Cosmic Evolution and Structure Growth

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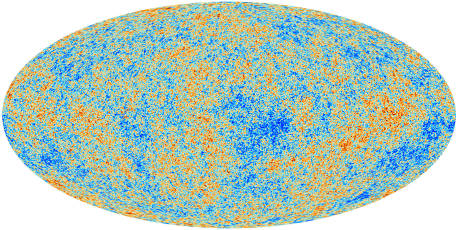
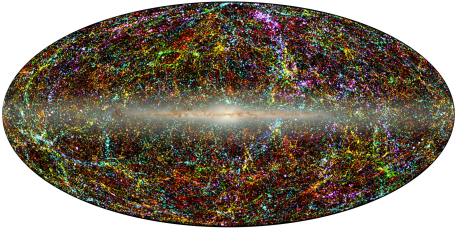
Cosmic structures

How to get from here. . .



Planck 2015

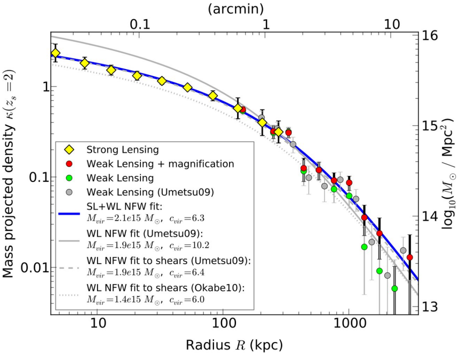
. . . to there?



2-MASS

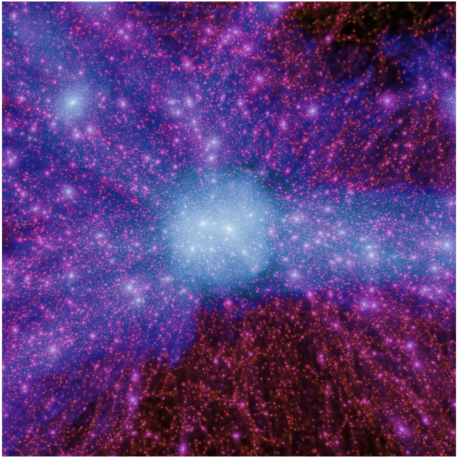
Problems in cosmic structure formation

Abell 2261, CLASH Project

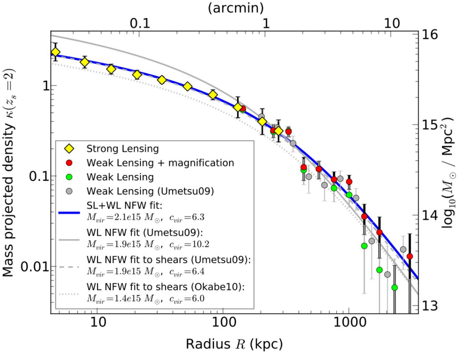


Coe et al. 2012

Problems in cosmic structure formation



Boylan-Kolchin et al.



Coe et al. 2012

Expansion, parameters, horizon

- $a(t)$ is described by Friedmann's equation

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_{\mathrm{r}0}a^{-4} + \Omega_{\mathrm{m}0}a^{-3} + \Omega_{\Lambda 0} + \Omega_{\mathrm{K}}a^{-2}\right]$$

- Backward in time, radiation overtakes matter density at

$$a_{\mathrm{eq}} = \frac{\Omega_{\mathrm{r}0}}{\Omega_{\mathrm{m}0}} = (8.3 \pm 1.1) \times 10^{-5}$$

- Important for structure formation is the horizon at $a = a_{\mathrm{eq}}$:

$$r_{\mathrm{H,eq}} = \frac{c}{H_0}\frac{a_{\mathrm{eq}}^{3/2}}{\sqrt{2\Omega_{\mathrm{m}0}}}$$

Perturbation equations

- Conventional approach: hydrodynamical equations, Newtonian gravity
- Linearized, in comoving coordinates:

$$\dot{\delta} + \vec{\nabla} \cdot \vec{u} = 0$$

$$\dot{\vec{u}} + H\vec{u} = -\frac{\vec{\nabla}\delta p}{a^2 \rho_0} + \frac{\vec{\nabla}\delta\phi}{a^2}$$

$$\nabla^2 \delta\phi = 4\pi G \rho_0 a^2 \delta$$

  for δ, $\vec{u}$ and $\delta\phi$

- Equation of state to relate pressure to density fluctuations (sound speed c_s):

$$\delta p = \delta p(\delta) = c_s^2 \delta\rho = c_s^2 \rho_0 \delta$$

Density perturbations

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- Combined Euler and continuity equations:

$$\ddot{\delta} + 2H\dot{\delta} = \delta\left(4\pi G\rho_0 - \frac{c_s^2 k^2}{a^2}\right)$$

  Oscillations for $k \geq k_J$: Jeans scale; H causes expansion drag

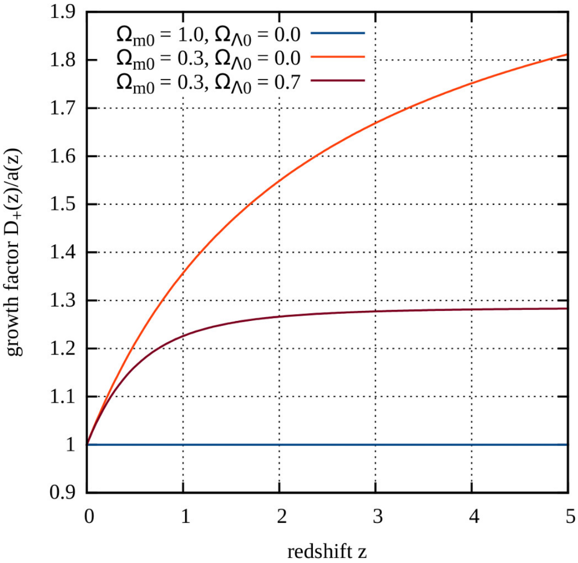
- For $k \ll k_J$, $\Omega_{m0} = 1$:

$$\ddot{\delta} + 2H\dot{\delta} = H^2\delta \cdot \begin{cases} 4 & \text{radiation era} \\ 3/2 & \text{matter era} \end{cases}$$

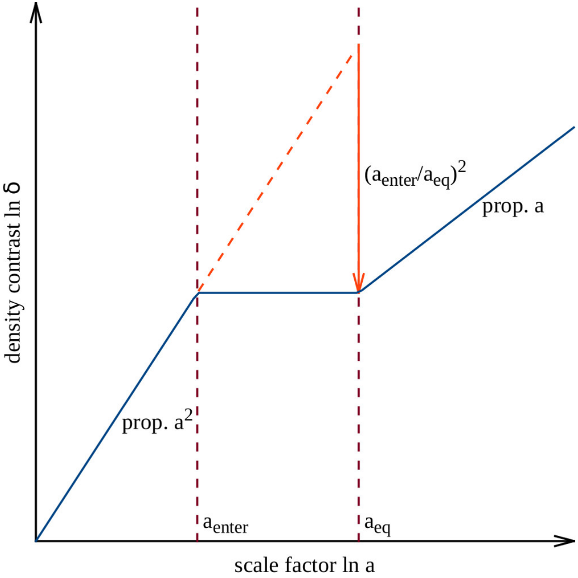
- Simple solutions for Einstein-de Sitter:

$$\delta_+ = \begin{cases} a^2 & (a < a_{eq}) \\ a & (a > a_{eq}) \end{cases}, \quad \delta_- = \begin{cases} a^{-2} & (a < a_{eq}) \\ a^{-3/2} & (a > a_{eq}) \end{cases}$$

Density perturbations



Growth suppression



CDM power spectrum

- Modes are suppressed by

$$f_{\rm sup} = \left(\frac{a_{\rm enter}}{a_{\rm eq}}\right)^2 = \left(\frac{k_0}{k}\right)^2$$

- Initial power spectrum $P_{\rm i}(k)$ is scale-free if

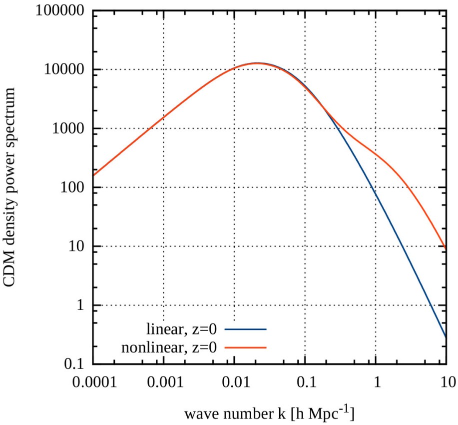
$$P_{\rm i}(k) \propto k$$

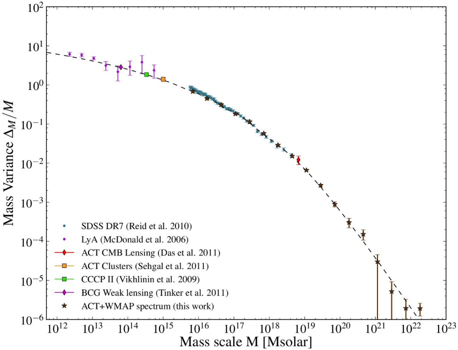
(Harrison-Zel'dovich-Peebles spectrum, inflation)

- Suppression leads to

$$P(k) = P_{\rm i}(k) f^2(k) \propto \begin{cases} k & (k < k_0) \\ k^{-3} & (k \gg k_0) \end{cases}$$

CDM power spectrum





Hlozek et al. 2011

Particle trajectories and pancakes

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- Zel'dovich approximation, particle trajectories:

$$\vec{r} = a\left(\vec{x} + D_+(a)\vec{f}\right) \ , \quad \vec{f} = \vec{\nabla}\psi$$

- Displacement field

$$\vec{f} = \frac{\vec{u}}{HD_+ f(\Omega_\mathrm{m})} \ , \quad \frac{\partial f_i}{\partial x_j} = f_{ij} = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$$

- For Gaussian random field, probability distribution of $\{\lambda_i\}$ is

$$p(\lambda_1, \lambda_2, \lambda_3) \propto |(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)(\lambda_2 - \lambda_1)|$$

- Probability for two equal eigenvalues of f_{ij} is zero: isotropic collapse is excluded!

Peculiar motion

- Density perturbations δ cause displacements via peculiar motion

$$\delta\vec{x} = \frac{\vec{r}}{a} - \vec{x} = \frac{\vec{u}}{Hf(\Omega_\mathrm{m})}$$

- Distance to a galaxy is inferred from its line-of-sight velocity

$$v = \vec{v}\cdot\vec{e}_z = a\left(H\vec{x} + \vec{u}\right)\cdot\vec{e}_z$$

- Interpreting $\vec{v}$ as Hubble flow implies apparent displacement:

$$\delta\vec{x}_\mathrm{app} = \delta\vec{x}_\mathrm{real} + f(\Omega_\mathrm{m})\left(\delta\vec{x}_\mathrm{real}\cdot\vec{e}_z\right)\,\vec{e}_z$$

- Displacements give apparent density contrast

$$\delta_\mathrm{app} = \delta_\mathrm{real}\left(1 + f(\Omega_\mathrm{m})\mu^2\right)\,, \quad \mu := \vec{k}\cdot\vec{e}_z/k$$

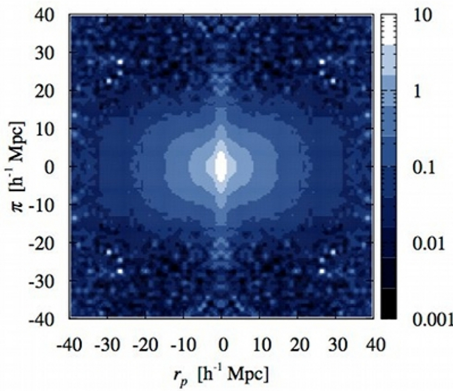
Redshift-space distortions

- Apparent density contrast in galaxy counts:

$$\delta_{\text{app}}^{\text{gal}} = \delta_{\text{real}}^{\text{gal}} \left(1 + \frac{f(\Omega_{\text{m}})\mu^2}{b}\right)$$

- Power spectra are related by

$$\frac{P_{\text{app}}}{P_{\text{real}}} = \left(1 + \beta\mu^2\right)^2 \,, \quad \beta := \frac{f(\Omega_{\text{m}})}{b}$$



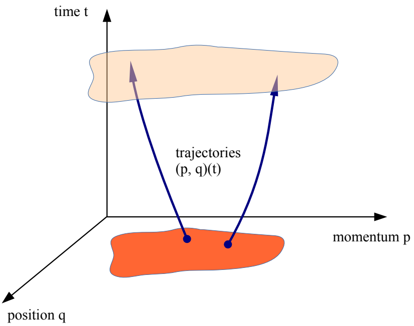
VIPERS

Analytic theory for cosmic structure formation

Conventional approaches:

- Standard perturbation theory: based on hydrodynamic equations, path-integral formalism, effective field theory
- Lagrangian perturbation theory: based on Zel'dovich approximation, higher orders

Analytic theory for cosmic structure formation



New approach:

- Non-equilibrium statistics of N classical particle trajectories
- Describe particle ensemble by partition sum (generating functional) Z
- Derive statistical properties by functional derivatives

Phase-space trajectories

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- Classical particles follow Hamiltonian equations of motion,

$$\dot{q} = \frac{\partial \mathcal{H}}{\partial p}\,, \qquad \dot{p} = -\frac{\partial \mathcal{H}}{\partial q}\,, \qquad x = (q,p)$$

- Trajectories are described by (retarded) Green's function $G_{\mathrm{R}}(t,t')$,

$$\bar{x}(t) = \underbrace{G_{\mathrm{R}}(t,0)x^{(\mathrm{i})}}_{\text{free motion}} - \underbrace{\int_0^t G_{\mathrm{R}}(t,t')K(t')\mathrm{d}t'}_{\text{interaction}}$$

- In static space, with potential v,

$$G_{\mathrm{R}}(t,t') = \begin{pmatrix} 1 & (t-t')/m \\ 0 & 1 \end{pmatrix}, \qquad K = \begin{pmatrix} 0 \\ \nabla v \end{pmatrix}$$

Principle

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Non-equilibrium, statistical theory for classical degrees of freedom:

$$Z_0[J,K] = \int \mathrm{d}\Gamma^{(\mathrm{i})}\, \mathrm{e}^{\mathrm{i}\int \langle J, \bar{x}\rangle \mathrm{d}t}$$

$$Z[H,J,K] = \mathrm{e}^{\mathrm{i}\hat{S}_\mathrm{I}} \mathrm{e}^{\mathrm{i}H\hat{\Phi}} Z_0[J,K]$$

$$\langle \rho(1)\rho(2)\rangle = \frac{\delta}{\mathrm{i}\delta H_\rho(1)} \frac{\delta}{\mathrm{i}\delta H_\rho(2)} Z$$

Principle

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Interaction operator:

$$\hat{S}_\mathrm{I} = \int \mathrm{d}1 \int \mathrm{d}2\, \hat{\rho}(1)\vec{\nabla} v(12)\hat{\rho}(2)$$

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Interaction operator:

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Perturbation theory:

$$\mathrm{e}^{\mathrm{i}\hat{S}_\mathrm{I}} = 1 + \mathrm{i}\hat{S}_\mathrm{I} - \frac{1}{2}\hat{S}_\mathrm{I} + \dots$$

Specialisation to cosmology

1. Choose initial phase-space measure

$$\mathrm{d}\Gamma^{(\mathrm{i})} = P(q,p)\,\mathrm{d}^{3N}q\,\mathrm{d}^{3N}p$$

   fully specified by initial power spectrum

2. Change time coordinate

$$t \to \tau = D_+(t) - D_+(t_\mathrm{i})$$

3. Adapt Green's function to expanding universe

Density power spectrum

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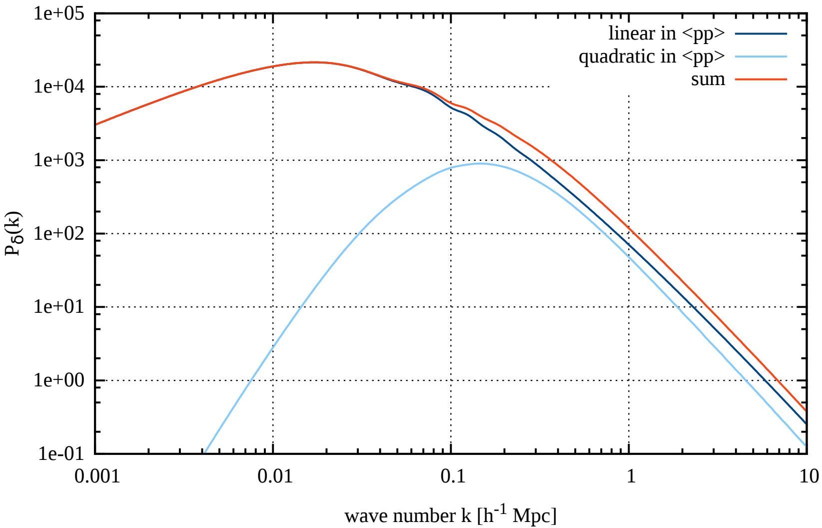
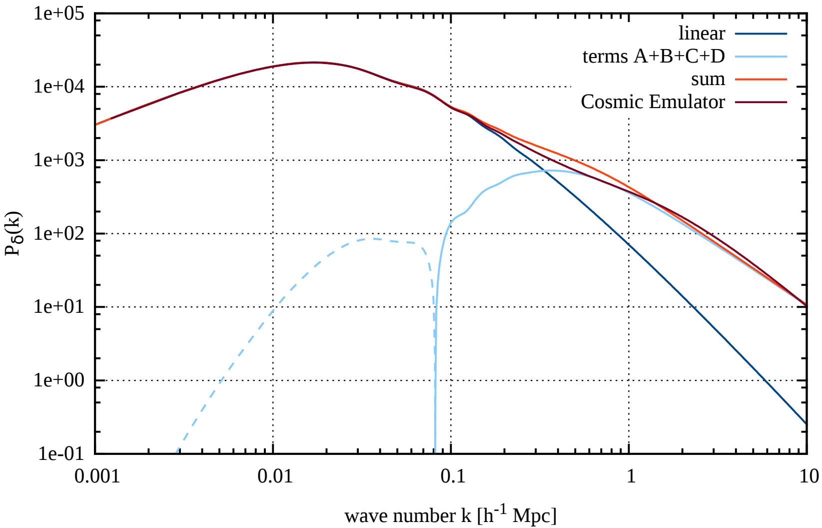


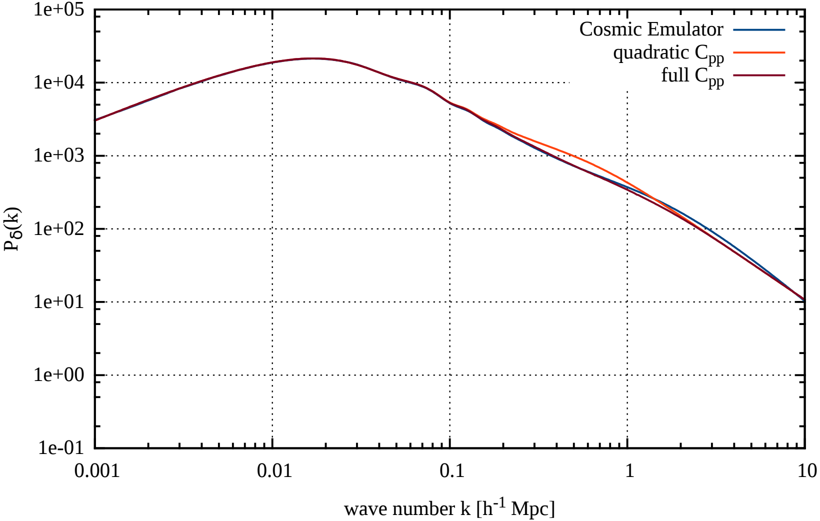
Improved Zel'dovich trajectories, no interaction

Density power spectrum



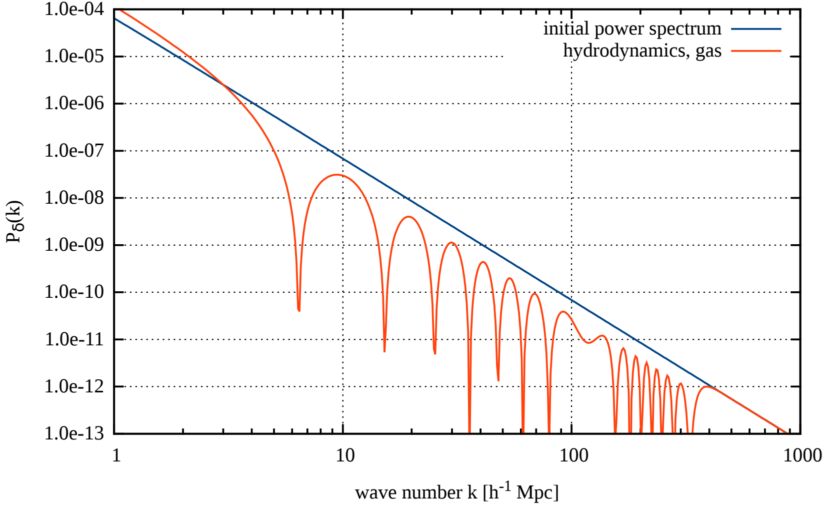
Improved Zel'dovich trajectories plus first-order interaction

Density power spectrum



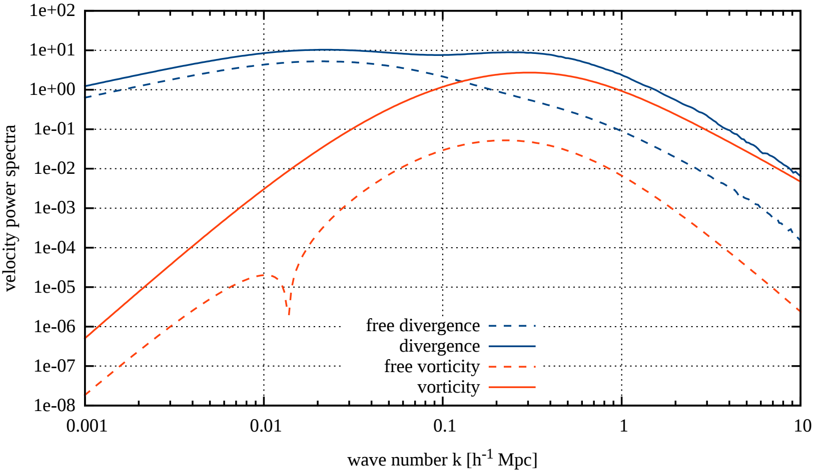
MB et al. 2016

Density power spectrum including gas



D. Geiss, F. Fabis, R. Lilow, C. Viermann 2017

Velocity power spectra



C. Littek et al. 2017

Factorization

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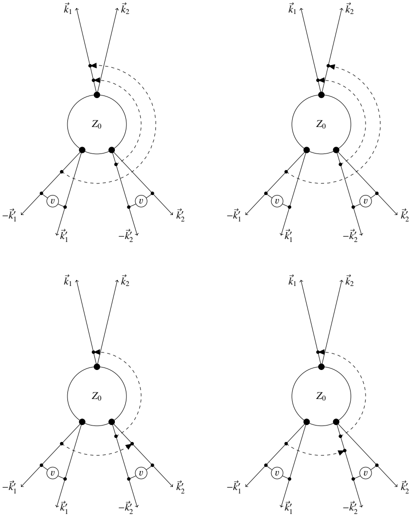
Free generating functional can be completely factorized:

$$Z_0[J,K] = \mathcal{N} \underbrace{\int \cdots \int}_{\vec{k}'} \prod (1+\mathcal{P})$$

Allows diagrammatic approach to ordering perturbation terms



MB et al. 2017

Summary

- Initial conditions and final state well known
- Linear structure formation well understood
- Non-linear structures numerically well accessible
- Analytic approach essential for fundamental understanding, predictions and interpretation of observations
- New approach based on non-equilibrium classical field theory
- First-order perturbation theory well reproduces numerical results
- Mixtures of gas and dark matter can be treated in the same way
- Velocity power spectra calculated
- Free generating functional completely factorized