Effective action and very early quantum Universe

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Introduction

Effective action – state of art

a) Quantum effective action – methods for local and nonlocal effects
b) Heat kernel method and gradient and curvature expansion
c) Coleman-Weinberg potential, conformal anomaly, nonlocal anomaly action, etc.

Effective action and inflation

a) Slow roll expansion
b) Higgs inflation and Starobinsky R^2 models
c) Higgs mass and CMB
d) Quantum mechanism of hill-shape inflaton potential

Effective action and quantum cosmology

a) Cosmological wavefunction and effective action
b) Microcanonical density matrix in CFT driven cosmology:
-- subplanckian energy scale and suppression of no-boundary states
-- new paradigm of inflationary scenario -- ‘’hill-top” inflation
c) Higher spin conformal fields, hierarchy problem and stabilization of graviton loops

Introduction: what is ''effective'' action?

Quantization vs "modification":

$$f(R)$$

local models

$$Rf\Big(-\frac{1}{\Box}R\Big)$$

nonlocal ''cosmology''

$$R+\alpha\, R^{\mu\nu}\frac{1}{\Box+\hat{P}}G_{\mu\nu}$$

$$R_{\mu\nu}\frac{M_P^2(\Box)}{2\,\Box}\Big(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\Big)$$

"filter function" degravitation models

$$\frac{1}{G(\Box)}=m_P^2\Big(1+\frac{1}{r_c\sqrt{-\Box}}\Big)$$

brane induced gravity models

Galileon models avoiding ghosts

....

Effective action – state of the art

Generating functional of the one-particle irreducible Green's functions

$$e^{-\Gamma[\phi]/\hbar} = \int D\varphi \, \exp\frac{1}{\hbar}\left\{-S[\varphi] + \int dx \left(\varphi(x) - \phi(x)\right)\frac{\delta\Gamma[\phi]}{\delta\phi(x)}\right\}$$

quantum field ($\varphi(x)$) — *mean field* ($\phi(x)$)

Physical setup:

Mean field (on shell)

$$\phi_0 = \langle \varphi \rangle \equiv \frac{\int D\varphi \, \varphi \, e^{-S[\varphi]/\hbar}}{\int D\varphi \, e^{-S[\varphi]/\hbar}}$$

Effective equation

$$\frac{\delta\Gamma[\phi_0]}{\delta\phi_0} = 0 \quad + \text{ physical boundary conditions}$$

Correlation function

$$\langle \zeta(x_1),,,\zeta(x_n) \rangle \equiv \frac{\int D\varphi \, \zeta(x_1),,,\zeta(x_n) \, e^{-S[\varphi]}}{\int D\varphi \, e^{-S[\varphi]/\hbar}}$$

Quantum fluctuation

$$\zeta(x) = \varphi(x) - \phi_0(x)$$

Loop expansion

$$\Gamma[\phi] = S[\phi] + \hbar\,\Gamma_{1-\text{loop}}[\phi] + \hbar^2 \Gamma_{2-\text{loop}}[\phi] + \ldots$$

One-loop order

$$\Gamma_{1-\text{loop}} = \frac{1}{2}\,\ln \mathrm{Det}\, F(\nabla) = \frac{1}{2}\,\mathrm{Tr}\,\ln F(\nabla)$$

$$F(\nabla)\,\delta(x,y) = \frac{\delta^2 S[\phi]}{\delta\phi(x)\,\delta\phi(y)}$$

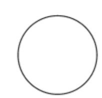
inverse propagator

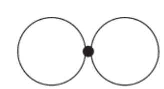
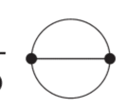
Two-loop order:

$$\Gamma_{2-\text{loop}} = \frac{1}{8}\int dx_1 dx_2 dx_3 dx_4\, G(x_1,x_2)\,\frac{\delta^4 S[\phi]}{\delta\phi(x_1)\delta\phi(x_2)\delta\phi(x_3)\delta\phi(x_4)}\,G(x_3,x_4)$$
$$+\frac{1}{12}\int dx_1 dx_2 dx_3\, dy_1 dy_2 dy_3\,\frac{\delta^3 S[\phi]}{\delta\phi(x_1)\delta\phi(x_2)\delta\phi(x_3)}$$
$$\times\, G(x_1,y_1)\,G(x_2,y_2)\,G(x_3,y_3)\,\frac{\delta^3 S[\phi]}{\delta\phi(y_1)\delta\phi(y_2)\delta\phi(y_3)},$$

$$F(\nabla)G(x,y) = -\delta(x,y)$$

Feynman diagrams in an external mean field **(Euclidean** **spacetime, Wick rotation –** ***IN-IN formalism later*****)**

$$\Gamma_{1-\text{loop}} = \frac{1}{2}$$ 

$$\Gamma_{2-\text{loop}} = \frac{1}{8}$$  $$+\ \frac{1}{12}$$ 

Heat kernel method

Heat kernel (proper time) representation

$$G \equiv -F^{-1}(\nabla) = \int_0^\infty ds\, K(s),$$

$$\frac{1}{2}\,\mathrm{Tr}\,\ln F(\nabla) = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\,\mathrm{Tr}\,K(s) = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\int dx\,\mathrm{tr}\,\hat{K}(s|\,x,x)$$

Heat equation

$$\frac{\partial}{\partial s}\hat{K}(s\,|\,x,y) = \hat{F}(\nabla)\,\hat{K}(s\,|\,x,y), \qquad \hat{K}(0\,|\,x,y) = \hat{1}\,\delta(x,y)$$

Scale, gap, mass parameter:

$$\hat{F}(\nabla) = \Box + \hat{P} - \frac{\hat{1}}{6}R - M^2\hat{1}, \qquad \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$$

Local curvature and gradient expansion

$$\mathit{\Gamma}_{\text{one-loop}} = \mathit{\Gamma}_{\text{div}} + \mathit{\Gamma}_{\log} - \frac{1}{2}\left(\frac{M^2}{4\pi}\right)^{d/2} \sum_{n=d/2+1}^{\infty} \frac{\Gamma(n-\frac{d}{2})}{(M^2)^n} \int dx\, g^{1/2} \mathrm{tr}\, \hat{a}_n(x,x)$$

$$\mathit{\Gamma}_{\text{div}} = \frac{1}{2(4\pi)^{d/2}} \sum_{n=0}^{d/2} \left[-\frac{1}{\frac{d}{2}-\omega} - \psi\left(\frac{d}{2}-n+1\right)\right] \frac{(-M^2)^{\frac{d}{2}-n}}{(\frac{d}{2}-n)!} \int dx\, g^{1/2} \mathrm{tr}\, \hat{a}_n(x,x),$$

$$\mathit{\Gamma}_{\log} = \frac{1}{2(4\pi)^{d/2}} \sum_{n=0}^{d/2} \frac{(-M^2)^{\frac{d}{2}-n}}{\left(\frac{d}{2}-n\right)!} \ln\frac{M^2}{\mu^2} \int dx\, g^{1/2} \mathrm{tr}\, \hat{a}_n(x,x)$$

Schwinger – DeWitt (Gilkey-Seely) coefficients:

$$\hat{a}_n(x,y) = a^{A}_{n\;B}(x,y), \qquad \mathrm{tr}\, \hat{a}_n(x,x) \equiv a^{A}_{n\;A}(x,x)$$

Lowest order coefficients

Reminder:

$$\hat{F}(\nabla) = \Box + \hat{P} - \frac{\hat{1}}{6} R - M^2 \hat{1}$$

$$\mathcal{R}_{\mu\nu} = [\nabla_\mu, \nabla_\nu]$$

$$\hat{a}_0(x,x) = \hat{1}$$

$$\hat{a}_1(x,x) = \hat{P}$$

$$\hat{a}_2(x,x) = \frac{1}{180}\left(R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - R_{\mu\nu}R^{\mu\nu}\right)\hat{1} + \frac{1}{12}\hat{\mathcal{R}}_{\mu\nu}\hat{\mathcal{R}}^{\mu\nu} + \frac{1}{2}\hat{P}^2 + \frac{1}{6}\Box\hat{P} + \frac{1}{180}\Box R\hat{1}$$

Coleman-Weinberg potential:

$$\Gamma_{\rm CW} = \int d^4x \, \frac{M^4(\varphi)}{64\pi^2} \ln \frac{M^2(\varphi)}{\mu^2}$$

$$M^2(\varphi) = \frac{d^2V(\varphi)}{d\varphi^2} \sim \hat{P}$$

Massless theories ⇨ nonlocal curvature expansion

A.B.& G.A.Vilkovisky

Special case of nonlocality in massless conformal theories:

$$d = 4$$

Conformal anomaly of massless fields conformally coupled to gravity

$$\langle T^\mu_\mu \rangle = \frac{2}{g^{1/2}} g_{\mu\nu} \frac{\delta \Gamma}{\delta g_{\mu\nu}}$$

$$= -\frac{1}{16\pi^2} \mathrm{tr}\, \hat{a}_2(x,x) = \frac{1}{32\pi^2} \Big(\alpha \Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta} \Big)$$

Gauss-Bonnet term (E) — Weyl term ($C^2_{\mu\nu\alpha\beta}$)

Riegert, Fradkin,Tseytlin Antoniadis,Mottola, Mazur

Nonlocal anomaly action

$$\Gamma_A[g] = \frac{1}{16(4\pi)^2} \int d^4x g^{1/2} \Big[\frac{1}{2} \gamma C^2_{\mu\nu\alpha\beta} + \frac{\beta}{2} (E - \frac{2}{3} \Box R) \Big] \frac{1}{\mathcal{D}} (E - \frac{2}{3} \Box R) \sim \int d^4x g^{1/2} R^2 .. \frac{1}{\Box^2} R^2 ..$$

$$\mathcal{D} = \Box^2 + 2R^{\mu\nu} \nabla_\mu \nabla_\nu - \frac{2}{3} R \Box + \frac{1}{3} (\nabla^\mu R) \nabla_\mu$$

Nonlocal -- strong in the infrared

Treatment of nonlocality in effective equations: Schwinger-Keldysh technique vs Euclidean QFT

$$\langle IN \,|\, \varphi(x) \,|\, IN \rangle \neq \langle OUT \,|\, \varphi(x) \,|\, IN \rangle = \phi_0$$

No **Wick rotation, *no* effective action for effective equations**

Wick rotation to EQFT, effective action and effective equations $\dfrac{\delta \Gamma}{\delta \phi_0(x)} = J(x)$

$$\Gamma[\,\phi\,] = \frac{1}{2} \int dx\, dy\, \phi(x)\, G(x,y)\, \phi(y) + \ldots$$

$$\frac{\delta \Gamma}{\delta \phi(x)} \propto \int dy \left[G(x,y) + G(y,x) \right] \phi(y) + \ldots$$

not causal: $\neq 0$ for $y^0 > x^0$

Physical observables are *IN-IN expectation values* $\langle IN \,|\, \hat{\mathcal{O}}(x) \,|\, IN \rangle$

Subject to Schwinger-Keldysh diagrammatic technique

$$\frac{\delta \langle IN \,|\, \hat{\mathcal{O}}(x) \,|\, IN \rangle}{\delta J(y)} = 0, \quad x^0 < y^0$$

Non-manifest corollary of locality and unitarity --- via a set of cancellations between nonlocal terms with chronological and anti-chronological boundary conditions

Euclidean version of Schwinger-Keldysh technique

IN-IN mean field $\phi(x) = \langle\, IN \,|\, \varphi(x) \,|\, IN \,\rangle$

Quantum effective action of Euclidean QFT (nonlocal) $\Gamma = \Gamma_{Euclidean}[\,\phi\,]$

IN state is the ***Poincare invariant vacuum*** associated with the past asymptotically flat infinity

Effective equations for *in-in* field

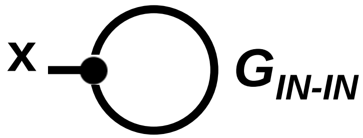
A.B. & G.A.Vilkovisky (1987)

$$\left.\frac{\delta \Gamma_{Euclidean}}{\delta \phi(x)}\right|_{++++\;\Rightarrow\;-+++}^{retarded} = 0.$$

Causal, diffeomorphism and gauge invariant !

One-loop effective equations for IN-IN mean field:

In-in Wightman Green's function in Poincare-invariant vacuum in asymptotically flat (AF) spacetime



$$G_{IN\text{–}IN}(x,y) = \langle IN \,|\, \varphi(x)\,\varphi(y) \,|\, IN \rangle$$

$$\underbrace{\frac{\delta S}{\delta \phi(x)} + \frac{i}{2}\int dy\, dz\, S_3(x,y,z)\, G_{IN\text{–}IN}(y,z)}$$

=

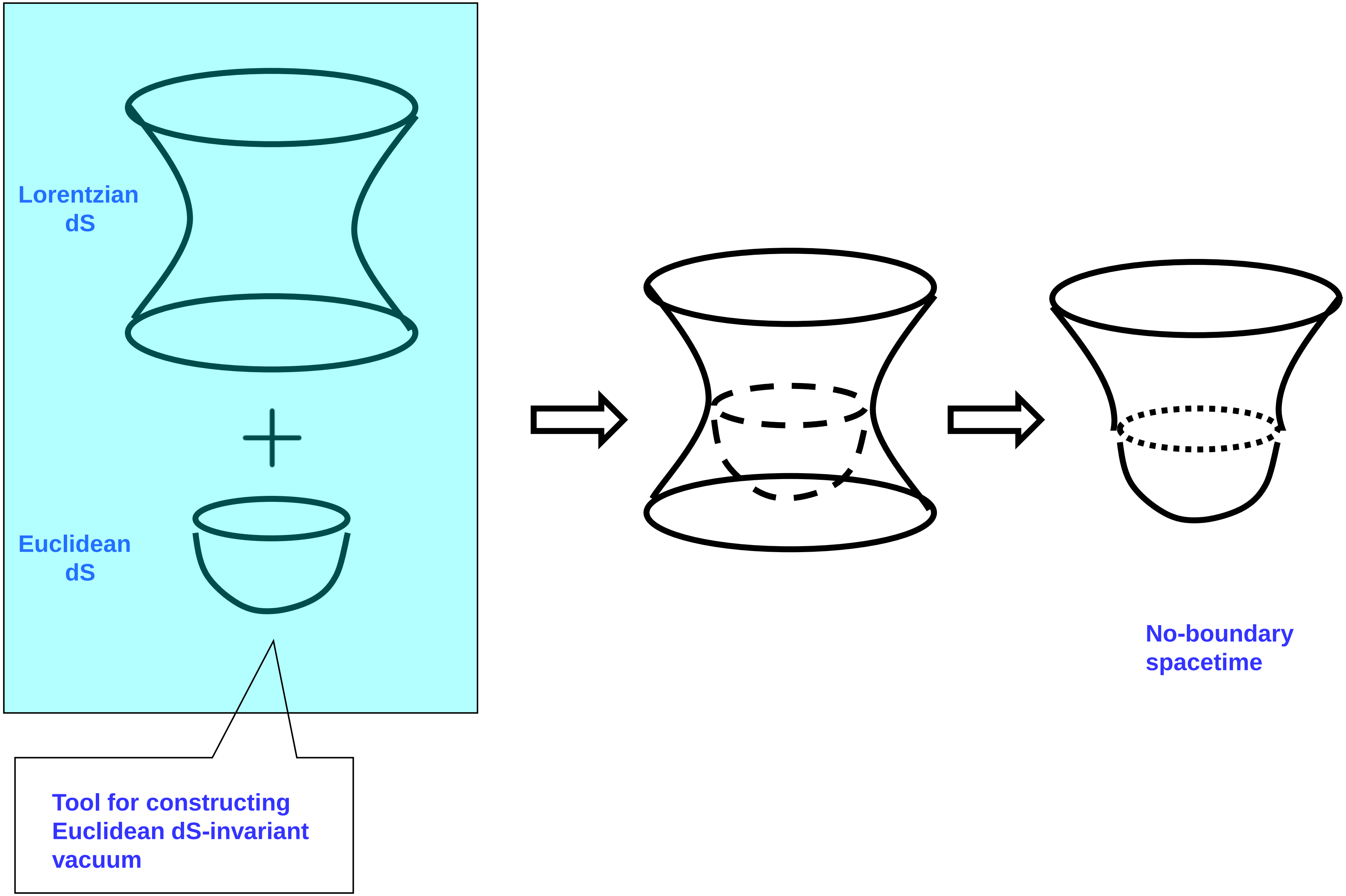
$$\left.\frac{\delta \Gamma^{1-loop}_{Euclidean}}{\delta \phi(x)}\right|^{retarded}_{++++ \Rightarrow -+++}$$

?

AF spacetime / **Poincare vacuum** ⇒ **(A)dS spacetime** / **Euclidean vacuum**

Higuchi, Marolf, Morrison (2011)

Korai, Tanaka (2012)


Lorentzian dS
Euclidean dS
Tool for constructing Euclidean dS-invariant vacuum
No-boundary spacetime

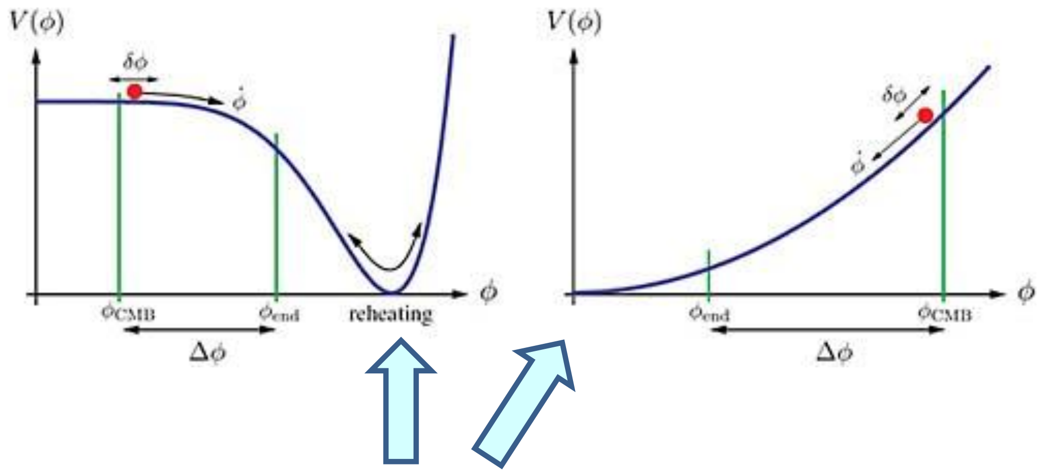
Effective action and inflation

$$\boldsymbol{L}(g_{\mu\nu},\phi)=\frac{1}{2}M_P^2R-\frac{1}{2}\nabla\phi^2-V(\phi)$$

Homogeneity, Friedmann metric, scale and Hubble factors $\quad a=a(t),\qquad H=\frac{\dot a}{a}$

$$\ddot\phi+3H\dot\phi+V'(\phi)=0,\qquad H^2=\frac{V(\phi)}{3M_P^2}$$

Slow roll approximation: $\quad \dot\phi\ll H\phi,\ \ H(\phi)\sim\text{const},\ \ a(t)\sim e^{Ht}$



End of inflation, inflaton oscillations $\rightarrow$ reheating of the matter $\rightarrow$ standard model of Big Bang
Shapes of potential -- types of inflation: “new” inflation, “chaotic” inflation, etc.

Higgs inflation model:

inflaton

non-minimal curvature coupling

$$L(g_{\mu\nu}, \Phi) = \frac{1}{2}\left(M_P^2 + \xi|\Phi|^2\right) R - \frac{1}{2}|\nabla\Phi|^2 - \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$$

inflaton-graviton sector of gravitating SM

$$\varphi^2 \equiv |\Phi|^2 = \Phi^\dagger\Phi$$

Starobinsky model of R^2 gravity:

$$S_\xi^{\rm Star}[\,g_{\mu\nu}\,] = \int d^4x\, g^{1/2}\left\{-\frac{M_P^2}{2}R - \frac{\xi}{4}R^2\right\}$$

$$\Longleftrightarrow$$

$$S_\xi^{\rm Star}[\,g_{\mu\nu}, \varphi\,] = \int d^4x\, g^{1/2}\left\{\frac{M_P^2 + \xi\varphi^2}{2}R - \frac{\xi\varphi^4}{4}\right\}$$

Non-minimal curvature coupling

$$\xi \gg 1, \qquad \frac{\Delta T}{T} \sim \frac{\sqrt{\lambda}}{\xi} \sim 10^{-5}$$

B. Spokoiny (1984),
D.Salopek, J.Bond & J. Bardeen (1989),
R. Fakir& W. Unruh (1990),
A.Barvinsky & A. Kamenshchik (1994, 1998)

F.Bezrukov & M.Shaposhnikov Phys.Lett. 659B (2008) 703:

Transcending the idea of non-minimal inflation to the Standard Model ground: Higgs boson as an inflaton – no new physics between TeV and inflation.

A.O.B, A.Kamenshchik, C.Kiefer, A.Starobinsky and C.Steinwachs (2008-2009):

Non-minimally gravitating SM can be probed beyond tree level by current and future CMB observations and LHC experiments. On account of *RG running* with the Higgs mass

$$M_H \simeq 136 \text{ GeV}$$

the SM Higgs can drive inflation with the observable CMB spectral index $n_s \simeq 0.96$ and a very low T/S ratio $r \simeq 0.0004$.

Bezrukov & Shaposhnikov (2009):

2-loop RG contribution leads to $M_{Higgs} \to M_{LHC} = 125$ GeV

Slow roll inflation is an ideal playground for the gradient and curvature expansion

$$M^2_{\text{eff}}(\varphi) = M_P^2 + \xi\varphi^2 \gg M_P^2 \quad \Longrightarrow \quad \text{expansion in } \frac{1}{M^2_{\text{eff}}(\varphi)}$$

$$S[g_{\mu\nu},\varphi] = \int d^4x\, g^{1/2}\left(-V(\varphi) + U(\varphi)\, R(g_{\mu\nu}) - \frac{1}{2} G(\varphi)\, (\nabla\varphi)^2 + \ldots\right)$$

One-loop coefficient Functions for large $\xi \gg 1$

$$\begin{cases} V(\varphi) = \dfrac{\lambda}{4}(\varphi^2 - v^2)^2 + \dfrac{\lambda\varphi^4}{128\pi^2}\boldsymbol{A}\ \ln\dfrac{\varphi^2}{\mu^2} \\ U(\varphi) = \dfrac{1}{2}(M_P^2 + \xi\varphi^2) + \dfrac{3\xi\lambda\varphi^2}{32\pi^2}\ \ln\dfrac{\varphi^2}{\mu^2} \\ G(\varphi) = 1 + \ldots \end{cases}$$

Anomalous scaling behavior constant A

Overall Coleman-Weinberg potential:

$$V(\varphi) = \sum_{\text{particles}} \frac{\pm m^4(\varphi)}{64\pi^2} \ln \frac{m^2(\varphi)}{\mu^2} = \frac{\lambda \boldsymbol{A}}{128\pi^2} \varphi^4 \ln \frac{\varphi^2}{\mu^2} + \ldots$$

Masses in terms of SU(2),U(1) and top-quark Yukawa constants

$$m_W^2 = \frac{1}{4} g^2 \varphi^2, \quad m_Z^2 = \frac{1}{4} (g^2 + g'^2) \varphi^2, \quad m_{top}^2 = \frac{1}{2} y_{top}^2 \varphi^2$$

$$\boldsymbol{A} = \frac{3}{8\lambda} \Big(2g^4 + \left(g^2 + g'^2\right)^2 - 16 y_{top}^4 \Big) + 6\lambda$$

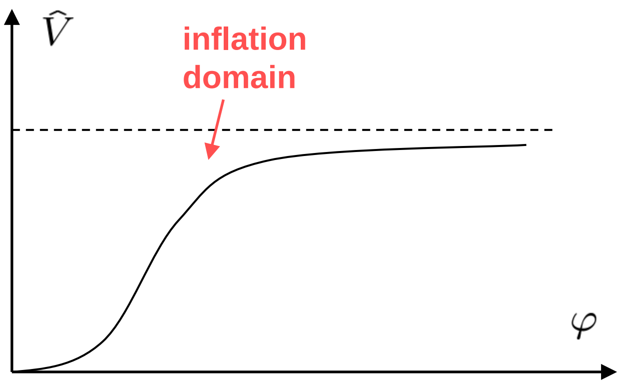
Transition to the Einstein frame -- Einstein frame potential:

$$(g_{\mu\nu}, \varphi) \to (\hat{g}_{\mu\nu}, \phi): \quad U \to \hat{U} \equiv \frac{M_P^2}{2}, \; G \to \hat{G} \equiv 1, \; V \to \hat{V}(\phi):$$

$$\hat{g}_{\mu\nu} = \frac{2U(\varphi)}{M_P^2} g_{\mu\nu}, \quad \left(\frac{d\phi}{d\varphi}\right)^2 = \frac{M_P^2}{2} \frac{GU + 3U'^2}{U^2}.$$

$$\hat{V}(\phi) = \left(\frac{M_P^2}{2}\right)^2 \frac{V(\varphi)}{U^2(\varphi)}\Bigg|_{\varphi=\varphi(\phi)}$$

$$\hat{V} = \left(\frac{M_P^2}{2}\right)^2 \frac{V(\varphi)}{U^2(\varphi)} \simeq \frac{\lambda M_P^4}{4\,\xi^2}\left(1 - \frac{2M_P^2}{\xi\varphi^2} + \frac{\boldsymbol{A_I}}{16\pi^2} \ln\frac{\varphi}{\mu}\right)$$



Inflationary anomalous scaling

$$\boldsymbol{A_I} = \boldsymbol{A} - 12\lambda = \frac{3}{8\lambda}\left(2g^4 + \left(g^2 + g'^2\right)^2 - 16y_{top}^4\right) - 6\lambda$$

Slow-roll smallness parameters

$$\hat{\varepsilon} \equiv \frac{M_P^2}{2}\left(\frac{1}{\hat{V}}\frac{d\hat{V}}{d\phi}\right)^2 = \frac{4M_P^4}{3\xi^2\varphi^4}\left(1-\frac{\varphi^2}{\varphi_0^2}\right)^2, \quad \hat{\eta} \equiv \frac{M_P^2}{\hat{V}}\frac{d^2\hat{V}}{d\phi^2} = -\frac{4M_P^2}{3\xi\varphi^2}$$

$$\varphi_0^2 = -\frac{64\pi^2 M_P^2}{\xi \boldsymbol{A_I}}$$

Effective equations in *Einstein frame* with the CW potential:

$$\ddot{\phi} + 3H\dot{\phi} + \hat{V}'(\phi) = 0, \quad H^2 = \frac{\hat{V}(\phi)}{3M_P^2}$$

⇩

end of inflation

e-folding #
$$N(\phi) = \int_{\phi}^{\phi_{\rm end}} d\phi' \frac{H(\phi')}{\dot{\phi}'} \simeq \frac{48\pi^2}{\boldsymbol{A_I}} \ln\left(1-\frac{\varphi^2}{\varphi_0^2}\right)$$

horizon crossing – formation of perturbation of wavelength related to *N*: $N(k) \simeq \ln(T_0/k)$

Inflationary CMB parameters

amplitude

$$\zeta^2(k) = \frac{N^2(k)}{72\pi^2}\frac{\lambda}{\xi^2}\left(\frac{e^x-1}{x\,e^x}\right)^2 \simeq 2.5\times 10^{-9}$$

WMAP normalization at
$k \simeq (500\,\mathrm{Mpc})^{-1}$
$N \simeq 60$

spectral index

$$n_s = 1 - \frac{2}{N}\frac{x}{e^x-1} \qquad 0.94 < n_s < 0.99$$

WMAP+BAO+SN
at 2σ

T/S ratio

$$r = \frac{12}{N^2}\left(\frac{xe^x}{e^x-1}\right)^2$$

quantum (1-loop) factors are in red with $x \equiv \frac{N\boldsymbol{A_I}}{48\pi^2}$

⇓ **CMB compatible range of the Higgs mass**

$$135.6\ \mathrm{GeV} \lesssim M_H \lesssim 184.5\ \mathrm{GeV}$$

A.O.B, A.Kamenshchik, C.Kiefer,A.Starobinsky and C.Steinwachs (2008-2009):

RG improvement

$$\lambda, \xi, g, g', y_{top}, A_I \rightarrow \lambda(t), \xi(t), g(t), g'(t), y_{top}(t), A_I(t)$$

running scale:

$$t = \ln(\varphi/M_t)$$

top quark mass

anomalous scaling

RG equations:

$$\frac{d}{dt}\left(\frac{\lambda}{\xi^2}\right) = \frac{A_I}{16\pi^2}\frac{\lambda}{\xi^2}$$

Bezrukov,Shaposhnikov 2008 (Einstein frame calculations),

one-loop RG

$$n_s \simeq 0.96$$

prediction of the theory

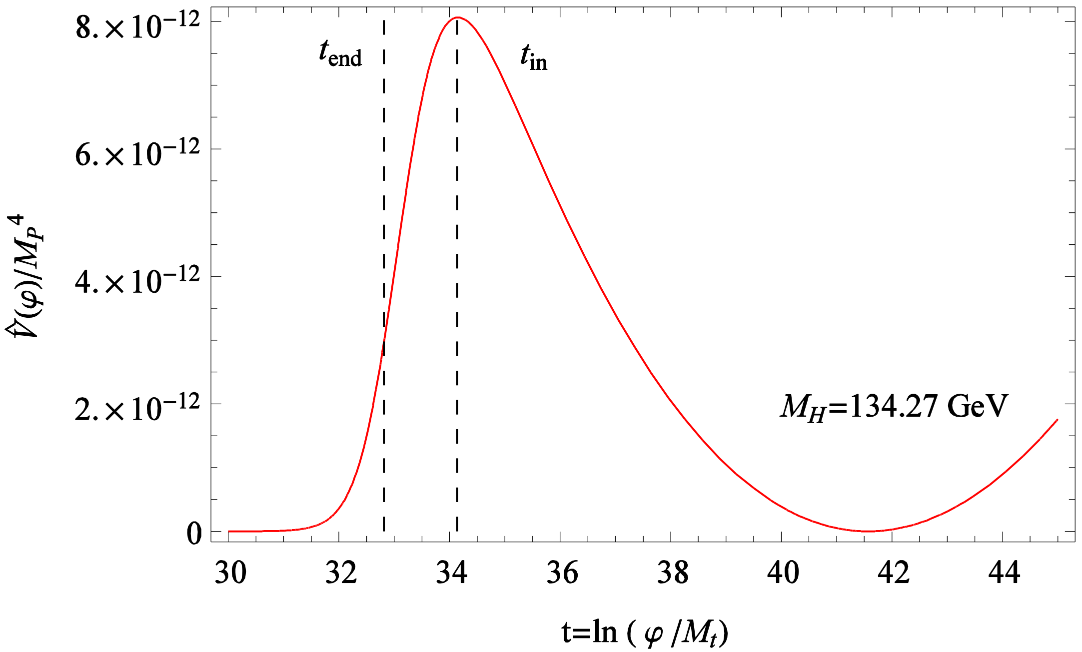
$$M_{\mathrm{Higgs}} \simeq 136 \text{ GeV}$$

A.Kamenshchik, A.Starobinsky & A.B (2008)
A.Kamenshchik, C.Kiefer, A.Starobinsky, C.Steinwachs & A.B (2009

2-loop RG : $M_{Higgs} \rightarrow M_{LHC}$ **= 125 GeV**

Bezrukov & Shaposhnikov, 2009

Resummation of the effective potential by renormalization group



One-loop RG improved effective potential. Inflationary domain for N=60 CMB perturbation is marked by dashed lines

A.Kamenshchik, C.Kiefer, A.Starobinsky, C.Steinwachs, & A.B., JCAP 12 (2009) 003, arXiv:0904.1698 – 1-loop RG approximation

Formation of ‘’*hill shape*’’ potential --- nonperturbative (?) effect of transition from the *Jordan* to *Einstein* frame:

Not in Einstein frame, IR instability and breakdown of grad. expansion!

non-minimal coupling

$$\Gamma[g_{\mu\nu},\varphi]=\int d^4x\, g^{1/2}\Big(V(\varphi)-U(\varphi)\,R(g_{\mu\nu})+\frac{1}{2}G(\varphi)\,(\nabla\varphi)^2+\ldots\Big)$$

$$V(\varphi)=\frac{\lambda}{4}\varphi^4+O\Big(\varphi^4\ln\frac{\varphi^2}{\mu^2}\Big),\qquad U(\varphi)=\frac{M_P^2+\xi\varphi^2}{2}+O\Big(\varphi^2\ln\frac{\varphi^2}{\mu^2}\Big),$$

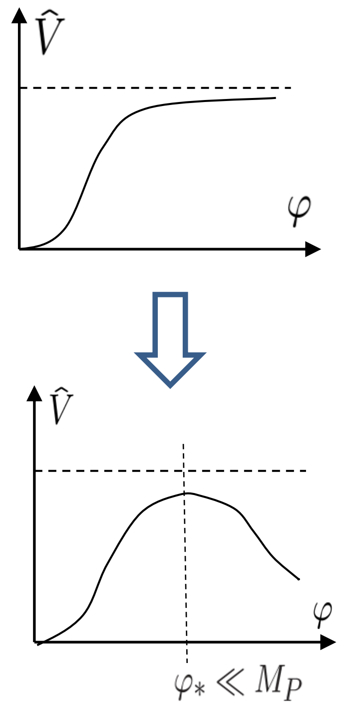
$$G(\varphi)=1+O\Big(\ln\frac{\varphi^2}{\mu^2}\Big)$$

Transition to the *Einstein* frame:

$$V(\varphi)\to\hat V(\phi)=\frac{M_P^4}{4}\frac{V(\varphi)}{U^2(\varphi)}\sim\frac{M_P^4}{\lambda\xi^2}\frac{\boldsymbol{A}\ln\frac{\varphi}{\mu}}{\ln^2\frac{\varphi}{\mu}}\sim\frac{1}{\ln\frac{\varphi}{\mu}}\to 0,\qquad \varphi\to\infty$$

Any *l*-th loop order:

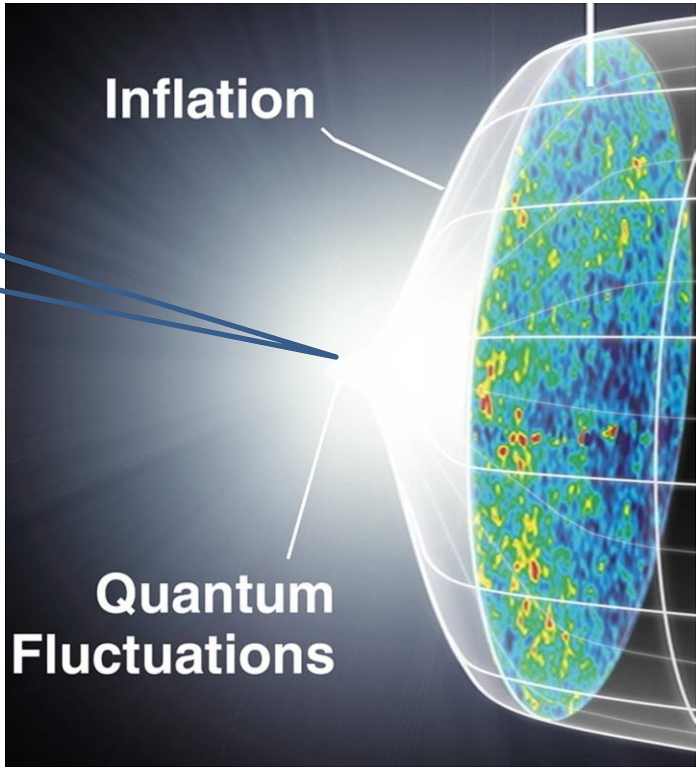
$$\frac{\ln^l\frac{\varphi}{\mu}}{\ln^{2l}\frac{\varphi}{\mu}}\sim\frac{1}{\ln^l\frac{\varphi}{\mu}}\to 0,\qquad \varphi\to\infty$$



Effective action and quantum cosmology

Current status of quantum cosmology: the Wheeler-DeWitt equation is the ''most useless'' equation in theoretical physics?

What was at the beginning?



The space and time had both one beginning . The space was made not in time but simultaneously with time.

Saint Augustin of Hippo

Origin of effective action in quantum cosmology: no-boundary (HH) and tunneling (T) states

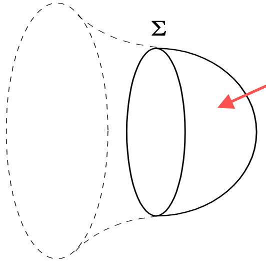
$$\boldsymbol{\Psi}_{HH,T}\big(\varphi,\Phi(\mathbf{x})\big) = \exp\left(\mp\frac{1}{2}S_E(\varphi)\right)\,\Psi_{\text{matter}}\big(\varphi,\Phi(\mathbf{x})\big)$$

inflaton (→ φ) other fields (→ $\Phi(\mathbf{x})$)

Euclidean action of quasi-de Sitter instanton with the effective Λ (slow roll): $\Lambda \simeq \dfrac{V(\varphi)}{M_P^2}$

$$S_E(\varphi) \simeq -\frac{24\pi^2 M_{\rm P}^4}{V(\varphi)} < 0$$

Euclidean spacetime



no tunneling, really: "birth from nothing"

Euclidean FRW

Analytic continuation
-- Lorentzian signature
dS geometry:

$$a(\tau) = \frac{1}{H}\sin(H\tau),\quad H = \sqrt{\frac{\Lambda}{3}}$$

⇒

$$\tau = \pi/2H + it$$
$$a_L(t) = \frac{1}{H}\cosh(Ht)$$

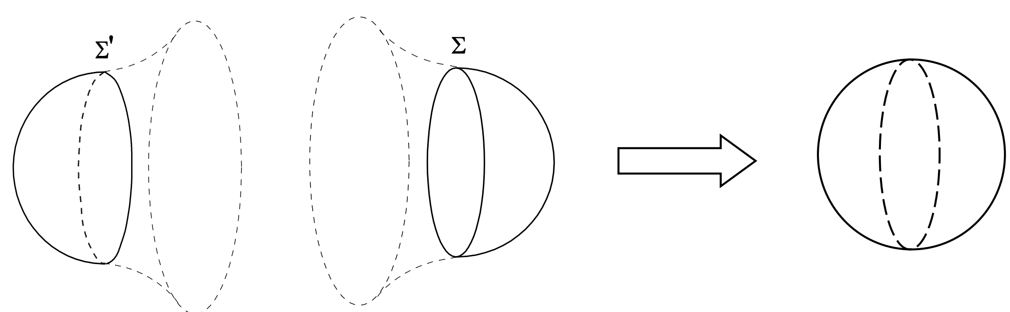
$\Psi_{\text{matter}}\big(\varphi,\Phi(\mathbf{x})\big)$ -- de Sitter invariant (Euclidean) vacuum of "other" fields

$$\rho_{HH,T}(\varphi) = \int d[\Phi(\mathbf{x})] \left| \boldsymbol{\Psi}_{HH,T}\big(\varphi, \Phi(\mathbf{x})\big) \right|^2$$

$$= \exp\left(-\Gamma_{1-\mathrm{loop}}(\varphi)\right) = \int D[\Phi(\tau,\mathbf{x})]\, e^{-S_E[\Phi(\tau,\mathbf{x})]}$$

Eucidean effective action on S^4

$$\Gamma_{1-\mathrm{loop}}(\varphi) = \frac{1}{2}\,\mathrm{Tr}\,\ln \frac{\delta^2 S_E[\Phi]}{\delta\Phi(x)\delta\Phi(y)}\Big|_{\Lambda(\varphi)}$$



Path integral in quantum cosmology

Physical states: $\hat{H}_\mu|\Psi\rangle = 0 \quad \hat{H}_\mu \equiv \underbrace{\hat{H}_\perp(\mathbf{x}),\ \hat{H}_i(\mathbf{x})}$

operators of the Wheeler-DeWitt equations

$\mu = (\perp\mathbf{x},\, i\mathbf{x}),\ i = 1,2,3$

$\mathbf{x}$ – spatial coordinates

$$\hat{H}_\mu \Psi(\,^3g) = 0 \quad \begin{cases} \Psi(\,^3g) = \int D[\,^4g]\, e^{iS[\,^4g]} \\ \Psi(\,^3g) = \int D[\,^4g]\, e^{-S_E[\,^4g]} \end{cases}$$

Leutwyler (1964)
A.B (1986)

Hartle
& Hawking (1983-1984)

Microcanonical ensemble in cosmology

$$|\Psi\rangle \to \hat{\rho}, \qquad \hat{H}_\mu \hat{\rho} = 0$$

Microcanonical density matrix – projector onto subspace of quantum gravitational constraints

$$\hat{\rho} = e^{\Gamma} \prod_\mu \delta(\hat{H}_\mu)$$

$$e^{-\Gamma} = \mathrm{Tr} \prod_\mu \delta(\hat{H}_\mu)$$

A.B., Phys. Rev. Lett. 99, 071301 (2007)

Motivation: aesthetic (minimum of assumptions – Occam razor)

A simple analogy -- a system with a conserved Hamiltonian in the microcanonical state of a fixed energy *E*

$$\hat{\rho} \sim \delta(\hat{H} - E) \quad \Rightarrow \quad \hat{\rho} \sim \prod_{\mu} \delta(\hat{H}_{\mu})$$

Spatially closed cosmology does not have *freely specifiable* constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_{μ}, all having a particular value --- zero

$$\hat{\rho} = \sum_{\text{all } |\boldsymbol{\Psi}\rangle} |\, \boldsymbol{\Psi} \,\rangle\langle\, \boldsymbol{\Psi} \,|$$

sum over "everything" that satisfies the Wheeler-DeWitt equation

An ultimate equipartition in the full set of states of the theory --- "*Sum over Everything*". Creation of the Universe from *Everything* is conceptually more appealing than creation from *Nothing*, because the democracy of the microcanonical equipartition better fits the principle of the Occam razor than the selection of a concrete state.

EQG path integral representation of the statistical sum: time arises as an operator ordering parameter

cf. Saint Augustine of Hippo

BFV/BRST method
A.B. JHEP 1310 (2013) 051,
arXiv:1308.3270

↓

$$e^{-\Gamma} \equiv \mathrm{Tr} \prod_{\mu} \delta(\hat{H}_\mu) = \int_{\mathrm{periodic}} D[\,g_{\mu\nu},\phi\,]\, e^{-S[\,g_{\mu\nu},\phi\,]}$$

↑

Euclidean metric

↓

$$-i\infty < N < i\infty, \quad g^{44} = +N^2$$

EQG density matrix
D.Page (1986)

important for convexity of the Euclidean action at saddle points – provides ``conformal" rotation

Lorentzian signature path integral
=
EQG path integral with integration over the *imaginary* lapse

$$ds^2_{\mathrm{Euclidean}} = N^2_{\mathrm{Euclidean}} dt^2 + g_{ab}(dx^a + N^a dt)\,(dx^b + N^b dt),$$

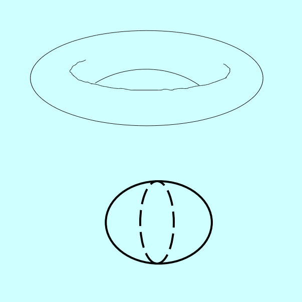
$$N_{\mathrm{Euclidean}} = i N_{\mathrm{Lorentzian}}$$

equivalent to
$x^0 \to -ix^4$

$$e^{-\Gamma} = \int_{\text{periodic}} D[\, g_{\mu\nu}, \Phi\,]\; e^{-S[\, g_{\mu\nu},\Phi\,]}$$

on $S^3 \times S^1$ (thermal)

including as a limiting (vacuum) case S^4



Application to CFT driven cosmology -- Universe dominated by quantum matter conformally coupled to gravity :

$$S[\, g_{\mu\nu}, \Phi\,] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}\,(R - 2\Lambda) + S_{CFT}[\, g_{\mu\nu}, \Phi\,]$$

Λ -- primordial cosmological constant

Omission of graviton loops

$$\Gamma[g_{\mu\nu}] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}(R - 2\Lambda) + \Gamma_{CFT}[g_{\mu\nu}],$$

$$e^{-\Gamma_{CFT}[\, g_{\mu\nu}]} = \int D\Phi\, e^{-S_{CFT}[\, g_{\mu\nu},\Phi\,]}$$

A.B. & A.Kamenshchik, JCAP, 09, 014 (2006) Phys. Rev. D74, 121502 (2006);

A.B., Phys. Rev. Lett. 99, 071301 (2007)

$$\Gamma_{CFT}[\, g_{\mu\nu}\,] = \ ?$$

Local conformal invariance of S_{CFT} $\rightarrow$

recovery of $\Gamma_{CFT}[\,g_{\mu\nu}^{FRW}\,] = \Gamma_{CFT}[\,a, N\,]$ on a generic FRW background by a conformal map onto static Einstein Universe:

i) contribution of the conformal anomaly associated with this map;
ii) contributions of the Casimir energy and free energy on a static periodically identified Einstein Universe

$$g_{\mu\nu}\frac{\delta \Gamma_{CFT}}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2} g^{1/2} \left(\alpha \Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

Gauss-Bonnet term

Weyl term

A.A.Starobinsky (1980);
Fischetty,Hartle,Hu;
Riegert; Tseytlin;
Antoniadis, Mazur, Mottola;
……
A.B. & A.Kamenshchik,
JCAP, 09, 014 (2006)
Phys. Rev. D74, 121502 (2006)

The coefficient of the topological Gauss-Bonnet term

$$\beta = \sum_s \beta_s \mathbb{N}_s, \qquad \mathbb{N}_s$$

-- number of fields of spin *s*,
β_s -- spin-dependent coefficients

⇩

Effective Friedmann equation for saddle points of the path integral:

$$\frac{\delta S_{\text{eff}}[\,a,N\,]}{\delta N(\tau)} = 0$$

anomaly contribution

$$\frac{1}{a^2} - \frac{\dot{a}^2}{a^2} - B\left(\frac{\dot{a}^4}{2a^4} - \frac{\dot{a}^2}{a^4}\right) = \frac{\Lambda}{3} + \frac{C}{a^4},$$

$$C = \frac{B}{2} + \frac{1}{6\pi^2 M_P^2}\frac{dF}{d\eta}$$

Casimir energy and radiation energy constant

$$F(\eta) = \sum_{\omega} \ln\left(1 \mp e^{-\omega\eta}\right)$$

$$\frac{dF}{d\eta} = \sum_{\omega} \frac{\omega}{e^{\omega\eta} \pm 1}$$

free energy and energy of CFT particles – sum over field oscillators with frequencies ω on S^3

$$\eta = \int_{S^1} \frac{d\tau N}{a}$$

Inverse temperature in units of conformal time period on S^1

$$B = \frac{\beta}{8\pi^2 M_P^2}$$

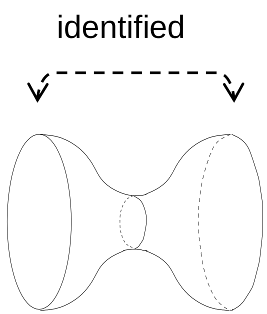
-- coefficient of the Gauss-Bonnet term in the conformal anomaly

FRW ansatz, conformal map onto static Einstein universe, recovery of the action from the conformal anomaly and RTF-AMM nonlocal action, solution of effective equations of motion

⇩

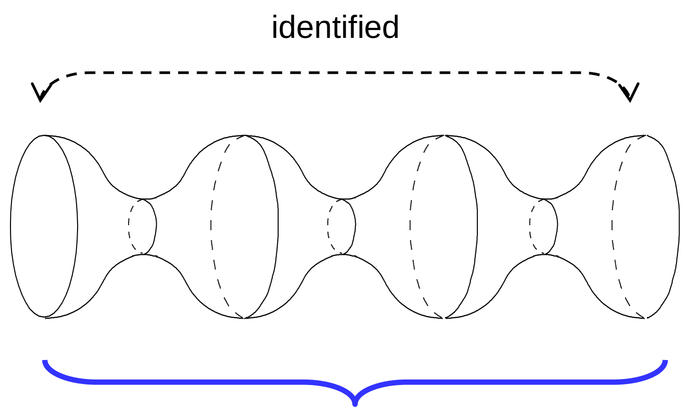
A.B. & A.Kamenshchik,
JCAP, 09, 014 (2006)
Phys. Rev. D74, 121502 (2006)

Saddle point solutions --- set of periodic (thermal) garland-type instantons with oscillating scale factor ($S^1 \times S^3$) and the vacuum Hartle-Hawking instantons (S^4)

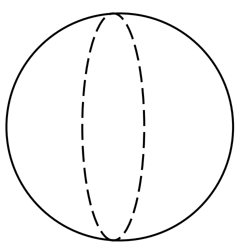


1- fold, *k*=1

, ...

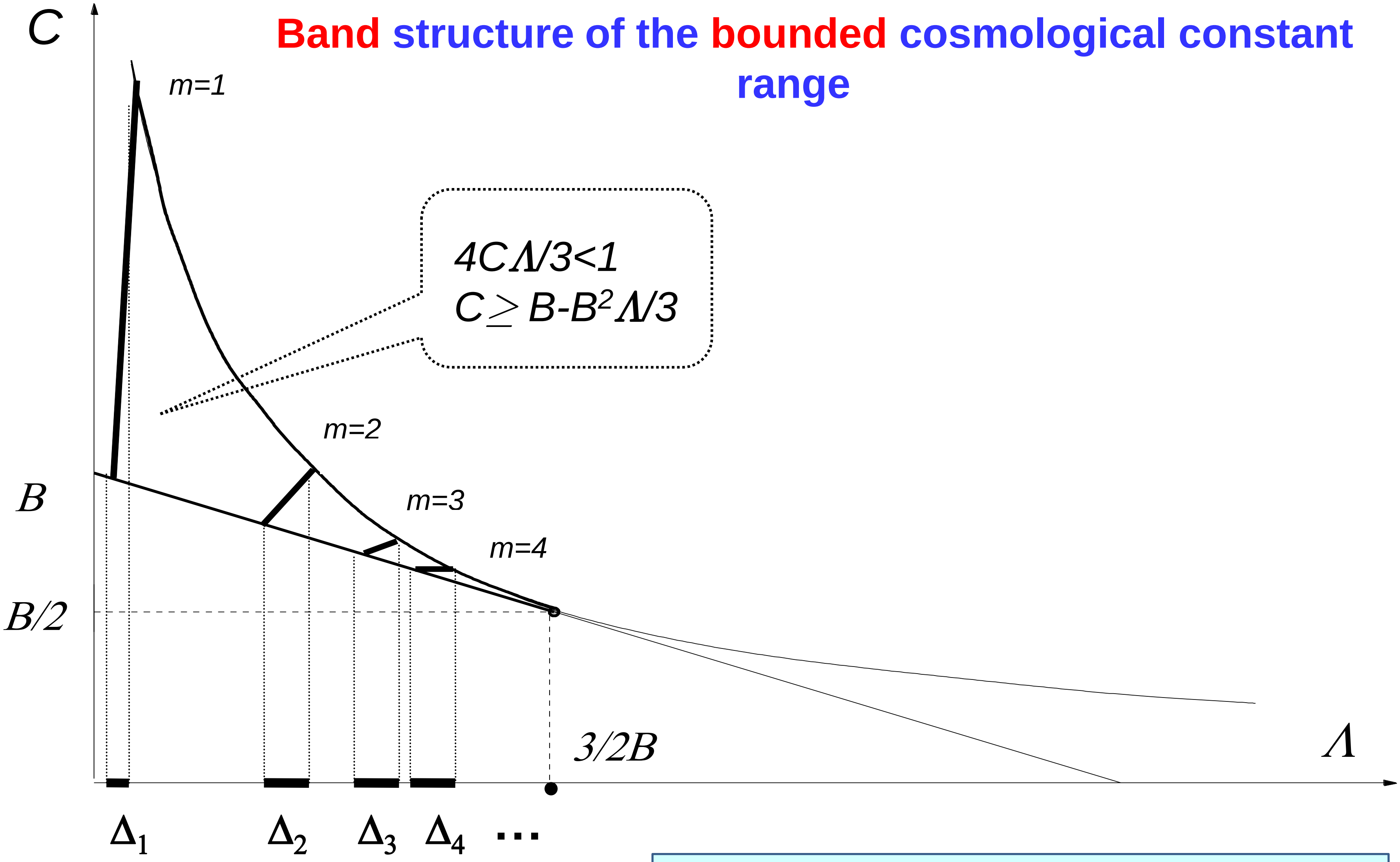


***k*- folded garland, *k*=1,2,3,…**



S^4

Band structure of the bounded cosmological constant range



$\Lambda_{min} < \Lambda < \Lambda_{max} = 12\pi^2 M^2_P/\beta$

$$\Lambda_{max} = \frac{12\pi^2 M_P^2}{\beta}$$

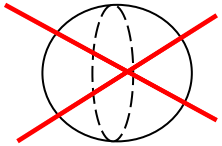
new quantum gravity scale

1) Limited range of Λ – subplanckian domain (limiting the string vacua landscape?):

$$\Lambda_{\rm min} \leq \Lambda \leq \Lambda_{\rm max} = \frac{12\pi^2 M_P^2}{\beta}$$

$$g_{\mu\nu}\frac{\delta\Gamma}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2} g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

2) No-boundary instantons S^4 are ruled out by ***infinite positive*** Euclidean action – elimination of infrared catastrophe



3) Generalization to inflationary model, $\Lambda \to V(\phi)$ – selection of inflaton potential $V(\phi)$ ***maxima*** (new type of hill-top inflation) – quantum origin of the Starobinsky model and Higgs inflation model at $V(\phi) \sim \Lambda_{max}$. Employs **the mechanism of hill shape** inflaton potential!

4) Thermal corrections to primordial power s

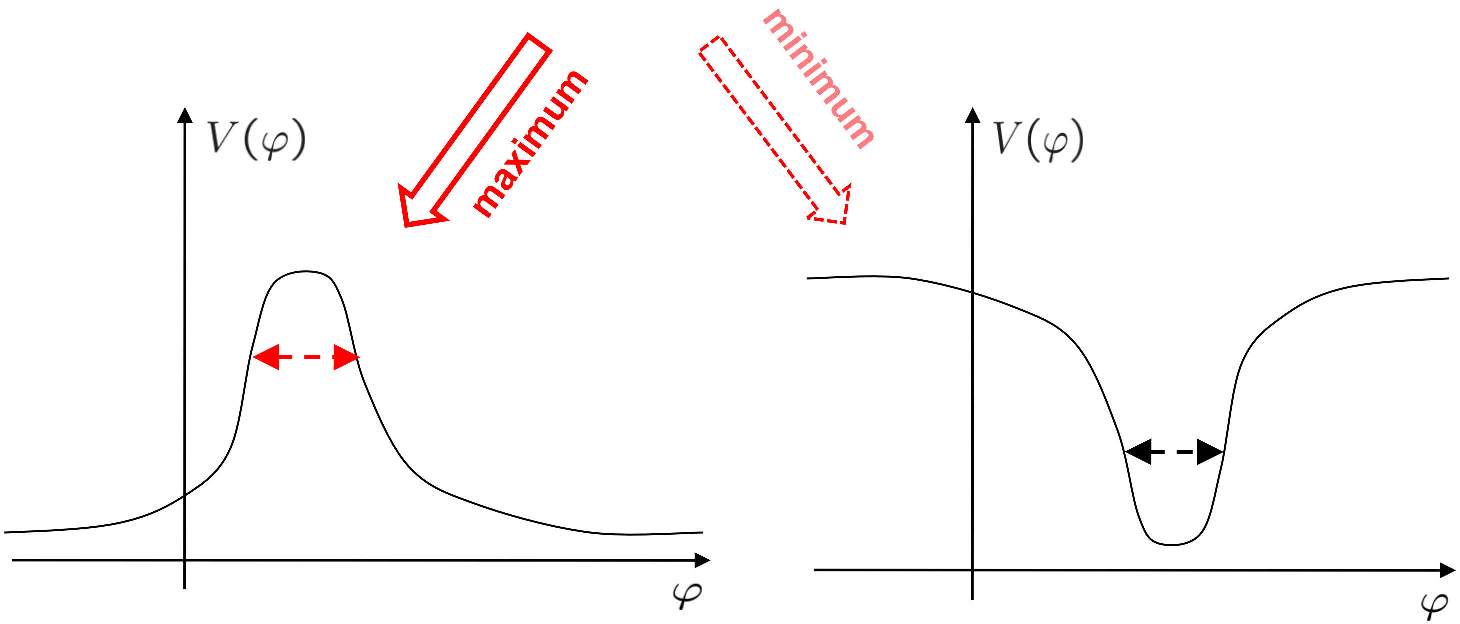
$$n_s(k) = n_s^{\rm vac}(k) + \Delta n_s^{\rm thermal}(k)$$ **additional red tilt** of the CMB spectrum

5) Hidden sector of conformal higher spin fields (CHS): solution of the hierarchy problem and stabilization of the theory against the inclusion of graviton loop corrections

Selection of inflaton potential *maxima* as initial conditions for inflation -- long standing problem of the no-boundary state

Critical point: $$\frac{d}{d\tau}a^3\dot{\phi} = a^3\,\frac{\partial V}{\partial \phi} \;\Rightarrow\; \oint d\tau\, a^3\,\frac{\partial V}{\partial \phi} = 0 \;\Rightarrow\; \frac{\partial V}{\partial \phi} \gtrless 0$$

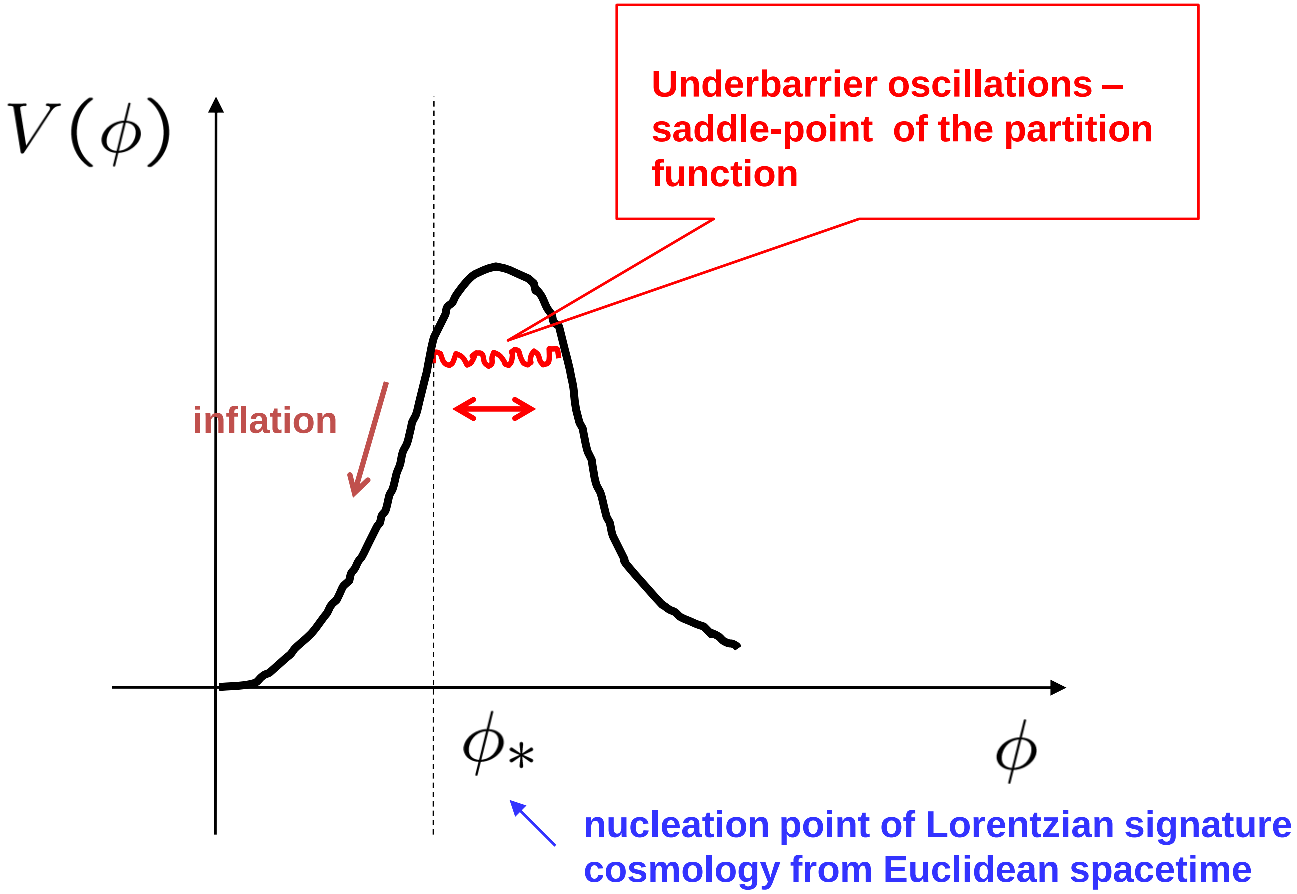
Potential extremum ``inside" instanton



classically forbidden (underbarrier) oscillation

classically allowed (overbarrier) oscillation --- ruled out because of *underbarrier* oscillations of scale factor

Hill-top inflation



Approximation of two coupled oscillators $\rightarrow$ slow roll parameters of inflation $\rightarrow$ parameters of the observable CMB characteristic of the Higgs inflation or Starobinsky models

Hierarchy problem

Starobinsky R^2-model and non-minimal Higgs inflation model at $V(\phi) \sim \Lambda_{max}$

$$10^{-11} M_P^4 \simeq V_{\text{inflation}} \sim \Lambda_{max} = \frac{12\pi^2}{\beta} M_P^4 \quad \Rightarrow \quad \boxed{\beta \simeq 10^{13}}$$

E-coefficient of total conformal anomaly

Impossible in Standard model with low spins ***s=0,1/2,1*** **and $N_s \sim 100$**

$$\beta = \frac{1}{180}\left(\mathbb{N}_0 + 11\mathbb{N}_{1/2} + 62\mathbb{N}_1\right)$$

Hidden sector of CHS fields: recent progress in HS field theory (Vasiliev) and CHS theory (Klebanov, Giombi, Tseytlin, etc) arXiv:1309.0785 – a-anomalies and #'s of polarizations

$$S_{CHS} = \int d^4x \left(h^{\mu_1 \ldots \mu_s} \Box^s h_{\mu_1 \ldots \mu_s} +,,,\right)$$

Vasiliev 1990, 1992, 2003

$$\beta_s = \frac{1}{360}\nu_s^2(3 + 14\nu_s), \quad \nu_s = s(s+1), \quad s = 1, 2, 3, \ldots$$

$$\beta_s = \frac{1}{720}\nu_s(12 + 45\nu_s + 14\nu_s^2), \quad \nu_s = -2\left(s + \frac{1}{2}\right)^2, \quad s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots.$$

Giombi, Klebanov, Pufu, Safdi, and Tarnopolsky 2013; Tseytlin 2013

A hidden sector of CHS fields up to $S \sim 100$ and # of polarizations $\sim 10^6$ solves the hierarchy problem

BUT !

Stability of quantum corrections and gravitational cutoff

Our inflation scale

Gravitational cutoff for $\mathbb{N} \gg 1$ quantum species (from smallness of graviton loops)

$$\Lambda_I = \frac{M_P}{\sqrt{\beta}} \ll M_P \quad \Leftrightarrow \quad \Lambda_{\rm cutoff} = \frac{M_P}{\sqrt{\mathbb{N}}}$$

Veneziano (2002); G.Dvali et al (2002);
G.Dvali and M.Redi (2008); G.Dvali (2010)

Critical feature of CHS fields providing smallness of graviton loop effects relative to quantum matter loops

$$\beta_s \sim s^6 \gg \mathbb{N} = \nu_s \sim s^2$$

$$\Downarrow$$

$$\Lambda_I \ll \Lambda_{\rm cutoff}$$

Justification of a special approximation scheme:
EFT for the nonrenormalizable graviton sector
and nonperturbative CHS matter sector

Conclusions

Effective action , effective equations and path integral in very early inflationary Universe and quantum cosmology

Unification of EW and cosmological energy scales (Higgs mass vs CMB data)

Microcanonical density matrix of the Universe – Sum over Everything

Application to the CFT driven cosmology with a large # of quantum species – a limited range of Λ -- elimination of IR dangerous no-boundary states

New initial conditions paradigm –hill-top inflation, mechanism of hill-shape potential, thermally corrected CMB spectrum -- cool Universe

Solution of hierarchy problem via CHS fields, stabilization of quantum corrections below the gravitational cutoff – origin of the Universe is the ‘’low energy” (subplanckian) phenomenon

SOME LIKE IT HOT

SOME LIKE IT COOL