Can effects of quantum gravity be observed in the cosmic microwave background?

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Introduction

Semiclassical approximation

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Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, "What is the question?"
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals, . . .)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, . . .)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, . . .);
  have partially grown out of the other approaches

Quantum gravitational corrections in the covariant approach

One-loop corrections to the non-relativistic potentials obtained from the scattering amplitude by calculating the non-analytic terms in the momentum transfer

- Quantum gravitational correction to the Newtonian potential

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + \underbrace{3\frac{G(m_1+m_2)}{rc^2}}_{\mathrm{GR-correction}} + \underbrace{\frac{41}{10\pi}\frac{G\hbar}{r^2c^3}}_{\mathrm{QG-correction}}\right)$$

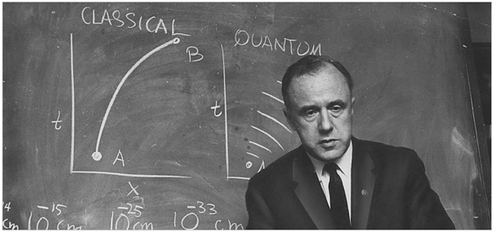
(Bjerrum-Bohr *et al.* 2003)

- Quantum gravitational effects to the Coulomb potential (scalar QED)

$$V(r) = \frac{Q_1Q_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{6}{\pi}\frac{G\hbar}{r^2c^3}\right) + \dots$$

(Faller 2008)

Quantum geometrodynamics



(a) John Archibald Wheeler

(b) Bryce DeWitt

- *Question*: what is the quantum wave equation that immediately gives Einstein's equations in the semiclassical limit?
- *Answer*: the Wheeler–DeWitt equation

$$\boxed{\hat{H}\Psi = 0}$$

Constraints of this type also occur in loop quantum gravity

Semiclassical (Born–Oppenheimer type) approximation

Ansatz:

$$|\Psi[h_{ab}]\rangle = C[h_{ab}]\mathrm{e}^{\mathrm{i}m_{\mathrm{P}}^2 S[h_{ab}]}|\psi[h_{ab}]\rangle$$

and expansion with respect to the Planck-mass squared.

Highest order: One evaluates $|\psi[h_{ab}]\rangle$ along a solution of the classical Einstein equations, $h_{ab}(\mathbf{x}, t)$, corresponding to a solution, $S[h_{ab}]$, of the Hamilton–Jacobi equations;

$$\dot{h}_{ab} = NG_{abcd}\frac{\delta S}{\delta h_{cd}} + 2D_{(a}N_{b)}$$

$$\frac{\partial}{\partial t}|\psi(t)\rangle := \int \mathrm{d}^3x\,\dot{h}_{ab}(\mathbf{x},t)\,\frac{\delta}{\delta h_{ab}(\mathbf{x})}|\psi[h_{ab}]\rangle$$

This leads to a functional Schrödinger equation for quantized matter fields in the chosen external classical gravitational field:

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial}{\partial t}|\psi(t)\rangle &= \hat{H}^{\mathrm{m}}|\psi(t)\rangle \\ \hat{H}^{\mathrm{m}} &:= \int \mathrm{d}^3x\left\{N(\mathbf{x})\hat{\mathcal{H}}^{\mathrm{m}}_{\perp}(\mathbf{x}) + N^a(\mathbf{x})\hat{\mathcal{H}}^{\mathrm{m}}_a(\mathbf{x})\right\} \end{aligned}$$

$\hat{H}^{\mathrm{m}}$: matter-field Hamiltonian in the Schrödinger picture, parametrically depending on (generally non-static) metric coefficients of the curved space–time background.

WKB time t controls the dynamics in this approximation

Quantum gravitational corrections

The next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^2} \times (\text{various terms})$$

(C.K. and T.P. Singh (1991); A. O. Barvinsky and C.K. (1998))

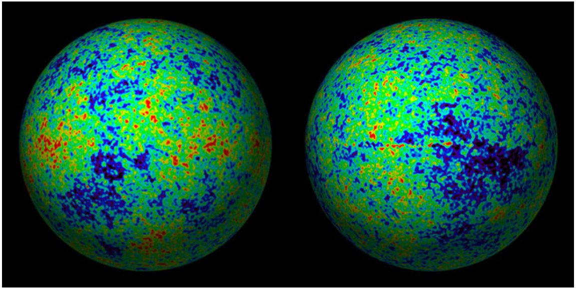
Example: Quantum gravitational correction to the trace anomaly in de Sitter space:

$$\delta\epsilon \approx -\frac{2G\hbar^2 H_{\mathrm{dS}}^6}{3(1440)^2\pi^3 c^8}$$

(C.K. 1996)

Observations

Does the anisotropy spectrum of the cosmic background radiation contain information about quantum gravity?



C.K. and M. Krämer, *Phys. Rev. Lett.*, **108**, 021301 (2012);

D. Bini, G. Esposito, C.K., M. Krämer, and F. Pessina, *Phys. Rev. D*, **87**, 104008 (2013); D. Brizuela, C.K., M. Krämer, *ibid.* **93**, 104035 (2016); *ibid.* **94**, 123527 (2016).

Perturbed inflationary universe

- flat Friedmann-Lemaître universe plus fluctuations
- massive scalar field ϕ with potential $\mathcal{V}(\phi)$
- use conformal time, $\mathrm{d}\eta/\mathrm{d}t = a^{-1}$
- $\hbar = c = 1$; $m_{\mathrm{P}} = \sqrt{3/4\pi G} \approx 0.60 \times 10^{19}$ GeV
- metric:

$$\begin{aligned} \mathrm{d}s^2 &= a^2(\eta)\Big\{-(1-2A)\,\mathrm{d}\eta^2 + 2\,(\partial_i B)\,\mathrm{d}x^i\mathrm{d}\eta \\ &\quad + \left[(1-2\psi)\,\delta_{ij} + 2\partial_i\partial_j E + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\Big\} \end{aligned}$$

- combine scalar perturbations $\varphi(\eta, \mathbf{x})$ with scalar metric perturbations to get the gauge-invariant Mukhanov-Sasaki variable:

$$v(\eta, \mathbf{x}) := a\left[\varphi + \frac{\phi'}{\mathscr{H}}\left(A + 2\,\mathscr{H}\,(B - E') + [B - E']'\right)\right],$$

where $\mathscr{H} := a'/a$

$$S = \frac{1}{2} \int \mathrm{d}\eta \left\{ \mathfrak{L}^3 \left[-\frac{3}{4\pi G} (a')^2 + a^2 (\phi')^2 - 2\, a^4\, \mathcal{V}(\phi) \right] \right.$$

$$+ \frac{1}{\mathfrak{L}^3} \sum_{\mathbf{k}} \left[v'_{\mathbf{k}} v^{*\prime}_{\mathbf{k}} + {}^{\mathsf{S}}\omega^2_{\mathbf{k}}\, v_{\mathbf{k}} v^*_{\mathbf{k}} \right]$$

$$\left. + \frac{1}{\mathfrak{L}^3} \sum_{\lambda=+,\times} \sum_{\mathbf{k}} \left[v^{(\lambda)\prime}_{\mathbf{k}} v^{(\lambda)*\prime}_{\mathbf{k}} + {}^{\mathsf{T}}\omega^2_{\mathbf{k}}\, v^{(\lambda)}_{\mathbf{k}} v^{(\lambda)*}_{\mathbf{k}} \right] \right\}$$

- $v_{\mathbf{k}}$: Fourier transform of $v(\eta, \mathbf{x})$; Fourier-transformed perturbation variable of the gauge-invariant tensor perturbations h_{ij} with polarization $\lambda \in \{+, \times\}$: $v^{(\lambda)}_{\mathbf{k}} := \frac{a\, h^{(\lambda)}_{\mathbf{k}}}{\sqrt{16\pi G}}$
-
$$ {}^{\mathsf{S}}\omega^2_{\mathbf{k}}(\eta) := k^2 - \frac{z''}{z}\,, \quad {}^{\mathsf{T}}\omega^2_{\mathbf{k}}(\eta) := k^2 - \frac{a''}{a}\,, $$

  where $z := a\, \phi' / \mathscr{H}$.

$\mathfrak{L}$: maximum length scale (IR cutoff); remove this by the following redefinitions:

$$a_{\mathsf{new}} = a_{\mathsf{old}}\,\mathfrak{L}\,,\ \eta_{\mathsf{new}} = \frac{\eta_{\mathsf{old}}}{\mathfrak{L}}\,,\ k_{\mathsf{new}} = k_{\mathsf{old}}\,\mathfrak{L}\,,\ v_{\mathsf{new}} = \frac{v_{\mathsf{old}}}{\mathfrak{L}^2}$$

Now, a has the dimension of a length, whereas η, k, and $v_{\mathbf{k}}$ are dimensionless.

Canonical quantization and choosing a product ansatz for the wave function leads to

$$\frac{1}{2}\left\{\mathrm{e}^{-2\alpha}\left[\frac{1}{m_{\mathsf{P}}^2}\,\frac{\partial^2}{\partial\alpha^2}-\frac{\partial^2}{\partial\phi^2}+2\,\mathrm{e}^{6\alpha}\,\mathcal{V}(\phi)\right]-\frac{\partial^2}{\partial v_{\mathbf{k}}^2}+\omega_{\mathbf{k}}^2(\eta)\,v_{\mathbf{k}}^2\right\}\Psi_{\mathbf{k}}(\alpha,\phi,v_{\mathbf{k}})=0,$$

where $\alpha := \ln(a/a_0)$

Born-Oppenheimer approximation

Rescale $\tilde{\phi} := m_{\mathrm{P}}^{-1}\phi$ and perform the expansion

$$\Psi_{\mathbf{k}} = \exp\Big[\mathrm{i}\left(m_{\mathrm{P}}^2\, S_0 + m_{\mathrm{P}}^0\, S_1 + m_{\mathrm{P}}^{-2}\, S_2 + \dots\right)\Big]$$

- $\mathcal{O}(m_{\mathrm{P}}^4)$: S_0 is independent of $v_{\mathbf{k}}$
- $\mathcal{O}(m_{\mathrm{P}}^2)$: S_0 obeys the Hamilton–Jacobi equation of the minisuperspace background
- $\mathcal{O}(m_{\mathrm{P}}^0)$: After the definition of WKB time according to

$$\frac{\partial}{\partial \eta} := \mathrm{e}^{-2\alpha}\left[-\frac{\partial S_0}{\partial \alpha}\frac{\partial}{\partial \alpha} + m_{\mathrm{P}}^2\,\frac{\partial S_0}{\partial \phi}\frac{\partial}{\partial \phi}\right],$$

one finds that each $\psi_k^{(0)}$ obeys a Schrödinger equation

$$\mathcal{H}_{\mathbf{k}}\psi_{\mathbf{k}}^{(0)} = \mathrm{i}\,\frac{\partial}{\partial \eta}\,\psi_{\mathbf{k}}^{(0)}.$$

- $\mathcal{O}(m_{\mathrm{P}}^{-2})$: corrected Schrödinger equation

$$\mathrm{i}\,\frac{\partial}{\partial\eta}\,\psi_{\mathbf{k}}^{(1)} = \mathcal{H}_{\mathbf{k}}\psi_{\mathbf{k}}^{(1)} - \frac{\psi_{\mathbf{k}}^{(1)}}{2\,m_{\mathrm{P}}^{2}\,\psi_{\mathbf{k}}^{(0)}}\left[\frac{\left(\mathcal{H}_{\mathbf{k}}\right)^{2}}{V}\,\psi_{\mathbf{k}}^{(0)} + \mathrm{i}\,\frac{\partial}{\partial\eta}\left(\frac{\mathcal{H}_{\mathbf{k}}}{V}\right)\psi_{\mathbf{k}}^{(0)}\right],$$

  where

$$V(a,\phi) := \frac{2\,a^{4}}{m_{\mathrm{P}}^{2}}\,\mathcal{V}(\phi),$$

  which has the dimension of a length squared.

- Gaussian ansatz:

$$\psi_{\mathbf{k}}^{(0,1)}(\eta, v_{\mathbf{k}}) = N_{\mathbf{k}}^{(0,1)}(\eta)\,\mathrm{e}^{-\frac{1}{2}\,\Omega_{\mathbf{k}}^{(0,1)}(\eta)\,v_{\mathbf{k}}^{2}}$$

- Initial condition: Bunch-Davies vacuum

Power spectrum

- Scalar perturbations

$$\mathcal{P}_{\mathsf{S}}^{(1)}(k) = \frac{4\pi G}{a^2\,\epsilon}\,\frac{k^3}{2\pi^2}\,\frac{1}{2\,\mathfrak{Re}\,{}^{\mathsf{S}}\Omega_{\mathbf{k}}^{(1)}} \approx \mathcal{P}_{\mathsf{S}}^{(0)}(k)\Big\{1+\Delta_{\mathsf{S}}\Big\},$$

with $\Delta_{\mathsf{S}} := -\frac{\mathfrak{Re}\,{}^{\mathsf{S}}\widetilde{\Omega}_{\mathbf{k}}^{(1)}}{\mathfrak{Re}\,{}^{\mathsf{S}}\Omega_{\mathbf{k}}^{(0)}}$ to be evaluated in the limit of super-Hubble scales (or late times) given by $k\eta \to 0^-$ ("frozen perturbations")

- Tensor perturbations

$$\mathcal{P}_{\mathsf{T}}^{(1)}(k) = \frac{64\pi G}{a^2}\,\frac{k^3}{2\pi^2}\,\frac{1}{2\,\mathfrak{Re}\,{}^{\mathsf{T}}\Omega_{\mathbf{k}}^{(1)}} \approx \mathcal{P}_{\mathsf{T}}^{(0)}(k)\Big\{1+\Delta_{\mathsf{T}}\Big\}$$

- Tensor-to-scalar ratio:

$$r^{(1)} := \frac{\mathcal{P}_{\mathsf{T}}^{(1)}(k)}{\mathcal{P}_{\mathsf{S}}^{(1)}(k)} \approx r^{(0)}\,(1+\Delta_{\mathsf{T}}-\Delta_{\mathsf{S}})$$

The slow-roll approximation

- Slow-roll parameters:

$$\epsilon := -\frac{\dot{H}}{H^2} = 1 - \frac{\mathscr{H}'}{\mathscr{H}^2}, \quad \delta := \epsilon - \frac{\dot{\epsilon}}{2H\epsilon} = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

- Mode frequencies:

$$\begin{aligned} {}^{\mathsf{S}}\omega_{\mathbf{k}}^2(\eta) &= k^2 - \frac{2+3\gamma}{\eta^2} + \mathcal{O}(2), \\ {}^{\mathsf{T}}\omega_{\mathbf{k}}^2(\eta) &= k^2 - \frac{2+3\epsilon}{\eta^2} + \mathcal{O}(2), \end{aligned}$$

where $\gamma := 2\epsilon - \delta$. Note that setting $\delta = \epsilon$, or equivalently $\gamma = \epsilon$, converts the equation for scalar perturbations into the one for tensor perturbations.

Uncorrected power spectra

The calculation gives the standard result for the power spectra:

- Scalar modes:

$$\mathcal{P}_{\mathsf{S}}^{(0)}(k) = \frac{G\,H_k^2}{\pi\,\epsilon}\left[1 - 2\epsilon + \gamma(4 - 2\gamma_{\mathsf{E}} - 2\ln(2))\right]$$

- Tensor modes:

$$\mathcal{P}_{\mathsf{T}}^{(0)}(k) = \frac{16\,G\,H_k^2}{\pi}\left[1 - 2\epsilon + \epsilon(4 - 2\gamma_{\mathsf{E}} - 2\ln(2))\right],$$

where $\gamma_{\mathsf{E}} \simeq 0.5772$ is the Euler–Mascheroni constant, $H_k = k/a$, and the result should be evaluated at the horizon exit of the modes. Note that this is already a quantum-gravitational effect (tree level)!

Corrected power spectra

The quantum-gravitational corrections to the power spectra can be put into the form

$$\Delta_{\mathsf{S}} = \frac{H_k^2}{k^3 m_{\mathsf{P}}^2} \left[\beta_{\mathsf{dS}} + \epsilon\, \beta_\epsilon + \gamma\, \beta_\gamma\right]$$

and

$$\Delta_{\mathsf{T}} = \frac{H_k^2}{k^3 m_{\mathsf{P}}^2} \left[\beta_{\mathsf{dS}} + \epsilon\, (\beta_\epsilon + \beta_\gamma)\right],$$

where the β's are k-independent numbers that have to be determined numerically (except β_{dS}, which can be calculated analytically). Note the breaking of scale invariance!

Results and observability

Recall

$$\mathcal{P}_{\mathsf{S}}^{(1)}(k) = \mathcal{P}_{\mathsf{S}}^{(0)}(k)\Big\{1 + \Delta_{\mathsf{S}}\Big\},$$
$$\mathcal{P}_{\mathsf{T}}^{(1)}(k) = \mathcal{P}_{\mathsf{T}}^{(0)}(k)\Big\{1 + \Delta_{\mathsf{T}}\Big\}$$

We find for the quantum-gravitational corrections

$$\Delta_{\mathsf{S}} = \frac{H_k^2}{m_{\mathsf{P}}^2}\left(\frac{\bar{k}}{k}\right)^3 \big(0.988 + 3.14\,\epsilon - 2.56\,\delta\big),$$
$$\Delta_{\mathsf{T}} = \frac{H_k^2}{m_{\mathsf{P}}^2}\left(\frac{\bar{k}}{k}\right)^3 \big(0.988 + 0.58\,\epsilon\big),$$

where $\bar{k} := 1/\mathfrak{L}$. This leads to an enhancement of power at large scales.

$$\begin{aligned}
\mathcal{P}_{\mathsf{S}}^{(1)}(k) &= A_{\mathsf{S}} \left(\frac{k}{k_*}\right)^{n_{\mathsf{S}}-1+\alpha_{\mathsf{S}}\ln(k/k_*)}, \\
\mathcal{P}_{\mathsf{T}}^{(1)}(k) &= A_{\mathsf{T}} \left(\frac{k}{k_*}\right)^{n_{\mathsf{T}}+\alpha_{\mathsf{T}}\ln(k/k_*)},
\end{aligned}$$

where k_* is the pivot scale.

Spectral index and r-parameter

$$n_{\mathsf{S}} - 1 \approx 2\delta - 4\epsilon - \frac{H_k^2}{m_{\mathsf{P}}^2}\left(\frac{\bar{k}}{k_*}\right)^3 (2.96 + 11.40\,\epsilon - 7.68\,\delta)\,,$$

$$n_{\mathsf{T}} \approx -\,2\epsilon - \frac{H_k^2}{m_{\mathsf{P}}^2}\left(\frac{\bar{k}}{k_*}\right)^3 (2.96 + 3.72\,\epsilon)\,,$$

$$r^{(1)} \approx 16\epsilon\left(1 + 2.56\,\frac{H_k^2}{m_{\mathrm{P}}^2}\left(\frac{\bar{k}}{k}\right)^3 (\delta - \epsilon)\right)$$

Magnitude of the correction

From observations one infers that

$$\frac{H_{\mathsf{inf}}}{m_{\mathrm{P}}} \lesssim 1.3 \times 10^{-5}, \quad n_{\mathsf{S}} \approx 0.968 \pm 0.006, \quad r \lesssim 0.11.$$

Using this and the (somewhat arbitrary) choice $\bar{k} = k_* = 0.05\,\mathsf{Mpc}^{-1}$, one finds

$$|\Delta_{\mathsf{S}}| \lesssim 2 \times 10^{-10}, \quad |\Delta_{\mathsf{T}}| \lesssim 2 \times 10^{-10}, \quad \frac{\Delta_{\mathsf{S}}}{\Delta_{\mathsf{T}}} \approx 1.02$$

The assumption that the corrections are smaller than the experimental errors of the CMB anisotropies leads to

$$\bar{k}_{\max} \approx 100\,\mathrm{Mpc}^{-1}$$

CMB temperature anisotropies

$$C_\ell^{(0,1)} = \int_0^\infty \frac{\mathrm{d}k}{k}\, \mathcal{P}_{\mathsf{S}}^{(0,1)}(k)\, \Theta_\ell^2(k),$$

where $\Theta_\ell(k)$ is the transfer function. The quantum-gravitational corrections are then given by

$$\Delta C_\ell := C_\ell^{(1)} - C_\ell^{(0)}$$

We find

$$\Delta C_\ell \approx \frac{3}{4\pi\epsilon}\left(\frac{H_k}{m_{\mathsf{P}}}\right)^4 \frac{|\bar{k}(\eta_{\mathsf{hor}} - \eta_{\mathsf{rec}})|^3}{(2\ell-3)(2\ell-1)(2\ell+1)(2\ell+3)(2\ell+5)}$$

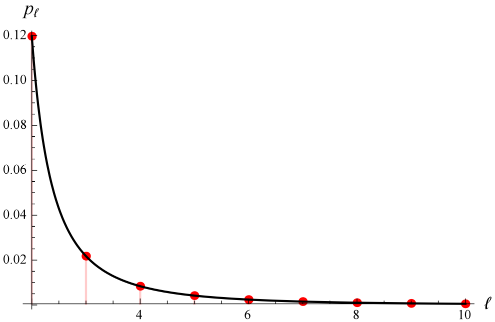


Figure: The ratio of the correction ΔC_ℓ to the uncorrected $C_\ell^{(0)}$ without the non-numerical prefactors, such that we have $p_\ell := 3\pi\,\ell(\ell+1)\,[(2\ell-3)(2\ell-1)(2\ell+1)(2\ell+3)(2\ell+5)]^{-1}$.

In order to get an effect bigger than cosmic variance, one would need $(H_k/m_{\mathsf{P}}) \gtrsim 10^{-2}$.

Summary and Outlook

- Concrete prediction from a conservative approach to quantum gravity (Wheeler-DeWitt equation); consistent with existing observational limits
- Enhancement of power on largest scales
- Corrections: k^{-3}-dependence resp. ℓ^{-3}-dependence
- In the present case, the effect is too small to be observable (main limit for accuracy: cosmic variance)
- Similar results (but different in detail) from a modified scheme put forward by Kamenshchik, Tronconi, and Venturi (2013–2016)
- Quantum gravitational corrections for galaxy–galaxy correlation functions?
- More general initial states?
- Non-slow roll models of inflations?