Loop Quantum Cosmology and the CMB

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LSU

Quantum Gravity and Gravitation: Testing General Relativity in Cosmology

Mainz, MITP, March 13 2017

Summary of work done in collaboration with several people:

Ashtekar, Bolliet, Gupt, Morris, Nelson, Parker, Shandera, Vijayakumar.

1. An invitation to LQC

LQG rests on Ashtekar's reformulation of GR in connexion variables

$$g_{\mu\nu} \longrightarrow A_i^I(\vec{x})\ ,\ E_J^j(\vec{x})$$

Ashtekar variables

$A_j^I(\vec{x})$ is a $SU(2)$ connection $I, J = 1, 2, 3$

Classical phase space of GR becomes same as in Yang-Mills theories, providing a unifying framework for all interactions

Quantum theory:

The quantum representation is chosen using symmetries: diffeomorphisms invariance $\longrightarrow$ unique kinematical Hilbert space: $\Psi(A_I^i)$

Dynamics: $\hat{H}\Psi(A_I^i) = 0$ Wheeler-De Witt-like equation

LQC is a mini-superspace version of LQG:
quantization of spacetimes with cosmological symmetries.

First: the simplest, homogeneous + isotropic model: FLRW

Classical system: gravity $a(t)$ + scalar field $\phi(t)$

$$A_i^I(t) = c(t)\, e_i^I \qquad E_I^i(t) = p(t)\, e_I^i$$

orthonormal triad in space

Analogy: homogeneous electromagnetic field

$$\vec{A}(\vec{x}) = c\,\hat{x}$$

$$\vec{E}(\vec{x}) = p\,\hat{x}$$

Canonical commutation relations: $\{c, -p\} = 1$

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$$A_i^I(t) = c(t)\, e_i^I \qquad E_I^i(t) = p(t)\, e_I^i$$

orthonormal triad in space

Again, diffeo. invariance picks a kinematical Hilbert space: $\Psi(c, \phi)$

Dynamics: $\hat{H}\Psi(c,\phi) = 0 \longrightarrow [\hbar^2 \partial_\phi^2 + H_0^2]\Psi(c,\phi) = 0$

This equation can be solved both numerical, and analytically. One can build the Hilbert space of physical states and physical observables in it.

This is a theory of quantum cosmology

Ashtekar, Bojowald, Corichi, Martin-Benito, Mena-Marugan, Olmedo, Pawloswki, Singh, Wilson-Ewing….

Analytical results:

(Ashtekar, Corichi, Pawlowski, Singh)

All physical observables (e.g. curvature invariants, energy density of ϕ) are bounded from above. No singularity in the entire Hilbert space. For instance:

$$\rho_{\rm sup} = \frac{18\pi}{G^2\hbar\Delta_o} \approx 0.4\,\rho_{Pl} \qquad\qquad R_{\rm sup} = 48\pi G\rho_{\rm sup}$$

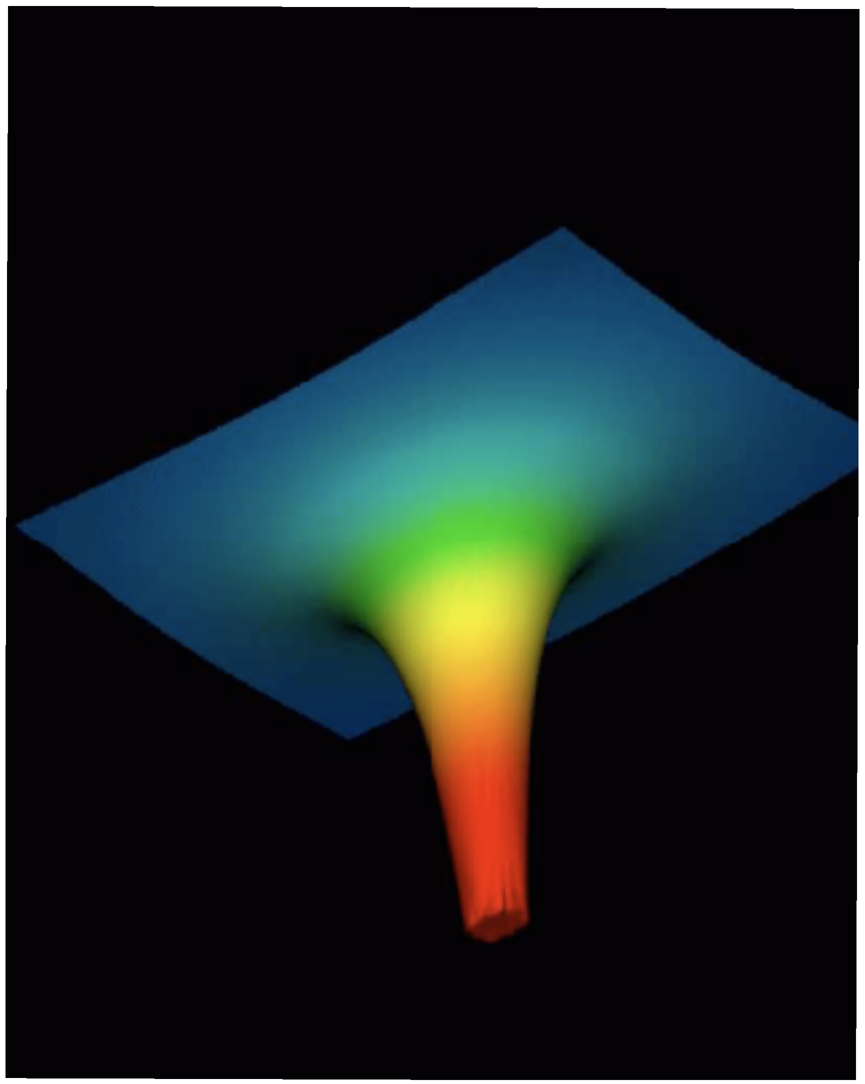
area gap in LQG: minimum area eigenvalue

Additionally:

All states during the evolution go through an instant (in ϕ-time) of minimum volume and maximum curvature: Bounce

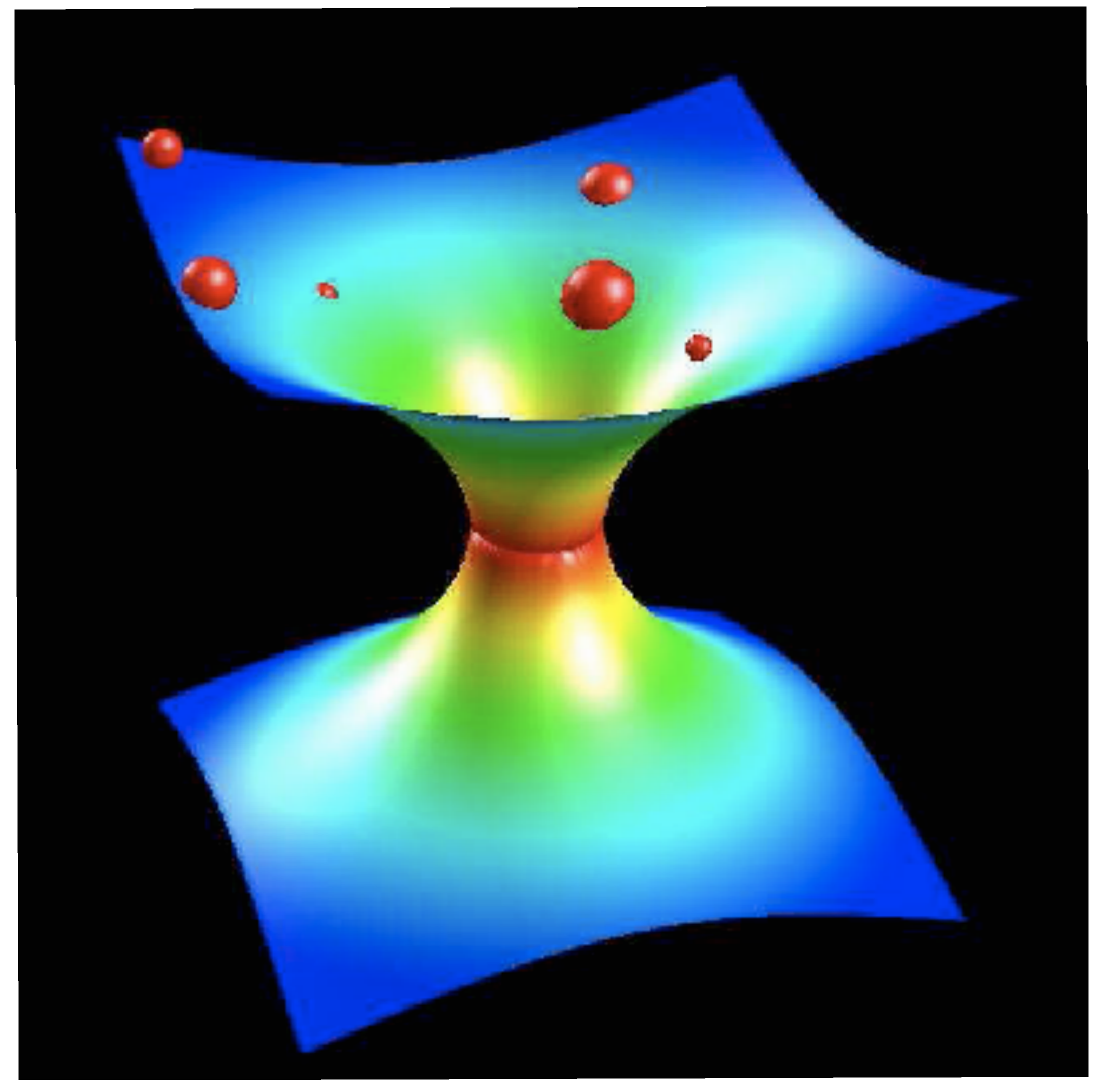
Artistic conceptions of the Big Bang and Big Bounce

Big Bang



Credits: Pablo Laguna

Big Bounce



Credits: Cliff Pikover

LQC effective eqns for “highly peaked” states $\Psi(c,\phi)$

Geometry well approximated by a smooth metric tensor with the FLRW symmetries:

$$H^2 = \frac{8\pi G}{3}\,\rho\left(1 - \frac{\rho}{\rho_{\rm sup}}\right)$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\,\rho\left(1 - 4\frac{\rho}{\rho_{\rm sup}}\right) - 4\pi G\,P\left(1 - 2\frac{\rho}{\rho_{\rm sup}}\right)$$

$$\ddot\phi + 3H\dot\phi + \frac{dV(\phi)}{d\phi} = 0$$

where, as usual: $\rho = \frac{1}{2}\dot\phi^2 + V(\phi)$ $\quad P = \frac{1}{2}\dot\phi^2 - V(\phi)$

Work has been extended to more complex cosmological models:

-with spatial curvature

-with cosmological constant

-Bianchi I, IX

-Gowdy

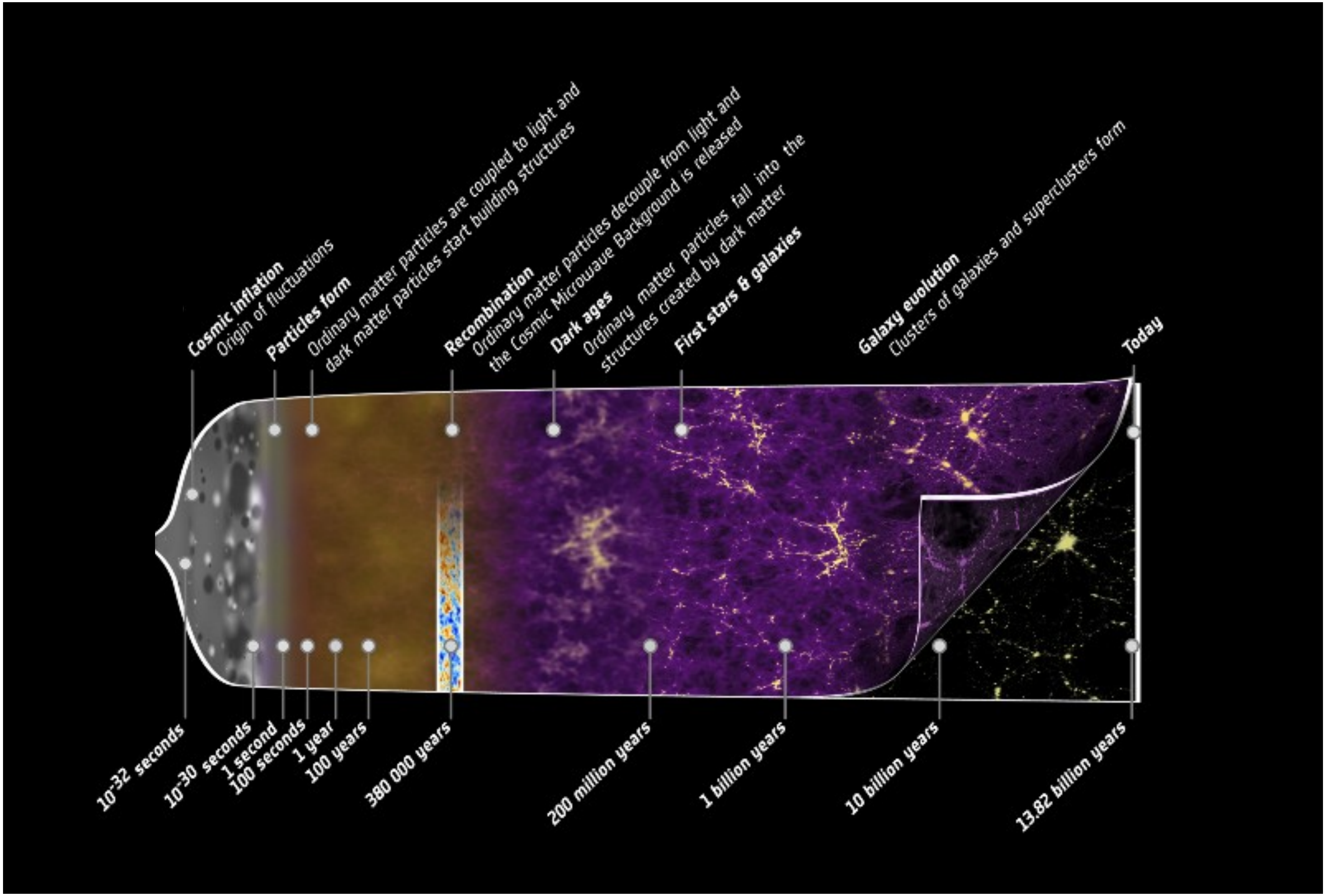
Results are robust

Lots of recent work on relating LQC to LQG in a more systematic way (symmetry reduction at the quantum level)

Alesci, Cianfrani, Engle, Brunnemann, Freishack

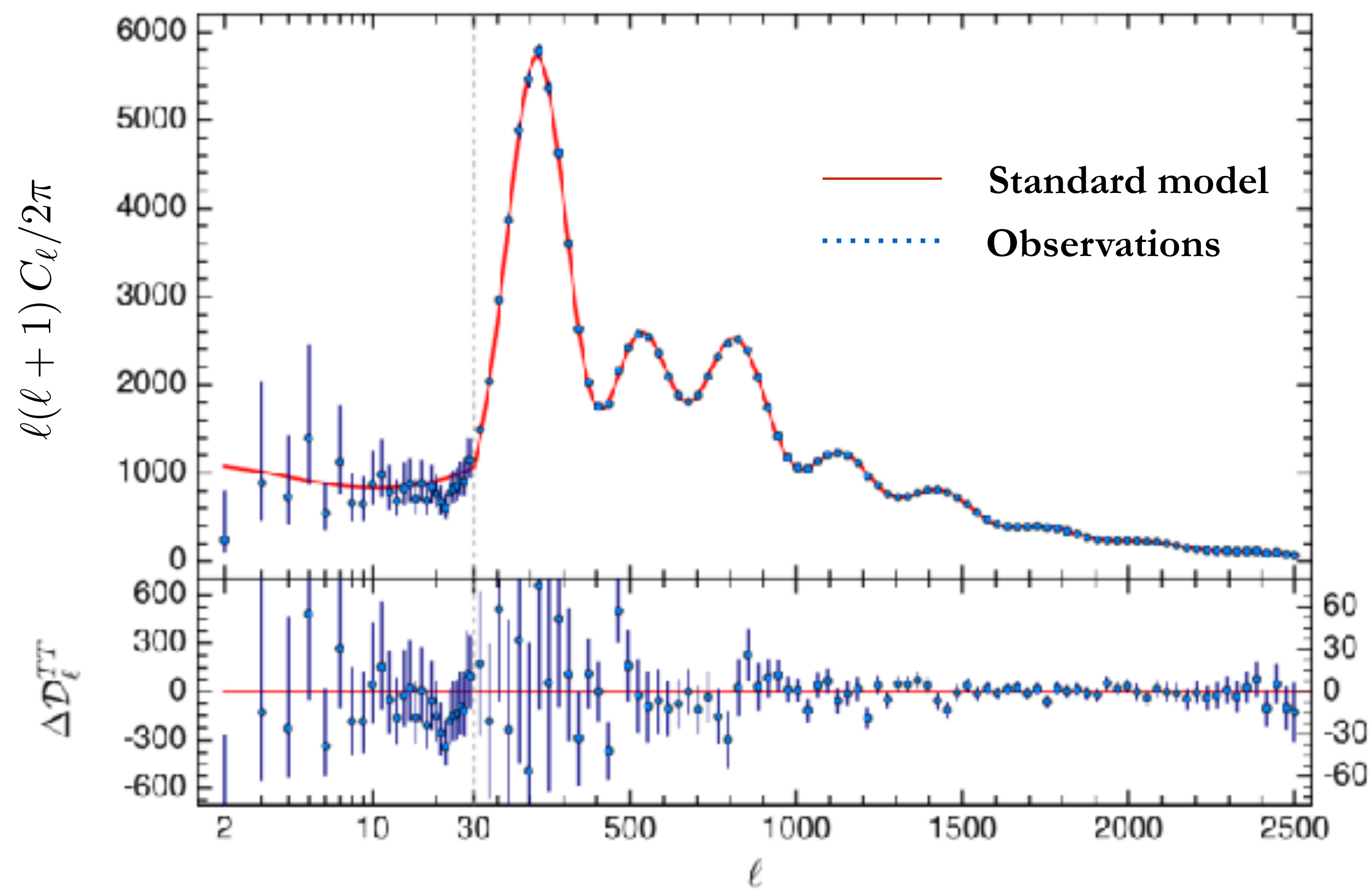
2. LQC and the standard model of cosmology

THE STANDARD MODEL OF COSMOLOGY



Standard Model: ΛCDM + inflation

Theory vs Observations (Planck 2015)



Deviations from theory

Spectacular agreement

Some deviations for $\ell \lesssim 30$

Planck 2015 results. XVI. Isotropy and statistics of the CMB

Planck Collaboration: P. A. R. Ade[89], N. Aghanim[60], Y. Akrami[65,103], P. K. Aluri[55], M. Arnaud[75], M. Ashdown[72,6], J. Aumont[60], C. Baccigalupi[88], A. J. Banday[100,9 *], R. B. Barreiro[67], N. Bartolo[32,68], S. Basak[88], E. Battaner[101,102], K. Benabed[61,99], A. Benoît[58], A. Benoit-Lévy[26,61,99], J.-P. Bernard[100,9], M. Bersanelli[35,49], P. Bielewicz[85,9,88], J. J. Bock[69,11], A. Bonaldi[70], L. Bonavera[67], J. R. Bond[8], J. Borrill[14,94], F. R. Bouchet[61,92], F. Boulanger[60], M. Bucher[1], C. Burigana[48,33,50], R. C. Butler[48], E. Calabrese[97], J.-F. Cardoso[76,1,61], B. Casaponsa[67], A. Catalano[77,74], A. Challinor[64,72,12],

ABSTRACT

We test the statistical isotropy and Gaussianity of the cosmic microwave background (CMB) anisotropies using observations made by the *Planck* satellite. Our results are based mainly on the full *Planck* mission for temperature, but also include some polarization measurements. In particular, we consider the CMB anisotropy maps derived from the multi-frequency *Planck* data by several component-separation methods. For the temperature anisotropies, we find excellent agreement between results based on these sky maps over both a very large fraction of the sky and a broad range of angular scales, establishing that potential foreground residuals do not affect our studies. Tests of skewness, kurtosis, multi-normality, N-point functions, and Minkowski functionals indicate consistency with Gaussianity, while a power deficit at large angular scales is manifested in several ways, for example low map variance. The results of a peak statistics analysis are consistent with the expectations of a Gaussian random field. The "Cold Spot" is detected with several methods, including map kurtosis, peak statistics, and mean temperature profile. We thoroughly probe the large-scale dipolar power asymmetry, detecting it with several independent tests, and address the subject of a posteriori correction. Tests of directionality suggest the presence of angular clustering from large to small scales, but at a significance that is dependent on the details of the approach. We perform the first examination of polarization data, finding the morphology of stacked peaks to be consistent with the expectations of statistically isotropic simulations. Where they overlap, these results are consistent with the *Planck* 2013 analysis based on the nominal mission data and provide our most thorough view of the statistics of the CMB fluctuations to date.

1. Introduction

foreground-cleaned CMB maps, it was generally considered that the case for anomalous features in the CMB had been strengthened. Hence, such anomalies have attracted considerable attention in the community, since they could be the visible traces of fundamental physical processes occurring in the early Universe.

However, the literature also supports an ongoing debate about the significance of these anomalies. The central issue in this discussion is connected with the role of a posteri-

To summarize

Inflation nice, but open issues of two kinds:

Theory:

- Big bang
- Trans-Planckian issues
- How inflation begins
- Initial conditions for inflation
- Where is ϕ and $V(\phi)$ coming from?
- Reheating
- …

Observations:

- CMB anomalies at large angles: dipole modulation (hence anisotropies) and power suppression

Goal of the program: use LQC to answer these questions

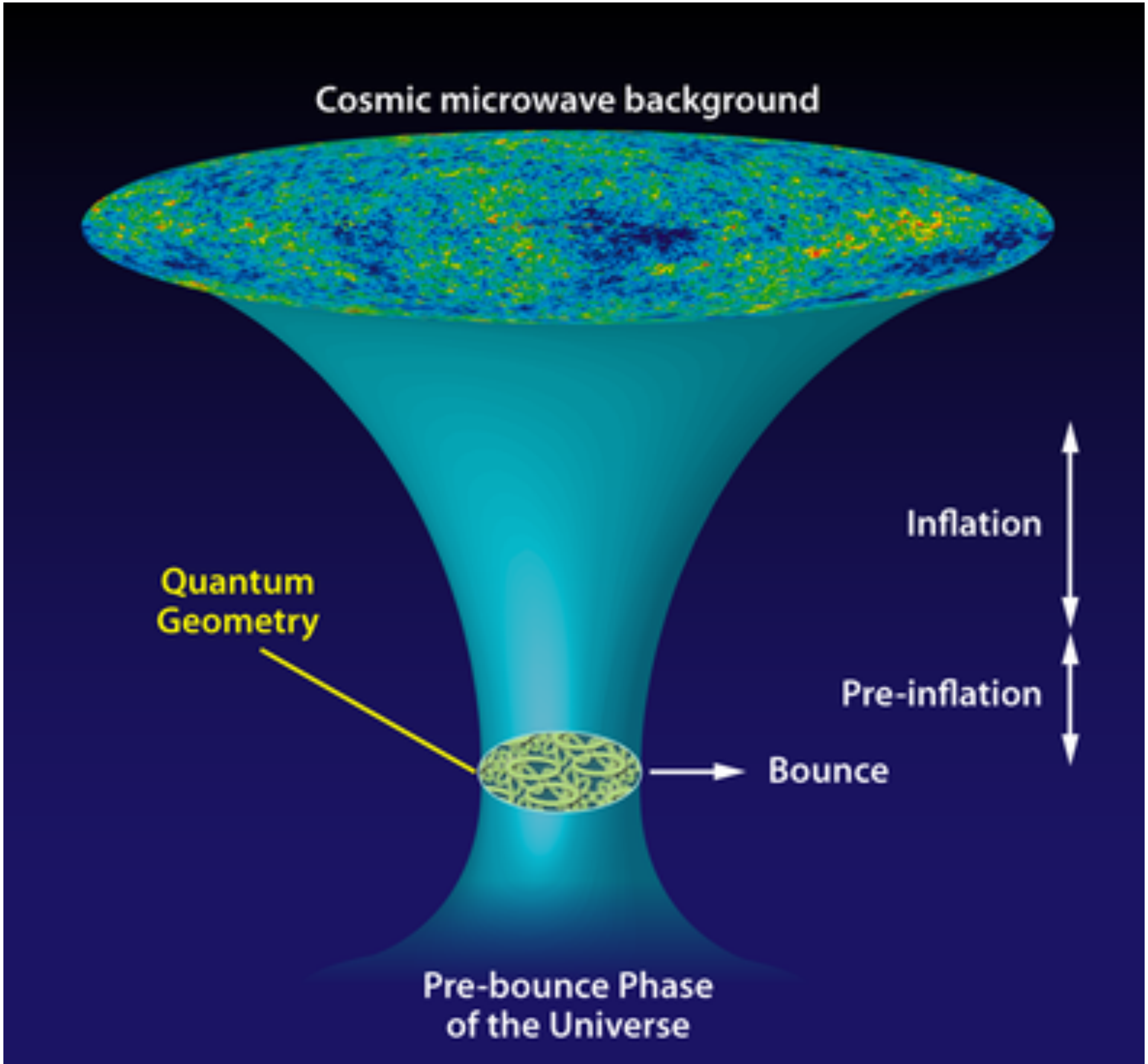


Fig. Credits:
P. Singh, Physics 5, 142 (2012)

3. Scalar and tensor perturbations in LQC

Ashtekar, Kaminski, Lewandowski 2010

I.A., Ashtekar, Nelson 2013

QFT in Quantum Spacetimes

Starting point: $\Psi(a, \phi, \delta\phi, \delta g_{\mu\nu})$

Perturbation theory $\Psi(a, \phi, \delta\phi, \delta g_{\mu\nu}) = \Psi_{\mathrm{FRW}}(a, \phi) \otimes \psi_{\mathrm{pert}}(a, \phi, \delta\phi, \delta g_{\mu\nu})$

Equations of motion:

$$\hat{H}\,\Psi(a, \phi, \delta\phi, \delta g_{\mu\nu}) = 0 \xrightarrow[\text{take expectation value in } \Psi_{\mathrm{FRW}}]{} \partial_t^2 \psi_{\mathrm{pert}} + f(\langle \hat{a}^n \rangle, \langle \hat{\phi}^m \rangle)\, \psi_{\mathrm{pert}} = 0$$

One obtains a QFT in a quantum spacetime

QFT in Quantum Spacetimes

The resulting equations are formally equivalent to the equations normally used in cosmology:

$$(\tilde{\Box} + \tilde{\mathcal{U}})\mathcal{Q}(x) = 0 \qquad\qquad \tilde{\Box}\,\mathcal{T}^{(+,\times)}(x) = 0$$

scalar pert **tensor perts (two polarizations)**

where the classical FRW metric has been replaced by

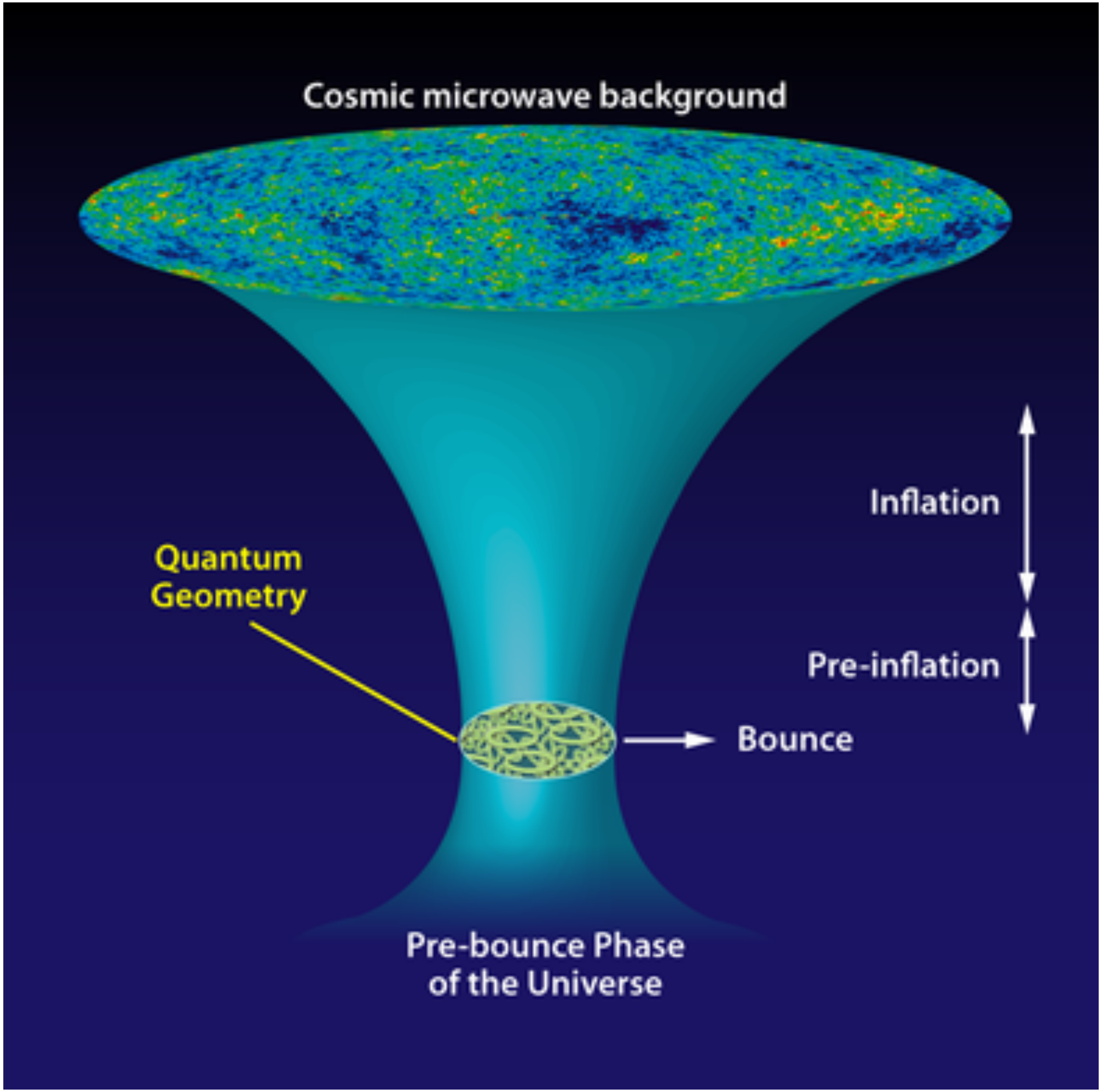
$$d\tilde{s}^2 = \tilde{a}^2\,(-d\tilde{\eta}^2 + d\vec{x}^2)$$ **Dressed, effective metric**

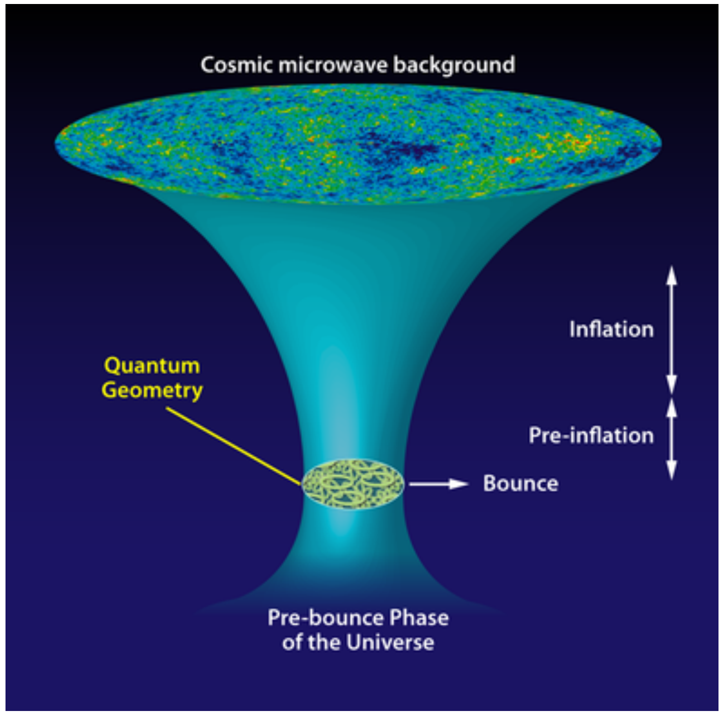
where $$\tilde{a}^4 = \frac{\langle \hat{H}_0^{-1}\hat{a}^4\hat{H}_0^{-1}\rangle_{\Psi_{\rm FRW}}}{\langle \hat{H}_0^{-1}\rangle_{\Psi_{\rm FRW}}} \qquad\qquad d\tilde{\eta} = \tilde{a}^2\,\langle H_0^{-1}\rangle_{\Psi_{\rm FRW}}\,d\phi$$

Perturbations only sensitive to a couple of "moments" of $\Psi_{\rm FRW}$

(simple result, although the specific moments are non-trivial)

4. Phenomenology of LQC


Cosmic microwave background
Inflation
Quantum
Geometry
Pre-inflation
Bounce
Pre-bounce Phase
of the Universe



Strategy:

1) Perturbations start in the vacuum at early times

2) Evolution across the bounce amplifies curvature perturbations

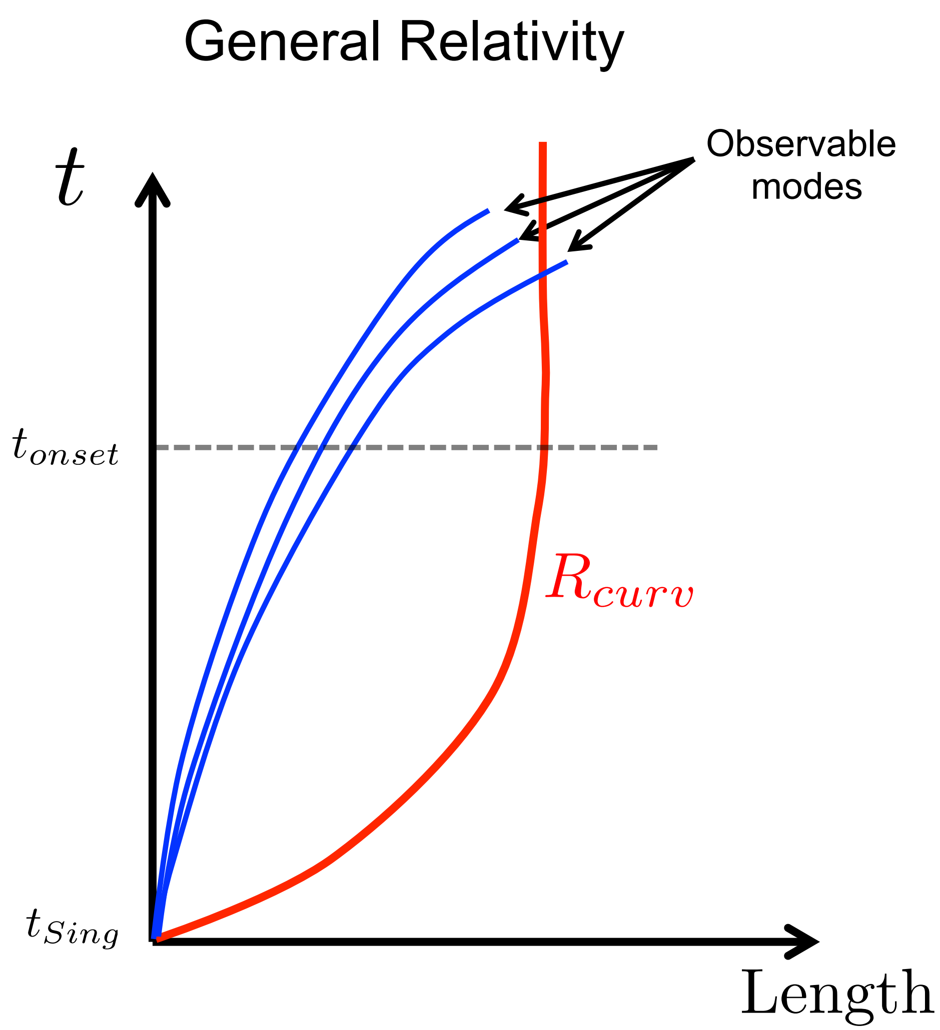
3) Then standard slow-roll inflation begins, but perturbations reach the onset of inflation in an excited state, rather than the Bunch-Davies vacuum

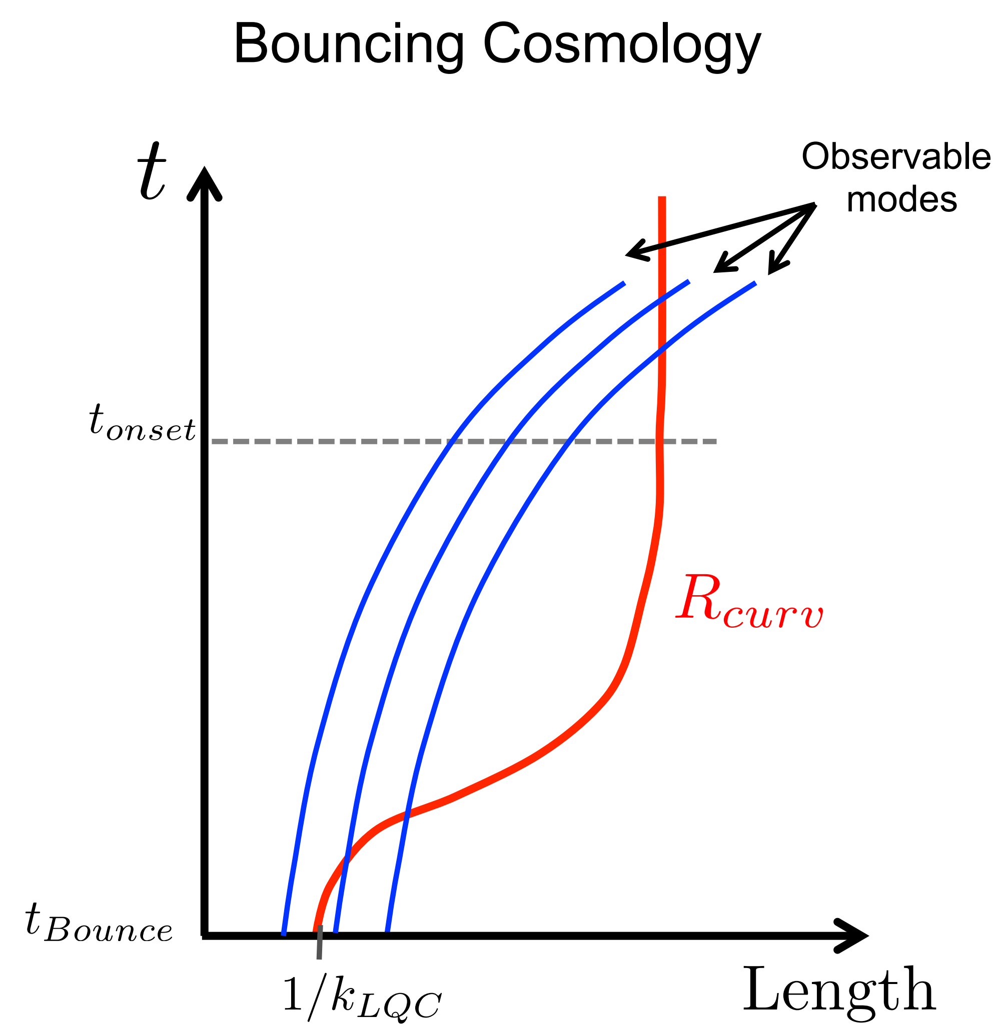
4) These excitations impact observables quantities

Remark: I'll use the $V(\phi) = \frac{1}{2}m^2\phi^2$ potential, but other choice are certainly possible and results have been shown to be robust (Bonga-Gupt 2015)

Why perturbations are affected by the bounce?

Qualitative discussion to gain intuition:

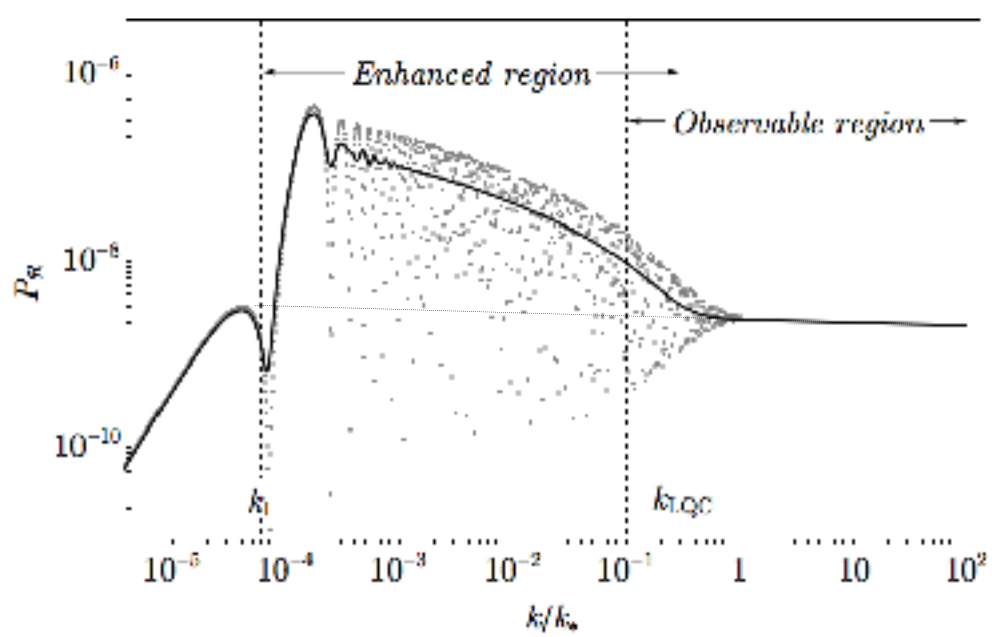




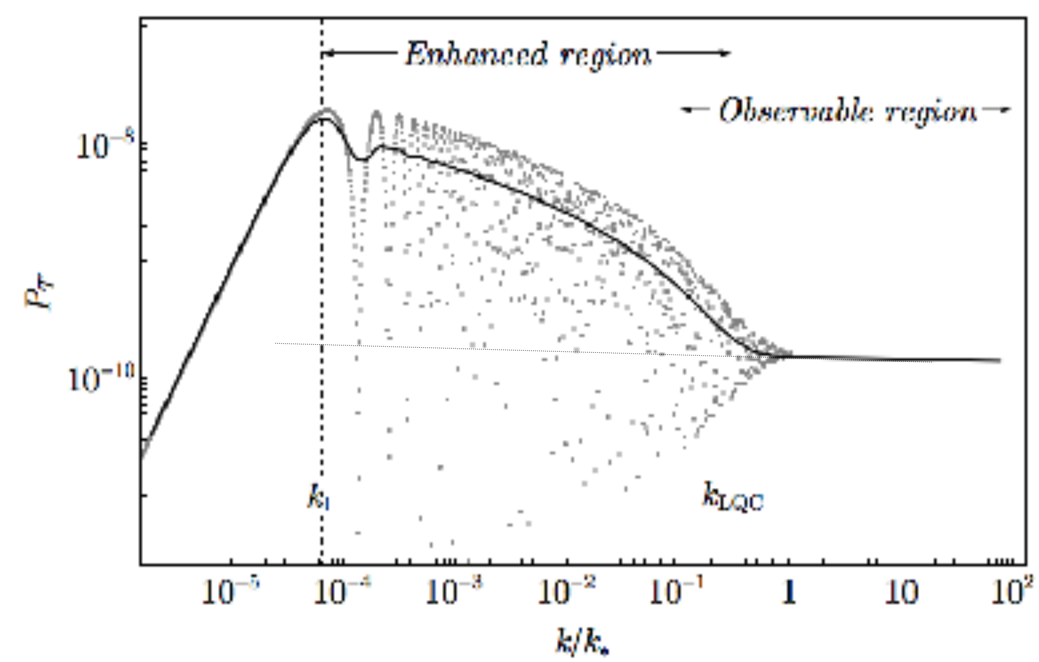
Results of numerical evolution

(I.A.-Ashtekar-Nelson 2012-13, I.A.-Morris 2015)

Scalar Power Spectrum



Tensor Power Spectrum



Choices to make:

- Inflation potential
- Initial data for ϕ e.g. at the bounce time: ϕ_B (= amount of expansion between bounce and onset inflation)
- Initial conditions for perturbations

In these plots: $\phi_B = 1.22$, $m = 1.1 \times 10^{-6}$, $k_\star/a_0 = 0.002\,\mathrm{Mpc}^{-1}$

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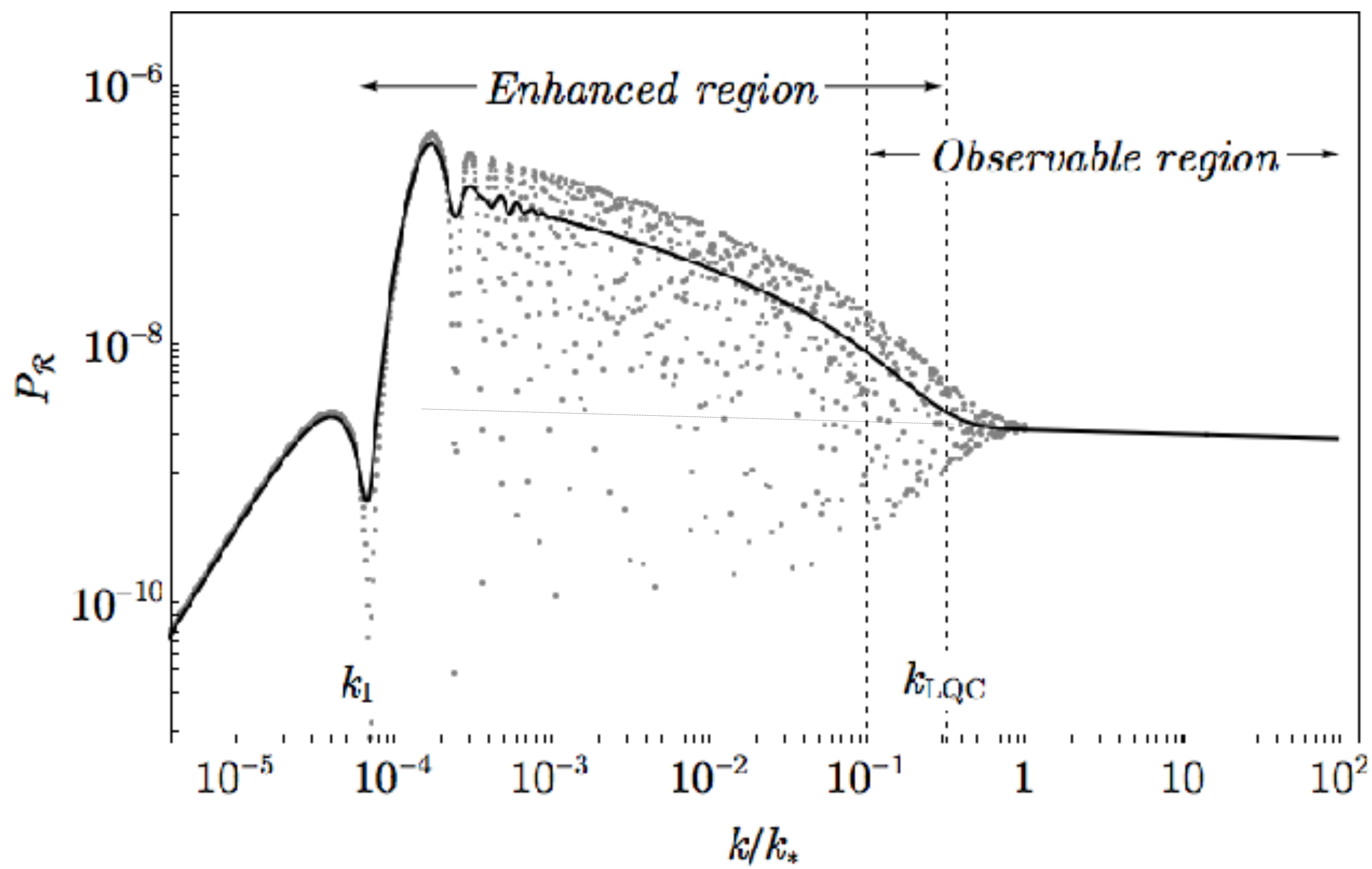
(I.A.-Ashtekar-Nelson 2012-13, I.A.-Morris 2015)

Scalar Power Spectrum



The LQC pre-inflationary evolution modifies the power for the lowest k-values (longest wavelengths) we can observe, and quite significantly for even longer wavelengths (super-Hubble modes)

- For large values of ϕ_B predictions are indistinguishable from standard inflation

$\longrightarrow$ QG extension of the inflationary scenario

- For smaller ϕ_B , QG corrections at large angles in CMB. In particular:

Most important:

- reduction of tensor-to-scalar ratio (slightly alleviates constrains on quadratic potential)
- modification of consistency relation $r < -8\, n_t$
- effects on spectral indices and runnings

Robustness tests:

- Conclusions robust against change in the potential (Bonga-Gutp 2015-16)
- Conclusions robust against choice of the quantum of FLRW geometry (I.A.,Ashtekat-Gutp 2016)
- Conclusions are robust against initial conditions (I.A.-Ashtekar-Nelson 2013, I.A.-Morris 2015)
- Extension to anisotropic bounces (I.A., Olmedo, Vijayakumar)
- Other approaches for perturbations (within LQC) produce quite similar results

5. LQC and CMB large scale “anomalies”

Two proposals so far:

1. Ashtekar and Gupt 2016 (see Gupt's talk on this conference)
   Martin de Blas-Olmedo (see Olmedo's talk in tis session)

Add physical principles that select for us an initial state for perturbations at the bounce.

Principles are related with:

1. Quantum generalization of Penrose Null Well curvature hypothesis
2. Relation between UV and IR physics in cosmology

The resulting power spectrum shows suppression at large angular scales that fits the data better than standard results

2. I.A.

Observed anomalies (WMAP, PLANCK)

- Hemispherical anomaly
- Quadrupole-octopole alignment
- Low power @ large scales
- Power asymmetry

...

Significance $\lesssim 3\sigma$

anisotropies

$$\Delta T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n}) \quad \text{with} \quad \langle a_{\ell m} a^{\star}_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_{\ell} + \text{non-diagonal terms}$$

Data indicate, if primordial origin, new physics at large scales needs to introduce anisotropies (e.g. remnants of Bianchi phase, etc)

But this is not sufficient. Effects only appears at large scales: we need a scale-dependent anisotropic modulation

Recent discussions: we do not need anisotropic physics to modify the statistics in our observable universe. Large correlations between modes can do the job

(Adhikari, Brahma, Bartolo, Bramante, Byrnes, Carrol, Dai, Deutsch, Dimastrogiovanni, Erickcen, **Hui**, Jeong, Kamionkowski, LoVerde, Matarrese, Mota, Nelson, Nurmi, Peloso, Pullen, Ricciardone, Shandera, **Schmidt**, Tasinato, Thorsrud, Urban,...)

Non-Gaussian modulation of the power spectrum

A typical realization shows larger anisotropies if the distribution is non-Gaussian

But there are strong limits on non-Gaussianity for the CMB (PLANCK)

$$|f_{NL}| \lesssim 10 \quad \text{for} \quad \ell \gtrsim 1000$$

We need a mechanism to produce strongly scale dependent non-Gaussianity

Non-Gaussian modulation of the power spectrum caused by a long wave length mode $\vec{k}_L$

In k-space:

Non-Gaussianity

$$\langle Q_{\vec{k}_1} Q_{\vec{k}_2} \rangle = P_Q(k_1)\Big[(2\pi)^3\delta(\vec{k}_1 - \vec{k}_2) + f_{NL}(\vec{k}_1, \vec{k}_L)\, Q_{\vec{k}_L}\Big]$$

In angular space:

Wigner 3j-symbols

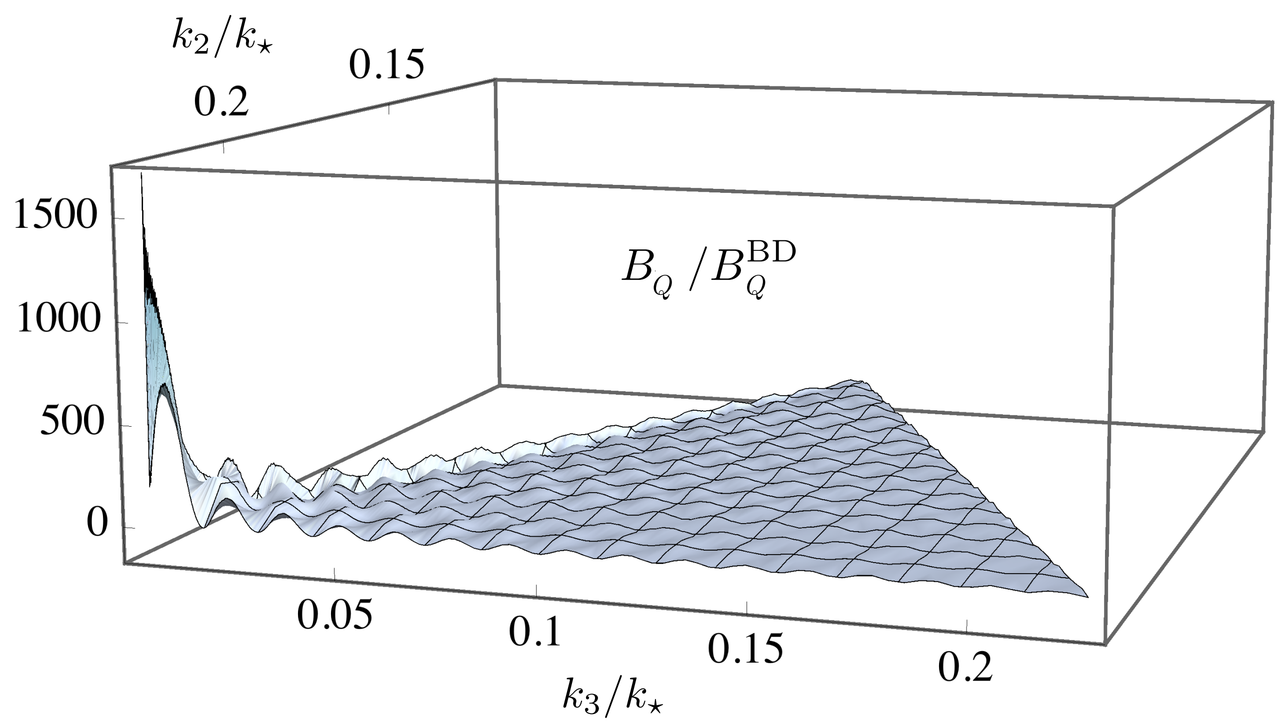
$$\boxed{\langle a_{\ell m} a^{\star}_{\ell' m'} \rangle = \delta_{\ell\ell'}\delta_{mm'}\, C_\ell + \underbrace{\sum_{LM} A_{LM}\, \mathcal{G}^{\ell\ell' L}_{-mm' M}\,(C_\ell + C_{\ell'})}_{\text{off-diagonal = anisotropies}}}$$

Goal: Compute the modulating amplitude $A_{\rm LM}$ using the bounce+inflation

First: Non-Gaussianity created during +inflation

I.A. 2015

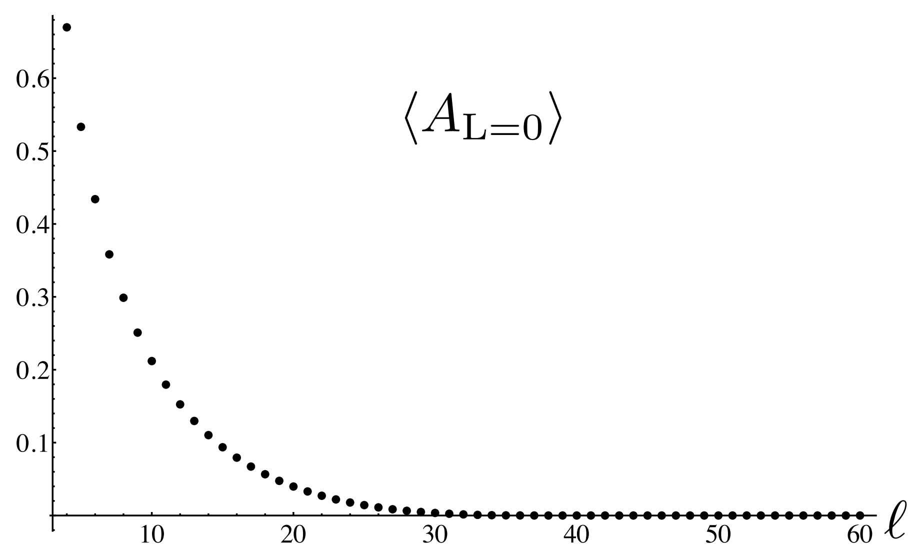
Ratio (inflation+LQC)/inflation Bispectrum:



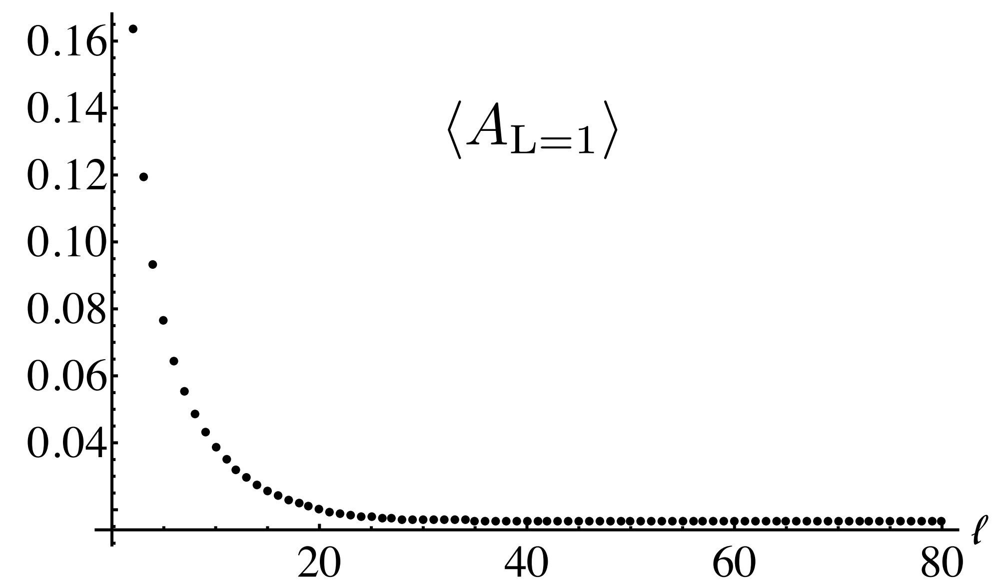
Two messages from this plot:

- Observable modes are not correlated among themselves: ok with observations
- But the longest wavelengths we can observe are strongly correlated with super-Hubble modes

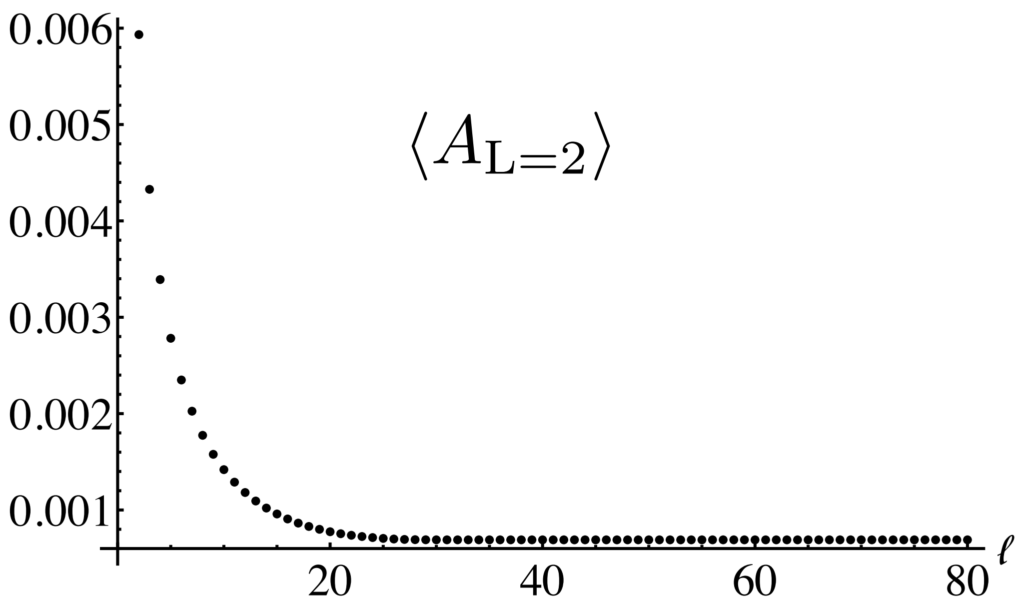
Amplitud modulation A_{LM} of a typical realization



Monopole:



Dipole:



Quadrupole:

Etc.

Conclussions

There is a choice of the parameters of the models for which:

- For the monopole: 1 every 6 simulated spectra show a suppression of at least 10% for $\ell < 30$
- A scale dependent dipole modulation in quantitative agreement with observations arises
- Negligible quadrupole, octopole, etc

In summary: the LQC bounce preceding inflation is a good candidate to account for the CMB large scale anomalies

- Prediction: tensor perturbations must also show the similar anomalies

6. Non-Gaussianity from the bounce

I.A., Bolliet, Vijayakumar: In Progess…

Challenging computation:

1. No slow-roll approximation available

Work in flat slicing gauge: $\delta\phi(\vec{x})$

$$H_{\rm int}(\delta\phi,\,\delta P_\phi) = \int d^3\vec{x}\,\Big\{\frac{9\,\kappa\,P_\phi^3}{4\,a^4\,\pi_a}\,\delta\phi^3 - \frac{3\,P_\phi}{2\,a^4\,\pi_a}\,\delta P_\phi^2\,\delta\phi + \frac{3\,P_\phi^2}{a\,\pi_a}\,\delta\phi^2\partial^2\chi$$

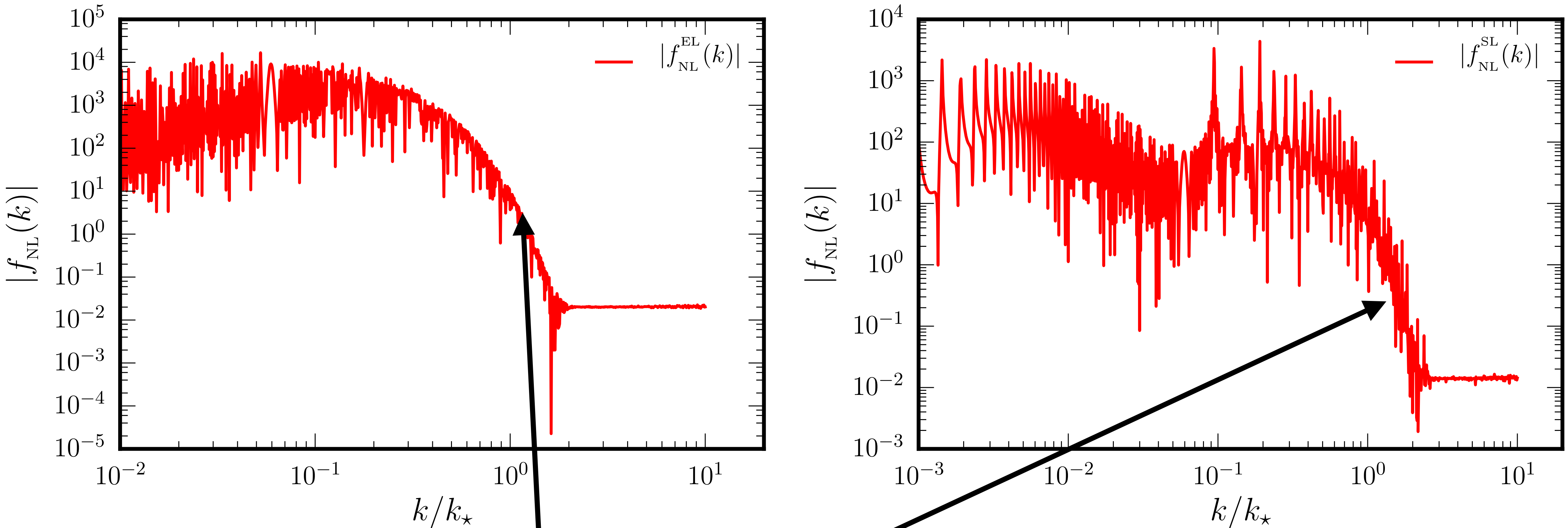
$$-\frac{3\,a^2\,P_\phi}{2\,\pi_a}\delta\phi\,\partial_i\delta\phi\,\partial^i\delta\phi + \delta P_\phi\,\partial_i\delta\phi\,\partial^i\chi + \frac{3\,a^2\,P_\phi}{2\,\kappa\,\pi_a}\,\delta\phi\,\partial^2\chi\,\partial^2\chi$$

$$-\frac{3\,a^2\,P_\phi}{2\,\kappa\,\pi_a}\,\delta\phi\,\partial_i\partial_j\chi\,\partial^i\partial^j\chi - \frac{3\,a^2\,P_\phi\,V_{\phi\phi}}{2\,\pi_a}\,\delta\phi^3 + \frac{a^3\,V_{\phi\phi\phi}}{6}\,\delta\phi^3\Big\}$$

Where $\partial^2\chi = \dfrac{\delta p_\phi^2}{2\,\pi_a^2}\,\dfrac{d}{dt}\left(-\dfrac{\delta p_\phi}{\pi_a}\delta\phi\right)$

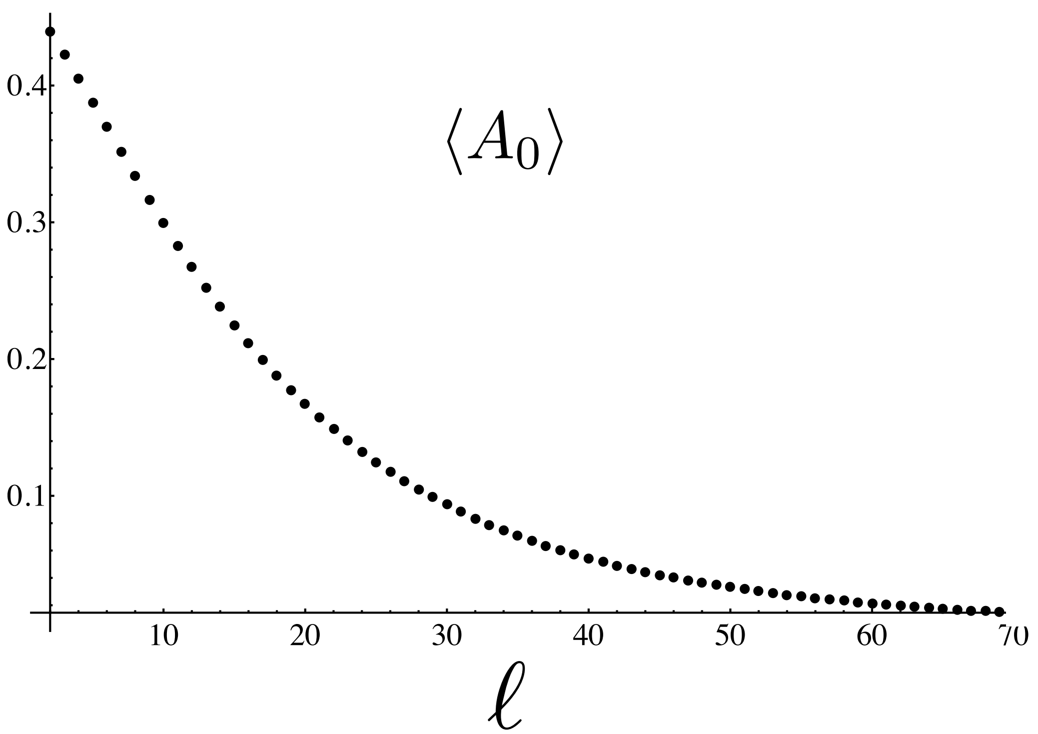
2. Challenging numerical integrals

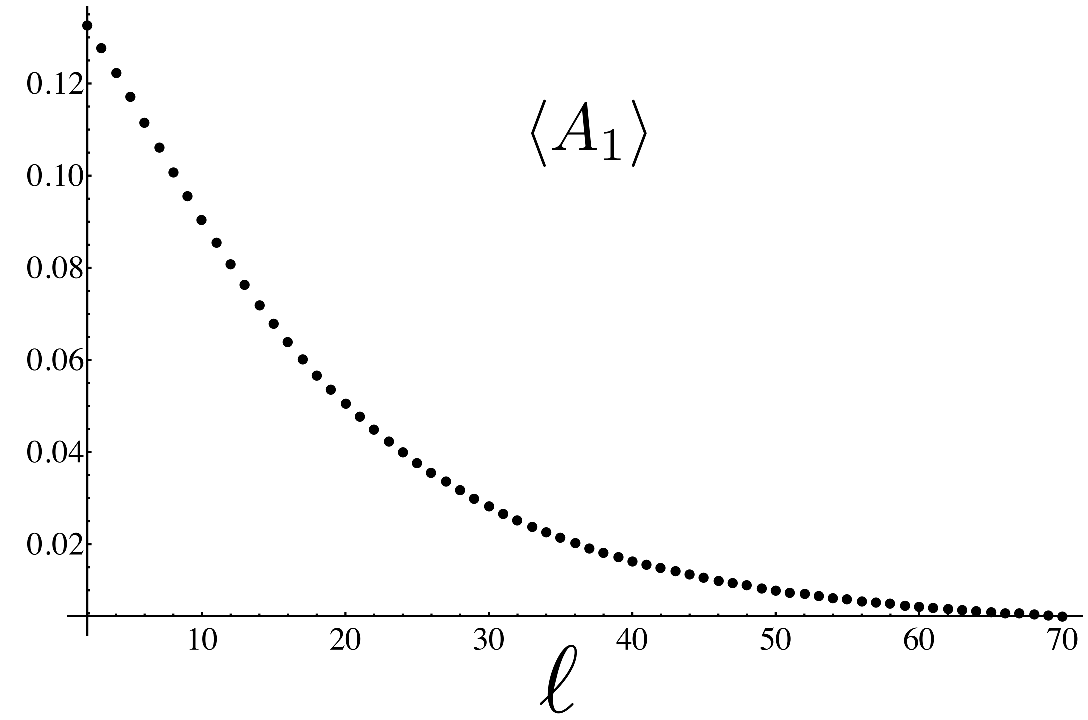
A sample of the result:

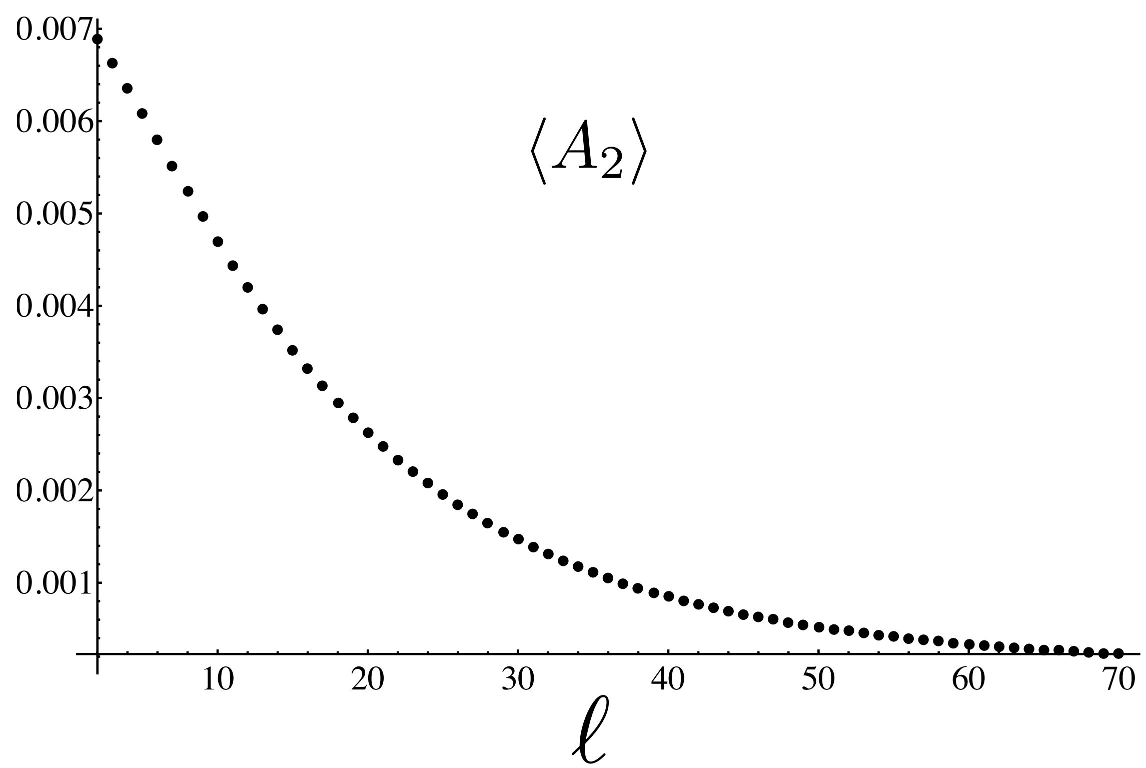


$$e^{-(k_1+k_2+k_3)/k_{LQC}}$$

A sample of results

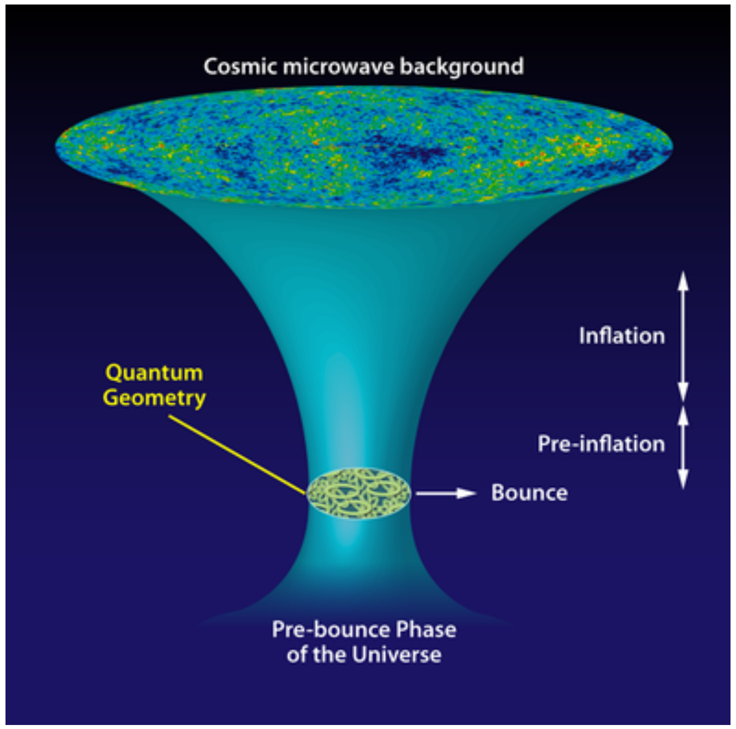




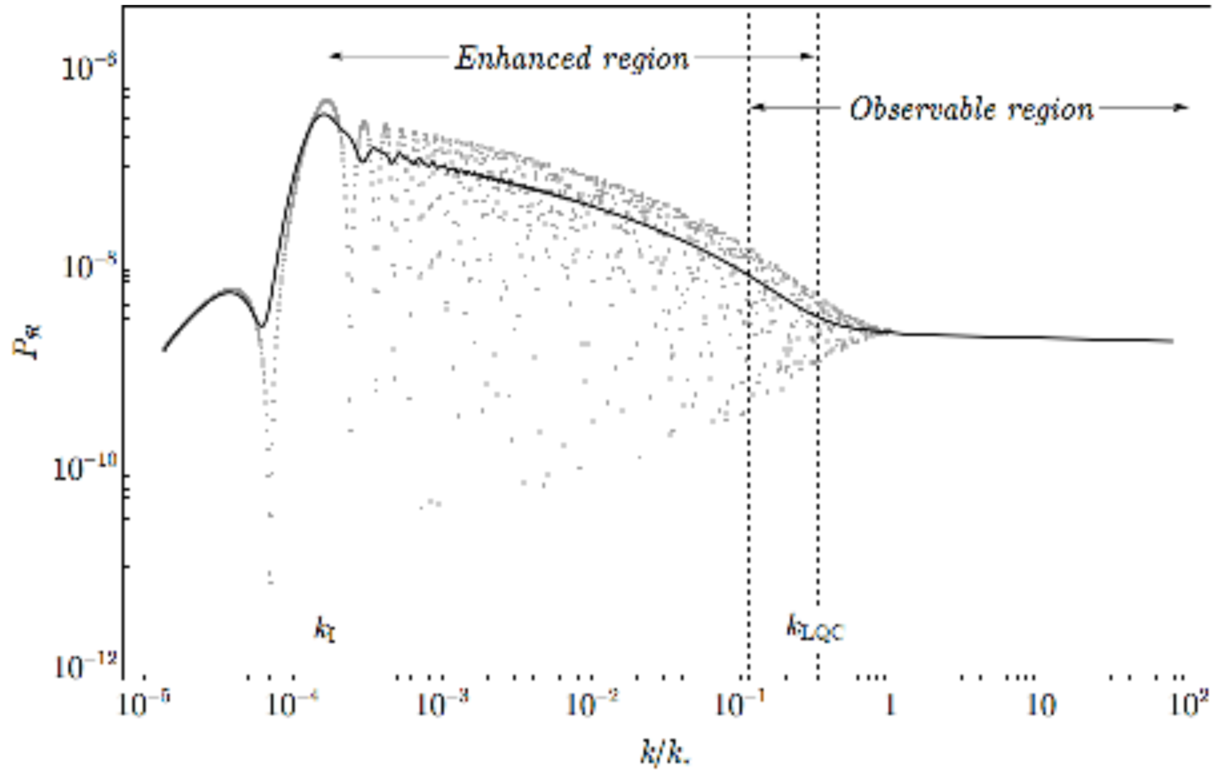


7. Summary

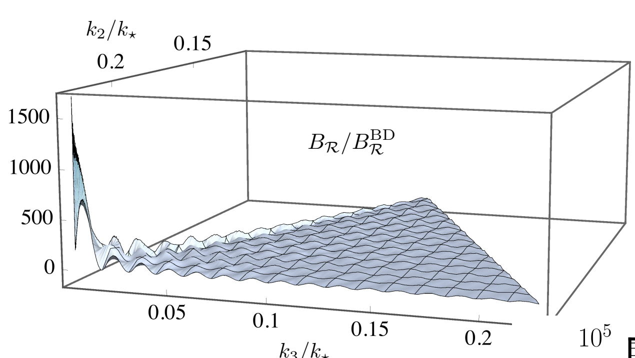
LQC and FRW space-time

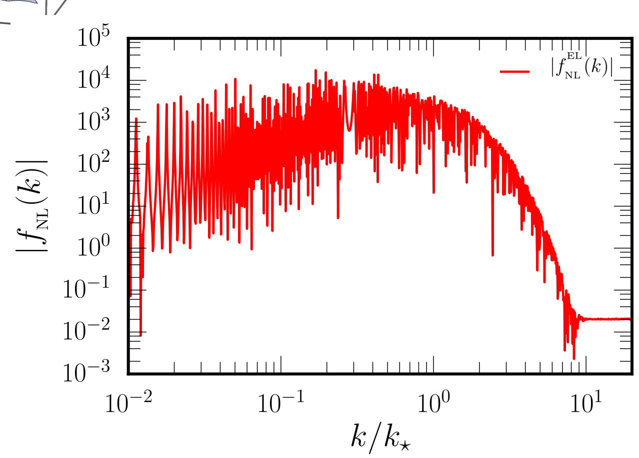


LQC and the Spectrum of primordial perturbations

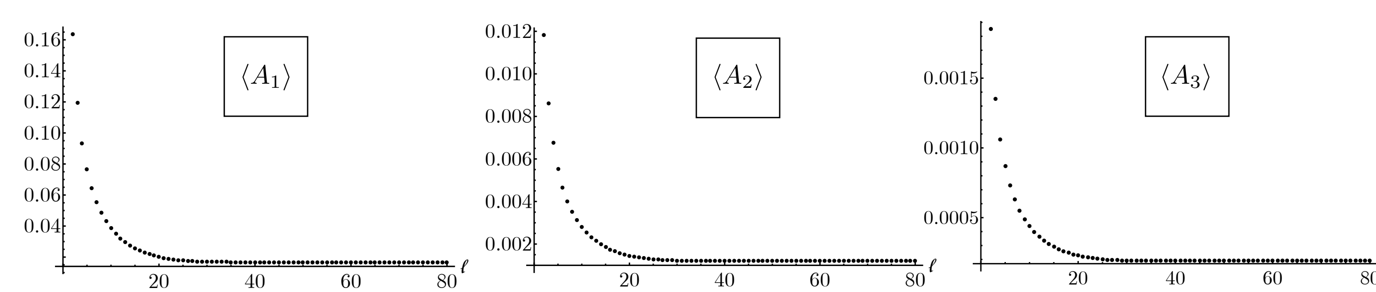


LQC and the spectrum of Non-Gaussianity





LQC and CMB anomalies



LQC has matured enormously in the last 10 years regarding both theory and connexion with observations:

- **Solid mathematical framework based on first principles**
- **Agreement with current observational constraints**
- **New mechanisms to account for phenomenology**
- **An opportunity to connect quantum gravity with observations**