Viscous Cosmology

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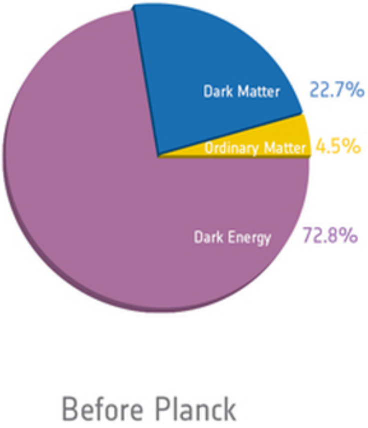
Dark matter and dark energy

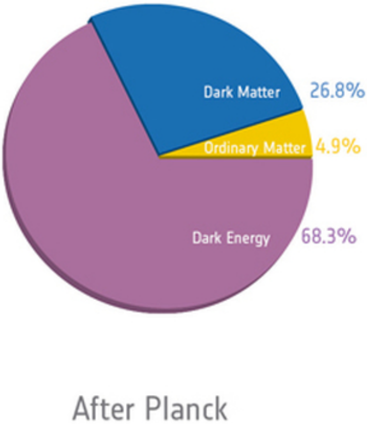
The cosmic budget

- The usual lore establishes that the universe contains five types of components:
  - Baryons (A jargon for particles formed by quarks and the massive leptons).
  - photons.
  - Neutrinos.
  - Dark matter.
  - Dark energy.

Dark matter and dark energy

The cosmic budget





Dark matter and dark energy

The success of the ΛCDM model

- It reproduces the CMB spectrum.
- It reproduces the matter power spectrum at linear level.
- It predicts an expansion accelerated phase without spoil the structure formation process.

Dark matter and dark energy

The success of the ΛCDM model

- The ΛCDM model is called the **Concordance model**, since it fits many observational data simultaneously for some range of its free parameters.
- But, is it just a simple *fitting model* as some times it is stated?

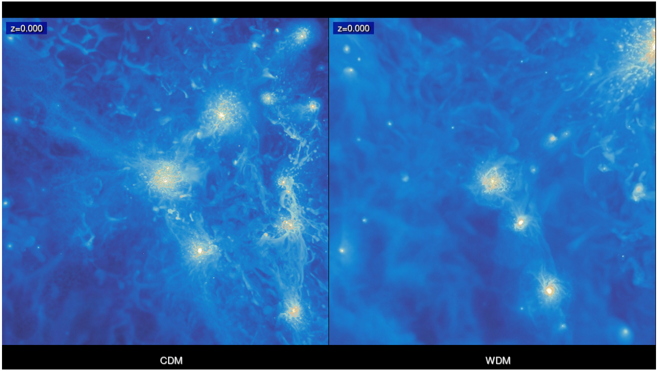
Dark matter and dark energy

The problems of the ΛCDM model

- It requires an important fine tunning on the value of $\Omega_{\Lambda 0}$ today.
- If the origin of he cosmological constant is from quantum fields in curved space-time, the theoretical value has a huge discrepancy with the observational value.
- At non-linear level simulations indicate an excess of power not observed in the local structures.

Dark matter and dark energy

The problems at non-linear level



Dark matter and dark energy

The problems of the ΛCDM model

- The NFW profile as deduced from simulations:

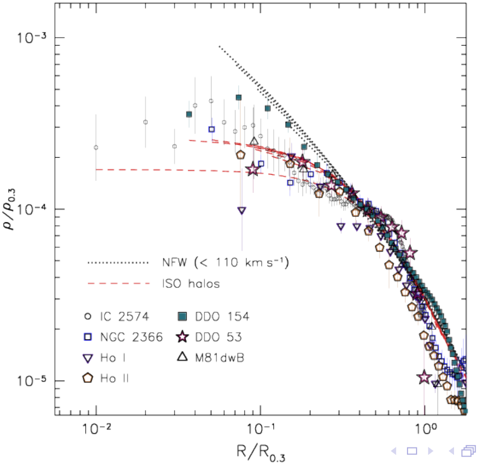
$$\rho = \frac{\rho_s}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)}.$$

- A phenomenological fit, the Burkert profile:

$$\rho = \frac{\rho_0}{\left(1 + \frac{r}{r_c}\right)\left[1+\left(\frac{r}{r_c}\right)^2\right]}.$$

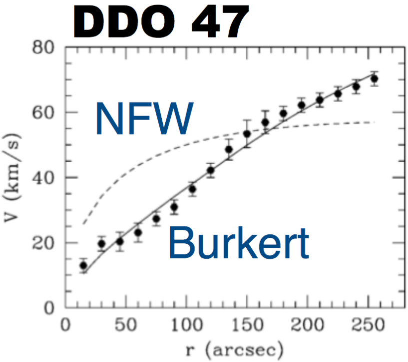
Dark matter and dark energy

The problems at non-linear level



Dark matter and dark energy

The problems at non-linear level



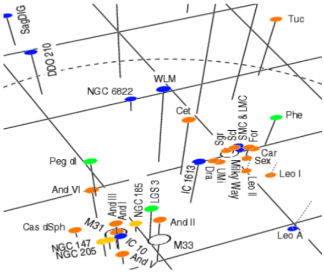
[Gentile et al ApJ (2005)]

Dark matter and dark energy

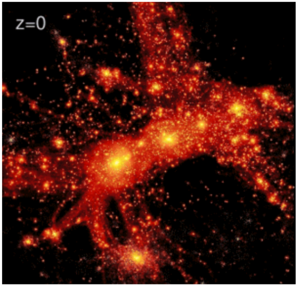
The problems at non-linear level

Defining the Missing Satellites Problem

(Kauffmann et al. 1993; Moore et al. 1999; Klypin et al. 1999)



The observed Local Group



The simulated Local Group

Dark matter and dark energy

Alternatives

- Among the possibilities to cure those problems, we can quote:
  - Effects of the baryons,
  - Warm dark matter,
  - New models for the dark sector.

Unified models for the dark sector

Chaplygin gas

- It is possible, in principle, to unify dark matter and dark energy into a single fluid.
- The traditional example: The Chaplygin gas model:

$$p = -\frac{A}{\rho}, \quad A = \text{constant}.$$
$$\rho = \sqrt{A + \frac{B}{a^6}}.$$

- Asymptotic behaviour:

$$a \rightarrow 0 \quad \Rightarrow \rho \propto a^{-3}, \quad \textbf{dust}.$$
$$a \rightarrow \infty \quad \Rightarrow \rho = \text{constant}, \quad \textbf{cosmological constant}.$$

Unified models for the dark sector

The generalized Chaplygin gas model

- The equation of state of the Chaplygin gas model can be generalized:

$$
\begin{aligned}
p &= -\frac{A}{\rho^{\alpha}}, \quad A = \text{constant}. \\
\rho &= \left\{A + \frac{B}{a^{3(1+\alpha)}}\right\}^{\frac{1}{3(1+\alpha)}}.
\end{aligned}
$$

- It has the same asymptotic behaviour as before, provided $\alpha > -1$.

Unified models for the dark sector

The generalized Chaplygin gas model

- One reason to such generalization:
  - The confrontation of the traditional Chaplygin gas model with observation implies that it is not competitive with the ΛCDM model.
- But, such generalization leads to a new free parameter, which it is not so desirable.

Unified models for the dark sector

Buk viscosity models

- Another possibility to unify dark matter and dark energy is to introduce bulk viscosity:

$$\begin{aligned} p^* &= p - \xi(\rho) u^\mu_{;\mu}. \\ u^\mu_{;\mu} &= 3H. \end{aligned}$$

- It is similar, from the background point of view, to the Chaplygin gas model if $p = 0$ and if,

$$\xi(\rho) = \xi_0 \rho^\nu.$$

Unified models for the dark sector

Buk viscosity models

- For one fluid model there is the following correspondence:

$$\nu + 1/2 = -\alpha.$$

- The perturbative behaviour is much more well-behaved, but the viscous pressure must be much more important than the adiabatic pressure.

Unified models for the dark sector

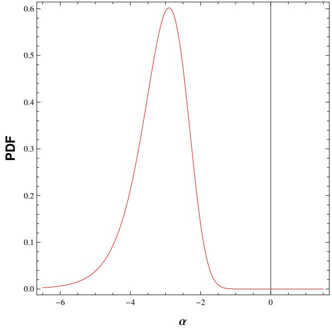
The main problems of the Chaplygin Gas

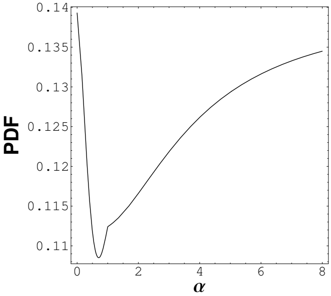
J.C. Fabris, T. Guio, M. Hamani Daouda, O. Piattella, G&C(2011)

- Tension in the estimations obtained from different observational set of data.
- Problems at perturbative level: the speed of sound.

Observational tests

SN versus power spectrum





Observational tests

The speed of sound

- The parameter α can not be negative:

$$v_s^2 = \frac{\partial p}{\partial \rho} = \alpha \frac{A}{\rho^{\alpha+1}}.$$

- $\alpha < 0 \quad \Rightarrow \quad v_s^2 < 0.$

Viscous models

Eckart formalism

- The viscous model constructed by using the Eckart's formalism give the same results as the Chaplygin gas model for the background if the bulk viscosity coefficient is chosen such that,

$$\xi(\rho) = \xi_0 \rho^{\nu}.$$

- But, at perturbative level new features appear.

Viscous models

Eckart formalism

- In the bulk viscosity case, the problem of the speed of sound is very different.
- This comes from the perturbation of viscous pressure.

Viscous models

Eckart formalism

- For a general barotropic fluid, with pressure given by $p = p(\rho)$,

$$\delta p = p'\delta\rho = \frac{\dot{p}}{\dot{\rho}}\delta\rho = v_s^2\delta\rho.$$

- For a viscous fluid in the Eckart formalism, with $p = -\xi(\rho)u^{\mu}_{;\mu}$, we find

$$\delta p = -3H\xi'\delta\rho - \xi\left(\theta - \frac{\dot{h}}{2}\right).$$

No direct relation to the sound speed.

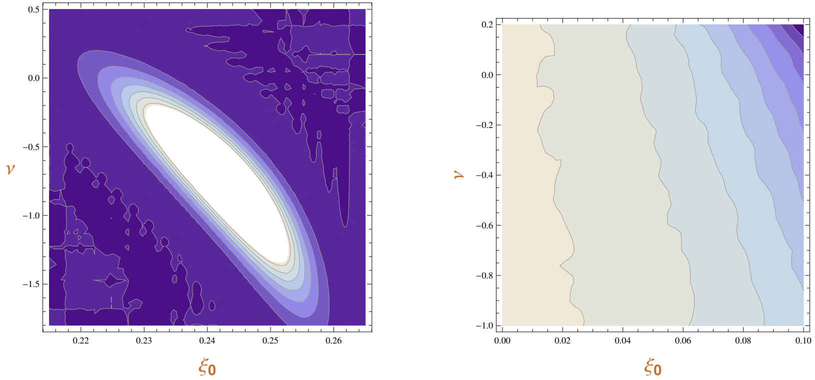
Viscous models

Eckart formalism

- However, at least in the Eckart formalism, the good region in the space parameter to fit the supernova data, does not overlap with the good region to have the agreement with the power spectrum data.

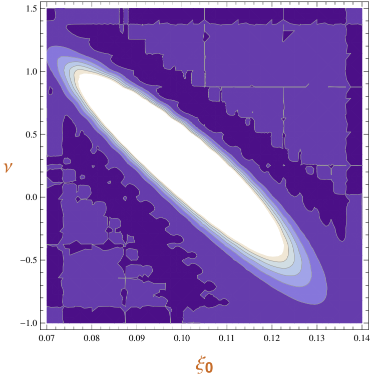
Viscous models

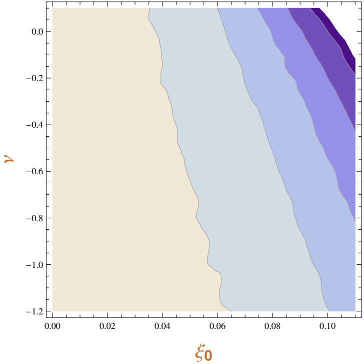
2 dimensional PDF - $\Omega_{\Lambda 0} = 0.0$



Viscous models

2 dimensional PDF - $\Omega_{\Lambda 0} = 0.7$





Viscous models

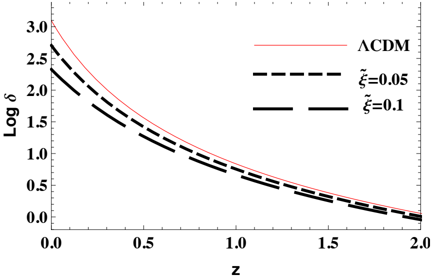
Non-linear matter agglomeration

H. Velten, T. Caramês, J.C. Fabris, R. Batista, PRD(2014).

- Viscous models suffer from difficulties to give a unified description of the dark sector.
- However, a viscous dark matter model alleviate the problems of excess of power of the ΛDCDM model.

Viscous models

Non-linear matter agglomeration



New possibilities for viscous models

Müller-Israel-Stewart formalism

J. Fabris, O. Piattella, W. Zimdahl, JCAP(2011).

- The Ecakart formalism is non-causal.
- A causal formalism is given by the MÏler-Israel-Stewart formalism.
- In this case, the viscosity pressure is defined by an equation:

$$\tau\Pi^{\bullet} + \Pi = -\theta\zeta - \frac{1}{2}\tau\Pi\left[\theta - \frac{(\zeta/\tau)^{\bullet}}{(\zeta/\tau)} - \frac{T^{\bullet}}{T}\right] ,$$

where τ is a relaxation time and T is the temperature. The bullet $\bullet$ denotes

$$\Pi^{\bullet} := u^{\mu}\nabla_{\mu}\Pi ,$$

i.e. derivation along the fluid wordline.

New possibilities for viscous models

Müller-Israel-Stewart formalism

- There is a truncated version of this formalism.
- The viscous pressure is then defined as,

$$\tau\Pi^{\bullet} + \Pi = -\theta\zeta \ .$$

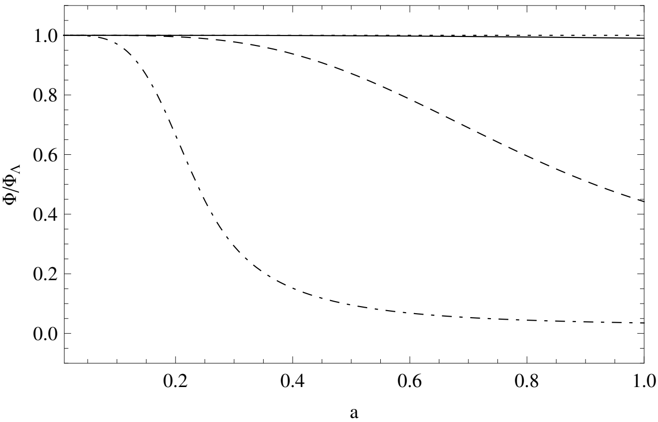
New possibilities for viscous models

Müller-Israel-Stewart formalism

- But there is still problems, mainly concerning the ISW, at least with some assumptions for the free parameters of the model.

New possibilities for viscous models

Müller-Israel-Stewart formalism



Shear viscosity

Introducing shear

- Until now we have considered only bulk viscosity.
- Shear viscosity does not contribute to the background equations for a homogeneous and isotropic universe.
- But, can it contribute significativily at perturbative level"

Shear viscosity

Introducing shear

C. Barbosa, J.C. Fabris, R. Ramos, H. Velten, arXiv:1702.07040

- Let us consider the following equations:

$$\begin{aligned} R_{\mu\nu} &- \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \\ T^{\mu\nu} &= \rho u^\mu u^\nu - p\left(g^{\mu\nu} - u^\mu u^\nu\right) + \Delta T^{\mu\nu}, \\ \Delta T^{\mu\nu} &= \eta\left[u^{\mu;\nu} + u^{\nu;\mu} - u^\rho \nabla_\rho \left(u^\mu u^\nu\right)\right] \\ &+ \left(\xi - \frac{2}{3}\eta\right)\left(g^{\mu\nu} - u^\mu u^\nu\right)\nabla_\rho u^\rho. \end{aligned}$$

Shear viscosity

Introducing shear

- We set the adiabatic pressure equal to zero:

$$p = 0.$$

- Hence, the bulk viscous pressure becomes, using the Eckart formalism,

$$p_B = -\xi u^{\mu}_{;\mu}.$$

Shear viscosity

Introducing shear

- Using a FLRW metric the Friedmann equation reads

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_v + \frac{\Lambda}{3},$$

- By defining the fractional densities,

$$\Omega_v = 8\pi G\rho_v/(3H_0^2), \quad \Omega_\Lambda = \Lambda/(3H_0^2),$$

where H_0 is the present value for the Hubble parameter, then the Friedmann equation becomes

$$H^2 = H_0^2\left(\Omega_v + \Omega_\Lambda\right).$$

Shear viscosity

Introducing shear

- Using now the fluid equation for ρ_v,

$$\dot{\rho}_v + 3H\left(\rho_v + p_v\right) = 0,$$

we can obtain an equation for the fractional density Ω_v as

$$a\frac{d\Omega_v}{da} + 3\Omega_v(1 + \omega_v) = 0,$$

where we have defined the fluid equation of state parameter for the viscous dark matter fluid, ω_v, as

$$\omega_v \equiv \frac{p_v}{\rho_v} = -\frac{3H\xi}{\rho_v}.$$

Shear viscosity

Introducing shear

- It will be useful to define dimensionless bulk and shear viscosities,

$$\tilde{\xi} = 24\pi G\xi/H_0, \quad \tilde{\eta} = 24\pi G\eta/H_0.$$

- We will also assume a general form for the viscous coefficients such that,,

$$\xi \equiv (\Omega_v/\Omega_{v0})^{\nu}\,\xi_0,$$
$$\eta \equiv (\Omega_v/\Omega_{v0})^{\lambda}\,\eta_0,$$

where the exponents ν and λ are real numbers, while ξ_0 and η_0 are constant parameters.

Perturbative dynamics

The metric

- The line element for scalar perturbations in an homogeneous and isotropic flat Universe is

$$ds^2 = a^2(\tau)\left[-(1+2\phi)d\tau^2 + (1-2\psi)\delta_{ij}dx^i dx^j\right],$$

where τ is the conformal time and ϕ and ψ are the metric perturbations, which are in general equal in the absence of anisotropic stresses, e.g., shear viscosity, but for dissipative processes, as it will be considered here, they are independent functions.

Perturbative dynamics

Perturbed equations

- The perturbed $(0,0)$-component of the field equations, in momentum space, are,

$$-k^2\psi - 3\mathcal{H}\left(\psi' + \mathcal{H}\phi\right) = \frac{3}{2}\Omega_v \mathcal{H}_0^2 a^2 \Delta.$$

- The $(0,i)$-component is

$$-k^2\left(\psi' + \mathcal{H}\phi\right) = \frac{3}{2}\mathcal{H}_0^2 \Omega_v \left(1 + w_v\right) a\theta.$$

  The symbol "$'$" corresponds to a derivative with respect to the conformal time and k is the (comoving) momentum.
- The following definitions were used:

$$\Delta = \delta\rho/\rho, \quad \mathcal{H} = \frac{a'}{a}.$$

Perturbative dynamics

Perturbed equations

- Finally, the evolution of the potentials ψ and ϕ are encoded in the $i-j$ component of the Einstein equation,

$$\left[\psi'' + \mathcal{H}\left(2\psi + \phi\right)' + \left(2\mathcal{H}' + \mathcal{H}^2\right)\phi + \frac{1}{2}\nabla\left(\phi - \psi\right)\right]\delta^i_j - \frac{1}{2}\partial_i\partial_j\left(\phi - \psi\right) = 4\pi G a^2 \delta T^i_j$$

where

$$\begin{aligned} \delta T^i_j &= \delta p \delta^i_j - \xi\left(\delta u^m_{,m} - \frac{3\mathcal{H}}{a}\phi - \frac{3\psi'}{a}\right)\delta^i_j - \left(\delta\xi\right)\frac{3\mathcal{H}}{a}\delta^i_j \\ &- \eta g^{ik}\delta^l_j\left(\delta u_{k,l} + \delta u_{l,k} - \frac{2}{3}a^2\delta u^m_{,m}\delta_{kl}\right). \end{aligned}$$

Perturbative dynamics

Perturbed equations

- For $i \neq j$ case of the above equation we find

$$-\frac{k^2}{2}(\phi - \psi) = \frac{3\mathcal{H}^2}{\rho}\eta\,\theta.$$

- One notices that $\phi \neq \psi$ if $\eta \neq 0$.
- This property is also verified for modified gravity ($f(R)$ for example).

Perturbative dynamics

Quasi-static approximation

- We are interested in the evolution of small scale (inside the Hubble radius) perturbations along the matter dominated period.
- In such situation, the Newtonian potentials ϕ and ψ almost do not vary in time.
- quasi-static approximation for these quantities is a good approximation.

Perturbative dynamics

Quasi-static approximation

- The continuity equation can be written as

$$\Delta' - 3\mathcal{H}\omega_v\Delta + (1+2\omega_v)(a\theta) - \frac{9\mathcal{H}^2(\delta\xi)}{\rho a} \approx 0.$$

- The (0-0) component of the Einstein's equation becomes

$$-k^2\psi \approx \frac{3}{2}\Omega_v\mathcal{H}_0^2 a^2\Delta.$$

Perturbative dynamics

Quasi-static approximation

- By combining the previous equation, we find,

$$(a\theta)' + \left[\mathcal{H}\left(1 - 3w_v\right) + \frac{w_v'}{1 + w_v} + \frac{k^2\left(\tilde{\xi} + \frac{4}{3}\tilde{\eta}\right)}{a\rho(1 + w_v)}\right](a\theta)$$

$$+ \frac{k^2 w_v \psi'}{\mathcal{H}(1 + w_v)} - \frac{k^2\phi}{1 + w_v} - \frac{w_v k^2(\delta\xi)}{\xi(1 + w_v)} = 0.$$

Perturbative dynamics

Quasi-static approximation

- Moreover,

$$\frac{a}{H}\frac{dH}{da} = -\frac{3}{2}(1+\omega_v)\Omega_v\frac{H_0^2}{H^2},$$

and

$$a\frac{d\omega_v}{da} = 3\omega_v(1+\omega_v)\left(1-\nu-\frac{\Omega_v}{2}\frac{H_0^2}{H^2}\right).$$

Perturbative dynamics

Quasi-static approximation

- In the quasi-static approximation, it is possible to have a unique equation for the density contrast:

$$a^2\frac{d^2\Delta}{da^2} + \left(3 - \frac{3}{2}\Omega_v\frac{H_0^2}{H^2} + A + k^2 B\right) a\frac{d\Delta}{da} + \left(C + k^2 D\right)\Delta = 0.$$

Perturbative dynamics

Quasi-static approximation

- The factors A, B, C and D appearing in the above equation are defined as

$$
\begin{aligned}
A &= \frac{3\omega_v}{1+2\omega_v}\Big[2\nu(1+\omega_v)+3 \\
&+ 4\omega_v - \omega_v\Omega_v\frac{H_0^2}{H^2}\Big] - \frac{2\omega_v}{1+\omega_v}\frac{R}{\Omega_v}, \\
B &= -\frac{w_v(1+\frac{4}{3}R)}{3H^2a^2(1+w_v)},
\end{aligned}
$$

Perturbative dynamics

Quasi-static approximation

$$
\begin{aligned}
C &= -\frac{3(1-\nu)\omega_v}{1+2\omega_v}\left[-3\nu(1-2\omega_v^2)+7\omega_v+5\right]+\frac{2(1-3\nu)\omega_v^2}{1+\omega_v}R \\
&+ \frac{3\Omega_v}{2}\frac{H_0^2}{H^2}\left[3(1-\nu)\frac{\omega_v(4\omega_v^2+5\omega_v+2)}{1+2\omega_v}-\frac{1+2w_v}{1+w_v}\right], \\
D &= \frac{w_v^2(1+\frac{4}{3}R)}{H^2a^2(1+w_v)}(1-\nu)+\frac{\nu\omega_v\,(1+2w_v)}{1+w_v}\left(\frac{\Omega_v}{\Omega_{v0}}\right)^{\nu}, \\
R &\equiv \tilde{\eta}/\tilde{\xi}.
\end{aligned}
$$

Perturbative dynamics

Quasi-static approximation

- In the absence of bulk viscosity, $\omega_v \to 0$, $\omega_v R \to -\tilde{\eta} H/(3H_0\Omega_v)$ and the above expressions for the factors A, B, C and D reduce to

$$
\begin{aligned}
A &= \frac{2\tilde{\eta}}{3\Omega_v^2}\frac{H}{H_0}, \\
B &= \frac{4\tilde{\eta}}{27a^2\Omega_v H H_0}, \\
C &= 0, \quad D = 0,
\end{aligned}
$$

and we can see explicitly how the differential equation for the density contrast depends on the shear.

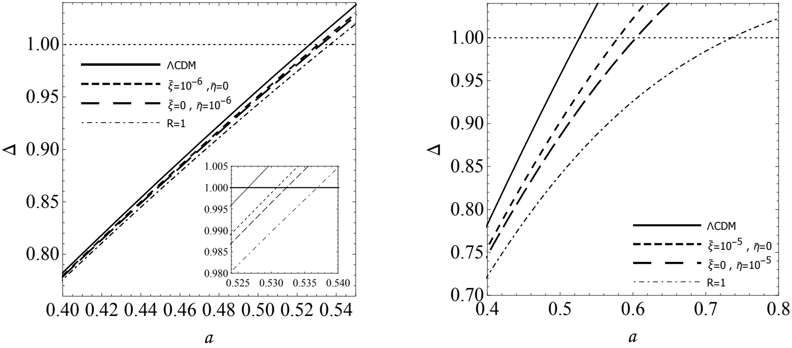
Perturbative dynamics

Linear growth of viscous dark matter halos

- we show the results for the linear evolution of the density contrast Δ considering the scale $k = 0.2h\,Mpc^{-1}$, which corresponds to the scale for typical galaxy clusters.
- According to the ΛCDM standard cosmology, such objects became nonlinear, i.e., the density contrast approaches $\Delta \sim 1$ at recent times, when the scale factor is $a_{nl} \sim 0.5$, or equivalently, at redshift $z_{nl} \sim 1$.
- In all of our results the initial conditions are taken at the matter-radiation equality and they are set with the help of the CAMB code.

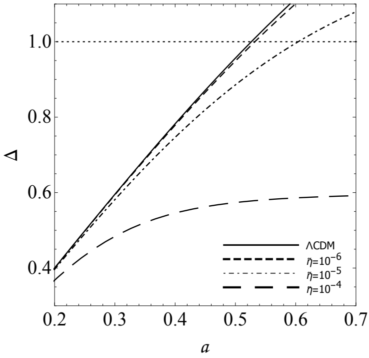
Perturbative dynamics

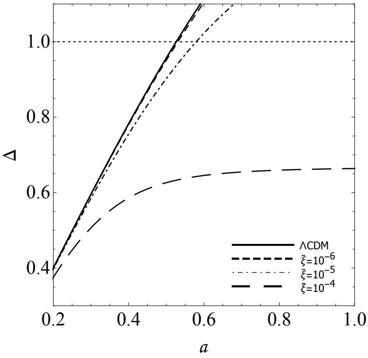
Linear growth of viscous dark matter halos



Perturbative dynamics

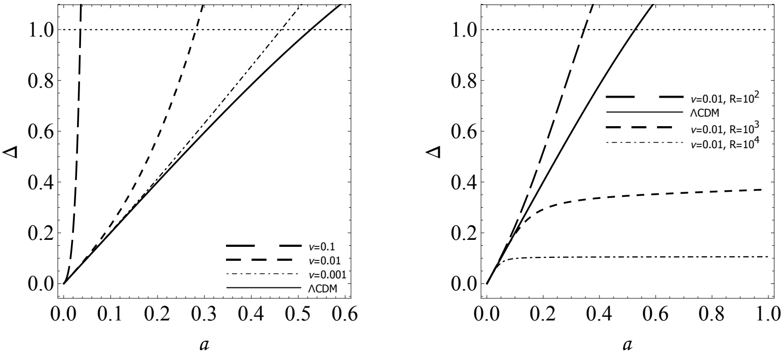
Linear growth of viscous dark matter halos





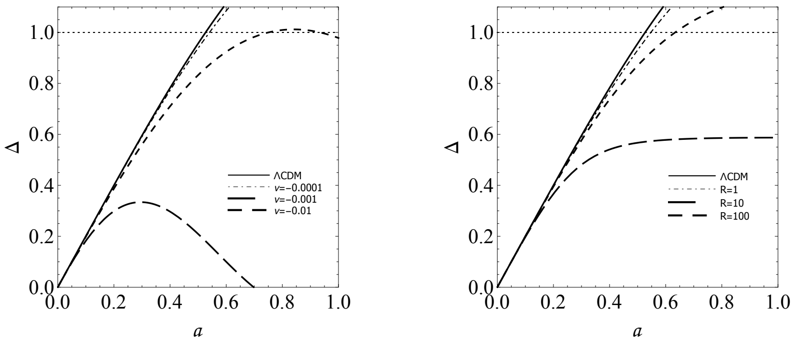
Perturbative dynamics

Linear growth of viscous dark matter halos



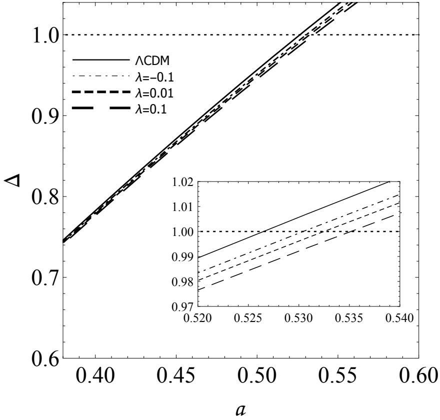
Perturbative dynamics

Linear growth of viscous dark matter halos



Perturbative dynamics

Linear growth of viscous dark matter halos



Perturbative dynamics

Constraints from redshift space distortions

- The features presented by dissipative effects on the linear perturbation theory can also be studied via the growth rate of matter fluctuations data.
- Observational projects have inferred from large scale clustering the redshift-space-distortion based $f(z)\sigma_8(z)$ at different redshifts.

Perturbative dynamics

Constraints from redshift space distortions

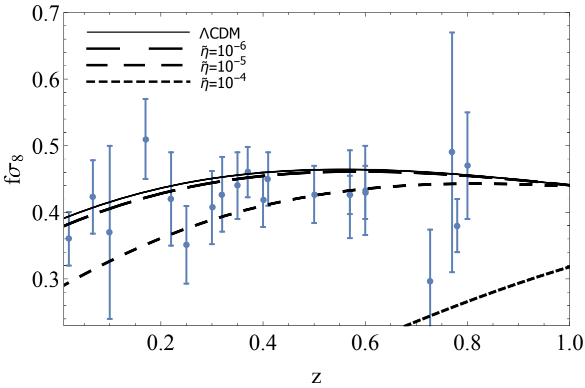
- The redshift space distortions observable combines the linear growth rate f,

$$D(a) = \frac{\Delta(a)}{\Delta(a_0)} \qquad \Rightarrow \qquad f(a) \equiv \frac{d \ln D(a)}{d \ln a},$$

with the variance σ^2 of the density field smoothed on $8h^{-1}$ Mpc scales.

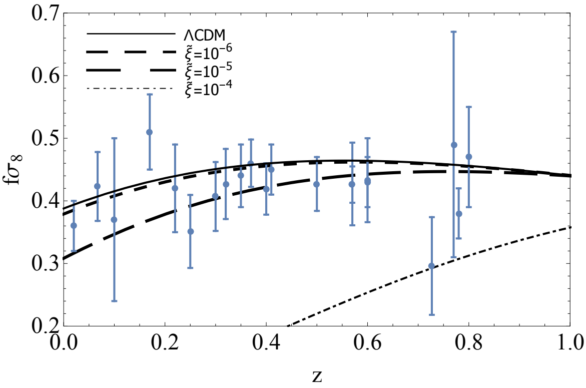
Perturbative dynamics

Constraints from redshift space distortions



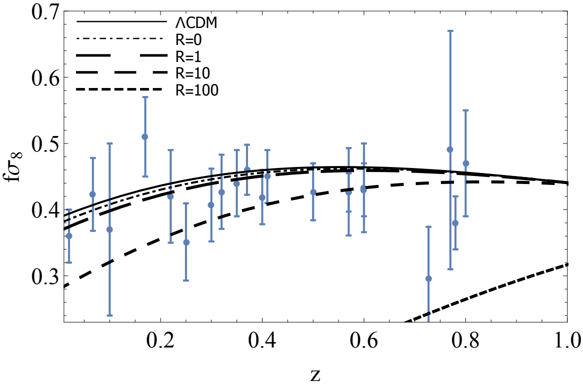
Perturbative dynamics

Constraints from redshift space distortions



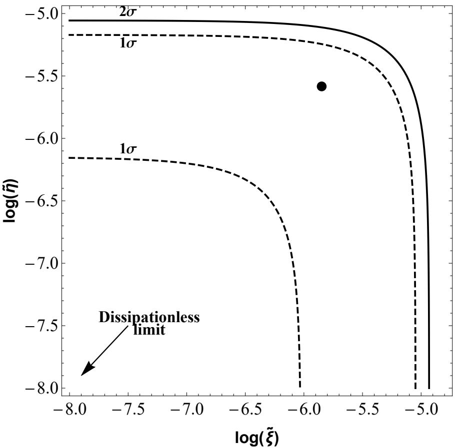
Perturbative dynamics

Constraints from redshift space distortions



Perturbative dynamics

Constraints from redshift space distortions



Conclusions

- Is the ΛCDM simply a fitting model for the dark sector of the universe?
- In any case, the anomalies at non-linear level remain an enormous challenge.
- Among the possibilities to cure these anomalies, a more realistic model for dark matter is one to be considered.
- It implies, among other aspects, to take into account dissipative effects.
- In general, good results if only dark matter is a viscous fluid, with $\tilde{\xi}_0 \stackrel{\sim}{<} 10^{-6}$. This is similar to warm dark matter with a particle mass $\stackrel{\sim}{<} 1\,keV$.
- Is there any degeneracy with modified gravity?