Infinite Derivative Theories of Gravity and potential ways of probing it

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Phys. Rev. Lett. (2012), JCAP (2012, 2011), JCAP (2006)

Class.Quant. Grav. (2013), Phys. Rev. D (2014), 1412.3467 (Class. Quant. Grav. 2014),

1503.05568 (Phys. Rev. Lett. 2015), 1509.01247 (Phys. Rev. D, 2015), 1602.08475,

1603.03440, 1604.01989, 1701.01009

Quantum Vacuum and Gravitation, Mainz 2017

Lessons from this workshop so far ...

Motivate Quantum Gravity from Observations

Cosmology

$$S \sim M_p^2 R + 10^8 R^2$$

Starobinsky Model

$$S \sim M_p^2 R + \left(\frac{M_p}{M_s}\right)^2 R^2$$

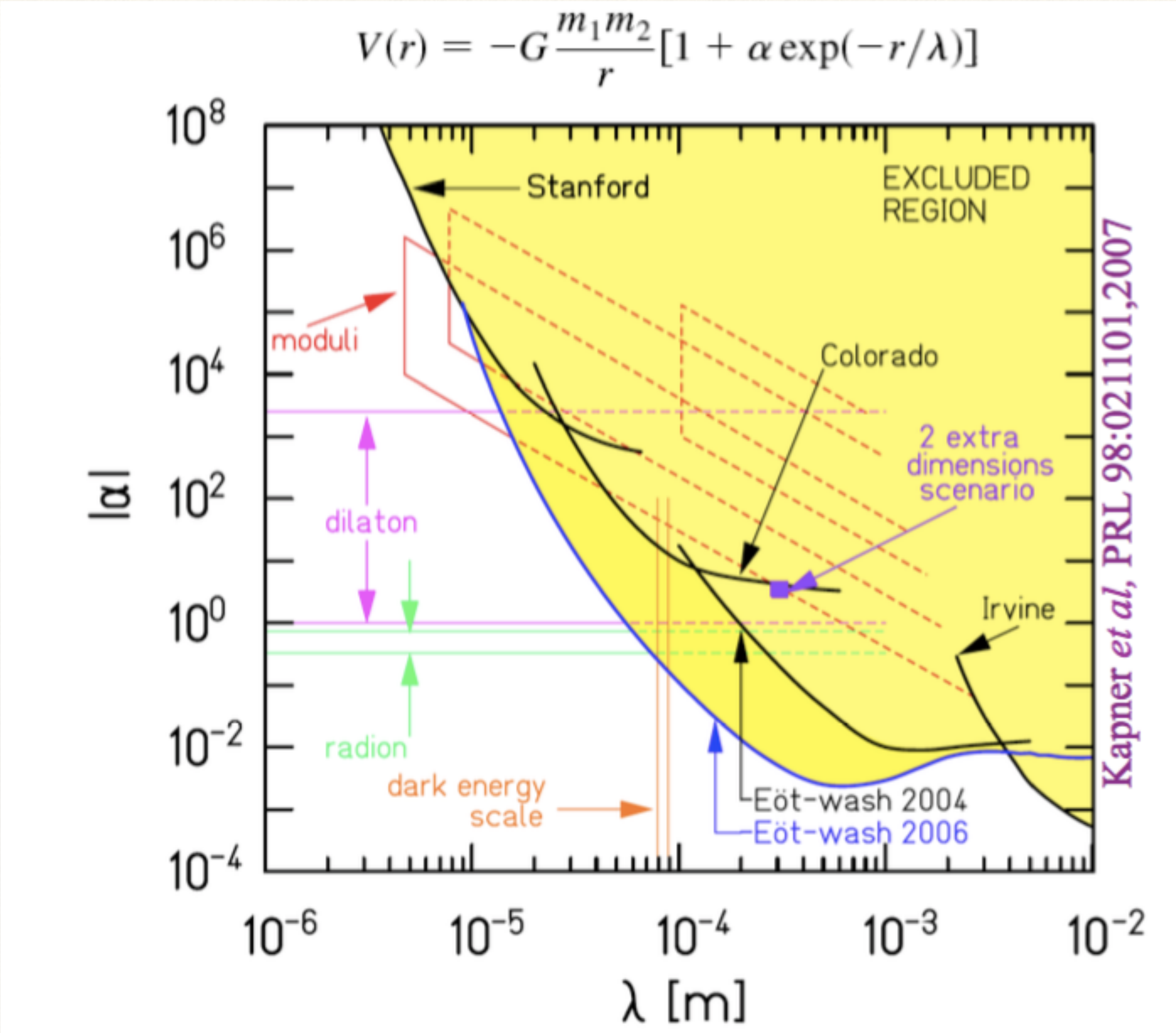
Quadratic Curvature Gravity

$$S \sim M_p^2 R + \alpha R^2 + \beta R^{mn} R_{mn} + \gamma C^{monp} C_{monp}$$

Numbers (α, β, γ)

Does a new scale ameliorates Blackholes and Cosmological Singularity Problems ?

Regarding Scales…



$(10^{27}\ \mathrm{eV})^4$

$(10^{-2}\ \mathrm{eV})^4$

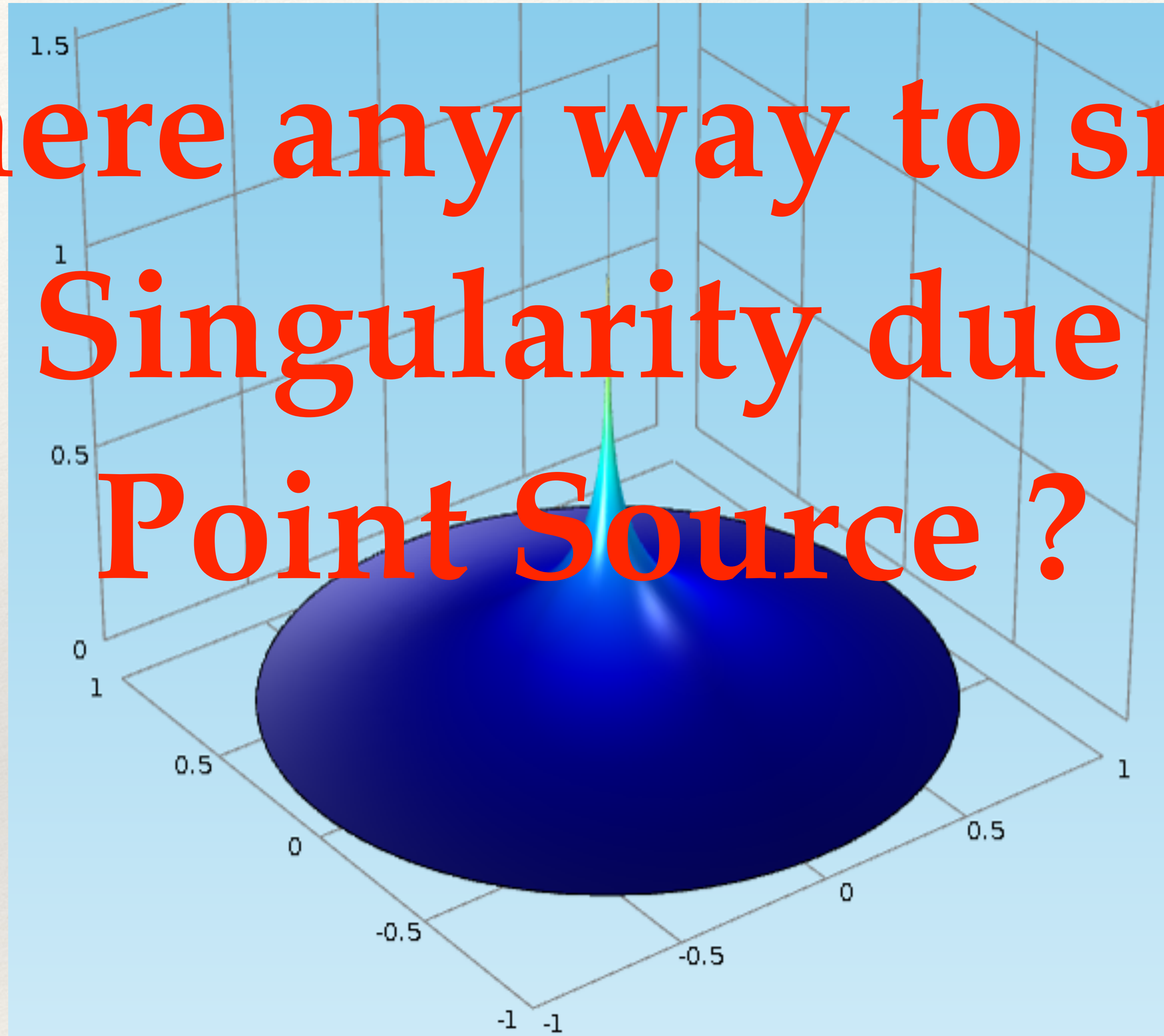
$(10^{-3}\ \mathrm{eV})^4$

No departure from Newtonian Gravity
up to

$10^{-5}\ \mathrm{m} \sim 100\ (\mathrm{eV})^{-1}$ or, $M \sim 10^{-2}$ eV

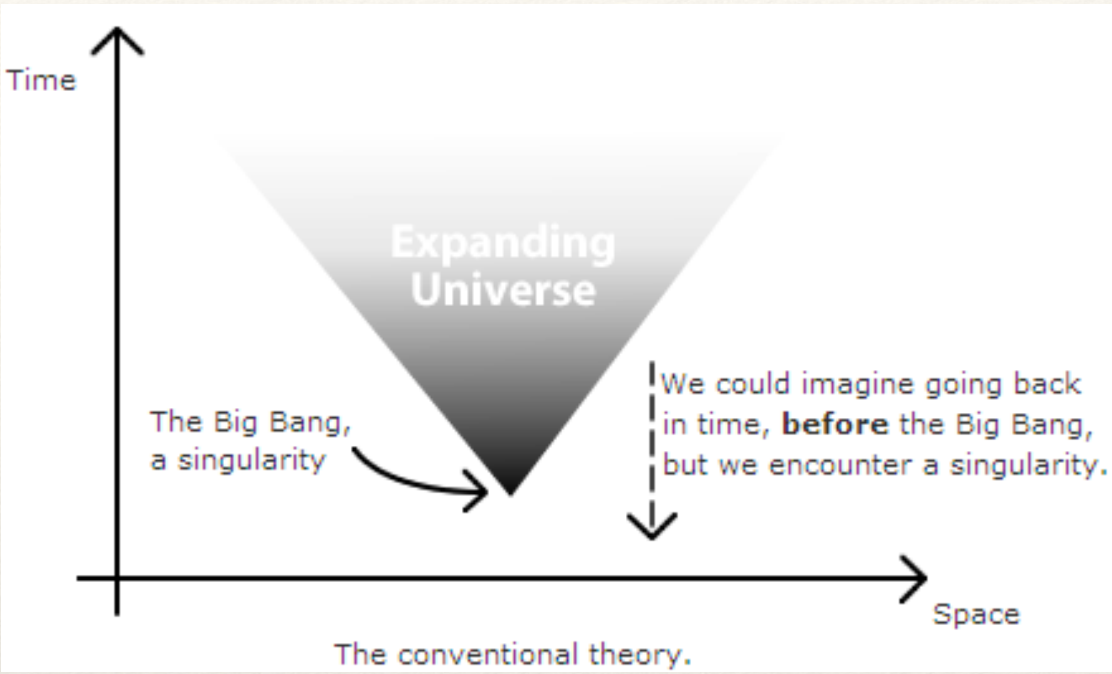
Einstein Gravity

Is there any way to smear the Singularity due to a Point Source ?

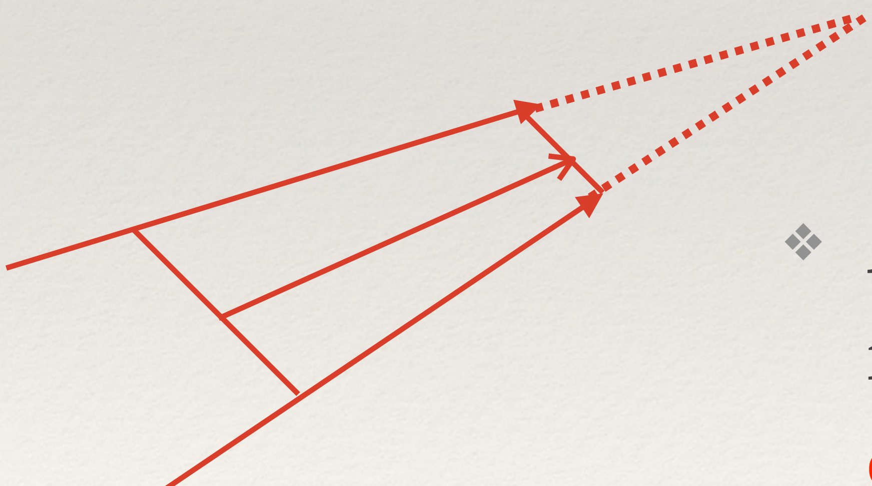


$$ds^2 = \left(1 - \frac{2Gm}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm}{r}\right)}$$

Cosmological Singularity



Big Bang Singularity, Space Time have an edge

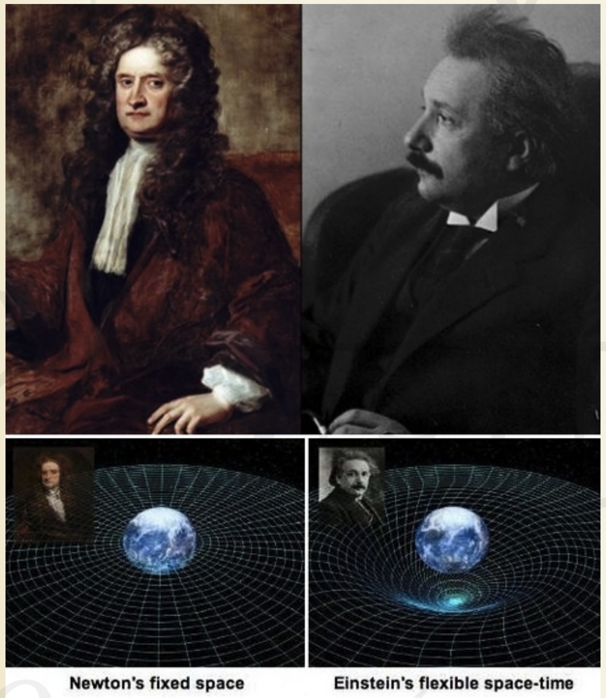


$$\rho + p \geq 0$$

- A singularity would always imply focusing of geodesics, but **focusing alone cannot imply a singularity**

"Inflation does not solve the singularity problem"

UV Modification of Gravity



UV is Pathological,

IR Part is Safe

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} + \cdots \right)$$

Gravity requires modification at small distances and at early times

While keeping the General Covariance

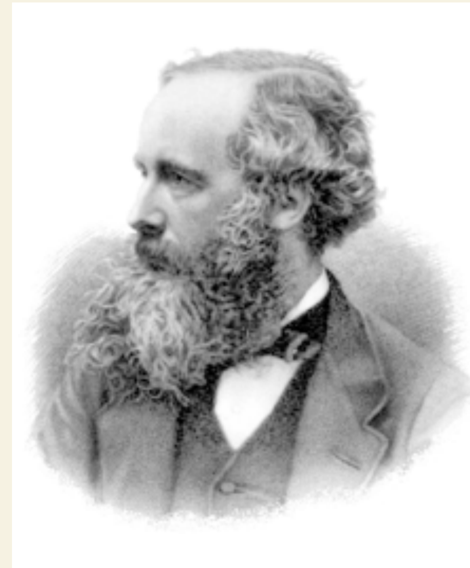
analogous to

Born-Infeld theory of E & M

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

Maxwell's Electromagnetism

Self energy of an electron is infinite in Maxwell's theory



1/r-fall of Coulomb's Potential

Quantum Electrodynamics (QED)

Classical approach: Born-Infeld

Born-Infeld resolves 1/r singularity in Coulomb Potential

$$\mathcal{L}_{\text{Born-Infeld}} = b^2 \left[1 - \sqrt{1 - (\boldsymbol{E}^2 - \boldsymbol{B}^2)/b^2 - (\boldsymbol{E}\cdot\boldsymbol{B})^2/b^4} \right]$$

$$b \to \infty \qquad\qquad \mathcal{L}_{\text{Born-Infeld}} \to \mathcal{L}_{\text{Maxwell}}$$

Maxwell	Born-Infeld
$E_{\text{tot}} = \frac{1}{2}\int (E.D + B.H) d^3r$ $D = e\hat{r}/4\pi r^2, \quad E = e\hat{r}/4\pi\epsilon r^2, \quad B = H = 0$ $E_{\text{tot}} = \frac{1}{32\pi^2}\int_0^\infty \frac{e^2}{r^4} 4\pi r^2 dr = \infty$	$\boldsymbol{\nabla}\cdot D = e\delta^{(3)}(\boldsymbol{r}) \qquad \boldsymbol{B} = 0$ $\boldsymbol{D} = \frac{e\boldsymbol{r}}{4\pi\lvert\boldsymbol{r}\rvert^3} \qquad \boldsymbol{D}^2/b^2 = \frac{q^2}{r^4}$ $E_{\text{tot}} = 4\pi b^2 \int_0^\infty dr\, r^2 \left(\sqrt{1 + q^2/r^4} - 1\right)$ $= \frac{4\Gamma^2(5/4)\sqrt{e^3 b}}{3\pi} = 1.2361\sqrt{e^3 b}$

Dealing Pure Gravity similar to QED is extremely hard

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

One loop pure gravitational action is renormalizable

Beyond two loops it is hard to compute, number of Feynman diagrams increases rapidly

Quadratic Curvature Gravity is renormalizable, but contains "Ghosts": Vacuum is Unstable

Utiyama, De Witt (1961), Stelle (1977)

Constructing Singularity Free & Ghost Free version for Gravity

- Consistent theory of Gravity around Constant Curvature Backgrounds
- Criteria for resolving Cosmological Singularity
- Divergence structures in 1 and 2-loops in a scalar Toy model

Without SUSY and SUGRA : SUSY is broken for a generic time dependent scenarios

Consistent General Covariant Quadratic Theories of Gravity with Constant Curvature Backgrounds

Spin-2

"Perturbative Unitarity"

&

"Ghost Free"

Spin-0

"Tachyon Free"

components of a Graviton Propagator

"Correct degrees of freedom in Graviton Propagator"

4th Derivative Gravity & Power Counting Renormalizability

$$I = \int d^4x \sqrt{g} \left[\lambda_0 + k\, R + a\, R_{\mu\nu} R^{\mu\nu} - \tfrac{1}{3}(b+a) R^2 \right]$$

$$D \propto \frac{1}{k^4 + A k^2} = \frac{1}{A}\left(\frac{1}{k^2} - \frac{1}{k^2 + A} \right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama, De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification of Graviton Propagator

Extra propagating degree of freedom

Challenge: to get rid of the extra dof

Ghosts

Higher Order Derivative Theory Generically Carry Ghosts (-ve Risidue) with real "m"(No-Tachyon)

$$S = \int d^4x \ \phi \Box (\Box + m^2) \phi \Rightarrow \Box (\Box + m^2) \phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$\Box e^{-\Box} \phi = 0$$

No extra states other than the original dof.

Moffat (1991), Tomboulis (1997), Tseytlin (1997), Siegel (2003), Biswas, Grisaru, Siegel (2004), Biswas, Mazumdar, Siegel (2006)

Higher order Construction of Gravity in Any Arbitrary Background

$$S = S_E + S_q$$

$$S_q = \int d^4x\sqrt{-g}R_{\mu_1\nu_1\lambda_1\sigma_1}\mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2}R^{\mu_2\nu_2\lambda_2\sigma_2}$$

Covariant derivatives

Unknown Infinite Functions of Derivatives

Well defined Minkowski Limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$R \sim \mathcal{O}(h) \qquad S_q \sim \int d^4x\sqrt{-g}\mathcal{O}(h^2)$$

Gravitational Form Factors

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

UV completion of Starobinsky Inflation up to quadratic in curvature

(1) GR
(2) Weyl Gravity
(3) F(R) Gravity
(4) Gauss-Bonnet Gravity
(5) Ghost free Gravity

Biswas, Mazumdar, Siegel, 2006,

Chialva, Mazumdar, 2013,

Koshelev, Modesto, Rachwal, Starobinsky, 2016

Linearised Equations of Motion around Minkowski

$$= \int d^4x \sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_q = -\int d^4x\Big[\frac{1}{2}h_{\mu\nu}a(\Box)\Box h^{\mu\nu} + h_\mu^\sigma b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu} \qquad (3)$$
$$+ hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} + \frac{1}{2}hd(\Box)\Box h + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\Big]$$

$$a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box$$
$$b(\Box) = -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box$$
$$c(\Box) = 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$d(\Box) = -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$f(\Box) = -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.$$

$$R_{\mu\nu\lambda\sigma} = \frac{1}{2}(\partial_{[\lambda}\partial_\nu h_{\mu\sigma]} - \partial_{[\lambda}\partial_\mu h_{\nu\sigma]})$$
$$R_{\mu\nu} = \frac{1}{2}(\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} - \partial_\nu\partial_\mu h - \Box h_{\mu\nu})$$
$$R = \partial_\nu\partial_\mu h^{\mu\nu} - \Box h$$

$$a + b = 0$$
$$c + d = 0$$
$$b + c + f = 0$$

Similar analysis has been derived for dS an AdS

Graviton Propagator around Minkowski

$$a(\Box)\Box h_{\mu\nu} + b(\Box)\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} + c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h)$$
$$+\eta_{\mu\nu}d(\Box)\Box h + \frac{1}{4}f(\Box)\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}$$

$$-\kappa\tau\nabla_\mu\tau^\mu_\nu = 0 = (c \overset{=0}{+} d)\Box\partial_\nu h + (a \overset{=0}{+} b)\Box h^\mu_{\nu,\mu} + (b + c \overset{=0}{+} f)h^{\alpha\beta}_{,\alpha\beta\nu}$$

Bianchi Identity

$$a + b = 0$$
$$c + d = 0$$
$$b + c + f = 0$$

$$\Pi^{-1\lambda\sigma}_{\mu\nu} h_{\lambda\sigma} = \kappa\tau_{\mu\nu} \qquad h = h^{TT} + h^L + h^T$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P^0_s}{(a-3c)k^2}$$

Spin projection operators

Let us introduce

$$\mathcal{P}^2 = \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma},$$
$$\mathcal{P}^1 = \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho}),$$
$$\mathcal{P}^0_s = \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \quad \mathcal{P}^0_w = \omega_{\mu\nu}\omega_{\rho\sigma},$$
$$\mathcal{P}^0_{sw} = \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}, \quad \mathcal{P}^0_{ws} = \frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma}, \qquad (16)$$

where the transversal and longitudinal projectors in the momentum space are respectively

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \qquad \omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}.$$

Note that the operators $\mathcal{P}^i$ are in fact 4-rank tensors, $\mathcal{P}^i_{\mu\nu\rho\sigma}$, but we have suppressed the index notation here.

Out of the six operators four of them, $\{\mathcal{P}^2, \mathcal{P}^1, \mathcal{P}^0_s, \mathcal{P}^0_w\}$, form a complete set of projection operators:

$$\mathcal{P}^i_a\mathcal{P}^j_b = \delta^{ij}\delta_{ab}\mathcal{P}^i_a \quad \text{and} \quad \mathcal{P}^2 + \mathcal{P}^1 + \mathcal{P}^0_s + \mathcal{P}^0_w = 1, \qquad (17)$$

$$\mathcal{P}^0_{ij}\mathcal{P}^0_k = \delta_{jk}\mathcal{P}^0_{ij}, \quad \mathcal{P}^0_{ij}\mathcal{P}^0_{kl} = \delta_{il}\delta_{jk}\mathcal{P}^0_k, \quad \mathcal{P}^0_k\mathcal{P}^0_{ij} = \delta_{ik}\mathcal{P}^0_{ij},$$

Ph.D. Thesis : K. J. Barnes, 1963

R. J. Rivers (1963)

P. Van Nieuwenhuizen,

Nucl.Phys. B60 (1973), 478.

For the above action, see:

Biswas, Koivisto, Mazumdar
1302.0532

Tree level Graviton Propagator

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

No new propagating degree of freedom other than the massless Graviton

$$a(\Box) = c(\Box) \Rightarrow 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0$$

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R\mathcal{F}_1(\Box)R - \frac{1}{2}R^{\mu\nu}\mathcal{F}_2(\Box)R_{\mu\nu}\right]$$

Without loss of generality either $\mathcal{F}_1,\ or\ \mathcal{F}_2,\ or\ \mathcal{F}_3 = 0$

Well known Higher Derivative limits

$$a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box$$
$$b(\Box) = -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box$$
$$c(\Box) = 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$d(\Box) = -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$f(\Box) = -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.$$

(1) GR:

$$a(0) = c(0) = -b(0) = -d(0) = 1$$

$$\lim_{k^2 \to 0} \Pi = (\mathcal{P}^2/k^2) - (\mathcal{P}_s^0/2k^2) \equiv \Pi_{GR}$$

(2) F(R) Gravity:

$$\mathcal{L}(R) = \mathcal{L}(0) + \mathcal{L}'(0)R + \frac{1}{2}\mathcal{L}''(0)R^2 + \cdots$$

$$a = -b = 1, \qquad c = -d = 1 - \mathcal{L}''(0)\Box$$

$$\Pi = \frac{\mathcal{P}^2}{k^2} - \frac{\mathcal{P}_s^0}{2k^2(1+3\mathcal{L}''(0)k^2)} \qquad \Pi = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}_s^0}{k^2+m^2}, \quad m^2 = \frac{1}{3\mathcal{L}''(0)}$$

(3) GB Gravity:

$$\mathcal{L} = R + \alpha(\Box)G,$$

$$a = c = -b = -d = 1$$

$$\Pi = \Pi_{GR}$$

(4) Weyl Gravity:

$$\mathcal{L} = R - \frac{1}{m^2}C^2 \qquad C^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$$a = -b = 1 - (k/m)^2$$

$c = -d = 1 - (k/m)^2/3$ and $f = -2(k/m)^2/3$

$$\Pi = \frac{\mathcal{P}^2}{k^2(1-(k/m)^2)} - \frac{\mathcal{P}_s^0}{2k^2} = \Pi_{GR} - \frac{\mathcal{P}^2}{k^2+m^2}$$

Biswas, Koivisto, Mazumdar
1302.0532

Complete Field Equations

Ghost-free gravity

2.3. The Complete Field Equations

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} + R\mathcal{F}_1(\Box)R + R^{\mu\nu}\mathcal{F}_2(\Box)R_{\mu\nu} + C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C_{\mu\nu\lambda\sigma}\right)$$

Following from this we find the equation of motion for the full action S in (1) to be a combination of S_0, S_1, S_2 and S_3 above

$$\begin{aligned} P^{\alpha\beta} = {} & G^{\alpha\beta} + 4G^{\alpha\beta}\mathcal{F}_1(\Box)R + g^{\alpha\beta}R\mathcal{F}_1(\Box)R - 4\left(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\Box\right)\mathcal{F}_1(\Box)R \\ & - 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\sigma}^{\sigma} + \bar{\Omega}_1) + 4R_\mu^\alpha\mathcal{F}_2(\Box)R^{\mu\beta} \\ & - g^{\alpha\beta}R_\nu^\mu\mathcal{F}_2(\Box)R_\mu^\nu - 4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\Box)R^{\mu\alpha}) + 2\Box(\mathcal{F}_2(\Box)R^{\alpha\beta}) \\ & + 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\sigma}^{\sigma} + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta} \\ & - g^{\alpha\beta}C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C_{\mu\nu\lambda\sigma} + 4C^{\alpha}_{\ \mu\nu\sigma}\mathcal{F}_3(\Box)C^{\beta\mu\nu\sigma} \\ & - 4(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\Box)C^{\beta\mu\nu\alpha}) - 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}^{\gamma} + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta} \\ = {} & T^{\alpha\beta}\,, \end{aligned} \tag{52}$$

where $T^{\alpha\beta}$ is the stress energy tensor for the matter components in the universe and we have defined the following symmetric tensors:

$$\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1_n} \sum_{l=0}^{n-1} \nabla^\alpha R^{(l)}\nabla^\beta R^{(n-l-1)}, \quad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1_n} \sum_{l=0}^{n-1} R^{(l)}R^{(n-l)}, \tag{53}$$

$$\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} R_\nu^{\mu;\alpha(l)}R_\mu^{\nu;\beta(n-l-1)}, \quad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} R_\nu^{\mu(l)}R_\mu^{\nu(n-l)}\,, \tag{54}$$

$$\Delta_2^{\alpha\beta} = \frac{1}{2}\sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} [R_\sigma^{\nu(l)}R^{(\beta|\sigma|;\alpha)(n-l-1)} - R_\sigma^{\nu;(\alpha(l)}R^{\beta)\sigma(n-l-1)}]_{;\nu}\,, \tag{55}$$

$$\Omega_3^{\alpha\beta} = \sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} C^{\mu;\alpha(l)}_{\ \nu\lambda\sigma}C_\mu^{\ \nu\lambda\sigma;\beta(n-l-1)}, \ \bar{\Omega}_3 = \sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} C^{\mu(l)}_{\ \nu\lambda\sigma}C_\mu^{\ \nu\lambda\sigma(n-l)}\,, \tag{56}$$

$$\Delta_3^{\alpha\beta} = \frac{1}{2}\sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} [C^{\lambda\nu(l)}_{\ \ \sigma\mu}C_\lambda^{\ (\beta|\sigma\mu|;\alpha)(n-l-1)} - C^{\lambda\nu\ \ ;(\alpha(l)}_{\ \ \sigma\mu}C_\lambda^{\ \beta)\sigma\mu(n-l-1)}]_{;\nu}\,. \tag{57}$$

The trace equation is often particularly useful and below we provide it for the general action (1):

$$\begin{aligned} P = {} & -R + 12\Box\mathcal{F}_1(\Box)R + 2\Box(\mathcal{F}_2(\Box)R) + 4\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu}) \\ & + 2(\Omega_{1\sigma}^{\sigma} + 2\bar{\Omega}_1) + 2(\Omega_{2\sigma}^{\sigma} + 2\bar{\Omega}_2) + 2(\Omega_{3\sigma}^{\sigma} + 2\bar{\Omega}_3) - 4\Delta_{2\sigma}^{\sigma} - 8\Delta_{3\sigma}^{\sigma} \\ = {} & T \equiv g_{\alpha\beta}T^{\alpha\beta}\,. \end{aligned} \tag{58}$$

It is worth noting that we have checked special cases of our result against previous work in sixth order gravity given in [24] and found them to be equivalent at least to the cubic order (see Appendix C for details).

$$R^{(m)} \equiv \Box^m R$$

Biswas, Conroy, Koshelev, Mazumdar
1308.2319 Class.Quant. Grav. (2014)

Cosmological Bouncing solution is known exactly

Biswas, Mazumdar, Siegel, 2006,

Stability of Hamiltonian

Hamiltonian Analysis for Infinite Derivative Field Theories and Gravity

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Abstract

Typically higher-derivative theories are unstable. Instabilities manifest themselves from *extra* propagating degrees of freedom, which are unphysical. In this paper, we will investigate *infinite derivative* field theories and study their *true* dynamical degrees of freedom via Hamiltonian analysis. In particular, we will show that if the infinite derivatives can be captured by a Gaussian kinetic term, i.e. *exponential of entire function*, then it is possible to prove that there are only *finite* number of dynamical degrees of freedom. This conclusion is similar to previous analyses which were performed in the context of Lagrangian analysis. We will further extend our investigation into infinite derivative theory of gravity, and in particular concentrate on *ghost free* and *singularity free* theory of gravity, which has been studied extensively in the Lagrangian approach. Here we will show from the Hamiltonian perspective that there are only finite number of degrees of freedom. For a homogeneous case, we will show that the Hamiltonian density can be bounded form below.

Consistent theories of Gravity around dS and Ads backgrounds

$$S = \int d^4x \sqrt{-g} \left[\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right]$$

Most generic action - "Parity Invariant" and "Torsion Free"

$$R = \bar{R} = \text{const}, \qquad R_{\mu\nu} = \frac{\bar{R}}{4} \bar{g}_{\mu\nu}, \qquad R^{\rho}_{\mu\sigma\nu} = \frac{\bar{R}}{12} (\delta^{\rho}_{\sigma} \bar{g}_{\mu\nu} - \delta^{\rho}_{\nu} \bar{g}_{\mu\sigma})$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \Lambda + \frac{\lambda}{2} \left(R \mathcal{F}_1(\Box) R + S_{\mu\nu} \mathcal{F}_2(\Box) S^{\mu\nu} + C_{\mu\nu\lambda\sigma} \mathcal{F}_3(\Box) C^{\mu\nu\lambda\sigma} \right) \right]$$

$$h_{\mu\nu} = h^{\perp}_{\mu\nu} + \bar{\nabla}_{\mu} A^{\perp}_{\nu} + \bar{\nabla}_{\nu} A^{\perp}_{\mu} + (\bar{\nabla}_{\mu} \bar{\nabla}_{\nu} - \frac{1}{4} \bar{g}_{\mu\nu} \bar{\Box}) B + \frac{1}{4} \bar{g}_{\mu\nu} h$$

For pure EH action, see D'Hoker, Freedman, Mathur, Matusis, Rastelli (hep-th/9902042)

Full Quadratic action, see: Biswas, Koshelev, Mazumdar, 1602.08475

Quadratic order Action for spin-2 and spin-0 components

$$S_2 \equiv \frac{1}{2}\int dx^4\sqrt{-\bar g}\,\widetilde{h^{\perp}}^{\mu\nu}\left(\bar\Box - \frac{\bar R}{6}\right)$$
$$\left\{1 + \frac{2}{M_p^2}\lambda c_{1,0}\bar R + \frac{\lambda}{M_p^2}\left[\left(\bar\Box - \frac{\bar R}{6}\right)\mathcal{F}_2(\bar\Box) + 2\left(\bar\Box - \frac{\bar R}{3}\right)\mathcal{F}_3\left(\bar\Box + \frac{\bar R}{3}\right)\right]\right\}\widetilde{h^{\perp}}_{\mu\nu}$$

$$S_0 \equiv -\frac{1}{2}\int dx^4\sqrt{-\bar g}\,\tilde\phi\left(\bar\Box + \frac{\bar R}{3}\right)$$
$$\left\{1 + \frac{2}{M_p^2}\lambda c_{1,0}\bar R - \frac{\lambda}{M_p^2}\left[2(3\bar\Box + \bar R)\mathcal{F}_1(\bar\Box) + \frac{1}{2}\bar\Box\mathcal{F}_2\left(\bar\Box + \frac{2}{3}\bar R\right)\right]\right\}\tilde\phi$$

Minkowski limit matches with our earlier propagator

$$\Pi_2 = \frac{i}{p^2\left\{1 - \frac{2p^2}{M_p^2}\left[\mathcal{F}_2(-p^2) + 2\mathcal{F}_3(-p^2)\right]\right\}},$$
$$\Pi_0 = \frac{-i}{p^2\left\{1 + \frac{2p^2}{M_p^2}\left[6\mathcal{F}_1(-p^2) + \frac{1}{2}\mathcal{F}_2(-p^2)\right]\right\}}$$

$$\widetilde{h^{\perp}}_{\mu\nu} = \frac{1}{2}M_p h^{\perp}_{\mu\nu}, \qquad \tilde\phi = \sqrt{\frac{3}{32}}M_p\phi$$

Biswas, Koshelev, Mazumdar
1602.08475

Most generic Ghost FreeGraviton Propagator in dS/AdS

$$\mathcal{T}(\bar{R},\bar{\Box}) \equiv 1 + \frac{4\bar{R}}{M_p^2}c_{1,0} + \frac{2}{M_p^2}\left[\left(\bar{\Box} - \frac{\bar{R}}{6}\right)\mathcal{F}_2(\bar{\Box}) + 2\left(\bar{\Box} - \frac{\bar{R}}{3}\right)\mathcal{F}_3\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\right]$$

$$\mathcal{S}(\bar{R},\bar{\Box}) \equiv 1 + \frac{4\bar{R}}{M_p^2}c_{1,0} - \frac{2}{M_p^2}\left[2(3\bar{\Box} + \bar{R})\mathcal{F}_1(\bar{\Box}) + \frac{1}{2}\bar{\Box}\mathcal{F}_2\left(\bar{\Box} + \frac{2}{3}\bar{R}\right)\right]$$

$$\mathcal{T}(\bar{R},\bar{\Box}) \equiv e^{\tau(\bar{\Box})},$$

$$\mathcal{S}(\bar{R},\bar{\Box}) \equiv \left(1 - \frac{\bar{\Box}}{m^2}\right)^{\epsilon} e^{\sigma(\bar{\Box})}$$

$\epsilon = 0$, No scalar propagating d.o.f.

Biswas, Koshelev & AM, 1602.08475

Newtonian Limit in Minkowski

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} \qquad a(\Box) = c(\Box) = e^{-\Box/M^2}$$

$$S = \int d^4x \, \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Psi)dr^2$$

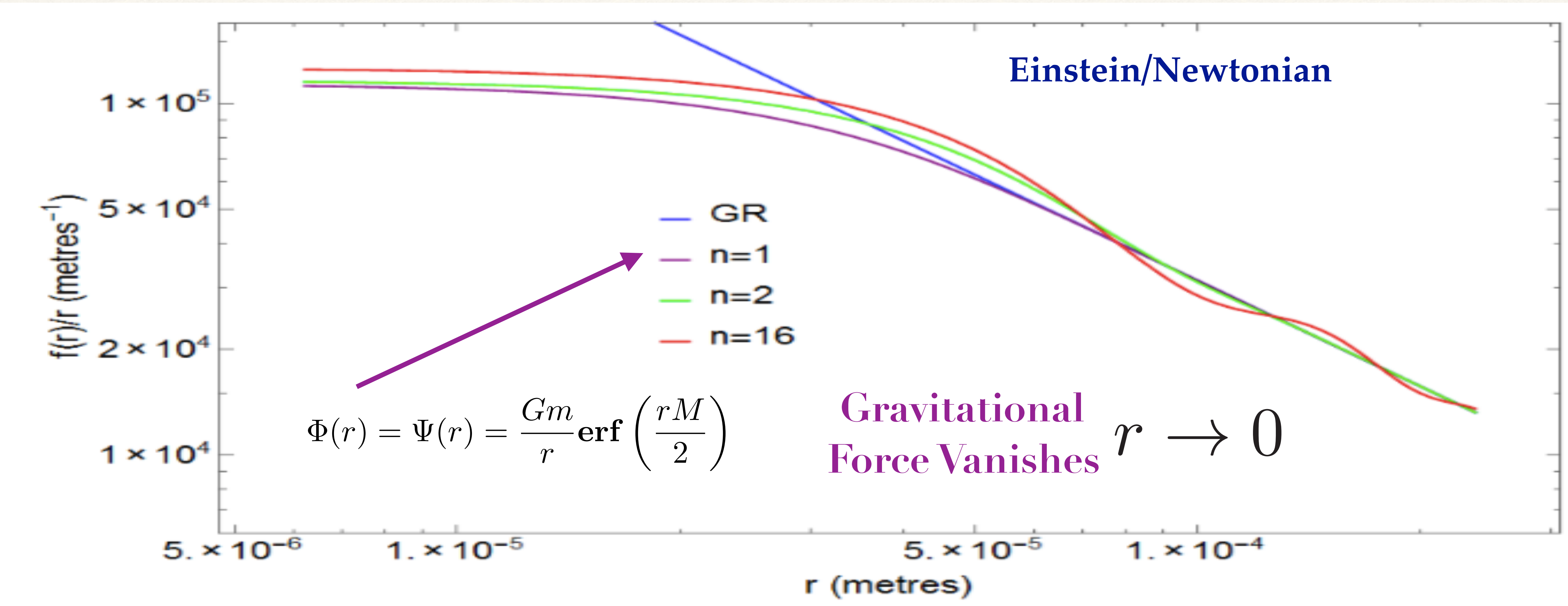
$$\Phi = \Psi = \frac{Gm}{r} \mathbf{erf}\left(\frac{rM}{2} \right)$$

Biswas, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. (2012) (gr-qc/1110.5249)

Resolution of Singularity at short distances

$$a(\Box) = e^{\gamma(\Box)}$$

Any Entire Function: $\gamma(\Box) = -\frac{\Box}{M^2} - \sum_N a_N \left(\frac{\Box}{M^2}\right)^N$



$$mM \ll M_p^2 \implies m \ll M_p$$

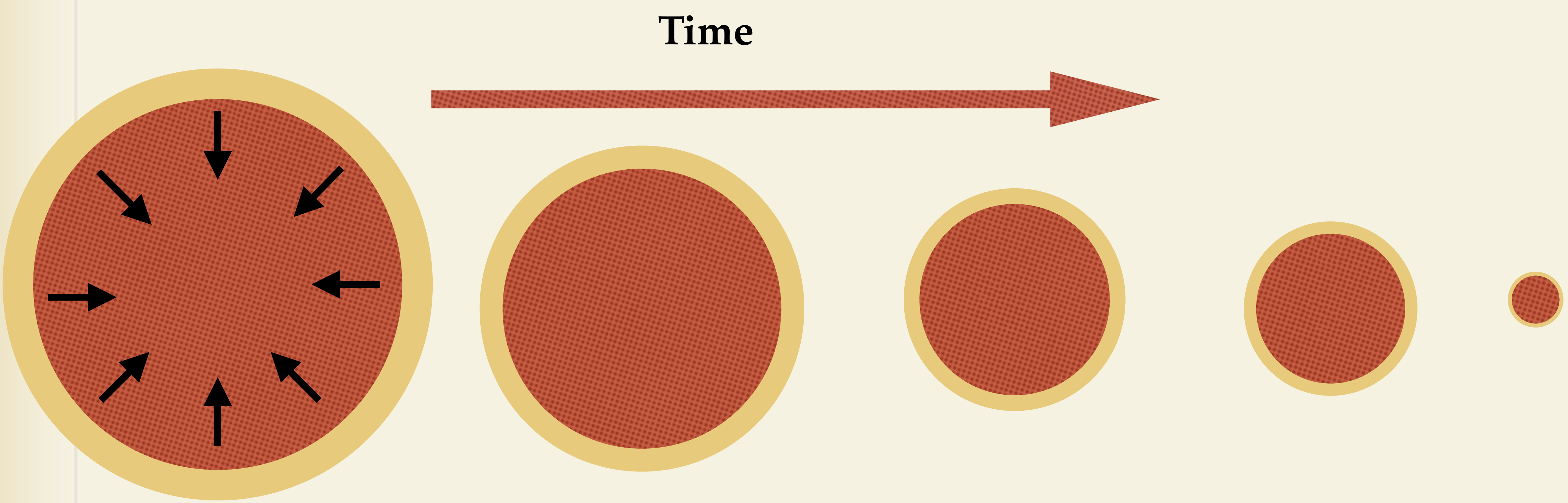
Current Bound : M > 0.01 eV

Edholm, Koshelev, Mazumdar (2016)

Frolov & Zelnikov (2015, 2016)

Dynamical Aspects

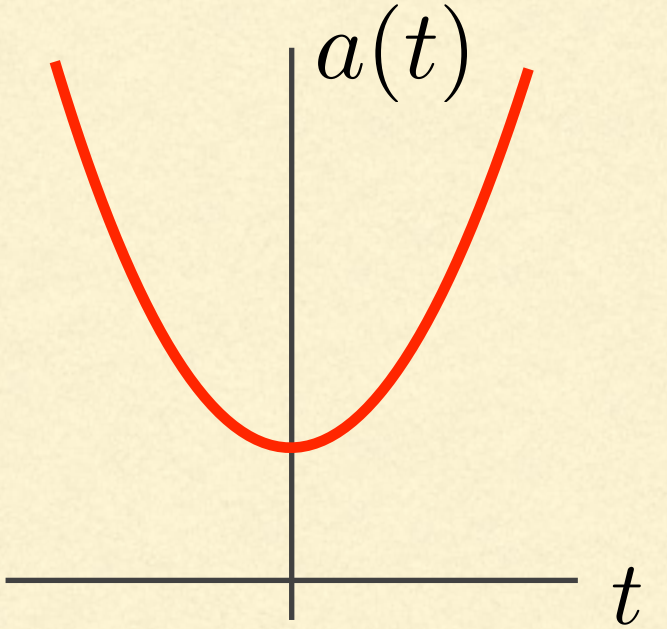
Valeri Frolov & Andrei Zelnikov
Studied various aspects in 5 papers
(2015, 2016)



Conclusion: A lump of matter without Horizon and without Singularity in a Linear regime

Cosmological Singularity can be resolved in a Full Non-linear Regime

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R + \Lambda \right]$$

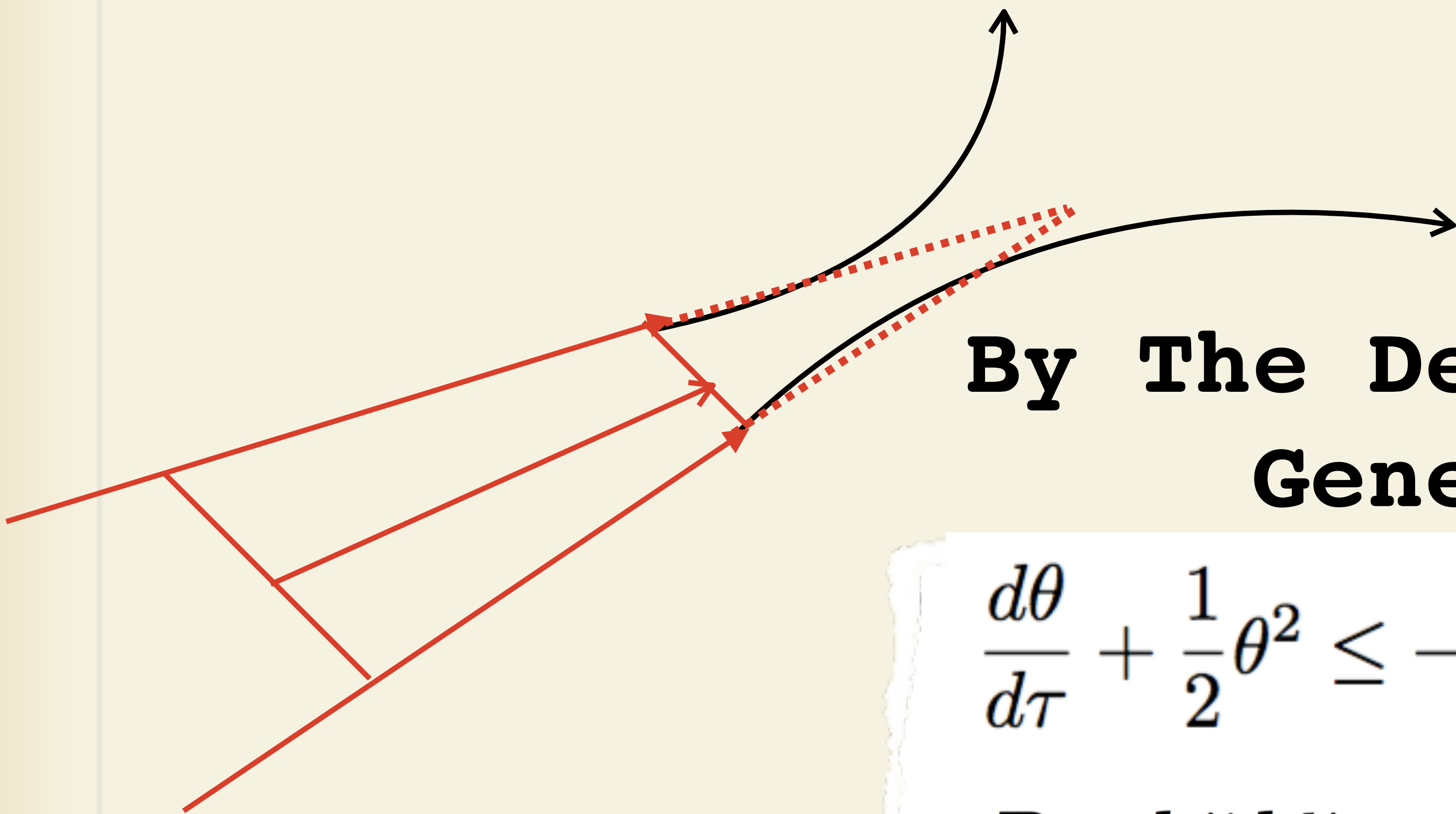


$$a(t) = a_0 \cosh \left(\sqrt{\frac{\Lambda}{6M_{pl}^2}} t \right)$$

$$\Box R = c_1 R + c_2$$

Biswas, Mazumdar, Siegel, JCAP (2006)

Defocusing Null rays



By The Defocusing Theorem of General Relativity

$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{\mu\nu}k^\mu k^\nu$$

Einstein-Hilbert:

$$R_{\mu\nu}k^\mu k^\nu = \kappa T_{\mu\nu}k^\mu k^\nu$$

$$R_{\mu\nu}k^\mu k^\nu \geq 0, \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq 0$$

Infinite derivative Gravity:

$$R_{\mu\nu}k^\mu k^\nu \leq 0, \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

Conroy, Koshelev, Mazumdar, 1604.01989

3 Criteria for Defocusing Null Congruences without Ghosts & Tachyons

$$\frac{f(\bar{\Box})\Box}{a(\bar{\Box})}R^{(L)} > 0 \Rightarrow \frac{a(\bar{\Box}) - c(\bar{\Box})}{a(\bar{\Box})}R^{(L)} > 0$$

$$c(\bar{\Box}) = \frac{a(\bar{\Box})}{3}\left[1 + 2\left(1 - \alpha M_P^{-2}\Box\right)\tilde{a}(\bar{\Box})\right]$$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_P^2 R + R\mathcal{F}_1(\bar{\Box})R\right]$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

Massless Graviton for : a=c

(1) Infinite Derivatives

Locality leads to Starobinsky Model, which requires Tachyonic massive Spin-0 states to resolve singularity, but it cannot give Inflation !

(2) Massless Spin-2,

(3) Non-Tachyonic Massive Spin-0

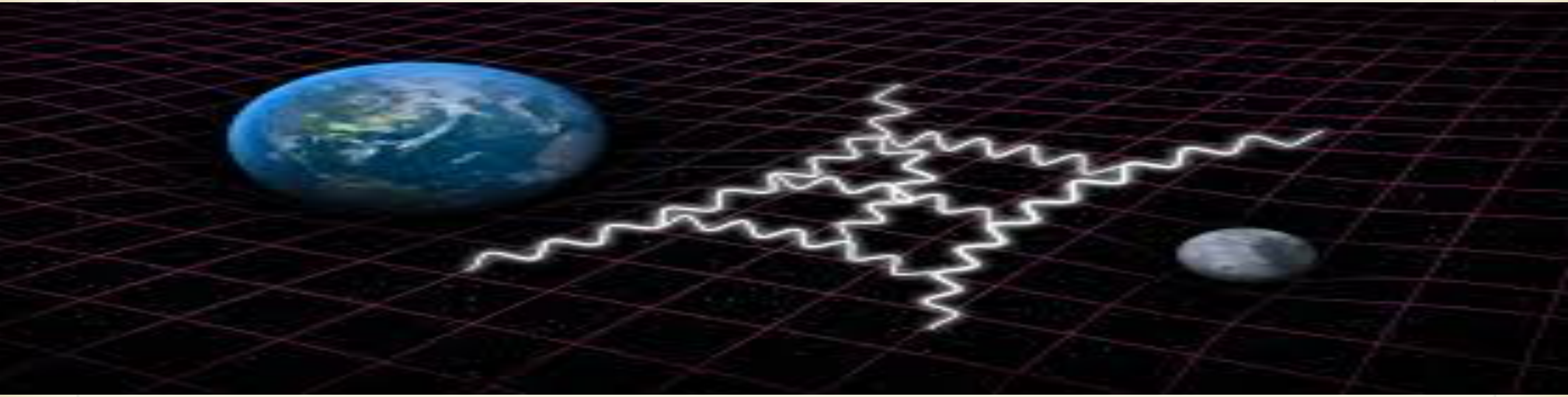
$$\Pi(-k^2) = \frac{1}{a(-k^2)}\left[\frac{\mathcal{P}^2}{k^2} - \frac{1}{2\tilde{a}(-k^2)}\left(\frac{\mathcal{P}_s^0}{k^2} - \frac{\mathcal{P}_s^0}{k^2+m^2}\right)\right]$$

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_p^2 R + cR^2\right]$$

$$\Pi_{R^2} = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}_s^0}{k^2+m^2}$$

Conroy, Koshelev, Mazumdar, 1604.01989

Quantum Aspects



How to make Gravity UV Finite ?

Could we make Gravity weak in UV?

Some interesting progress have been made:

Gravitational entropy, Boundary action, Hamiltonian, Quantum loop corrections, Ultra high energy scatterings, etc.

Quantum aspects

- **Superficial degree of divergence goes as**

$$E = V - I. \ \ \text{Use Topological relation}: \ L = 1 + I - V$$

$$E = 1 - L \qquad\qquad E < 0, \ \text{for} \ L > 1$$

- **At 1-loop, the theory requires counter term, the 1-loop, 2 point function yields Λ^4 divergence**
- **At 2-loops, the theory does not give rise to additional divergences, the UV behaviour becomes finite, at large external momentum, where dressed propagators gives rise to more suppression than the vertex factors**

Talaganis, Biswas, Mazumdar, (2014)

Toy model based on Symmetries

GR e.o.m : $g_{\mu\nu} \rightarrow \Omega\, g_{\mu\nu}$

Around Minkowski space the e.o.m are invariant under: $h_{\mu\nu} \rightarrow (1+\epsilon)h_{\mu\nu} + \epsilon\eta_{\mu\nu}$

Construct a scalar field theory with infinite derivatives whose e.o.m are invariant under

$$\phi \rightarrow (1+\epsilon)\phi + \epsilon$$

$$S_{free} = \frac{1}{2}\int d^4x(\phi\Box a(\Box)\phi)$$

$$a(\Box) = e^{-\Box/M^2}$$

$$S_{int} = \frac{1}{M_p}\int d^4x\left(\frac{1}{4}\phi\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}\phi\Box\phi a(\Box)\phi - \frac{1}{4}\phi\partial_\mu\phi a(\Box)\partial^\mu\phi\right)$$

$$\Pi(k^2) = -\frac{i}{k^2 e^{\bar{k}^2}}$$

Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity

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arXiv:1412.3467v1 [hep-th] 10 Dec 2014

Abstract

In this paper we will consider quantum aspects of a non-local, infinite derivative scalar field theory - a *toy model* depiction of a covariant infinite derivative, non-local extension of Einstein's general relativity which has previously been shown to be free from ghosts around the Minkowski background. The graviton propagator in this theory gets an exponential suppression making it *asymptotically free*, thus providing strong prospects of resolving various classical and quantum divergences. In particular, we will find that at 1-loop, the 2-point function is still divergent, but once this amplitude is renormalized by adding appropriate counter terms, the ultraviolet (UV) behavior of all other 1-loop diagrams as well as the 2-loop, 2-point function remains well under control. We will go on to discuss how one may be able to generalize our computations and arguments to arbitrary loops.

Ultra High Energy Scatterings of Scalar Gravitons

High-Energy Scatterings in Infinite-Derivative Field Theory and Ghost-Free Gravity

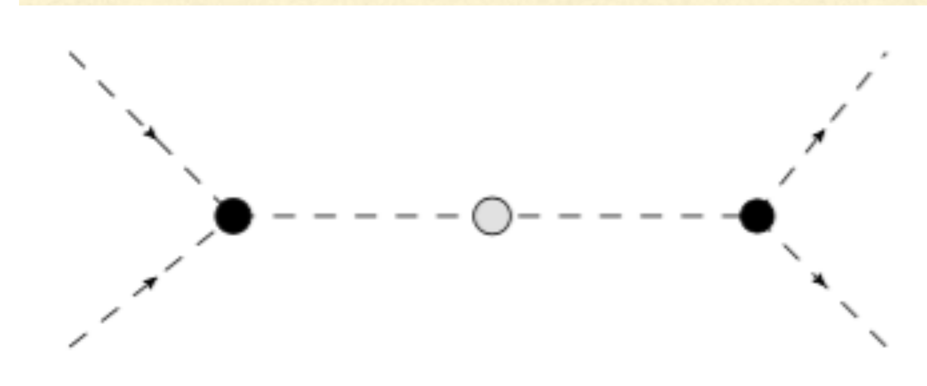
Spyridon Talaganis and Anupam Mazumdar

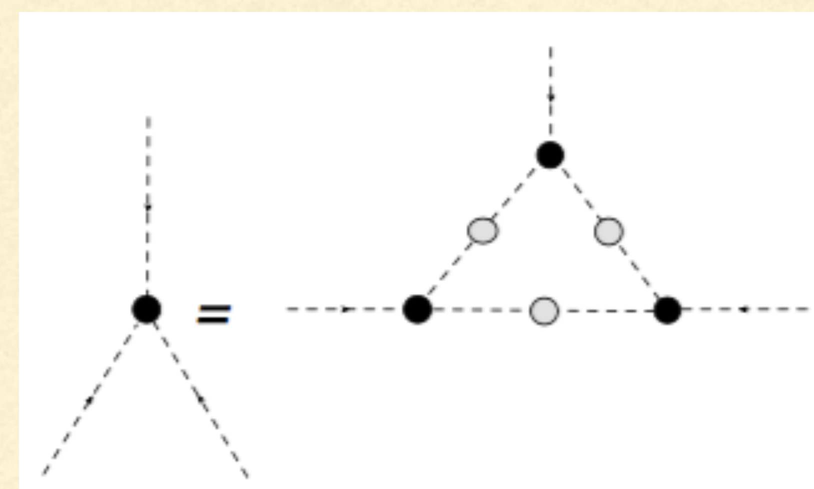
Consortium for Fundamental Physics, Lancaster University, LA1 4YB, UK

March 14, 2016

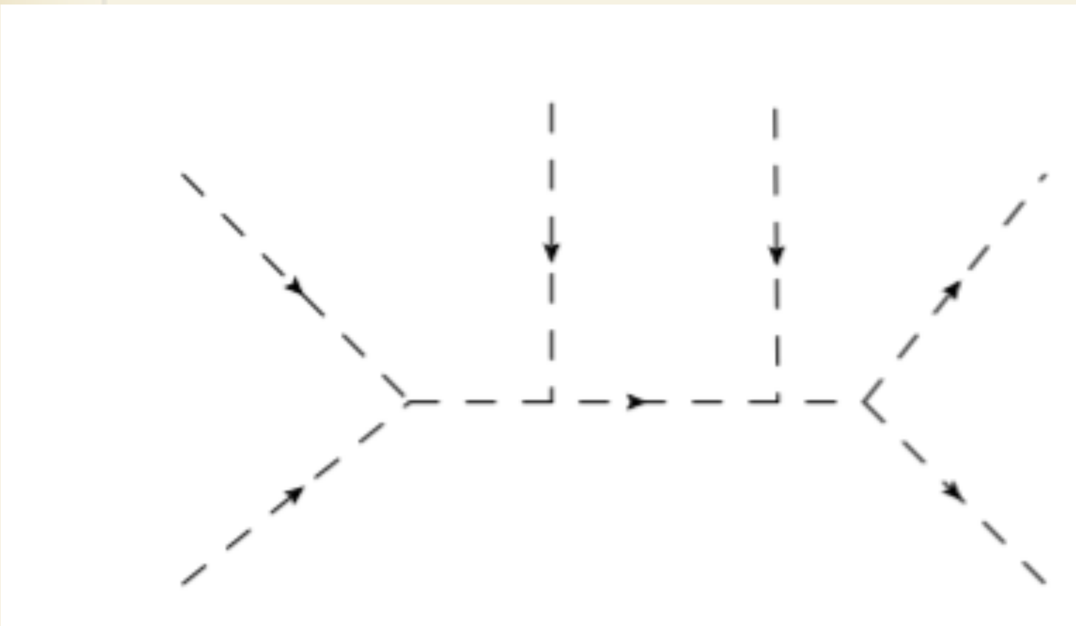
Abstract

In this paper, we will consider scattering diagrams in the context of infinite-derivative theories. First, we examine a finite-order higher-derivative scalar field theory and find that we cannot eliminate the external momentum divergences of scattering diagrams in the regime of large external momenta. Then, we employ an infinite-derivative scalar toy model and obtain that the external momentum dependence of scattering diagrams is convergent as the external momenta become very large. In order to eliminate the external momentum divergences, one has to dress the bare vertices of the scattering diagrams by considering renormalised propagator and vertex loop corrections to the bare vertices. Finally, we investigate scattering diagrams in the context of a scalar toy model which is inspired by a *ghost-free* and *singularity-free* infinite-derivative theory of gravity, where we conclude that infinite derivatives can eliminate the external momentum divergences of scattering diagrams and make the scattering diagrams convergent in the ultraviolet.





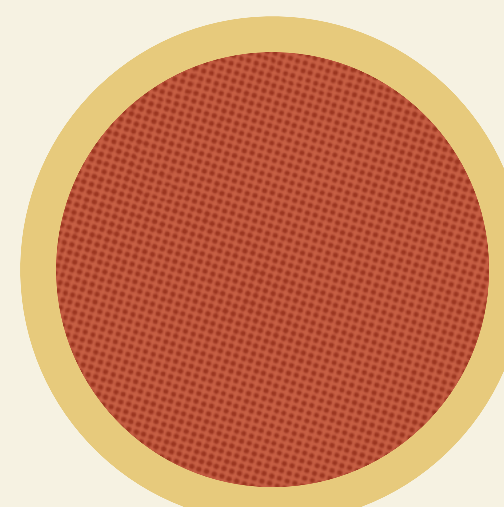
Ultra High Energy Scatterings of many Scalar Gravitons :



$$\mathcal{M}' \sim e^{-\left(\frac{p}{M_{eff}}\right)^2},$$

$$M_{eff} = \left(\frac{54}{125n - 206}\right)^{1/2} M$$

Non-Locality can be spread out on Event Horizon scale !

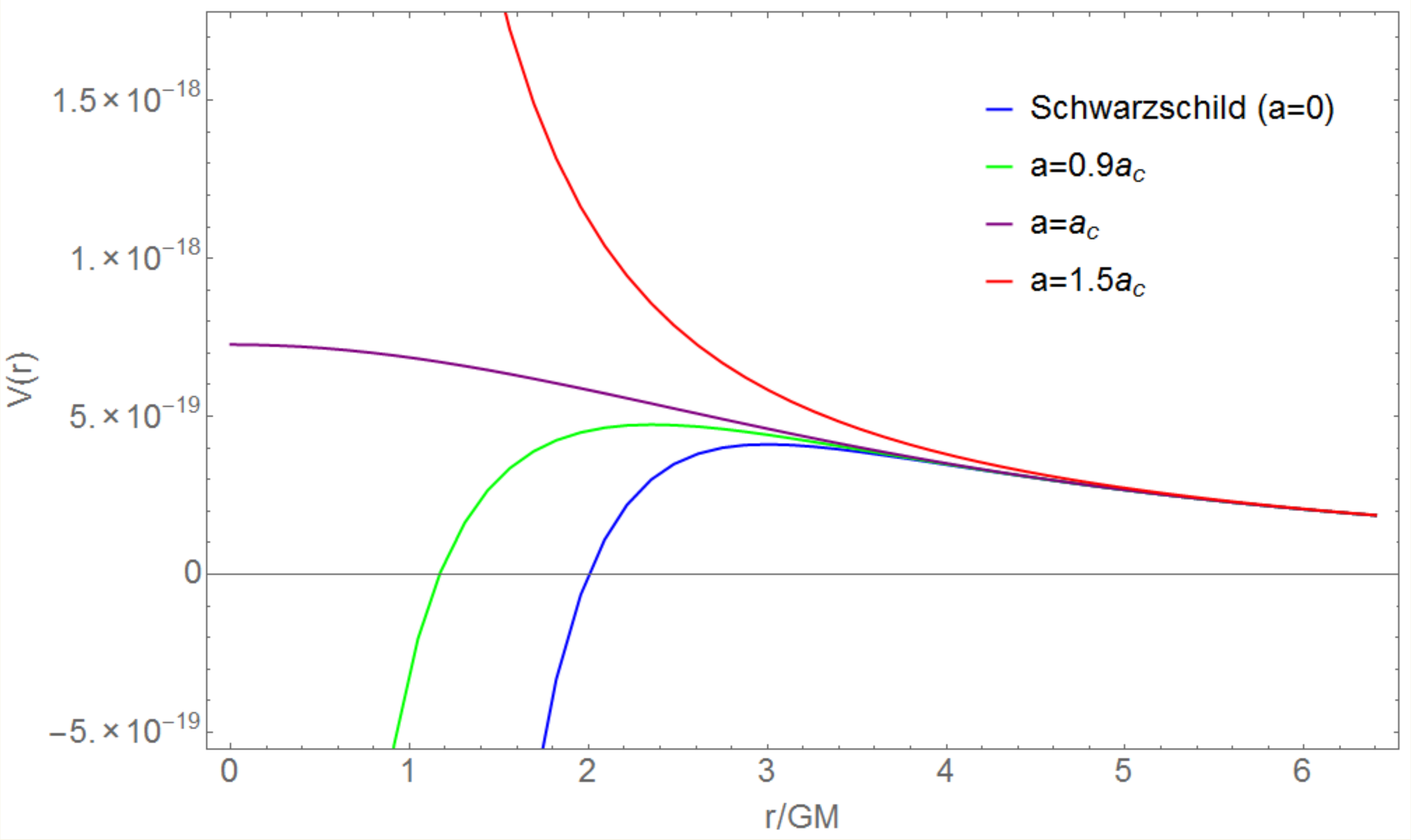


$$r \sim r_* \sim (2GM_{star}) \gg M$$

Conjecture : Gravity can be made weak not to form a trapped surface

Photon Potential

$$ds^2 = -\left(1 - \frac{r_*}{r}\mathrm{Erf}\left(\frac{r}{ar_*}\right)\right)dt^2 + \frac{1}{1 - \frac{r_*}{r}\mathrm{Erf}\left(\frac{r}{ar_*}\right)}dr^2 + r^2 d\theta^2 + r^2\sin^2(\theta)d\phi^2$$



Conclusions

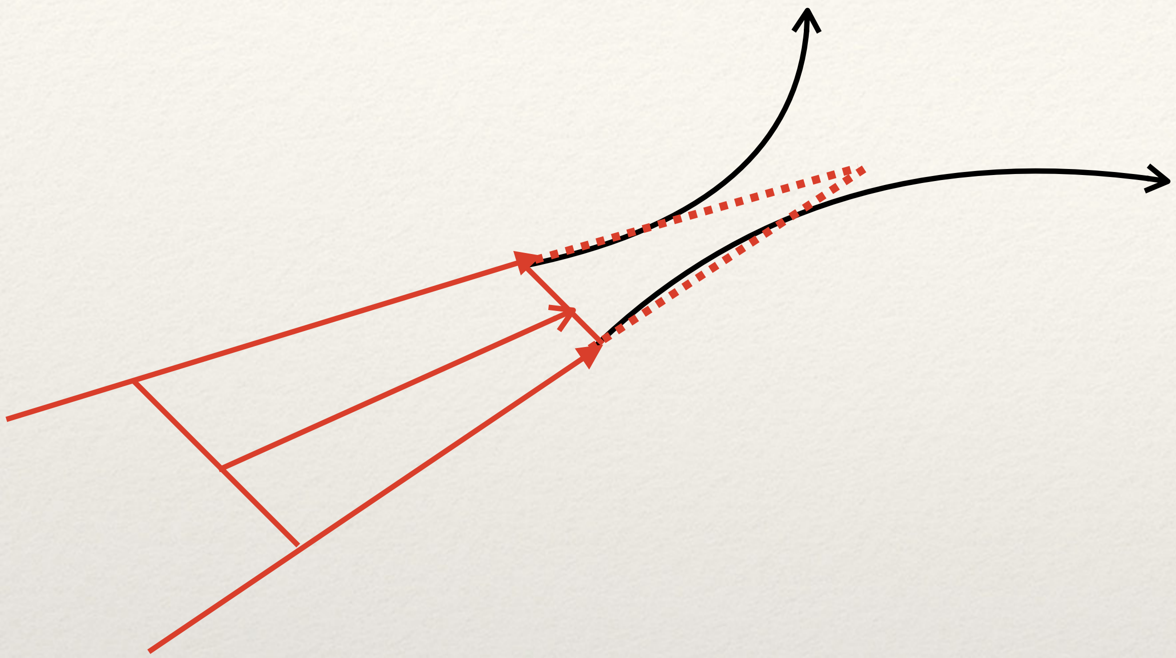
- We have constructed a Ghost Free & Singularity Free Theory of Gravity.
- Studying singularity theorems, Hawking radiation, Non-Singular Bouncing Cosmology , ..., many interesting problems has been studied in this framework.
- Quantum computations also show that Infinite Derivative Gravity can ameliorate UV behaviour.
- Ultra-High energy graviton scatterings do not blow up.
- Quantum effects can be seen on Macroscopic scale.

All these consequences have ramifications for Blackhole, Inflation & Quantum aspects of Gravity:
Both are Time Dependent Problems

Extra Slides

Non-Singular Bouncing Solutions: UV completion of Starobinsky Inflation

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}}-1}{\Box}\right]R + \Lambda\right]$$



Linear Solution

$$h \sim \mathtt{diag}(0, A\sin\lambda t, A\sin\lambda t, A\sin\lambda t) \text{ with } A \ll 1$$

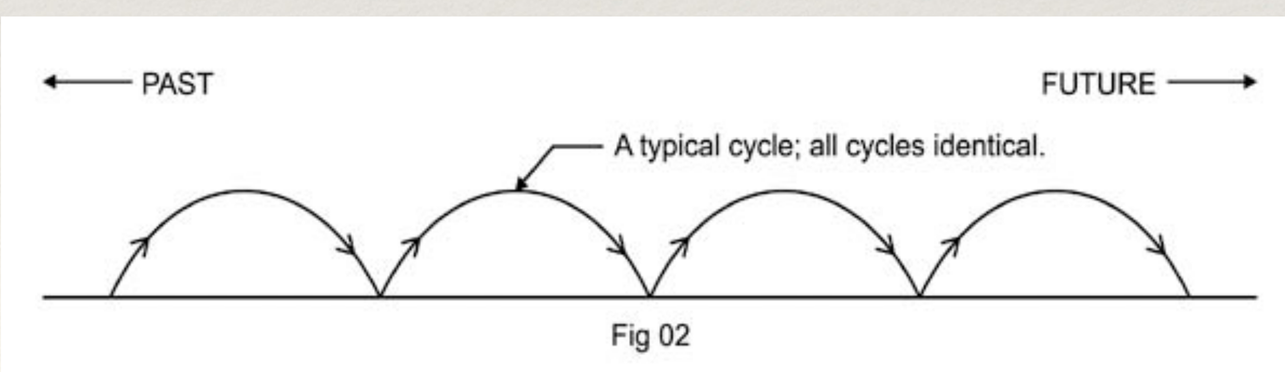
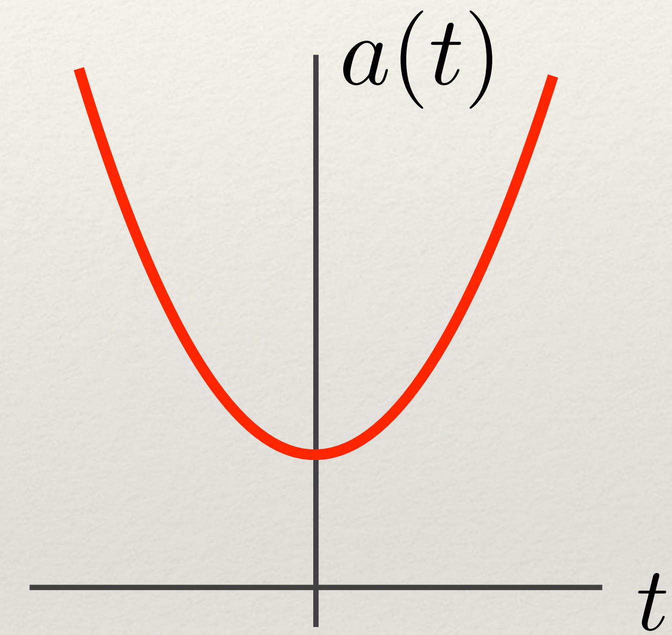


Fig 02

Biswas, Gerwick, Koivisto, Mazumdar,
Phys. Rev. Lett. (gr-qc/1110.5249)



Non-Linear Solution

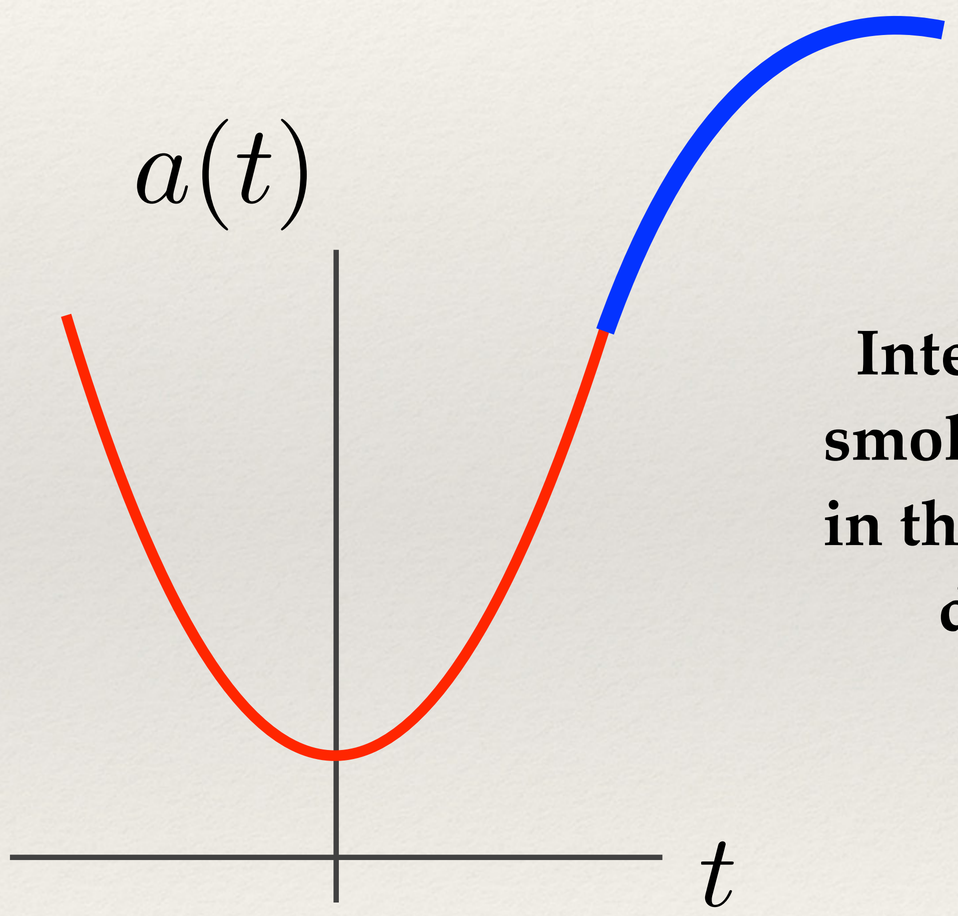
$$a(t) = \cosh\left(\sqrt{\frac{r_1}{2}}t\right)$$

Biswas, Mazumdar, Siegel, JCAP (2006)

Nonlocal Gravity & Cosmological Singularity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R + \Lambda \right]$$

$$a(t) = a_0 \cosh \left(\sqrt{\frac{\Lambda}{6M_{pl}^2}} t \right)$$



Cosmological Constant at Bounce

$$M \sim \Lambda^{1/4}$$

Interesting smoking gun in the Planck data !

Biswas, AM, PRD (2014)

"Einstein Gravity Does Not Permit Such Solution"

Hawking-Penrose Singularity Theorems & RayChaudhuri Equation

$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{\mu\nu}k^\mu k^\nu \qquad \theta = \nabla_\mu k^\mu$$

$$R_{\mu\nu}k^\mu k^\nu = \kappa T_{\mu\nu}k^\mu k^\nu$$

General Relativity

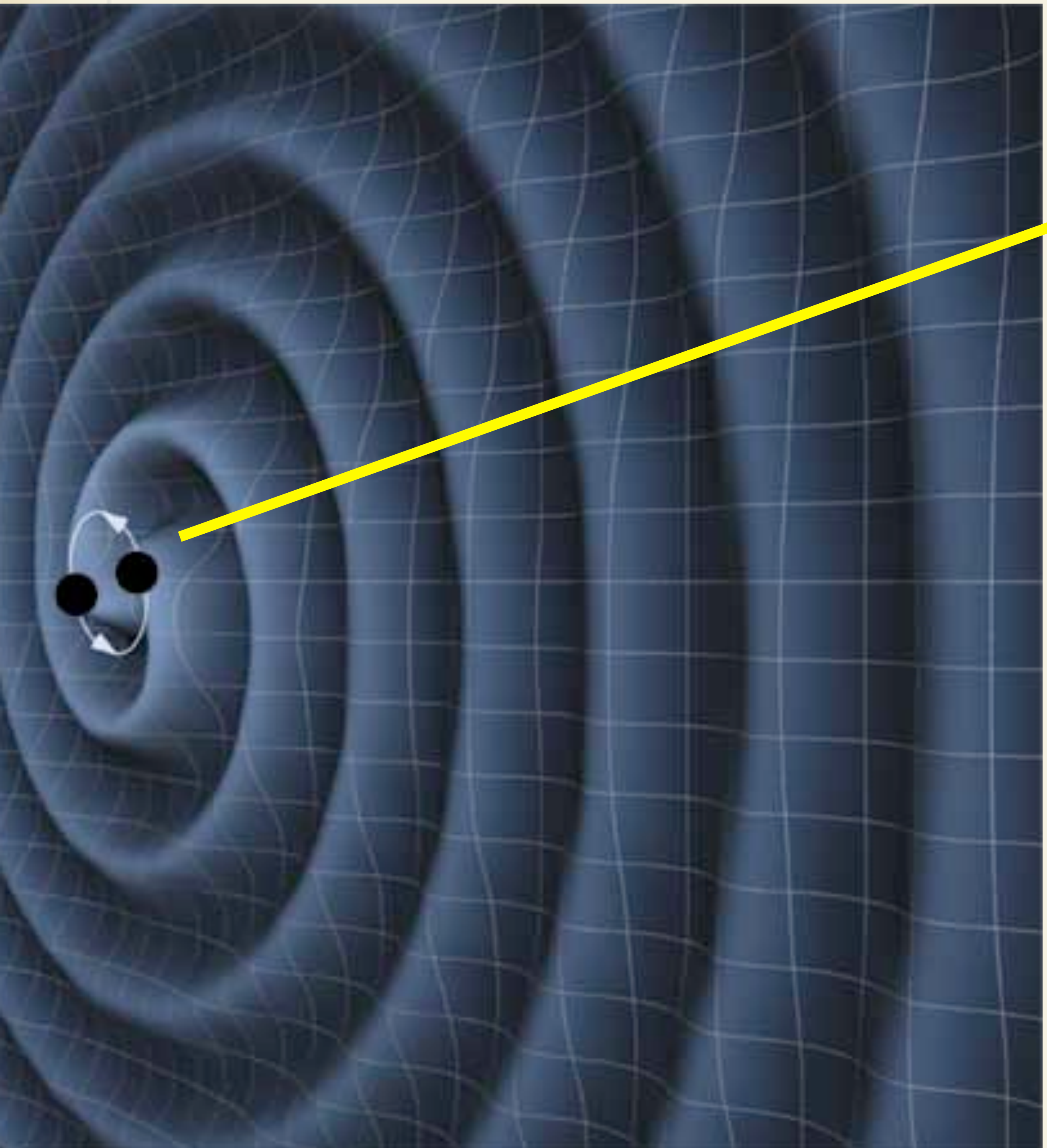
$$R_{\mu\nu}k^\mu k^\nu \geq 0, \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq 0$$

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$R^{(L)}_{\mu\nu}k^\mu k^\nu = \frac{1}{a(\bar{\Box})}\left[\kappa T_{\mu\nu}k^\mu k^\nu - \frac{(k^0)^2}{2}f(\bar{\Box})\Box R^{(L)}\right]$$

Defocusing: $R^{L}_{\mu\nu}k^\mu k^\nu \leq 0$

Gravitational Waves



$$\bar{h}_{jk} \approx G\frac{\omega^2(ML^2)}{r}$$

Large r limit

$$\bar{h}_{jk} \approx G\frac{\omega^2(ML^2)}{r}\mathrm{erf}\left(\frac{rM_P}{2}\right)$$

$r \Longrightarrow 0,$ **No Singularity**

Biswas, Gerwick, Koivisto, AM,
Phys. Rev. Lett. (gr-qc/1110.5249)