# The $\theta$ -dependent vacuum energy. The application to cosmology and axion search experiments

Ariel Zhitnitsky

University
of British Columbia,
Vancouver, Canada

*Quantum Vacuum and Gravitation,*
*Mainz, March 13-24, 2017*

# 1. Motivation. Structure of the talk.

- The main motivation of this talk is to argue that there is a novel type of energy. This energy has "non-dispersive" nature, and can not be expressed in terms of conventional scattering amplitudes.

- All these novel effects are due to the nontrivial topological sectors in the gauge systems and tunnelling transitions between them.

- The effect is non-local in nature, and can not be expressed in terms of local curvature. It is expressed in terms of a non-local characteristics of the system such as the holonomy.

- It should be contrasted with the "dispersive" terms computed from the dispersion relations (see Tolley's talk).

- We want to test the ideas in a tabletop experiment where there is an <u>extra contribution</u> to the <u>Casimir vacuum pressure.</u> A novel extra term can not be expressed in terms of propagating physical photons with two transverse polarizations (S-matrix).

- The effect (based exclusively on the SM physics) is highly sensitive to the $\theta$ parameter. It motivated few new ideas on the axion search experiments.

- The talk is based on applications of this new type of energy to:
  1.cosmology- DE, inflation (PRD, arxiv:1505.05151);
  2.testing of these ideas in a tabletop experiment (PRD-2017, arxiv; 1605.01411);
  3.axion search experiments (arxiv:1702.00012).
  New opportunities emerge due to high sensitivity of these novel effects to the axion $\theta(x)$ field

# 2. Topological susceptibility

- A convenient way to explain the nature of new type of vacuum energy is to study the topologically susceptibility ( *it is the key element in the resolution of the so-called U(1) problem in QCD, Witten, Veneziano, 1979* ).

$$\chi_{YM} = \int d^4x \, \langle q(x), q(0) \rangle \neq 0 \qquad \frac{\partial^2 E_{\rm vac}(\theta)}{\partial \theta^2} = \chi_{YM}$$

*To avoid confusion: This is the Wick's T-product, not Dyson's*

- $\chi_{YM}$ does not vanish, though $q(x) \sim \partial_\mu K^\mu(x)$ . It has ``wrong sign", see below. It can not be related to any physical propagating degrees of freedom. Furthermore, it has a pole in momentum space

$$\lim_{k \to 0} \int d^4x e^{ikx} \langle K_\mu(x), K_\nu(0) \rangle \sim \frac{k_\mu k_\nu}{k^4}$$

- There is a <u>massless</u> pole, but there are <u>no</u> any <u>physical massless</u> states in the system.

$$\chi_{dispersive} \sim \lim_{k\to 0} \sum_n \frac{\langle 0|q|n\rangle\langle n|q|0\rangle}{-k^2 - m_n^2} < 0,$$

- Conventional physical degrees of freedom always contribute with sign (-) while one needs sign (+) to satisfy WI and resolve the U(1) problem

$$\chi_{non-dispersive} = \int d^4x \, \langle q(x), q(0)\rangle = \frac{1}{N^2}|E_{vac}| > 0$$

- Conventional terms (related to propagating degrees of freedom) always produce $\exp\left(-\Lambda_{QCD} L\right)$ behaviour at large distances.

- Witten simply postulated this term, while Veneziano assumed the unphysical field, the so-called the "Veneziano ghost" to saturate "wrong" sign in $\chi$.

- In some models this contact non-dispersive term with "wrong" sign (+) can be explicitly computed. It is originated from the tunnelling effects between the <u>degenerate topological sectors of the theory</u>.

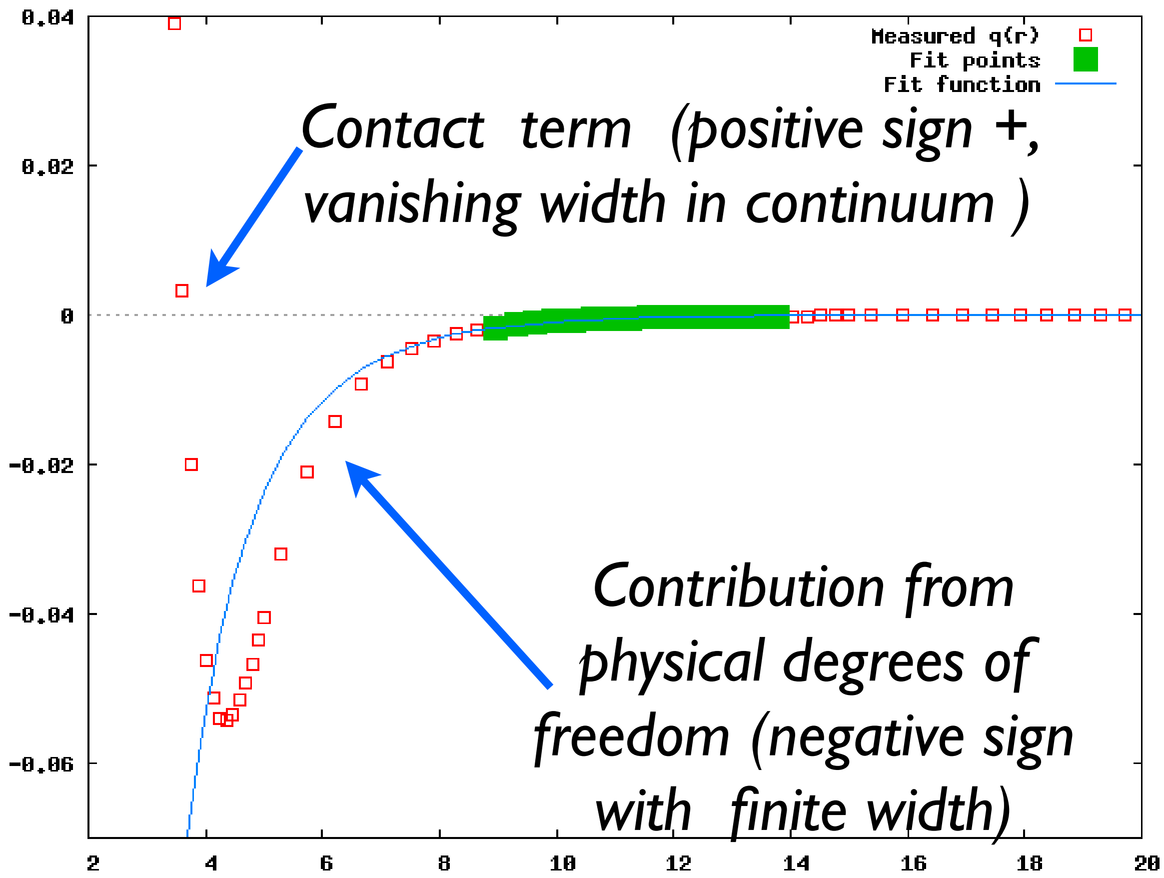
- These contributions can not be described in terms of conventional degrees of freedom (wrong sign);

- They are inherently non-local in nature as they are related to the tunnelling processes which are formulated in terms of the non-local large gauge transformation operator and holonomy;

- These terms may exhibit the long range features even through QCD has a gap (similar to the CM topologically ordered systems);

- The effects have been explained in terms $\chi_{YM}$ . However, the $\theta$ -dependent portion of energy $E_{\rm vac}(\theta)$ (which is generated due to the tunnellings) has all these unusual features due to the relation

$$\frac{\partial^2 E_{\rm vac}(\theta)}{\partial \theta^2} = \chi_{YM}$$




The topological susceptibility $\chi(r)$ as a function of r. Wrong sign for $\chi$ is well established phenomenon; it has been tested on the lattice (plot above is from C. Bernard et al, LATTICE 2007). This $\chi(r=0)$ contribution is not related to any physical degrees of freedom, and can be interpreted as a contact term.

# 3. Warm up example: Maxwell system in 2D

- 2D Maxwell theory is exactly solvable model. It is an empty theory as it does not support any propagating dof. Still, it has non-trivial dynamics.

- The partition function for $\theta$ vacua is known (in Hamiltonian approach):

$$E_n(\theta) = \frac{1}{2}\left(n + \frac{\theta}{2\pi}\right)^2 e^2 L,$$

$$\mathcal{Z}(V,\theta) = \sum_{n\in\mathbb{Z}} e^{-\frac{e^2 V}{2}\left(n+\frac{\theta}{2\pi}\right)^2}$$

- We want to reproduce $\mathcal{Z}(V,\theta)$ using path integral computations as it can be generalized to 4D system. Instanton configurations, topological charge density Q, classical action are:

$$\int \mathrm{d}^2x\, Q(x) = k, \quad eE^{(k)} = \frac{2\pi k}{V}, \quad Q = \frac{e}{2\pi}E \quad \frac{1}{2}\int \mathrm{d}^2x E^2 = \frac{2\pi^2 k^2}{e^2 V}.$$

$$\mathcal{Z}(\theta) = \sum_{k\in\mathbb{Z}} \int \mathcal{D}A^{(k)} e^{-\frac{1}{2}\int d^2x E^2 + i\frac{e\theta}{2\pi}\int d^2x\ E(x)}$$

- Partition function is quadratic and can be easily evaluated. The Euclidean computations (with boundary conditions up to large gauge transformations) produce identically the same results as the Hamiltonian approach (with conventional periodic boundary conditions)

$$\mathcal{Z}(\theta) = \sum_{n\in\mathbb{Z}} e^{-\frac{e^2V}{2}\left(n+\frac{\theta}{2\pi}\right)^2} = \sqrt{\frac{2\pi}{e^2V}} \sum_{k\in\mathbb{Z}} e^{-\frac{2\pi^2k^2}{e^2V} + ik\theta}.$$

- Topological susceptibility is <u>finite</u> (in infinite volume limit) and saturated by very large topological winding numbers, $k \sim \sqrt{e^2V} \gg 1.$ Infrared regularization (boundary conditions) are essential at every step (I.Sachs , A. Wipf)

$$\chi \equiv \lim_{k\to 0} \int \mathrm{d}^2x\ e^{ikx} \langle TQ(x)Q(0)\rangle = -\frac{1}{V}\cdot\frac{\partial^2 \ln \mathcal{Z}(\theta)}{\partial\theta^2}\Big|_{\theta=0} = \frac{e^2}{4\pi^2}.$$

- THE INTERGRAND $\left(\delta^2(x) - \text{function}\right)$ FOR THE TOPOLOGICAL SUSCEPTIBILITY IS SATURATED BY UNIFORM FLUXES FILLING THE ENTIRE SPACE-TIME VOLUME (IR <u>NOT UV</u> PHYSICS). THIS "NON-DISPERSIVE" CONTACT TERM IS NOT RELATED TO ANY PROPAGATING DEGREES OF FREEDOM

$$\langle Q(x)Q(0)\rangle = \frac{e^2}{4\pi^2}\delta^2(x). \qquad \int d^2x \langle Q(x), Q(0)\rangle \sim \int d^2x\delta^2(x) \sim \int d^2x\ \partial_\mu\left(\frac{x_\mu}{x^2}\right)$$

- IS THIS CONSTANT NON-DISPERSIVE CONTRIBUTION TO THE VACUUM ENERGY PHYSICALLY OBSERVABLE?

- THE ULTIMATE ANSWER IS "YES" AS THE ANOMALOUS WARD IDENTITIES $\chi = 0$ (WHEN PHYSICAL MASSLESS FERMIONS ARE INTRODUCED INTO THE SYSTEM) CAN BE ONLY SATISFIED IF THE CONTACT TERM IS NOT ZERO.

$$\chi = \frac{e^2}{4\pi^2}\int d^2x\left[\delta^2(x) - \frac{e^2}{2\pi^2}K_0(\mu|x|)\right] = \frac{e^2}{4\pi^2}\left[1 - \frac{e^2}{\pi}\frac{1}{\mu^2}\right] = \frac{e^2}{4\pi^2}[1-1] = 0.$$

# 4. Applications to Cosmology

- We assume (see next few slides) that the non-dispersive $\theta$ - dependent portion of the vacuum energy $E_{\rm vac}(\theta)$ shows the linear correction with respect to IR regulator "L" of the background, i.e.

$$E(L) = c_0\Lambda^4_{\rm QCD} + c_1 L^{-1}\Lambda^3_{\rm QCD} + \mathcal{O}(L^{-2}\Lambda^2_{\rm QCD}) + ...$$

- We also assume that the relevant (gravitating) energy which enters the Friedman's equation is the difference $\Delta E = [E(L) - E_{\rm Mink}]$ similar to computations of the Casimir energy, when the difference $\Delta E$ is observed. This assumption was, in fact, originally formulated by *Zeldovich in 1967*.

- In this formula the dimensional parameter "L" should be interpreted as <u>any</u> IR parameter of the system. e.g. curvature in hyperbolic space $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$ or temporal/spatial size of $\mathbb{T}^3 \times \mathbb{S}^1$ or $\mathbb{S}^3 \times \mathbb{S}^1$

- Historical comments: many people from different fields had advocated (after *Zeldovich*) a similar idea on the RHS for the Friedman's equation

$$\Delta E(L) = [E(L) - E_{\rm Mink}]$$

- $$E(L) \equiv -(\beta V)^{-1} \ln \mathcal{Z}$$

- *James Bjorken (particle physics), 2001,*
  *Ralf Schuetzhold (GR), PRL, 2002;*
  *Grisha Volovik (CM physics), 2008 +many more*

- I personally adopted this idea in 2009, mostly due to the intense discussions with Grisha Volovik in the relation with his COSLAB (Cosmology in a Laboratory) activities.

- This "L"-dependent energy has the same "non-dispersive" nature, which can not be expressed in terms of any local propagating degrees of freedom (such as "inflaton" or DE-scalar fields). Dyson T-product formulation cannot describe this physics.

- For numerical estimates we take $L \sim H^{-1}$, where "H" is the Hubble constant. This estimate obviously does not contradict any observations.It should not be interpreted as $\cancel{L^{-1} \sim \sqrt{R}}$ as it is formulated in terms of a different characteristic, the holonomy (not expressible as local curvature)

- With these assumptions the non-dispersive contribution to energy is amazingly close to the observed values (without any fitting parameters)

$$L^{-1} \sim H \sim 10^{-33}\text{eV}, \qquad \rho_{\text{DE}} = \Delta E \sim L^{-1}\Lambda^3_{\text{QCD}} \sim (10^{-3}\text{eV})^4$$

- *Q: How a system with a gap could be ever sensitive to arbitrary large distances?*

- A1: The long range order in gapped QCD is similar to Aharonov -Casher effect. If one inserts an external charge into superconductor when electric field is screened $\exp(-r/\lambda)$ a neutral magnetic fluxon will be still sensitive to external charge at arbitrary large distances.

- A2: Long range order in the system emerges because the large gauge transformation operator $\mathcal{T}$ and holonomy are non-local operators sensitive to far IR-physics, similar to "modular operator" in Aharonov -Casher effect.

# 5. Applications to inflation

- We assume a scaled up version of QCD with the scale $M_{PL} \gg \Lambda_{QCD} \gg \sqrt[3]{M_{EW}^2 M_{PL}} \sim 10^8 \text{ GeV}$ to avoid interference with EW physics.

- The Friedman equation has a de Sitter solution after the phase transition to the confined phase when the topological susceptibility is generated

$$H^2 = \frac{8\pi G}{3}\left(\rho_{\rm Inf} + \rho_R\right) = \frac{8\pi G}{3}\left(\overline{\alpha} H \Lambda_{QCD}^3 + \rho_R\right), \quad H_0 \sim \frac{8\pi G}{3}(\overline{\alpha}\Lambda_{QCD}^3)$$

- This non-dispersive type of energy (the contact term) is linear in "H" and drives the Universe into the de Sitter phase

- The relevant dynamics is governed by some non-propagating auxiliary topological fields without canonical kinetic term; it can not be expressed in terms of any local fields like "inflaton"

- This regime would be the final destination of our Universe if the interaction with SM fields is switched off.

- When the coupling is switched back on, the end of inflation is triggered precisely by this interaction which itself is unambiguously fixed by triangle anomaly.

$$\mathcal{L}_{b\gamma\gamma} = \frac{\alpha(H_0)}{8\pi} NQ^2 \left[\theta - b(x)\right] \cdot F_{\mu\nu}\tilde{F}^{\mu\nu} \ ,$$

- where $b(x)$ is topological non-propagating field and $\alpha(H_0) \sim \alpha_{EW}(H_0) \sim \alpha_s(H_0) \sim 0.1$ is the coupling at inflationary scale

- The number of e-folds in this framework is related to the gauge coupling constant and not to some ad hoc inflaton potential, i.e. $N_{\mathrm{e-folds}} \sim \alpha_s^{-2}(H_0) \sim 10^2$

# *Concluding comments on cosmological applications:*

- We speculate that a liner in "L" correction to the energy could be generated as a result of dynamics of topological configurations with nontrivial holonomy. The idea is <u>tested</u> in "deformed QCD" and in the system defined on hyperbolic space $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$

- It produces an order of magnitude estimate which is consistent with the observations for DE:

$$\rho_{\mathrm{DE}} = \Delta E \sim L^{-1}\Lambda^3_{\mathrm{QCD}} \sim (10^{-3}\mathrm{eV})^4$$

- The same idea can be applied to describe inflation.

- Technically: effect is similar to Aharonov -Bohm effect when the gauge potential $A_\mu$ (rather than $F_{\mu\nu}$) is physically observable. Effect can not be expressed in terms of propagating DoF

*Proposal: Instead of theoretical speculations I suggest to conduct a real tabletop experiment to study this new type of energy:*

- When the Maxwell system is formulated on a non-simply connected manifold there will be an extra contribution to the Casimir pressure, not related to the physical propagating photons with two transverse polarizations

- This setting should be contrasted with conventional setting when the Casimir energy is generated between two conducting plates (trivial holonomy).

- The Maxwell system on a non-trivial manifold shows all signs (degeneracy, etc) which are normally attributed to the topologically ordered systems.

# 6. Maxwell system in 4 dimensions

- It is normally assumed that topology plays no role for abelian Maxwell theory because $\pi_3(U(1))$ is trivial.

- However, if we consider a non-simply connected manifold (for example, a cylinder), or consider external field which enforces a nontrivial boundary conditions than $\pi_1[U(1)] \cong \mathbb{Z}$ plays a role.

- In this case the problem is reduced to the previously studied 2D case when the "instanton fluxes" describe the tunnelling transitions between distinct topological $|k\rangle$ sectors. In particular, instanton-fluxes along z-direction are:

$$\vec{B}_{\rm top} = \vec{\nabla} \times \vec{A}_{\rm top} = \left(0,\ 0,\ \frac{2\pi k}{eL_1 L_2}\right), \qquad \frac{1}{2}\int \mathrm{d}^4x \vec{B}^2_{\rm top} = \frac{2\pi^2 k^2 \beta L_3}{e^2 L_1 L_2}$$

- The action is quadratic such that the cross term vanishes and the quantum fluctuations do not depend on topological sectors "k", i.e. partition function decouples:

$$\int \mathrm{d}^4x\ \vec{B}\cdot\vec{B}_{\text{top}} = \frac{2\pi k}{eL_1L_2}\int \mathrm{d}^4x\ B_z = 0 \qquad\qquad \mathcal{Z} = \mathcal{Z}_0 \times \mathcal{Z}_{\text{top}}.$$

- The topological part of the partition function is non-analytical in coupling $\exp(-1/e^2)$

$$\mathcal{Z}_{\text{top}} = \sqrt{\frac{2\pi\beta L_3}{e^2L_1L_2}}\sum_{k\in\mathbb{Z}} e^{-\frac{2\pi^2k^2\beta L_3}{e^2L_1L_2}} = \sqrt{\pi\tau}\sum_{k\in\mathbb{Z}} e^{-\pi^2\tau k^2} \qquad\qquad \tau \equiv 2\beta L_3/e^2L_1L_2.$$

- Our computations are based on <u>Euclidean</u> path integral, in which case we impose the boundary conditions up to large gauge transformations.

- These fields saturate the path integral but should not be confused with real <u>Minkowski</u> fields with conventional boundary conditions in Hamiltonian approach. Both approaches lead to the same results as consequence of the Poisson summation formula

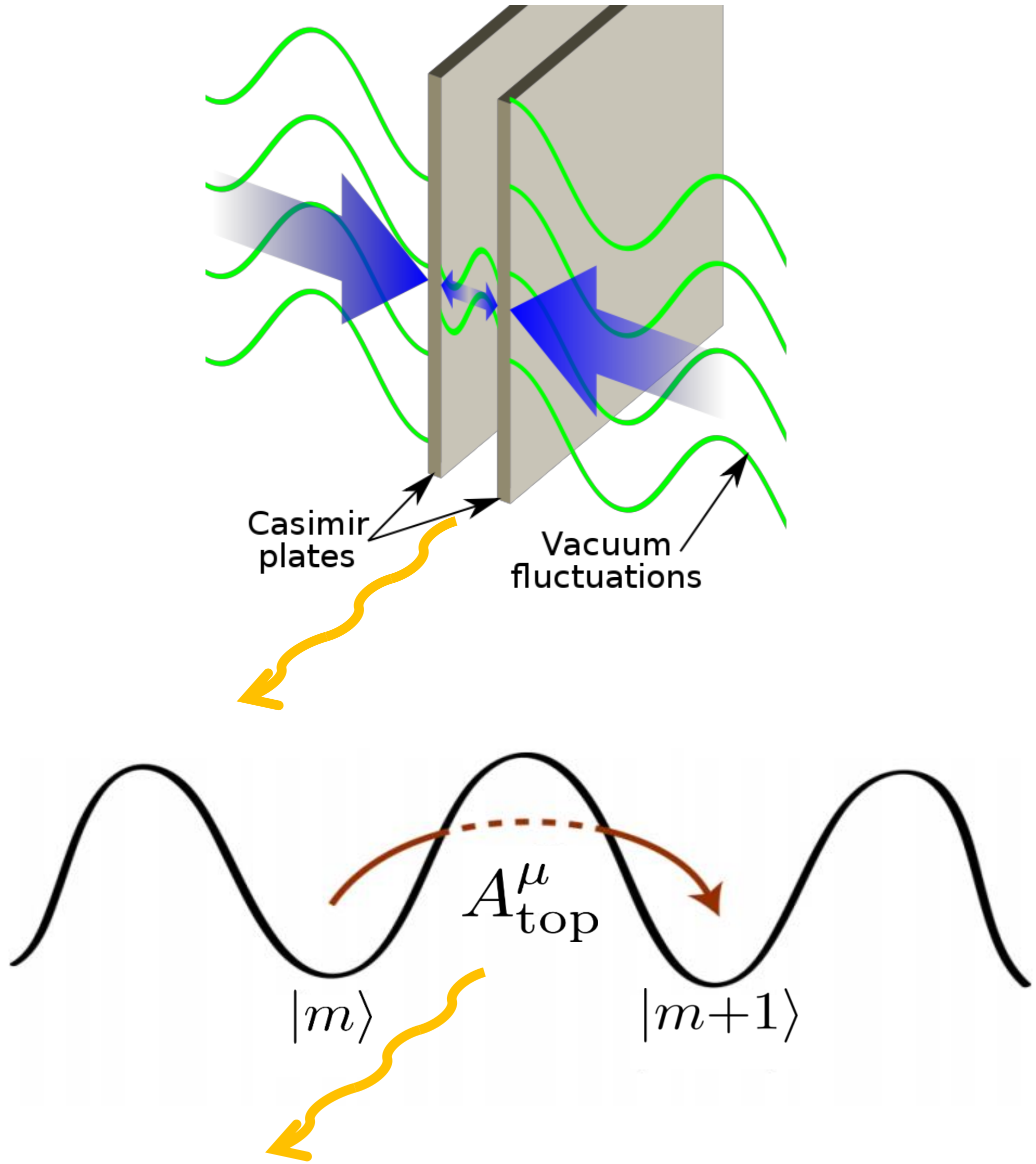
# 7. Topological Casimir effect in a time-dependent background

- The dynamical Casimir Effect (DCE): real physical photons can be radiated from the vacuum due to the time- dependent boundary conditions.

- Our case: the extra energy is generated due to the tunnelling transitions between topological sectors $|m\rangle \to |m+1\rangle$. The emission occurs from E&M configurations describing these transitions.

- Technical obstacle: tunnelling transitions are formulated in terms of Euclidean configurations, while radiation of physical photons is inherently a real-time Minkowskian phenomenon.

- The problem has been resolved by introducing auxiliary topological fields + Wick rotation.




Radiation of real photons due to the tunnelling transitions in a time-dependent background. The $\gamma$ are emitted from E&M configurations interpolating between $|m\rangle$ sectors in contrast with conventional DCE when virtual $\gamma$ become real $\gamma$

- This is exactly solvable Maxwell system with topological terms and auxiliary fields.

- One can explicitly see how the vacuum transfers its "non-dispersive" vacuum energy to emit real physical photons in a time dependent background.

- Technically this problem was motivated by inflation when the vacuum energy should be transferred to the real physical propagating particles, so called "reheating epoch".

- For this simplified system this problem has been solved. The Euclidean tunnelling transitions are reformulated in terms of the auxiliary fields. Their anomalous interaction with physical fields induce the radiation from vacuum ("reheating").

# 8. Applications to the axion search experiments and $\theta_{\rm QED}$ in Maxwell theory

- It is normally assumed that $\theta_{\rm QED}$ in the abelian Maxwell Electrodynamics (in contrast with QCD) is <u>unphysical parameter</u>, and can be always removed from the system (trivial $\pi_3(U(1))$ ).

- A conventional argument is based on observation that $\theta_{\rm QED}(\vec{E}\cdot\vec{B})$ is total derivative for $\theta_{\rm QED} = const$ , does not change the equations of motion.

- This argument is flamed for non-simply connected manifolds (rings with $\mathcal{Z}_{\rm top}(\theta)\neq 1$ as discussed above). It is also incorrect if an external magnetic field enforces the nontrivial boundary conditions.

- it opens up a new perspective with axion searches when effects are proportional to $\theta$ , not $\dot{\theta}\sim m_a$

- Indeed, the $\theta$ term in the background of the magnetic field in the given topological sector $|\kappa\rangle$ can be represented as

$$S_\theta \sim \theta e^2 \int d^4x\ \vec{E}\cdot\vec{B} = \theta \left[e\int d^2x_\perp B_z\right]\cdot\left[e\int dzdtE_z\right] = 2\pi\kappa\ \theta\cdot\left[e\int dzdtE_z\right].$$

- Non-trivial topological sector $\oint_\Gamma A_\mu dx_\mu = 2\pi\kappa$ is enforced by external field similar to AB phase.

- 4d formula for $\theta$ -term is reduced to 2d Schwinger model where $\theta$ is obviously a physical parameter of the system due to the nontrivial $\pi_1[U(1)] = \mathbb{Z}$

- The effect is similar to Witten effect when $\theta$ becomes a physical parameter in the monopole sector and the monopole becomes the dyon with electric charge $e' = -(e\theta/2\pi)$

- In both cases the $\theta$ parameter becomes a physically observable parameter not in vacuum but in a heavy topological sector (monopole's charge in Witten's case, magnetic flux in our case)

- In both cases the topology is enforced by some external fields, in contrast with our previous discussions when the topology is enforced by non-simply connected manifolds (rings).

- The effect is proportional to $\theta$ without tunnelling suppression factor $\exp(-1/e^2)$

- it opens up a new perspective with axion searches because the effects are proportional to static $\theta$ and because it is proportional to strength of the magnetic field (topological sector $\kappa$ )

- In particular, if we take a cylinder in the background of the magnetic field, than in the presence of the passing axion $\theta(x)$ the electric field will be induced along the magnetic field

$$\langle \vec{E} \rangle_{\rm ind} = -\frac{K_{a\gamma\gamma}\alpha}{\pi}\theta \vec{B}_{\rm ext},$$

- If we place the plates at the ends of the cylinder this induced field will induce the charges on plates

$$\langle Q \rangle \sim \frac{e\theta(t)}{2\pi} K_{a\gamma\gamma} \cdot \left[\frac{eB_{\rm ext}L_1L_2}{2\pi}\right].$$

- This charge separation effect due to $\theta \neq 0$ generates the potential difference $\langle \Delta V \rangle$

$$\langle \Delta V \rangle \simeq \frac{e\theta K_{a\gamma\gamma}L_3}{2\pi L_1 L_2} \cdot \left[\frac{eB_{\rm ext}L_1L_2}{2\pi}\right] \sim 0.2 K_{a\gamma\gamma}\theta \cdot \left(\frac{L_3}{\rm mm}\right) \cdot \left(\frac{B_{\rm ext}}{\rm Gauss}\right) ({\rm volt}).$$

- If we connect two plates with a wire there will be induced current due to induced charges

$$\langle J \rangle \sim \frac{\langle Q \rangle c}{L_3}\left(\frac{v}{c}\right) \approx 10^{-6}\cdot\left(\frac{\theta K_{a\gamma\gamma}}{10^{-14}}\right)\cdot\left(\frac{L_1 L_2/L_3}{\mathrm{mm}}\right)\cdot\left(\frac{B_{\mathrm{ext}}}{1\mathrm{T}}\right)\cdot\left(\frac{v}{c}\right)\mathrm{nA}$$

- The dual picture with electric external field suggests that the magnetic field will be induced in the presence of $E_{\mathrm{ext}}$,

$$\langle B \rangle_{\mathrm{ind}} = \frac{\theta K_{a\gamma\gamma}\alpha}{\pi}E_{\mathrm{ext}},$$

- The induced magnetic field can be interpreted as the surface persistent current on the ring

$$\langle J \rangle \simeq \frac{\theta K_{a\gamma\gamma}\alpha}{\pi}L_3 E_{\mathrm{ext}} \sim 10^{-6}\cdot\left(\frac{\theta K_{a\gamma\gamma}}{10^{-14}}\right)\cdot\left(\frac{E_{\mathrm{ext}}}{10^{5}\,\frac{\mathrm{V}}{\mathrm{cm}}}\right)\cdot\left(\frac{L_3}{\mathrm{mm}}\right)\mathrm{nA}.$$

# 9. Conclusion. Speculations.

- <u>There is a fundamentally new type of the vacuum energy</u> which <u>can not</u> be expressed in terms of the scattering amplitudes (the S-matrix elements).

- It emerges as a result of tunnelling processes between degenerate topological sectors, and formulated in terms of the "non-dispersive" contact terms and nonlocal holonomy.

- We identify this new type of energy (generated in QCD) with cosmological vacuum energy, e.g.

$$\rho_{\mathrm{DE}} = \Delta E \sim L^{-1} \Lambda^3_{\mathrm{QCD}} \sim (10^{-3}\mathrm{eV})^4$$

- Interpretation: the QCD vacuum energy is slightly different from its Minkowski value due to expansion or/and topology. This difference $\Delta E$ is what we observe (similar to the Casimir vacuum energy)

- We suggest to test these ideas in a tabletop experiments in 4D Maxwell system defined on a nontrivial manifold.

- A deep reason of why all these new effects emerge is that a conventional quantization procedure does not remove all unphysical dof: instead, the so-called Gribov's ambiguities will also emerge in QED (phenomena is known to mathematicians) when it is formulated on a <u>nontrivial manifold</u>.

- $\theta_{\rm QED}$ is <u>physical parameter</u> (similar to $\theta$ in QCD) in the presence of external magnetic flux

- This 4d maxwell system is highly sensitive to the $\theta$ parameter (axion). It opens up a new perspective in the axons search experiments as the effects are directly proportional to the $\theta$.