Remarks on non-singular black hole models

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Based on:

"Information loss problem and a 'black hole` model with a closed apparent horizon", V.F., JHEP 1405 (2014) 049;

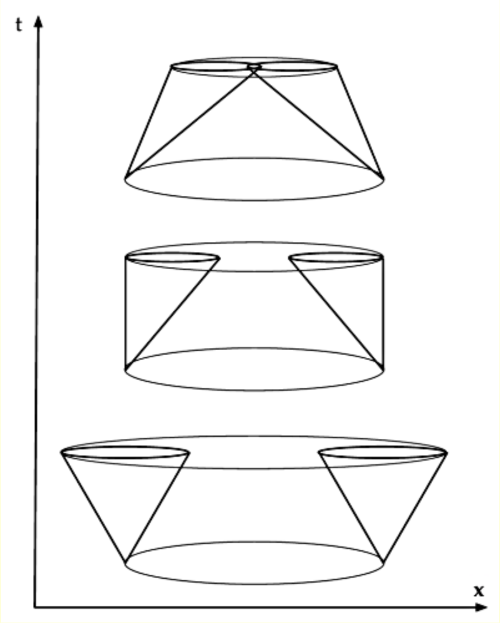
"Notes on non-singular models of black holes", V. F., Phys.Rev. D94 (2016) no.10, 104056;

"Quantum radiation from a sandwich black hole", V. F. and A. Zelnikov, Phys.Rev. D95 (2017) no.4, 044042;

"Quantum radiation from an evaporating non-singular black hole", V. F. and A. Zelnikov (under preparation).

Event horizon vs.apparent horizon

`Quasi-local definition' of BH: Apparent horizon



A compact smooth surface *B* is called a trapped surface if both, in- and out-going null surfaces, orthogonal to *B*, are non-expanding .

A trapped region is a region inside *B*.

A boundary of all trapped regions is called an apparent horizon.

According to GR: Singularity exists inside a black hole.

Theorems on singularities: Penrose and Hawking.

Penrose theorem: Assume

1. The null energy condition holds $T_{\mu\nu} l^{\mu} l^{\nu} \geq 0$;

2. There exists a noncompact connected Cauchy surface.

3. There exist a closed trapped null surface .

Then, we either have null geodesic incompleteness, or closed timelike curves.

Schwarzschild ST has a spacelike singularity.

RN and Kerr ST have a timelike singularity.

In both cases this is a curvature singularity.

Expectations: When curvature becomes high (reaches the Planckian value) the classical GR should be modified. Singularities of GR would be resolved.

The geometry near singularity inside Schwarzschild BH approaches Kasner type contracting universe geometry. To make it regular one needs to supress growing of the Weyl rensor. This makes this case quite different from the homogeneous cosmology where Weyl tensor vanishes.

Modified gravity: Options:

(i) Vacuum polarization and particle creation $\rightarrow$
Effective action (higher derivatives and
non-locality);

(ii) Modified fundamental gravity (higher derivatives,
$f(R)$ theory, etc.);

(iii) Non-local modification (Ghost-free gravity);

(iv) Gravity as an emergent phenomenon
(strings, loops, etc.)

Phenomenological description

(i) There exist two energy scale parameters $\mu < \mu^*$. The corresponding fundamental lengths are $\ell = \dfrac{\hbar}{\mu c} > \ell^* = \dfrac{\hbar}{\mu^* c}$;

(*ii*) In the domain where $E < \mu^*$ the gravitational field is describes by the effective metric *g*;

(iii) In the domain where $\Re \ll \ell^{-2}$ the metric obeys the Einstein equations with small corrections;

(iv) In the domain where $\Re \sim \ell^{-2}$ the Einstein equations should be modified;

(v) Limiting curvature condition: $|\Re| \leq \dfrac{C}{\ell^2}$. *C* is a universal constant, defined by the theory and independent of the parameters of the solution.

[Markov, JETP Lett. 36, 265 (1982)]

Remark on inflation theory.

General form of SS metric in advanced time coordinates

$$ds^2 = -\alpha^2 f\, dv^2 + 2\alpha\, dv\, dr + r^2 d\Omega^2, \qquad f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}\ .$$

$f(v,r)|_{r\to\infty} = 1.$ Apparent horizon: $f = 0.$

Red-shift function: α(v,r). In a static ST: $\xi_{\mathrm{t}}^2 = -\alpha^2 f.$

ST is regular at $r = 0,$ if curvature invariants are finite there:

$f = 1 + \frac{1}{2} f_2(v) r^2 + \dots\ ,\ \ \alpha = \alpha_0(v)[1 + \frac{1}{2}\alpha_2(v) r^2 + \dots].$

We use normalization: $\alpha(v,r)|_{r\to\infty} = 1,$ then the rate of the proper time at the center, τ, and the rate of the Killing time at infinity, v, are connected as: $d\tau = \alpha_0(v) dv.$

(i) An apparent horizon in a regular metric cannot cross $r = 0$.

(ii) It has two branches: outer- and inner-horizons.

(iii) Non-singular BH: regular metric obeying limiting curvature condition.

(iv) Non-singular BH model with a closed apparent horizon

[V.F. and G.Vilkovisky, Phys. Lett., 106B, 307 (1981)]

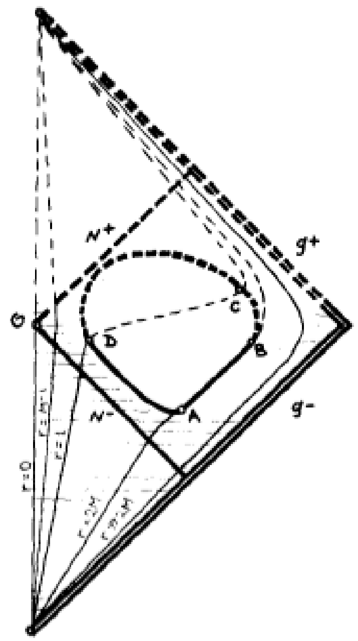


Fig. 1. Penrose diagram for the collapse of the null shell ($M \gg 1$). Solid (dashed) lines are used for the known (hypothetical) details of the picture. The shaded region is the region of validity of the obtained asymptotic solution. The line $N^- \cup N^+$ is the world line of the null shell. The closed and dashed bold line $ABCD$ is the apparent horizon. The light lines are the level lines r = const.

$$\Delta u \sim 2M, \ \Delta v \sim M^3$$

Static SS non-singular black-hole metrics

Remark: All stationary BH solutions in General Relativity can be written in the form, where the metric coefficients are rational functions of the coordinates.

We assume that $f = \dfrac{P_n(r)}{Q_n(r)}$. It has the form $= \dfrac{r^n + \ldots}{r^n + \ldots} \to 1$ at $r \to \infty$.

Example 1: Bardeen regular black hole [1968]

$$f = 1 - \frac{2Mr^2}{(r^2 + q^2)^{3/2}} \to 1 - \frac{2M}{|q|^3} r^2 + \ldots . \quad \Re \sim \frac{M}{|q|^3}.$$

Neither this metric nor its modification $(q \to \ell)$ satisfies the limiting curvature condition.

Example 2: Metric with $f = 1 - \dfrac{2Mr}{r^2 + \ell^2} \sim 1 - \dfrac{2M}{\ell^2} r + \ldots$

is non-regular.

Metrics with $n \leq 2$ cannot be consistent metrics of a non-singular black hole. [V.F. PR D94,104056 (2016)].

Example 3 (n=3): (Improved) Hayward metric:

$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}$$

Non-singular evaporating black hole: $M = M(v)$.

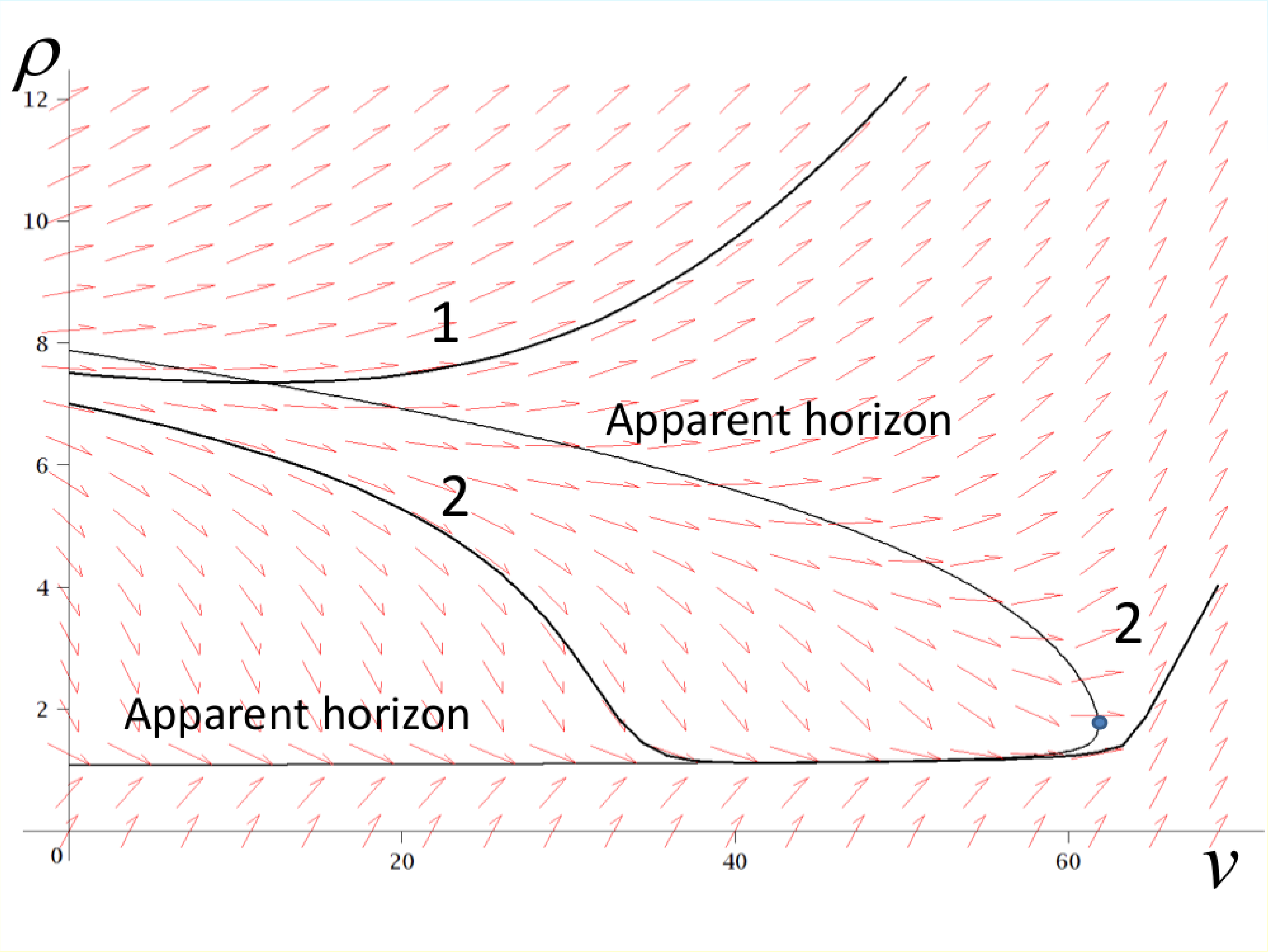
$$f \sim 1 - \frac{2M}{r} + \frac{2M(2M\ell^2 + \ell^3)}{r^4} + \dots, \text{ at large } r;$$

$$f \sim 1 - \frac{2Mr^2}{\ell^2(2M + \ell)} \dots, \text{ at small } r.$$

Change of regimes at $r \sim \ell(2M/\ell)^{1/3}$, where $\Re \sim \ell^{-2}$.

Non-singular evaporating SS black hole

Created paticles, that are able to penetrate the potential barrier in the BH exterior, form the Hawking radiation. Their patners are propagated in the BH interior. There exists the polarization negative energy flux through the horizon, which reduces the BH mass. In the quasi-stationary regime it is the same (with the minus sign) as the positive energy flux at infinity. To take the evaporation effect into account one usually simply put $M = M(v)$.



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60
v
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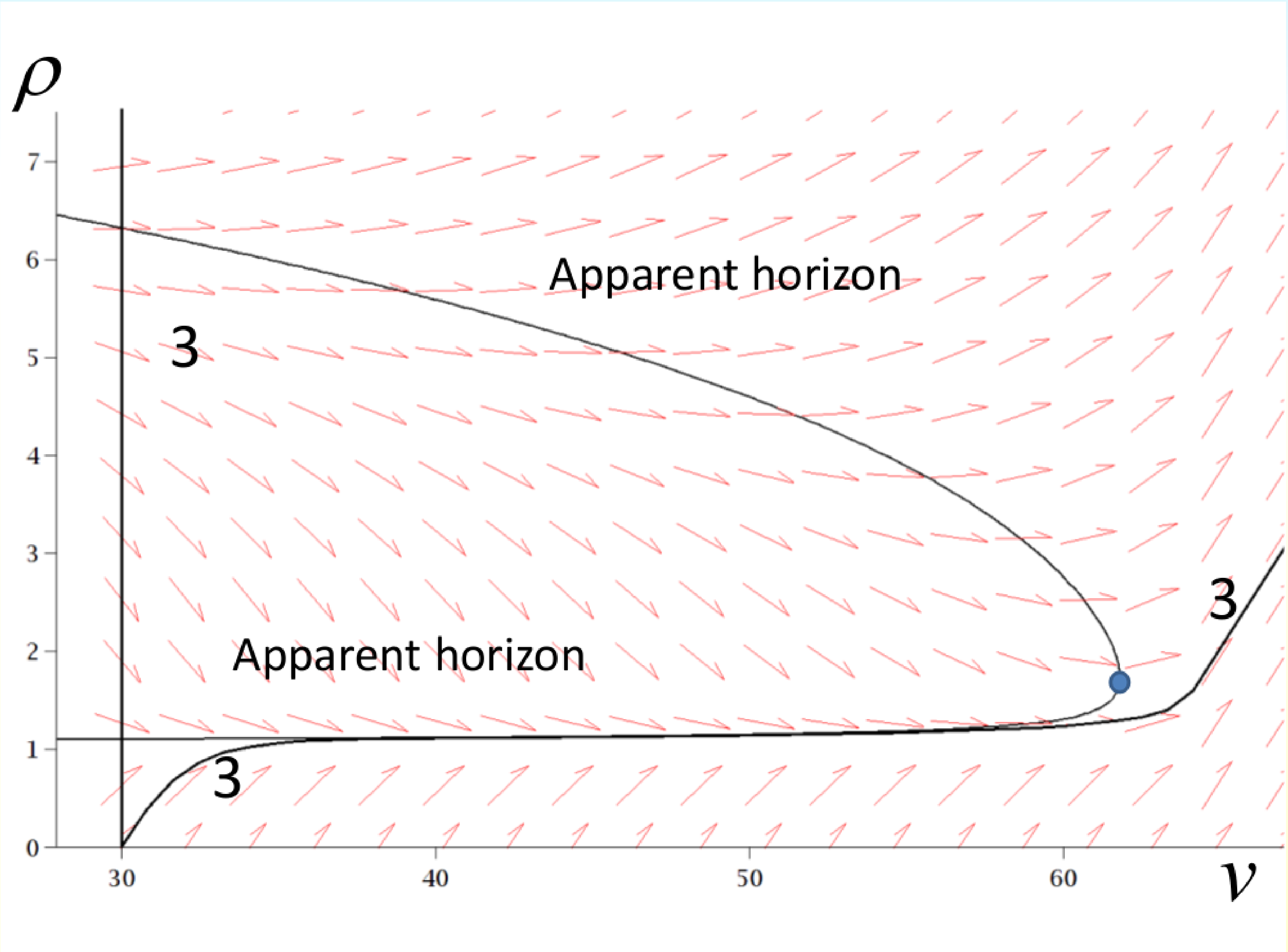
2
2
Apparent horizon
Apparent horizon



ρ
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4
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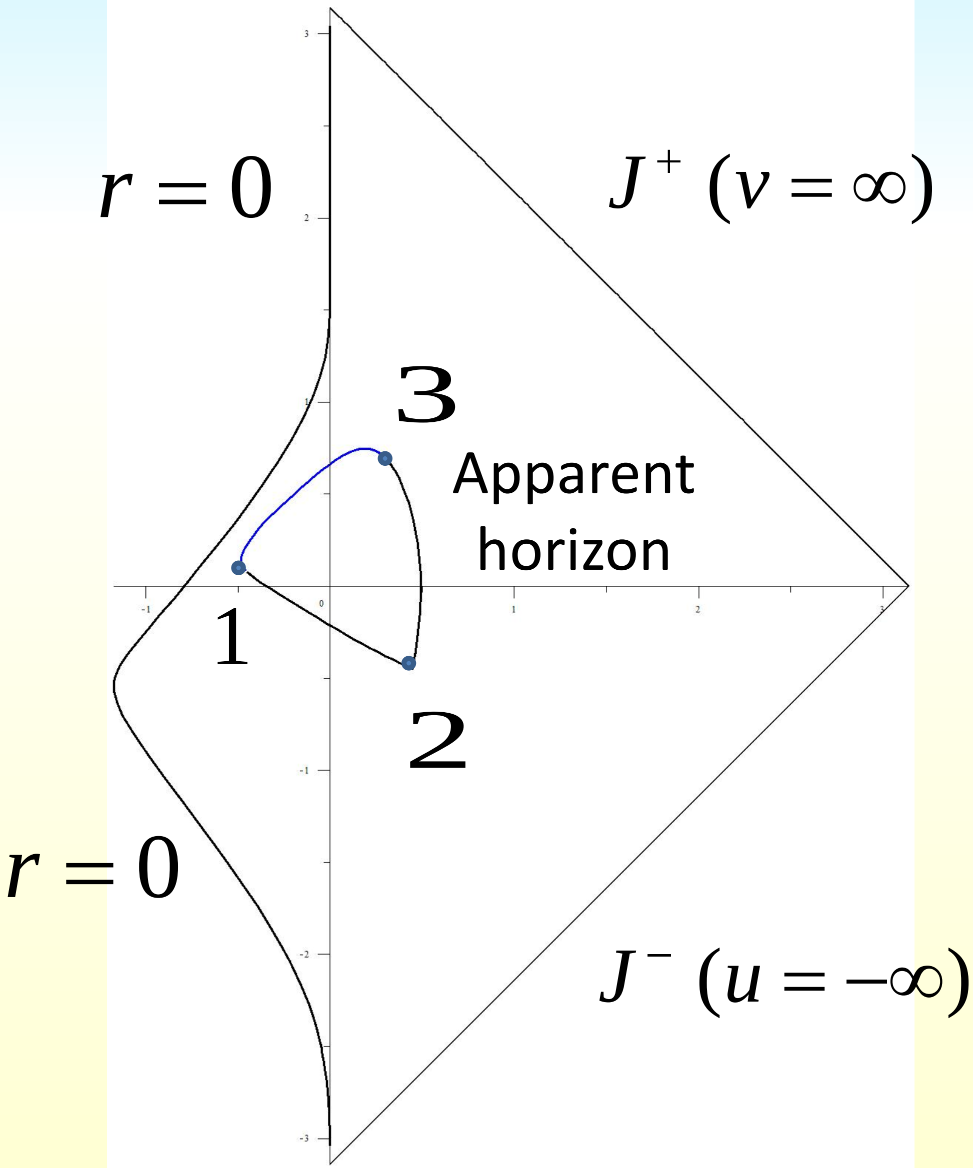
v
Apparent horizon
Apparent horizon
3
3
3



$r = 0$
$J^{+}\ (v = \infty)$
3
Apparent
horizon
1
2
$r = 0$
$J^{-}\ (u = -\infty)$

Quantum effects

We consider a quantum massless scalar field, propagating in the background of a non-singular black hole. We use 2D approximation. The corresponding expectation value of the stress-energy tensor can be easily obtained from the known conformal anomaly.

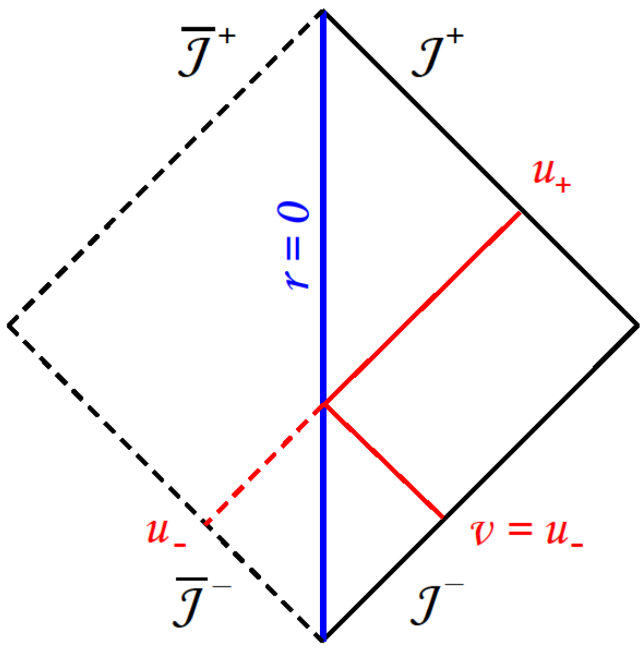
[Christensen and Fulling, PD, D15, 2088 (1977)].

It can also be derived from Polyakov effective action.

[V.F. and Vilkovisky, in Quantum Gravity, p.267 (1984)]

Radial null rays provide us with maps:

$$\mathfrak{I}^- \leftrightarrow \mathfrak{I}^+ : \quad u_- = u_-(u_+) \text{ and } u_+ = u_+(u_-)$$



$$\dot{E} = \frac{1}{48\pi}\left[-2\frac{d^2P}{du_+^2} + \left(\frac{dP}{\mathrm{d}u_+}\right)^2\right],$$

$$P = \ln\left(\frac{du_-}{\mathrm{d}u_+}\right).$$

Sandwich model (α=1)

Spherical collapse of a null shell of mass M at $v=0$, and later, at time $v=q$, a collapse of another shell with mass $-M$. The ST between the shells is static and non-singular (Hayward metric with or without α).

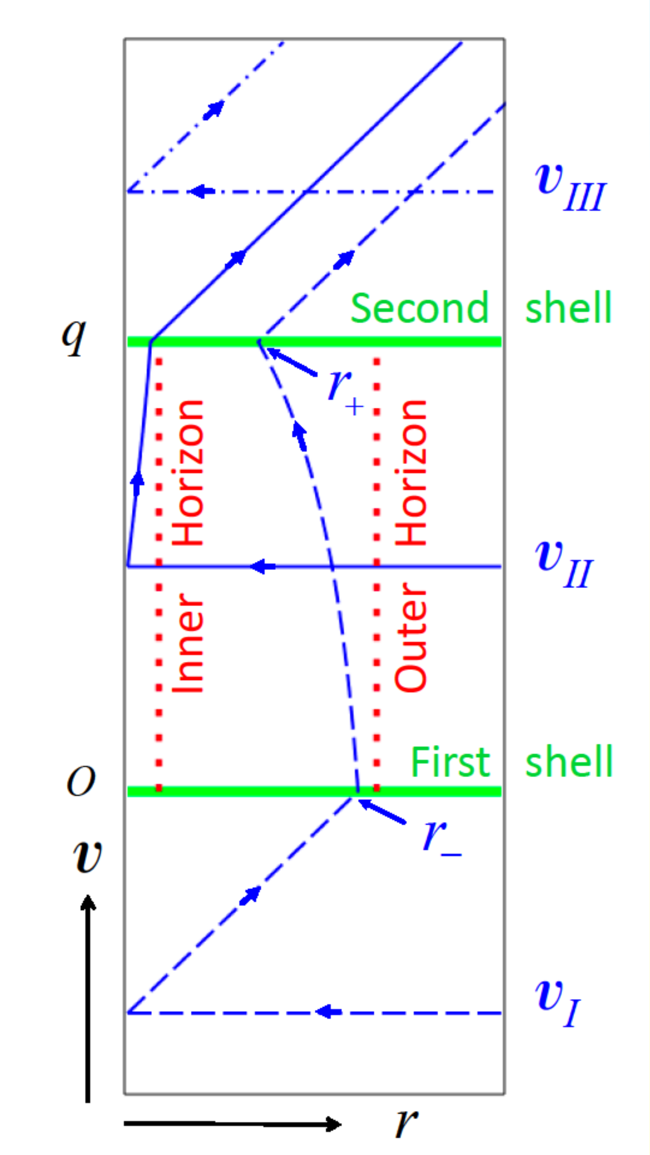
Useful parametrization of the Hayward metric: $(M,\ell)\to(r_{in},r_{out})$.

We denote $p=\dfrac{r_{out}}{r_{in}}$, and use r_{in} as a scale parameter:

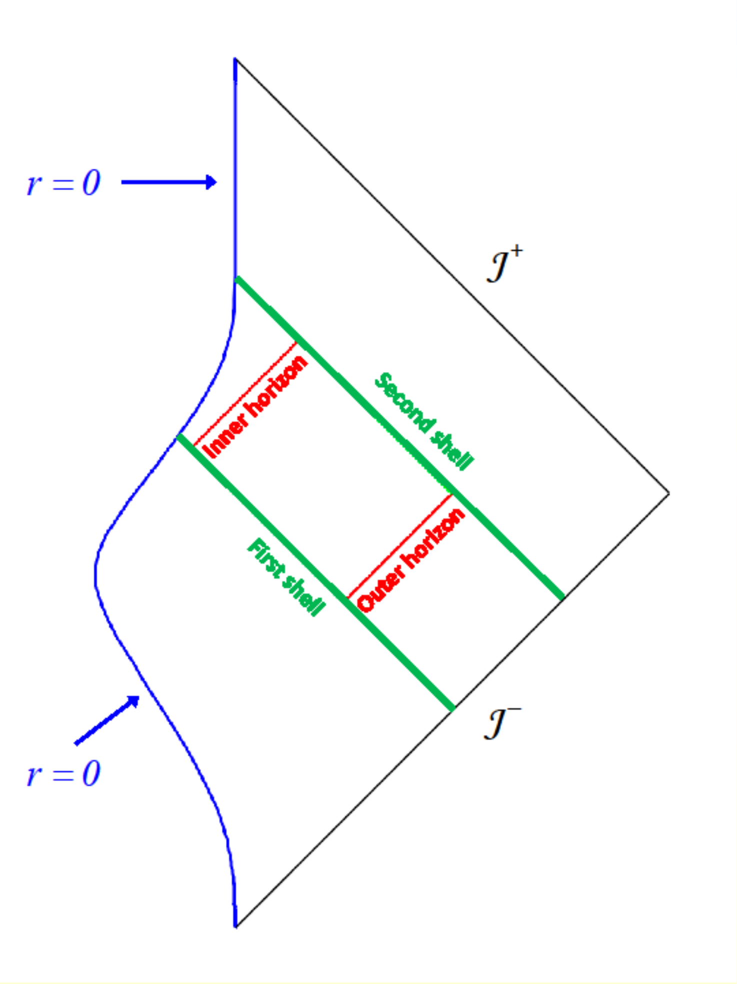
$$f=\frac{(r-p)(r-1)[r+p/(p+1)]}{r^3+p^2/(p+1)}.$$

For $M\gg\ell$, $p\approx 2M$, $r_{in}\approx\ell$, $\kappa_-\approx -1$ $(\approx-\ell^{-1})$, $\kappa_+\approx\dfrac{1}{2p}$ $(\approx\dfrac{1}{4M})$.

$$u_+ = v - 2r_+$$

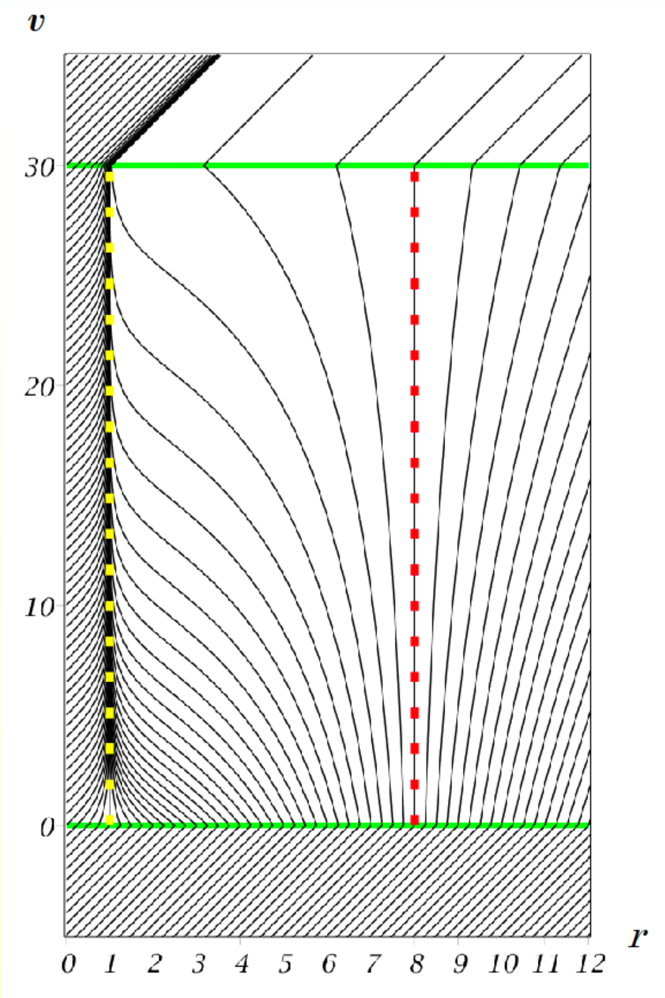


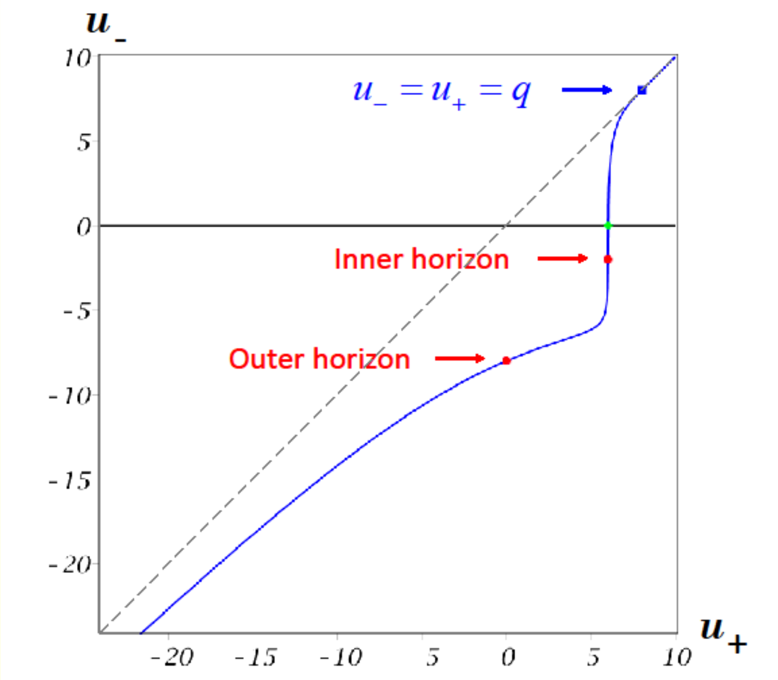
v_{III}
Second shell
q
r_+
Inner Horizon
Outer Horizon
v_{II}
First shell
O
r_-
v
v_I
r

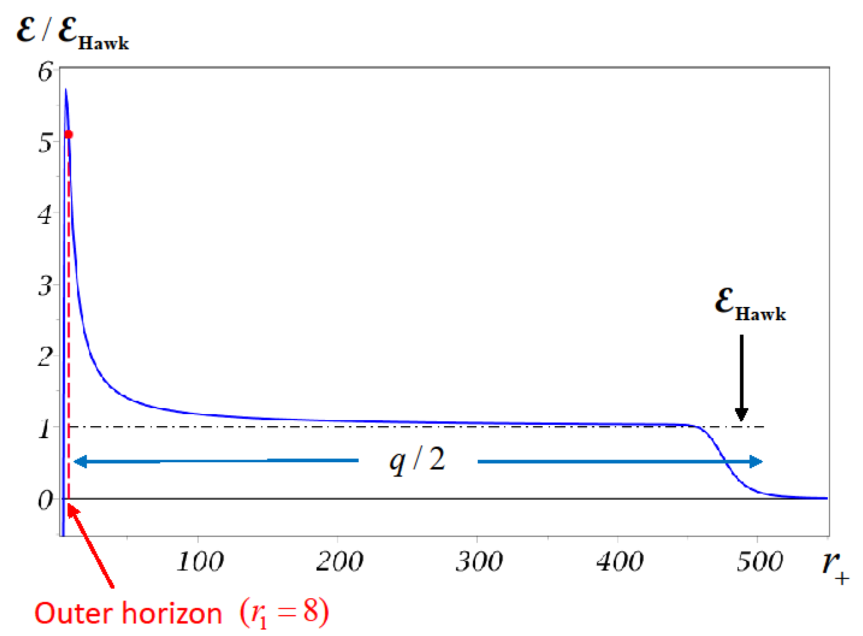


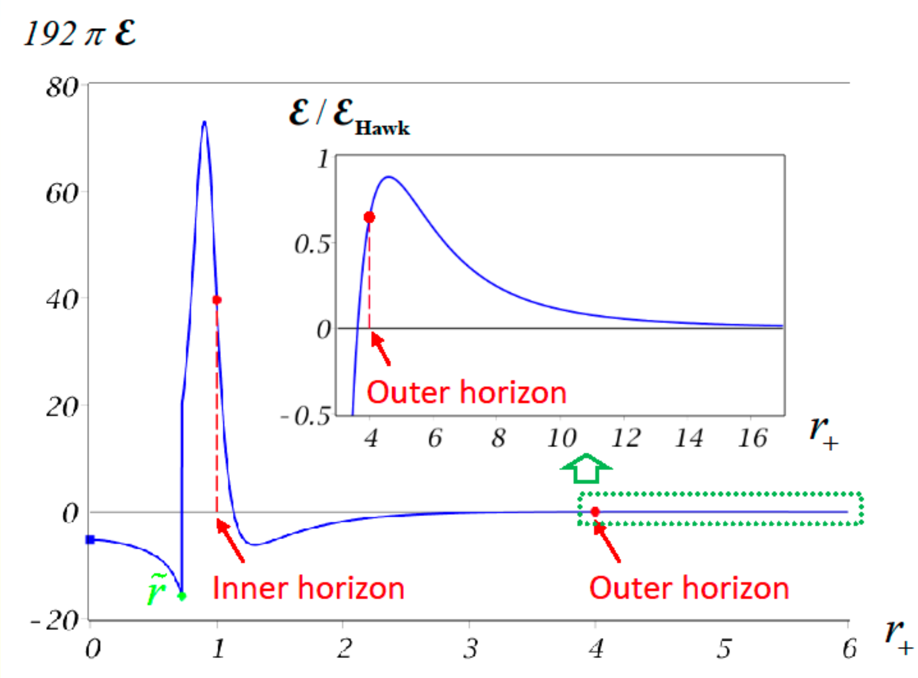
$r = 0$
$\mathcal{J}^+$
Inner horizon
Second shell
First shell
Outer horizon
$\mathcal{J}^-$
$r = 0$

The equation for out-going null rays in the static domain between the shells can be solved analytically.









$p = 8, q = 1000$

$p = 4, q = 2$

Main results:

(i) Properly reproduced Hawking radiation from the outer horizon (for $q \gg M$);

(ii) Huge outburst of the quantum radiation from the inner horizon: $\Delta \mathrm{E} \sim \exp(\mathrm{q}) \sim \exp(\Delta \mathrm{v}/\ell)$;

(iii) This radiation comes from the inner horizon during time interval $\Delta u_{+} \sim \exp(\mathrm{-q}) \sim \exp(-\Delta \mathrm{v}/\ell)$.

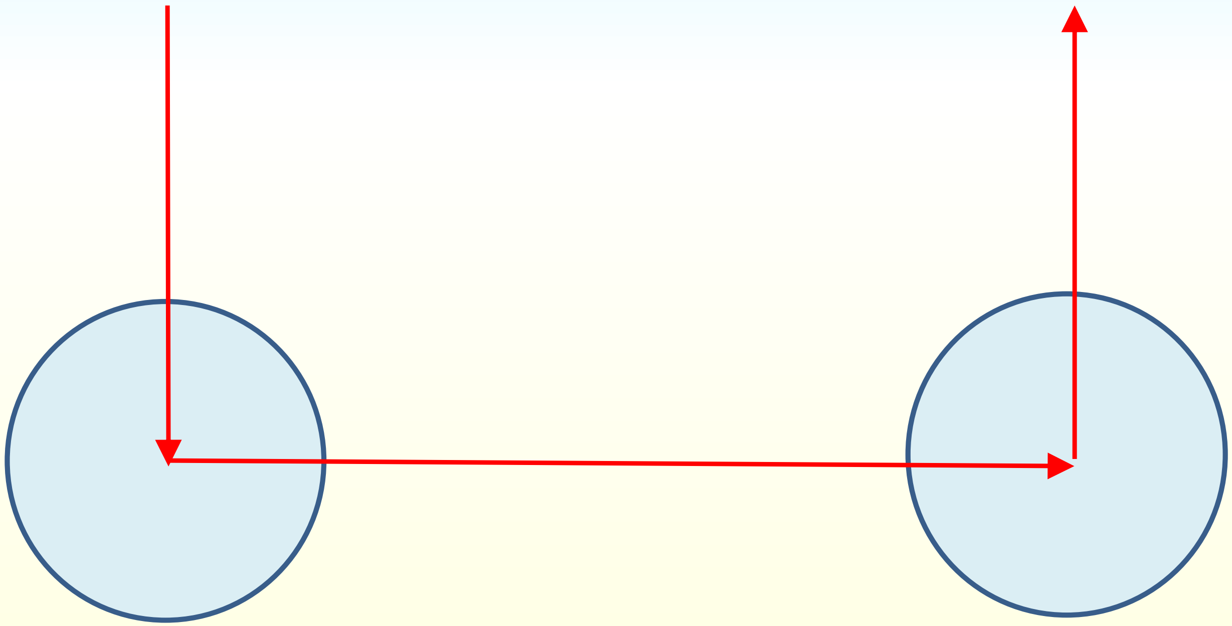
(iv) Self-consistency problem [Bolashenko and V.F. (1986)].

Two mechanisms of the energy amplification by evaporating black holes:

(i) BH as a "gravitational accelerator",

(ii) Mass inflation mechanism.

$$\Delta E = m(\varphi_1 - \varphi_0) = \frac{G(M_0 - M_1)m}{R}$$



$$\varphi_0 = -\frac{GM_0}{R} \qquad \varphi_1 = -\frac{GM_1}{R}$$

Gravitational accelerator

Near horizon geometry

$$ds^2 = -f(r)\,dv^2 + 2dvdr$$

$$\kappa = \tfrac{1}{2} f'|_{f=0}, \quad f \sim 2\kappa(r - r_0);$$

$$ds^2 = 0 \rightarrow v = const,$$

$$u = v - \kappa^{-1} \ln|r - r_0| = const;$$

$$U = \exp(-\kappa u) = \exp(-\kappa v)(r - r_0),$$

$$U^{,\mu} = [\exp(-\kappa v), \kappa \exp(-\kappa v)(r - r_0)],$$

$$\mathbf{U}^2 = 0, \quad U^{,\mu} \nabla_\mu U^{,\nu} = 0,$$

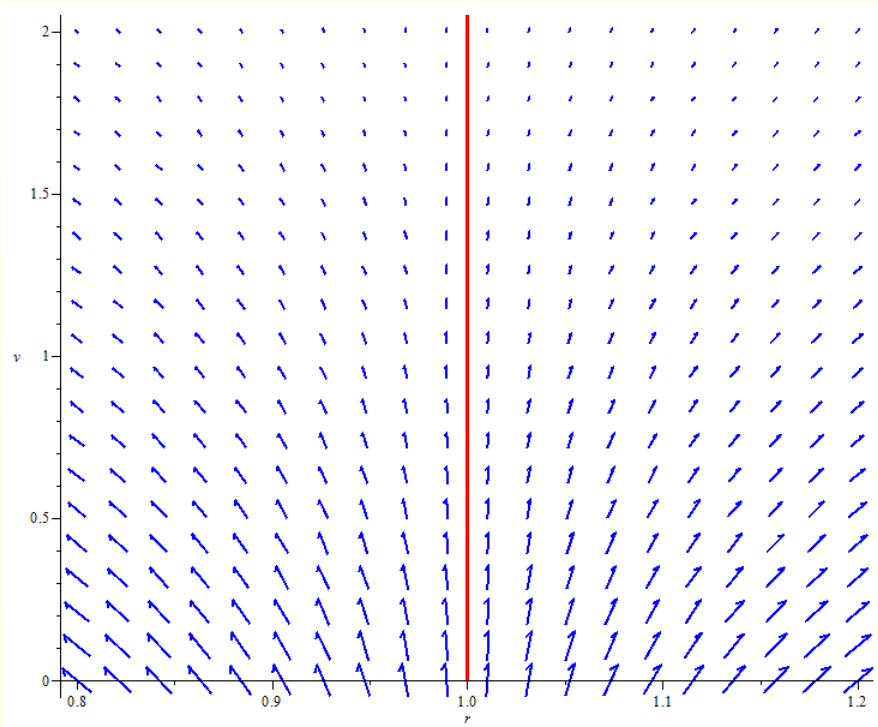
$$V_\mu = v_{,\mu} = [1, 0], \quad (\mathbf{U}, \mathbf{V}) = \exp(-\kappa v).$$

Outer horizon: $\kappa>0$

Red-shift $\sim$ exp(-κv),

$(\mathbf{U},\mathbf{V}) = \exp(-\kappa v)$,

Beam cross-section increases

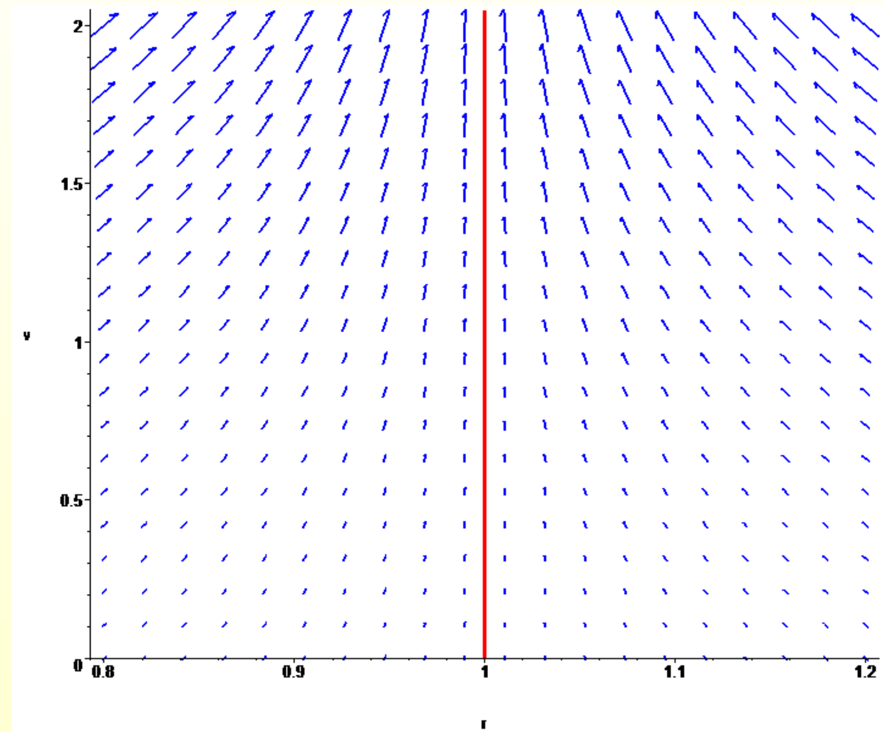


Inner horizon: $\kappa < 0$

Blue-shift $\sim$ exp(|κ}v),

$(\mathbf{U},\mathbf{V}) = \exp(|\kappa| v)$,

Beam cross-section decreases



Killing vector: $\xi^{\mu} = [1,0]$

Energy: $E = -\xi_{\mu} U^{\mu}$

$= \kappa \exp[-\kappa(v - v_0)(r - r_0)] \equiv \kappa U = const$

Suppose $r_0 \to r_1$ at $v = v_1$, then

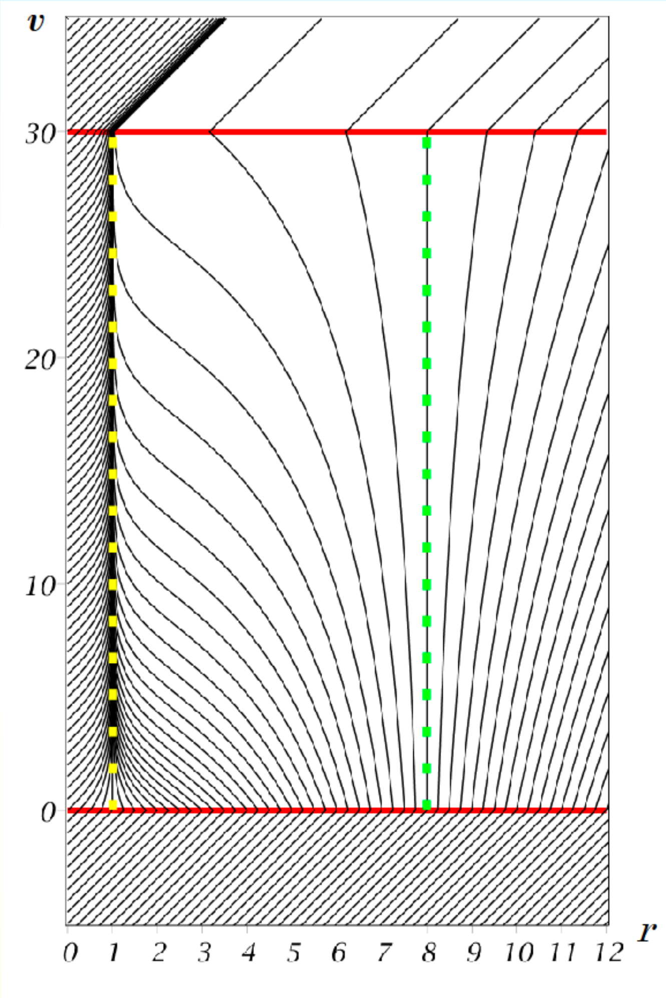
$$E_1 = \frac{r - r_1}{r - r_0} E_0 \sim \exp(-\kappa(v_1 - v_0)),$$

For inner horizon κ<0, $\to$

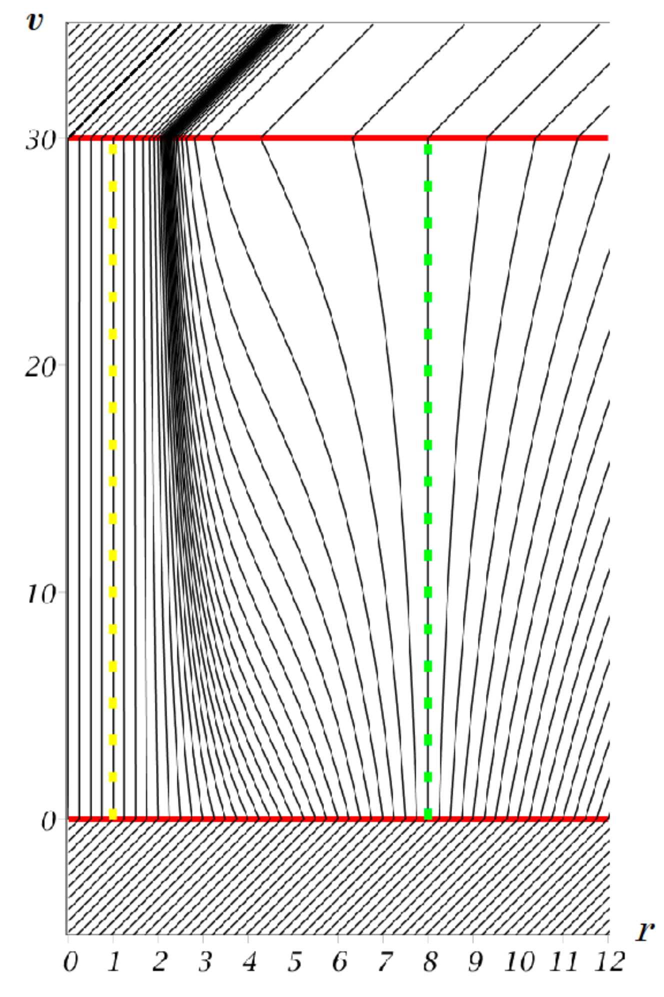
Mass inflation [Israel, Poisson (1990)]

"Fighting" with mass inflation.

In the presence of the non-trivial red-shift function, the surface gravity of the horizons is modified: $\kappa_\alpha = \alpha\,\kappa$. If α_0 is small the surface gravity of the inner horizon becomes small as well.

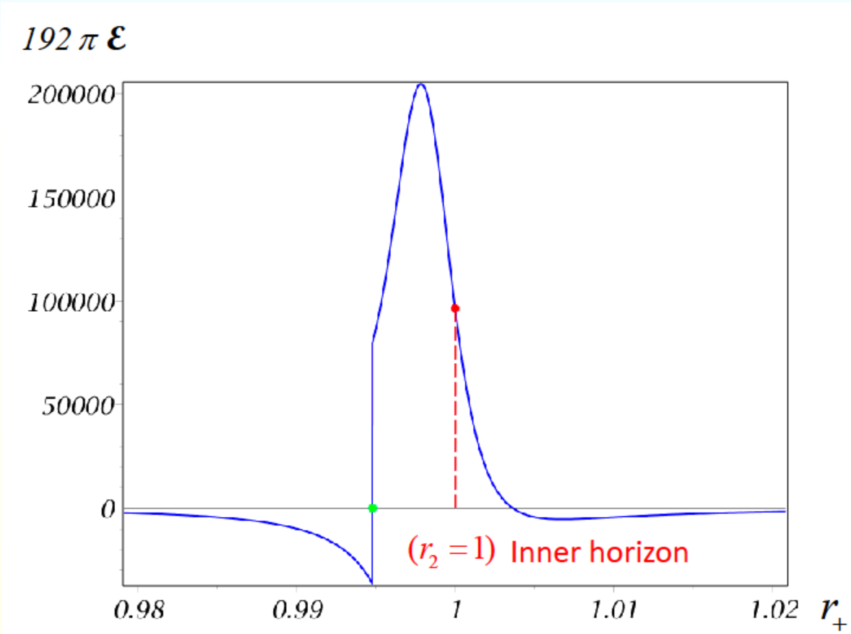


$p = 4, q = 30, \alpha_0 = 1$

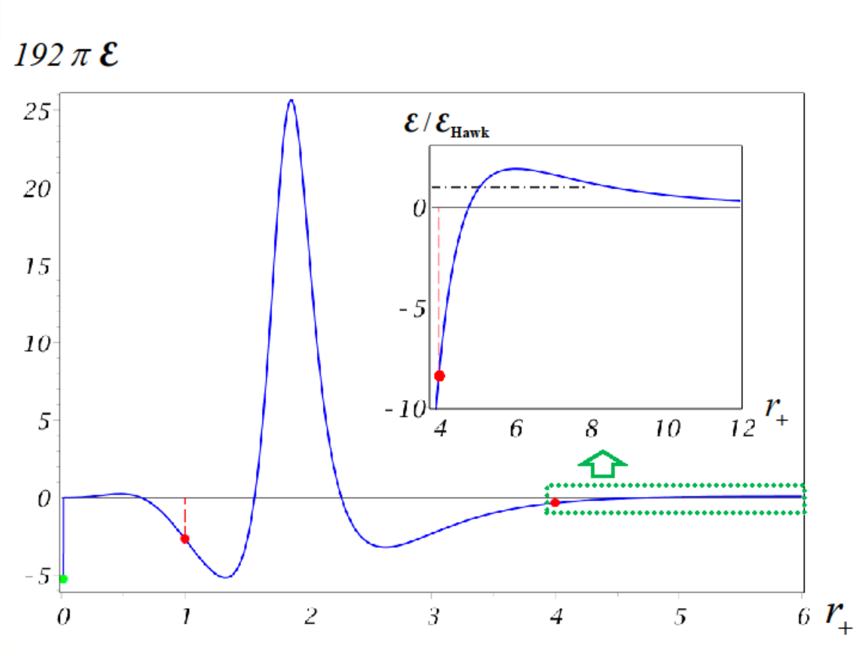


$p = 4, q = 30, \alpha_0 \sim p^{-4}$

$p = 4, q = 30, \alpha_0 = 1$



$p = 4, q = 30, \alpha_0 \sim p^{-4}$



$q \sim p^3, \quad \Delta E \sim \exp(p^3)$

$q \sim p^3, \quad \Delta E \sim p^3$

"Mass inflation" is cured, however "gravity acceleration" mechanism still works. Self-consistency problem.

Quantum radiation from evaporating non-singular BHs

(i) "Realistic" model of avaporating non-singular BH;

(ii) Formalism for calculation of quantum energy
flux in an arbitrary time dependent metric.
Numerical calculation of the map function $u_{-}(u_{+})$
is rather trivial, but obtaining third derivatives of
it generates big numerical errors.

(iii) Sharp behavior of energy flux near inner horizon.

(iv) We developed "bracket" formalist to solve these problems.

"Bracket" formalism

Let $y(x)$ be a function, and $x(y)$ is its inverse.

$$[y,x]=\ln|y'|,\quad <y,x>=\frac{y''}{y'},\quad \{y,x\}=\frac{y'''}{y'}-\frac{3}{2}\left(\frac{y''}{y'}\right)^2.$$

$$[y,x]=-[x,y],\quad <y,x>=-y'<x,y>,\quad \{y,x\}=-(y')^2\{x,y\}.$$

$$f\circ g(z)=f(g(z))\Rightarrow$$

$$[f\circ g,z]=[f,g]\big|_{g=g(z)}+[g,z],$$

$$<f\circ g,z>=<f,g>\big|_{g=g(z)}g'(z)+<g,z>,$$

$$\{f\circ g,z\}=\{f,g\}\big|_{g=g(z)}(g'(z))^2+\{g,z\}.$$

Observables on $\mathfrak{I}^+$ in terms of "brackets".

Gain function: $\beta = \dfrac{du_-}{du_+}, \quad P = \ln\beta = [u_-, u_+],$

"Radiation entropy" $S(v) = -\dfrac{1}{12}P,$

[Bianchi, DeLorenzo, Smerlak, JHEP, 06, 1280 (2015)]

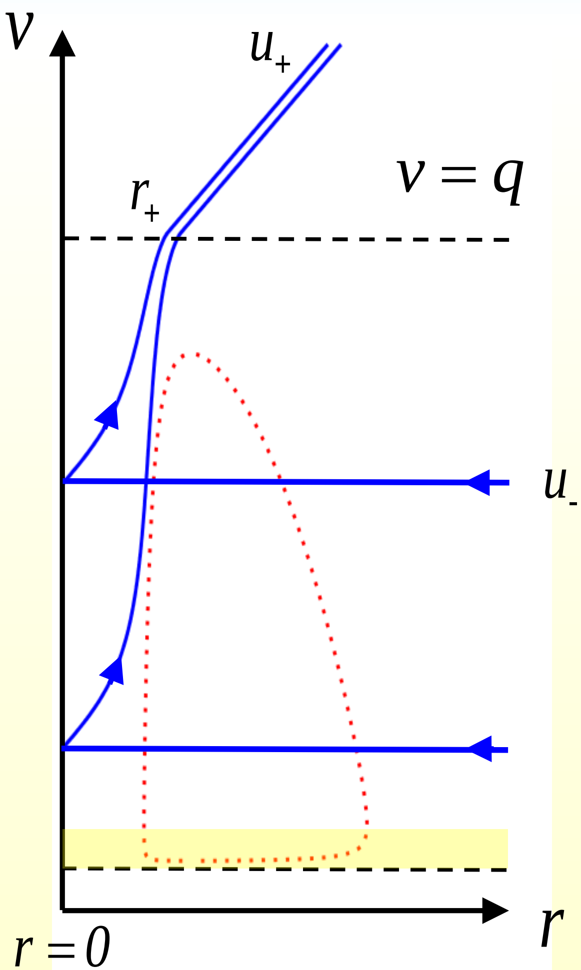
Density of out-going trajectories: $W = \langle u_-, u_+ \rangle.$

Energy flux: $\dot{E} = -\dfrac{1}{24\pi}\{u_-, u_+\},$

$\{u_-, u_+\}$ - Schwarz derivative,

[M.Reuter, CQG, 6, 1149 (1989)]

Beam of out-going null rays



$$r = r(v,x),\quad r(v,z) = r_0(v) + \sum_{n=1} \frac{z^n}{n!} r_n(v).$$

$$\frac{dr}{dv} = Z(v,r),\quad Z = \frac{1}{2}\alpha f,\quad Z_m(v) = \left.\frac{\partial^m Z(v,r)}{\partial r^m}\right|_{r=r_0(v)}.$$

$$p(v) = \ln r_1(v),\quad w(v) = \frac{r_2(v)}{r_1(v)},$$

$$\varepsilon(v) = \frac{r_3(v)}{r_1(v)} - \frac{3}{2}\left(\frac{r_2(v)}{r_1(v)}\right)^2.$$

$$\frac{dr_0}{dv} = Z_0,\quad \frac{dp}{dv} = Z_1,$$

$$\frac{dw}{dv} = Z_2 e^p,\quad \frac{d\varepsilon}{dv} = Z_3 e^{2p}.$$

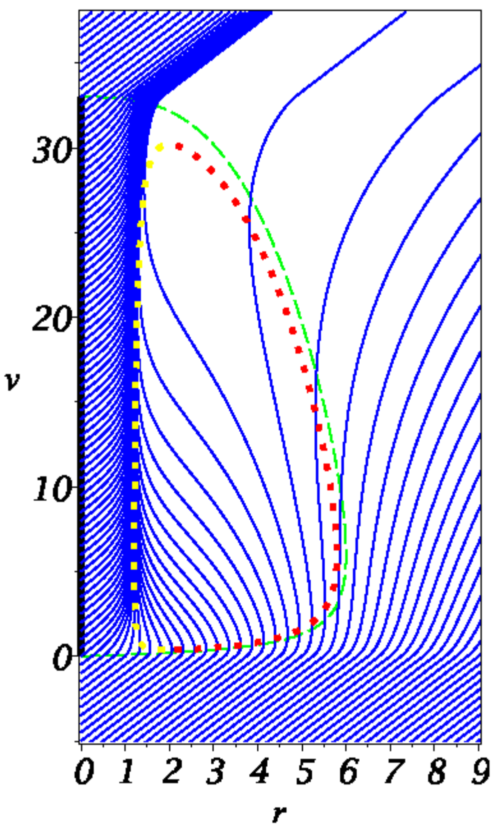
$$p(u\_) = w(u\_) = \varepsilon(u\_) = 0$$

$$P = \ln \beta = \ln\left(\frac{du_-}{du_+}\right) = [u_-, u_+] = -p(q) - \ln \alpha_0 \,,$$

$$W = < u_-, u_+ > = e^{-p(q)}\left[\frac{1}{2}w(q) - \frac{\alpha_0}{\alpha_0^2}\right],$$

$$-24\pi \dot{E} = \{u_-, u_+\} = -e^{-2p(q)}\left[\frac{1}{4}\varepsilon(q) + \frac{1}{\alpha_0^2}\{x, u_-\}\right],$$

$$\{x, u_-\} = \frac{1}{2}\alpha_0^2 a_2 + \frac{\alpha_0}{\alpha_0} - \frac{3}{2}\left(\frac{\alpha_0}{\alpha_0}\right)^2.$$



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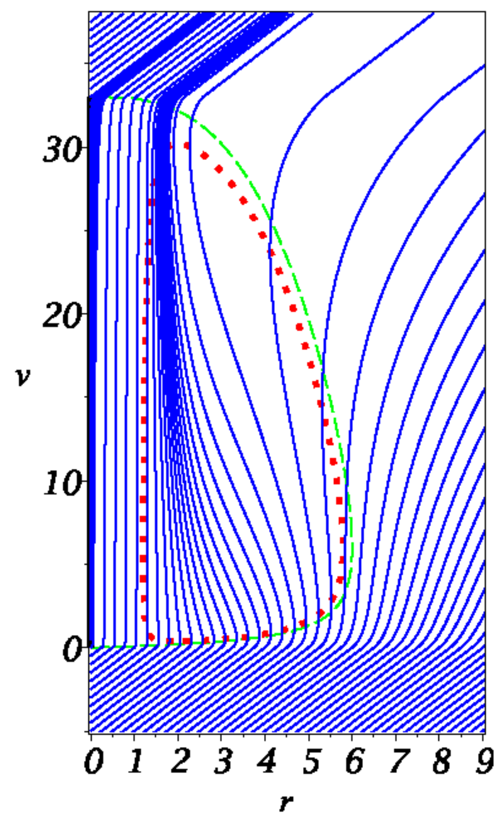

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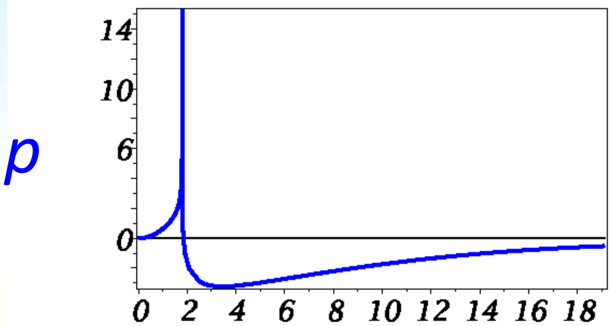
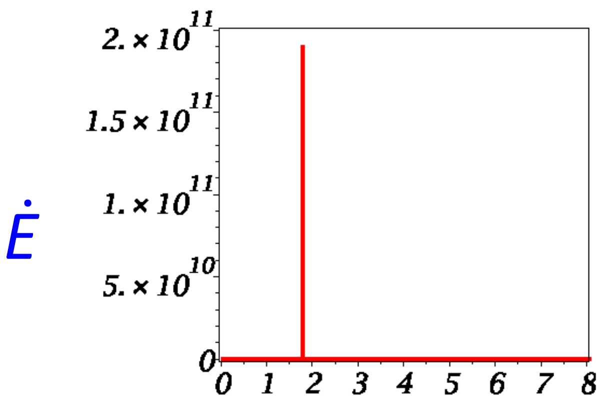
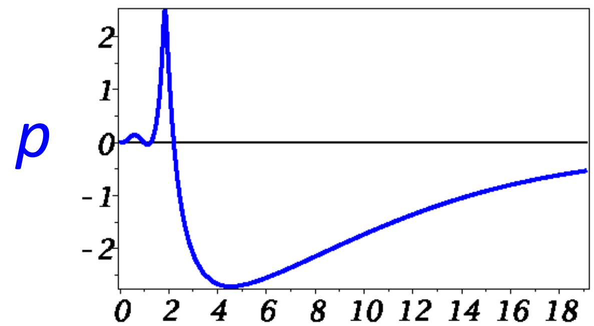
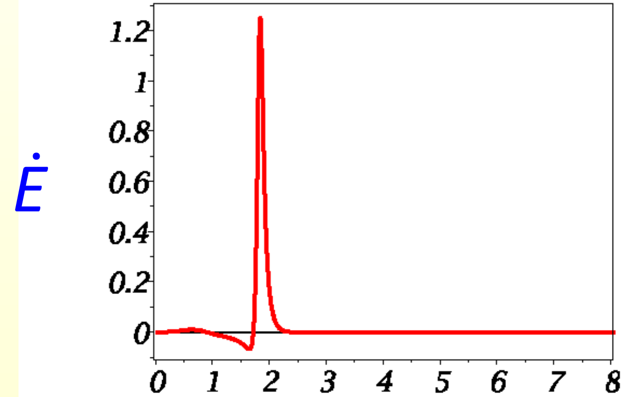




$M = 3, \ \ \alpha = 1$





$M = 3, \ \ \alpha_0 \sim M^{-3}$

Summary and Discussion

(i) Non-singular models of evaporating BHs

(ii) Quantum radiation from BH interior

2D approximation (its validity?)

(iii) Sandwich model: 2 shells, 2 parameres (p,q).

How good is this rough approximation?

(iv) Two mechanisms of energy aplification:

"Gravity accelerator" vs "Mass inflation";

(v) Non-trivial red-shift factor helps to cure

mass inflation problem;

(vi) "Realistic" non-singular models;

(vii) "Bracket" formalism and results;

(viii) Self-consistency problem;

(ix) Back-reaction of created radiation?!