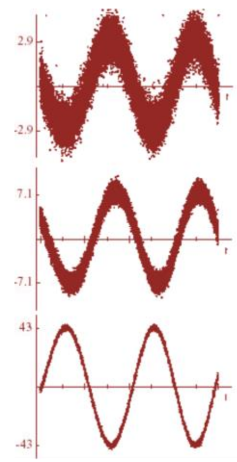
Quantum Vacuum and Gravitation: Testing General Relativity in Cosmology.
Mainz 13-24 March 2017

Cosmology with arbitrary-spin coherent fields



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In collaboration with J.A.R. Cembranos and S. J. Núñez Jareño

Introduction

Energy content

Particles

Radiation
Baryonic matter
Dark matter

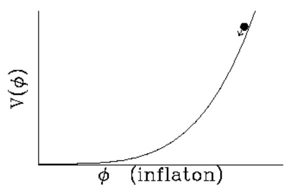
“Classical” fields

Inflation
Dark matter
Dark energy

Introduction

“Classical” scalar fields

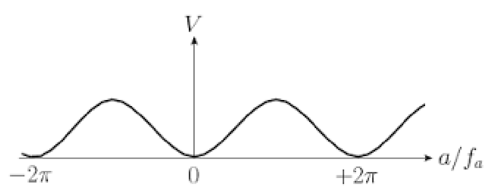
Inflation



Inflaton

Slow roll

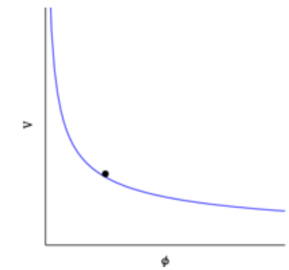
Dark matter



Axion

Fast oscillating

Dark energy



Quintessence

Slow roll

Introduction

- "Classical fields": classical limit of coherent states
  (large occupation numbers)

- Coherent field description: new features compared to the more standard particle description, e.g. in ultra-light DM models Heisenberg uncertainty principle could help solving the small-scale problems of CDM.

- Simplest models based on scalar fields, but in principle

any bosonic field could work

Introduction

Spin 1

Inflation

- **Vector inflation.**
  Ford, 1989.
  Golovnev, et al 2008
- **Gauge-flation.**
  Maleknejad, 2013
- **Chromonatural inflation.**
  Adshead, 2012

Slow evolution

Dark matter

- **Dark light.**
  Nelson et al, 2011

Fast oscillating

THIS TALK

Arbitrary spin

Perturbations

Dark energy

- **Vector dark energy (potential).**
  Armendáriz, 2004.
- **Einstein-Aether.**
  Zlosnik et al , 2007
- **Vector dark energy (kinetic)**
  Beltrán, A.L.M. 2008

Slow evolution

Outline

- Particle (Fock) states vs. coherent states
- A simple example of coherent field: axion
- Coherent scalar fields: wave(fuzzy) dark matter
- Extension to higher-spin fields
- Coherent vectors as ultra-light dark matter

Particle (Fock) states vs. coherent states

A simple example:

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\mathbf{p}}}} \left(a_{\mathbf{p}} e^{-ipx} + a^\dagger_{\mathbf{p}} e^{ipx}\right) \qquad E_{\mathbf{p}} = \mathbf{p}^2 + m^2$$

For every Fourier mode:

$$N_{\mathbf{p}}|n_{\mathbf{p}}\rangle = n_{\mathbf{p}}|n_{\mathbf{p}}\rangle$$

Fock states:

well-defined particle number

$$\langle n_{\mathbf{p}}|\phi(x)|n_{\mathbf{p}}\rangle = 0$$

$$z_{\mathbf{p}} = |z_{\mathbf{p}}|e^{i\theta_{\mathbf{p}}}$$

$$a_{\mathbf{p}}|z_{\mathbf{p}}\rangle = z_{\mathbf{p}}|z_{\mathbf{p}}\rangle$$

$$|z_{\mathbf{p}}\rangle = e^{\frac{-|z_{\mathbf{p}}|^2}{2}} \sum_{n_{\mathbf{p}}=0}^{\infty} \frac{z_{\mathbf{p}}^n}{\sqrt{n_{\mathbf{p}}!}}|n_{\mathbf{p}}\rangle$$

Coherent states:

not well-defined particle number

$$\langle z_{\mathbf{p}}|\phi(x)|z_{\mathbf{p}}\rangle = \frac{2|z_{\mathbf{p}}|}{(2\pi)^3\sqrt{2E_{\mathbf{p}}}}\cos(px - \theta_{\mathbf{p}})$$

Particle (Fock) states vs. coherent states

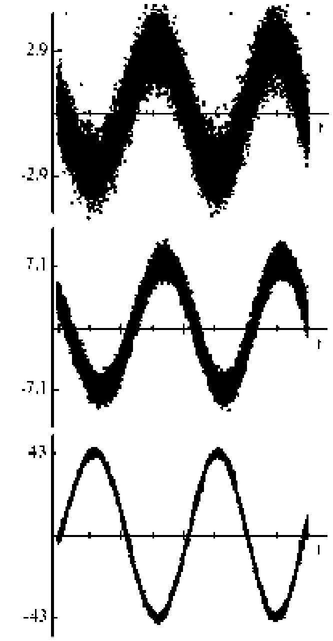
Coherent states

$$\langle z_{\mathbf{p}} | N_{\mathbf{p}} | z_{\mathbf{p}} \rangle = |z_{\mathbf{p}}|^2$$

$|z_{\mathbf{p}}|^2 \gg 1$ Large occupation numbers

$$\boxed{\frac{\left(\langle z_{\mathbf{p}} | \phi^2(x) | z_{\mathbf{p}} \rangle - \langle z_{\mathbf{p}} | \phi(x) | z_{\mathbf{p}} \rangle^2\right)^{1/2}}{\langle z_{\mathbf{p}} | \phi(x) | z_{\mathbf{p}} \rangle} \sim \frac{1}{|z_{\mathbf{p}}|} \to 0}$$

Classical limit



$|z_{\mathbf{p}}|$ increases

A simple example of coherent field: axions

Strong CP problem

$$S_\theta = \frac{\theta}{32\pi^2}\int d^4x\,\epsilon^{\mu\nu\lambda\rho}\mathrm{Tr}\,G_{\mu\nu}G_{\lambda\rho}$$

Induces an electric dipole moment for the neutron:

$$\bar{\theta} = \theta + \arg\det m_q \lesssim 10^{-10}$$

Solution: Introduce a global $U(1)_{PQ}$ symmetry which is spontaneosly broken at a f_a scale. Axion pseudo-NG boson.

$$S_a = \int d^4x\left(\frac{1}{2}(\partial_\mu a)^2 + \frac{a}{32\pi^2 f_a}\epsilon^{\mu\nu\lambda\rho}\mathrm{Tr}\,G_{\mu\nu}G_{\lambda\rho}\right)$$

Axions

Non-perturbative QCD effects gives a potential for the axion

$$V(a) = \Lambda_{QCD}^4 \left(1 - \cos\left(\frac{a}{f_a}\right)\right)$$

with mass

$$m_a \sim \frac{\Lambda_{QCD}^2}{f_a} \sim 6 \times 10^{-10}\mathrm{eV}\left(\frac{10^{16}\mathrm{GeV}}{f_a}\right)$$

For typical scales $f_a = 10^{12}$ GeV, $m_a = 10^{-5}$ eV

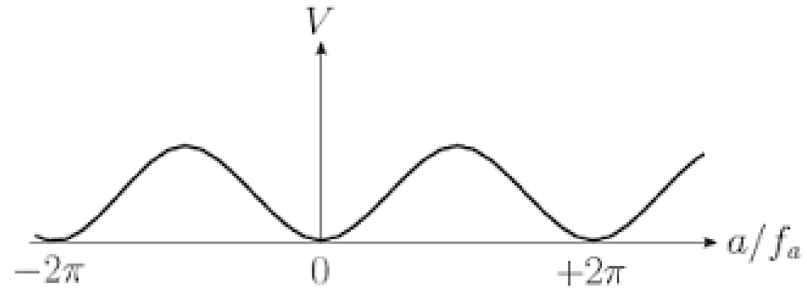
Minimizing the potential solves the problem $\theta_{eff} \equiv \frac{\langle a(x) \rangle}{f_a} + \bar{\theta} = 0$

Axions

Misalignment mechanism

$$\theta \equiv \frac{a(x)}{f_a}$$

Angular variable [-π,π]



Shift symmetry protects small axion mass

- Random initial value displaced from the origin

$$\ddot{\theta}_k + 3H\dot{\theta}_k + \frac{k^2}{a^2}\theta_k + m_a^2(T)\theta_k = 0$$

- Inflation suppresses the spatial derivatives. This means that when $m_a(T) \sim H(T)$ all the modes start oscillating coherently

$$\boxed{\rho_a \propto a^{-3}} \qquad m_a \gg H \qquad \textbf{DM behaviour}$$

Axions

Axion density $$\Omega_a \sim \left(\frac{f_a}{10^{11-12}\ \mathrm{GeV}}\right)^{7/6}$$

$$f_a < 3 \times 10^{11}\mathrm{GeV} \quad \text{or} \quad m_a > 2.1 \times 10^{-5}\mathrm{eV}.$$ Planck limit

Axion occupation number

$$n_{\mathrm{gal}} = \frac{\rho_{\mathrm{gal}}}{m} \approx \frac{\mathrm{GeV/cm^3}}{10^{-5}\ \mathrm{eV}} = \frac{10^{14}}{\mathrm{cm^3}}$$

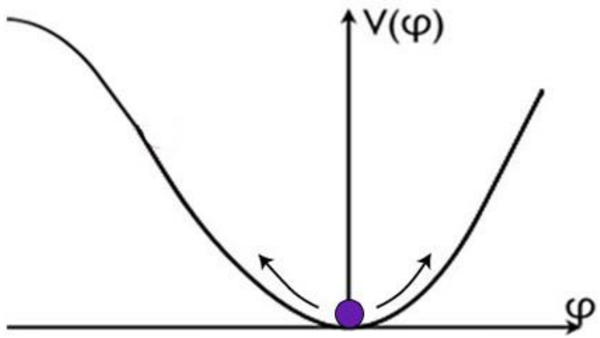
For virialized axions

$$\lambda_{dB} = \frac{2\pi}{mv} \approx \frac{2\pi}{10^{-5}\ \mathrm{eV} \times 10^{-3}} \approx 10^4\ \mathrm{cm}$$

$$\boxed{\mathcal{N} \sim n_{\mathrm{gal}} \lambda_{dB}^3 \approx 10^{26}}$$ >> 1 classical field

Coherent scalar fields

$$S = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$



$$\nu_{eff} \gg H$$

$$\boxed{V(\phi) = a\phi^n}$$

(Turner, 1983)

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0}$$

Homogeneous RW background

Coherent scalar fields

Virial theorem: average equation of state

$$G = \dot{\phi}\phi \qquad \boxed{\text{bounded}}$$

$$\dot{G} = \dot{\phi}^2 + \ddot{\phi}\phi = \dot{\phi}^2 - V'(\phi)\phi = \dot{\phi}^2 - nV(\phi)$$

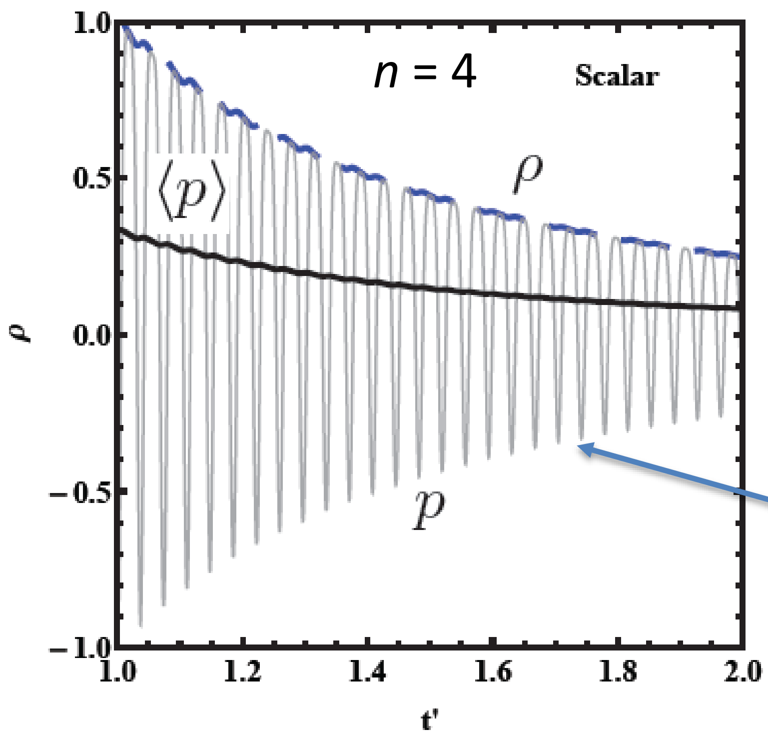
$$\langle \dot{G} \rangle = \frac{1}{T}\int_0^T \dot{G}dt = \frac{G(T) - G(0)}{T} \xrightarrow{T \gg \omega^{-1}} 0$$

$$\boxed{\begin{gathered}\langle \dot{\phi}^2 - nV(\phi) \rangle = 0 \\ \langle \rho + p - n(\rho - p) \rangle = 0\end{gathered}}$$

$$\boxed{\langle p \rangle = \frac{n-2}{n+2}\langle \rho \rangle}$$

Johnson and Kamionkowski (2008)

Coherent scalar fields



Average equation of state

Detectable in pulsar timing array
Khmelnitsky and Rubakov 2014

$$\omega = \frac{n-2}{n+2} \begin{cases} n = 2 & \text{matter (axions)} \\ n = 4 & \text{radiation} \end{cases}$$

Coherent scalar fields

Perturbations

$$\phi(\eta,\vec{x}) = \phi(\eta) + \delta\phi(\eta,\vec{x})$$

$$ds^2 = a^2(\eta)\left((1+2\Phi(\eta,\vec{x}))\,d\eta^2 - (1-2\Psi(\eta,\vec{x}))\,d\vec{x}^2\right)$$

Equations

$$\boxed{\begin{aligned} \ddot{\delta\phi}_k \;&+\; 2\mathcal{H}\dot{\delta\phi}_k - 3\dot{\Psi}_k\dot{\phi} - \dot{\Phi}_k\dot{\phi} \\ &+\; (V''(\phi)\,a^2 + k^2)\delta\phi_k + 2V'(\phi)\,a^2\Psi_k = 0 \\[2em] \delta G_{\mu\nu} &= 8\pi G\langle\delta T_{\mu\nu}\rangle \end{aligned}}$$

Coherent scalar fields

Adiabatic approximation

$$\epsilon = \frac{\mathcal{H}}{\nu_{eff}} \ll 1$$

Effective sound speed

$$c^2_{\rm eff}(k) \equiv \frac{\langle \delta p_k \rangle}{\langle \delta \rho_k \rangle} = \frac{\left\langle \frac{k^2}{a^2}\delta\phi_k\, \phi - V'(\phi)\delta\phi_k + V''(\phi)\phi\delta\phi_k \right\rangle}{\left\langle \frac{k^2}{a^2}\delta\phi_k\, \phi + 3V'(\phi)\delta\phi_k + V''(\phi)\phi\delta\phi_k \right\rangle} + \mathcal{O}(\epsilon)$$

Gauge-invariant

If $\nu_{eff} \gg k$

$$\boxed{c^2_{\rm eff} = \frac{n-2}{n+2} = \omega}$$

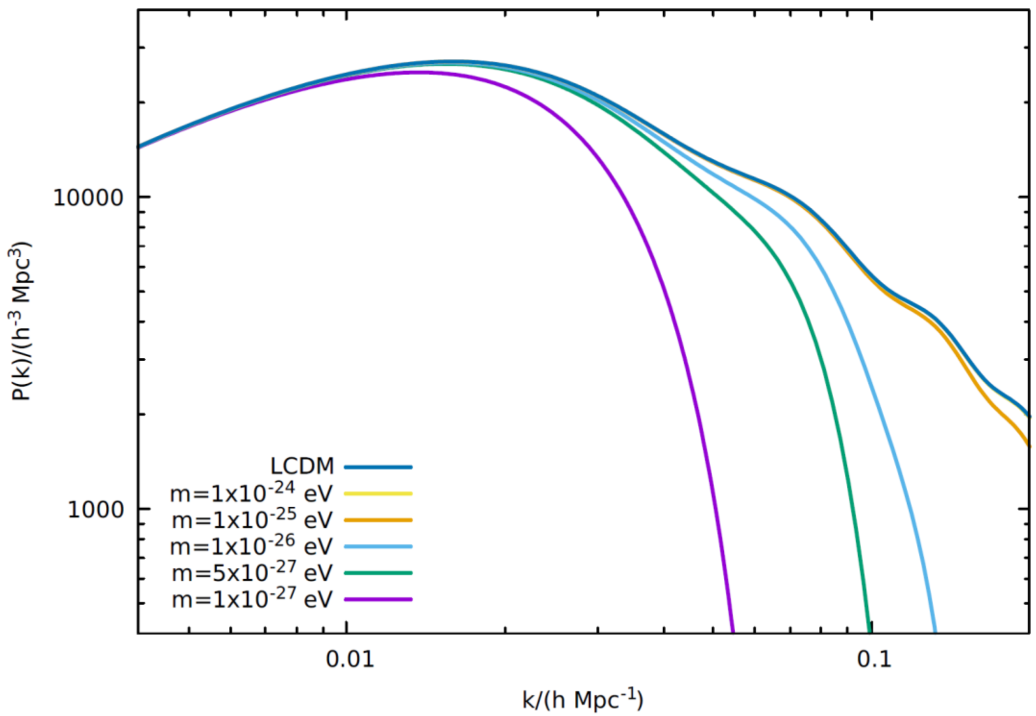
Coherent scalar fields

Harmonic case n=2

$$c_{\text{eff}}^2(k) \equiv \frac{\langle \delta p_k \rangle}{\langle \delta \rho_k \rangle} \begin{cases} 0 & k < \sqrt{\mathcal{H} m a} \\ \dfrac{k^2}{4m^2a^2} & k > \sqrt{\mathcal{H} m a} \end{cases}$$

Particle regime

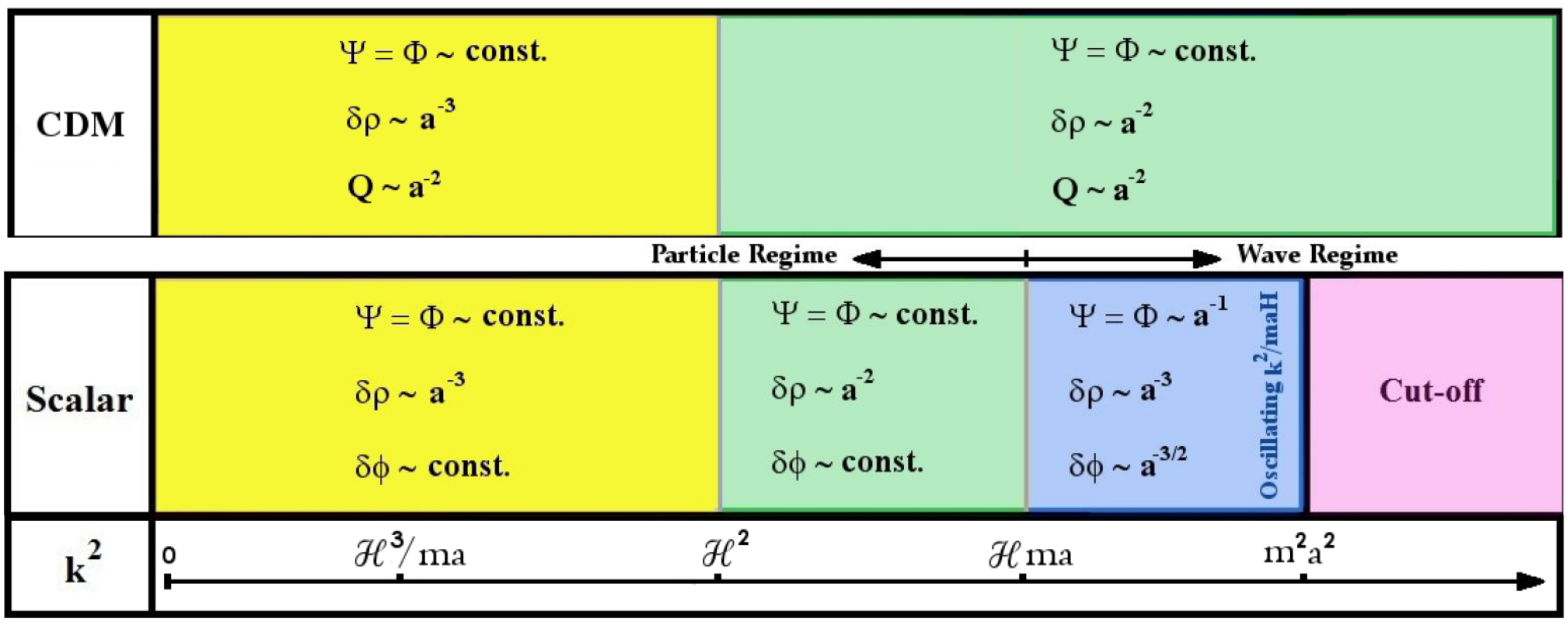
Wave regime



Matter power spectrum suppression on small scales

Coherent scalar fields

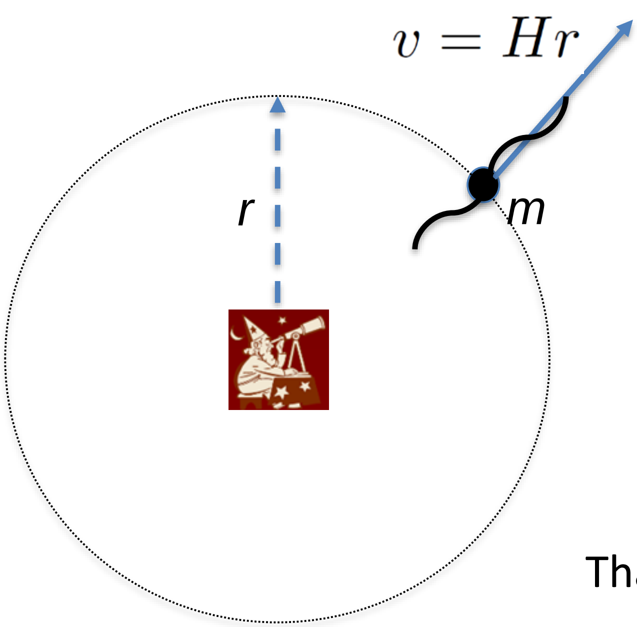
Harmonic case n=2



Particle DM vs. Wave DM

Heuristic interpretation (Hu et al, PRL85, 1158 (2000), Hlozek et al, PRD 91 103512 (2015))

Consider a particle of mass *m* << 1 eV moving with the Hubble flow *H*



The corresponding de Broglie wavelength:

$$\lambda_{\rm dB} = \frac{1}{mv} = \frac{1}{mHr}$$

Thus, the particle can be localized only in a sphere with radius:

$$r \geq \lambda_{\rm dB} \Longrightarrow \boxed{r \geq \frac{1}{\sqrt{Hm}}}$$

That corresponds to a (physical) wavenumber k=π/r

$$k_\star = \pi\sqrt{mH}$$

Particle DM vs. Wave DM

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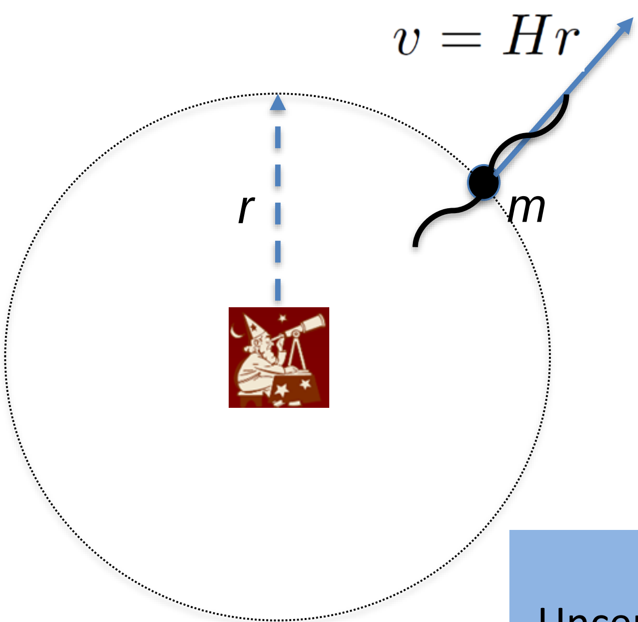


Thus, we have:

$k < \pi\sqrt{Hm}$ particle-like behaviour

$k > \pi\sqrt{Hm}$ wave-like behaviour

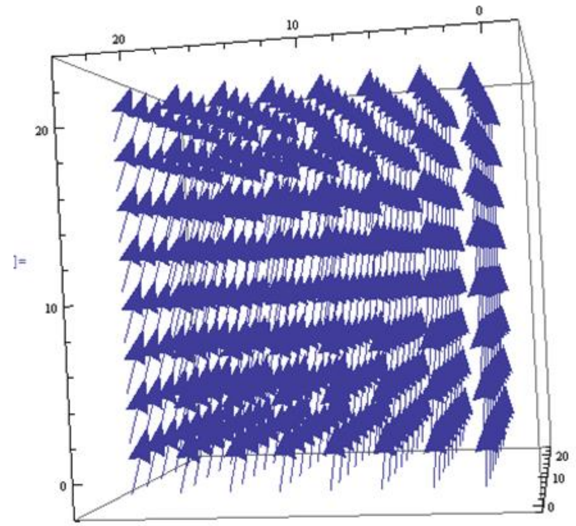
Jeans scale = de Broglie wavelength
Uncertainty principle avoids the small-scale problems of CDM:
a) missing satellite, b) too-big-to-fail, c) cusp-core

Ultralight scalar dark matter
(Sin 1994, Guzmán-Matos, 2000, Hu 2000)

$$m = 10^{-22}\ \mathrm{eV}$$

Higher-spin coherent fields

Problems at the background level



The anisotropy problem

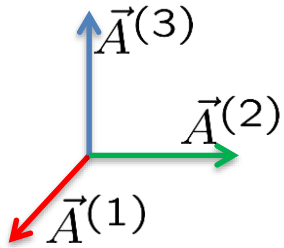
• Homogeneous vectors or other higher-spin fields are generically anisotropic

Higher-spin coherent fields

Anisotropy problem

There are different solutions in the literature:

- **Particular solutions**: Triads of orthogonal vectors.



$$A_i^{(a)} \propto \delta_i^a,\ \mathrm{a}=1,2,3$$

$$T^i{}_j \propto \delta^i{}_j$$

Cervero, Jacobs, 1978

- Large number, N, of **randomly oriented fields**.

$$T^i_j/p_k \sim 1/\sqrt{N}$$

Golovnev, Mukhanov, Vanchurin, 2008

Coherent vectors (abelian case)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(A^2)$$

$$A^2 = A_\mu A^\mu$$

$$\boxed{\ddot{A}_i + H\dot{A}_i - 2V'(A^2)A_i = 0}$$

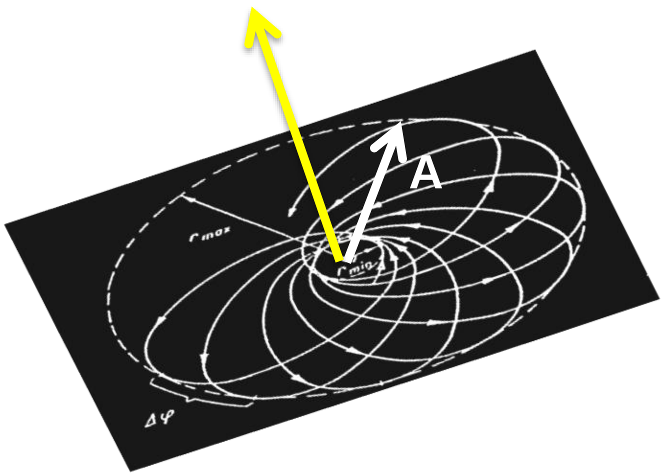
Homogeneous RW background

$$\boxed{\omega_i \sim |\dot{A}_i/A_i|}$$

$$\boxed{\ddot{A}_i = 2V'(A^2)A_i}$$

$$\omega_i \gg H$$

rapid oscillations



$$\boxed{G_{ij} = \frac{\dot{A}_i A_j}{a^2}}$$

bounded

$$\vec{L} = \vec{A} \times \dot{\vec{A}} = \text{const.}$$

Coherent vectors (abelian case)

Virial theorem: average energy-momentum tensor

$$\boxed{\frac{G_{ij}(T)-G_{ij}(0)}{T}=\left\langle 2V'(A^2)\frac{A_iA_j}{a^2}\right\rangle+\left\langle\frac{\dot{A}_i\dot{A}_j}{a^2}\right\rangle}$$

$$H^{-1}\gg T\gg\omega_i^{-1} \Rightarrow \boxed{\left\langle\frac{\dot{A}_i\dot{A}_j}{a^2}\right\rangle=-\left\langle 2V'(A^2)\frac{A_iA_j}{a^2}\right\rangle}$$

$$T^i{}_j=\frac{\dot{A}_i\dot{A}_j}{a^2}+2V'(A^2)\frac{A_iA_j}{a^2},\quad i\neq j$$

$$\boxed{\langle T^i_j\rangle=0,\quad i\neq j}$$

diagonal stress

Coherent vectors (abelian case)

$$\boxed{\left\langle \frac{\dot{A}_i \dot{A}_j}{a^2} \right\rangle = - \left\langle 2V'(A^2) \frac{A_i A_j}{a^2} \right\rangle}$$

Pressures:

$$\begin{aligned} p_k \ &\equiv \ -T^k{}_k = \frac{1}{2} \frac{\dot{A}_i \dot{A}_j}{a^2} \delta^{ij} - \frac{\dot{A}_k \dot{A}_k}{a^2} \\ &- \ V(A^2) - 2V'(A^2) \frac{A_k A_k}{a^2}, \ k = 1, 2, 3 \end{aligned}$$

$$\begin{aligned} \langle p_k \rangle \equiv - \langle T^k_k \rangle \ = \ \left\langle \frac{1}{2} \frac{\dot{A}_i \dot{A}_j}{a^2} \delta^{ij} \right\rangle - \langle V(A^2) \rangle \,, \\ k = 1, 2, 3 \,; \end{aligned}$$

$$\boxed{\langle T^i{}_j \rangle = - \langle p \rangle \ \delta^i{}_j}$$

Isotropic average energy-momentum tensor

Coherent vectors (abelian case)

Virial theorem: average equation of state $V = \lambda (A_\mu A^\mu)^n$

$$\left\langle \frac{\dot{A}_i \dot{A}_j}{a^2} \right\rangle = -\left\langle 2V'(A^2) \frac{A_i A_j}{a^2} \right\rangle \Rightarrow \left\langle \frac{1}{2} \frac{\dot{A}_i \dot{A}_j}{a^2} \delta^{ij} \right\rangle = n \left\langle V(A^2) \right\rangle$$

$$\boxed{\langle \rho \rangle = (n+1) \left\langle V(A^2) \right\rangle}$$

$$\boxed{\langle p \rangle = (n-1) \left\langle V(A^2) \right\rangle}$$

$$\boxed{\omega = \frac{\langle p \rangle}{\langle \rho \rangle} = \frac{n-1}{n+1}}$$

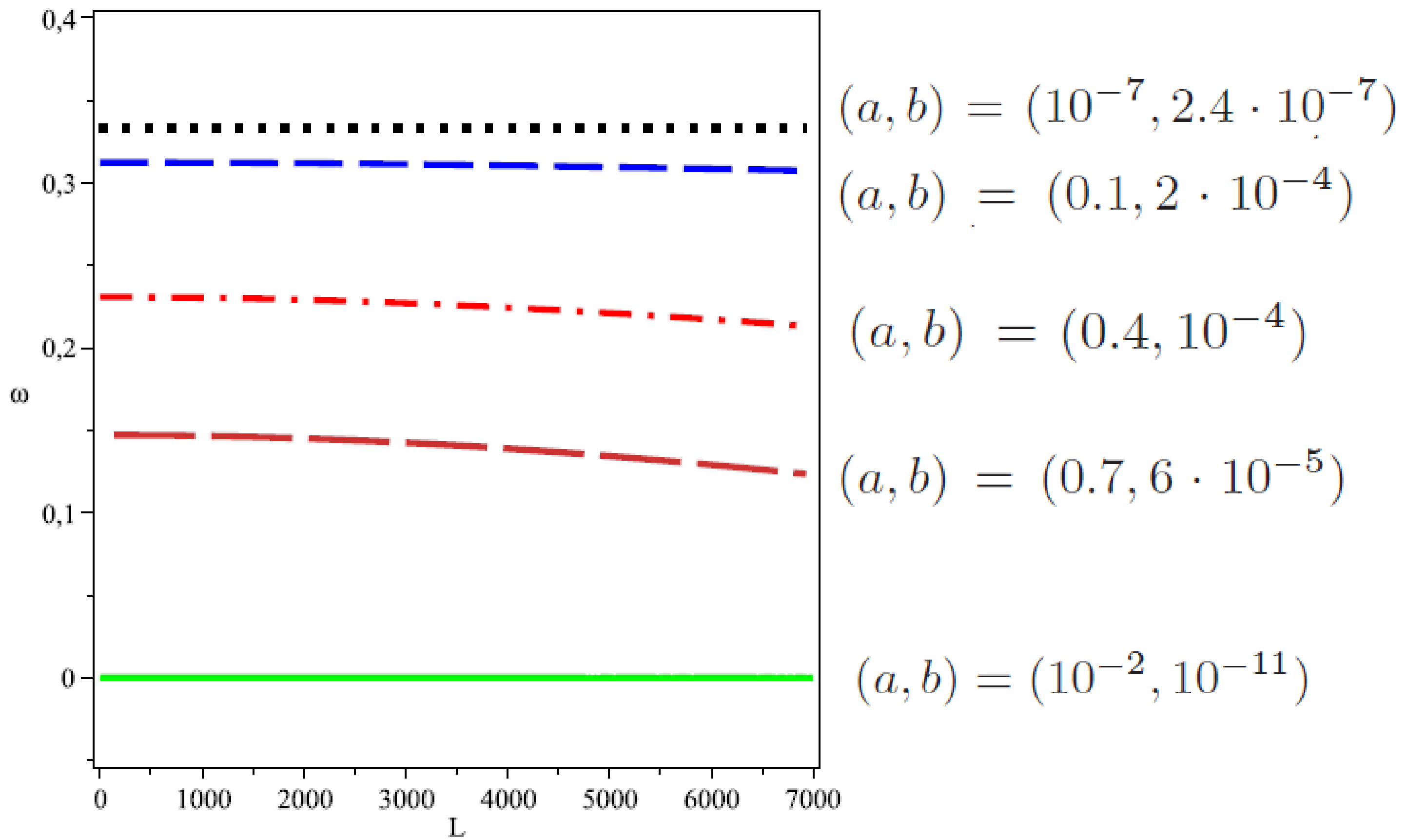
Agrees with the scalar case for power law potentials

L independent

Coherent vectors (abelian case)

Non power-law potentials

$$V = -aA_\mu A^\mu + b(A_\mu A^\mu)^2$$



Solving the anisotropy problem (abelian case)

1.- Average energy-momentum tensor is diagonal and isotropic

$$\langle T^i{}_0 \rangle = 0 \qquad \langle T^i{}_j \rangle = -\langle p \rangle \delta^i{}_j$$

2.- Average equation of state for $V = \lambda (A_\mu A^\mu)^n$:

$$\omega = \frac{\langle p \rangle}{\langle \rho \rangle} = \frac{n-1}{n+1}$$

Coherent vectors (non-abelian case)

$$\boxed{\mathcal{S} = \int d^4x\sqrt{g}\left(-\frac{1}{4}F^a_{\mu\nu}F^{a\ \mu\nu} - V(M_{ab}A^a_\rho A^{b\rho})\right)}$$

$$\boxed{\begin{aligned} F_{\mu\nu} &\equiv -igF^a_{\mu\nu}T^a \\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g c_{abc} A^b_\mu A^c_\nu \end{aligned}}$$

$$[T^a, T^b] = i c_{abc} T^c$$

c_{abc} totally antisymmetric
semi-simple Lie group

$$\boxed{F^{a\ ;\nu}_{\mu\nu} - g c_{abc} F^b_{\mu\nu} A^{c\ \nu} + 2V' M_{ab} A^b_\mu = 0}$$

M_{ab} symmetric constant matrix

Isotropy theorem for Yang-Mills theories

Yang-Mills theories for semi-simple Lie groups:

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{4} F^a{}_{\mu\nu} F^{a\,\mu\nu} - V\left(A^a{}_\mu A^{a\,\mu}\right) \right)$$

If the **field evolves rapidly** and $\boldsymbol{A^a{}_i}$ **,** $\boldsymbol{\dot{A}^a{}_i}$ **are bounded** during its evolution,

1.- **The energy momentum tensor is diagonal and isotropic in average.**

2.- **Without potential, the equation of state parameter is w = 1/3 i.e. it behaves as radiation.**

Cembranos, ALM, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Example: SU(2) theory

The self-interaction for non-Abelian theories changes the average equation of state. For high energy densities or large coupling constants it will behave as radiation, in the opposite limit, the Abelian behavior is recovered.

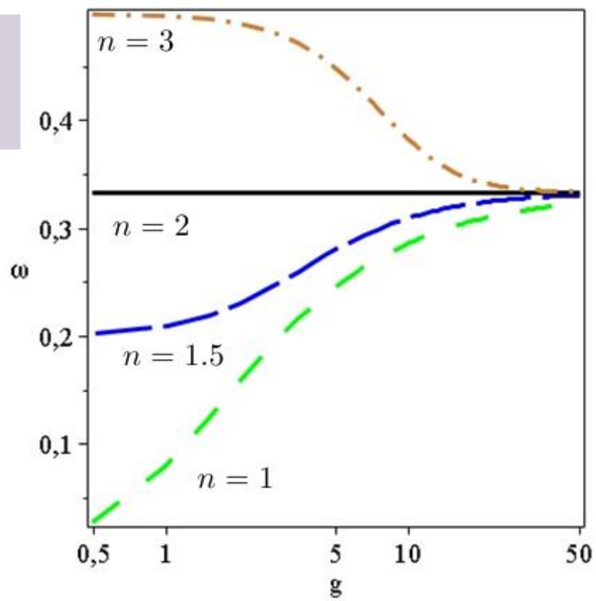
$$V = \frac{1}{2}(-M^2 A^a_\rho A^{a\,\rho})^n$$

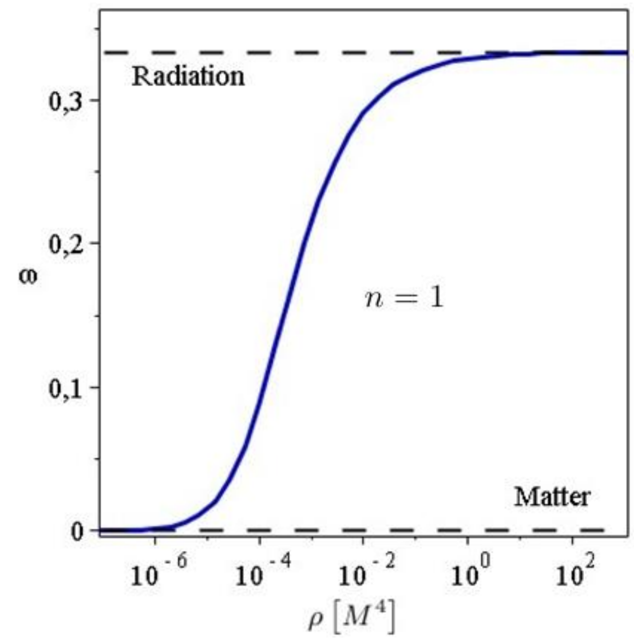
$\boldsymbol{g} \downarrow,\ \boldsymbol{\rho} \downarrow$

$$\boldsymbol{\omega} = \frac{\boldsymbol{n-1}}{\boldsymbol{n+1}}$$

$\boldsymbol{g} \uparrow,\ \boldsymbol{\rho} \uparrow$

$$\boldsymbol{\omega} = \frac{\boldsymbol{1}}{\boldsymbol{3}}$$





Arbitrary-spin fields

Consider a generic field ϕ_A with general Lagrangian of the form:

$$\mathcal{L} \equiv \mathcal{L}\left[\phi^A, \partial_\mu \phi^A\right] \qquad \phi_A \text{ and } \dot{\phi}_A \text{ bounded}$$

Canonical energy-momentum tensor

$$\Theta^{\mu\nu} = -\eta^{\mu\nu}\mathcal{L} + \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi^A\right)}\partial^\nu \phi^A$$

Belinfante-Rosenfeld energy momentum tensor

$$T^{\mu\nu} = \Theta^{\mu\nu} \overbrace{- \frac{1}{2}\nabla_\rho\left(S^{\rho\mu\nu} + S^{\mu\nu\rho} - S^{\nu\rho\mu}\right)}^{\nabla_\rho \tilde{\Theta}^{\nu\rho;\mu}}$$

$$S^{\mu\nu\rho} = \Pi^\mu_A \Sigma^{\nu\rho}\phi^A$$

Arbitrary-spin fields

Virial theorem: $H^{-1} \gg T \gg \omega^{-1}$

$$\left\langle g^{\rho\gamma}\nabla_\rho \tilde{\Theta}_{\nu\gamma;\mu}\right\rangle \approx \frac{1}{T}\int_t^{t+T} dt'\partial_0 \tilde{\Theta}_{\nu 0;\mu}(t') = \frac{\tilde{\Theta}_{\nu 0;\mu}(t+T)-\tilde{\Theta}_{\nu 0;\mu}(t)}{T}$$

Diagonal and isotropic energy-momentum tensor

$$\begin{aligned}
\langle T^{00}\rangle &= \langle \Pi^0_A \partial_0 \phi^A - \mathcal{L}\rangle \; ; \\
\langle T^{0j}\rangle &= T^{0j} = 0 \; ; \\
\langle T^{jj}\rangle &= \langle -g^{jj}\mathcal{L}\rangle \; ; \\
\langle T^{jk}\rangle &= 0 \; ; k \neq j \; ,
\end{aligned}$$

Average equation of state:

$$\omega = \frac{\langle p\rangle}{\langle \rho\rangle} = \frac{\langle \mathcal{L}\rangle}{\langle \Pi^0_A \partial_0 \phi^A - \mathcal{L}\rangle} = \frac{\langle \mathcal{L}\rangle}{\langle \mathcal{H}\rangle}$$

Arbitrary-spin fields

Average equation of state:

Virial theorem:

$$\langle \partial_0 \left(\Pi^0_A \phi^A \right) \rangle = 0$$

Power-law theories:

$$\mathcal{H} = \left(\lambda^{AB} g_{00} \Pi^0_A \Pi^0_B \right)^{n_T} + \left(M_{AB} \phi^A \phi^B \right)^{n_V}$$

$$\langle T \rangle = \frac{n_V}{n_T} \langle V \rangle$$

Average equation of state:

$$\boxed{\omega = \frac{2\, n_V}{1 + \frac{n_V}{n_T}} - 1}$$

Coherent spin 2 fields

Massive gravitons as wave DM

Cembranos, A.L.M., Núñez Jareño, JCAP 1403 (2014) 042

Fierz-Pauli Lagrangian

$$\mathcal{L} = \frac{M_{Pl}^2}{8}\Big[\nabla_\alpha h^{\mu\nu}\nabla^\alpha h_{\mu\nu} - 2\nabla_\alpha h_\mu^\alpha \nabla_\beta h^{\mu\beta} + 2\nabla_\alpha h_\mu^\alpha \nabla^\mu h_\beta^\beta - \nabla_\alpha h_\mu^\mu \nabla^\alpha h_\nu^\nu - m_g^2\left(h_{\mu\nu}h^{\mu\nu} - \left(h_\mu^\mu\right)^2\right)\Big].$$

Average equation of state:

$$\omega = \frac{2n_V}{1+\frac{n_V}{n_T}} - 1 = 0$$

General space-time geometries

For a general background metric. Riemann normal coordinates:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \frac{1}{3} R_{\mu\alpha\nu\beta} x^\alpha x^\beta + \ldots$$

1. The Lagrangian depends only on the fields and their gradients.

2. The field evolves rapidly:

$$|R^{\gamma}_{\lambda\mu\nu}| \ll (\omega_A)^2, \text{ and } |\partial_j S^{\mu\nu\rho}| \ll |\partial_0 S^{\mu\nu\rho}|,$$
$$\text{for } \; j = 1, 2, 3 \;;$$

3. $S^{\mu\nu\rho}$, i.e. ϕ^A and Π^0_A, remains bounded in the evolution.

The average energy-momentum tensor takes the perfect fluid form for any locally inertial observer.

Coherent vectors as ultralight dark matter

Massive abelian vector field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu \ .$$

Unlike scalar fields no extra (shift) symmetry required to protect small masses

$$A_\mu \ = \ \left(\delta A_0(\eta,\vec{x}),\ \vec{A}(\eta) + \delta\vec{A}(\eta,\vec{x})\right)$$

3 independent perturbations (δA_0 is fixed)

$$ds^2 \ = \ a(\eta)^2\left[(1+2\Phi(\eta,\vec{x}))\,d\eta^2 - ((1-2\Psi(\eta,\vec{x}))\,\delta_{ij} + h_{ij}(\eta,\vec{x}))\,dx^i dx^j - 2Q_i(\eta,\vec{x})d\eta dx^i\right]$$

Scalar-vector-tensor mixing

Coherent vectors as ultralight dark matter

Equations

$$\delta\ddot{A}_i + ik_i\delta\dot{A}_0 - \left(\dot{\Phi} + \dot{\Psi}\right)\dot{A}_i - 2\Phi\ddot{A}_i - i\left(\vec{k}\dot{\vec{A}}\right)Q_i - \dot{h}_{ij}\dot{A}_j + \left(m^2a^2 + k^2\right)\delta A_i - k_i\left(\vec{k}\delta\vec{A}\right) = 0$$

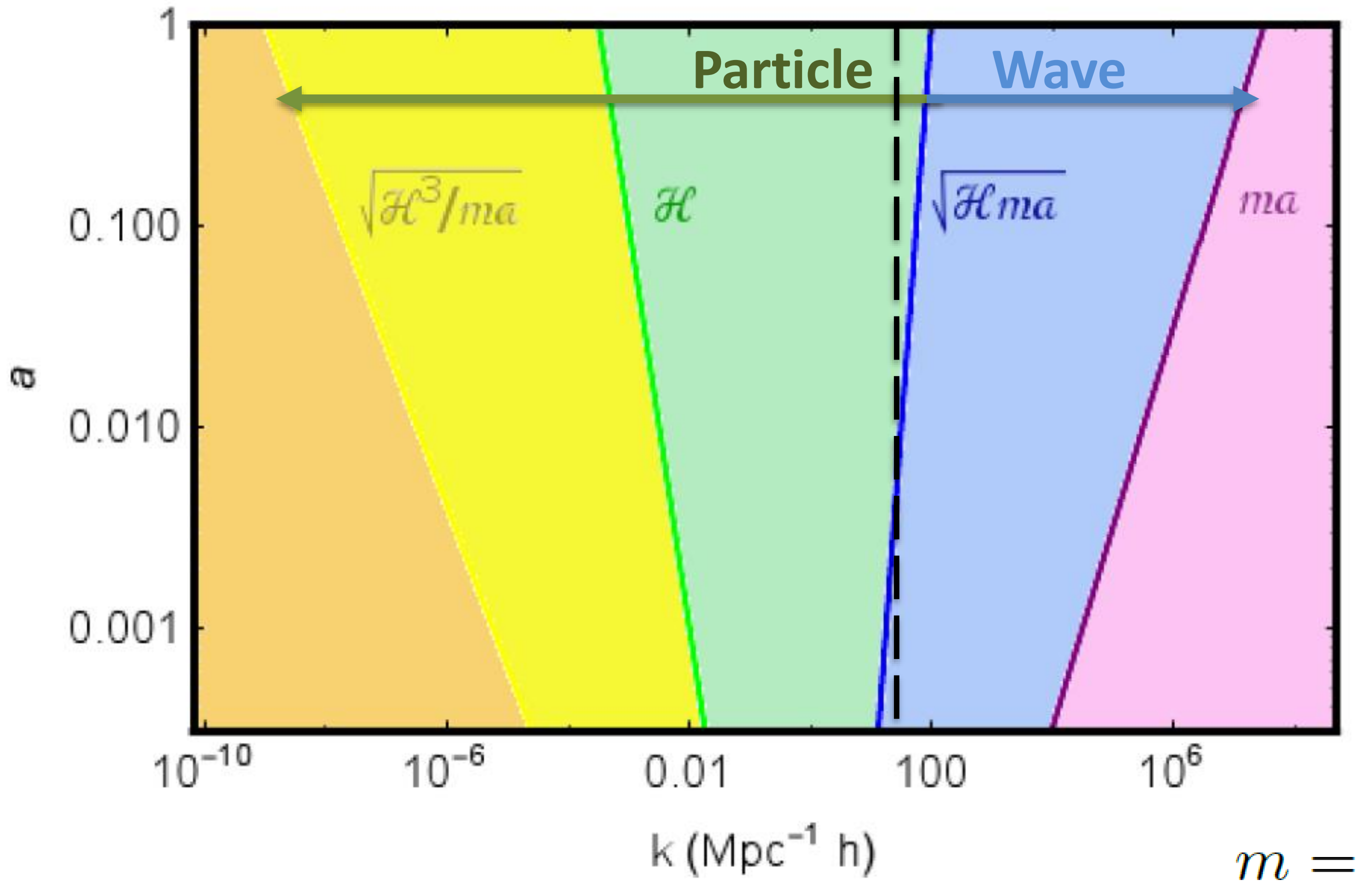
$$\delta G_{\mu\nu} = 8\pi G\langle\delta T_{\mu\nu}\rangle$$

Adiabatic approximation:

- Three comoving scales in the problem: $\boxed{ma,\ \mathcal{H}\ \text{and}\ k}$
- Adiabatic approximation: $ma \gg \{\mathcal{H}, k\}$

Coherent vectors as ultralight dark matter

Regimes



$m = 10^{-22}$ eV

Coherent vectors as ultralight dark matter

Particle regime (2 scalar and 4 vector modes)

- Same behaviour as CDM:
  - Scalar-vector-tensor decoupled evolution
  - No tensor sources
  - No anisotropic stress

Wave regime (2 scalar-tensor and 4 vector-tensor modes)

Scalar-tensor modes

$$\cos\theta \equiv \hat{k}\cdot\hat{u}_A$$

- Speed of sound

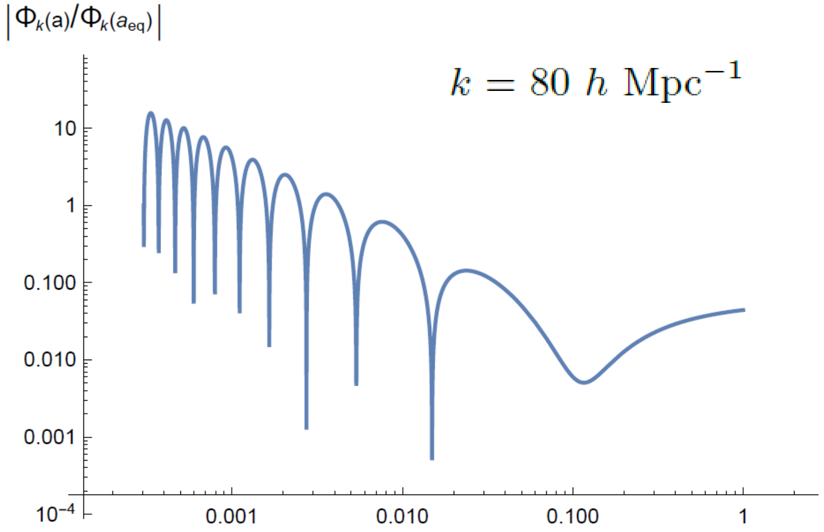
$$c_{eff}^2 \equiv \frac{\langle\delta p\rangle}{\langle\delta\rho\rangle} = -\frac{k^2}{4m^2a^2}\cos(2\theta)$$

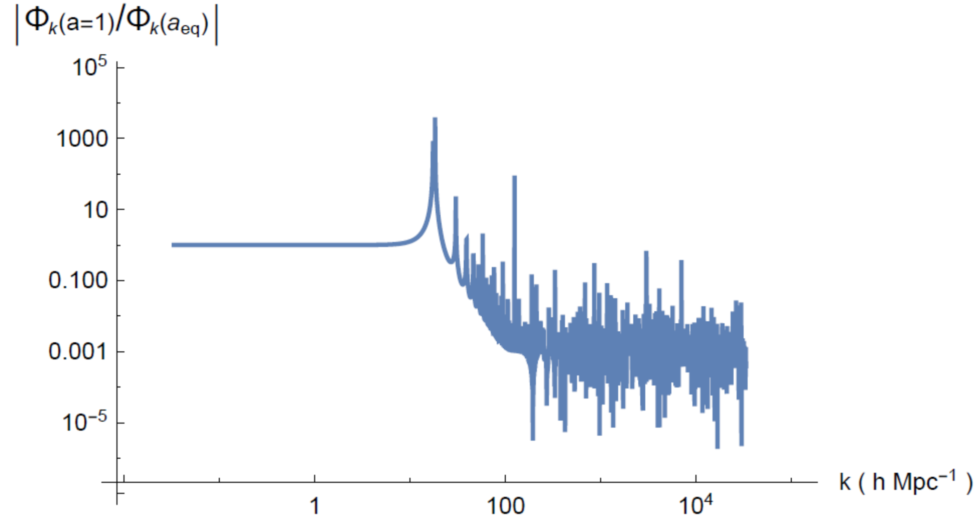
- Gravitational slip

$$\frac{\Psi(\eta,\vec{k})-\Phi(\eta,\vec{k})}{\Phi(\eta,\vec{k})} = \frac{k^2}{2m^2a^2}(1+\cos^2\theta)$$

Coherent vectors as ultralight dark matter

Scalar-tensor modes $m = 10^{-22}\ \mathrm{eV}$





Linear transfer function suppression for $k > 10h\ \mathrm{Mpc}^{-1}$

Coherent vectors as ultralight dark matter

Scalar-tensor modes

$$h_{ij}(\eta, \vec{k}) \equiv \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{h_+(\eta_{eq}, \vec{k})}{\Phi(\eta_{eq}, \vec{k})} = \frac{k^2}{2m^2 a_{eq}^2} \sin^2(\theta) \left(1 - 4\cos^2(\theta)\right)$$

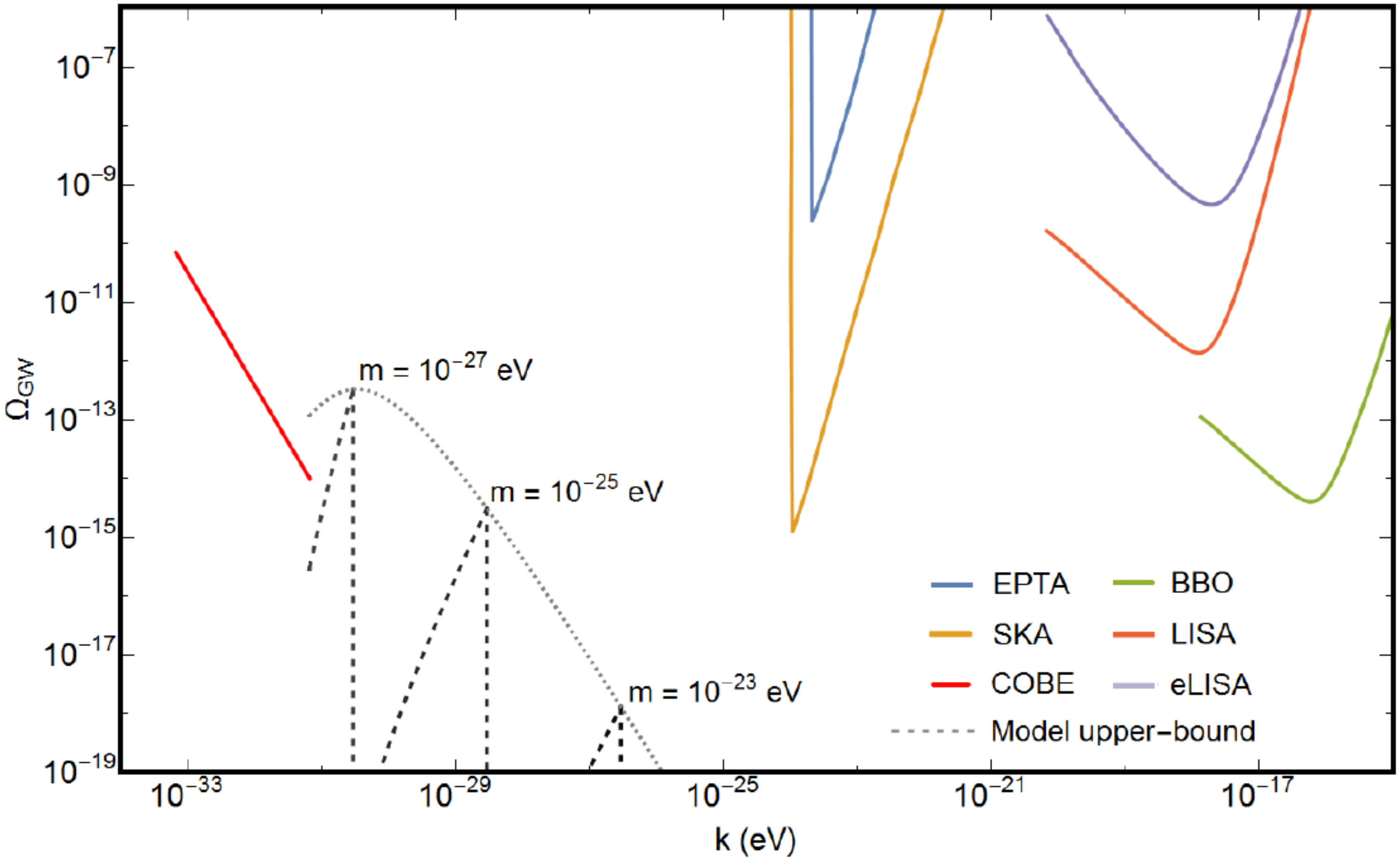
Gravity wave abundance

$$\Omega_{\mathrm{GW}}(k, \eta_0) = \int d\Omega \frac{k^5 |h_+|^2}{48\pi^3 H_0^2} = 1.605 A_s \frac{k^2}{H_0^2} \left(\frac{k_{\mathrm{eq}}^2}{m^2 a_{\mathrm{eq}}} \ln\left(\frac{k}{8k_{\mathrm{eq}}}\right)\right)^2 \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_{eq} \ll k \ll m a_{eq}$$

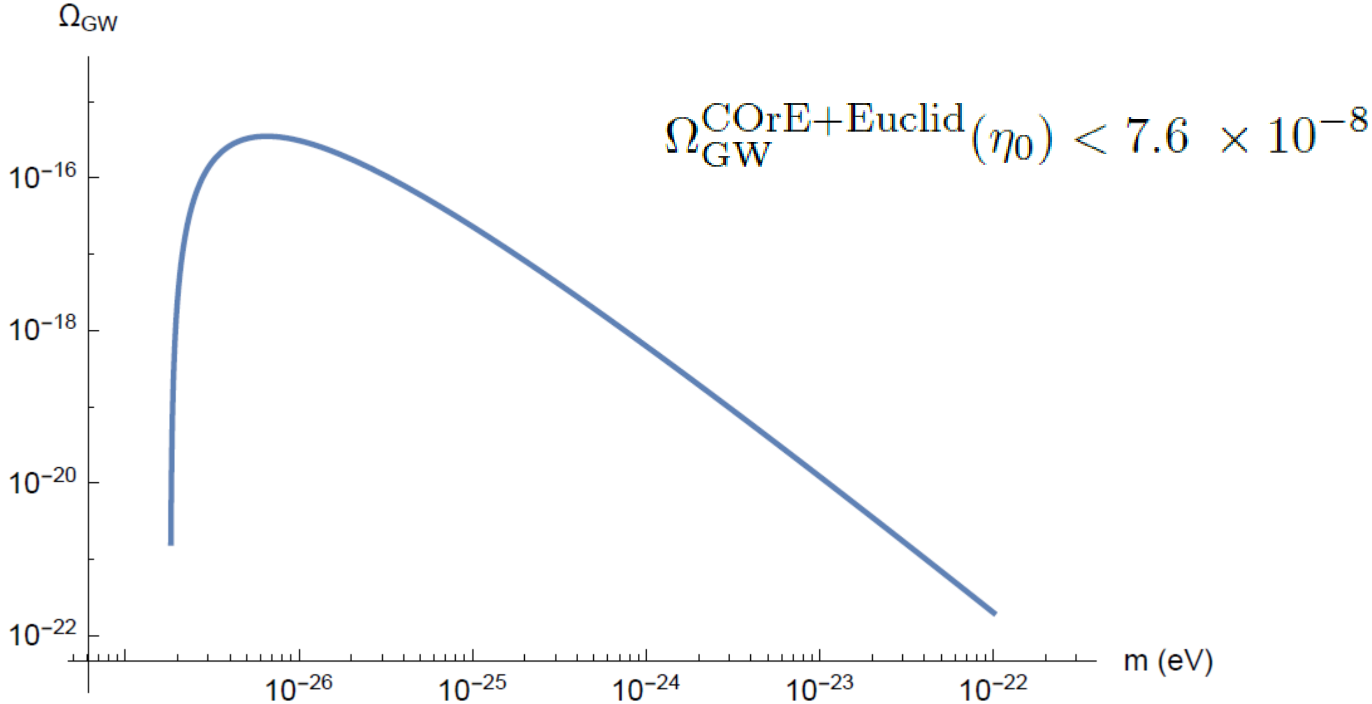
Coherent vectors as ultralight dark matter

Scalar-tensor modes



Coherent vectors as ultralight dark matter

Scalar-tensor modes



Coherent vectors as ultralight dark matter

CDM	$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-3}$ $Q \sim a^{-2}$	$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-2}$ $Q \sim a^{-2}$

Particle Regime ⟵ ⟶ Wave Regime

Scalar	$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-3}$ $\delta\phi \sim$ const.		$\Psi = \Phi \sim$ const. $\delta\rho \sim a^{-2}$ $\delta\phi \sim$ const.	$\Psi = \Phi \sim a^{-1}$ $\delta\rho \sim a^{-3}$ $\delta\phi \sim a^{-3/2}$ Oscillating k^2/maH	Cut-off
Vector	Averaging fails	$\Psi = \Phi \sim$ const. $\frac{\Psi - \Phi}{\Psi} = 0$ $\delta\rho \sim a^{-3}$ $\delta A_a \sim a$ $Q \sim a^{-2}$ $h_{ij} = 0$	$\Psi = \Phi \sim$ const. $\frac{\Psi - \Phi}{\Psi} = 0$ $\delta\rho \sim a^{-2}$ $\delta A_a \sim a$ $Q \sim a^{-2}$ $h_{ij} = 0$	$\Psi \sim \Phi \sim a^{-1}$ $\frac{\Psi - \Phi}{\Psi} \sim a^{-2}$ $\delta\rho \sim a^{-3}$ $\delta A_a \sim a^{-1/2}$ $Q \sim a^{-2}$ $h_{ij} \sim a^{-1}$ Oscillating k^2/maH	Cut-off
k^2	0	$\mathcal{H}^3/ma$	$\mathcal{H}^2$	$\mathcal{H}ma$	m^2a^2

Conclusions

- Cosmological coherent fields of arbitrary-spin do not present anisotropy or instability problems if they are fast oscillating

- Fields with power-law Hamiltonians behave as perfect fluids with average equation of state: $$\omega = \frac{2\,n_V}{1+\frac{n_V}{n_T}} - 1$$

- Higher-spin fields can play the role of wave(fuzzy) DM

- Ultralight vectors are indistinguishable from scalars in the particle regime, however in the wave regime they generate scalar-vector-tensor mixing, anisotropic stress and GW.

Example: SU(2) theory

The self-interaction for non-Abelian theories changes the average equation of state. For high energy densities or large coupling constants it will behave as radiation, in the opposite limit, the Abelian behavior is recovered.

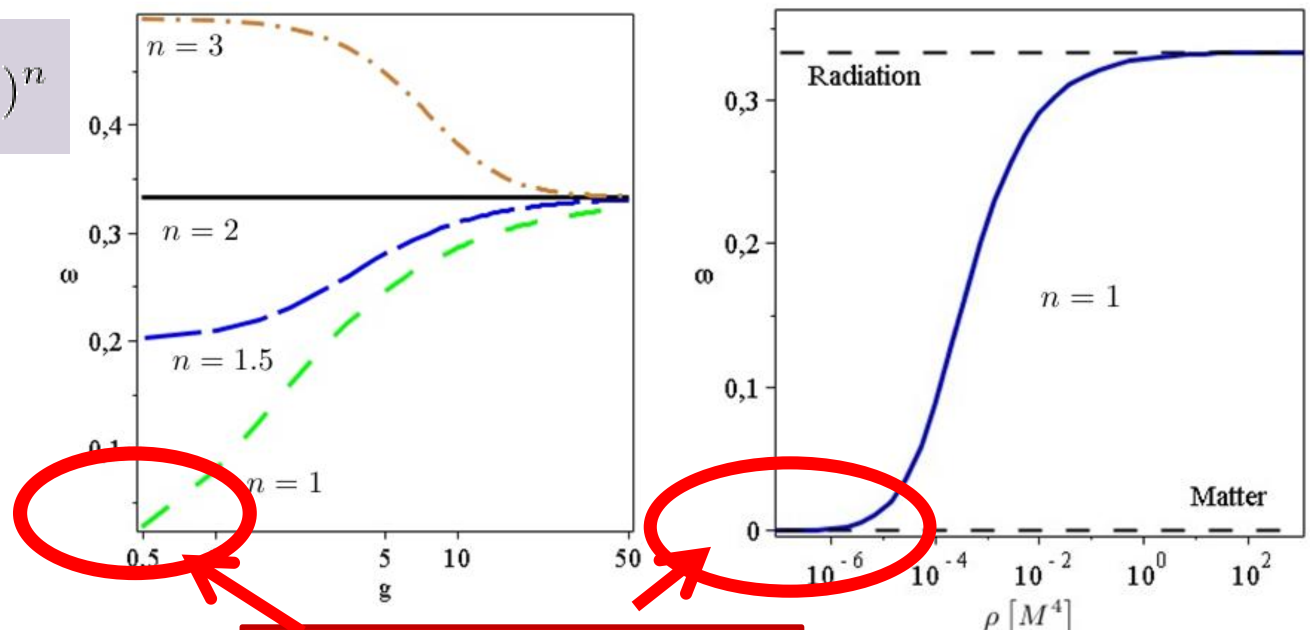
$$V = \frac{1}{2}(-M^2 A^a_\rho A^{a\,\rho})^n$$

$g \downarrow,\ \rho \downarrow$

$$\omega = \frac{n-1}{n+1}$$

$g \uparrow,\ \rho \uparrow$

$$\omega = \frac{1}{3}$$



DARK MATTER BEHAVIOR

JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Higher-spin coherent fields

Problems at the perturbation level

- Instabilities in vector theories with spatial VEVs:

  (Himmetoglu, Contaldi, Peloso 2009)

$$\boxed{S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{F^2}{4} - V(A^2) + \frac{\xi}{2}RA^2\right].}$$

$$\boxed{M^2 = 2\frac{\partial V}{\partial A^2} - \xi R}$$

$M^2 > 0$

to avoid instabilities in

the perturbations

Instabilities for abelian vectors

$$S = \int d^4x \sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{F^2}{4} - V(A^2) + \frac{\xi}{2}RA^2\right].$$

$$M^2 = 2\frac{\partial V}{\partial A^2} - \xi R$$

Himmetoglu, Contaldi, Peloso (2009)

$|R^{\gamma}_{\lambda\mu\nu}| \ll (\omega_i^a)^2$ + bounded evolution

$\Downarrow$

$M^2 > 0$ no instabilites

Coherent vectors (non-abelian case)

FRW background

$$\boxed{ds^2 = a^2(\eta)\left(d\eta^2 - d\vec{x}^2\right)}$$

$\mu = 0$

$$\boxed{g c_{abc} \dot{A}_i^b A_i^c + g^2 c_{abc} c_{bde} A_0^d A_i^e A_i^c \; + \; 2V' M_{ab} a^2(\eta) A_0^b = 0}$$

No $\ddot{A}_0^a$ term

$\mu = i$

$$\boxed{\begin{aligned} \ddot{A}_i^a \; &- \; g c_{abc}\left(2\dot{A}_i^b A_0^c + A_i^b \dot{A}_0^c\right) + g^2 c_{abc} c_{bde}\left(A_i^d A_0^e A_0^c\right. \\ &- \left. A_i^d A_j^e A_j^c\right) - 2V' M_{ab} a^2(\eta) A_i^b = 0\,, \end{aligned}}$$

Coherent vectors (non-abelian case)

Generalized virial theorem

$$\boxed{G_{ij}^{ab} = \frac{\dot{A}_i^a A_j^b}{a^4(\eta)}, \quad i,j = 1,2,3; \;\; a,b = 1\ldots N}$$

bounded

$$\boxed{\begin{aligned} \dot{G}_{ij}^{ab} = \frac{\ddot{A}_i^a A_j^b}{a^4(\eta)} + \frac{\dot{A}_i^a \dot{A}_j^b}{a^4(\eta)}, \quad i,j &= 1,2,3; \\ a,b &= 1\ldots N \end{aligned}}$$

$$\omega_i^{(a)} \gg H$$

Rapid evolution

$$\frac{G_{ij}^{ab}(T) - G_{ij}^{ab}(0)}{T} = \left\langle \frac{\ddot{A}_i^a A_j^b}{a^4(\eta)} + \frac{\dot{A}_i^a \dot{A}_j^b}{a^4(\eta)} \right\rangle = 0$$

$$H^{-1} \gg T \gg \omega^{-1}$$