Electrodynamic effects of inflationary gravitons

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with Shun-Pei Miao, Tomislav Prokopec, and Richard Woodard

DG, Miao, Prokopec, Woodard accepted in Class.Quant.Grav. [arXiv:1609.00386]
DG, Miao, Prokopec, Woodard Class.Quant.Grav. 32 (2015) no.19, 195014 [arXiv:1504.00894]
DG, Miao, Prokopec, Woodard Class.Quant.Grav. 31 (2014) 175002 [arXiv:1308.3453]

Electrodynamics in inflation

Why study photons?

- conformally coupled to gravity
- not produced due to rapid expansion during inflation

$\Rightarrow$ no big quantum corrections?

True if photons only interact with classical background

There are non-conformally coupled fields:

- minimally coupled massless scalars
- gravitons (metric fluctuation)

They are constantly produced during inflation
(primordial scalar and tensor power spectrum!)

Electrodynamics in inflation

Gravitons couple universally to all matter...

CAN INFLATIONARY GRAVITONS INDUCE LARGE QUANTUM LOOP CORRECTIONS TO PHOTONS?

Theoretical motivation

- Novel perturbative quantum gravitational effects
- Perturbative quantum gravitational observables & gauge dependence
- Computational practicalities and subtleties

Observational motivation

- Seeds of astrophysical magnetic fields

Outline

- Concrete electrodynamic systems
- Gravitons & graviton loops
- What and how to compute (effective field equations, diagrams, propagators...)
- Results (loop corrections)
- Conclusions

FLRW & de Sitter

FLRW line element:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^{\,2} = a^2(\eta)(-d\eta^2 + d\vec{x}^{\,2})\,, \qquad g_{\mu\nu} = a^2\eta_{\mu\nu} \quad (1)$$

Primordial inflation:

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1\,, \qquad \dot{\epsilon} \approx 0 \quad (2)$$

de Sitter space (Poincaré patch):

$$\epsilon = 0\,, \quad H = \text{const.}\,, \quad a(\eta) = -\frac{1}{H\eta}\,, \quad -\infty < \eta < 0 \quad (3)$$

Initial time:

$$a(\eta_0) = 1\,, \qquad \eta_0 = -\frac{1}{H} \quad (4)$$

de Sitter invariant length function

$$y(x;x') = aa'H^2\big[\|\vec{x}-\vec{x}\,'\|^2 - (\eta-\eta')^2\big]\,, \qquad y = 4\sin^2\big[\tfrac{H}{2}\ell\big] \quad (5)$$

Two concrete systems

One loop graviton corrections in electrodynamics on de Sitter

(I) POINT SOURCES
What is the quantum gravitational loop correction to the Coulomb force between two (free falling) point charges in de Sitter space?

(II) DYNAMICAL PHOTONS
How are electric and magnetic forces of a free photon in de Sitter space altered by propagation through a vast ansamble of (virtual) inflationary gravitons?

Perturbative quantum gravity

$\rightarrow$ Semiclassical expansion around de Sitter background

$$S = S_{EH} + S_M = \int d^D x \sqrt{-\boldsymbol{g}} \left[\frac{\boldsymbol{R} - (D-2)\Lambda}{\kappa^2} - \frac{1}{4} \boldsymbol{g}^{\mu\nu} \boldsymbol{g}^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} \right] \tag{6}$$

Gravitons – fluctuations of the metric

$$\boldsymbol{g}_{\mu\nu} = a^2 \eta_{\mu\nu} + \kappa h_{\mu\nu} \, , \qquad \kappa = \sqrt{16\pi G_N} \tag{7}$$

Free gravitons:

$$S_h = \frac{\kappa^2}{2} \int d^D x \, d^D x' \frac{\delta^2 S_{EH}}{\delta g_{\mu\nu}(x) \delta g_{\rho\sigma}(x')} h_{\mu\nu}(x) h_{\rho\sigma}(x')$$
$$= \frac{1}{2} \int d^D x \, h_{\mu\nu}(x) L^{\mu\nu\rho\sigma} h_{\rho\sigma}(x) \tag{8}$$

Perturbative quantum gravity

Free photons:

$$S_{ph} = -\frac{1}{4} \int d^D x \, a^{D-4} \eta^{\mu\nu} \eta^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} \tag{9}$$

Cubic interaction

$$S_3 = \int d^D x \tfrac{\kappa \delta S_M}{\delta g_{\mu\nu}(x)} h_{\mu\nu}(x) = \frac{\kappa}{2} \int d^D x \, a^{D-6} V^{\mu\rho\kappa\lambda\alpha\beta} (\partial_\kappa A_\mu)(\partial_\lambda A_\rho)(h_{\alpha\beta}) \tag{10}$$

Quartic interaction

$$S_4 = \frac{\kappa^2}{2} \int d^D x \, a^{D-8} U^{\mu\rho\kappa\lambda\alpha\beta\gamma\delta} (\partial_\kappa A_\mu)(\partial_\lambda A_\rho)(h_{\alpha\beta})(h_{\gamma\delta}) \tag{11}$$

Perturbative quantum gravity

$$V^{\mu\rho\kappa\lambda\alpha\beta} = \eta^{\alpha\beta}\eta^{\kappa[\lambda}\eta^{\rho]\mu} + 4\eta^{\alpha)[\mu}\eta^{\kappa][\rho}\eta^{\lambda](\beta} \tag{12}$$

$$\begin{aligned} U^{\mu\rho\kappa\lambda\alpha\beta\gamma\delta} = & \Big[\frac{1}{4}\eta^{\alpha\beta}\eta^{\gamma\delta} - \frac{1}{2}\eta^{\alpha(\gamma}\eta^{\delta)\beta}\Big]\eta^{\kappa[\lambda}\eta^{\rho]\mu} + \eta^{\alpha\beta}\eta^{\gamma)[\mu}\eta^{\kappa][\rho}\eta^{\lambda](\delta} + \eta^{\gamma\delta}\eta^{\alpha)[\mu}\eta^{\kappa][\rho}\eta^{\lambda](\beta} \\ & + \eta^{\kappa(\alpha}\eta^{\beta)[\lambda}\eta^{\rho](\gamma}\eta^{\delta)\mu} + \eta^{\kappa(\gamma}\eta^{\delta)[\lambda}\eta^{\rho](\alpha}\eta^{\beta)\mu} + \eta^{\kappa(\alpha}\eta^{\beta)(\gamma}\eta^{\delta)[\lambda}\eta^{\rho]\mu} + \eta^{\kappa(\gamma}\eta^{\delta)(\alpha}\eta^{\beta)[\lambda}\eta^{\rho]\mu} \\ & + \eta^{\kappa[\lambda}\eta^{\rho](\alpha}\eta^{\beta)(\gamma}\eta^{\delta)\mu} + \eta^{\kappa[\lambda}\eta^{\rho](\gamma}\eta^{\delta)(\alpha}\eta^{\beta)\mu} \end{aligned} \tag{13}$$

Loop expansion parameter: $\kappa^2 H^2 \sim 10^{-10}$

Quartic interaction enough for one loop

$$S_{int} = S_3 + S_4 + \mathcal{O}(\kappa^3) \tag{14}$$

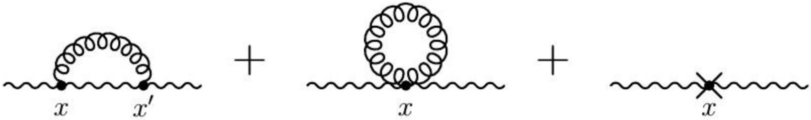
Effective field equation: vacuum polarization

Jordan Phys.Rev. D33 (1986) 444-454

Effective field equation (descending from effective action):

$$\partial_\nu\left[\sqrt{-g}g^{\nu\rho}g^{\mu\sigma}F_{\rho\sigma}(x)\right] + \int d^4x'\left[{}^\mu\Pi^\nu\right](x;x')A_\nu(x') = J^\mu(x') \quad (15)$$

Vacuum polarization $\left[{}^\mu\Pi^\nu\right](x;x')$:



$$i\left[{}^\mu\Pi^\nu_4\right](x;x') = \partial_\kappa\partial'_\lambda\left\{-i\kappa^2 a^{D-8}U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta}i\left[{}_{\alpha\beta}\Delta_{\gamma\delta}\right](x;x')\delta^D(x-x')\right\} \quad (16)$$

$$i\left[{}^\mu\Pi^\nu_3\right](x;x') = \partial_\kappa\partial'_\lambda\Big\{i\kappa a^{D-6}V^{\mu\rho\kappa\lambda\alpha\beta}i\left[{}_{\alpha\beta}\Delta_{\gamma\delta}\right](x;x')$$

$$\times\, i\kappa a'^{D-6}V^{\nu\sigma\theta\phi\gamma\delta}\partial_\lambda\partial'_\phi i\left[{}_\rho\Delta_\sigma\right](x;x')\Big\} \quad (17)$$

Effective field equation: perturbative solution

(1) Compute the one-loop renormalized diagrams contributing to vacuum polarization

(2) Use vacuum polarization to perturbatively correct the Maxwell field equation

Correction to classical fields:

$$A_\mu = A_\mu^{(0)} + \kappa^2 A_\mu^{(1)} + \mathcal{O}(\kappa^2)\,, \quad F_{\mu\nu} = F_{\mu\nu}^{(0)} + \kappa^2 F_{\mu\nu}^{(1)} + \mathcal{O}(\kappa^2) \quad (18)$$

Perturbative equations:

$$\partial_\nu \Big[\sqrt{-g} g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}^{(0)}(x) \Big] = J^\mu(x') \quad (19)$$

$$\partial_\nu \Big[\sqrt{-g} g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}^{(1)}(x) \Big] = - \int d^4x' \big[{}^\mu\Pi^\nu \big](x;x') A_\nu^{(0)}(x') \quad (20)$$

Schwinger-Keldysh formalism

- Study initial value problem in QFT $\rightarrow$ nonequilibrium QFT
- Compute genuine (in-in) expectation values, not transition elements (in-out)
- Causal and real field equations
- Diagrammatic technique:

  Schwinger-Keldysh path integral formalism
  (canonical formalism just the same)

$$\langle\psi|\hat{O}(t)|\psi\rangle \qquad \text{vs} \qquad \frac{\langle\psi|\hat{U}(t_{out},t_{in})\hat{O}(t)|\psi\rangle}{\langle\psi|\hat{U}(t_{out},t_{in})|\psi\rangle} \tag{21}$$

Effective field equation: gauge invariance

- Effective field equations are gauge dependent
- Propagators are gauge dependent
- Flat space:
  - S-matrix gauge invariant
  - compare results of in-out computation with in-in computation (e.g. correction to point charge potentials)
- What are gauge-invariant observables in cosmological QFT?!

Miao, Woodard JCAP 07 (2012) 008 [arXiv:1204.1784]

Conjecture: leading secular dependence of the solution to the effective equations is gauge independent - check!

General problem: how to define gauge-independent, UV renormalizable, IR finite observables in cosmological QFT? (beyond tree level)

Propagators

- Gauge-invariance of the final result is not guaranteed – computations in multiple gauges necessary!
- Gauge fixing and "gauge fixing"...
- Canonical gauge fixing – Dirac-Bergmann procedure:
  - Derive the phase space action and the complete set of first-class constraints $\phi_i \approx 0$
  - Impose gauge conditions (constraints) $\chi_i \approx 0$ such that $\{\phi_i, \chi_j\}$ is invertible
  - construct Dirac brackets, and impose all constraints strongly
  - reduced phase space quantization
- Inconvenient for practical computations

Propagators

- Preferred gauge fixing in QFT computation:
  adding a gauge-fixing term
- Not a gauge in the canonical sense (in Fadeev-Popov path integral arises as a weighted average over gauges)
- Allows for manifest preservation of symmetries (e.g. Lorentz)
- How it works:
  - Modify a gauge theory to an unconstrained theory, $\overline{S} = S + S_{gf}$
  - impose the gauge theory constraints on the unconstrained theory, $\phi_i \approx 0$ (on kinematic space of states)
  - gauge theory a subset of unconstrained theory
  - e.g. Gupta-Bleuler procedure in flat space
- Much more convenient to compute with
- Propagators computed from EOM - IR structure obscured!

Propagators: photon propagator

(i) covariant gauge-fixing term

Tsamis, Woodard J.Math.Phys. 48 (2007) 052306 [arXiv:gr-qc/0408002]

Fröb, Higuchi J.Math.Phys. 55, 062301 (2014) [arXiv:1305.3421]

$$\begin{aligned} S_{gf} &= -\frac{1}{2\xi}\int d^D x\sqrt{-g}\big[g^{\mu\nu}\nabla_\mu A_\nu\big]^2 \qquad (\xi\to 0) \\ &= -\frac{1}{2\xi}\int d^D x\, a^{D-4}\big[\eta^{\mu\nu}\partial_\mu A_\nu - (D-2)aHA_0\big]^2 \end{aligned} \tag{22}$$

- de Sitter space generalization of the Landau gauge
- de Sitter invariant and transverse
- covariant – expect covariant counterterms
- general structure:

$$i\big[{}_\mu\Delta_\nu\big](x;x')B(y)\frac{\partial^2 y}{\partial x^\mu \partial x'^\nu} + C(y)\frac{\partial y}{\partial x^\mu}\frac{\partial y}{\partial x'^\nu} \tag{23}$$

Propagators: photon propagator

(ii) non-covariant gauge-fixing term

Woodard [arXiv:gr-qc/0408002]

$$S_{gf} = -\frac{1}{2}\int d^D x\, a^{D-4}\Big[\eta^{\mu\nu}\partial_\mu A_\nu - (D-4)aHA_0\Big]^2 \tag{24}$$

- reduces to Feynman gauge in flat space
- not covariant – non-covariant counterms will be required
- preserves conformal coupling in $D=4 \Rightarrow$ relatively simple

$$i\big[{}_\mu\Delta_\nu\big](x;x') = (\eta^{\mu\nu} + \delta^\mu_0\delta^\nu_0)aa' i\Delta_B(x;x') - \delta^\mu_0\delta^\nu_0 aa' i\Delta_C(x;x') \tag{25}$$

- structure functions dS invariant scalar propagators (with different masses)

Propagators: graviton propagator

(i) covariant gauge-fixing term

Mora, Tsamis, Woodard J.Math.Phys. 53 (2012) 122502 [arXiv:1205.4468]

$$S_{gf} = -\frac{1}{2\xi}\int d^Dx\sqrt{-g}g^{\mu\nu}F_\mu F_\nu\,, \qquad (\xi\to\infty) \tag{26}$$

$$F_\mu = \nabla^\nu h_{\mu\nu} - \frac{\beta}{2}\nabla_\mu h \tag{27}$$

- generalized de Donder gauge
- free gauge parameter $\beta \neq 2 \Rightarrow$ check for gauge dependence!
- relatively complicated to use
- propagator not de Sitter invariant (IR divergences for dS invariant state), but respects isotropy and homogeneity
- splits into "spin 0" and "spin 2" parts

Propagators: graviton propagator

$$i\left[{}_{\mu\nu}\Delta_{\rho\sigma}\right] = i\left[{}_{\mu\nu}\Delta^{0}_{\rho\sigma}\right] + i\left[{}_{\mu\nu}\Delta^{2}_{\rho\sigma}\right] \tag{28}$$

$$i\left[{}_{\mu\nu}\Delta^{2}_{\rho\sigma}\right] = \frac{2}{H^4}\left(\frac{D-2}{D-3}\right)^2 \times \mathbf{P}_{\mu\nu}{}^{\alpha\beta}(x) \times \mathbf{P}_{\rho\sigma}{}^{\gamma\delta}(x')\times \left[\partial_\alpha\partial'_\gamma y(x;x') \times \partial_\beta\partial'_\delta y(x;x') \times i\Delta_{AAABB}(x;x')\right] \tag{29}$$

$$i\left[{}_{\mu\nu}\Delta^{0}_{\rho\sigma}\right] = \frac{-2(D\beta-2)^2}{(D-1)(D-2)(\beta-2)^2}\times\mathcal{P}_{\mu\nu}(x)\times\mathcal{P}_{\rho\sigma}(x')\times\left[i\Delta_{WNN}(x;x')\right] \tag{30}$$

Propagators: graviton propagator

(ii) non-covariant gauge-fixing term

Tsamis, Woodard Commun.Math.Phys. 162 (1994) 217-248

$$S_{gf} = -\frac{1}{2}\int d^D x a^{D-2}\,\eta^{\mu\nu} F_\mu F_\nu\,, \qquad [\boldsymbol{g} = a^2(\eta^{\mu\nu} + \kappa h_{\mu\nu})] \quad (31)$$

$$F_\mu = \eta^{\rho\sigma}\big(\partial_\sigma h_{\mu\rho} - \tfrac{1}{2}\partial_\mu h_{\rho\sigma} + (D-2)Hah_{\mu\rho}\delta^0_\sigma\big) \quad (32)$$

Propagator much simpler:

- only three tensor structures made out of Minkowski metric
- relatively simple structure functions: scalar propagators

$$i\big[_{\mu\nu}\Delta_{\rho\sigma}\big](x;x') = \sum_{I=A,B,C} [_{\mu\nu}\mathcal{T}^I_{\rho\sigma}] \times i\Delta_I(x;x') \quad (33)$$

Renormalization

- Gravity + EM not renormalizable
- Renormalizable in the EFT sense: at each order in perturbation one writes counterterms that absorb the divergence (BPHZ)
- Counterterms:

$$S_{ct}^{cov} = \int d^D x \sqrt{-g} \Big\{ C_1 F_{\mu\nu} F^{\mu\nu} R + C_2 F_{\mu\nu} F^{\mu}{}_{\rho} R^{\nu\rho} + C_3 F_{\mu\nu} F_{\rho\sigma} R^{\mu\nu\rho\sigma} + C_4 \nabla_\alpha F_{\mu\nu} \nabla^\alpha F^{\mu\nu} \Big\} \tag{34}$$

$$S_{ct}^{non} = \int d^D x \sqrt{-g}\, \Delta C H^2 F_{ij} F_{kl} g^{ik} g^{jl} \tag{35}$$

Vacuum polarization: representation

Leonard, Prokopec, Woodard Phys.Rev. D87 (2013) no.4, 044030 [arXiv:1210.6968]

Leonard, Prokopec, Woodard J.Math.Phys. 54 (2013) 032301 [arXiv:1211.1342]

- Simple, non-invariant representation ($\overline{\eta}^{\mu\nu} = \eta^{\mu\nu} + \delta^{\mu}_{0}\delta^{\nu}_{0}$),

$$i\big[{}^{\mu}\Pi^{\nu}\big](x;x') = \partial_\rho \partial'_\sigma \Big\{ \big[{}^{\mu\rho}T^{\nu\sigma}\big](x;x') \Big\} , \tag{36}$$

$$\begin{aligned} \big[{}^{\mu\rho}T^{\nu\sigma}\big](x;x') = {} & \big(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\big) F(x;x') \\ & + \big(\overline{\eta}^{\mu\nu}\overline{\eta}^{\rho\sigma} - \overline{\eta}^{\mu\sigma}\overline{\eta}^{\nu\rho}\big) G(x;x') \end{aligned} \tag{37}$$

- First part survives in flat space, second part de Sitter breaking
- There is a way to switch between this and an invariant representation
- Extracting derivatives very useful because it simplifies renormalization

Vacuum polarization: results

$$\partial_\nu F^{\mu\nu}(x) + \partial_\nu \int d^4x' \Big\{ iF(x;x')F^{\nu\mu}(x') + iG(x;x')\overline{F}^{\nu\mu}(x') \Big\} = J^\mu(x) \tag{38}$$

Flat space result:

Leonard, Woodard Phys.Rev. D85 (2012) 104048 [arXiv:1202.5800]

non-covariant dS result:

Leonard, Woodard Class.Quant.Grav. 31 (2014) 015010 [arXiv:1304.7265]

covariant dS result

DG, Miao, Prokopec, Woodard Class.Quant.Grav. 32 (2015) no.19, 195014 [arXiv:1504.00894]

surprise: despite dimensional regularization and covariant gauge fixing non-covariant counterterm required

why does this happen? – conditions need to be met:

- two derivative coupling (photons couple derivatively)
- coincident graviton propagator does not vanish in dimensional regulatization ($\sim 1/(D-4)$)

Vacuum polarization: results

$$F_2(x;x') = \frac{85\kappa^2 H^2}{72\pi^2}\ln(a) i\delta^4(x-x') - \frac{\kappa^2 H^2}{16\pi^4}\Big[\ln\Big(\frac{aa'}{4}\Big) + \frac{1}{3} - 2\gamma\Big]\nabla^2\Big(\frac{1}{\Delta x^2}\Big)$$
$$+\frac{5\kappa^2 H^2}{144\pi^4}\partial^2\Big(\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big) - \frac{5\kappa^2 H^6 (aa')^2}{144\pi^4}\Big\{\frac{\mathcal{L}(y)}{2} + \frac{2(2-y)\ln(\frac{y}{4})}{4y-y^2} + \frac{2}{y}\Big\}. \quad (132)$$

$$G_2(x;x') = -\frac{5\kappa^2 H^2}{4\pi^2}\ln(a) i\delta^4(x-x') + \frac{\kappa^2 H^2}{24\pi^4}\Big[\ln\Big(\frac{aa'}{4}\Big) + \frac{1}{3} - 2\gamma\Big]\nabla^2\Big(\frac{1}{\Delta x^2}\Big)$$
$$+\frac{\kappa^2 H^4 aa'}{96\pi^4}(\partial_0^2 + \nabla^2)\ln(H^2\Delta x^2) + \frac{5\kappa^2 H^6 (aa')^2}{72\pi^4}\Big\{\frac{(1-y)\mathcal{L}(y)}{4} + \frac{(y-3)\ln(\frac{y}{4})}{4-y}\Big\}. \quad (133)$$

$$F^{\rm ren}_{0d}(y) = \Big(\frac{2b-1}{b-2}\Big)^2\Big\{\frac{\kappa^2}{48\pi^2}\frac{\ln(a)}{aa'}\partial^2 i\delta^4(x-x') - \Big(\frac{b-8}{b-2}\Big)\frac{\kappa^2 H^2}{72\pi^2} i\delta^4(x-x')$$
$$-\frac{\kappa^2 H}{48\pi^2 a}\partial_0 i\delta^4(x-x') + \frac{\kappa^2}{384\pi^4}\frac{\partial^4}{aa'}\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big]$$
$$+\Big(\frac{b-8}{b-2}\Big)\frac{\kappa^2 H^2}{576\pi^4}\partial^2\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] - \frac{\kappa^2 H^6}{6\pi^4}(aa')^2\mathcal{N}_F(y)\Big\}, \quad (202)$$

$$G^{\rm ren}_{0d}(y) = \Big(\frac{2b-1}{b-2}\Big)^2\Big\{\frac{\kappa^2 H^2}{24\pi^2}\Big[1 - \ln(a)\Big] i\delta^4(x-x')$$
$$-\frac{\kappa^2 H^2}{192\pi^4}\partial^2\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] + \frac{\kappa^2 H^6}{12\pi^4}(aa')^2\mathcal{N}_G(y)\Big\}. \quad (203)$$

Digression: Flat space results

In-out computation in SQED:

Bjerrum-Bohr Phys.Rev. D66 (2002) 084023 [hep-th/0206236]

Inverse scattering method:

$$\Phi = \frac{q}{4\pi r}\left\{1 + \frac{3\kappa^2}{8\pi^2 r^2}\right\} \qquad (39)$$

Schwinger-Keldysh computation:

Leonard, Woodard Phys.Rev. D85 (2012) 104048 [arXiv:1202.5800]

No effect for dynamical photons

Correction to the potential of the point charge:

$$\Phi = \frac{q}{4\pi r}\left\{1 + \frac{\kappa^2}{24\pi^2 r^2} \times C(\alpha,\beta)\right\} \qquad (40)$$

Conjecture: quantum correcting the source eliminates gauge dependence

Point charge in dS: results

DG, Miao, Prokopec, Woodard Class.Quant.Grav. 31 (2014) 175002 arXiv:1308.3453

Correction to the electric potential ($F^{i0} = -\partial_i \Phi$) of the charge q at the origin:

$$\Phi = \Phi_0 \left[1 + \frac{\kappa^2 H^2}{8\pi^2} f(\eta, x) \right], \qquad \Phi_0 = \frac{q}{4\pi x} \tag{41}$$

$$f(\eta, x) = \theta(\Delta\eta_0 - x) \left[\boxed{\frac{1}{3a^2 H^2 x^2}} + \boxed{\ln(aHx)} + \alpha - 2\ln\left(\frac{a(1+Hx)-1}{a(1-Hx)-1} \right) \right] + \theta(x - \Delta\eta_0) \left[\ln(a) + \alpha - 4 - \frac{2a-1}{3(a-1)^2} - 3\ln\left(1 - \frac{1}{a}\right) \right] \tag{42}$$

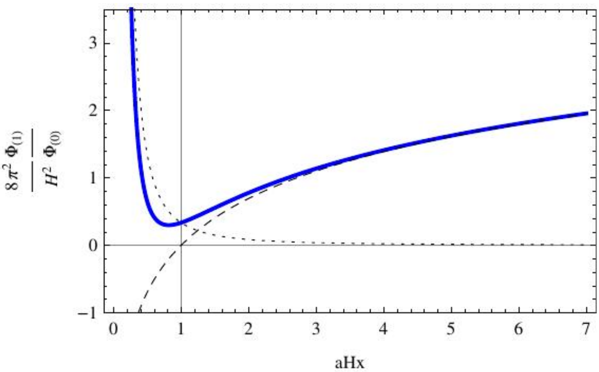
Initial surface divergences & light cone divergence

$\rightarrow$ initial state corrections (decay at late times)

Point charge in dS: results

Time dependent charge renormalization

$$f(\eta, x) = \theta(\Delta\eta_0 - x)\left[\boxed{\frac{1}{3a^2H^2x^2}} + \boxed{\ln(aHx)}\right] \tag{43}$$



Dynamical photons in dS: results

Plane wave photon solution valid to all orders ($k_i \varepsilon^i = 0$)

$$F^{0i}_{ph}(x) = -\partial_0 u(\eta, k) \times \varepsilon^i \, e^{i\vec{k}\cdot\vec{x}} \,, \quad F^{ij}_{ph} = u(\eta, k) \times i[k^i \varepsilon^j - k^j \varepsilon^i] e^{i\vec{k}\cdot\vec{x}} \tag{44}$$

Effective equation of motion for mode function:

$$(\partial_0^2 + k^2) u(\eta, k) = -\partial_0 \int d^4x' \, iF(x; x') \partial'_0 u(\eta', k) e^{-i\vec{x}\cdot\Delta\vec{x}} \\ - k^2 \int d^4x \left[iF(x; x') + iG(x; x') \right] u(\eta', k) e^{-i\vec{k}\cdot\Delta\vec{x}} \tag{45}$$

Only late time behaviour reliable – initial surface divergences (initial state not perturbatively corrected)

Dynamical photons in dS: results

No effect for magnetic field

Secular enhancement of electric field:

$$F_{0i} = F_{0i}^{(0)} \left[1 + \frac{\kappa^2 H^2}{8\pi^2} f(k,\eta) \right], \qquad u_0 = \frac{e^{-ik\eta}}{\sqrt{2k}} \tag{46}$$

noncovariant vac. pol.

Wang, Woodard Phys.Rev. D91 (2015) no.12, 124054 [arXiv:1408.1448]

$$f(k,\eta) = \boxed{\ln(a)} + \mathcal{O}(1) \tag{47}$$

covariant vac.pol.

DG, Miao, Prokopec, Woodard [arXiv:1609.00386]

$$f(k,\eta) = \boxed{\ln(a)} \left[\frac{45}{6} - \frac{ik}{3H} + \frac{5}{6} e^{2ik/H} \right] + \mathcal{O}(1) \tag{48}$$

no dependence on gauge parameter β, but disagreement with non-covariant gauge

Conclusions

- Effects derive from photons scattering off gravitons
  - Inflation produces a vast ensamble of IR gravitons
  - however weakly, but photons must scatter and pick up momentum
  - spin interaction allows for redshifted photons to interact
- Secular effects $\rightarrow$ eventual breakdown of perturbation theory
  - huge window of applicability of pert. theory
  - interesting nonperturbative effects (but worry about gauge first)
- What are good observables in cosmological QFT? (perturbative quantum gravity)