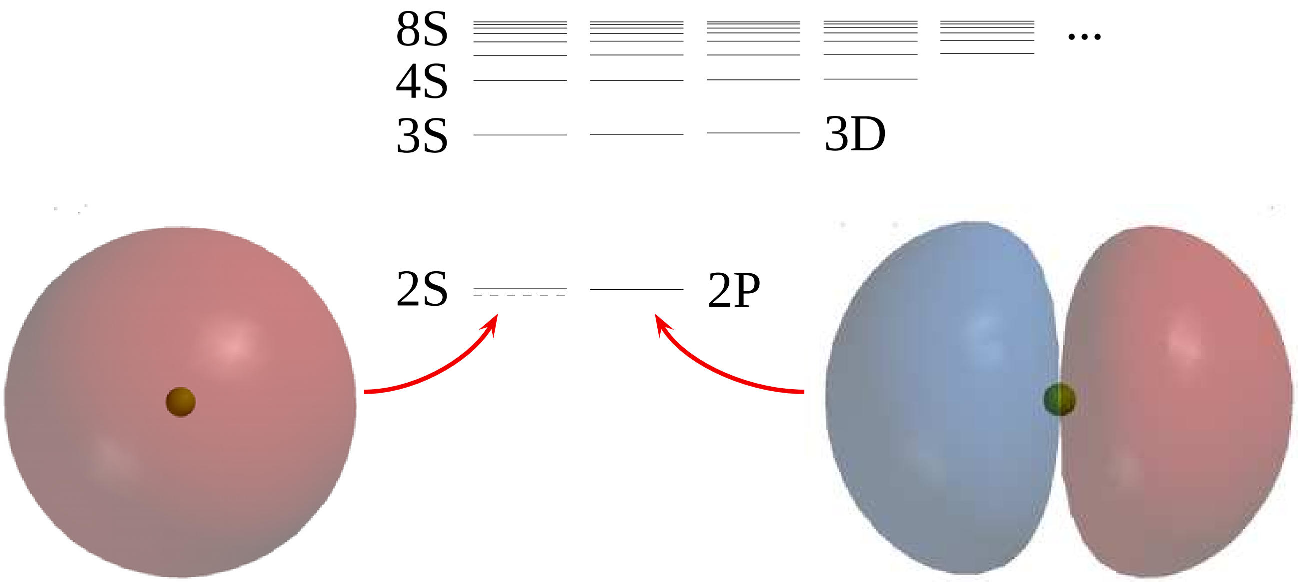
P states: zero at r=0

Electron is not inside the proton.

Orbital pictures from Wikipedia

Atomic physics





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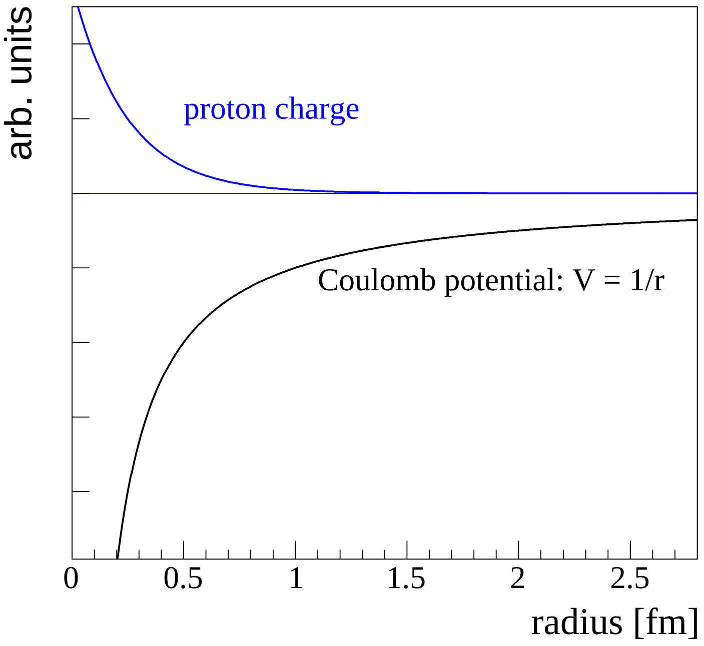
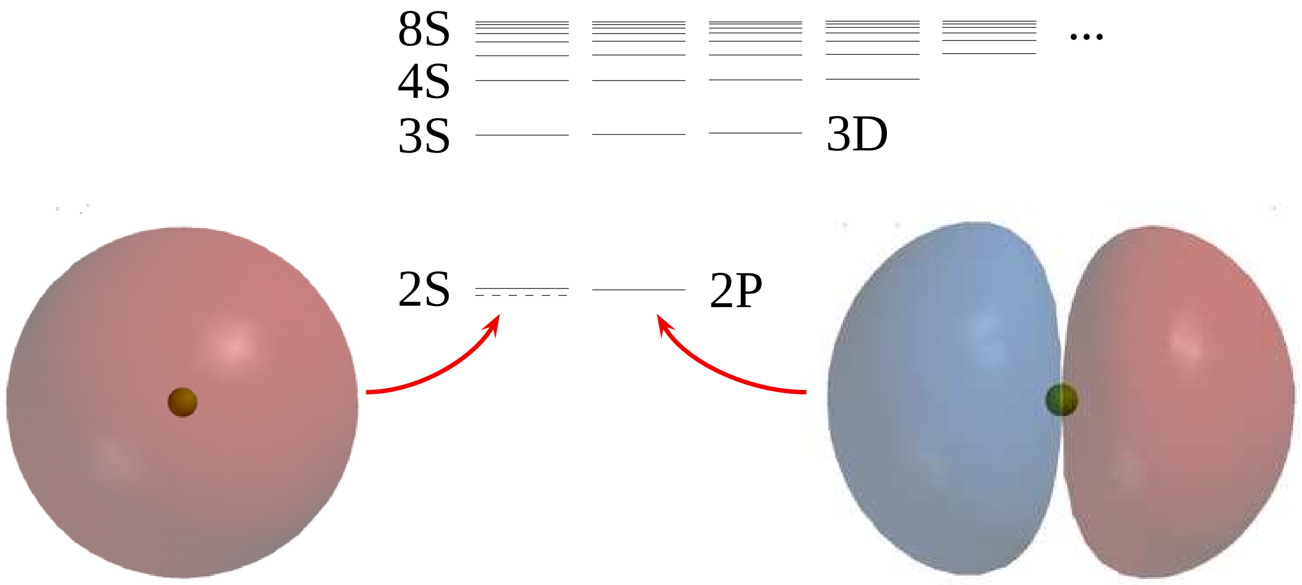
Orbital pictures from Wikipedia

Atomic and nuclear physics





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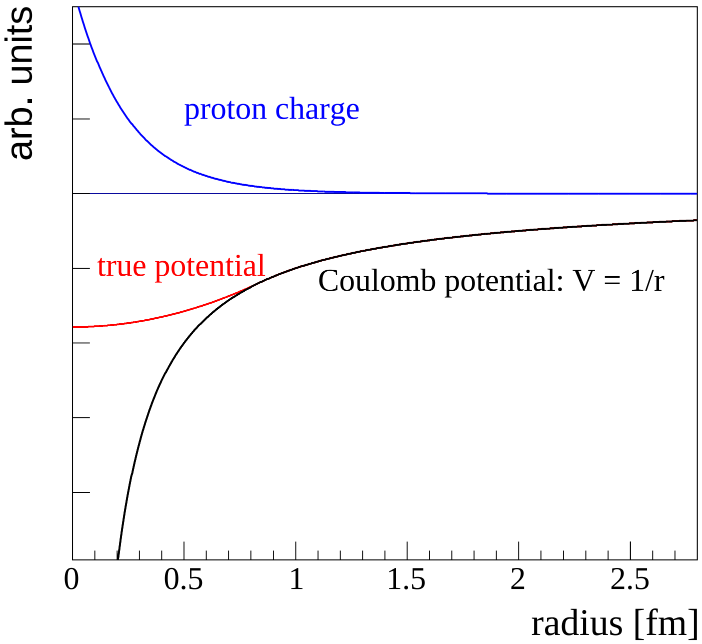

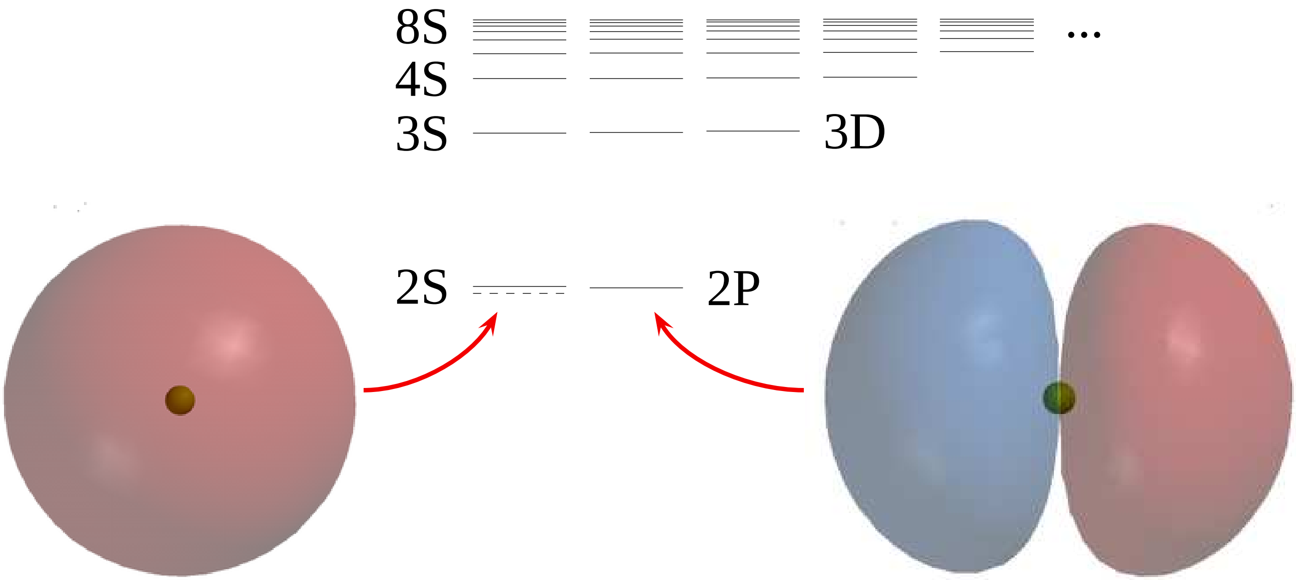

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Electron sometimes inside the proton.

Electron is not inside the proton.

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Orbital pictures from Wikipedia

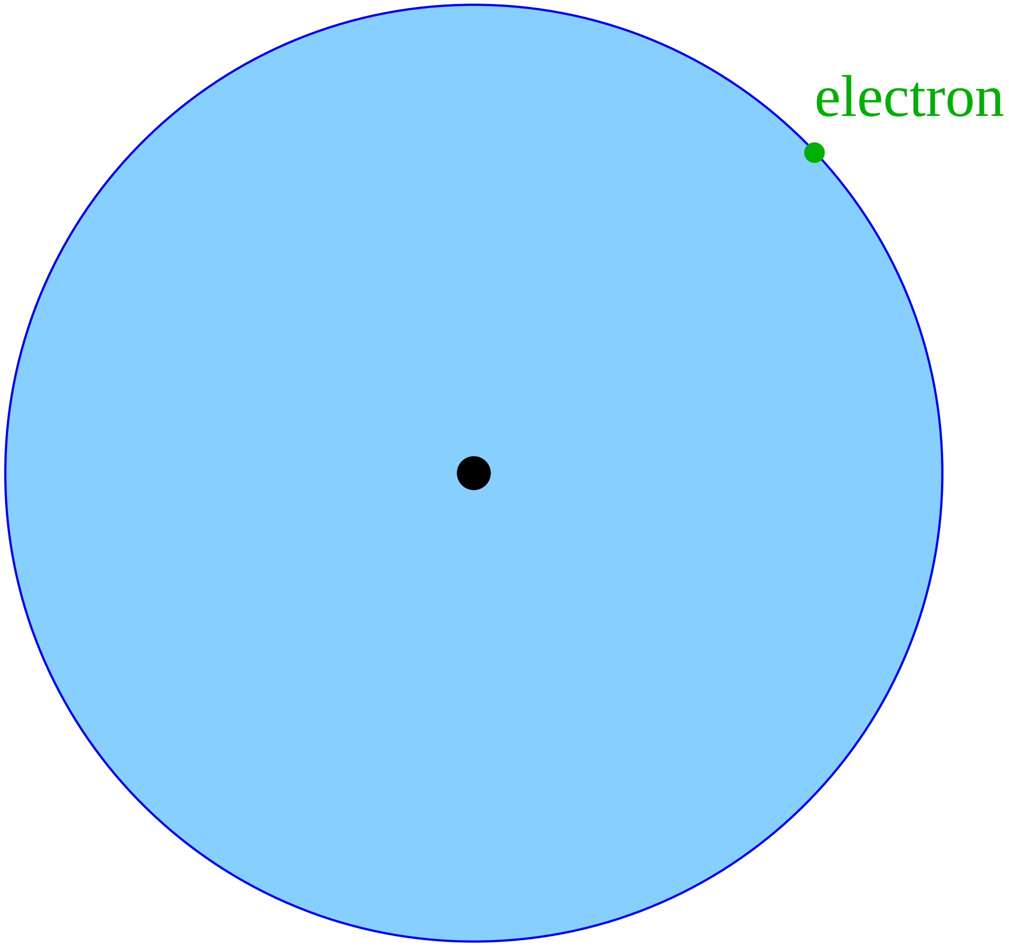
Muonic hydrogen

Regular hydrogen:

electron e^- + proton p

Muonic hydrogen:

muon μ^- + proton p



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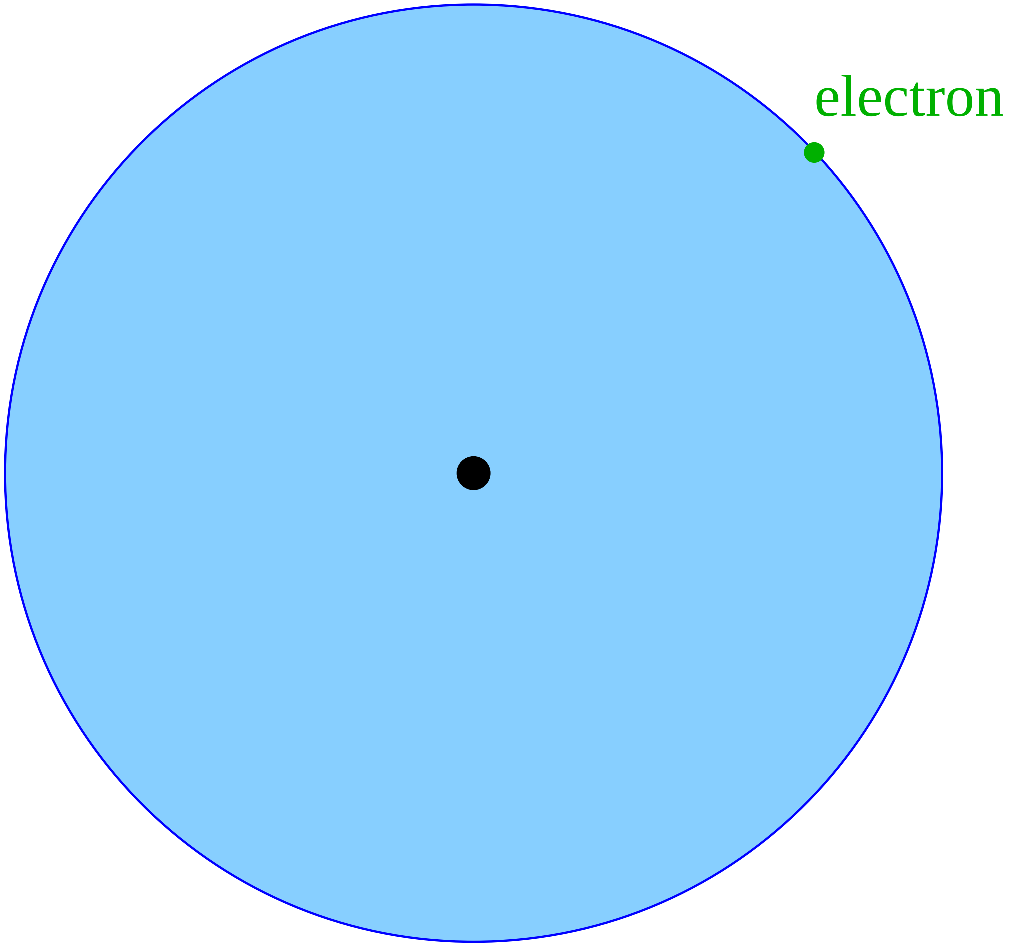

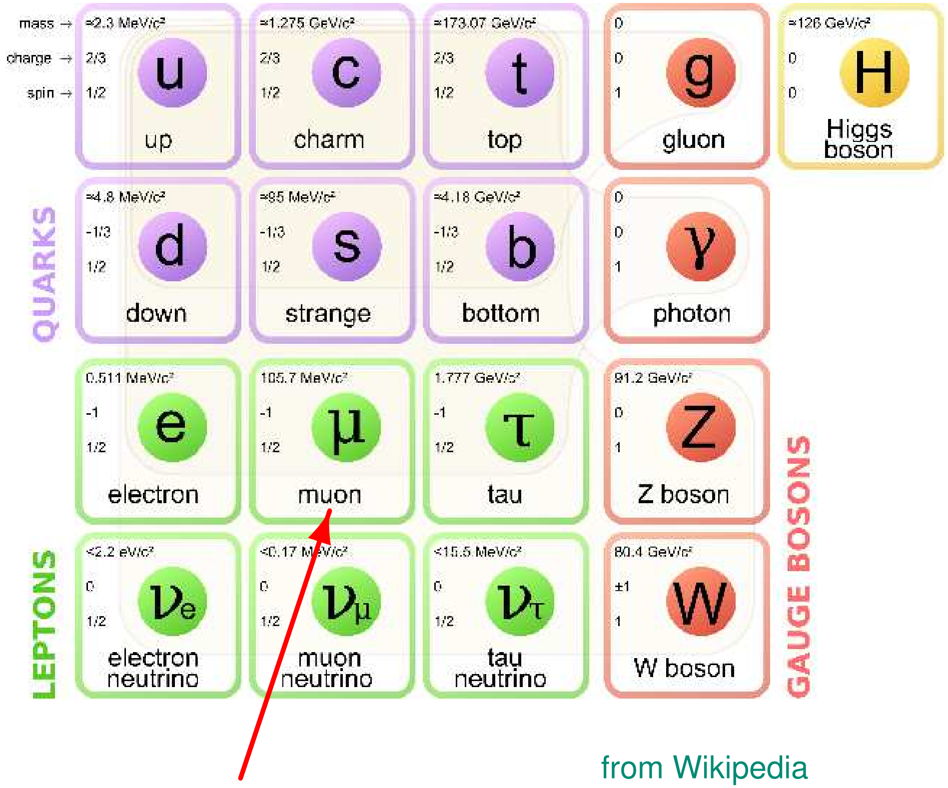

from Wikipedia

Muonic hydrogen

Regular hydrogen:

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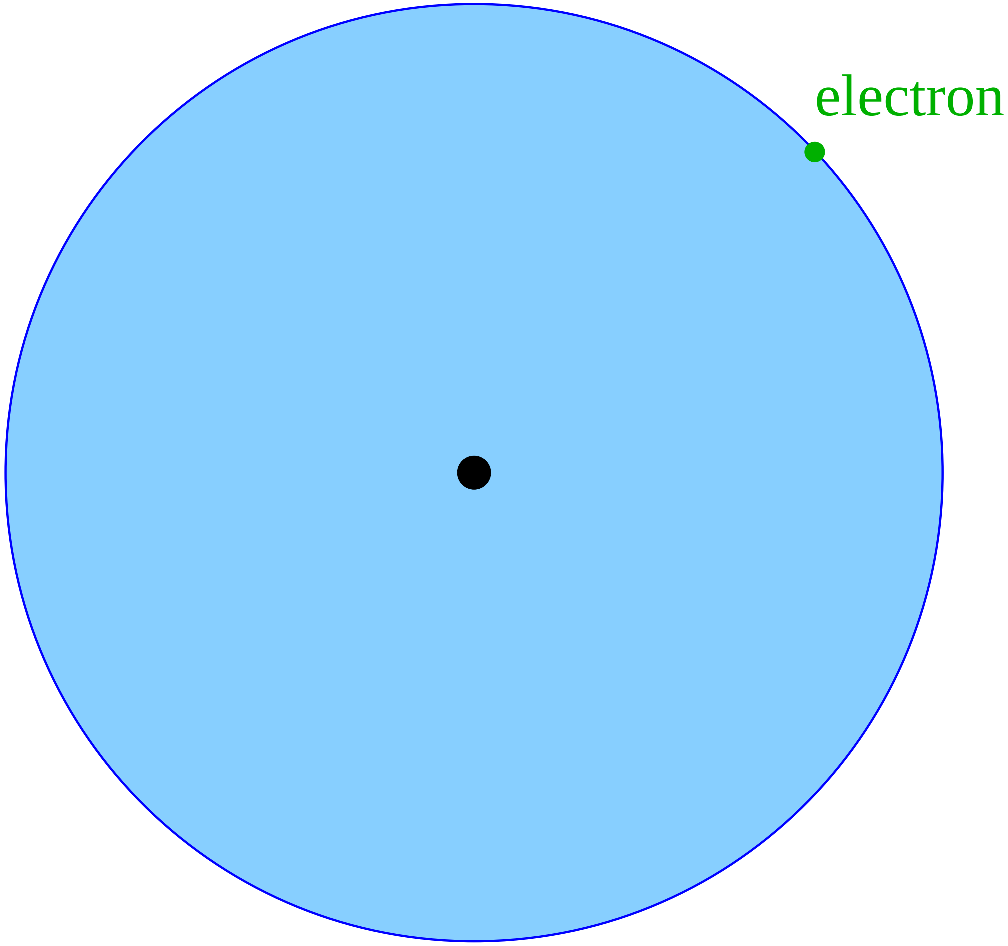


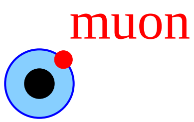
Muonic hydrogen:

muon μ^- + proton p

muon mass $m_\mu \approx$ 200 $\times m_e$

Bohr radius $r_\mu \approx$ 1/200 $\times r_e$

μ inside the proton: $200^3 \approx 10^7$



muon **much** is more sensitive to r_p

Proton charge radius and muonic hydrogen

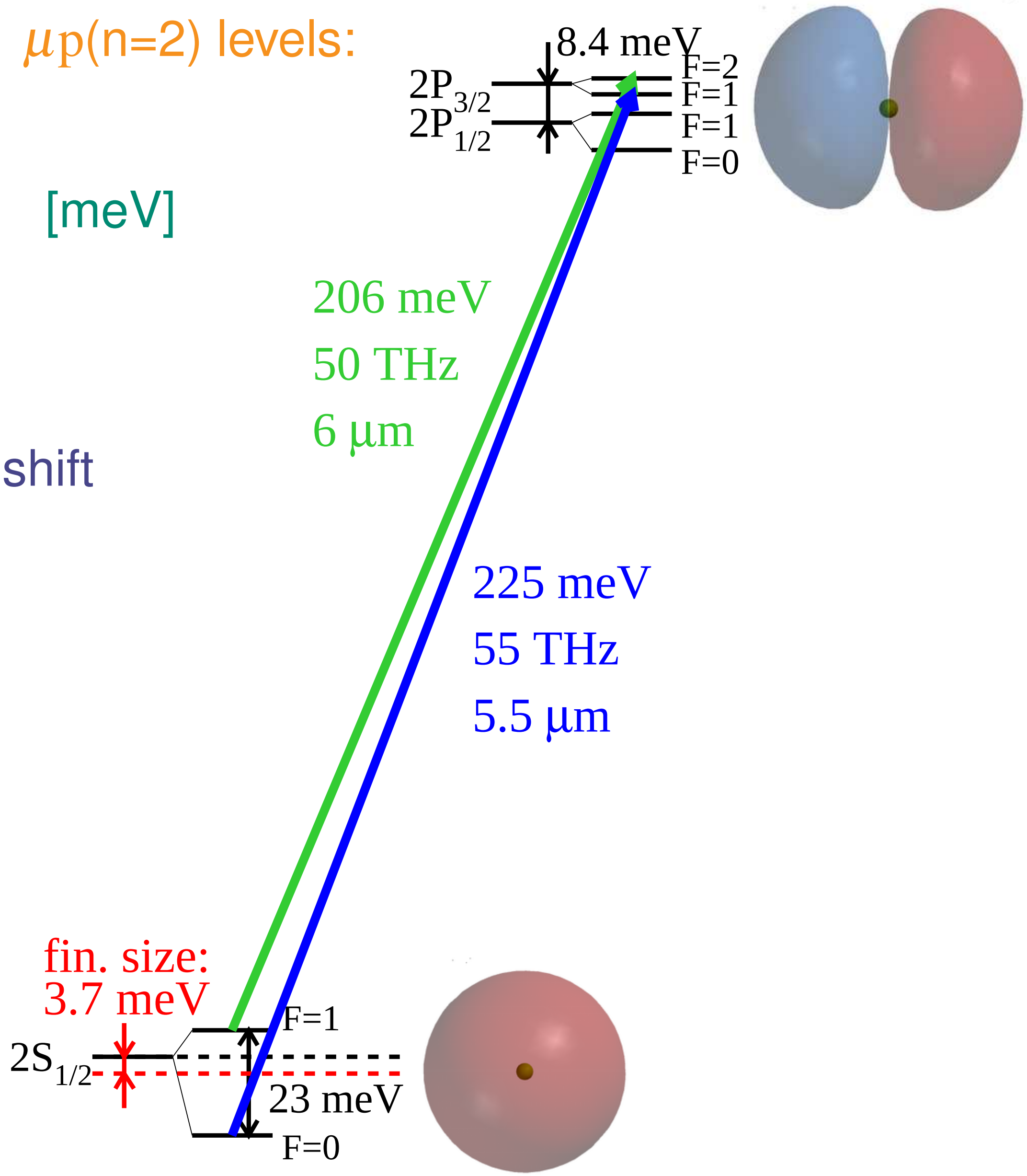
Lamb shift in μp [meV]:

$$\Delta E = 206.0668(25) - 5.2275(10)\, r_p^2 \quad [\text{meV}]$$

Proton size effect is 2% of the μp Lamb shift

Measure to 10^{-5} ⇒ r_p to 0.05 %

μp(n=2) levels:



Experiment:
R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini, RP *et al.*, Science 339, 417 (2013).

Theory summary:
A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013).

Lamb shift in μp 1: r_p independent

Table 1

All known radius-*independent* contributions to the Lamb shift in μp from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The "finite-size to relativistic recoil correction" (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit–Pauli)	0.0169[a]					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[80] Eq. (54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2)[b]	−0.0041	−0.0041		−0.00208[c]	[77,78]
4	Two-loop eVP (Källén–Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[80] Eq. (57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[80] Eq. (60)
7	eVP corr. to Källén–Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[80] Eq. (62), [87]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[87,88]
9	Wichmann–Kroll, "1:3" LBL		−0.00103	−0.00102	−0.00102	−0.00102	[80] Eq. (64), [89]
10	Virtual Delbrück, "2:2" LBL		0.00135	0.00115		0.00115	[74,89]
New	"3:1" LBL			−0.00102		−0.00102	[89]
20	μSE and μVP	−0.6677	−0.66770	−0.66788	−0.66761	−0.66761	[80] Eqs. (72) + (76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	−0.005(1)	−0.00500	−0.004924[d]		−0.00254	[85] Eq. (29a)[e]
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	−0.001	−0.00150			[f]	[74,90–92]
21	Higher order corr. to μSE and μVP		−0.00169	−0.00171[g]		−0.00171	[86] Eq. (177)
13	Mixed eVP + μVP		0.00007	0.00007		0.00007	[74]
New	eVP and μVP in two Coulomb lines				0.00005	0.00005	[80] Eq. (78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[93–95]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[94,95]
16	Rad corr. to hadronic VP		−0.000015			−0.000015	[94,95]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[80] Eq. (88)
22	Rel. RC $(Z\alpha)^5$	−0.045	−0.04497	−0.04497	−0.04497	−0.04497	[80] Eq. (88), [74]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[80] Eq. (86)+Tab.II

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A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 1

Lamb shift in μp 1: r_p independent

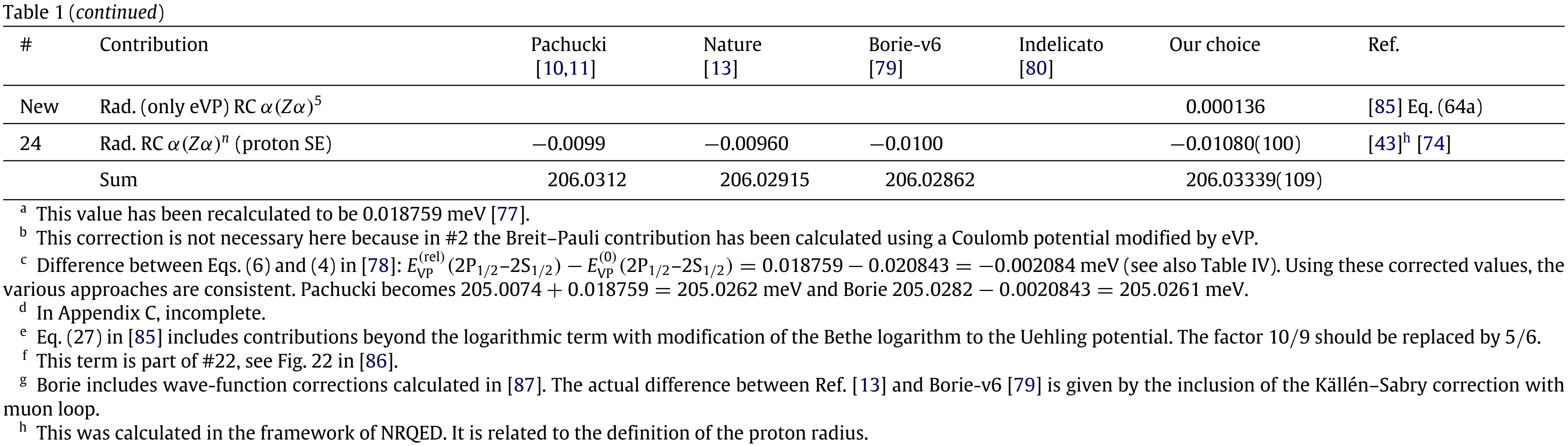

Table 1 (*continued*)

#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
New	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.000136	[85] Eq. (64a)
24	Rad. RC $\alpha(Z\alpha)^n$ (proton SE)	−0.0099	−0.00960	−0.0100		−0.01080(100)	[43][h] [74]
	Sum	206.0312	206.02915	206.02862		206.03339(109)	

[a] This value has been recalculated to be 0.018759 meV [77].

[b] This correction is not necessary here because in #2 the Breit–Pauli contribution has been calculated using a Coulomb potential modified by eVP.

[c] Difference between Eqs. (6) and (4) in [78]: $E_{\mathrm{VP}}^{(\mathrm{rel})}(2\mathrm{P}_{1/2}\text{–}2\mathrm{S}_{1/2}) - E_{\mathrm{VP}}^{(0)}(2\mathrm{P}_{1/2}\text{–}2\mathrm{S}_{1/2}) = 0.018759 - 0.020843 = -0.002084$ meV (see also Table IV). Using these corrected values, the various approaches are consistent. Pachucki becomes $205.0074 + 0.018759 = 205.0262$ meV and Borie $205.0282 - 0.0020843 = 205.0261$ meV.

[d] In Appendix C, incomplete.

[e] Eq. (27) in [85] includes contributions beyond the logarithmic term with modification of the Bethe logarithm to the Uehling potential. The factor 10/9 should be replaced by 5/6.

[f] This term is part of #22, see Fig. 22 in [86].

[g] Borie includes wave-function corrections calculated in [87]. The actual difference between Ref. [13] and Borie-v6 [79] is given by the inclusion of the Källén–Sabry correction with muon loop.

[h] This was calculated in the framework of NRQED. It is related to the definition of the proton radius.

43 R.J. Hill, G. Paz, Phys. Rev. Lett. 107, 160402 (2011)
74 M.I. Eides, H. Grotch, V.A. Shelyuto, Phys. Rep. 342, 63 (2001)
77 U.D. Jentschura, Phys. Rev. A 84, 012505 (2011)
78 S.G. Karshenboim, V.G. Ivanov, E.Y. Korzinin, Phys. Rev. A 85, 032509 (2012)
79 E. Borie, Ann. Phys. 327, 733 (2012); arXiv:1103.1772-v6
80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]
85 U.D. Jentschura, B.J. Wundt, Eur. Phys. J. D 65, 357 (2011)
86 E. Borie, G.A. Rinker, Rev. Mod. Phys. 54, 67 (1982)
87 V.G. Ivanov, E.Y. Korzinin, S.G. Karshenboim, Phys. Rev. D 80, 027702 (2009)
88 T. Kinoshita, M. Nio, Phys. Rev. Lett. 82, 3240 (1999)
89 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, V.A. Shelyuto, JETP Lett. 92, 8 (2010)
90 R. Barbieri, M. Caffo, E. Remiddi, Lett. Nuovo Cimento 7, 60 (1963)
91 H. Suura, E.H. Wichmann, Phys. Rev. 105, 1930 (1957)
92 A. Petermann, Phys. Rev. 105, 1931 (1957)
93 J. Friar, J. Martorell, D. Sprung, Phys. Rev. A 59, 4061 (1999)
94 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 63, 845 (2000)
95 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 64, 1282 (2001)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 1

Lamb shift in μp 2: r_{p}-dependent

Table 2

Proton-structure-dependent contributions to the Lamb shift in μp from different authors and the one we selected. Values are in meV, $\langle r^2\rangle$ in fm^2. The entry # in the first column refers to Table 1 in Ref. [13] supplementary information [9]. Entry # 18 is under debate. TPE: two-photon exchange, VP: vacuum polarization, SE: self-energy, Rel: relativistic.

#	Contribution	Borie-v6 [79]	Karshenboim [78]	Pachucki [10,11]	Indelicato [80]	Carroll [84]	Our choice
	Non-rel. finite-size	$-5.1973\ \langle r^2\rangle$	$-5.1975\ \langle r^2\rangle$	$-5.1975\ \langle r^2\rangle$			
	Rel. corr. to non-rel. finite size	$-0.0018\ \langle r^2\rangle$		-0.0009 meV[a]			
	Rel. finite-size						
	Exponential				$-5.1994\ \langle r^2\rangle$	$-5.2001\ \langle r^2\rangle$	$-5.1994\ \langle r^2\rangle$
	Yukawa					$-5.2000\ \langle r^2\rangle$	
	Gaussian					$-5.2001\ \langle r^2\rangle$	
	Finite size corr. to one-loop eVP	$-0.0110\ \langle r^2\rangle$	$-0.0110\ \langle r^2\rangle$	$-0.010\ \langle r^2\rangle$	$-0.0282\ \langle r^2\rangle$		$-0.0282\ \langle r^2\rangle$
	Finite size to one-loop eVP-it.	$-0.0165\ \langle r^2\rangle$	$-0.0170\ \langle r^2\rangle$	$-0.017\ \langle r^2\rangle$	(incl. in -0.0282)		
	Finite-size corr. to Källén–Sabry	[b]			$-0.0002\ \langle r^2\rangle$		$-0.0002\ \langle r^2\rangle$
New	Finite size corr. to μ self-energy	(0.00699)[c]			$0.0008\ \langle r^2\rangle$		$0.0009(3)\ \langle r^2\rangle$[d]
	ΔE_{TPE} [46]						0.0332(20) meV
	Elastic (third Zemach)[e]						
	Measured $R^3_{(2)}$	$0.0365(18)\ \langle r^2\rangle^{3/2}$					(incl. above)
	Exponential			$0.0363\ \langle r^2\rangle^{3/2}$	$0.0353\ \langle r^2\rangle^{3/2}$ [f]	$0.0353\ \langle r^2\rangle^{3/2}$	
	Yukawa					$0.0378\ \langle r^2\rangle^{3/2}$	
	Gaussian					$0.0323\ \langle r^2\rangle^{3/2}$	
25	Inelastic (polarizability)	0.0129(5) meV [101]		0.012(2) meV			(incl. above)
New	Rad. corr. to TPE	$-0.00062\ \langle r^2\rangle$					$-0.00062\ \langle r^2\rangle$
26	eVP corr. to polarizability						0.00019 meV [95]

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A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 2

Lamb shift in μp 2: r_p-dependent

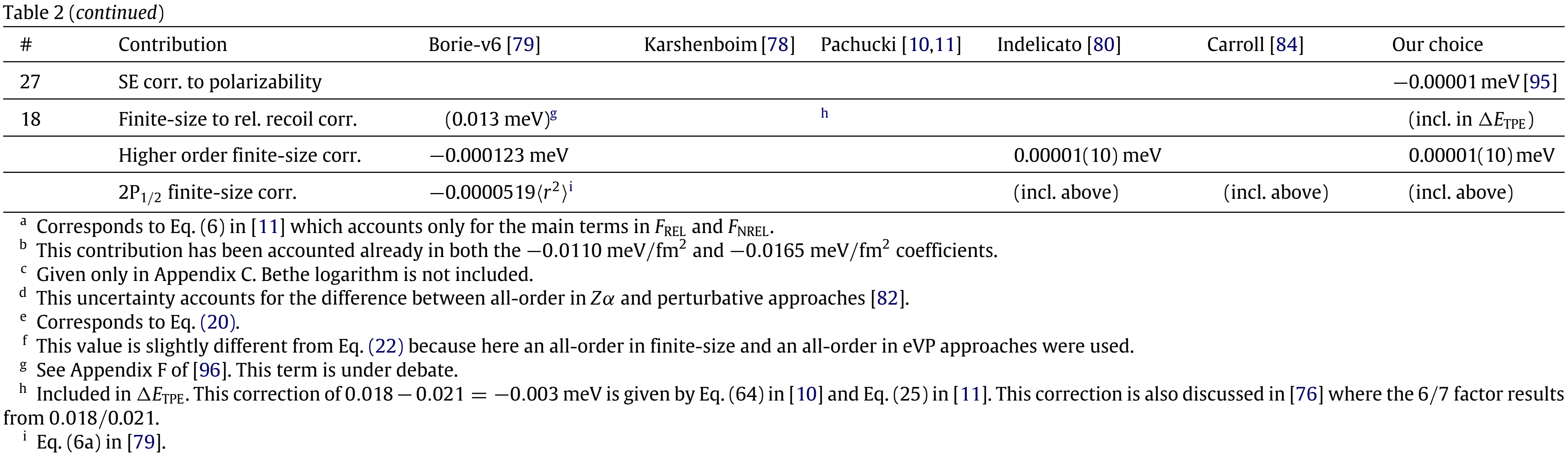


Table 2 (*continued*)

#	Contribution	Borie-v6 [79]	Karshenboim [78]	Pachucki [10,11]	Indelicato [80]	Carroll [84]	Our choice
27	SE corr. to polarizability						−0.00001 meV [95]
18	Finite-size to rel. recoil corr.	(0.013 meV)[g]		[h]			(incl. in ΔE_{TPE})
	Higher order finite-size corr.	−0.000123 meV			0.00001(10) meV		0.00001(10) meV
	$2\text{P}_{1/2}$ finite-size corr.	$-0.0000519\langle r^2\rangle$[i]			(incl. above)	(incl. above)	(incl. above)

[a] Corresponds to Eq. (6) in [11] which accounts only for the main terms in F_{REL} and F_{NREL}.

[b] This contribution has been accounted already in both the -0.0110 meV/fm^2 and -0.0165 meV/fm^2 coefficients.

[c] Given only in Appendix C. Bethe logarithm is not included.

[d] This uncertainty accounts for the difference between all-order in $Z\alpha$ and perturbative approaches [82].

[e] Corresponds to Eq. (20).

[f] This value is slightly different from Eq. (22) because here an all-order in finite-size and an all-order in eVP approaches were used.

[g] See Appendix F of [96]. This term is under debate.

[h] Included in ΔE_{TPE}. This correction of $0.018 - 0.021 = -0.003$ meV is given by Eq. (64) in [10] and Eq. (25) in [11]. This correction is also discussed in [76] where the 6/7 factor results from 0.018/0.021.

[i] Eq. (6a) in [79].

46 M.C. Birse, J.A. McGovern, Eur. Phys. J. A 48, 120 (2012); arXiv:1206.3030
76 U.D. Jentschura, Ann. Phys. 326, 500 (2011)
79 E. Borie, Ann. Phys. 327, 733 (2012); arXiv:1103.1772-v6
82 P. Indelicato, P.J. Mohr, 2012 (in preparation)
95 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 64, 1282 (2001)
96 J.L. Friar, Ann. Phys. 122, 151 (1979)
101 C.E. Carlson, M. Vanderhaeghen, Phys. Rev. A 84, 020102 (2011)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 2

HFS in μp

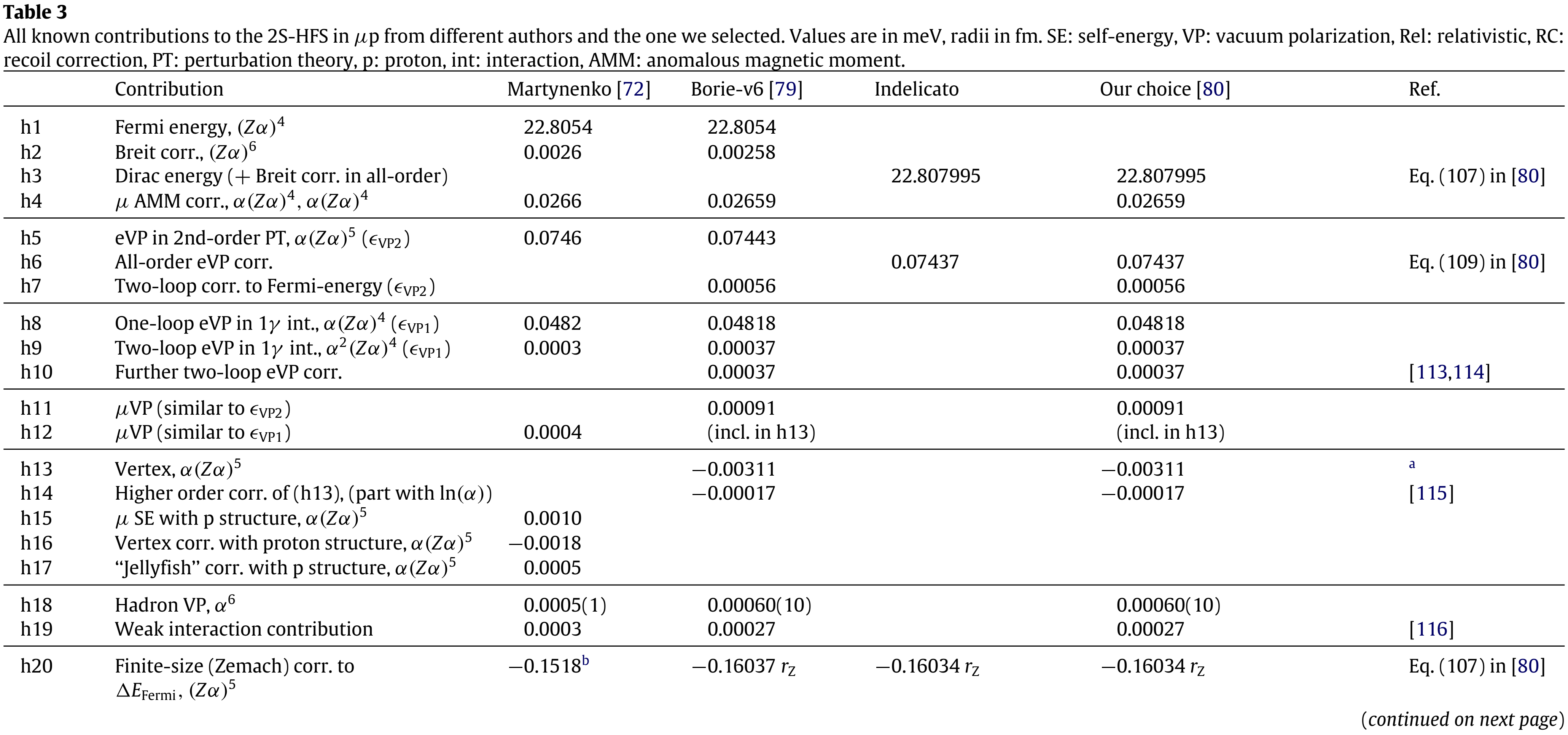


Table 3
All known contributions to the 2S-HFS in μp from different authors and the one we selected. Values are in meV, radii in fm. SE: self-energy, VP: vacuum polarization, Rel: relativistic, RC: recoil correction, PT: perturbation theory, p: proton, int: interaction, AMM: anomalous magnetic moment.

	Contribution	Martynenko [72]	Borie-v6 [79]	Indelicato	Our choice [80]	Ref.
h1	Fermi energy, $(Z\alpha)^4$	22.8054	22.8054			
h2	Breit corr., $(Z\alpha)^6$	0.0026	0.00258			
h3	Dirac energy (+ Breit corr. in all-order)			22.807995	22.807995	Eq. (107) in [80]
h4	μ AMM corr., $\alpha(Z\alpha)^4$, $\alpha(Z\alpha)^4$	0.0266	0.02659		0.02659	
h5	eVP in 2nd-order PT, $\alpha(Z\alpha)^5$ (ϵ_{VP2})	0.0746	0.07443			
h6	All-order eVP corr.			0.07437	0.07437	Eq. (109) in [80]
h7	Two-loop corr. to Fermi-energy (ϵ_{VP2})		0.00056		0.00056	
h8	One-loop eVP in 1γ int., $\alpha(Z\alpha)^4$ (ϵ_{VP1})	0.0482	0.04818		0.04818	
h9	Two-loop eVP in 1γ int., $\alpha^2(Z\alpha)^4$ (ϵ_{VP1})	0.0003	0.00037		0.00037	
h10	Further two-loop eVP corr.		0.00037		0.00037	[113,114]
h11	μVP (similar to ϵ_{VP2})		0.00091		0.00091	
h12	μVP (similar to ϵ_{VP1})	0.0004	(incl. in h13)		(incl. in h13)	
h13	Vertex, $\alpha(Z\alpha)^5$		−0.00311		−0.00311	[a]
h14	Higher order corr. of (h13), (part with $\ln(\alpha)$)		−0.00017		−0.00017	[115]
h15	μ SE with p structure, $\alpha(Z\alpha)^5$	0.0010				
h16	Vertex corr. with proton structure, $\alpha(Z\alpha)^5$	−0.0018				
h17	"Jellyfish" corr. with p structure, $\alpha(Z\alpha)^5$	0.0005				
h18	Hadron VP, α^6	0.0005(1)	0.00060(10)		0.00060(10)	
h19	Weak interaction contribution	0.0003	0.00027		0.00027	[116]
h20	Finite-size (Zemach) corr. to ΔE_{Fermi}, $(Z\alpha)^5$	−0.1518[b]	$-0.16037\ r_Z$	$-0.16034\ r_Z$	$-0.16034\ r_Z$	Eq. (107) in [80]

(*continued on next page*)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 3

HFS in μp

Table 3 (*continued*)

	Contribution	Martynenko [72]	Borie-v6 [79]	Indelicato	Our choice [80]	Ref.
h21	Higher order finite-size corr. to $\Delta E_{\rm Fermi}$			$-0.0022\ {r_{\rm E}}^2 + 0.0009$	$-0.0022\ {r_{\rm E}}^2 + 0.0009$	Eq. (107) in [80]
h22	Proton polarizability, $(Z\alpha)^5$, $\Delta E^{\rm pol}_{\rm HFS}$	0.0105(18)	0.0080(26)		0.00801(260)	[117,118]
h23	Recoil corr.	(incl. in h20)	0.02123		0.02123	[112]
h24	eVP + proton structure corr., α^6	-0.0026				
h25	eVP corr. to finite-size (similar to $\epsilon_{\rm VP2}$)		-0.00114	$-0.0018\ r_{\rm Z} - 0.0001$	$-0.0018\ r_{\rm Z} - 0.0001$	Eq. (109) in [80]
h26	eVP corr. to finite-size (similar to $\epsilon_{\rm VP1}$)		-0.00114		$-0.00114(20)$	
h27	Proton structure corr., $\alpha(Z\alpha)^5$	-0.0017				
h28	Rel. + radiative RC with p AMM, α^6	0.0018				
	Sum	22.8148(20)[c]	22.9839(26) $- 0.1604\ r_{\rm Z}$		22.9858(26) $-$ 0.1621(10) $r_{\rm Z} - 0.0022(5)\ r_{\rm E}^2$	
	Sum with $r_{\rm E} = 0.841$ fm, $r_{\rm Z} = 1.045$ fm [28]	22.8148 meV	22.8163 meV		22.8149 meV	

[a] Includes a correction $\alpha(Z\alpha)^5$ due to μVP.

[b] Calculated using the Simon et al. form factor.

[c] The uncertainty is 0.0078 meV if the uncertainty of the Zemach term (h20) is included (see Table II of [72]).

28 M.O. Distler, J.C. Bernauer, T. Walcher, Phys. Lett. B 696, 343 (2011)

80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]

112 C.E. Carlson, V. Nazaryan, K. Griffioen, Phys. Rev. A 78, 022517 (2008)

113 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, JETP Lett. 88, 641 (2008)

114 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, JETP Lett. 89, 216 (2009)

115 S.J. Brodsky, G.W. Erickson, Phys. Rev. 148, 26 (1966)

116 M.I. Eides, Phys. Rev. A 85, 034503 (2012)

117 C.E. Carlson, V. Nazaryan, K. Griffioen, Phys. Rev. A 83, 042509 (2011)

118 E. Cherednikova, R. Faustov, A. Martynenko, Nuclear Phys. A 703, 365 (2002)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 3

A nice hierarchy

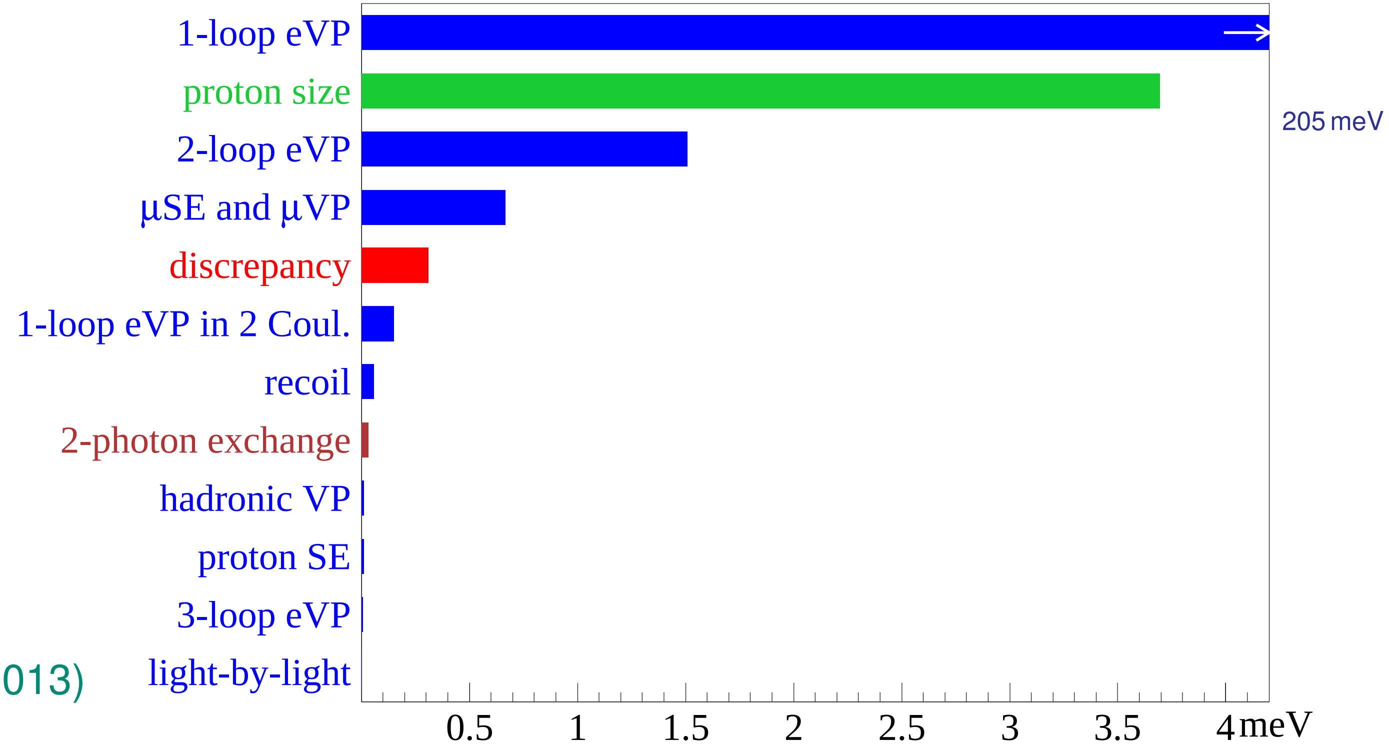
$$\boxed{\Delta E = 206.0668(25) - 5.2275(10)\, r_p^2 \;\; [\mathrm{meV}]}$$

Discrepancy = 0.33 meV
Theory uncert. = 0.0025 meV

$\Longrightarrow$ 120δ(theory) deviation

Some contributions to the μp Lamb shift

double-checked by many groups



Theory summary:

A. Antognini, RP *et al.*
Annals of Physics 331, 127 (2013)

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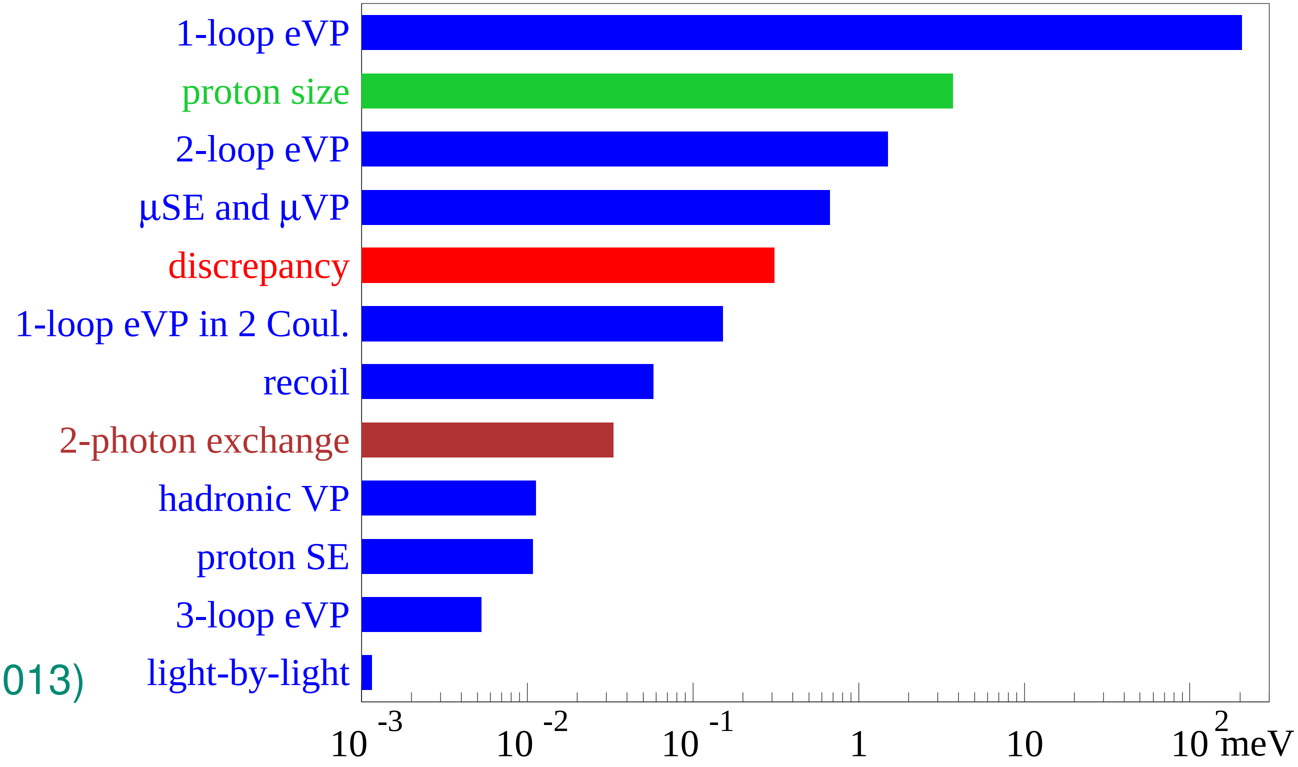
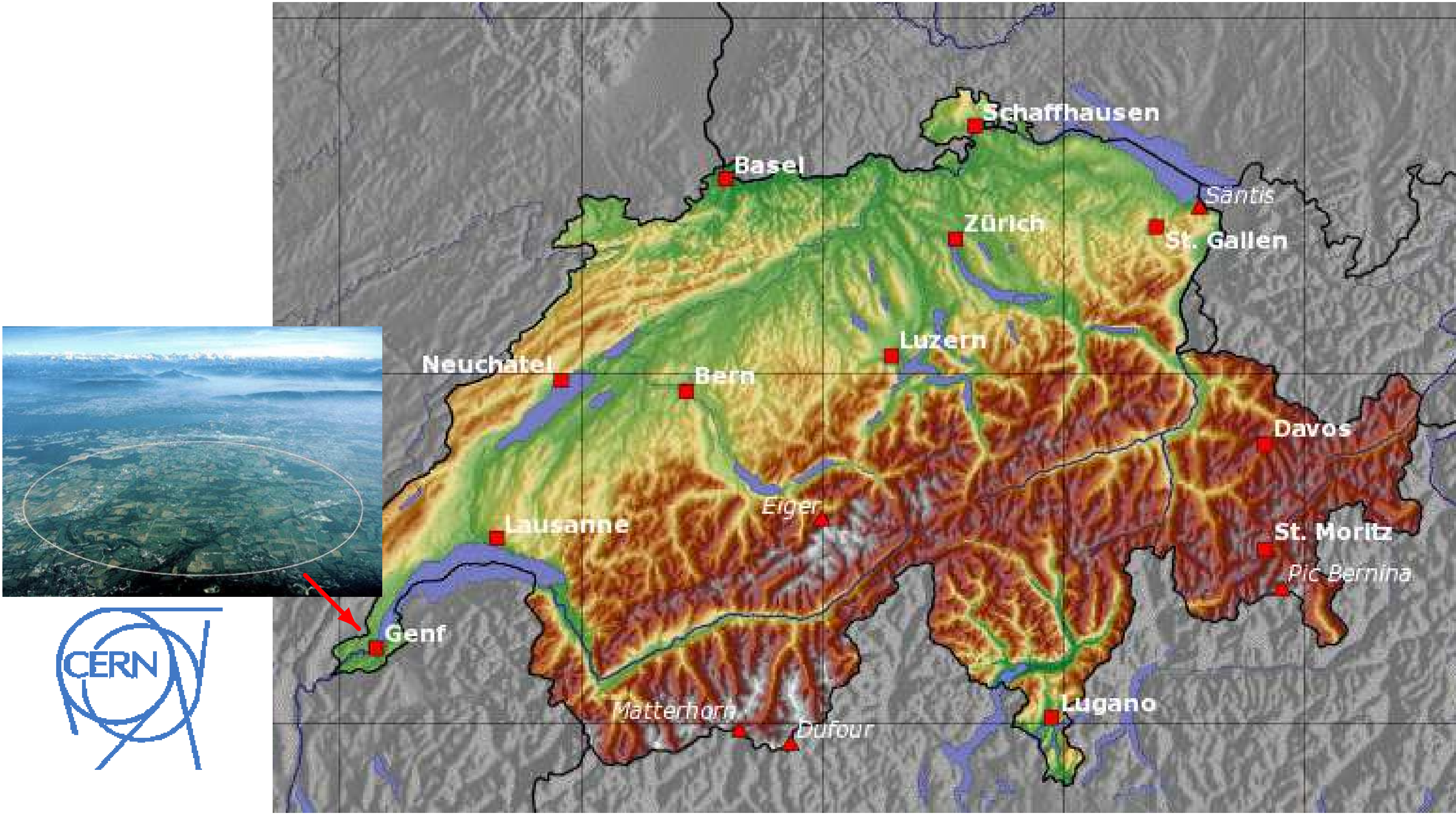


Theory summary:

A. Antognini, RP *et al.*

Annals of Physics 331, 127 (2013)

Swiss muons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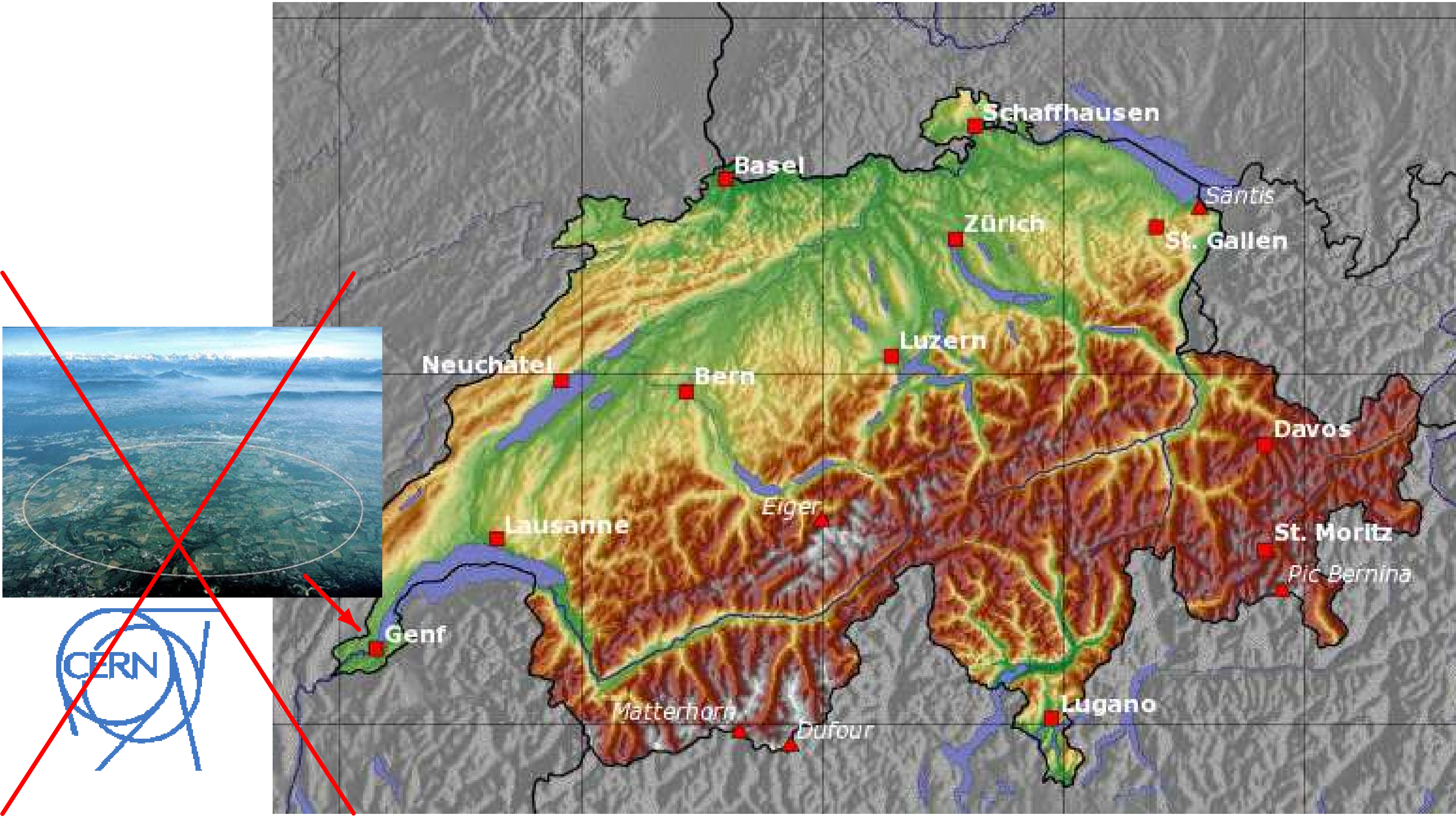
Lugano
Matterhorn
Dufour
CERN

Swiss muons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Säntis
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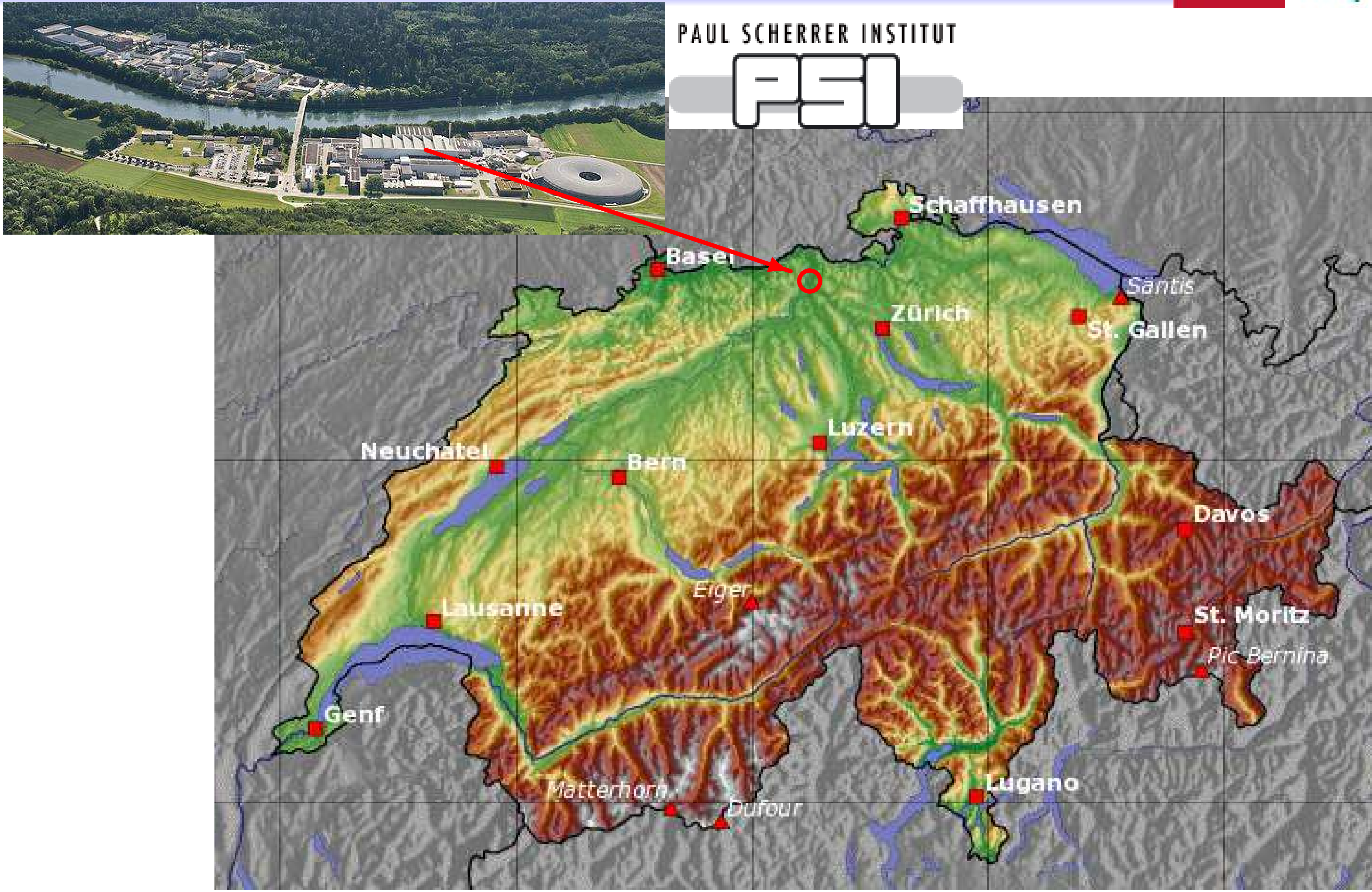
Dufour
CERN

Swiss muons


PAUL SCHERRER INSTITUT
PSI
Schaffhausen
Basel
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Lausanne
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Genf
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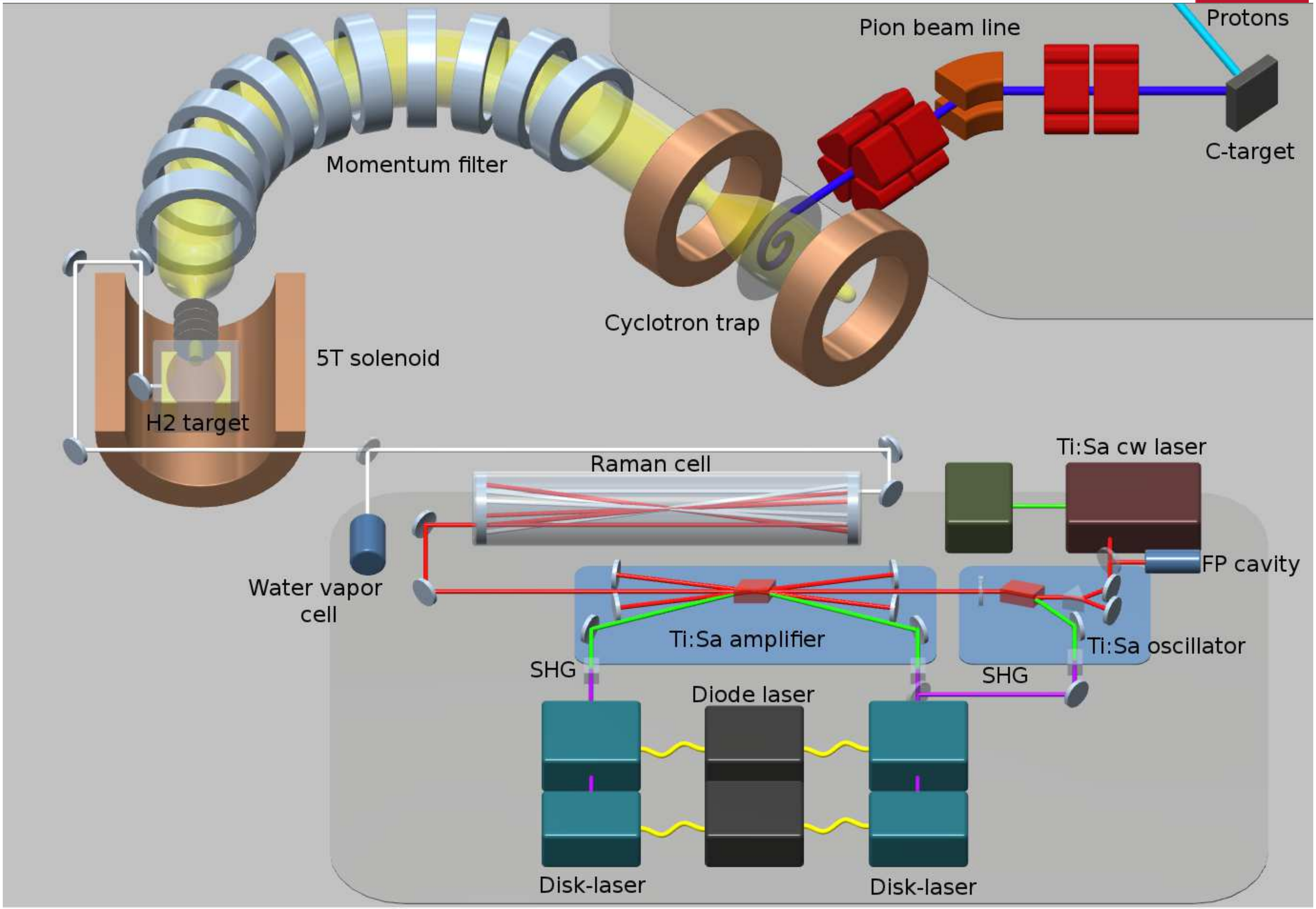
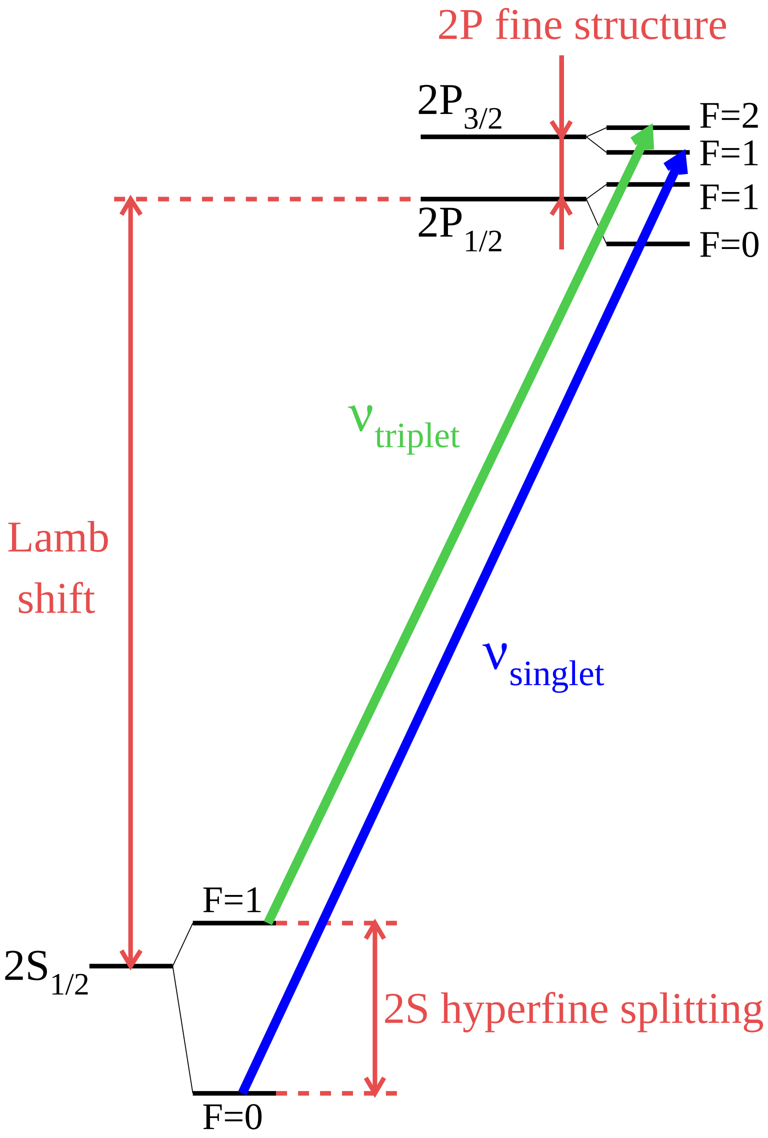
Dufour

Setup

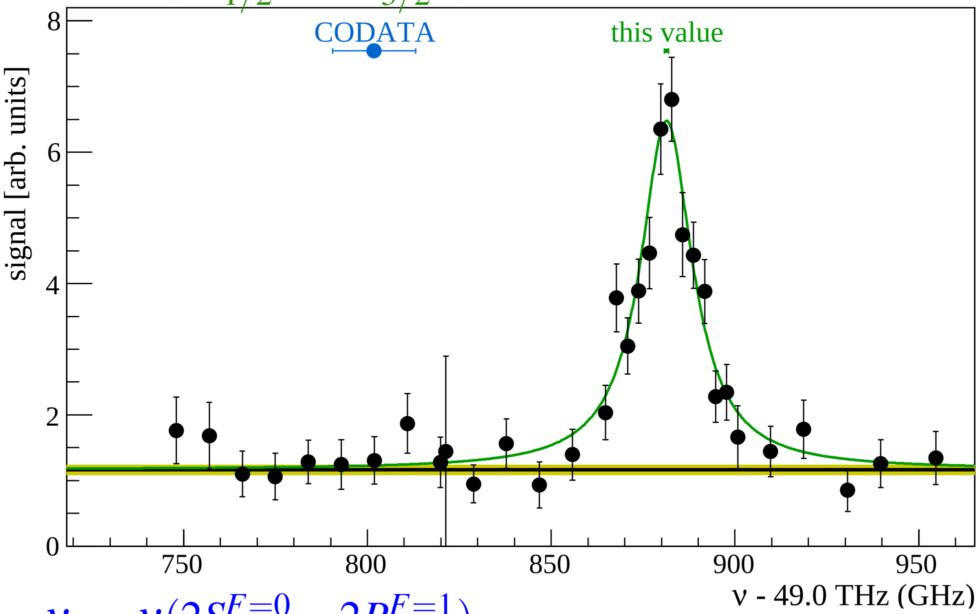

Pion beam line
Protons
C-target
Momentum filter
Cyclotron trap
5T solenoid
H2 target
Raman cell
Ti:Sa cw laser
FP cavity
Water vapor cell
Ti:Sa amplifier
Ti:Sa oscillator
SHG
SHG
Diode laser
Disk-laser
Disk-laser

Muonic hydrogen

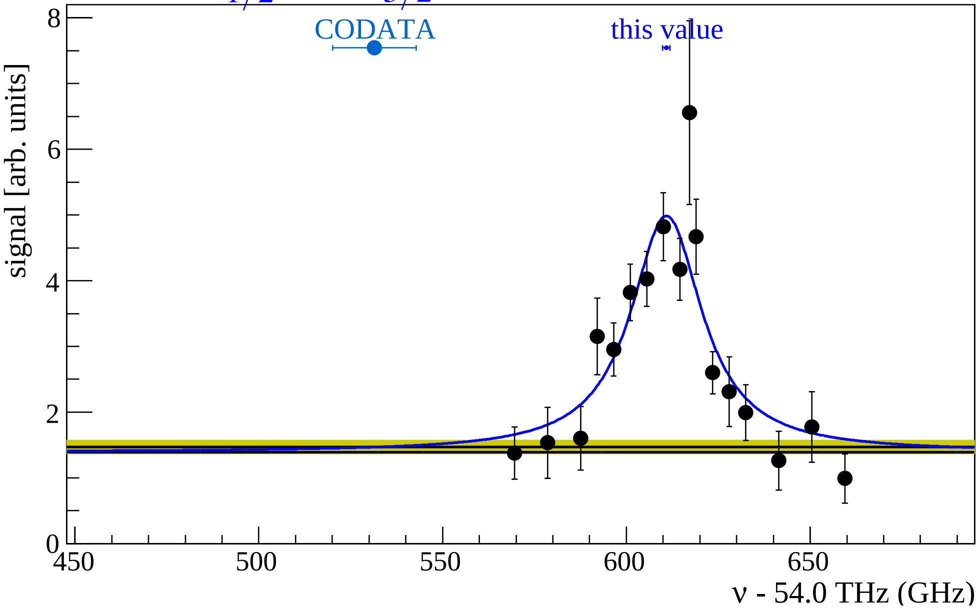
Muonic hydrogen results



$\nu_t = \nu(2S^{F=1}_{1/2} - 2P^{F=2}_{3/2})$

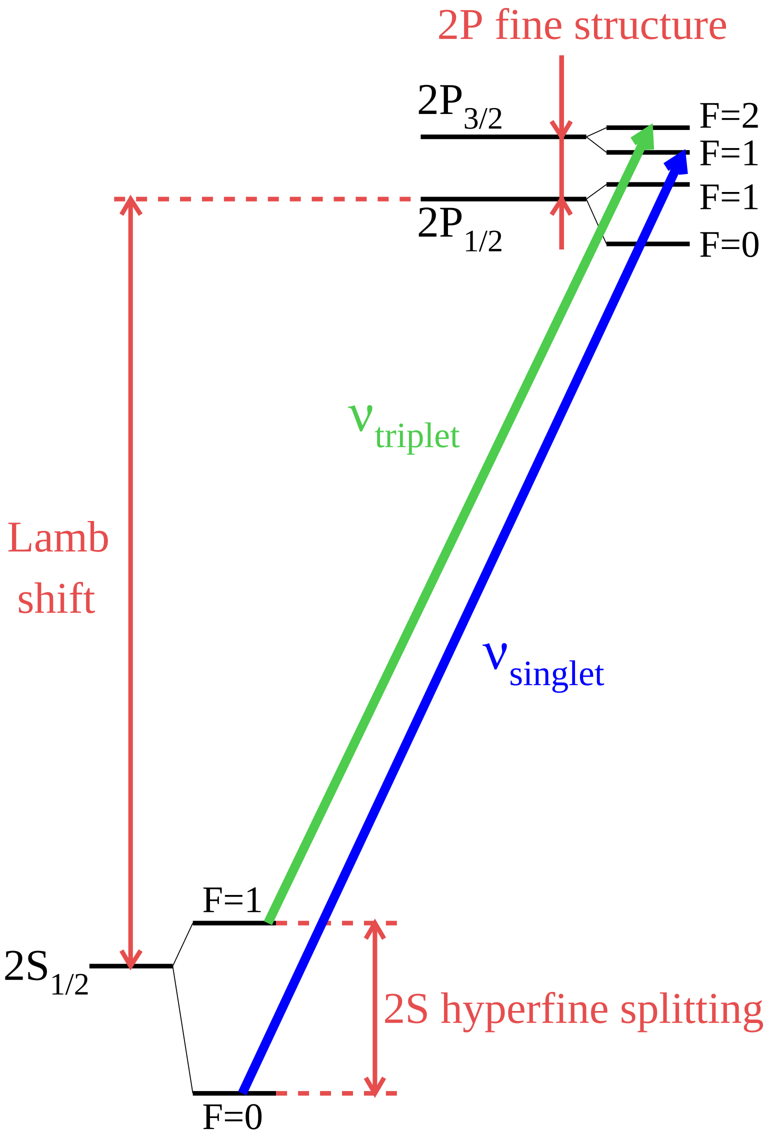


$\nu_s = \nu(2S^{F=0}_{1/2} - 2P^{F=1}_{3/2})$



Exp.: R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini, RP *et al.*, Science 339, 417 (2013).
Theo: A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013).

Muonic hydrogen results



- two transitions measured

$$\nu_t = 49881.35(\ \ 65)\ \text{GHz}$$

$$\nu_s = 54611.16(1.05)\ \text{GHz}$$

- Lamb shift ⇒ **charge radius**

$$\Delta E_{\text{LS}} = 206.0668(25) - 5.2275(10)\ r_{\text{E}}^2 \quad [\text{meV, fm}]$$

$$r_{\text{E}}^2 = \int d^3r\, r^2\, \rho_E(r)$$

$$\boxed{r_{\text{E}} = 0.84087\,(26)_{\text{exp}}\ (29)_{\text{th}}\ \text{fm} = 0.84087\ (39)\ \text{fm}}$$

10x more precise than CODATA-2010
4% smaller (7σ)
proton radius puzzle

Exp.: R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini, RP *et al.*, Science 339, 417 (2013).
Theo: A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013).

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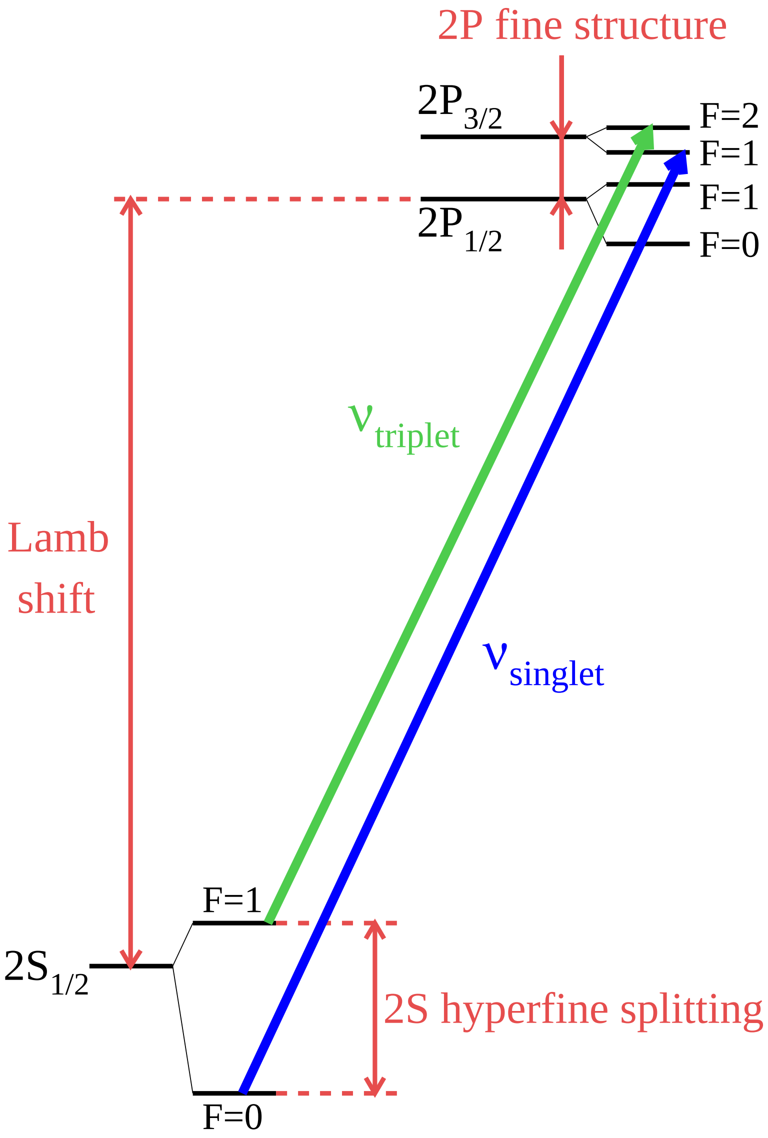

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- 2S-HFS ⇒ **Zemach radius**

$$\Delta E_{\text{HFS}} = 22.9843(30) - 0.1621(10)\, r_{\text{Z}} \quad \text{[meV, fm]}$$

$$r_{\text{Z}} = \int d^3r \int d^3r'\ r\, \rho_E(r)\, \rho_M(r-r')$$

$$\boxed{r_{\text{Z}} = 1.082\ (31)_{\text{exp}}\ (20)_{\text{th}}\ \text{fm} = 1.082\ (37)\ \text{fm}}$$

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A. Antognini, RP *et al.*, Science 339, 417 (2013).
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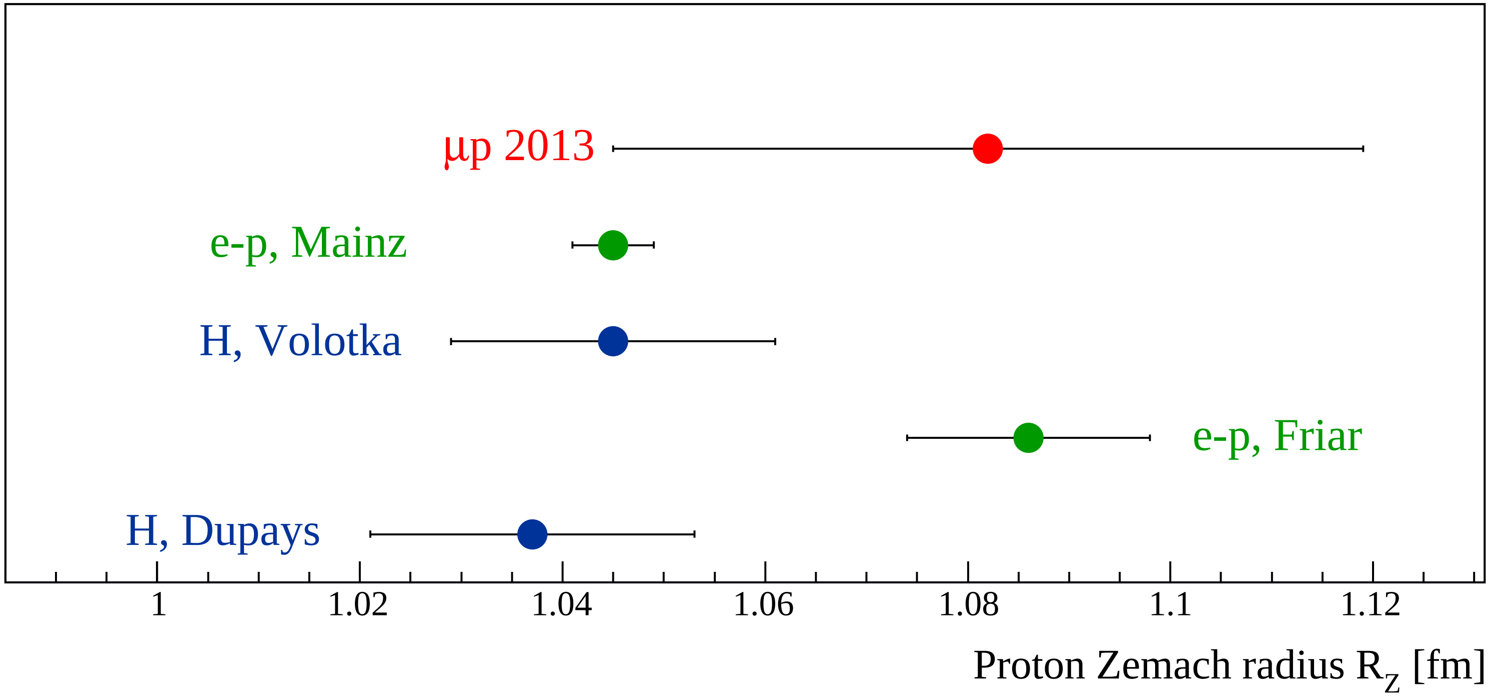
Proton Zemach radius

2S hyperfine splitting in μp is: $\Delta E_{\mathrm{HFS}} = 22.9843(30) - 0.1621(10)\, r_Z\, [\mathrm{fm}]$ meV

with $r_Z = \int d^3r \int d^3r'\, r\, \rho_E(r)\, \rho_M(r - r')$

We measured $\Delta E_{\mathrm{HFS}} = 22.8089(51)$ meV

This gives a proton Zemach radius r_Z = 1.082 $(31)_{\mathrm{exp}}$ $(20)_{\mathrm{th}}$ = 1.082 (37) fm



A. Antognini, RP *et al.*, Science 339, 417 (2013)

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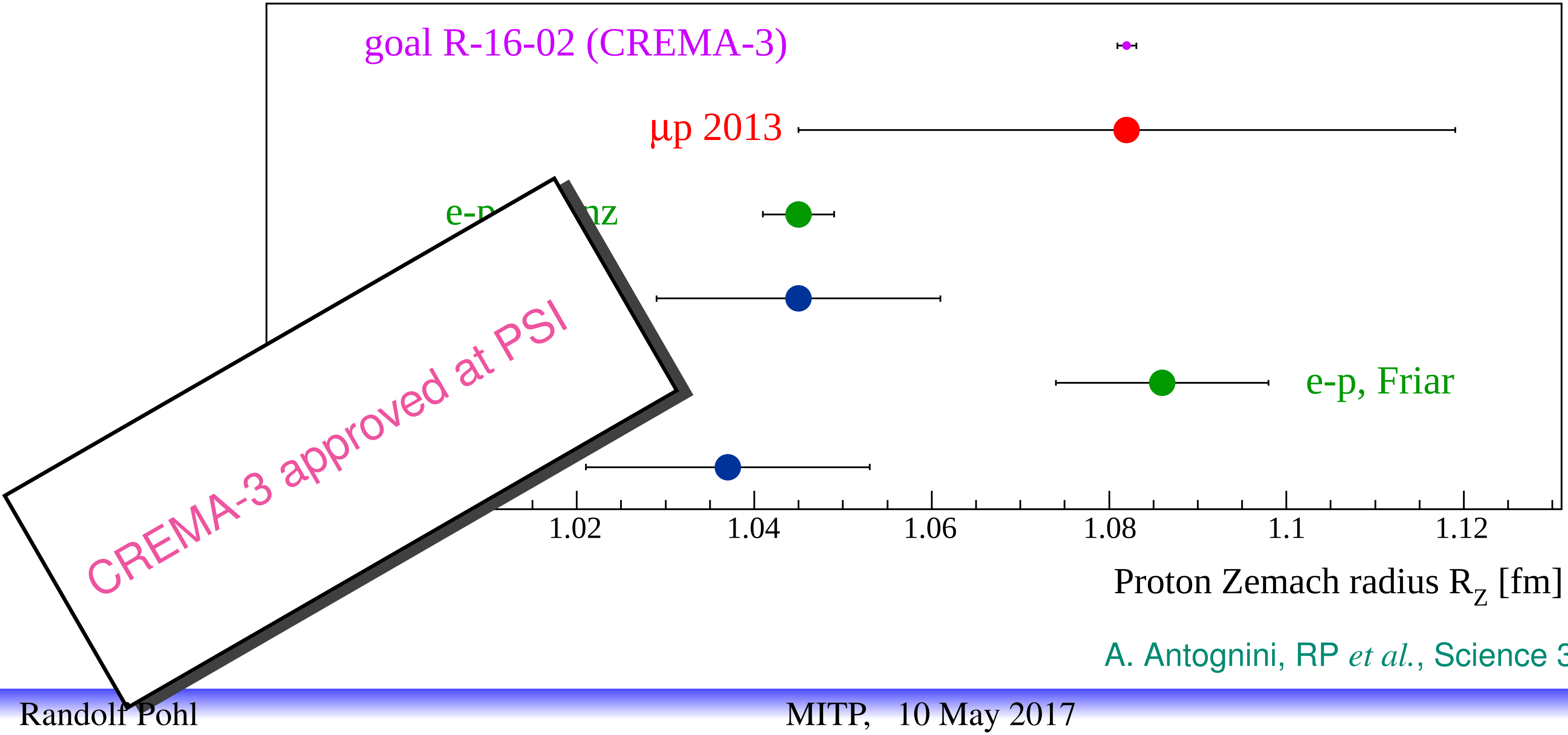
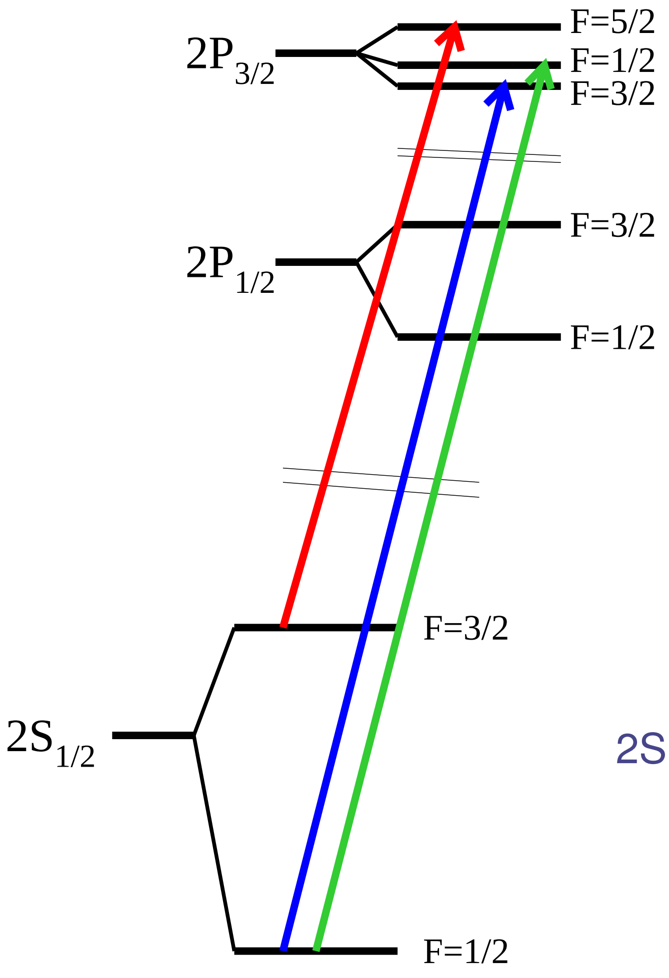


CREMA-3 approved at PSI

A. Antognini, RP *et al.*, Science 339, 417 (2013)

Muonic deuterium



0.7534 meV

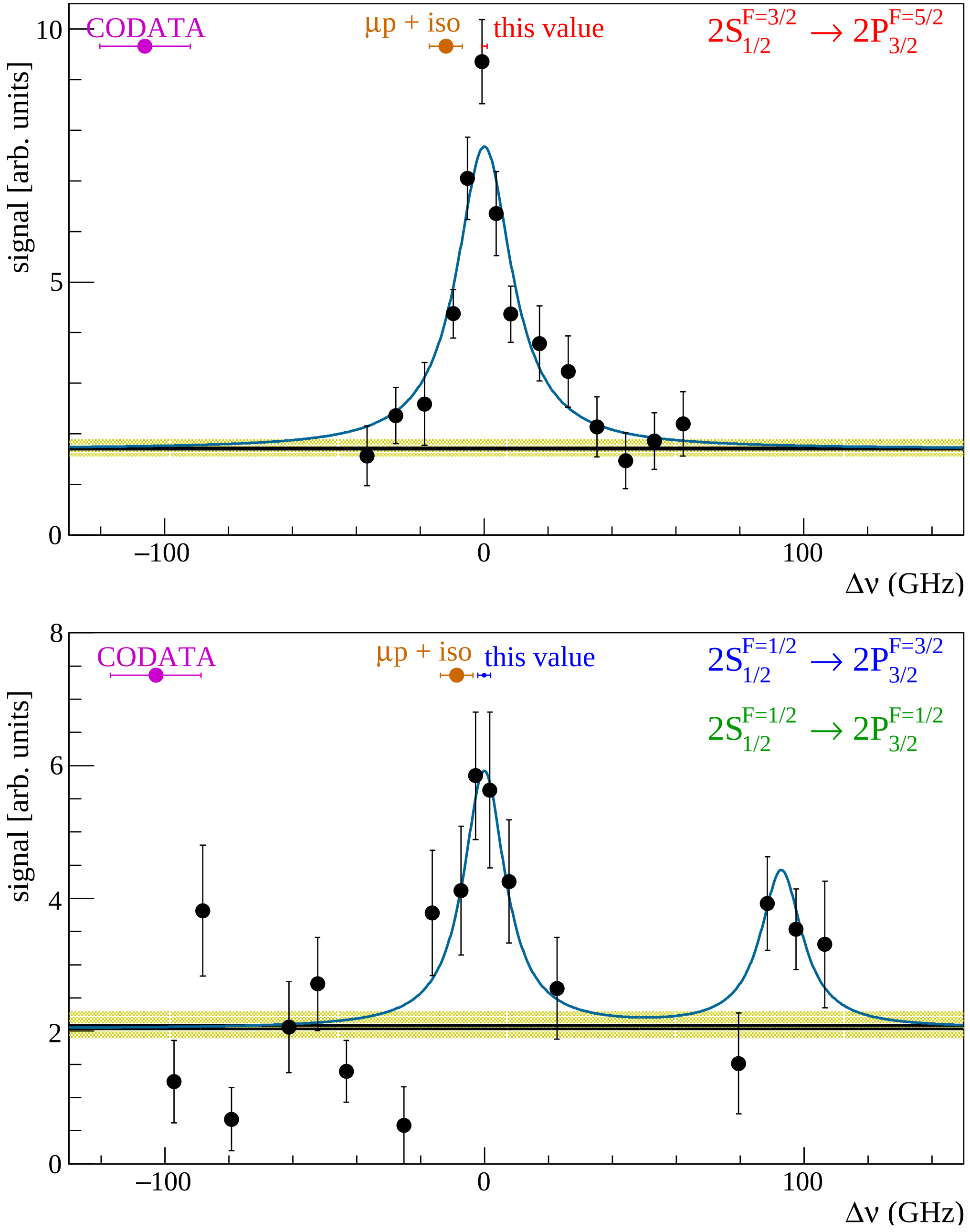
0.3634 meV

FS: 8.86412 meV

LS: 202.88 meV

2S-HFS: 6.27 meV

Muonic DEUTERIUM

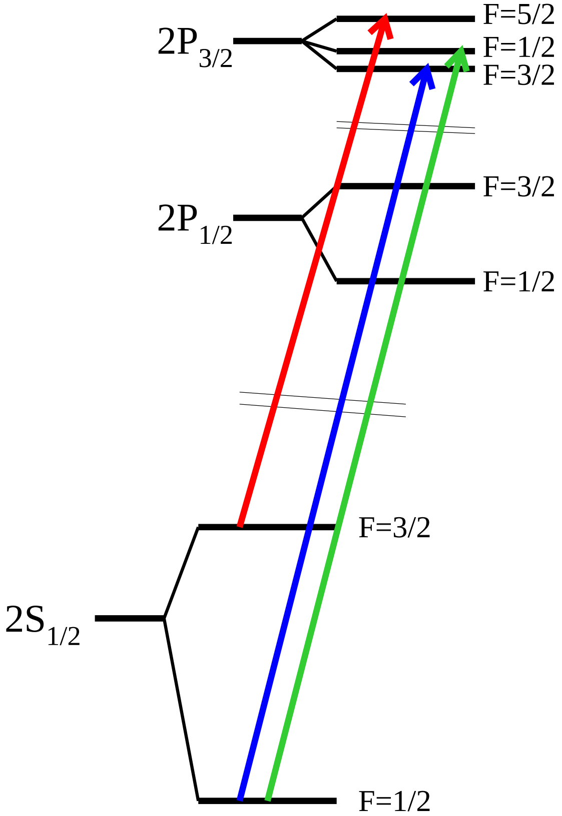


Experiment:

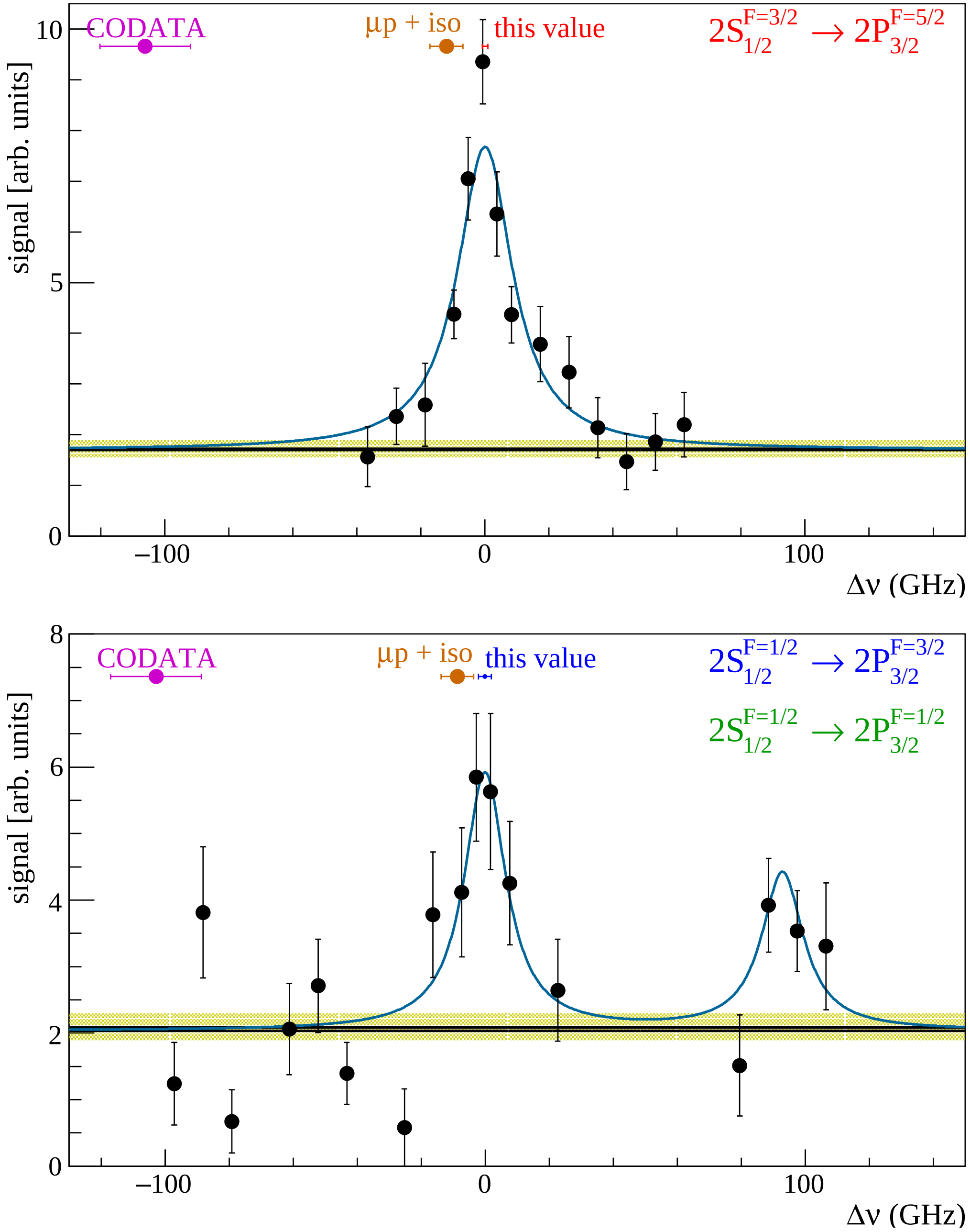
RP *et al.* (CREMA), Science **353**, 417 (2016).

$$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.8785\,(31)_{\mathrm{stat}}(14)_{\mathrm{syst}}\ \text{meV}$$

$$\boxed{\Rightarrow r_{\mathrm{d}} = 2.12562(13)_{\mathrm{exp}}(77)_{\mathrm{theo}}\ \text{fm}}$$



Muonic DEUTERIUM



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$$\boxed{\Rightarrow r_d = 2.12562(13)_{exp}(77)_{theo}\ \text{fm}}$$

Theory:

$$\begin{aligned}\Delta E_{LS}^{theo} = {}& 228.7766(\ \ 10)\,\text{meV}\ (\text{QED})\\ & + 1.7096(200)\,\text{meV}\ (\text{TPE})\\ & - 6.1103(\ \ \ 3)\,r_d^2\ \text{meV/fm}^2,\end{aligned}$$

Krauth, RP *et al.*, Ann. Phys. **366**, 168 (2016) [arXiv 1506.01298]

based on papers and communication from

Bacca, Barnea, Birse, Borie, Carlson, Eides, Faustov, Friar, Gorchtein, Hernandez, Ivanov, Jentschura, Ji, Karshenboim, Korzinin, Krutov, Martynenko, McGovern, Nevo Dinur, Pachucki, Shelyuto, Sick, Vanderhaeghen *et al.*

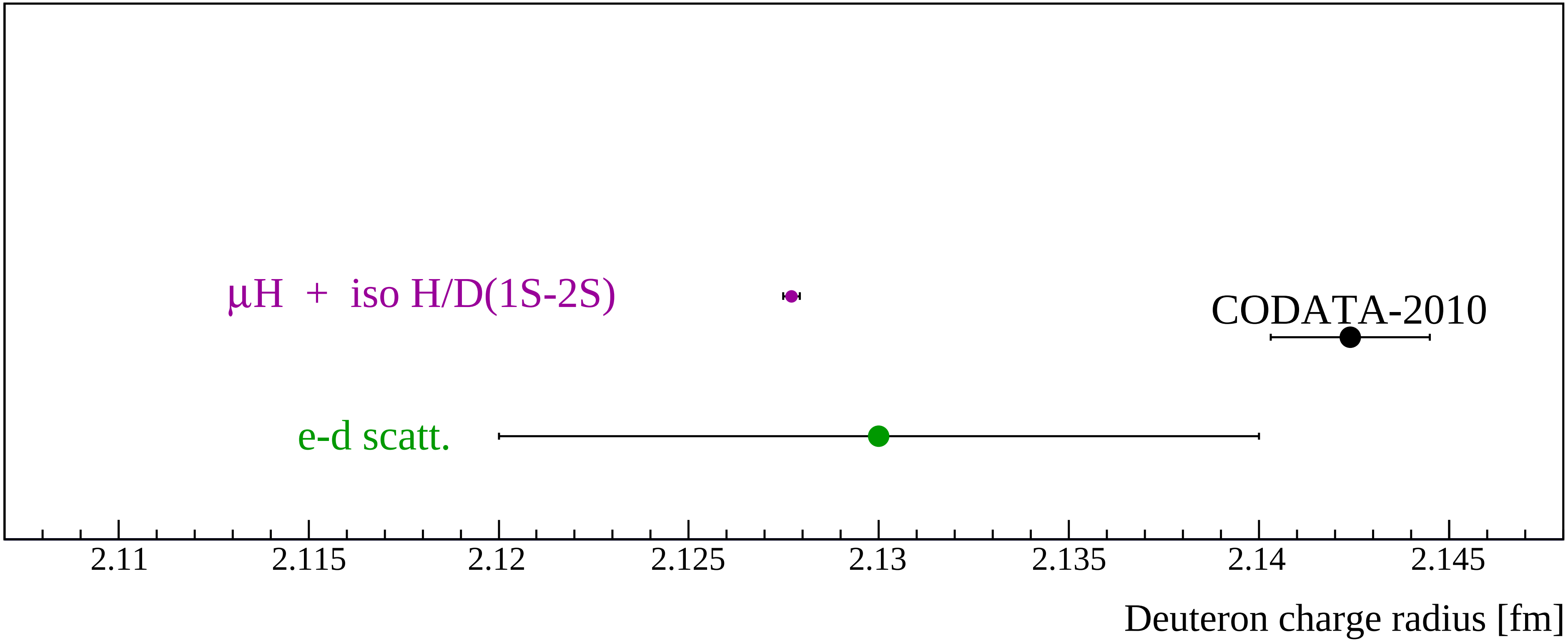
THANK YOU!

Deuteron charge radius

H/D isotope shift: $r_\mathrm{d}^2 - r_\mathrm{p}^2 = 3.82007(65)$ fm^2 C.G. Parthey, RP *et al.*, PRL **104**, 233001 (2010)

CODATA 2014 $r_\mathrm{d} = 2.14130(250)$ fm

$r_\mathrm{p}$from μH gives $r_\mathrm{d} = 2.12771(\ \ 22)$ fm $\leftarrow 5.4\sigma$ from r_p



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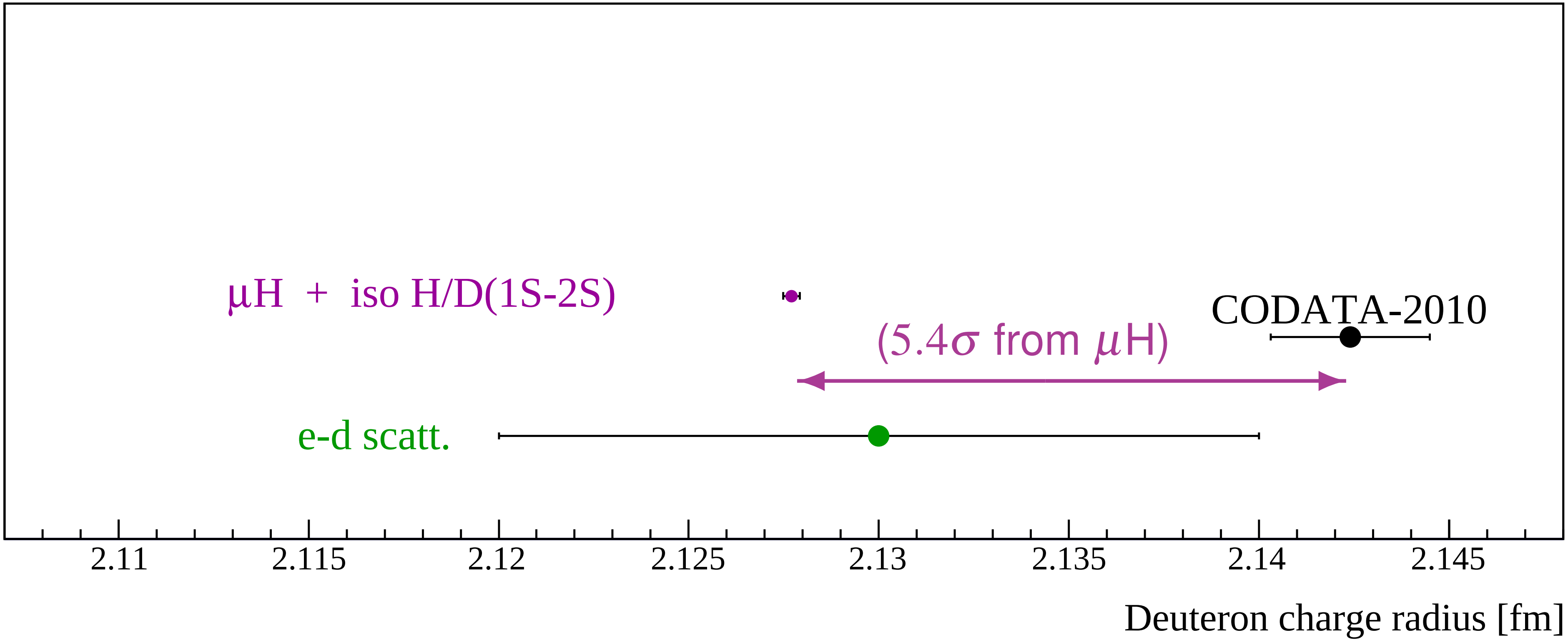

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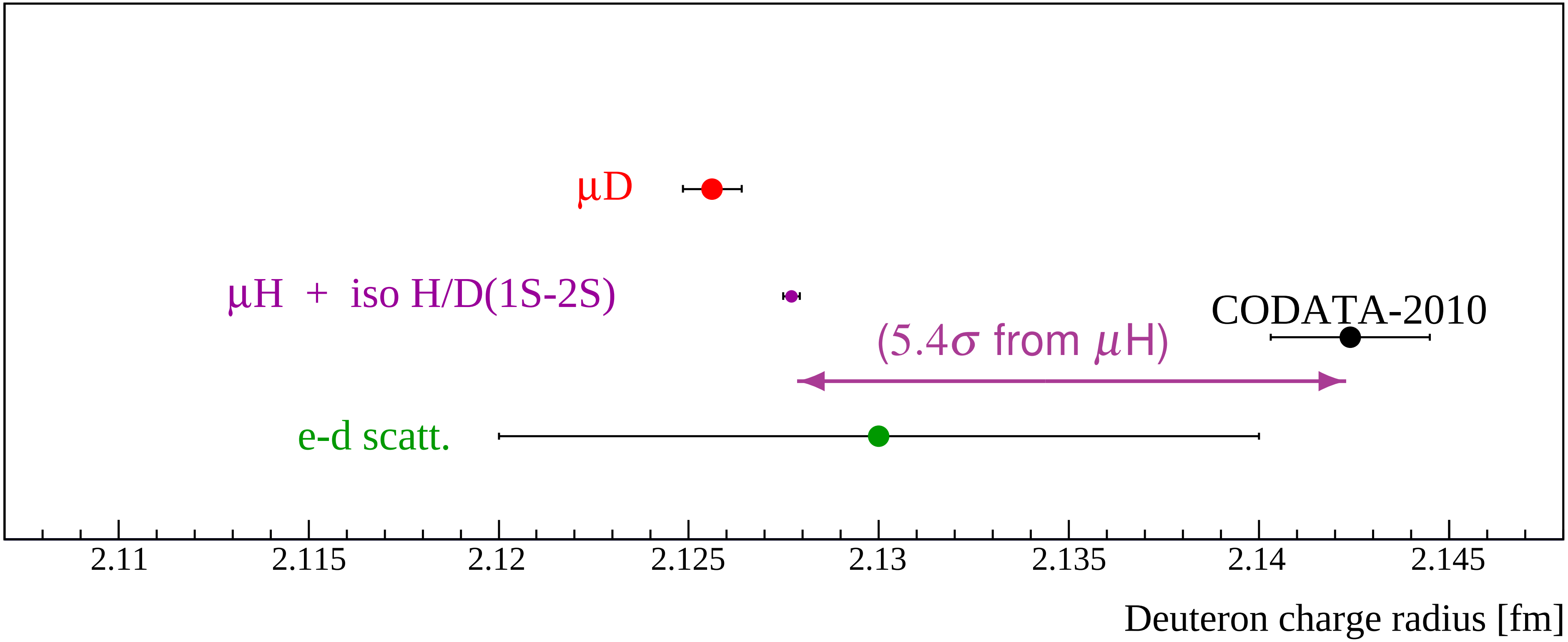
Muonic DEUTERIUM $r_d = 2.12562(\ 13)_{\text{exp}}\ (77)_{\text{theo}}$ fm RP *et al.*, Science **353**, 417 (2016)



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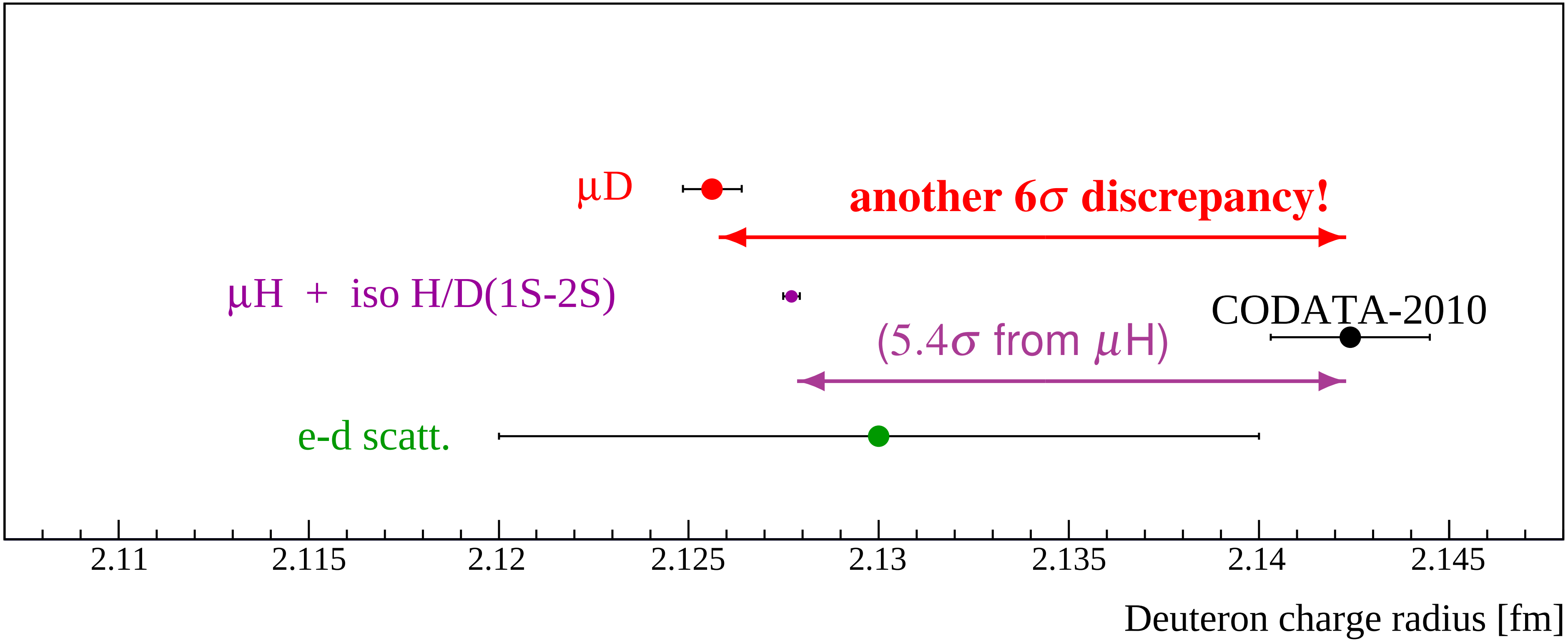
Muonic DEUTERIUM $r_{\mathrm{d}} = 2.12562(\ \ 13)_{\mathrm{exp}}\,(77)_{\mathrm{theo}}\ \mathrm{fm}$ RP *et al.*, Science **353**, 417 (2016)



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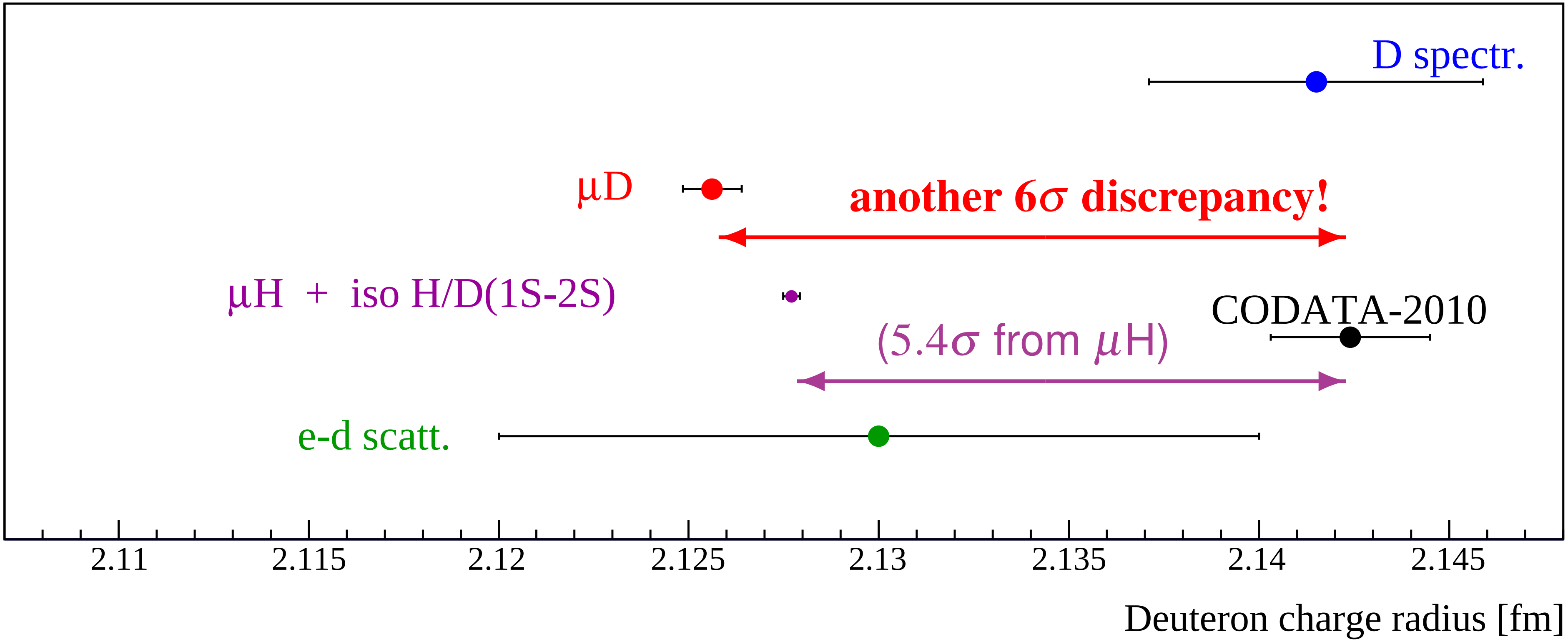
Muonic DEUTERIUM	$r_\mathrm{d} = 2.12562(\ \ 13)_\mathrm{exp}\,(77)_\mathrm{theo}$ fm	RP *et al.*, Science **353**, 417 (2016)
electronic D (r_p indep.)	$r_\mathrm{d} = 2.14150(450)$ fm	RP *et al.* Metrologia **54**, L1 (2017)



Deuteron charge radius

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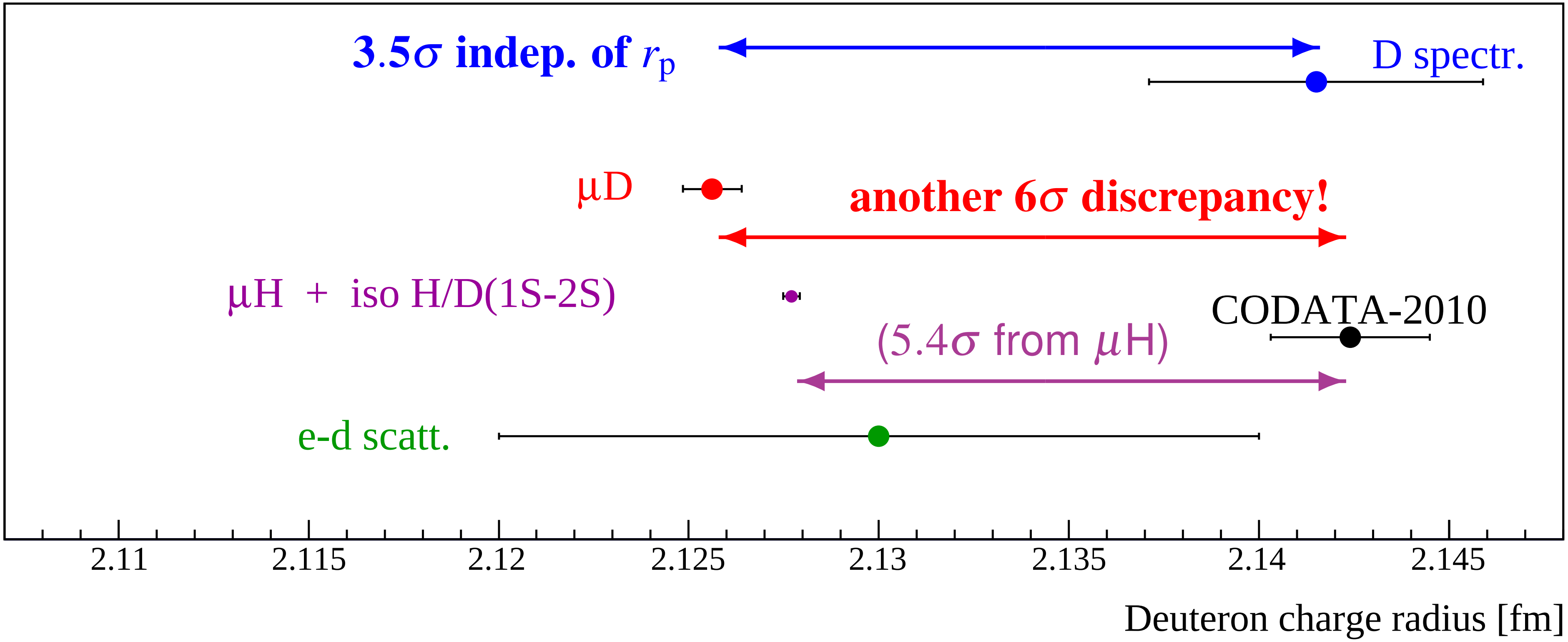
$r_\mathrm{p}$from μH gives $r_\mathrm{d} = 2.12771(\ \ 22)\ \mathrm{fm} \quad \leftarrow 5.4\sigma$ from r_p

Muonic DEUTERIUM $r_\mathrm{d} = 2.12562(\ \ 13)_\mathrm{exp}\,(77)_\mathrm{theo}\ \mathrm{fm}$ RP *et al.*, Science **353**, 417 (2016)

electronic D (r_p indep.) $r_\mathrm{d} = 2.14150(450)\ \mathrm{fm} \leftarrow 3.5\sigma$ RP *et al.* Metrologia **54**, L1 (2017)



Results from muonic deuterium

Lamb shift in muonic deuterium:

$$\Delta E_{\mathrm{LS}}^{\mathrm{theo}} = 228.7766(10)\,\mathrm{meV} + \Delta E^{\mathrm{TPE}} - 6.1103(3)\, r_{\mathrm{d}}^2\ \mathrm{meV/fm}^2$$

with deuteron polarizability (TPE) $\Delta E^{\mathrm{TPE}}(\mathrm{theo}) = 1.7096(200)\,\mathrm{meV}$

J.J. Krauth *et al.*, Ann. Phys. **366**, 168 (2016) [1506.01298]

compilation of original results from:
Borie, Martynenko *et al.*, Karshenboim *et al.*, Jentschura,
Bacca, Barnea, Nevo Dinur *et al.*, Pachucki *et al.*, Friar,
Carlson, Gorchtein, Vanderhaeghen, and others

$r_d(\mu \mathrm{d}) = 2.12562(\ 13)_{\mathrm{exp}}(77)_{\mathrm{theo}}\ \mathrm{fm}$ RP *et al.*, Science **353**, 417 (2016)

$r_d(\mu \mathrm{p} + \mathrm{iso}) = 2.12771(\ 22)\ \mathrm{fm}$ from $r_{\mathrm{p}}(\mu \mathrm{p})$ and H/D(1S-2S) 2.6σ

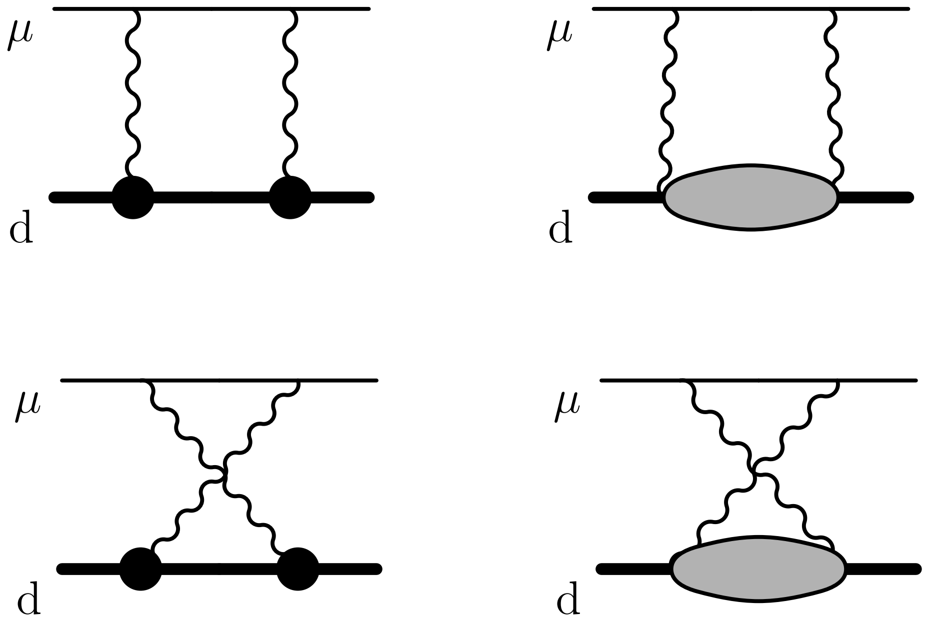
$r_d(\mathrm{CODATA}) = 2.14130(250)\ \mathrm{fm}$ 6.0σ

Disprepancy to $\Delta E_{\mathrm{LS}}(r_d(\mathrm{CODATA}))$ = 0.409(68) meV

("proton radius puzzle" (μp discrepancy) = 0.329(47) meV)

Theory in μd: TPE

Deuteron structure contributions to the Lamb shift in muonic deuterium.



Cancellation between elastic “Friar” (a.k.a. 3rd Zemach) terms and part of inelastic “polarizability” contributions.

Friar & Payne, PRA 56, 5173 (1997) ; Pachucki, PRL 106, 193007 (2011) ; Friar, PRC 88, 034003 (2013) ; Hernandez *et al.*, PLB 736, 344 (2014)

J.J. Krauth, RP *et al.*, Ann. Phys. **366**, 168 (2016) [1506.01298]

Theory in μd: TPE

Table 3: Deuteron structure contributions to the Lamb shift in muonic deuterium. Values are in meV.

Item	Contribution	Pachucki [55]		Friar [60]		Hernandez *et al.* [58]			Pach.& Wienczek [65]		Carlson *et al.* [64]	Our choice	
		AV18		ZRA		AV18	N^{3}LO †		AV18		data	value	source
	Source	1		2		3	4		5		6		
p1	Dipole	1.910	$\delta_0 E$	1.925	Leading C1	1.907	1.926	$\delta_{D1}^{(0)}$	1.910	$\delta_0 E$		1.9165 ± 0.0095	3-5
p2	Rel. corr. to p1, longitudinal part	−0.035	$\delta_R E$	−0.037	Subleading C1	−0.029	−0.030	$\delta_L^{(0)}$	−0.026	$\delta_R E$			
p3	Rel. corr. to p1, transverse part					0.012	0.013	$\delta_T^{(0)}$					
p4	Rel. corr. to p1, higher order								0.004	$\delta_{HO} E$			
sum	Total rel. corr., p2+p3+p4	−0.035		−0.037		−0.017	−0.017		−0.022			−0.0195 ± 0.0025	3-5
p5	Coulomb distortion, leading	−0.255	$\delta_{C1} E$						−0.255	$\delta_{C1} E$			
p6	Coul. distortion, next order	−0.006	$\delta_{C2} E$						−0.006	$\delta_{C2} E$			
sum	Total Coulomb distortion, p5+p6	−0.261				−0.262	−0.264	$\delta_C^{(0)}$	−0.261			−0.2625 ± 0.0015	3-5
p7	El. monopole excitation	−0.045	$\delta_{Q0} E$	−0.042	C0	−0.042	−0.041	$\delta_{R2}^{(2)}$	−0.042	$\delta_{Q0} E$			
p8	El. dipole excitation	0.151	$\delta_{Q1} E$	0.137	Retarded C1	0.139	0.140	$\delta_{D1D3}^{(2)}$	0.139	$\delta_{Q1} E$			
p9	El. quadrupole excitation	−0.066	$\delta_{Q2} E$	−0.061	C2	−0.061	−0.061	$\delta_Q^{(2)}$	−0.061	$\delta_{Q2} E$			
sum	Tot. nuclear excitation, p7+p8+p9	0.040		0.034	C0 + ret-C1 + C2	0.036	0.038		0.036			0.0360 ± 0.0020	2-5
p10	Magnetic	−0.008 ◊	$\delta_M E$	−0.011	M1	−0.008	−0.007	$\delta_M^{(0)}$	−0.008	$\delta_M E$		−0.0090 ± 0.0020	2-5
SUM_1	Total nuclear (corrected)	1.646		1.648		1.656	1.676		1.655			1.6615 ± 0.0103	
p11	Finite nucleon size			0.021	Retarded C1 f.s.	0.020 ◊	0.021 ◊??	$\delta_{NS}^{(2)}$	0.020	$\delta_{FS} E$			
p12	n p charge correlation			−0.023	pn correl. f.s.	−0.017	−0.017	$\delta_{np}^{(1)}$	−0.018	$\delta_{FZ} E$			
sum	p11+p12			−0.002		0.003	0.004		0.002			0.0010 ± 0.0030	2-5
p13	Proton elastic 3rd Zemach moment	} 0.043(3)	$\delta_P E$	0.030	$\langle r^3 \rangle_{(2)}^{\rm pp}$				} 0.043(3)	$\delta_P E$		0.0289 ± 0.0015	Eq.(13)
p14	Proton inelastic polarizab.					} 0.027(2)		$\delta_{\rm pol}^N$ [64]			} 0.028(2)$\Delta E^{\rm hadr}$	} 0.0280 ± 0.0020	6
p15	Neutron inelastic polarizab.								0.016(8)	$\delta_N E$			
p16	Proton & neutron subtraction term											−0.0098 ± 0.0098	Eq.(15)
sum	Nucleon TPE, p13+p14+p15+p16	0.043(3)		0.030		0.027(2)			0.059(9)			0.0471 ± 0.0101	
SUM_2	Total nucleon contrib.	0.043(3)		0.028		0.030(2)			0.061(9)			0.0476 ± 0.0105	
	Sum, published	1.680(16)		1.941(19)		1.690(20)			1.717(20)		2.011(740)		
	Sum, corrected			1.697(19)		1.714(20)			1.707(20)		1.748(740)	**1.7096 ± 0.0147**	

J.J. Krauth *et al.*, Ann. Phys. **366**, 168 (2016) [1506.01298]

$$\Delta E^{\rm TPE}(\text{theo}) = 1.7096 \pm 0.0200 \text{ meV}$$

vs. $\pm\, 0.0034$ meV exp. uncertainty

Experimental TPE in μd

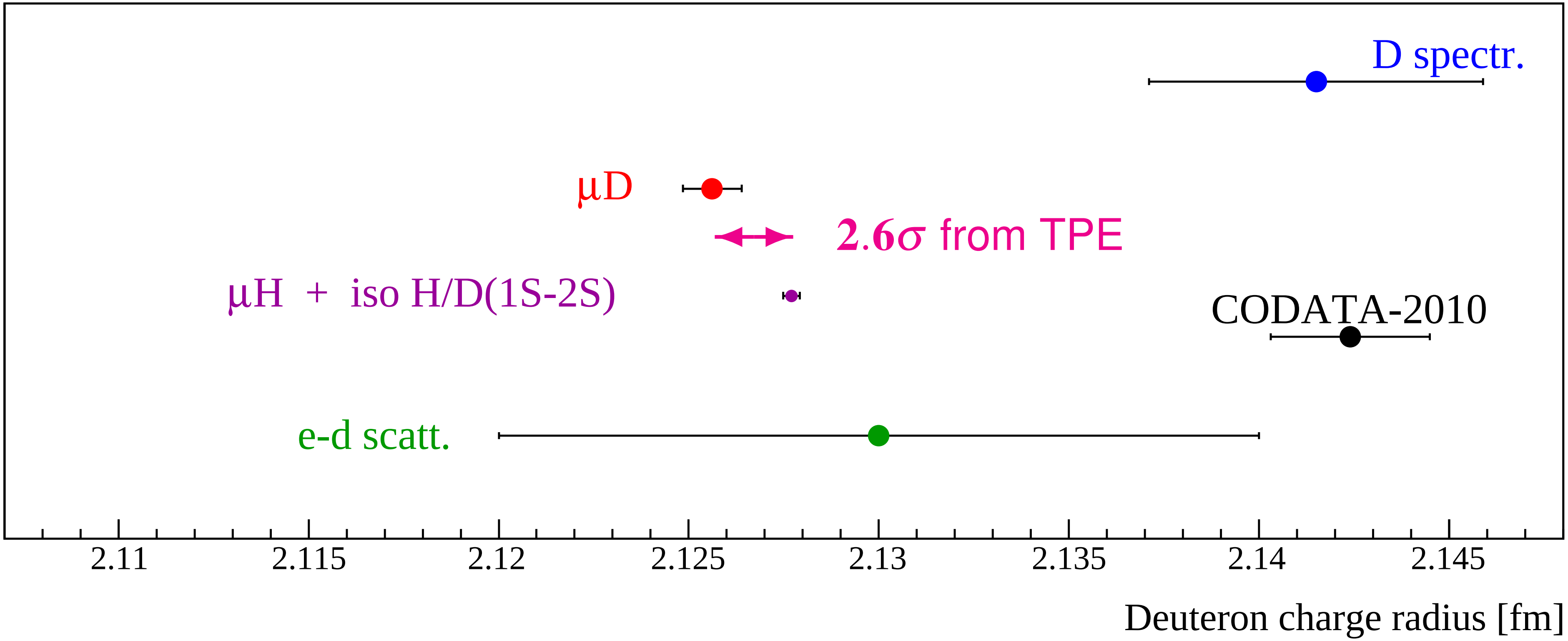
$$\Delta E^{\mathrm{TPE}}(\mathrm{theo}) = 1.7096 \pm 0.0200\ \mathrm{meV}$$

$$\Delta E^{\mathrm{TPE}}(\mathrm{exp}) = 1.7638 \pm 0.0068\ \mathrm{meV}$$ **2.6σ**, 3x more accurate

$$\Delta E_{\mathrm{LS}} = 228.7766(10)\,\mathrm{meV}\ (\mathrm{QED}) + \Delta E^{\mathrm{TPE}} - 6.1103(3)\ r_{\mathrm{d}}^2\ \mathrm{meV/fm}^2,$$

- $\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.8785\,(31)_{\mathrm{stat}}\,(14)_{\mathrm{syst}}$ meV from μD exp.
- $r_d = 2.12771(22)$ fm

from $r_{\mathrm{d}}^2 - r_{\mathrm{p}}^2 = 3.82007(65)\ \mathrm{fm}^2$ [H/D(1S-2S) isotope shift]
using $r_{\mathrm{p}}(\mu\mathrm{H})$ = 0.84087(39) fm



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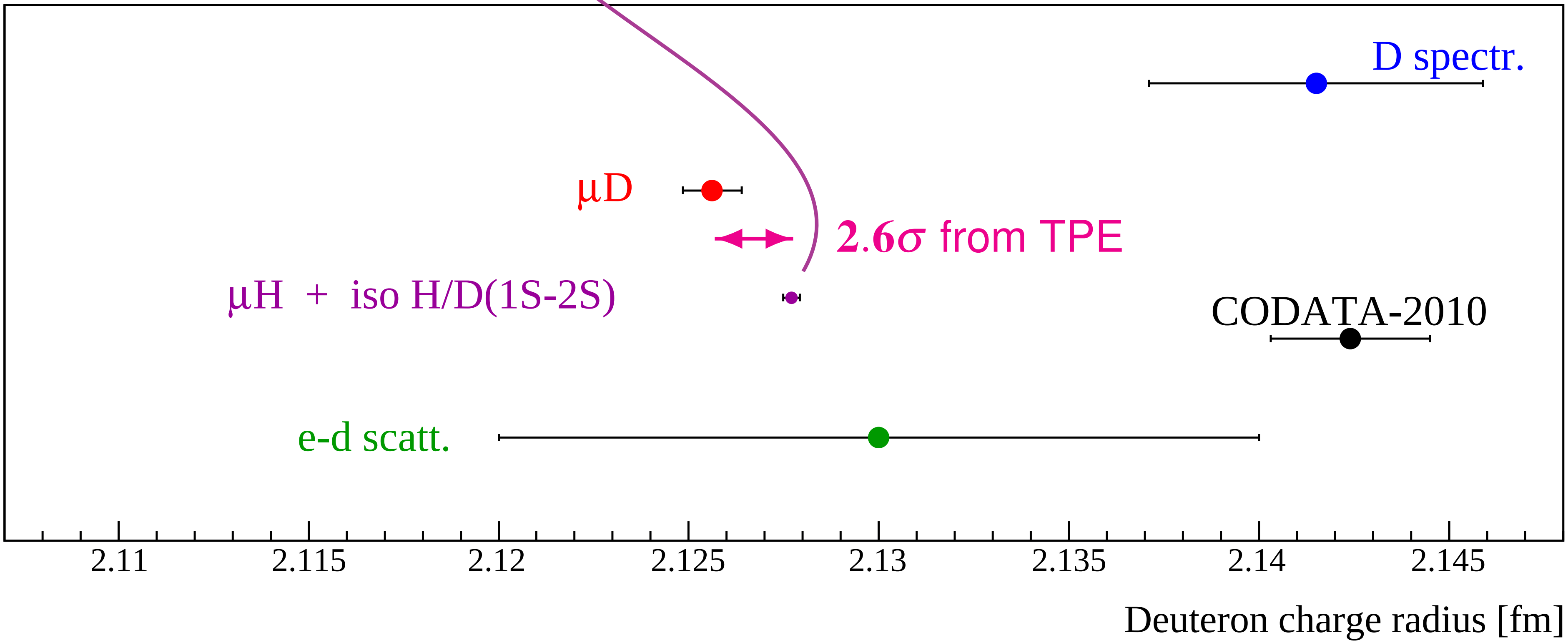

Conclusions μp and μd

Pohl *et al.*, Nature 466, 213 (2010).
Antognini *et al.*, Science 339, 417 (2013).
Pohl *et al.*, Science 353, 669 (2016).
Antognini *et al.*, Ann. Phys. 331, 127 (2013).
Krauth *et al.*, Ann. Phys. 366, 168 (2016).
Pohl *et al.*, Metrologia 54, L1 (2017).

- Proton charge radius: r_p= 0.84087 (39) fm
- Proton Zemach radius: R_Z = 1.082 (37) fm
- Rydberg constant, using H(1S-2S):
  $R_\infty = 3.289\,841\,960\,249\,5\ (10)^\mathrm{radius}\ (25)^\mathrm{QED}\ \times 10^{15}\ \mathrm{Hz/c}$
- Deuteron charge radius: r_d= 2.12771 (22) fm using H/D(1S-2S)
- r_p is $\sim 7\sigma$ smaller than CODATA-2010
  4.0σ smaller than r_p(H spectrosopy)
- r_d is 7.5σ smaller than CODATA-2010 (99% correlated with r_p!)
  3.5σ smaller than r_d(D spectrosopy)
- Proton and deuteron are consistently too small:

$$r_\mathrm{d}^2 = r_\mathrm{struct}^2 + r_\mathrm{p}^2 + r_n^2 + \frac{3\hbar^2}{4m_p^2c^2}$$

Conclusions μp and μd

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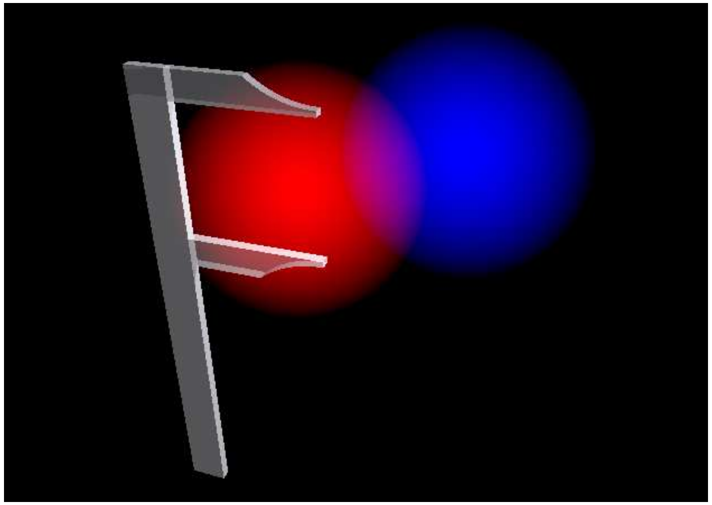
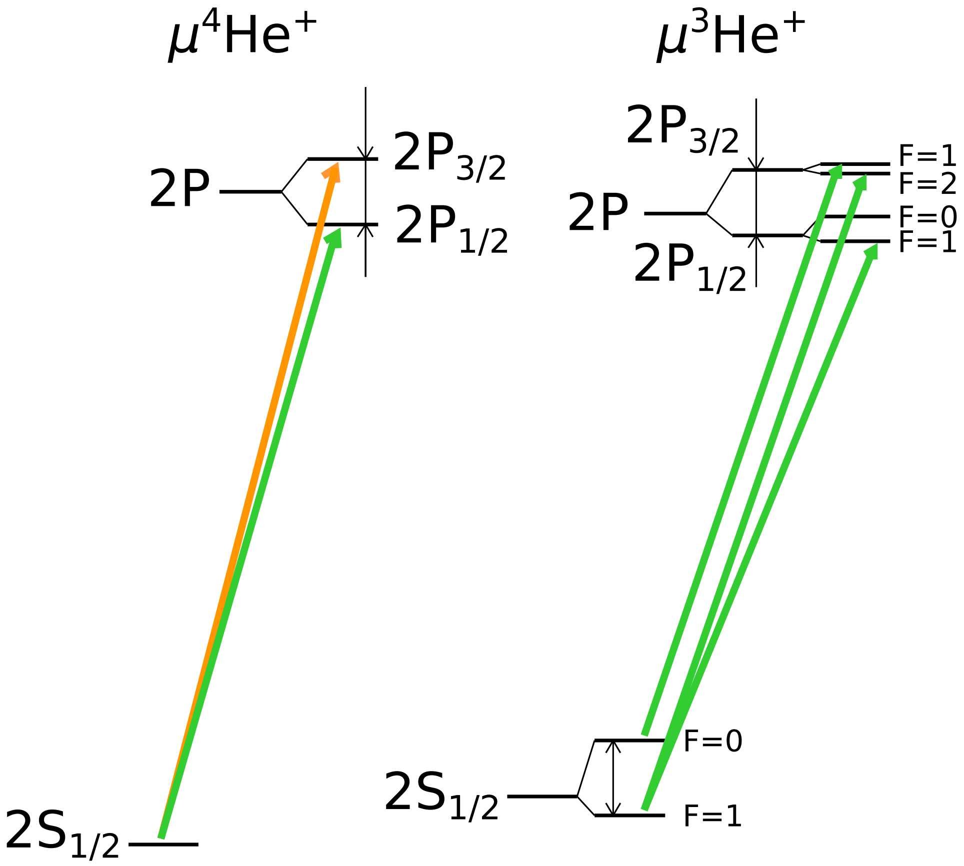
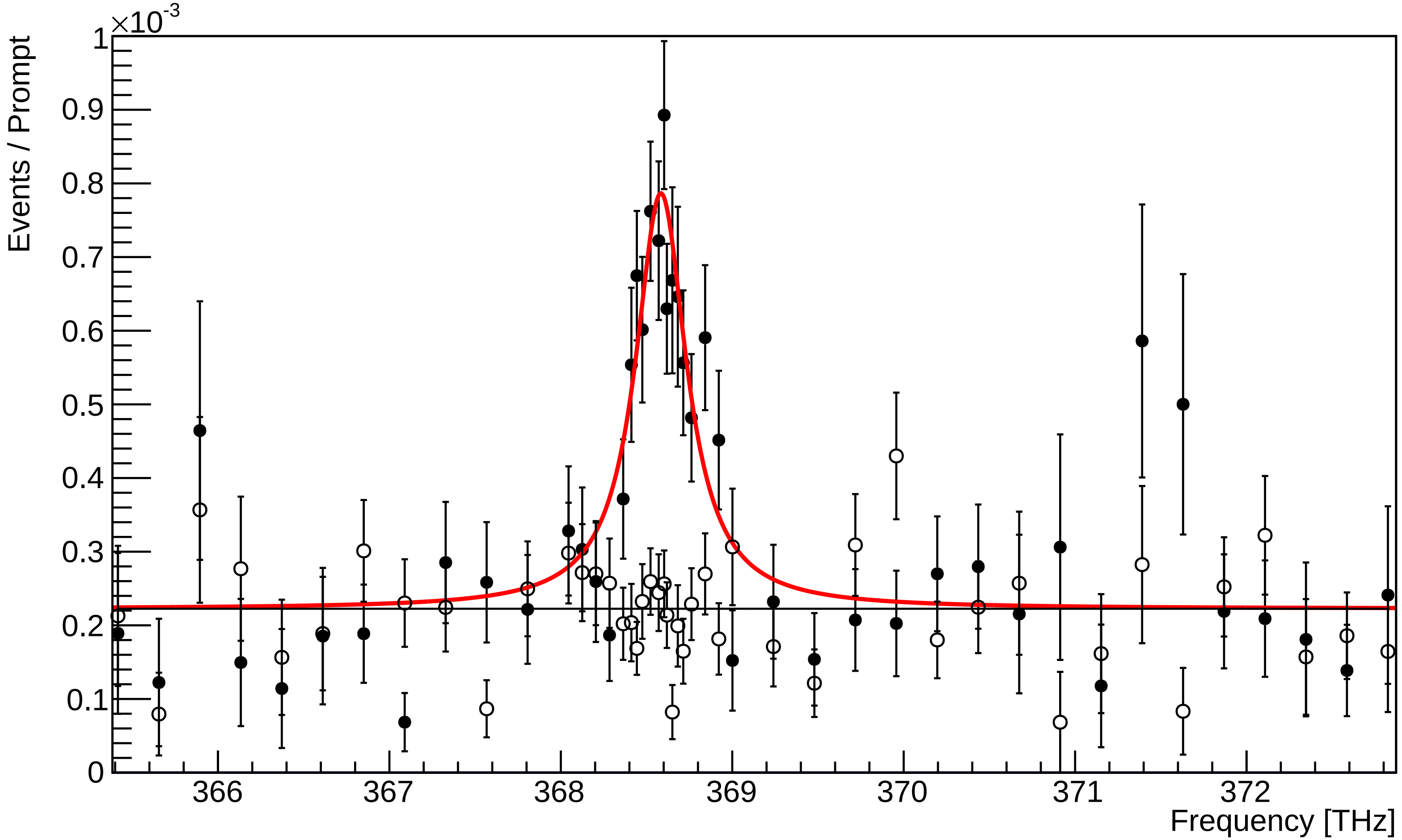


Muonic helium ions



$\mu^4\mathrm{He}^+(2S_{1/2} \rightarrow 2P_{3/2})$

1st μ^4He-ion resonance at $\sim$ 812 nm wavelength



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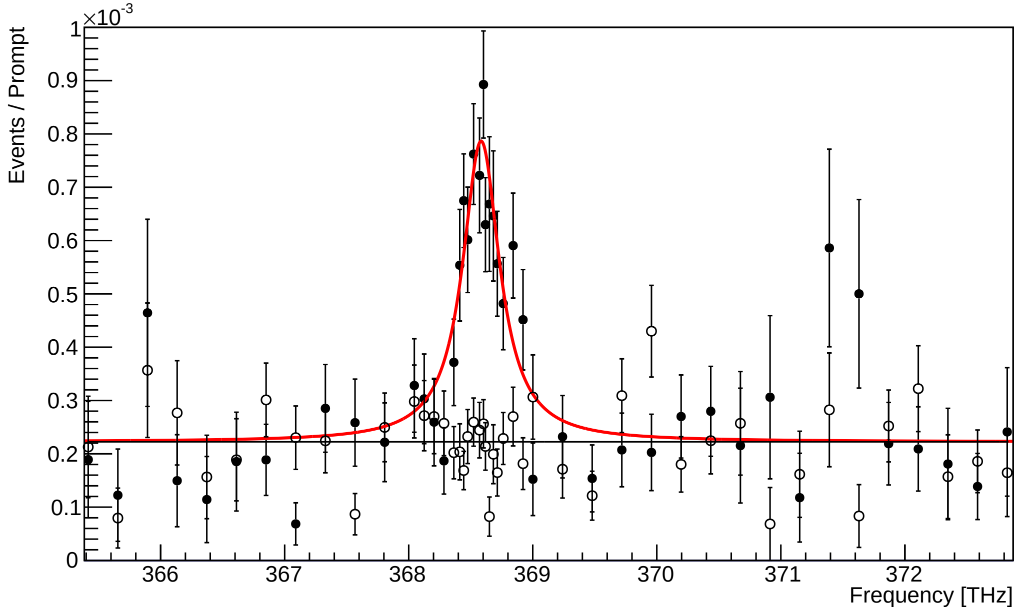


$$\begin{aligned}\Delta E(2S-2P) \quad = \quad & 1668.487(\ \ 14)\,\mathrm{meV}_{\mathrm{(QED)}} \\ & -106.358(\ \ \ 7)\,\mathrm{meV/fm}^2 \cdot \langle r^2 \rangle \\ & +6.761(\ \ 77)\,\mathrm{meV}_{\mathrm{(Friar)}} \\ & +3.296(189)\,\mathrm{meV}_{\mathrm{(polarizability)}} \\ & +146.197(\ \ 12)\,\mathrm{meV}_{\mathrm{(fine\ structure)}}\end{aligned}$$

Diepold et al., 1606.05231

Thanks to the theorists!

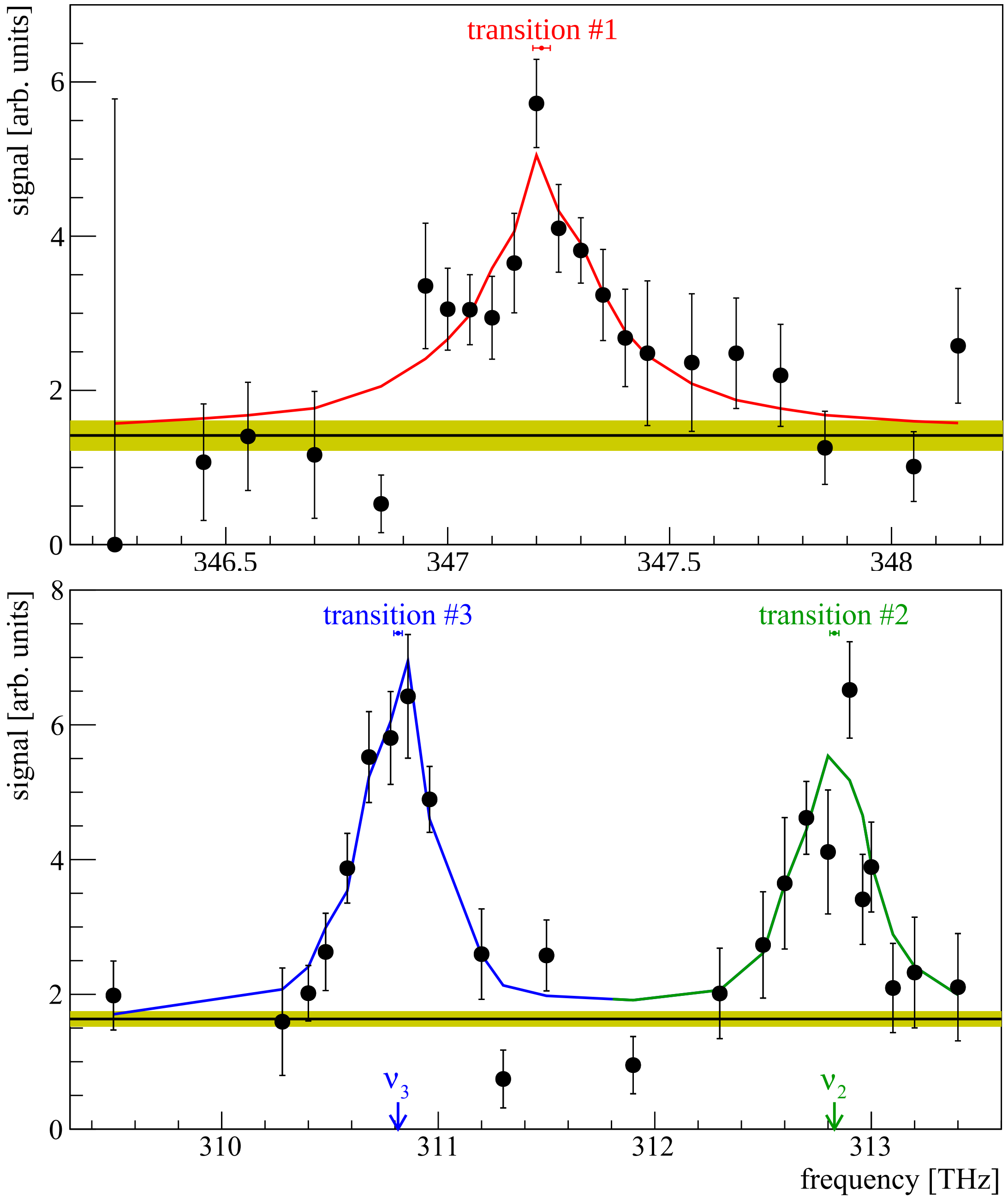
expt'l accuracy: 17 GHz $\equiv$ 0.066 meV

r(^{4}He) = 1.68xxx (19)$_{\mathrm{exp}}$ (58)$_{\mathrm{theo}}$ fm **PRELIMINARY**

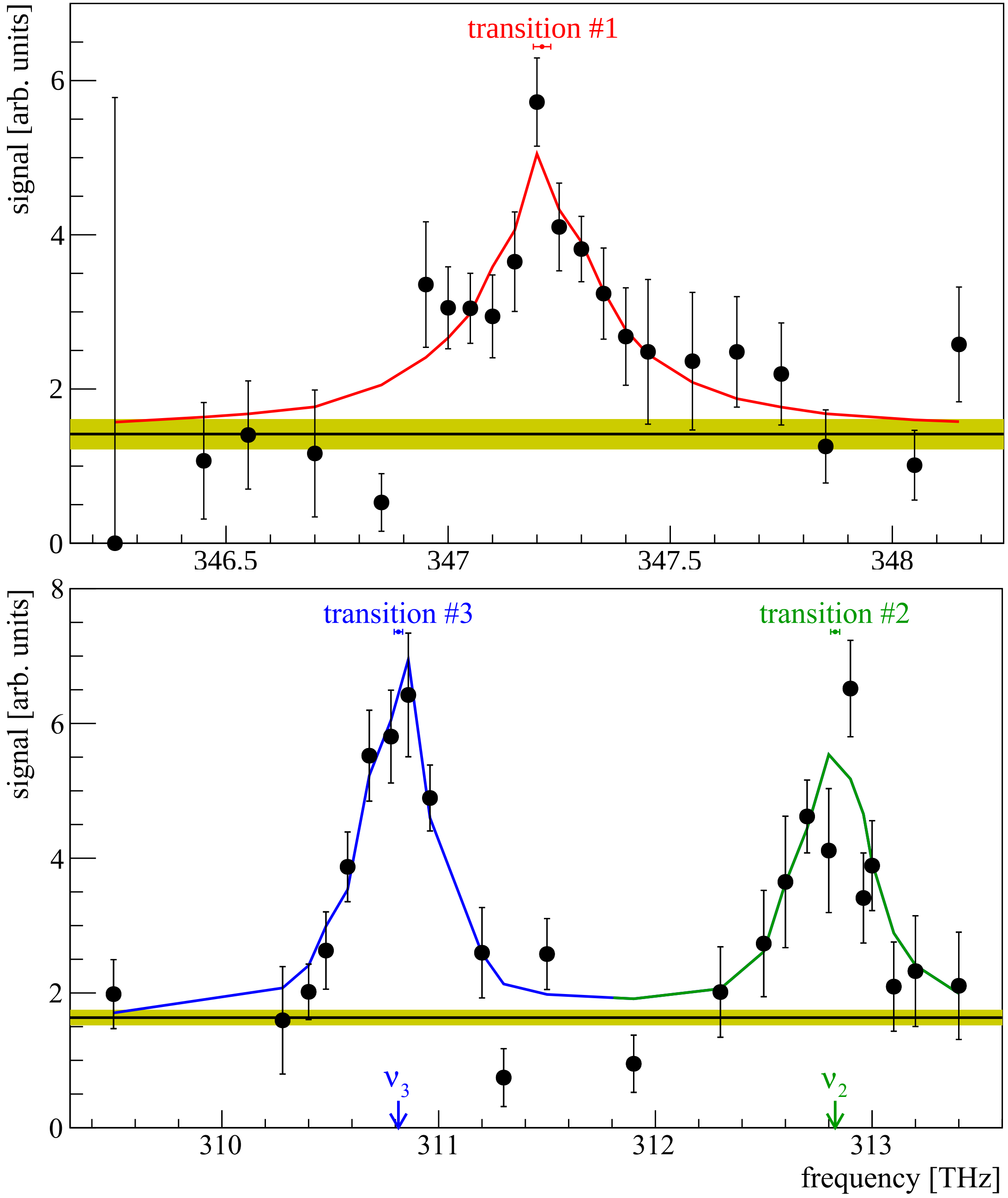
vs. 1.68100 (400) fm from e-He scattering

(plus the other transition $\mu^4\mathrm{He}^+(2S_{1/2} \to 2P_{\mathbf{1/2}})$)

μ^3He^+ resonances



$\mu^3\mathrm{He}^+$ resonances



$$\begin{aligned} \Delta E(2S-2P_{1/2}) = {} & \\ & 1644.347\,(\ 15)\,\mathrm{meV}_{(\mathrm{QED})} \\ - {} & 103.518\,(\ 10)\,\mathrm{meV/fm}^2\cdot\langle r^2\rangle \\ + {} & 0.118\,(\ \ 3)\,\mathrm{meV}_{(\mathrm{radius})} \\ + {} & 15.300\,(520)\,\mathrm{meV}_{(\mathrm{polarizability})} \end{aligned}$$

Franke, Krauth et al., 1705.00352

Thanks to the theorists!

expt'l accuracy: each $\sim$ 20 GHz

$\Rightarrow 0.082/\sqrt{3}\,\mathrm{meV} \ = \ 0.050\,\mathrm{meV}$

r(^{3}He) = 1.97xxx (12)$_{\mathrm{exp}}$ (128)$_{\mathrm{theo}}$ fm

PRELIMINARY

Theory in μ^3He^+: TPE

$\Delta E^{\rm LS}_{\rm TPE}$(nucl. potentials)	=	15.46(39) meV	Bacca, Barnea, Hernandez, Ji, Nevo Dinur
$\Delta E^{\rm LS}_{\rm TPE}$(disp. relations)	=	15.14(49) meV	Carlson, Gorchtein, VDH
$\Delta E^{\rm LS}_{\rm TPE}$(our value)	=	15.30(52) meV	

using the center of both original values
and the adding in quadrature the larger error and half the spread.

Too conservative??

Nevo Dinur *et al.*, Phys. Lett. B 755, 380 (2016)
Hernandez *et al.*, Hyp. Int. 237, 158 (2017)

Carlson *et al.*, PRA 95, 012506 (2017)

Muonic summary

- Muonic hydrogen gives:
  - Proton charge radius: $r_{\rm p}$= 0.84087 (39) fm

    7σ away from electronic average (CODATA: H, e-p scatt.)
  - Deuteron charge radius: $r_d = 2.12771\,(22)$ fm from μH + H/D(1S-2S)
- Muonic deuterium:
  - Deuteron charge radius: $r_d = 2.12562\,(13)_{\rm exp}\,(77)_{\rm theo}$ fm

    consistent with muonic proton radius, but

    again 7σ away from CODATA: $2.14240\,(210)$ fm
- “Proton” Radius Puzzle is in fact “Z=1 Radius Puzzle”
- muonic helium-3 and -4 ions: No big discrepancy (PRELIMINARY)

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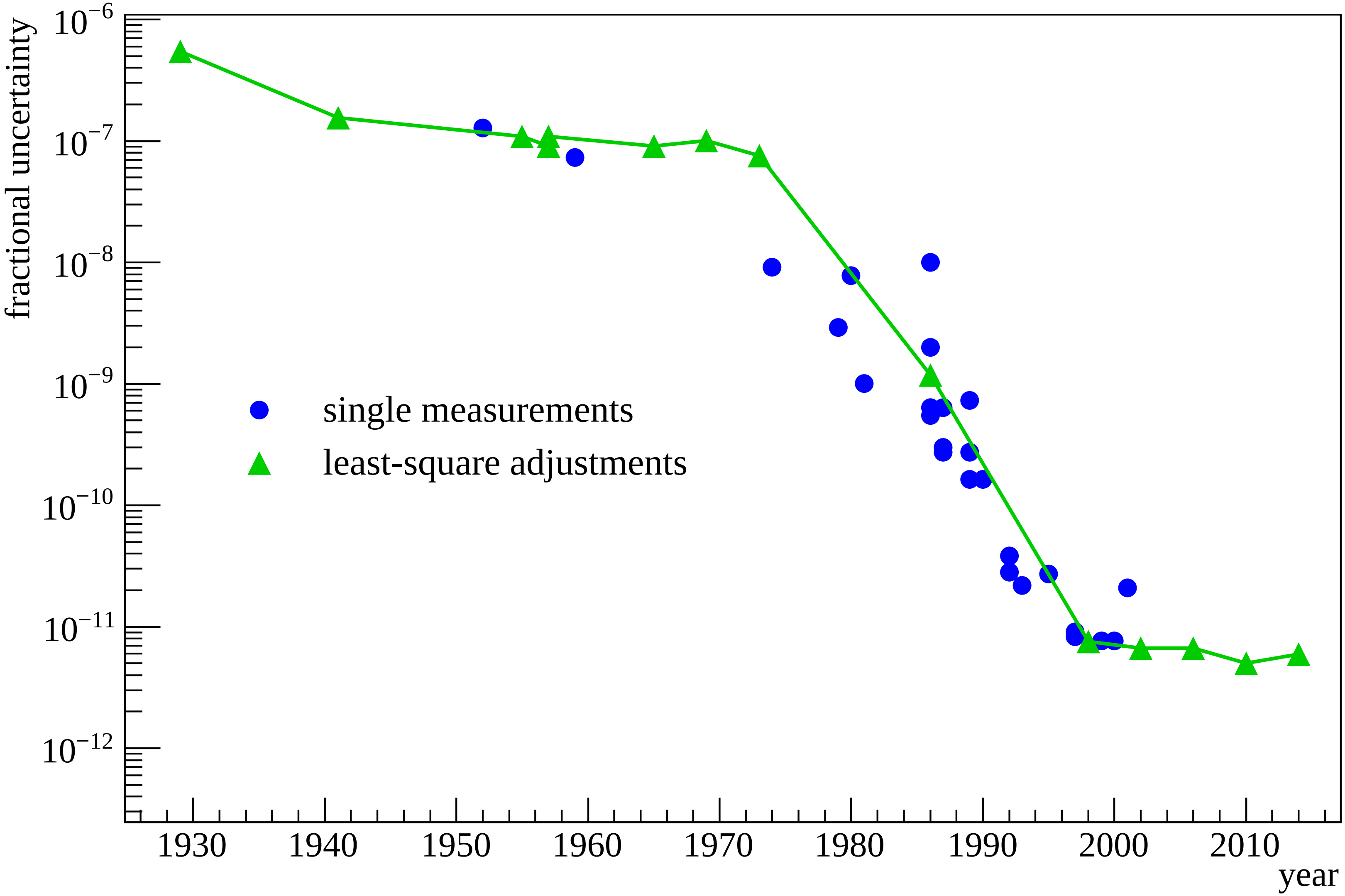
- "Proton" Radius Puzzle is in fact "Z=1 Radius Puzzle"
- muonic helium-3 and -4 ions: No big discrepancy (PRELIMINARY)
- Could **ALL** be solved if the **Rydberg constant** $[$ and hence the (electronic) proton radius $]$ was wrong.
  Plus $\sim 2.6\sigma$ change in deuteron polarizabililty.
  Plus: accept dispersion fits of e-p scattering
- Or: BSM physics, e.g. Tucker-Smith & Yavin (2011)

(Electronic) hydrogen.

Rydberg constant

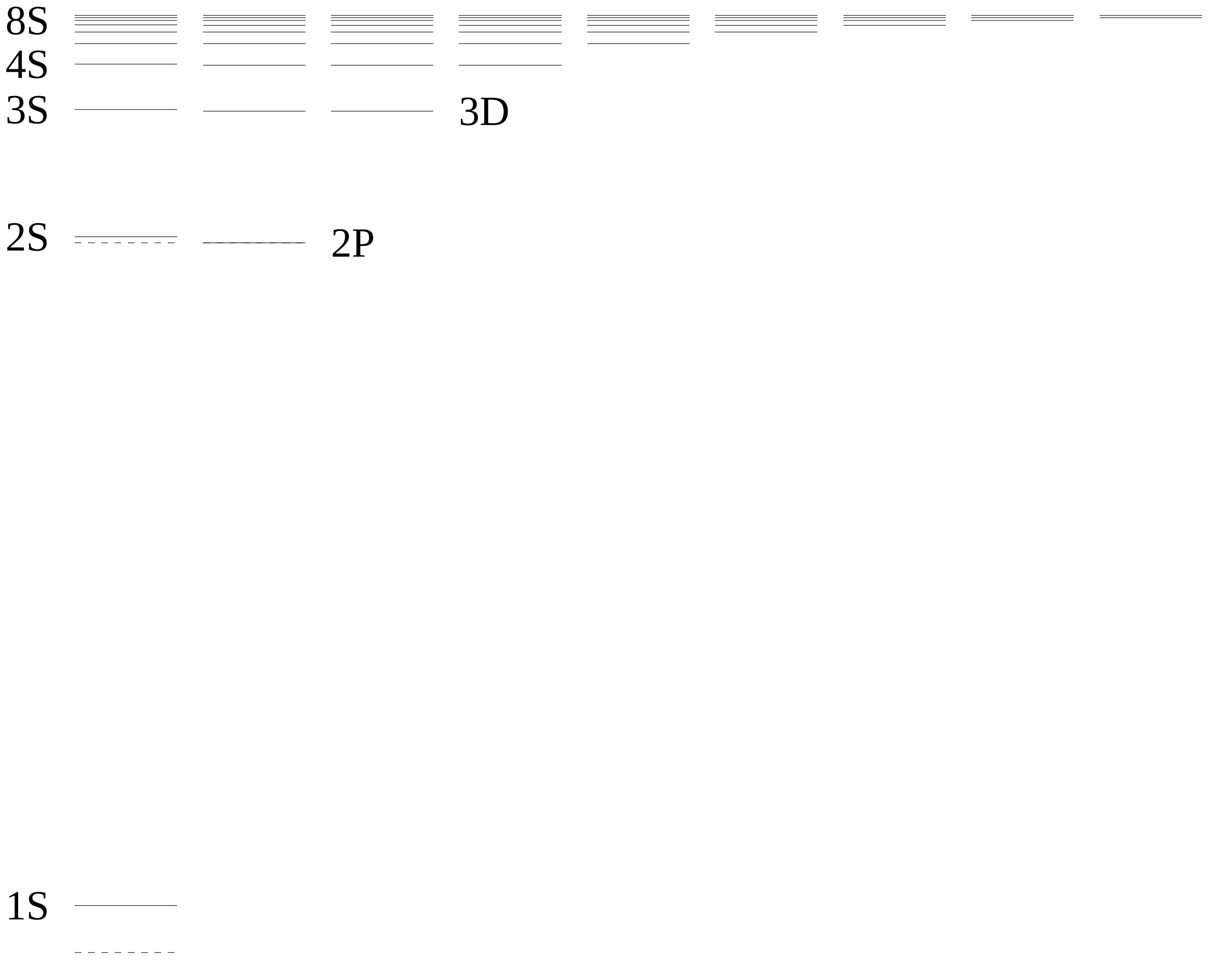
$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$



Hydrogen spectroscopy

$$\text{Hydrogen}: \quad E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

$$\text{Lamb shift}: \quad L_{1S}(r_\mathrm{p}) = 8171.636(4) + 1.5645\,\langle r_\mathrm{p}^2 \rangle \ \text{MHz}$$

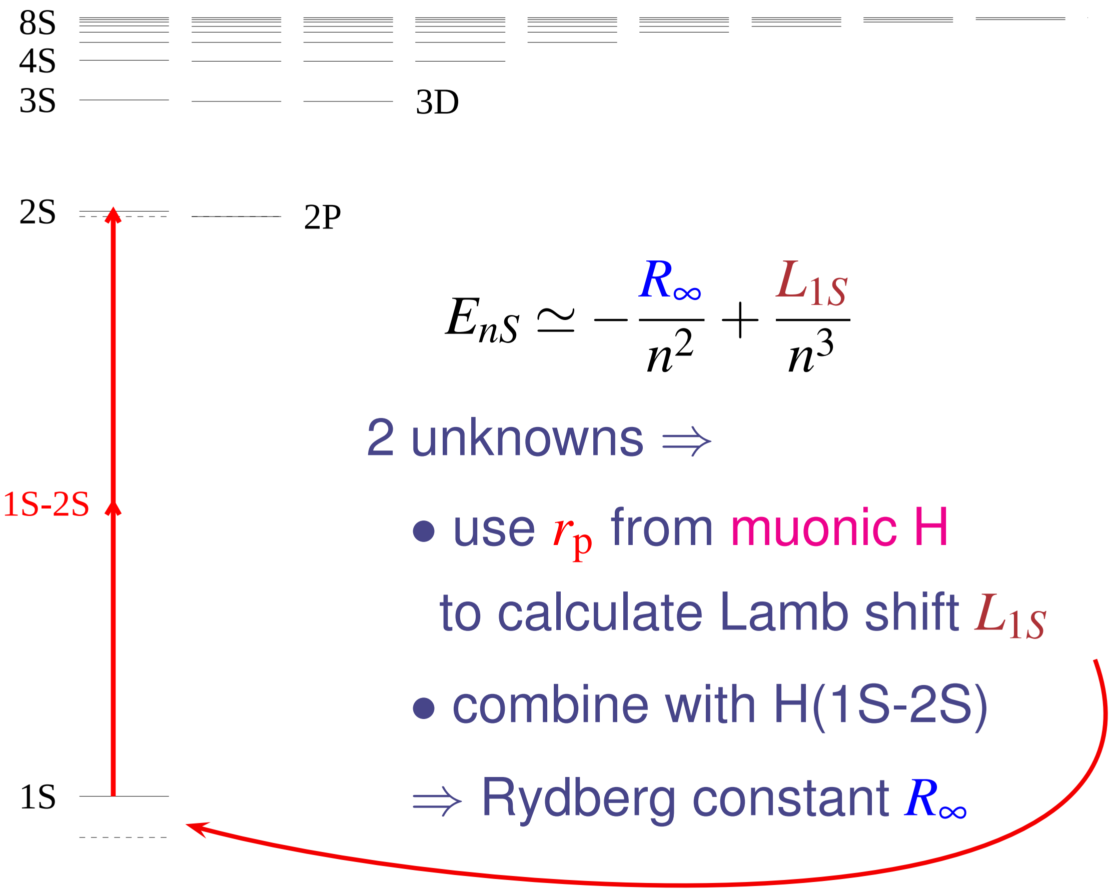


RP *et al.* arXiv 1607.03165

Rydberg constant

Hydrogen spectroscopy (Lamb shift):

$$\boxed{L_{1S}(r_\mathrm{p}) = 8171.636(4) + 1.5645\,\langle r_\mathrm{p}^2\rangle \quad \text{MHz}}$$



$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

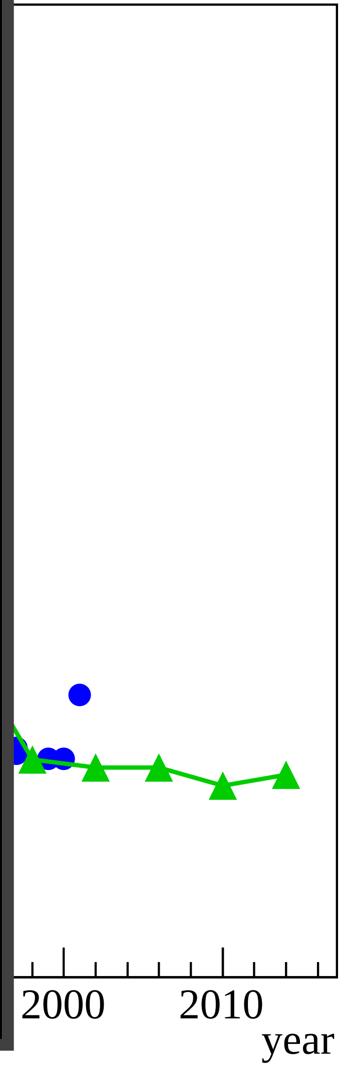
2 unknowns ⇒

- use r_p from muonic H

  to calculate Lamb shift L_{1S}

- combine with H(1S-2S)

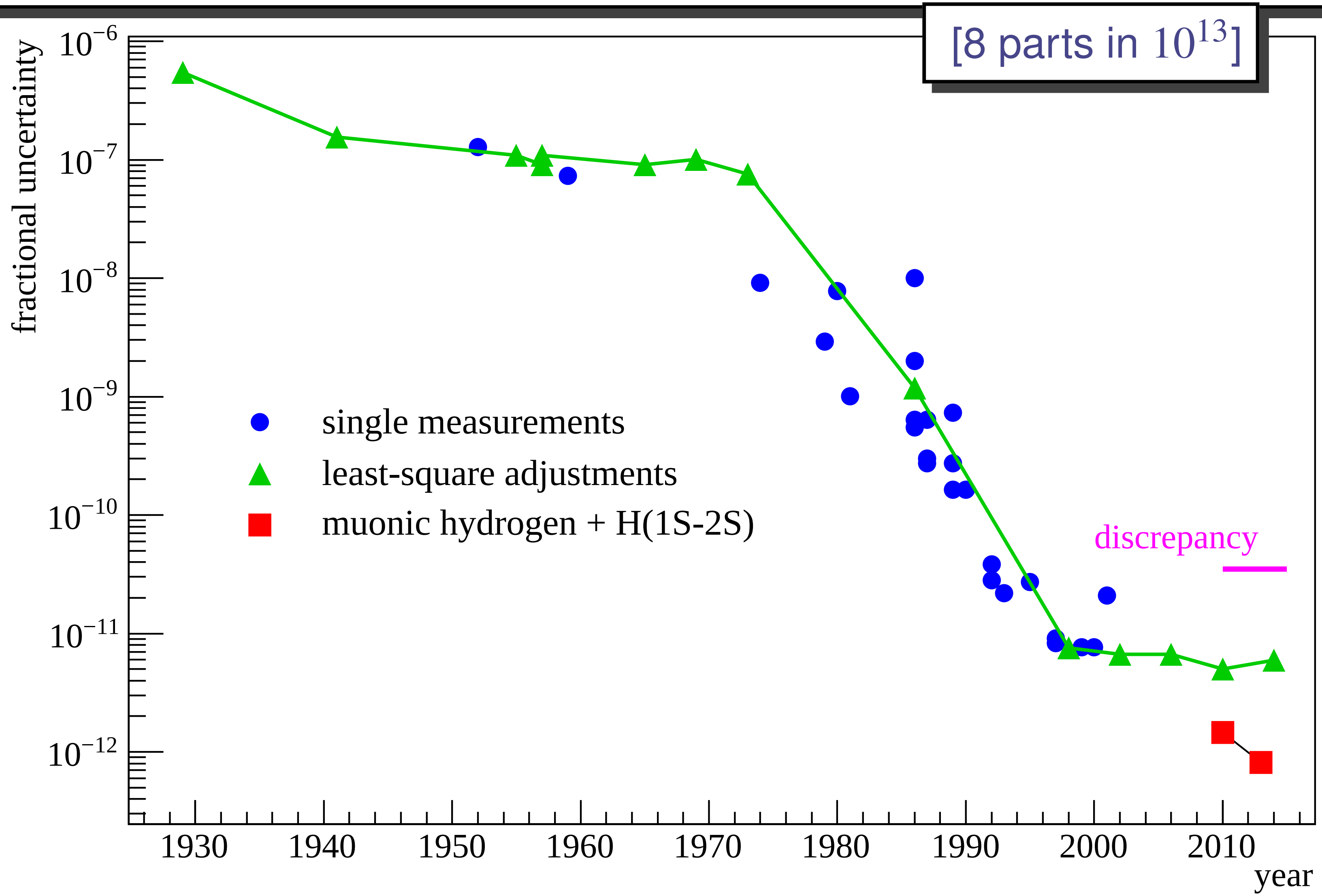
$\Rightarrow$ Rydberg constant R_∞



Rydberg constant

$$R_\infty = 3.289\,841\,960\,249\,5\,(10)^{r_p}\,(25)^{\text{QED}} \times 10^{15}\ \text{Hz/c}$$

[8 parts in 10^{13}]



H(1S-2S): C.G. Parthey, RP *et al.*, PRL 107, 203001 (2011).

r_p: A. Antognini, RP *et al.*, Science 339, 417 (2013).

Hydrogen spectroscopy

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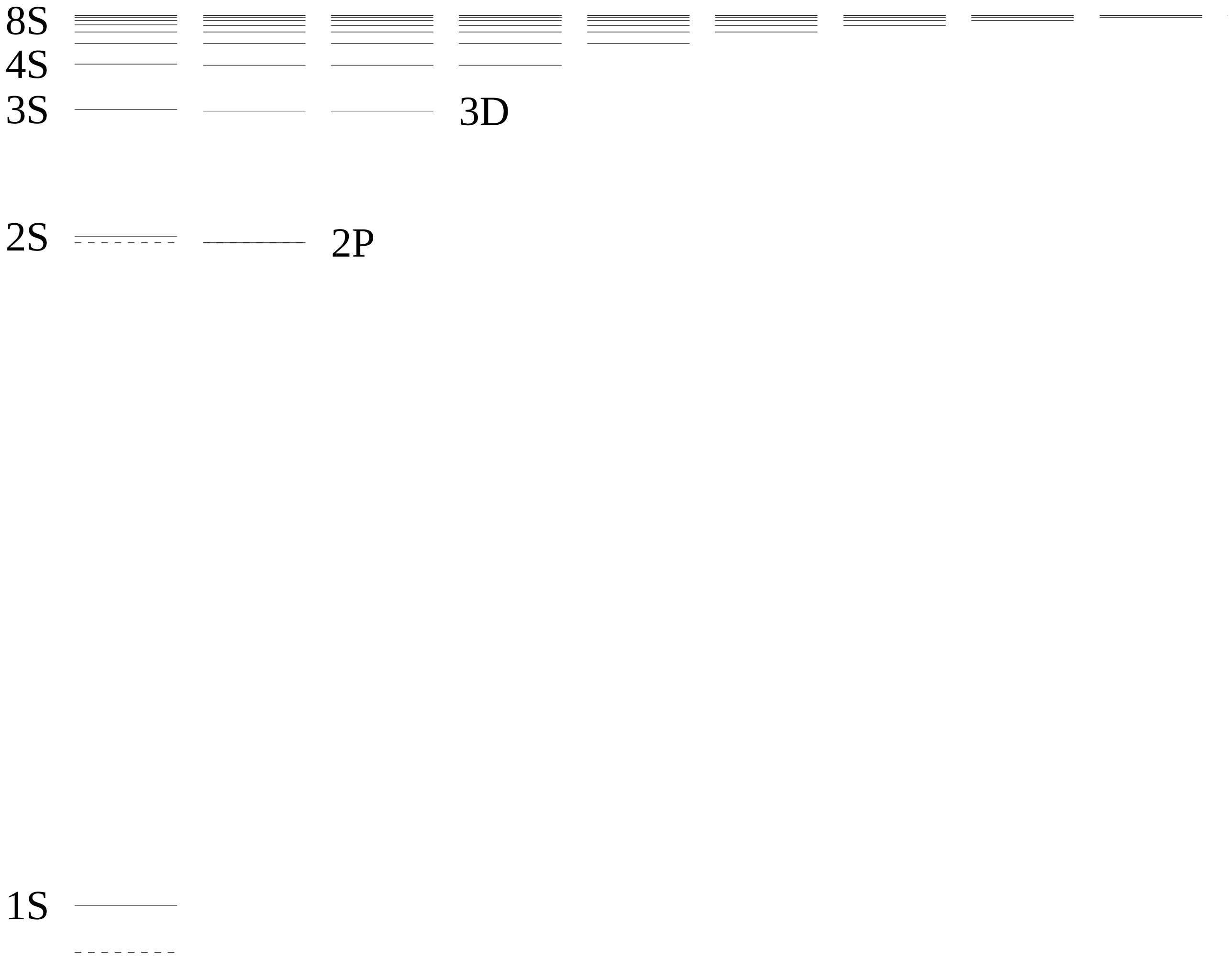
$$L_{nS} \simeq \frac{L_{1S}}{n^3}$$



RP *et al.* arXiv 1607.03165

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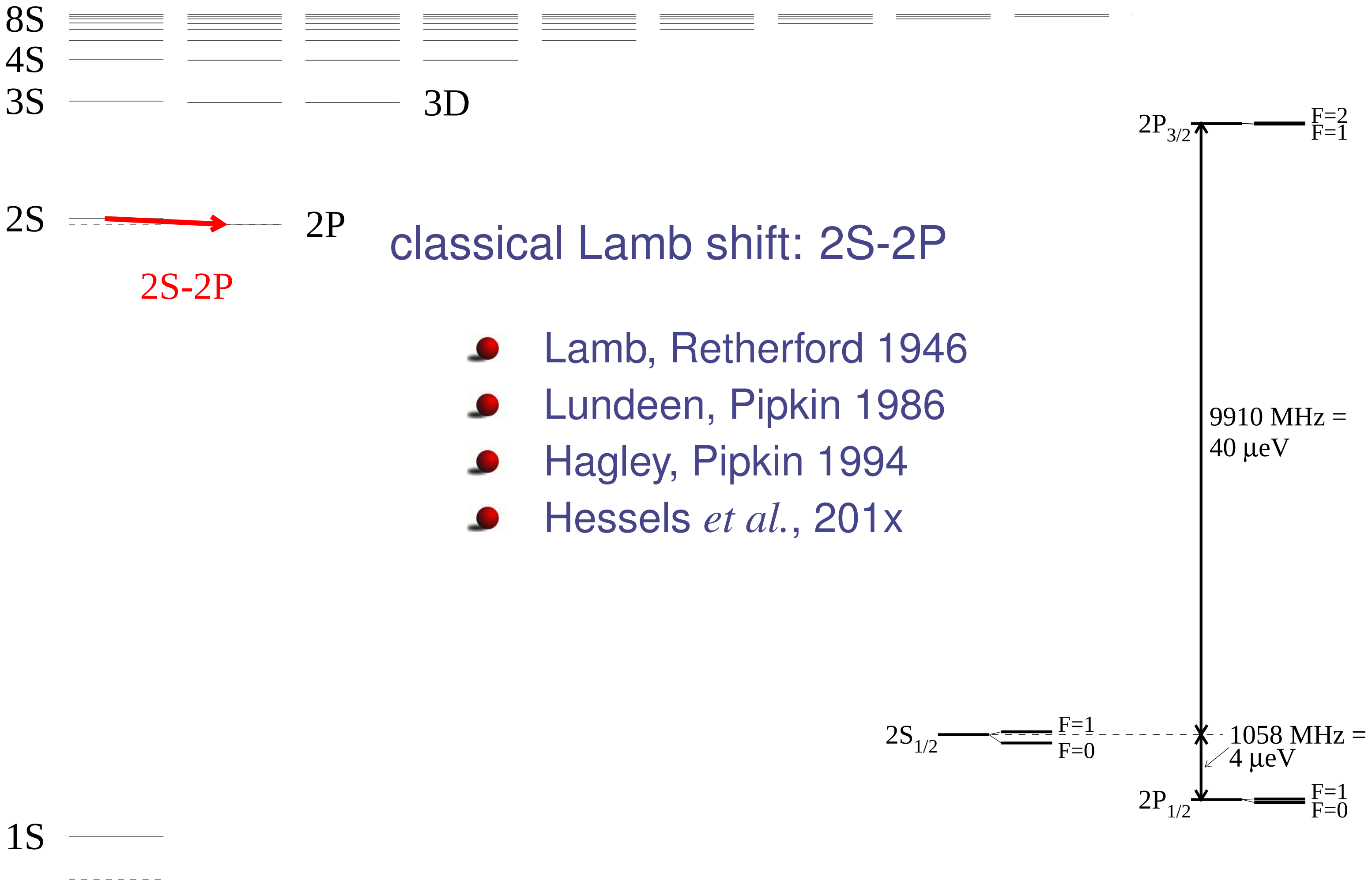

classical Lamb shift: 2S-2P

- Lamb, Retherford 1946
- Lundeen, Pipkin 1986
- Hagley, Pipkin 1994
- Hessels *et al.*, 201x

RP *et al.* arXiv 1607.03165

Hydrogen spectroscopy

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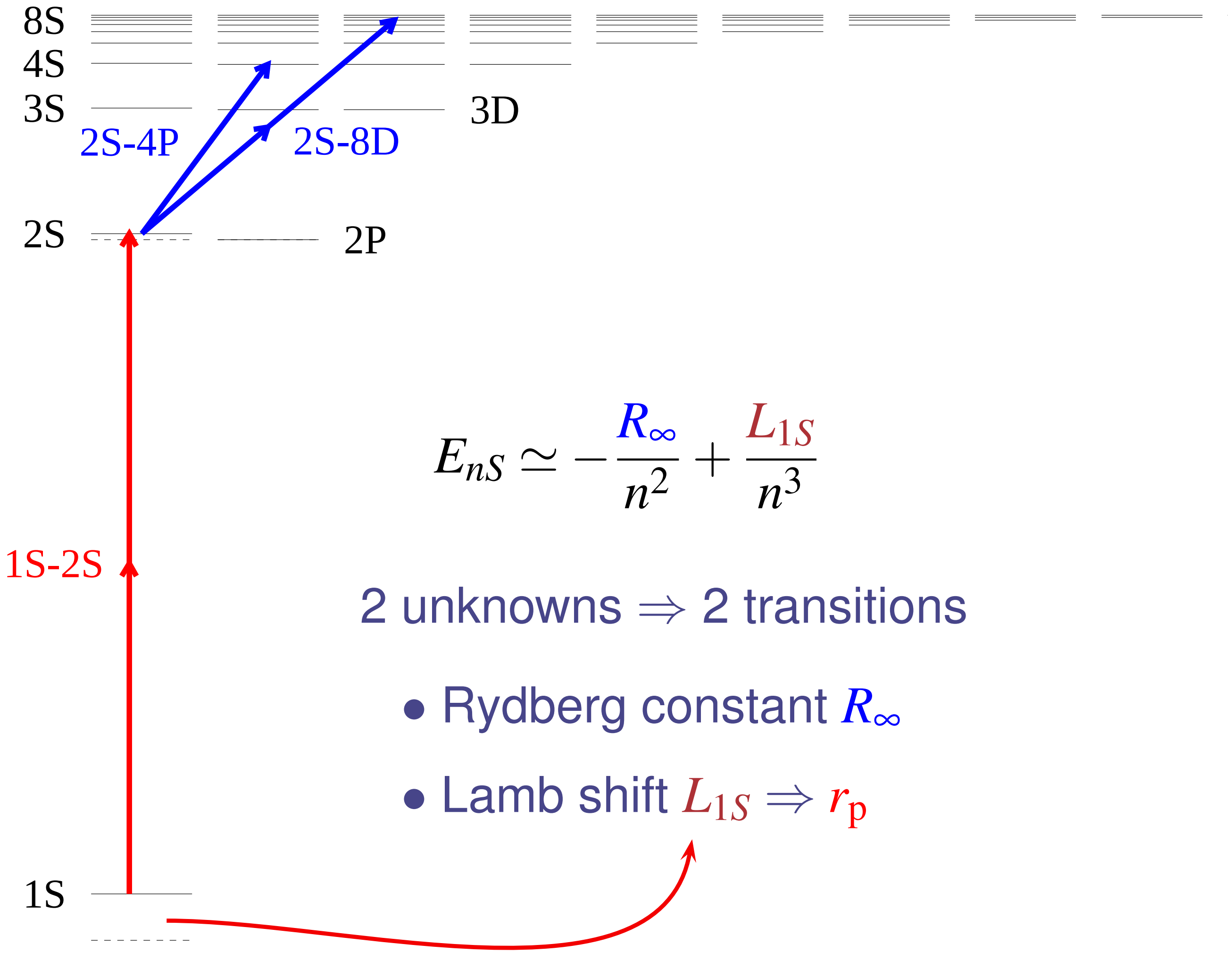
$$L_{nS} \simeq \frac{L_{1S}}{n^3}$$



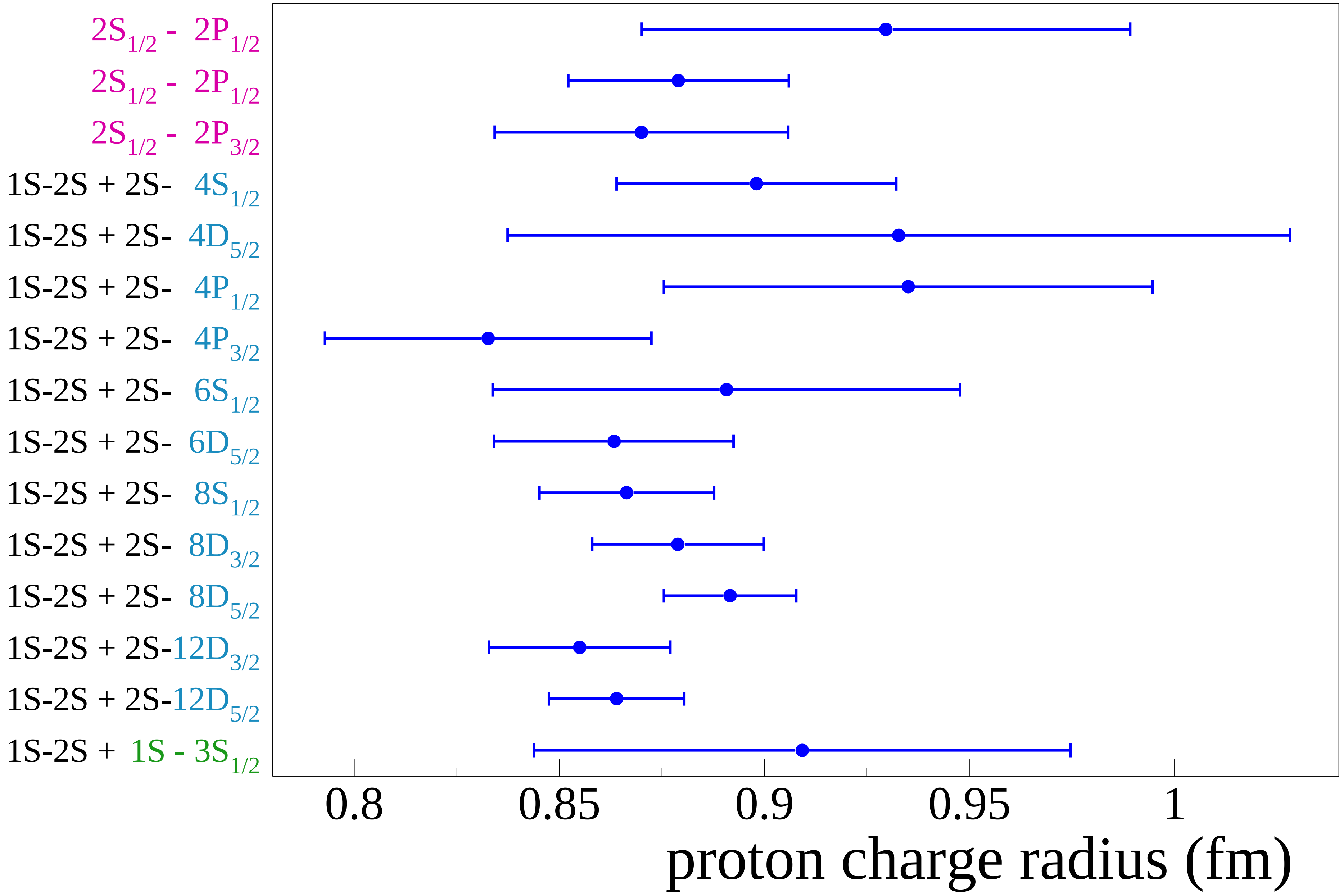
$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

2 unknowns ⇒ 2 transitions

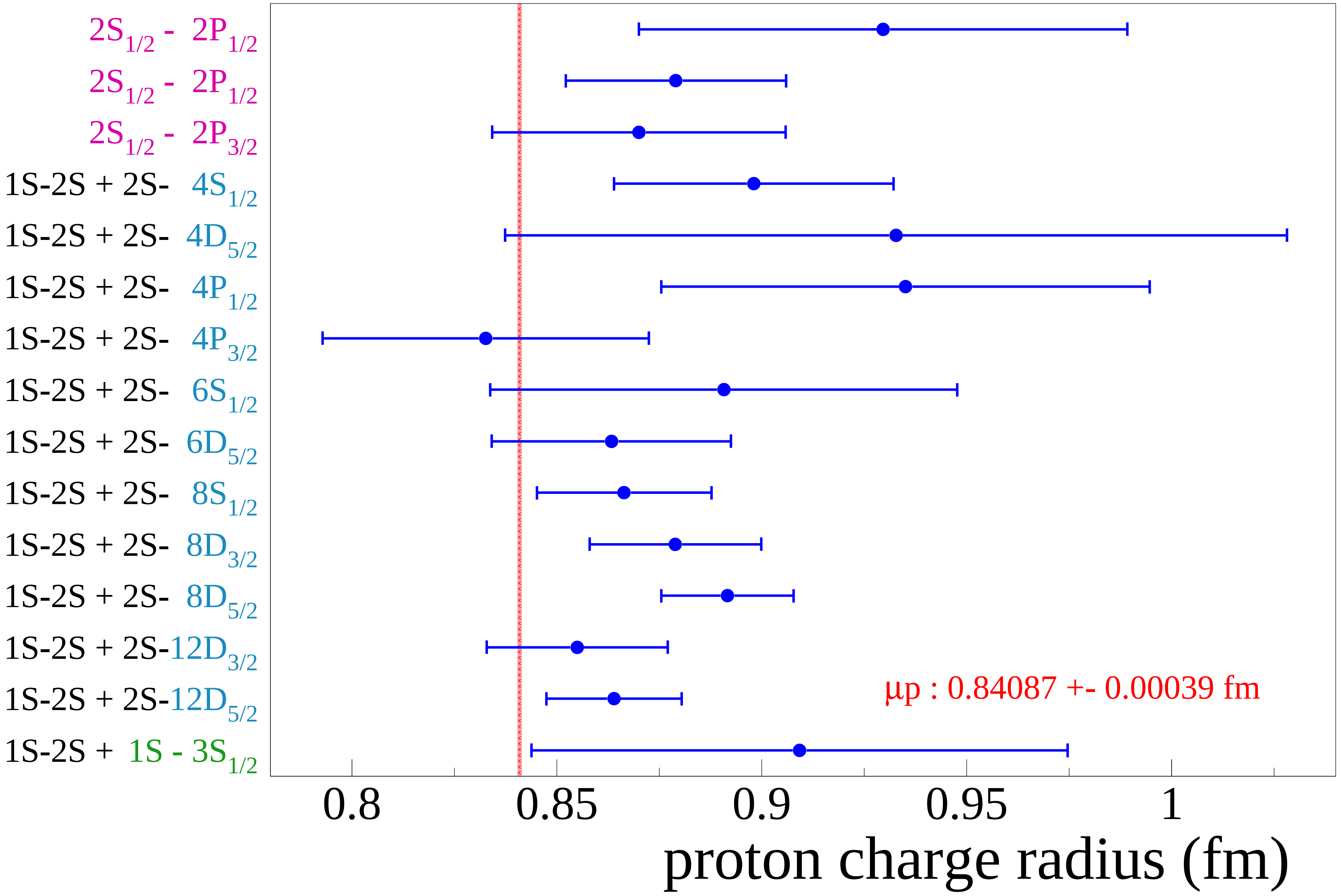
- Rydberg constant R_∞
- Lamb shift $L_{1S} \Rightarrow r_p$

RP *et al.* arXiv 1607.03165

Hydrogen spectroscopy



Hydrogen spectroscopy



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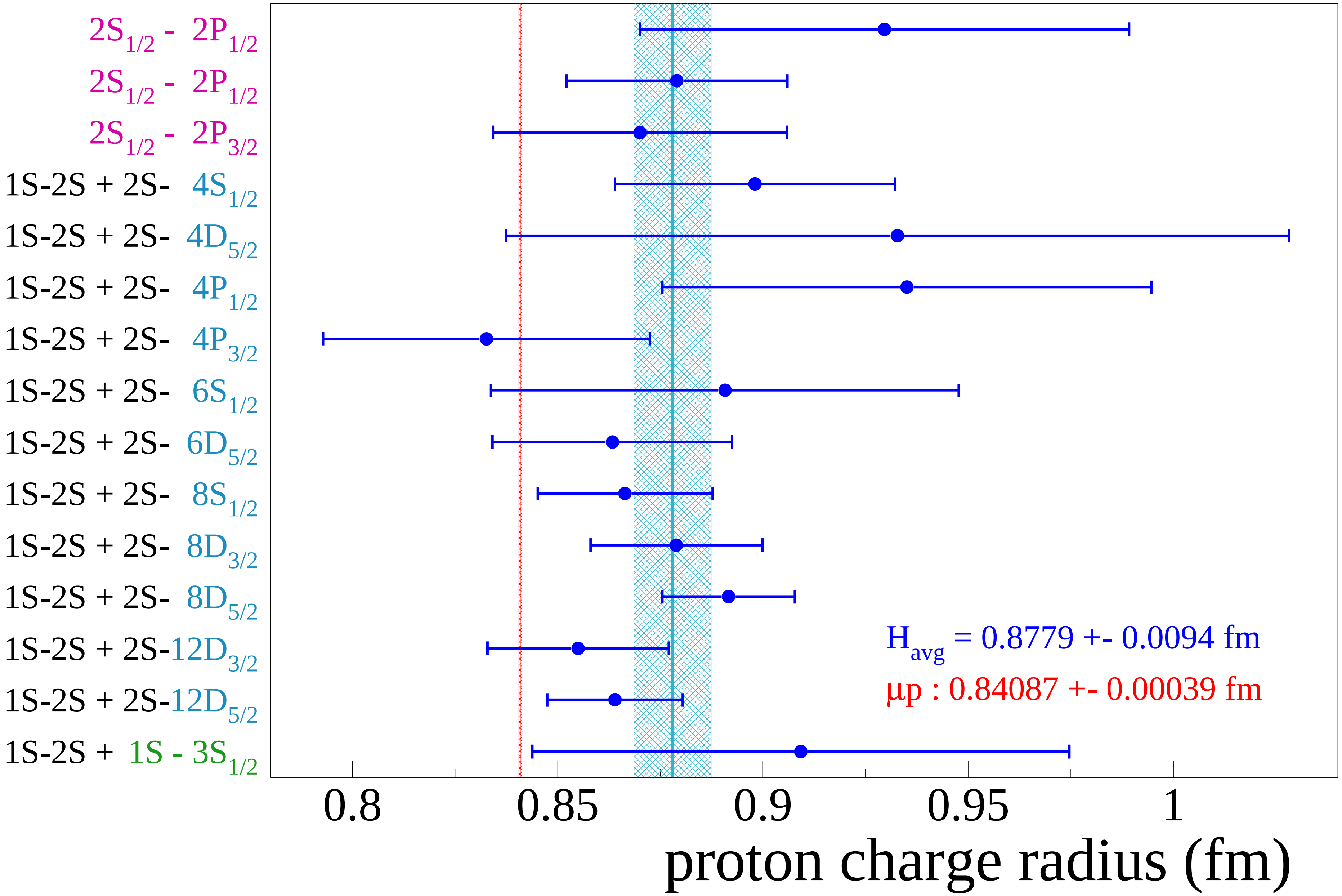

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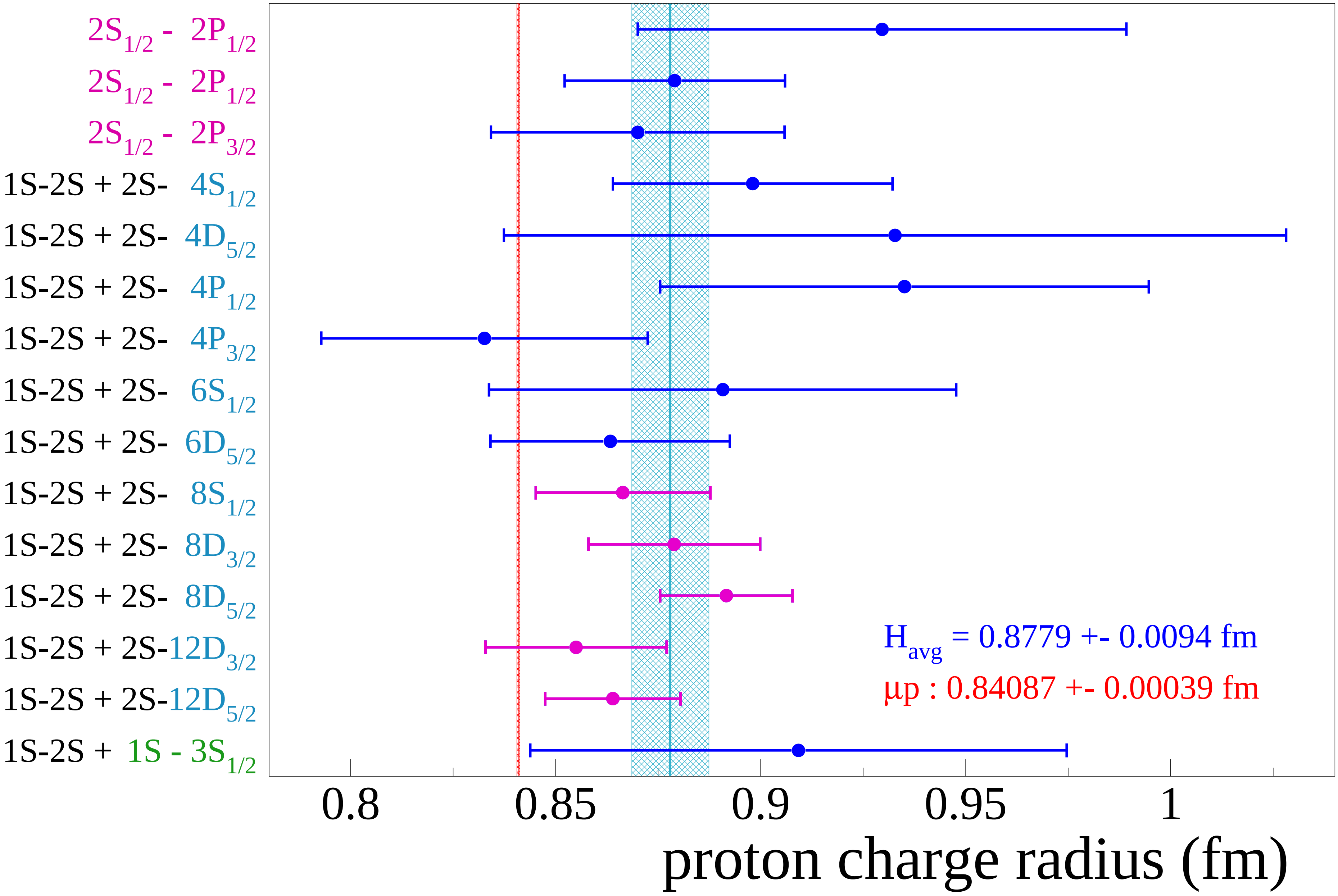
μp : 0.84087 +- 0.00039 fm

Hydrogen spectroscopy



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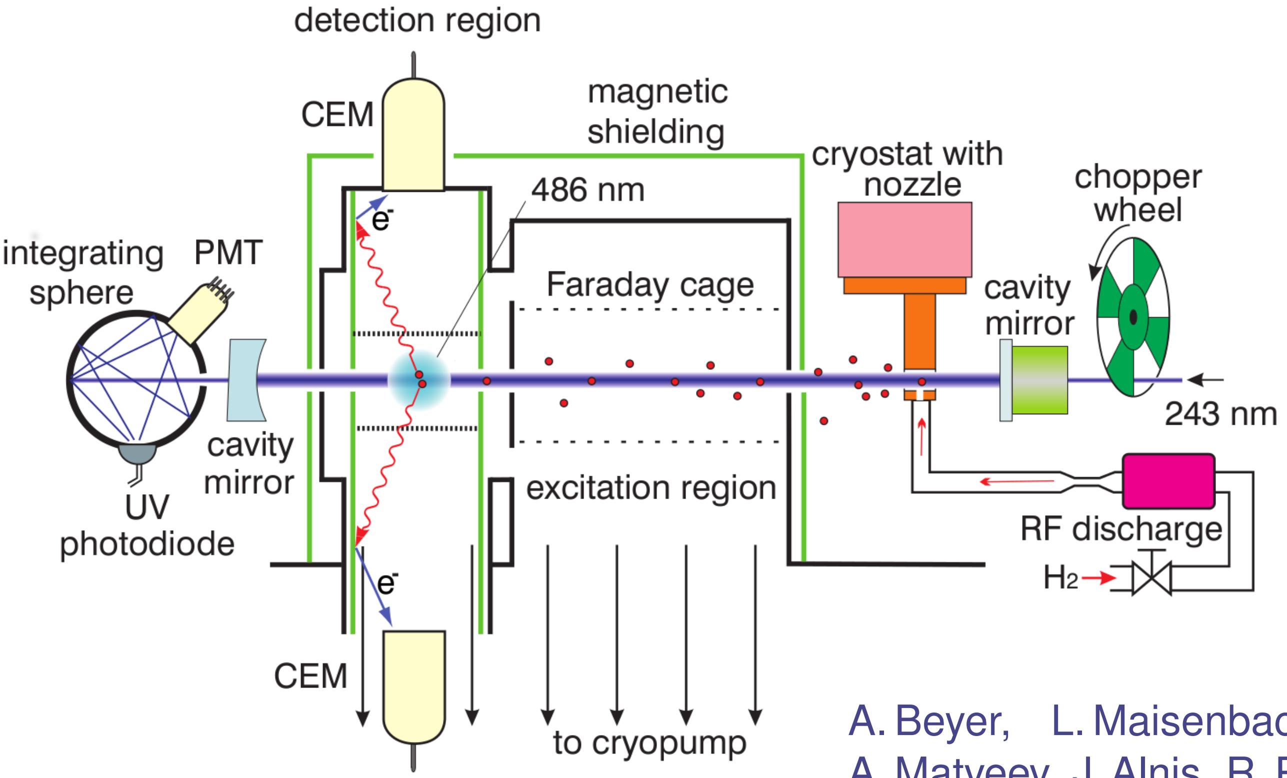
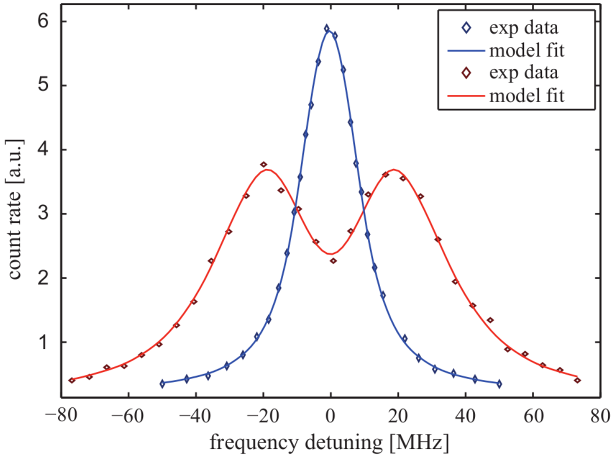
Rydberg constant from hydrogen





2S – 4P resonance at

88 ± 0.5 ° and 90 ± 0.08 °

A. Beyer, L. Maisenbacher, K. Khabarova, C.G. Parthey, A. Matveev, J. Alnis, R. Pohl, N. Kolachevsky, Th. Udem and T.W. Hänsch

- Apparatus used for H/D(1S-2S)

C.G. Parthey, RP *et al.*, PRL **104**, 233001 (2010)
C.G. Parthey, RP *et al.*, PRL **107**, 203001 (2011)

- 486 nm at 90° + Retroreflector ⇒ Doppler-free 2S-4P excitation
- 1st oder Doppler vs. ac-Stark shift
- ~ 2.5 kHz accuracy (vs. 15 kHz Yale, 1995)
- **cryogenic** H beam, **optical** excitation to 2S

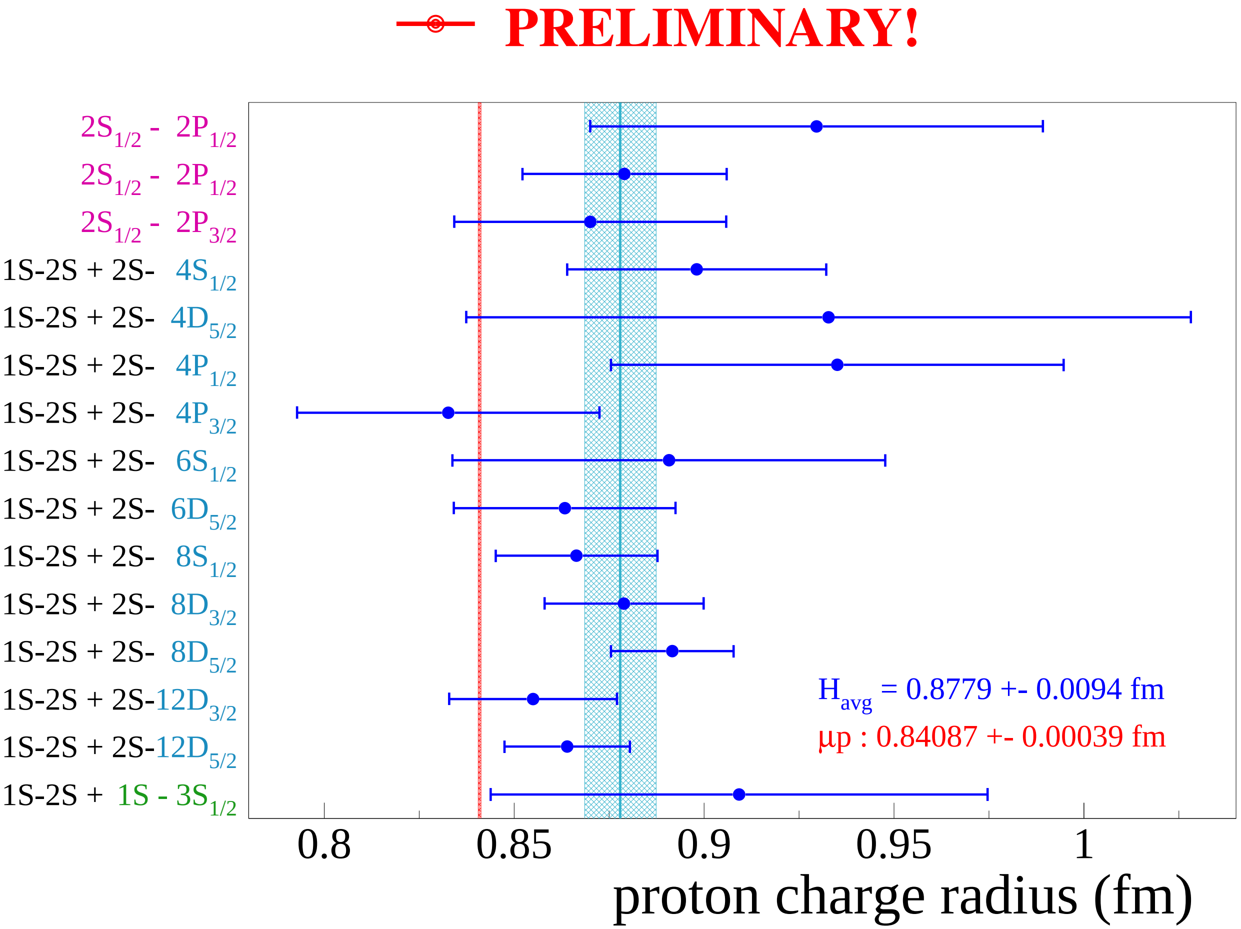
A. Beyer, RP *et al.*, Ann. d. Phys. **525**, 671 (2013)

New hydrogen 2S→4P at MPQ!

Proton is small in regular hydrogen, too!

Proton radius puzzle is NOT “solved”.

Our main systematics do NOT affect the previous measurements.



$2S \rightarrow 4P_{1/2}$ and $4P_{3/2}$

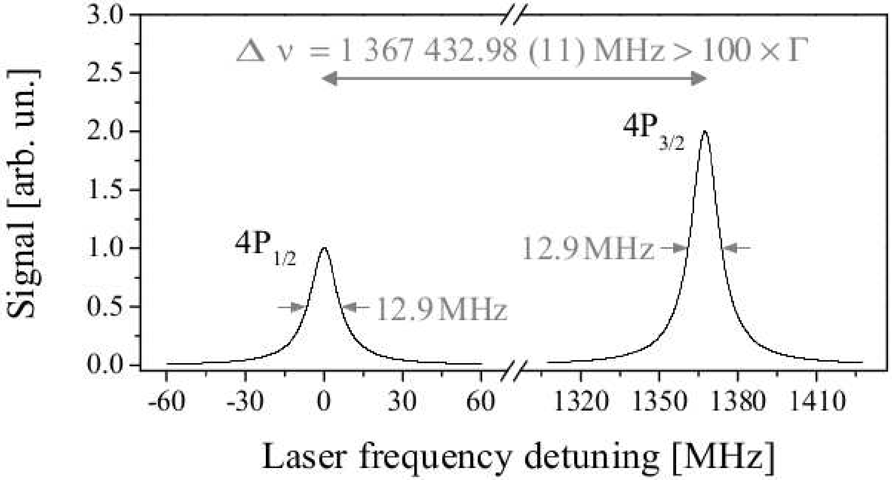
cold H(2S) beam

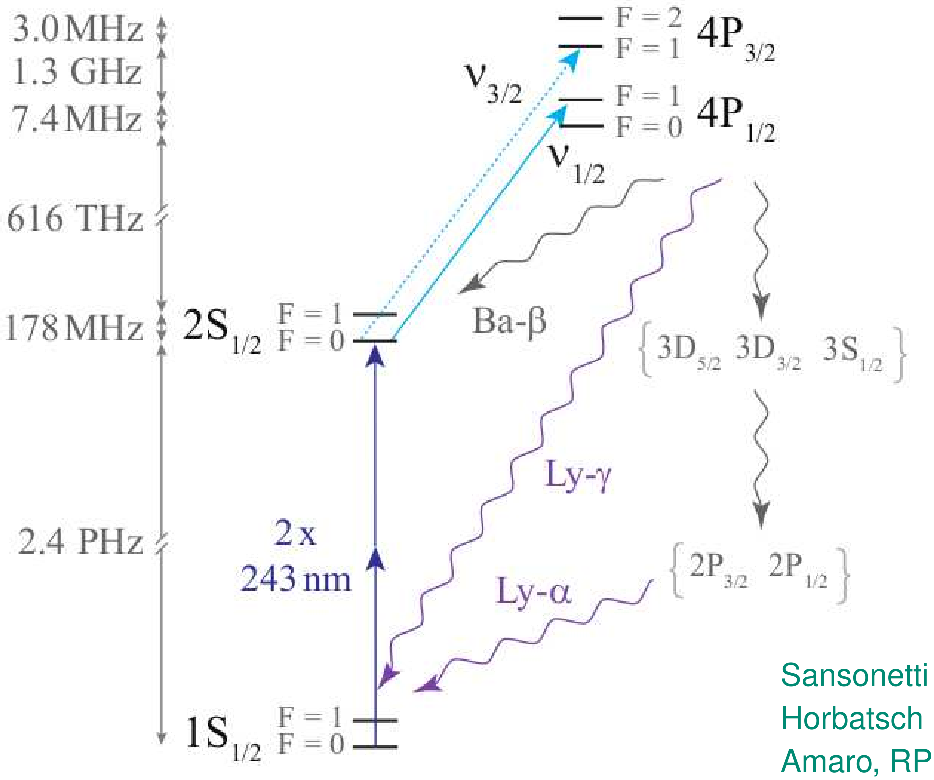
optically excited $(1S \rightarrow 2S)$

$\Delta\nu \sim 2\,\text{kHz} \quad \equiv \Gamma/10'000$!!!

Beyer, Maisenbacher, Matveev, RP,
Khabarova, Grinin, Lamour, Yost,
Hänsch, Kolachevsky, Udem,
submitted (2016)

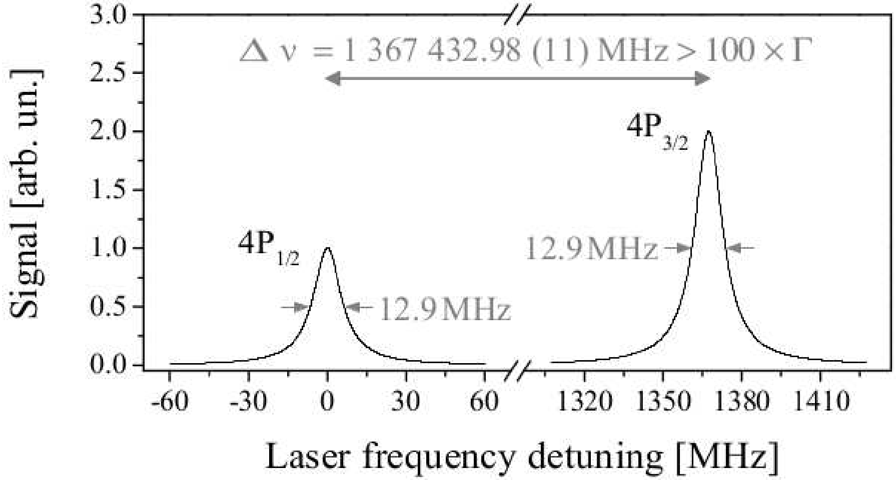
Quantum interference shifts

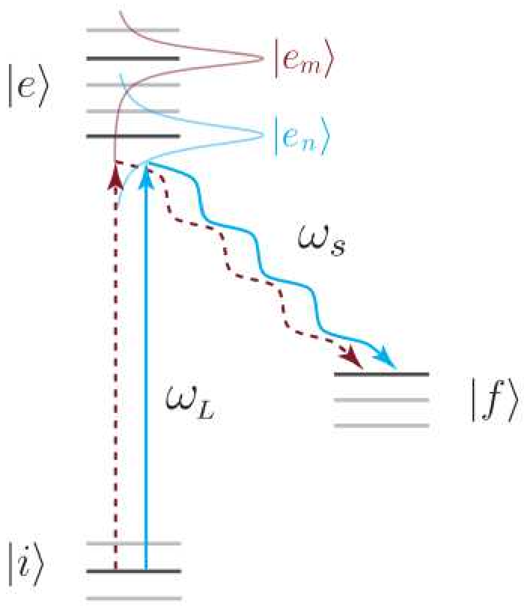


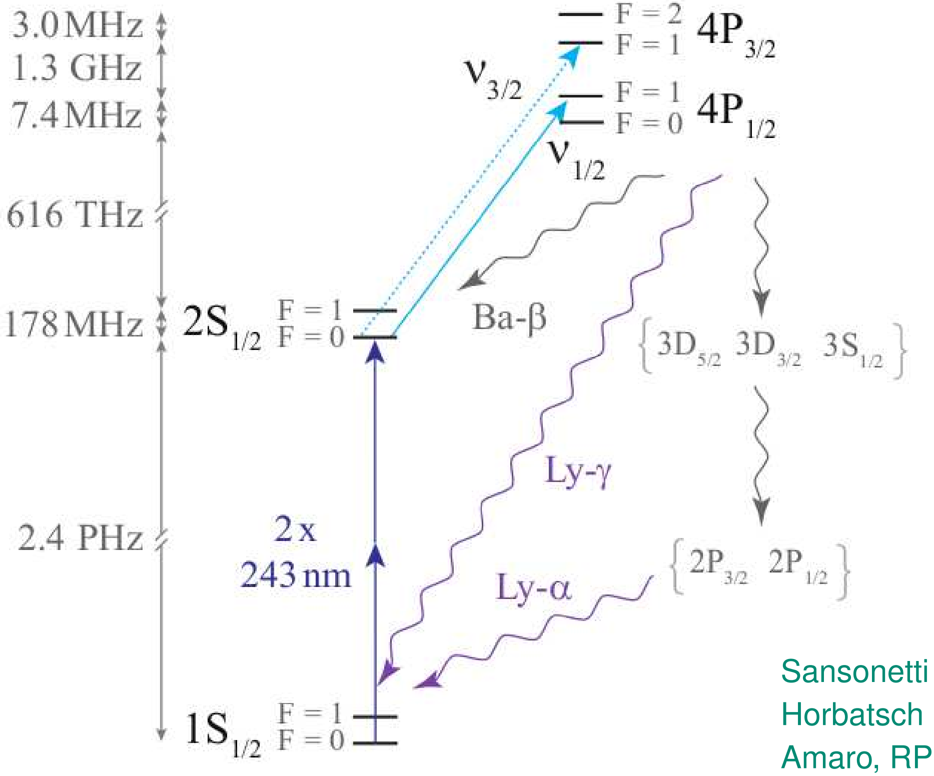


Sansonetti *et al.*, PRL 107, 023001 (2011); Brown *et al.*, PRA 87, 032504 (2013)
Horbatsch & Hessels, PRA 82, 052519 (2010); PRA 84, 032508 (2011), etc.
Amaro, RP *et al.*, PRA 92, 022514 (2015); PRA 92, 062506 (2015)

Quantum interference shifts







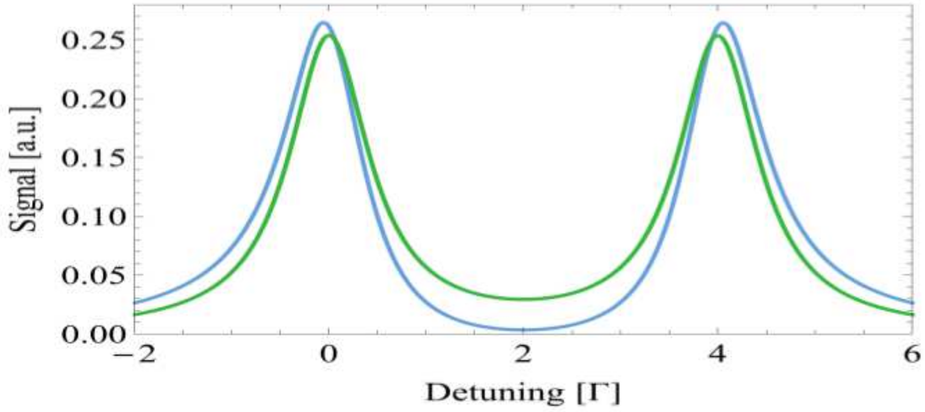
$$P(\omega) \propto \left| \frac{(\vec{d}_1\cdot\vec{E}_0)\vec{d}_1}{\omega_1-\omega_L+i\gamma_1/2} + \frac{(\vec{d}_2\cdot\vec{E}_0)\vec{d}_2 e^{i\Delta\phi}}{\omega_2-\omega_L+i\gamma_2/2} \right|^2$$

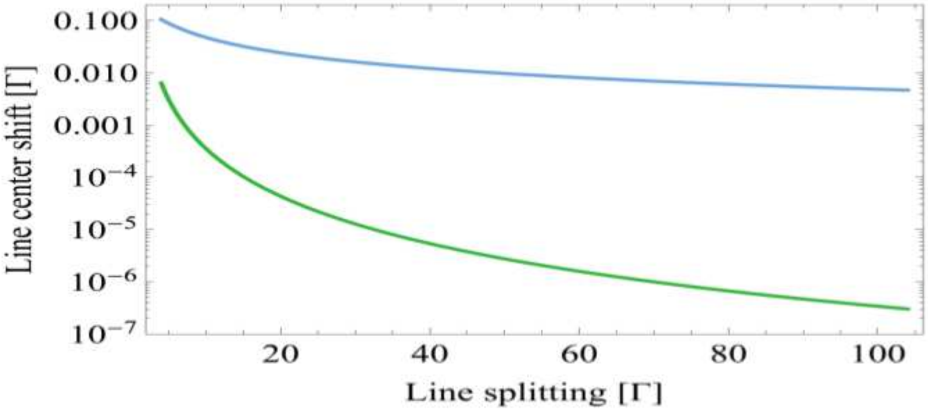
= Lorentzian(1) + Lorentzian(2) +

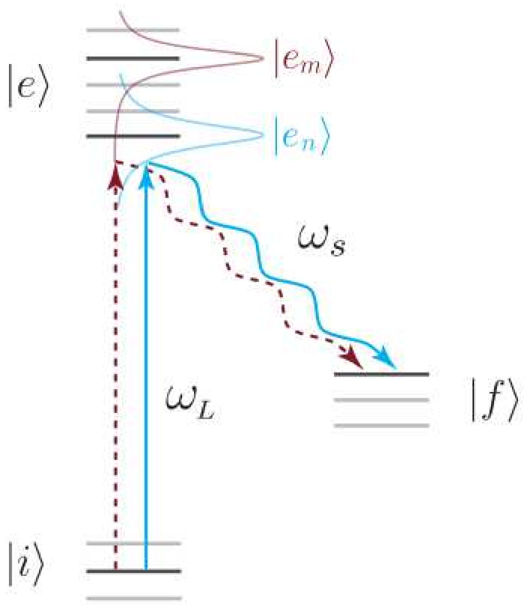
cross-term (QI)

Sansonetti *et al.*, PRL 107, 023001 (2011); Brown *et al.*, PRA 87, 032504 (2013)
Horbatsch & Hessels, PRA 82, 052519 (2010); PRA 84, 032508 (2011), etc.
Amaro, RP *et al.*, PRA 92, 022514 (2015); PRA 92, 062506 (2015)

Quantum interference shifts







$$P(\omega) \propto \left| \frac{(\vec{d}_1 \cdot \vec{E}_0)\vec{d}_1}{\omega_1 - \omega_L + i\gamma_1/2} + \frac{(\vec{d}_2 \cdot \vec{E}_0)\vec{d}_2 e^{i\Delta\phi}}{\omega_2 - \omega_L + i\gamma_2/2} \right|^2$$

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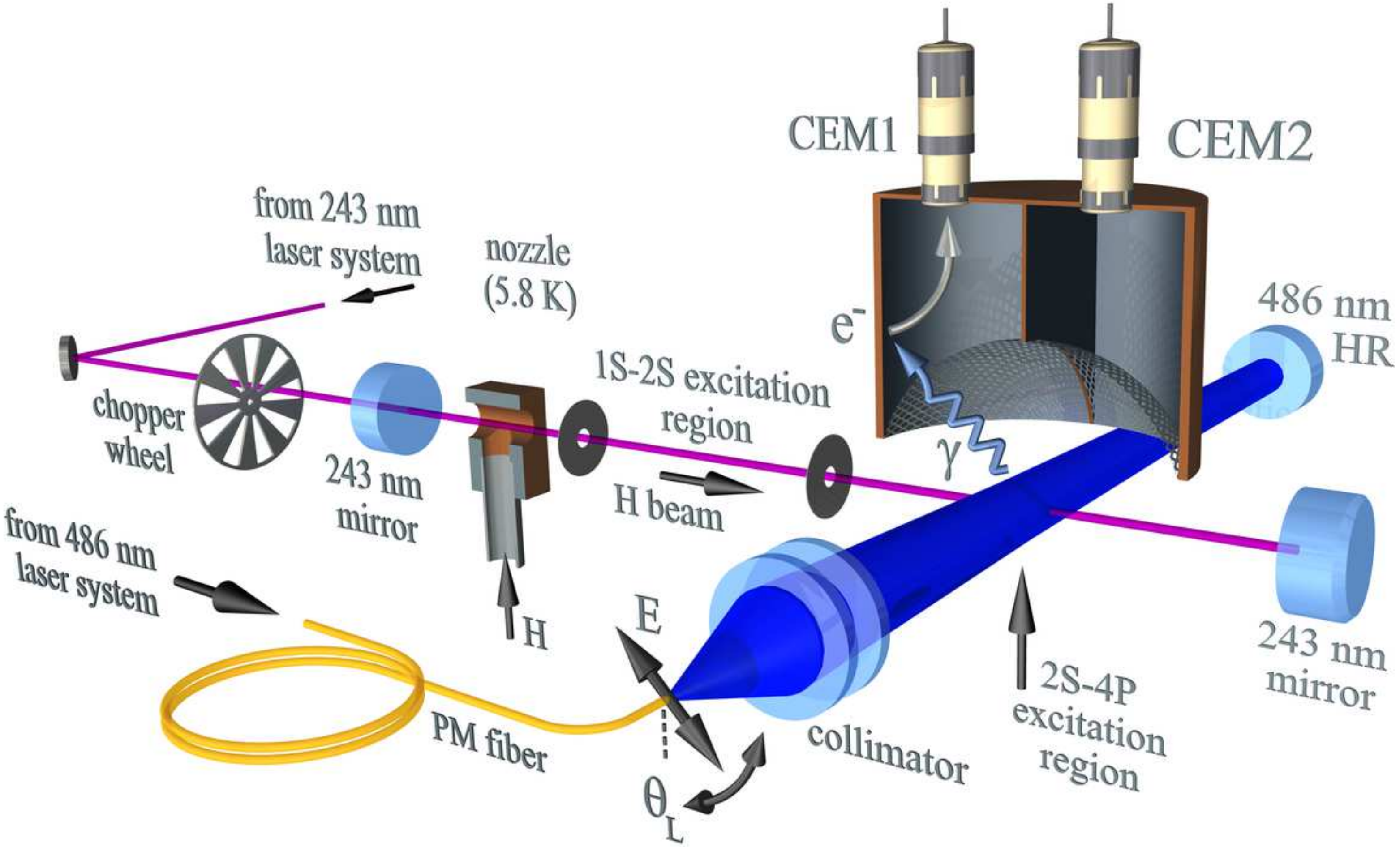
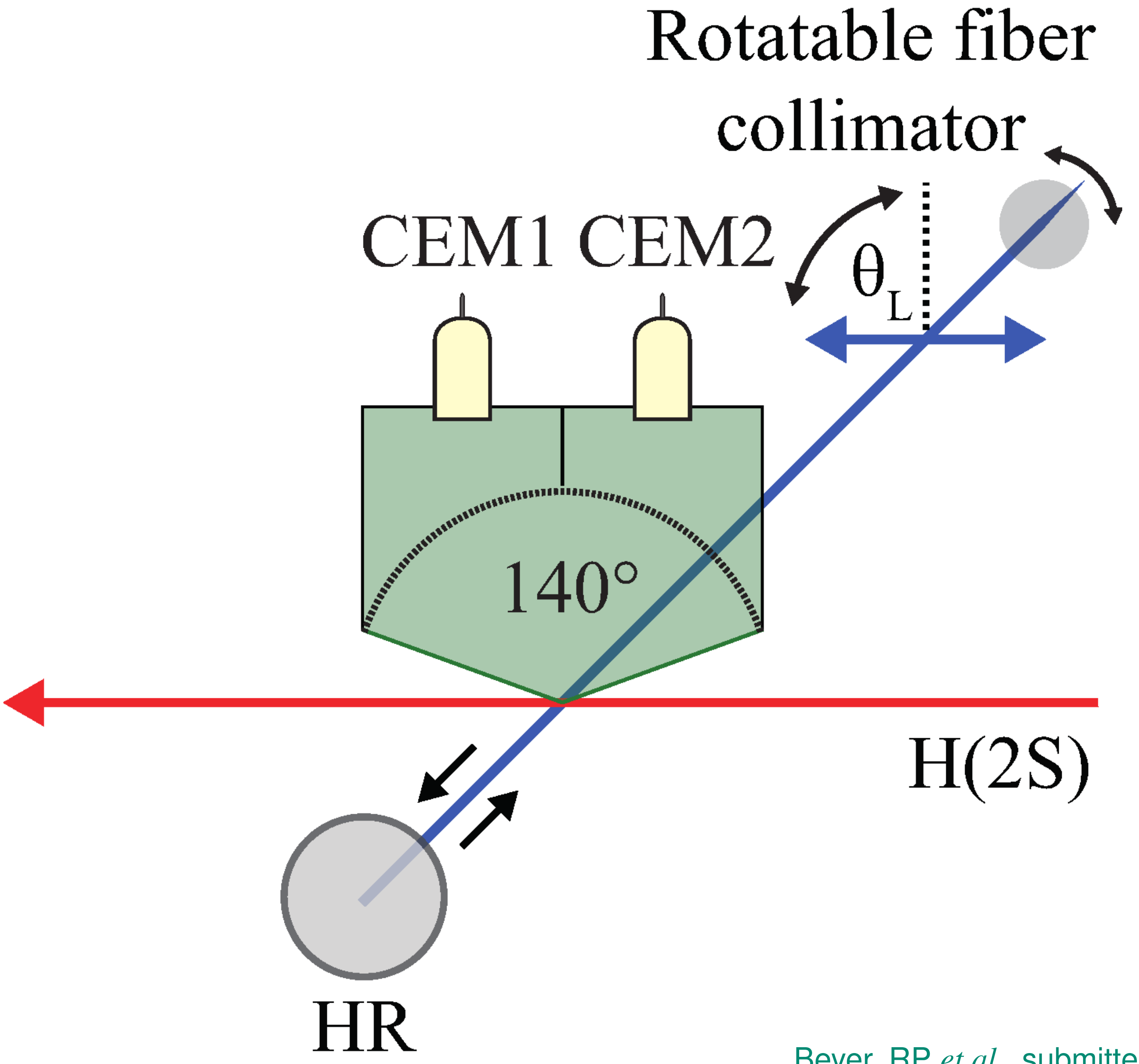
Quantum interference shifts

2S-4P setup



Beyer, RP *et al.*, submitted (2016)

Quantum interference shifts



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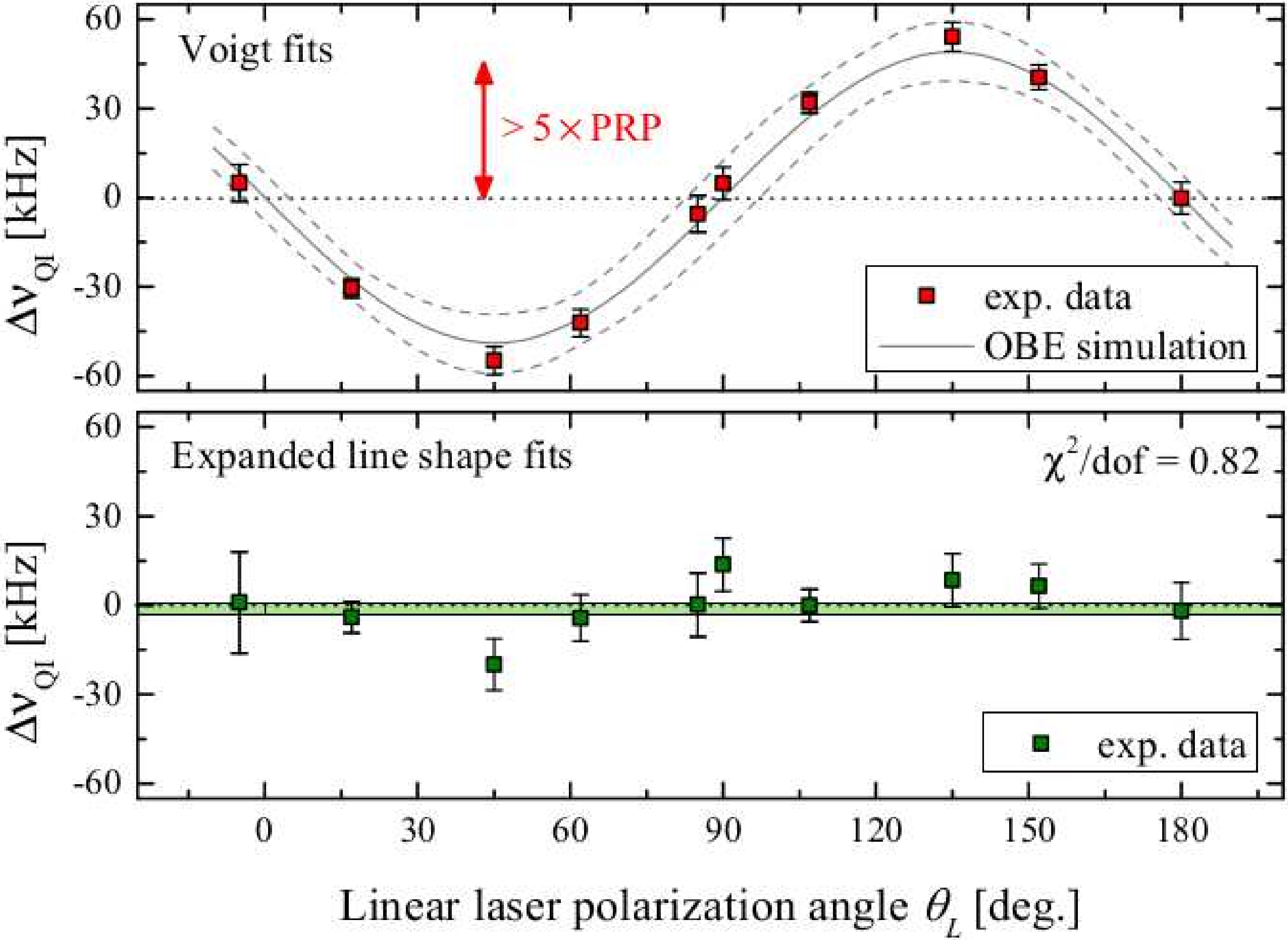
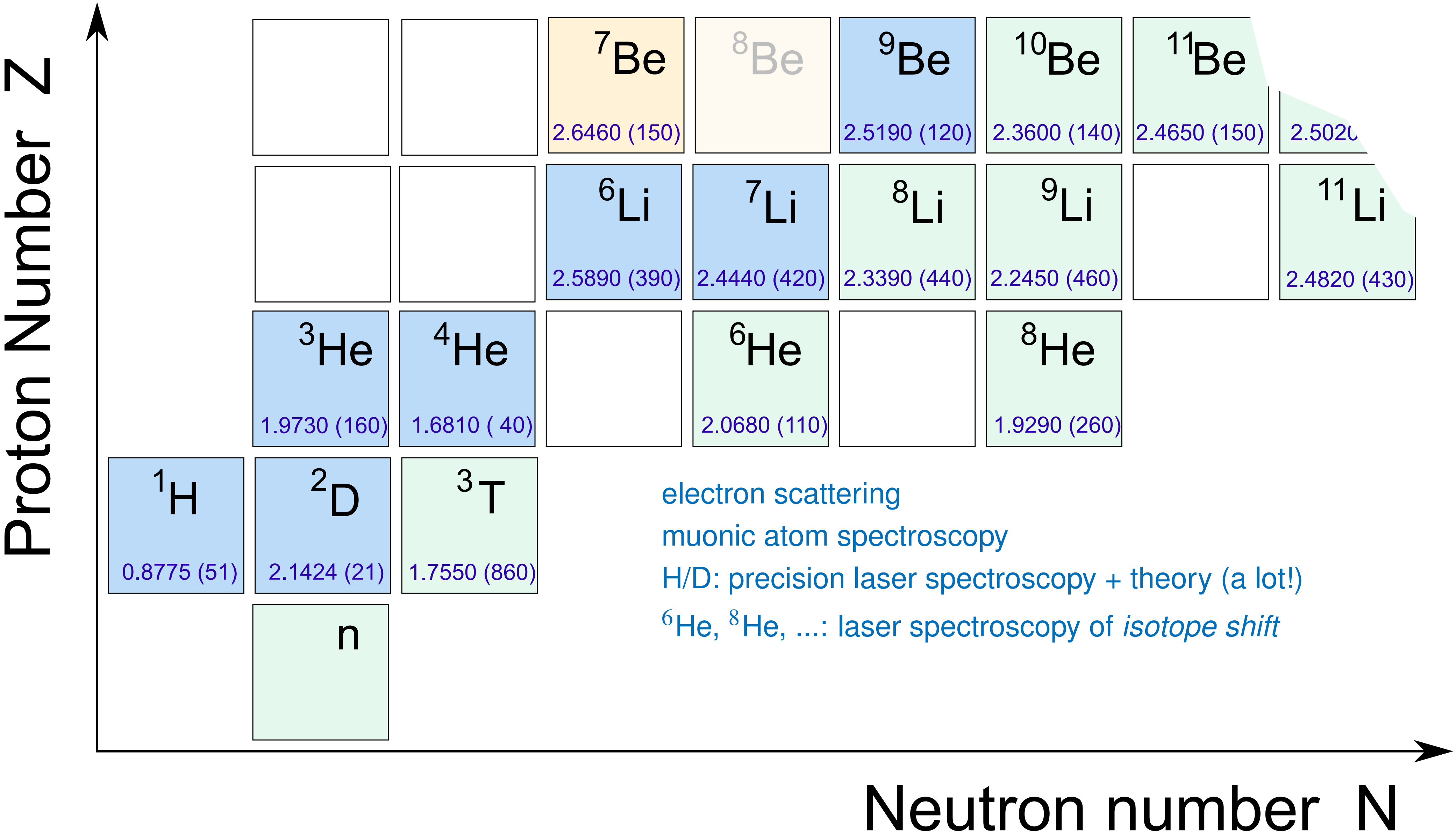


Beyer, RP *et al.*, submitted (2016)

The nuclear chart



			^{7}Be 2.6460 (150)	^{8}Be	^{9}Be 2.5190 (120)	^{10}Be 2.3600 (140)	^{11}Be 2.4650 (150)	2.502
			^{6}Li 2.5890 (390)	^{7}Li 2.4440 (420)	^{8}Li 2.3390 (440)	^{9}Li 2.2450 (460)		^{11}Li 2.4820 (430)
	^{3}He 1.9730 (160)	^{4}He 1.6810 (40)		^{6}He 2.0680 (110)		^{8}He 1.9290 (260)		
^{1}H 0.8775 (51)	^{2}D 2.1424 (21)	^{3}T 1.7550 (860)						
	n							

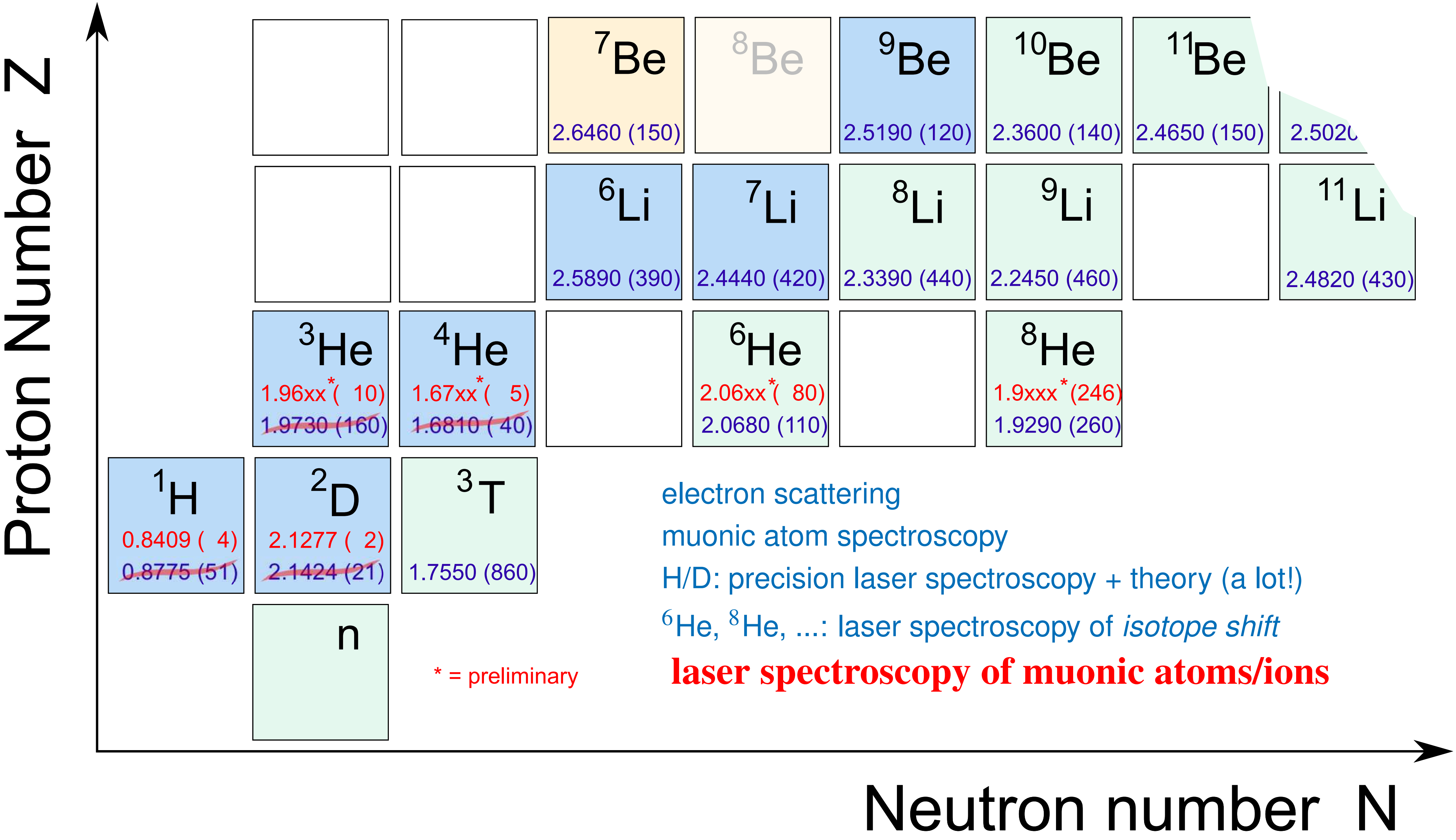
electron scattering

muonic atom spectroscopy

H/D: precision laser spectroscopy + theory (a lot!)

^{6}He, ^{8}He, ...: laser spectroscopy of *isotope shift*

The nuclear chart - new charge radii



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			^{6}Li 2.5890 (390)	^{7}Li 2.4440 (420)	^{8}Li 2.3390 (440)	^{9}Li 2.2450 (460)		^{11}Li 2.4820 (430)
	^{3}He 1.96xx* (10) ~~1.9730 (160)~~	^{4}He 1.67xx* (5) ~~1.6810 (40)~~		^{6}He 2.06xx* (80) 2.0680 (110)		^{8}He 1.9xxx* (246) 1.9290 (260)		
^{1}H 0.8409 (4) ~~0.8775 (51)~~	^{2}D 2.1277 (2) ~~2.1424 (21)~~	^{3}T 1.7550 (860)						
	n							

* = preliminary

electron scattering

muonic atom spectroscopy

H/D: precision laser spectroscopy + theory (a lot!)

^{6}He, ^{8}He, ...: laser spectroscopy of *isotope shift*

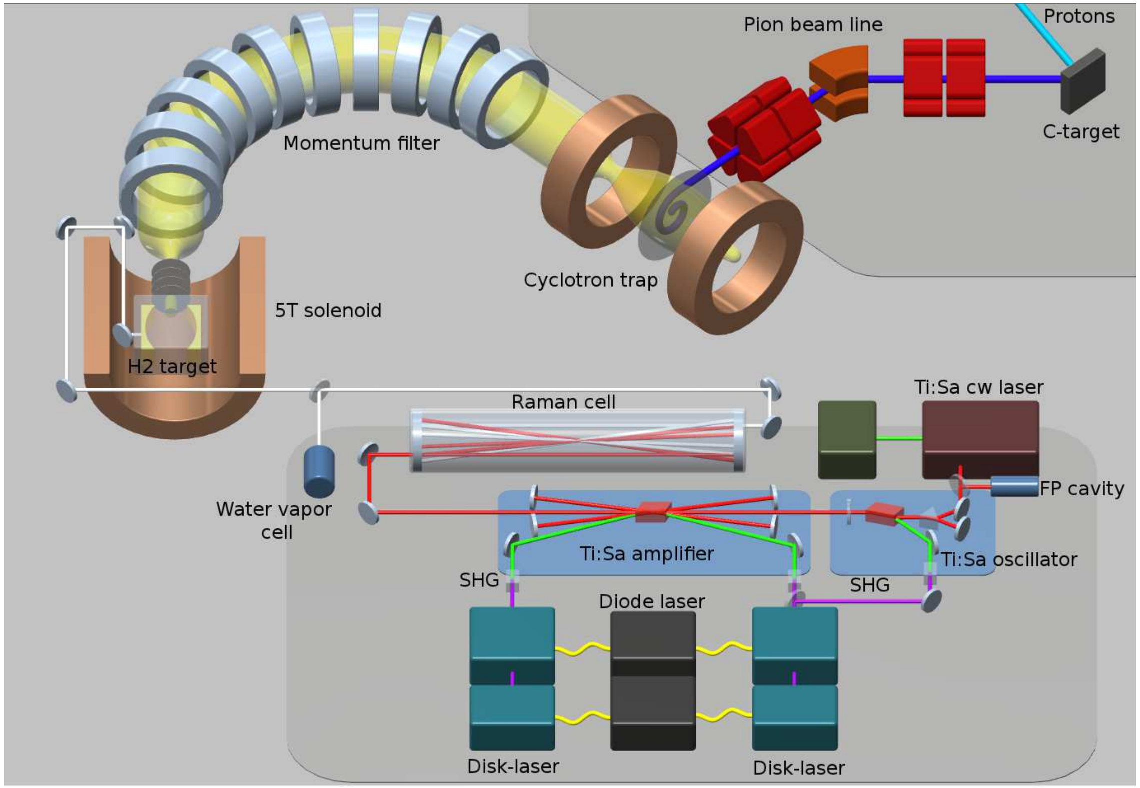
laser spectroscopy of muonic atoms/ions

Summary

- Results from muonic hydrogen and deuterium:
  - Proton charge radius: r_p= 0.84087 (39) fm
  - Proton Zemach radius: R_Z = 1.082 (37) fm
  - Rydberg constant: $R_\infty = 3.289\,841\,960\,249\,5\ (10)^{r_\mathrm{p}}\ (25)^{\mathrm{QED}}\ \times 10^{15}\ \mathrm{Hz/c}$
  - Deuteron charge radius: r_d = 2.12771 (22) fm from μH + H/D(1S-2S)
  - The "Proton radius puzzle"
- muonic helium-3 and -4: charge radius 10x more precise. No big discrepancy
- H(2S-4P) gives revised Rydberg ⇒ small r_p PRELIMINARY
- New projects:
  - 1S-HFS in muonic hydrogen / ^{3}He ⇐ PSI, J-PARC, RIKEN-RAL, ...
  - LS in muonic Li, Be, B, T, ...; muonic high-Z, ...
  - 1S-2S and 2S-$n\ell$ in Hydrogen/Deuterium/Tritium, He$^+$
  - He, H$_2$, HD$^+$,...
  - Positronium $\equiv e^+e^-$, Muonium $\equiv \mu^+e^-$
  - Electron scattering: H at lower Q^2, D, He
  - Muon scattering: MUSE @ PSI
  - ...

Future: Muonic

The **world's most intense beam** for **low-energy μ^-**

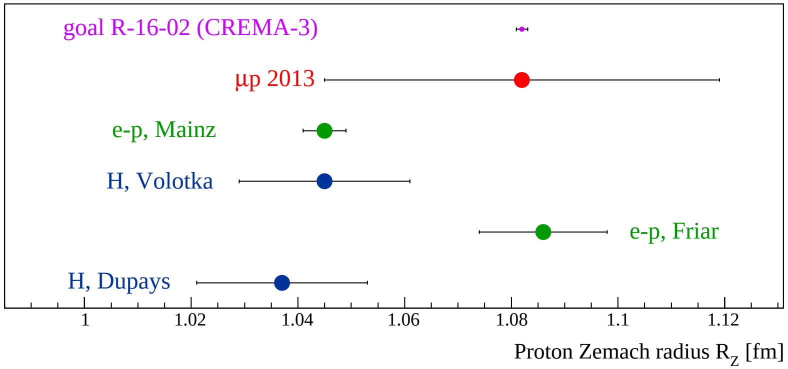


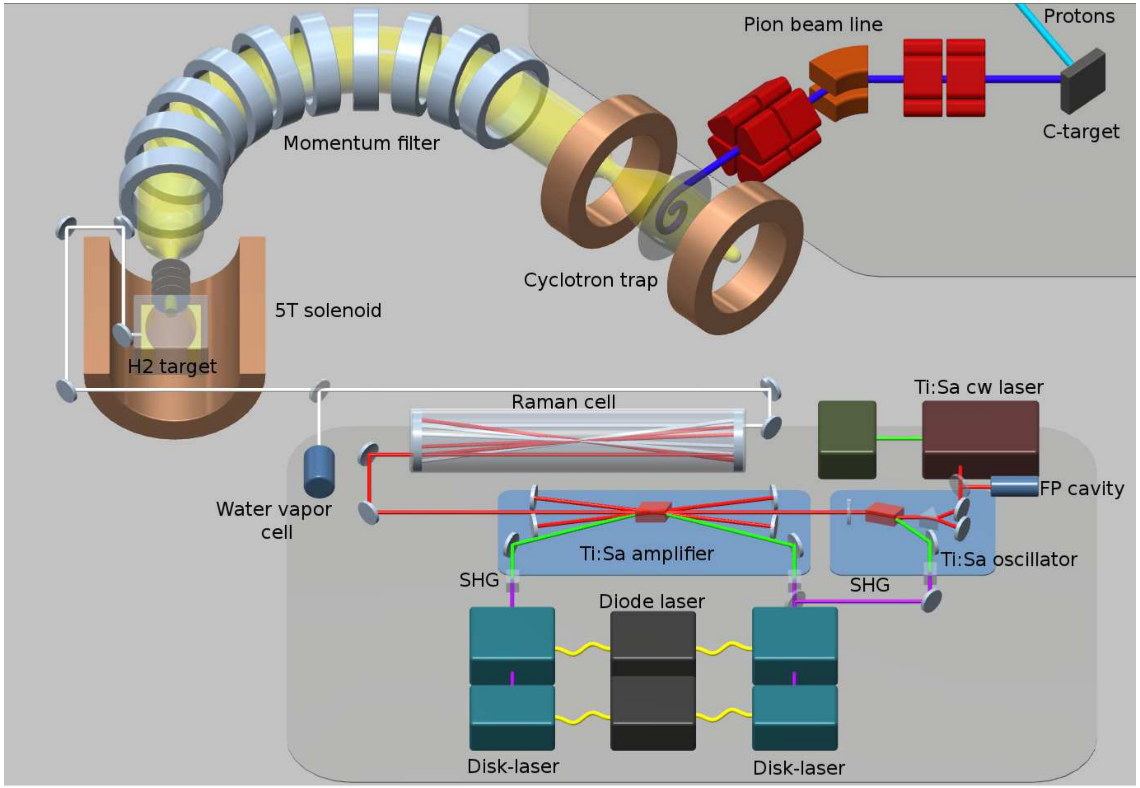
Future: Muonic

The **world's most intense beam** for **low-energy μ^-**

1S-HFS in μp, μ^3He

→ Zemach (magnetic) radius





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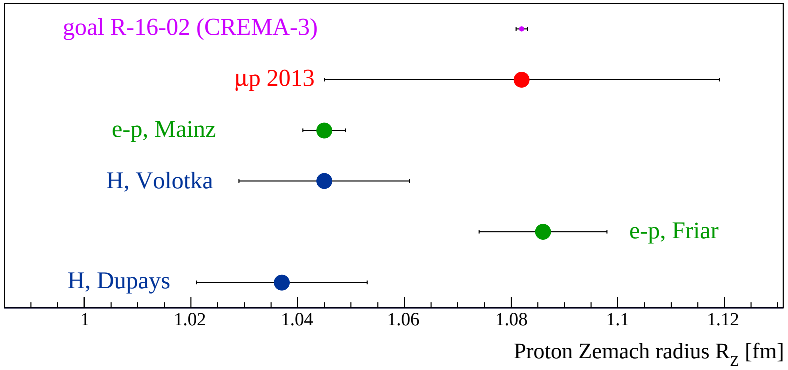
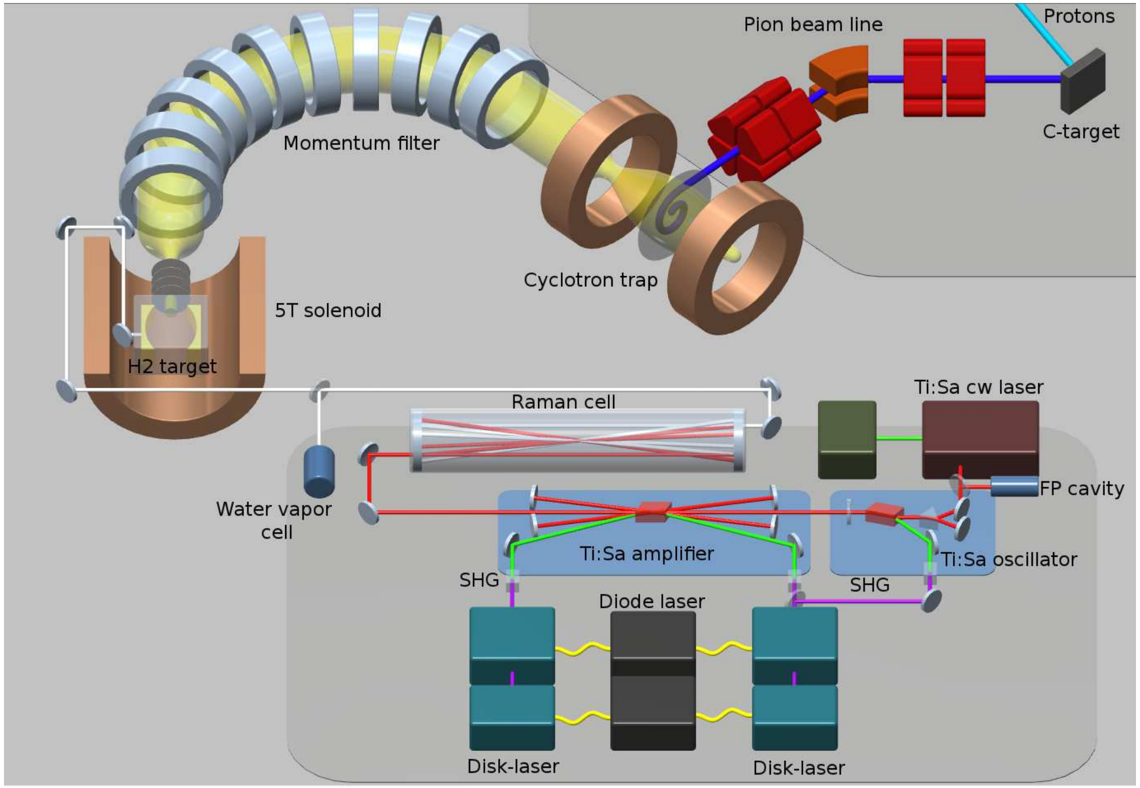
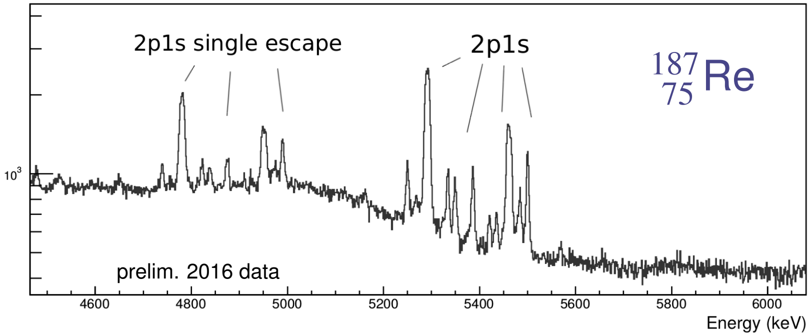




stop in μg of (radioactive) material

→ charge radii of higher Z

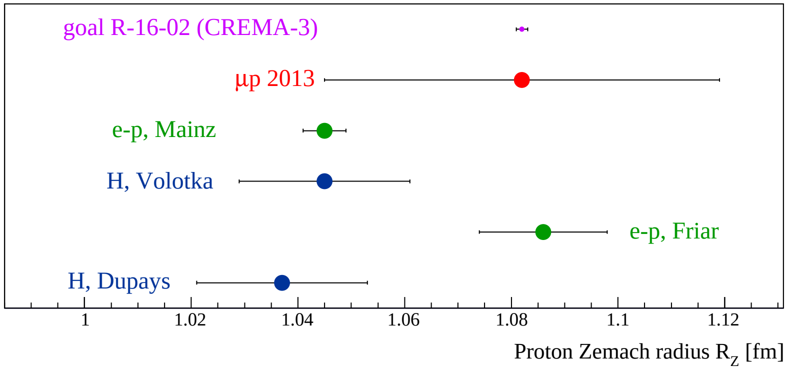
muX Collab @ PSI



Future: Muonic

1S-HFS in μp, μ^3He

→ Zemach (magnetic) radius



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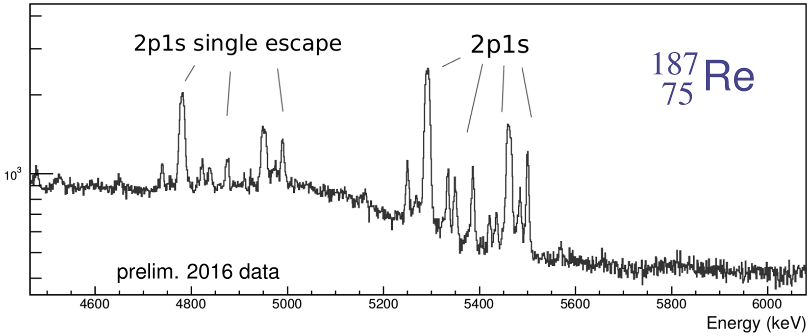
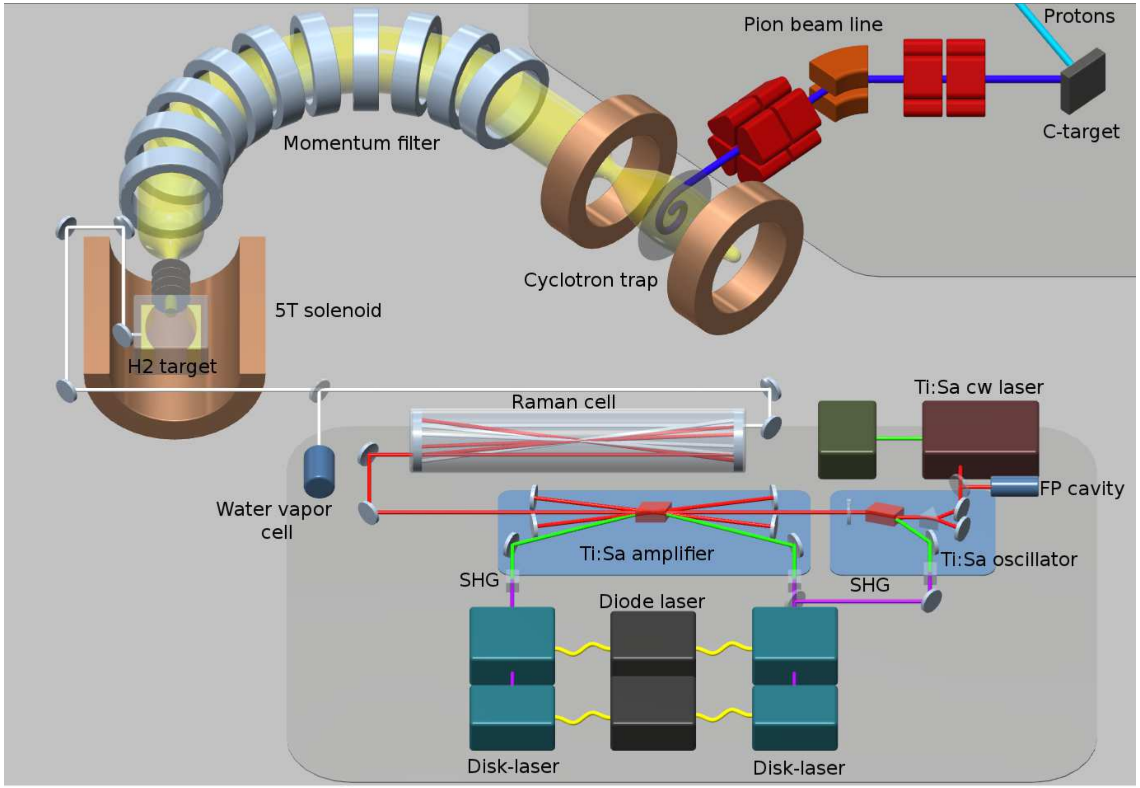
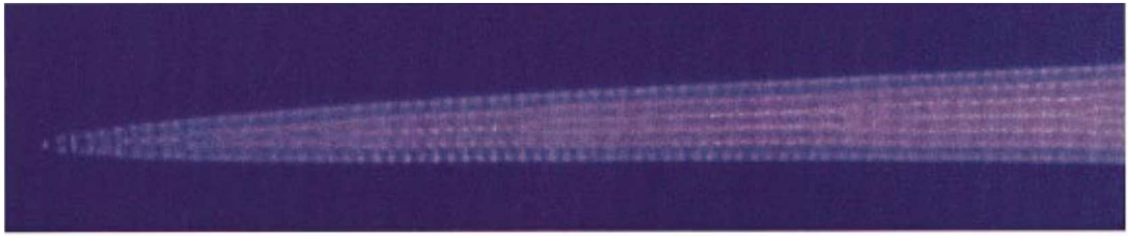


The **world's most intense beam** for **low-energy μ^-**



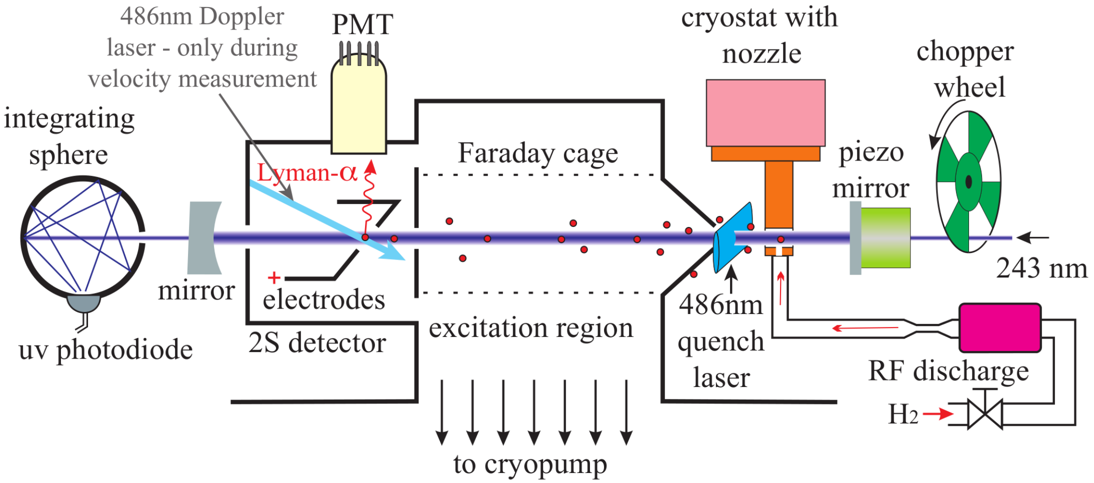
stop μ^- in Penning trap

→ charge radii of Li, Be, B, T



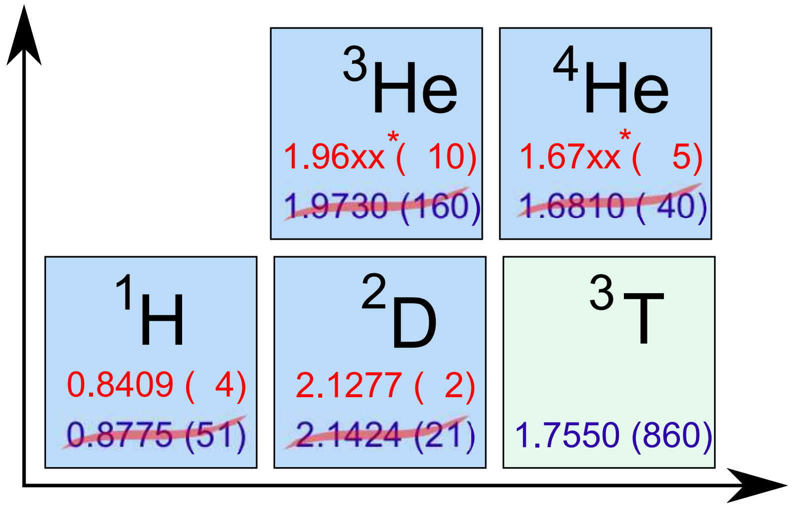
Future: Electronic

Hydrogen apparatus in Garching



Future: Electronic

Tritium = “missing link”



^{3}He
1.96xx*(10)
~~1.9730 (160)~~

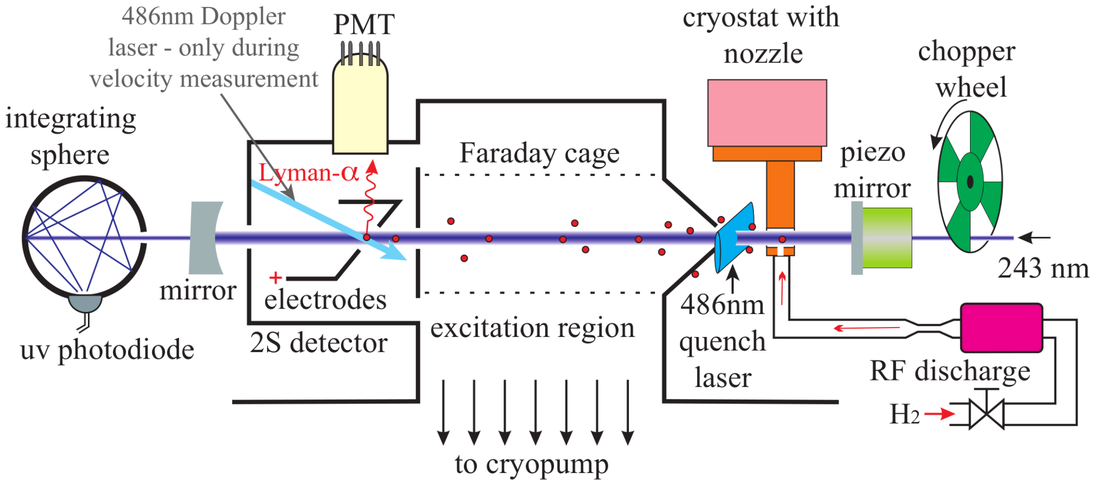
^{4}He
1.67xx*(5)
~~1.6810 (40)~~

^{1}H
0.8409 (4)
~~0.8775 (51)~~

^{2}D
2.1277 (2)
~~2.1424 (21)~~

^{3}T
1.7550 (860)

Hydrogen apparatus in Garching



$r_{\mathrm{p}} = 0.8775(51)\,\mathrm{fm} \rightarrow 0.8409(4)\,\mathrm{fm}$

$r_{\mathrm{d}} = 2.1424(21)\,\mathrm{fm} \rightarrow 2.1277(2)\,\mathrm{fm}$

$r_{\mathrm{t}} = 1.7550(860)\,\mathrm{fm} \Rightarrow$ potential improvement by 400!

$r_{\mathrm{d}}^2 - r_{\mathrm{p}}^2 = 3.82007(65)\,\mathrm{fm}^2$ H/D(1S-2S) isotope shift to 15 Hz

limit from **theory**: 1 kHz

r_{t} from T(1S-2S) to 10 kHz, later 1 kHz

CREMA in 2009...

Proton Size Investigators thank you for your attention

... and 2014