Bottom-up EFT for Lepton Flavour Violation

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Thanks to:
V. Cirigliano, A. Crivellin, M. Elmer, M. Gorbahn, G. Isidori, Y. Kuno, M. Pruna, A.Signer, ...

1. Introduction
   - LFV$\equiv$ contact interaction changing (charged) lepton flavour
   - NP required for m_ν, neccessarily generates LFV! (I $assume$ heavy NP)
   - What do we know (experimentally)?

2. Can I learn anything with bottom-up EFT?

3. observations from $\mu \leftrightarrow e$:

   - do we care about SM loops?
   - sensitivity vs exclusions
   - do we need dimension 8?
   - wee details/devils
   - ...

What do we know? (experimentally)

some processes	current constraints	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	2×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (2018, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	10^{-16} (Mu2e,COMET)
		10^{-18} (PRISM/PRIME)
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to e\phi$	$< 3.1 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)

$\mu A \to eA \equiv \mu^-$ bound in $1s$ state of nucleus A converts to e

What can a theorist do with those numbers?

For $\mu \to e$ processes at scale $\sim m_\mu$:

Can describe 3 or 4-point μ-e interactions involving e and μ, and 1 or 2 gauge fields, or 2(same-flavour) fermions $\in u, d, s, e$ with $QED * QCD$ invariant operators:

$$em_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad dim\ 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad dim\ 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{u}\gamma_\alpha u) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{u}\gamma_\alpha\gamma_5 u)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{d}\gamma_\alpha d) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{d}\gamma_\alpha\gamma_5 d)$$

$$(\overline{e}P_Y\mu)(\overline{u}u) \qquad (\overline{e}P_Y\mu)(\overline{u}\gamma_5 u)$$

$$(\overline{e}P_Y\mu)(\overline{d}d) \qquad (\overline{e}P_Y\mu)(\overline{d}\gamma_5 d)$$

$$(\overline{e}\sigma P_Y\mu)(\overline{d}\sigma d)$$

$$(\overline{e}\sigma P_Y\mu)(\overline{u}\sigma u)$$

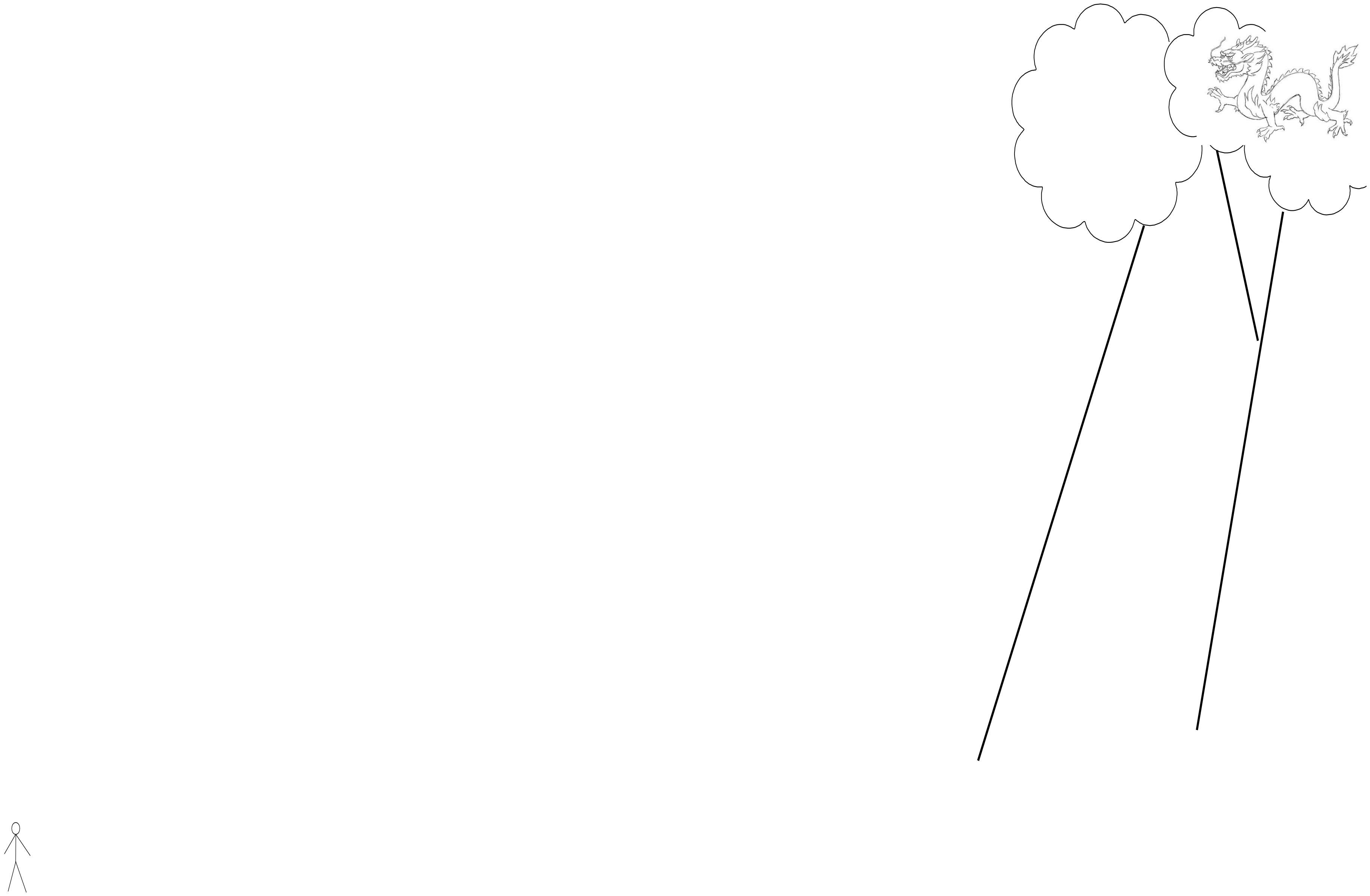
$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad dim\ 7$$

...zzz...

(plus operators with $d \leftrightarrow s$). $\quad (P_X, P_Y = (1 \pm \gamma_5)/2)$

Can express rates for $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and $\mu - e$ conv. in terms of sums of coefficients of such operators.

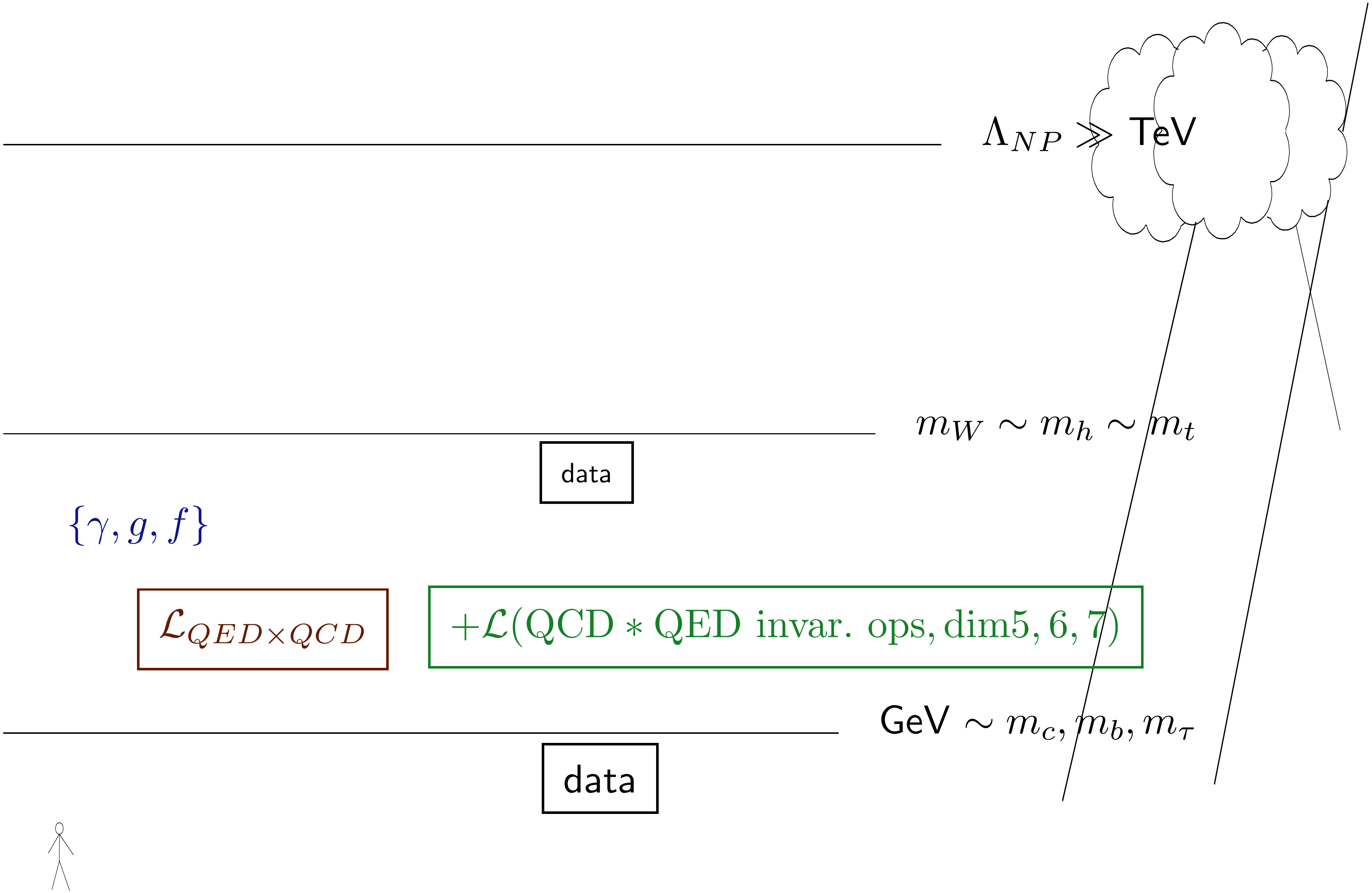
What can a theorist do with those constraints



Not gaze at mountain-tops from valley-bottom and hypothesize about the NP who lives there, instead, ask SM to carry me and exptal constraints as far up as possible...

Georgi, EFT, ARNPP 43(93) 209 (one of my all-time favourite papers)

EFT for CLV induced by heavy NP ($\Lambda_{NP} \gg m_W$)



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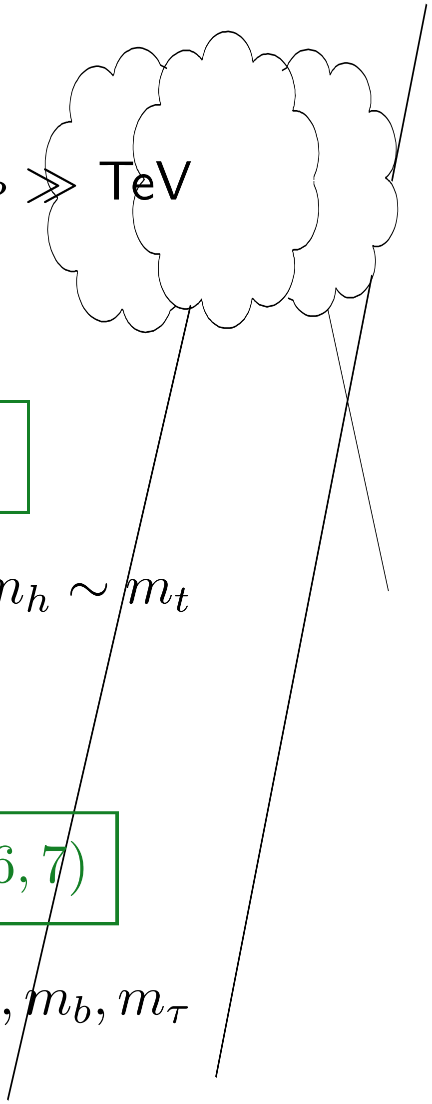


$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$

$\boxed{\mathcal{L}_{SU(3)*SU(2)*U(1)}}$ $\boxed{+\mathcal{L}(\text{SM invar. operators, dim6})}$

$m_W \sim m_h \sim m_t$

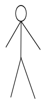
$\boxed{\text{data}}$

$\{\gamma, g, f\}$

$\boxed{\mathcal{L}_{QED\times QCD}}$ $\boxed{+\mathcal{L}(\text{QCD} * \text{QED invar. ops, dim5, 6, 7})}$

GeV $\sim m_c, m_b, m_\tau$

$\boxed{\text{data}}$



In practise, need operator basis + recipe to change scale

1. relate EFT to another theory(other EFT, model,data...):
   match Greens functions with same external legs

2. Within an EFT: operator coefficients $\{C_I\}$ evolve with scale according to Renormalisation Group Eqns.
   Below m_W**:**

Davidson,CrivellinDPS

$$\mu\frac{\partial}{\partial\mu}(C_I,...C_J,...) = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}^e$$

boring $\mathbf{\Gamma}^s$ rescales coefficients, interesting $\mathbf{\Gamma}^e$ transforms one coeff to another

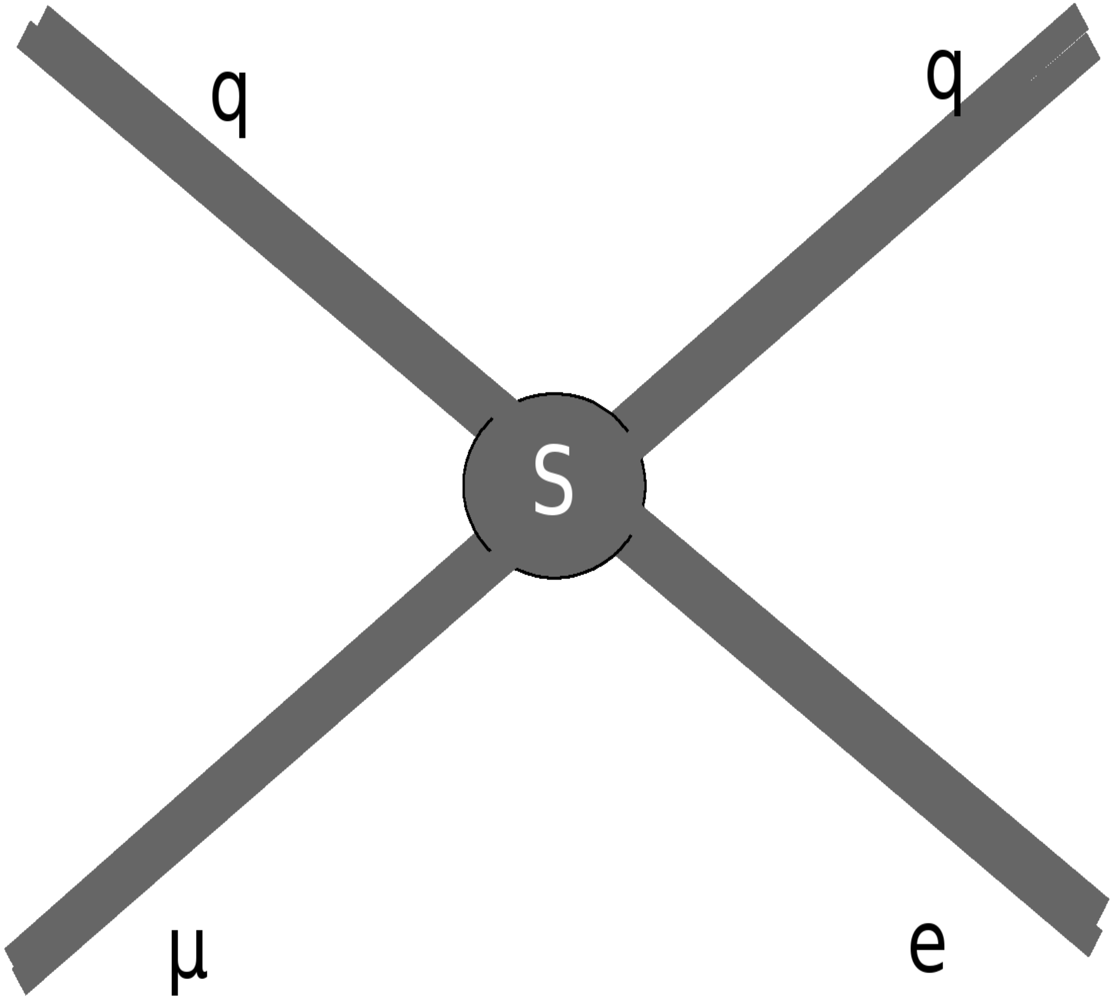
JenkinsManoharTrott

Above m_W :$\mathbf{\Gamma}$ for $SU(3)\times SU(2)\times U(1)$

??to what order in the multitude of SM perturbative expansions(α_i, y_j loops)??

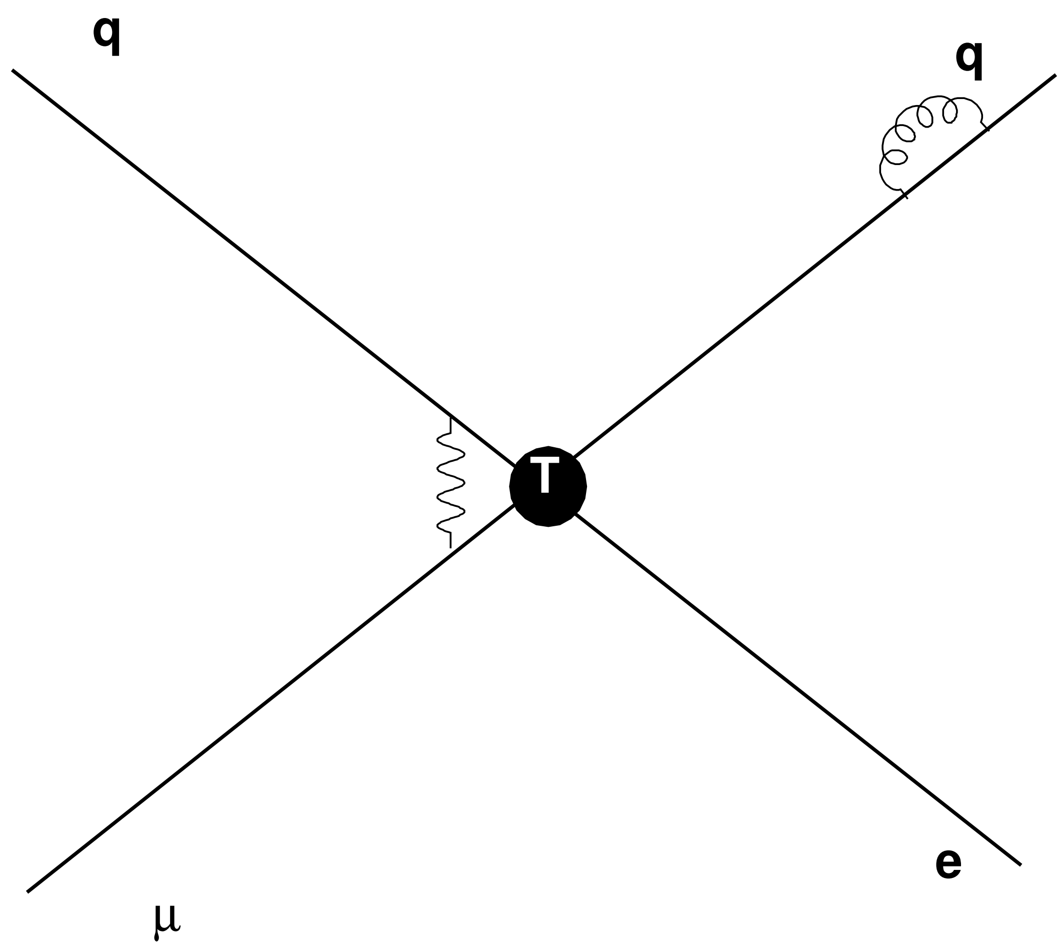
Want to "peel off" SM coating of loop corrections

expt measures operator coefficient $c(\mu_{exp})$ at exptal energy scale $\sim \mu_{exp} \sim m_\tau$



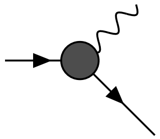
Peeling off SM loops

But if I look on shorter distance scale ($\sim 1/m_W$) I might see



Loop effects...is there sensitivity?

Two dipole operators contribute to $\mu \to e\gamma$:



$$\begin{aligned}
\delta\mathcal{L}_{meg} &= -\frac{4G_F}{\sqrt{2}} m_\mu \left(c_L^D \overline{\mu_R}\sigma^{\alpha\beta} e_L F_{\alpha\beta} + c_R^D \overline{\mu_L}\sigma^{\alpha\beta} e_R F_{\alpha\beta}\right) \\
BR(\mu \to e\gamma) &= 384\pi^2(|c_R^D|^2 + |c_L^D|^2) < 4.2 \times 10^{-13} \\
&\Rightarrow \quad |c_X^D| \lesssim 10^{-8}
\end{aligned}$$

MEG expt, PSI

How big does one expect c to be?

Is there sensitivity to loop effects ?

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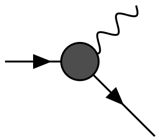

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$$

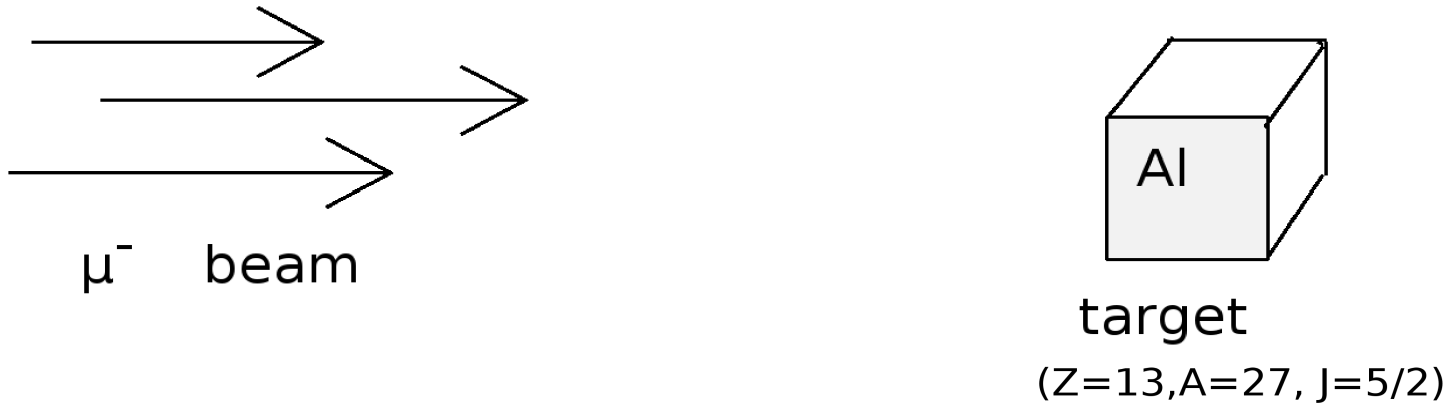
MEG expt, PSI

How big does one expect c to be? Suppose operator coefficient

$$
\begin{array}{llll}
 & & n = 1 & n = 2 \\
c\frac{m_\mu}{v^2} \sim \frac{ev}{(16\pi^2)^n \Lambda^2} \quad \Rightarrow & \text{probes} & \Lambda \lesssim 3000 \text{ TeV} & 300 \text{ TeV} \\
c\frac{m_\mu}{v^2} \sim \frac{em_\mu}{(16\pi^2)^n \Lambda^2} \quad \Rightarrow & \text{probes} & \Lambda \lesssim 100 \text{ TeV} & 10 \text{ TeV}
\end{array}
$$

$\Rightarrow \mu \to e$ expts probe multi-loop effects in NP theories with $\Lambda_{NP} \gg$ reach of LHC

RGEs, mixing and all that... does it matter? Consider $\mu \to e$ conversion



- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$

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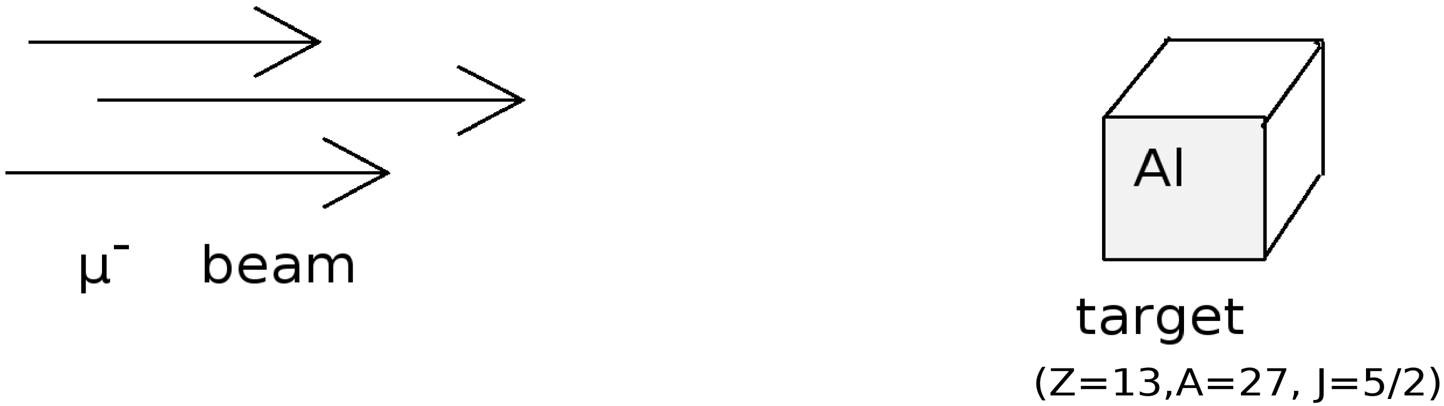

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- μ converts to e $(E_e \approx m_\mu)$ via

$$\delta\mathcal{L} = C_T^{uu}(m_W)(\overline{e}\sigma P_R\mu)(\overline{u}\sigma u) + C_A^{uu}(m_W)(\overline{e}\gamma P_L\mu)(\overline{u}\gamma\gamma_5 u)$$

- nuclear expectation value of quark currents like for WIMP scattering (at $q^2 = 0$):

V,S quark currents $\longrightarrow$ Spin-Indep, A,T quark currents $\longrightarrow$ Spin-Dep conversion.

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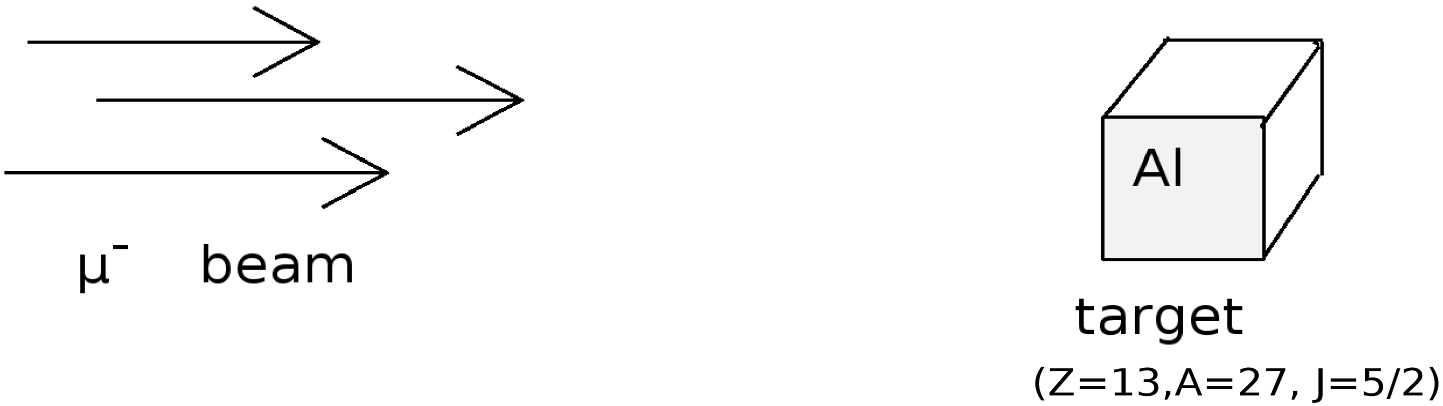

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- nuclear expectation value of quark currents like for WIMP scattering (at $q^2 = 0$):

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- Neglecting RG loops, get

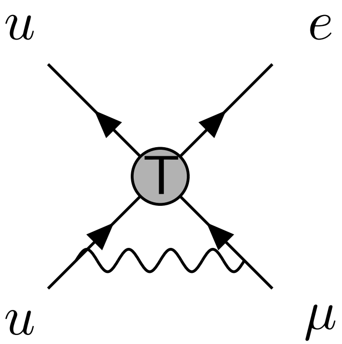
CiriglianoDavidsonKuno

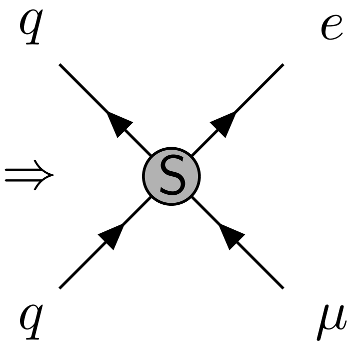
$$BR(\mu Al \to eAl)_{SD} \sim 8B\frac{J_{Al}+1}{J_{Al}}S_p^2|C_A^{uu} + 2C_T^{uu}|^2$$

$S_p \equiv \langle Al|\vec{S}_p|Al\rangle \sim .3,\ B \sim .33$

EngelRTO, KlosMGS

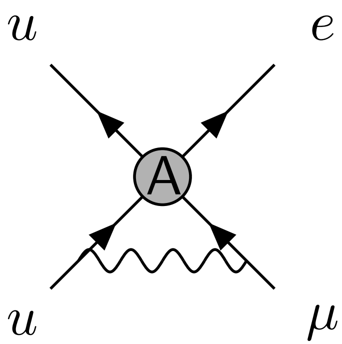
Include QED loops between $m_W \leftrightarrow m_\mu$

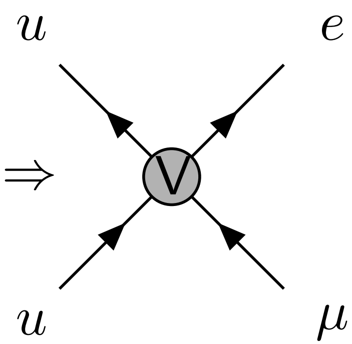
$C_T(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$ 

$+\ldots \quad \Rightarrow$ 

$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\; C_T\; (\overline{u}u)(\overline{e}P_Y\mu)$$

$$\Delta C_S(m_\tau) \sim \tfrac{1}{7}C_T(m_W)$$

$C_A(\overline{u}\gamma\gamma_5 u)(\overline{e}\gamma P_Y \mu)$ 

$+\ldots \quad \Rightarrow$ 

$$8\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}C_A(\overline{u}\gamma u)(\overline{e}\gamma P_Y\mu)$$

$$\Delta C_V(m_\tau) \sim \tfrac{1}{50}C_A(m_W)$$

Including the loop effects...

Recall $\Delta C_S^{uu} \sim 1/7 C_T^{uu}$ from RG mixing,
then $\langle p|\bar{u}u|p\rangle \sim 10\langle p|\bar{u}\sigma u|p\rangle$, so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$, and

$$BR(\mu Al \to eAl)_{SI} \sim 0.33(27)^2|.03C_A^{uu} + 2C_T^{uu}|^2$$

(A $= 27$ for Al)
(Recall that the BR_{SD} induced directly was $BR(\mu Al \to eAl)_{SD} \sim 0.1|C_A^{uu} + 2C_T^{uu}|^2$)

$\Rightarrow$ loop effects change $BR(\mu Al \to eAl)$ by $\left\{ \begin{array}{ll} \mathcal{O}(10^3) & \text{for } u, d \text{ tensor} \\ \mathcal{O}(\text{few}) & \text{for axial} \end{array} \right.$

“Constraints” = sensitivities or Exclusions?

(or: How small can we see *vs* How big could it be?)

$sensitivity \equiv$ how small a coefficient could one see?

$\Leftrightarrow$ “setting bounds one operator at a time”

1. put a coefficient, eg C_T^{uu} at m_W
2. compute observables, obtain:

$$C_T^{uu} \lesssim \epsilon$$

$\Leftrightarrow$ can’t see C_T^{uu} if its smaller than ϵ.

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constraints and exclusions $\equiv$ what values of a coefficient are excluded by the data?

- models induce numerous operators
- observables often depend on linear combinations of operators coefficents...
  ... all coefficients run and mix with scale

$\Rightarrow$ a given expt constrains a linear combination of coefficients

Example: should the LHC look for $h \to \mu^{\pm} e^{\mp}$?

At Λ_{NP}: $\mathcal{L}_{SM} \quad + \quad \frac{C_h}{\Lambda_{NP}^2} H^\dagger H \overline{\ell_\mu} H e \qquad + \quad \frac{C_{meg}}{\Lambda_{NP}^2} \overline{\ell_\mu} H \sigma \cdot F e$

At m_h: h decays to $\mu^{\pm} e^{\mp}$; LHC $excludes \sim \frac{C_h v^2}{\Lambda_{NP}^2} \lesssim 10^{-3}$ (at 1-loop $C_h(m_h) \approx C_h(\Lambda_{NP})$)

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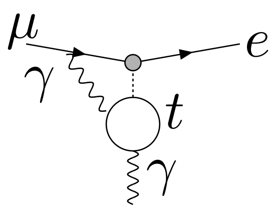
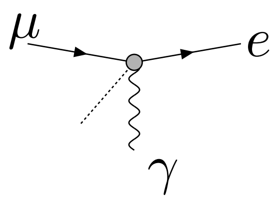
At m_μ:

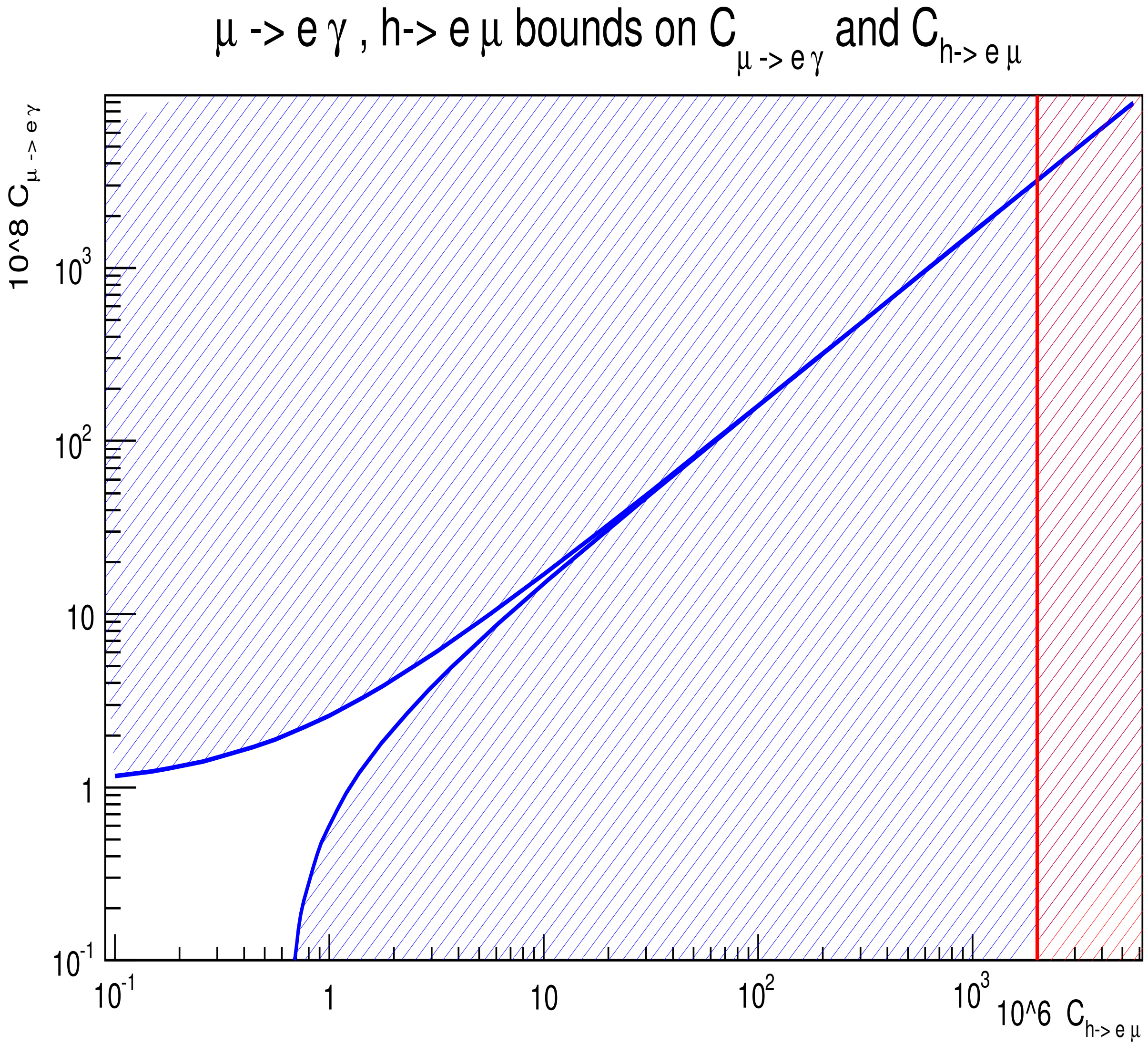


$+$



$$BR(\mu \to e\gamma) \Rightarrow \left| \frac{e\alpha}{8\pi^3 Y_\mu} C_h + C_{meg} \right| \lesssim 10^{-8} \frac{\Lambda^2}{v^2} \quad , \quad \frac{e\alpha}{8\pi^3 Y_\mu} \sim 10^{-2}$$

$\mu \to e\gamma$ *sensitive* to $C_h v^2/\Lambda^2 \gtrsim 10^{-6}$...



How much "tuning" is allowed between operator ceofficients?
Can one define "*natural*" in EFT?

(Parenthese...so are there as many constraints as operators?)

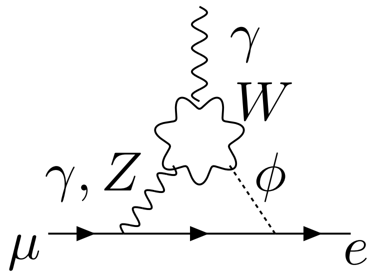
1. $\mu \to e\gamma$ mediated by 2 non-interfering dipoles $\bar{e}\sigma P_Y \mu F$ $\leftrightarrow$ 2 bds

2. $\mu \to e\bar{e}e$ mediated by 6 4f operators $+$ 2 dipoles, 6 bds.

3. $\mu - e$ conv. mediated by 2 dipoles,2 GG operators and 20 4f operators...
   exptal bds in 2 nuclei (Ti, Au) $\Rightarrow$ 4 bds (if independent?)
   (or maybe 8, if allow for spin-dep scattering).

$\Rightarrow$ 16 - 20 "flat directions" in operator basis made with $\{\gamma, g, u, d, s, e\}$

Are dimension eight operators negligeable?



(?no answer in EFT? Ask in many models?)

Consider 2HDM in decoupling limit, heavy doublet mass $M \sim 10v$.

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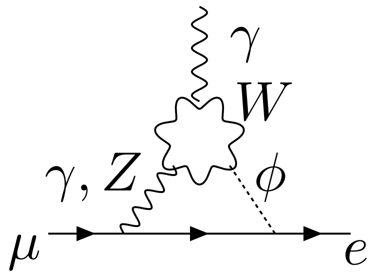

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$\mu \to e\gamma$ calculated at one and two (electroweak) loops.

Bjorken-Weinberg

Extract and compare $1/M^2$ (= dim6) and $1/M^4$ (= dim8) parts:

$$\frac{\text{dim8}}{\text{dim6}} \sim \frac{m_W^2}{M^2} \ln^2 \frac{m_W^2}{M^2} \quad , \quad \lambda_i \tan\beta \frac{v^2}{M^2}$$

(NB: $z \ln^2 z \sim 0.2$ for $z \sim 0.01$!)

(the dominant W contribution is log^2 enhanced at dim8, not dim6)

$\Rightarrow$ $1/M^2$ terms $>$ $1/M^4$ terms, but need dimension 8 to get numerically reliable result?

In b loops, dim6 $>$ dim8 if reasonable Higgs potential parameters $\{\lambda_i\}$, and $\cot\beta$, $\tan\beta \lesssim 50$.

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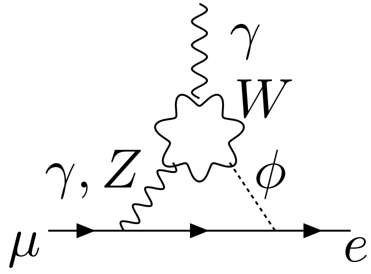

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... in a model where the $\tan\beta$ and log enhancements were combined, dim 8$>$ dim6

Summary: its not just about "a sufficiently high scale"; also need "sufficiently non-hierarchical coefficients", and cooperative logs.

Wee details and other nightmares: what order in what expansions?

EFT in kindergarten (N=0): run at N+1 loop, match at N loop

(the wee problem: at $N > 0$, can appear terms depending on operator renorm. scheme. Must cancel, because operators are just an approx to the renormalisable NP theory. But do they cancel?)

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...but in SM, several expansions: $\left\{ \begin{array}{l} \text{loops} \\ \alpha_s, \alpha_2, \alpha_{em} \\ y_q, y_\ell \end{array} \right\} \frac{y_t}{(16\pi^2)^2} \gg \frac{y_\mu}{(16\pi^2)}$

SM is part of what we $know$, in the EFT calculation: there is only one right answer. When dominant contributions come from loop matching, multi-loop running, need to include....

So what to do?

?? Full calculation at 2 or three loop?

...or want numerically largest contribution of every operator to every observable?

(tbc if is gauge invar and scheme indep...)

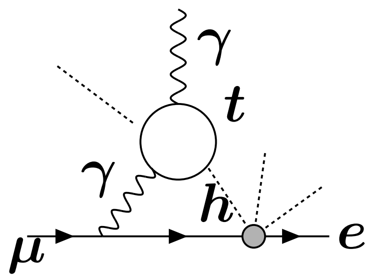
What goes wrong at m_W?

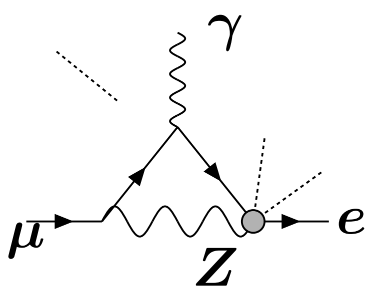
The problem: there are (one or two) loop matching diagrams that give the largest contribution of a coefficient to a observable, with no corresponding diagrams in the RGEs.

Arises because operator dimensions change at m_W (Higgs field becomes vev)
rule of thumb: if run with 1-loop RGEs, then match at tree
reasonable if same diagram gives matching and running
...but... Dim 6 LFV Higgs and Z vertices:

$$H^\dagger H \overline{L}_\mu H E_e \quad , \quad i(\overline{L}_e \gamma^\alpha L_\mu)(H^\dagger \overleftrightarrow{D}_\alpha H) \quad , \quad i(\overline{E}_e \gamma^\alpha E_\mu)(H^\dagger \overleftrightarrow{D}_\alpha H)$$

contribute in loops to *dim 8* dipole $H^\dagger H(\overline{L}_e H \sigma \cdot F E_\mu)$, so not mix in RG running above m_W to the dim6 dipole, but do contribute in matching at m_W.





Summary

BackUp

In practise, need operator basis + recipe to change scale

A basis...is a boring tool? Of doubtful physical significance?

(?? Is there anything like "Jarlskog invariants" for EFT ??)

$\Rightarrow$ choose convenient basis(and not change during calculation)

Most CLV operators induce processes absent in the SM $\Rightarrow$ no contributions to SM observables $\Rightarrow$ basis choice simpler than *eg* for Higgs-EFT.

Some more operators for $\mu \to e$ at all scales $< m_W$

(That was only operators with one μ and lighter fermions...). At higher scales there are also operators containing μ, τ, c, b bilinears: :

$$
\begin{array}{rcl}
(\overline{e}\gamma^\alpha P_Y \mu)(\bar{l}\gamma^\alpha P_Y l) & , & (\overline{e}\gamma^\alpha P_Y \mu)(\bar{l}\gamma^\alpha P_X l) \\
(\overline{e} P_Y \mu)(\bar{l} P_Y l) & & (\overline{e} P_Y \mu)(\overline{\tau} P_X \tau) \\
(\overline{e}\sigma P_Y \mu)(\overline{\tau}\sigma P_Y \tau) & &
\end{array}
$$

$$
\begin{array}{rcl}
(\overline{e}\gamma^\alpha P_Y \mu)(\overline{q}\gamma^\alpha P_Y q) & , & (\overline{e}\gamma^\alpha P_Y \mu)(\overline{q}\gamma^\alpha P_X q) \\
(\overline{e} P_Y \mu)(\overline{q} P_Y q) & , & (\overline{e} P_Y \mu)(\overline{q} P_X q) \\
(\overline{e}\sigma P_Y \mu)(\overline{q}\sigma P_Y q) & &
\end{array}
$$

where $l \in \{\mu, \tau\}$, $q \in \{c, b\}$, $X, Y \in \{L, R\}$, and $X \neq Y$.

(notice: only lepton tensors with τ bilinear, and $(\overline{e}\sigma P_L \mu)(\overline{\tau}\sigma P_R \tau) = 0$)

Then **more operators if allow flavour non-diagonal quark bilinears...**
eg mediate $K \to \bar{\mu}e$....

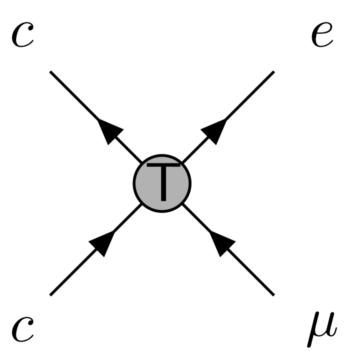
And **different operators above m_W...**

BuchmullerWyler
GrzadkowskiIMR

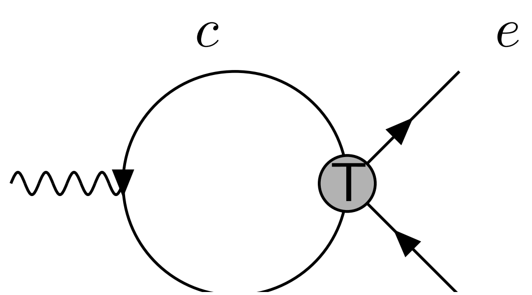
Does one need the loops, part 3? Of the tensor and the dipole...

suppose at $\sim m_W$: $\delta\mathcal{L} \supset C_T^{cc}(\bar{c}\sigma^{\alpha\beta}P_L c)(\bar{e}\sigma_{\alpha\beta}P_L\mu) + ...$
(eg from doublet leptoquark S with interactions $\lambda_L(\overline{\nu}s_L^c - \overline{\mu}c_L^c)S + \lambda_R\overline{e}c_R^c S$)

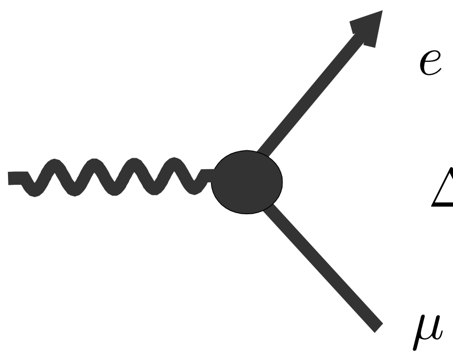
?How to observe that operator at tree level??



$\Rightarrow$



$\frac{16m_c}{em_\mu}\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\; C_T^{cc}m_\mu(\overline{e}\sigma\cdot FP_L\mu)$



$\Delta c_{D,L} \sim 1.2C_T^{cc}m_\mu(\overline{e}\sigma\cdot FP_L\mu)$

recall MEG bound : $c_{D,Y} \lesssim 10^{-8}$ at m_μ

at m_W : $|C_{D,L} - C_{T,L}^{cc} + C_{T,L}^{\tau\tau} + 1.8C_{T,L}^{bb} + \mathcal{O}(10^{-3})C| \lesssim 10^{-8}$

excellent sensitivity of $\mu \to e\gamma$ to mid-weight-fermion tensor operators

Why to do EFT

EFT $\Leftrightarrow$ add (yet more) perturbative expansions(in SM, already loops, gauge cplgs, Yukawas...).

Two perspectives in EFT:
top-down: EFT as the simple way to get the answer to desired accuracy
know the high-scale theory = can calculate operator coeffs
EFT simplifies (loop) calculations: expand in scale ratios ($eg\ m_B/m_W$)
rather than calculate dynamics at different scales

bottom-up: EFT as a parametrisation of ignorance
unknowable accuracy...

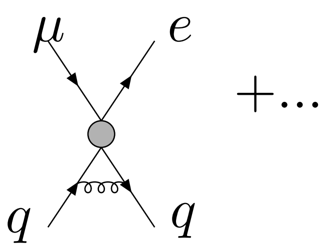
So in practise, EFT ...
1) gives a parametrisation of NP $\Leftrightarrow$ an operator basis
2) reorganises SM loop calculations involving those operators

need a basis, and need a recipe to include loops

Step 3: Run up to m_W with *one-loop* RGEs of QCD+QED

$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}$$

Step 3: Run up to m_W with *one-loop* RGEs of QCD+QED

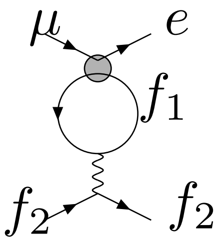


$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_s}{4\pi} \vec{C} \mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi} \vec{C} \mathbf{\Gamma}$$

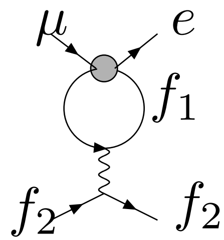
QCD: not mix ops, should resum $\Rightarrow$ multiplicative renorm S,T ops

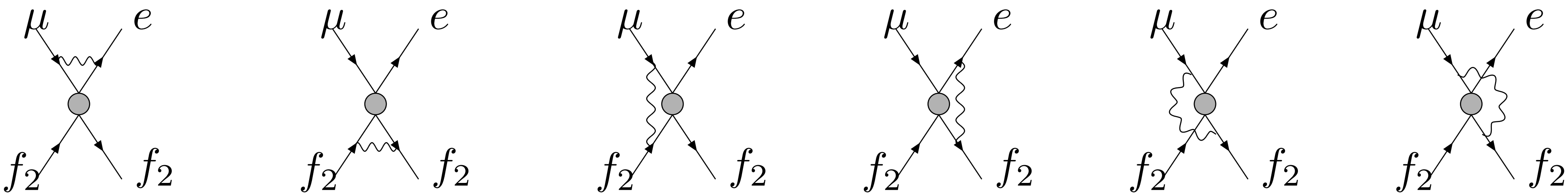
QED:

$$C_A(m_W) \left(\left[\frac{\alpha_s(m_W)}{\alpha_s(m_\tau)} \right]^{\frac{\gamma_A^s}{2\beta_0}} \delta_{AB} - \frac{\alpha_{em}}{4\pi} [\Gamma]_{AB} \log \frac{m_W}{m_\tau} + \frac{\alpha_{em}^2}{32\pi^2} [\Gamma\Gamma]_{AB} \log^2 \frac{m_W}{m_\tau} + .. \right) = C_B(m_\tau)$$



3: Run up to m_W with *one-loop* RGEs of QED





$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}$$

QCD: not mix ops, should resum $\Rightarrow$ multiplicative renorm S,T ops
QED: *does* mix ops, $\alpha_{em} \ll \Rightarrow$ mixing in pert theory, neglect renormalisation:

$$C_A(m_W)\left[\frac{\alpha_s(m_W)}{\alpha_s(m_\tau)}\right]^{\frac{\gamma_A^s}{2\beta_0}}\left(\delta_{AB} - \frac{\alpha_{em}}{4\pi}[\Gamma]_{AB}\log\frac{m_W}{m_\tau} + \frac{\alpha_{em}^2}{32\pi^2}[\Gamma\Gamma]_{AB}\log^2\frac{m_W}{m_\tau} + ..\right) = C_B(m_\tau)$$

DegrassiGiudice

NB: at one loop: $\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}_V & 0 \\ 0 & \mathbf{\Gamma}_{STD} \end{bmatrix}$ ... V $\rightarrow$dipole mixing arises at 2-loop

(neglect vectors in this talk! Better bounds from $\mu \to e\bar{e}e$,$\mu - e$ conv.... but thats not a reason!)

Why bother to match at m_W to QED$\times$QCD invar theory?

Why not use SMEFT everywhere?
Could work in full SM all the way down to m_μ with SM-invar operators?
Then only have to match operators to observables.

Answer 1:Because its more difficult.
Quark flavour people use EFT below m_W because replacing EW dynamics with contact interactions allows to focus on the complexities of QCD.

Answer 2: Using SMEFT everywhere doesn't simplify anything.
All the curiosities and difficulties of matching at m_W still arise; just now appear when match to observables.

Answer 3: Does SMEFT-everywhere give the right logs?
EFT is supposed to be a simple recipe to get the right answer. Its simple to regularise with dim reg, but $\overline{MS}$ resums the wrong logs (massless renorm scheme:doesn't know how many quark flavours in the QCD β-fn...)
EFT recipe for "matching out" puts the right logs back!