CONFORMAL SYMMETRY IN STANDARD MODEL AND GRAVITY

Tomislav Prokopec, ITP, Utrecht University

Stefano Lucat and T. Prokopec, arxiv:1606.02677 [hep-th] & 1512.06074 [gr-qc], in prep.

T. Prokopec, Leonardo da Rocha, Michael Schmidt, Bogumila Swiezewska, in preparation

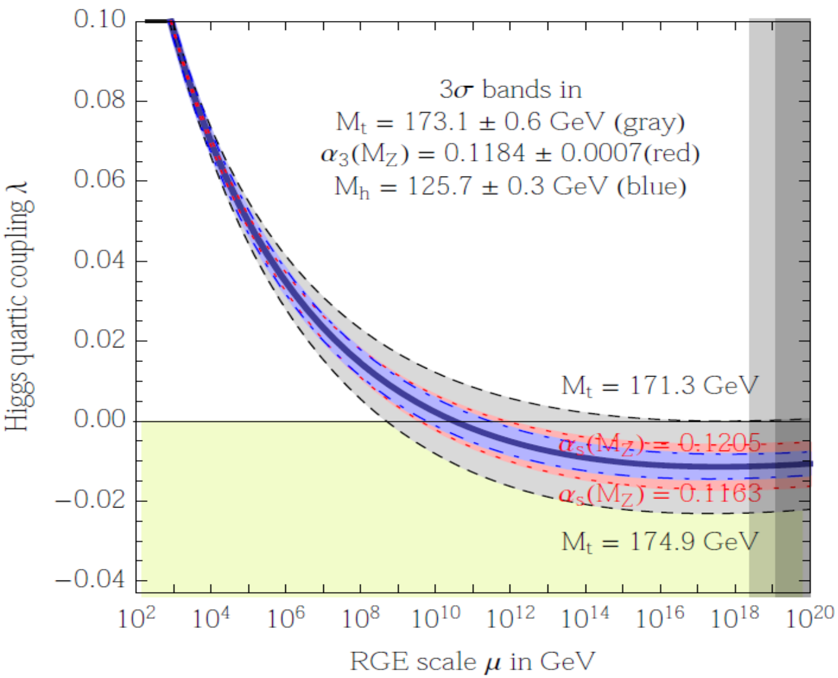
CONTENTS

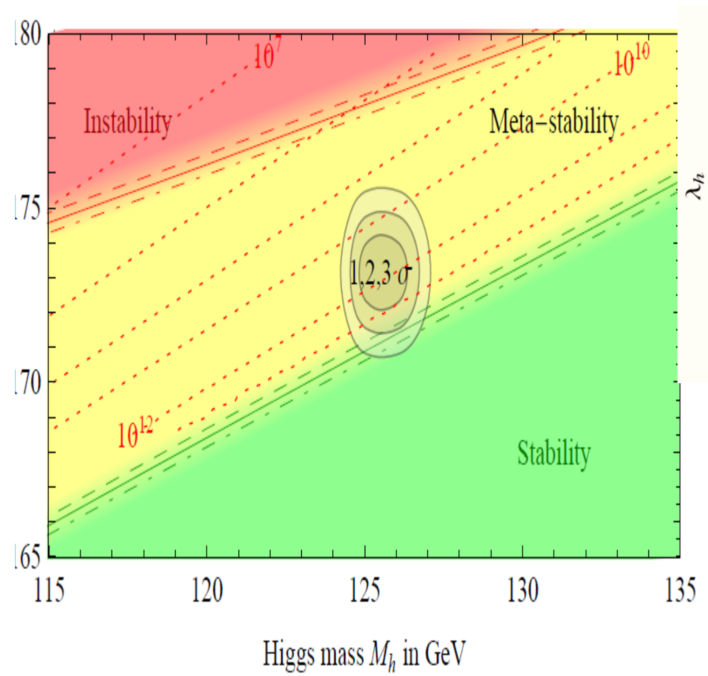
PHYSICAL MOTIVATION

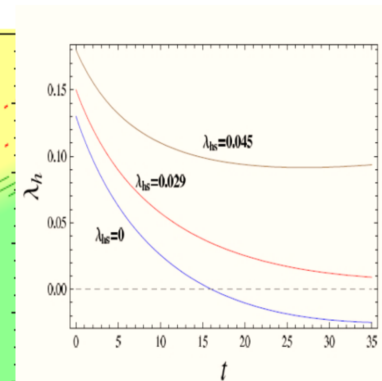
PHYSICAL MOTIVATION

- AT LARGE ENERGIES THE STANDARD MODEL IS ALMOST CONFORMALLY INVARIANT.
- HIGGS MASS AND KINETIC TERMS BREAK THE SYMMETRY
- OBSERVED HIGGS MASS: $m_H = 125.9\text{GeV}$ is close to the stability bound
- STABILITY BOUND: $m_H \approx 130\text{GeV}$: CAN BE ATTAINED BY ADDING SCALAR

Oleg Lebedev, e-Print: arXiv:1203.0156 [hep-ph]







Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia, 1205.6497 [hep-ph]

THEORETICAL MOTIVATION

THEORETICAL MOTIVATION IN SM

- HIGGS MASS TERM RESPONSIBLE FOR GAUGE HIERARCHY PROBLEM

- IF WE COULD FORBID IT BY SYMMETRY, THE GHP WOULD BE SOLVED

- THIS SYMMETRY COULD BE WEYL SYMMETRY IMPOSED CLASSICALLY

- HIGGS MASS COULD BE GENERATED DYNAMICALLY BY
  THE COLEMAN-WEINBERG (CW) MECHANISM (?)

THEORETICAL MOTIVATION IN GRAVITY

- THE SYMMETRY IS BROKEN BY THE NEWTON CONSTANT AND COSMOLOGICAL TERM, G & Λ.

- G & Λ ARE RESPONSIBLE FOR GRAVITATIONAL HIERARCHY PROBLEM.

- SCALAR DILATON & CARTAN TORSION CAN RESTORE WEYL SYMMETRY IN CLASSICAL GRAVITY.

- G & Λ CAN BE GENERATED BY DILATON CONDENSATION INDUCED BY QUANTUM EFFECTS akin to THE COLEMAN-WEINBERG MECHANISM.

- IF GRAVITY IS CONFORMAL IN UV, IT MAY BE FREE OF SINGULARITIES (BOTH COSMOLOGICAL AND BLACK HOLE).

WEYL SYMMETRY IN CLASSICAL GRAVITY

CARTAN EINSTEIN THEORY

- POSITS THAT FERMIONS (& SCALARS) SOURCE SPACETIME TORSION.

- TORSION IS CLASSICALLY A CONSTRAINT FIELD
  (NOT DYNAMICAL, DOES NOT PROPAGATE)

$\Rightarrow$ CARTAN EQUATION CAN BE INTEGRATED OUT,
RESULTING IN THE KIBBLE-SCIAMA THEORY

Lucat, Prokopec, e-Print: arXiv:1512.06074 [gr-qc]

$\Rightarrow$ THIS THEORY PROVIDES ADDITIONAL SOURCE TO STRESS-ENERGY,
WHICH CAN CHANGE BIG-BANG SINGULARITY TO A BOUNCE.

- CARTAN-EINSTEIN THEORY CAN BE MADE CLASSICALLY CONFORMAL!

Lucat & Prokopec, arxiv:1606.02677 [hep-th]

CLASSICAL WEYL SYMMETRY

- WEYL TRANSFORMATION ON THE METRIC TENSOR

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{2\theta(x)} g_{\mu\nu} \qquad \mathrm{d}\tau \to \mathrm{d}\tilde{\tau} = e^{\;\theta(x)} \mathrm{d}\tau$$

- GENERAL CONNECTION Γ, TORSION TENSOR T, CHRISTOFFEL CON $\mathring{\Gamma}$

$$\Gamma^{\lambda}{}_{\mu\nu} = T^{\lambda}{}_{\mu\nu} + T_{\mu\nu}{}^{\lambda} + T_{\nu\mu}{}^{\lambda} + \mathring{\Gamma}^{\lambda}{}_{\mu\nu}$$

$$\delta\Gamma^{\mu}_{\alpha\beta} = \delta^{\mu}_{\alpha}\partial_{\beta}\theta\,, \qquad \delta T^{\mu}{}_{\alpha\beta} = \delta^{\mu}_{[\alpha}\partial_{\beta]}\theta \qquad 2\delta T_{(\alpha\beta)}{}^{\mu} = g_{\alpha\beta}\partial^{\mu}\theta - \delta^{\mu}_{(\alpha}\partial_{\beta)}\theta$$

- RIEMANN TENSOR IS INVARIANT: $\delta R^{\alpha}{}_{\beta\gamma\delta} = 0$

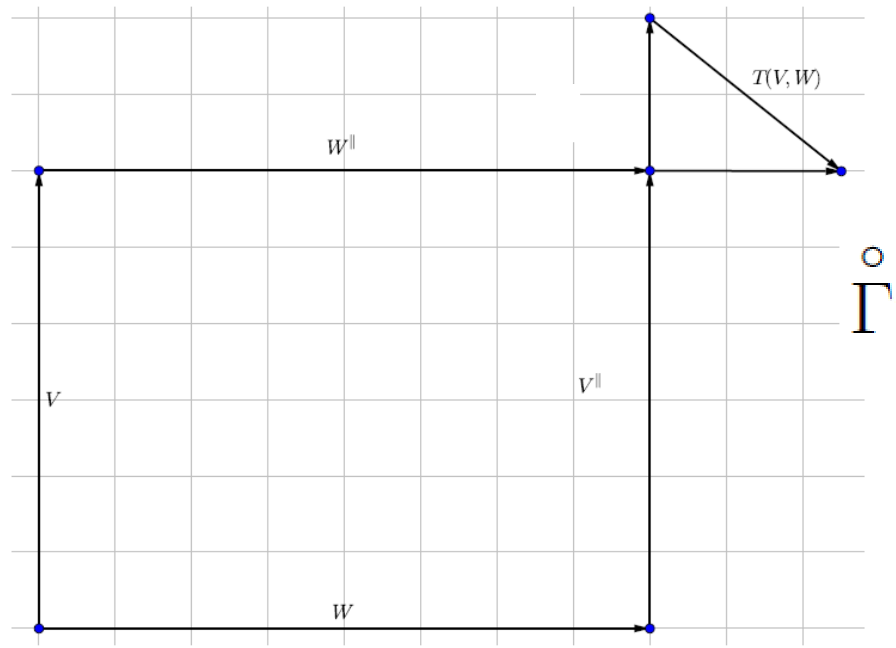
- THAT IMPLIES THAT THE VACUUM EINSTEIN EQUATION IS WEYL INV:

$$G_{\mu\nu} = 0, \qquad \delta G_{\mu\nu} = 0$$

GEOMETRIC VIEW OF TORSION

- (VECTORIAL) TORSION TRACE 1-FORM:

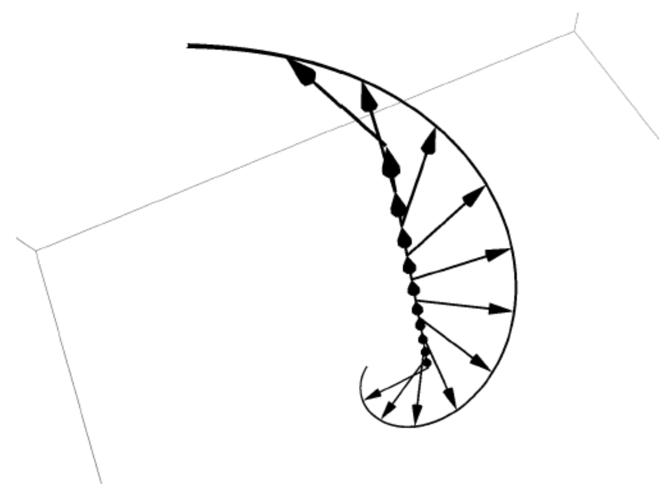
$$\mathcal{T} \equiv \mathcal{T}_\mu \mathrm{d}x^\mu = \frac{2}{D-1} T^\lambda{}_{\lambda\mu} \mathrm{d}x^\mu$$



- TRANSFORMS AS A VECTOR FIELD:

$$\mathcal{T} \to \mathcal{T} + \mathrm{d}\theta$$

- WHEN A VECTOR IS PARALLEL-TRANSPORTED, TORSION TRACE INDUCES A LENGTH CHANGE: CRUCIAL IN WHAT FOLLOWS



A TANGENT VECTOR $\dot{\gamma}$ AND JACOBI FIELD J PARALLEL-TRANSPORTED. J EXHIBITS A ‘CARTAN STAIRCASE.’

PARALLEL TRANSPORT AND JACOBI EQUATION

- GEODESIC EQUATION:

$$\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau} \equiv \frac{dx^\lambda}{d\tau}\nabla_\lambda\frac{dx^\mu}{d\tau} = 0$$

$$\Gamma^\lambda{}_{\mu\nu} = T^\lambda{}_{\mu\nu} + T_{\mu\nu}{}^\lambda + T_{\nu\mu}{}^\lambda + \mathring{\Gamma}^\lambda{}_{\mu\nu}$$

$\mathring{\Gamma}$ = LEVI-CIVITA

$$T[X,Y] = -\frac{1}{2}(\nabla_X Y - \nabla_Y X - [X,Y])$$

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{[\mu\nu]} = \frac{1}{2}\left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}\right)$$

→ TRANSFORMS MULTIPLICATIVELY (as $1/d\tau^2$)

$$\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau} = 0 \Rightarrow e^{-2\theta(x)}\nabla_{\dot\gamma}\frac{dx^\mu}{d\tau} = 0$$

NB: TRANSFORMATION OF $d\tau$ COMPENSATED BY TRANSFORMATION OF Γ !

- JACOBI EQUATION (JACOBI FIELDS J ⊥ $\dot\gamma$) AND RAYCHAUDHURI EQ:

$$\nabla_{\dot\gamma}\nabla_{\dot\gamma}J + 2\nabla_{\dot\gamma}T[\dot\gamma, J] = R[\dot\gamma, J]\dot\gamma$$

→ ALSO TRANSFORMS MULTIPLICATIVELY (as $1/d\tau^2$) UNDER WEYL TR

- SUGGESTS TO DEFINE A GAUGE INVARIANT PROPER TIME:

$(d\tau)_{g.i.} = \exp\left(-\int_{x_0}^{x} T_\mu dx^\mu\right) d\tau :=$ PHYSICAL TIME OF COMOVING OBSERVERS!

T. Prokopec, het Lam, e-Print: arXiv:1606.01147

WEYL SYMMETRY IN MATTER SECTOR

SCALAR MATTER

- CONFORMAL WEIGHT w_ϕ OF A CANONICAL SCALAR:

$$\phi \to e^{-\frac{D-2}{2}\theta}\phi \quad \Rightarrow w_\phi = -\frac{D-2}{2}$$

- CONFORMAL (WEYL) COVARIANT DERIVATIVE:

$$\overset{w}{\nabla}_\mu \phi = \partial_\mu \phi + \frac{D-2}{2}\mathcal{T}_\mu \phi$$

TORSION TRACE: $\mathcal{T} \equiv \mathcal{T}_\mu \mathrm{d}x^\mu = \frac{2}{D-1} T^\lambda{}_{\lambda\mu} \mathrm{d}x^\mu$

ACTS AS A GAUGE CONNECTION!

(no i - the group is non-compact)

- CONFORMALLY INVARIANT SCALAR ACTION:

KINETIC/GRADIENT TERMS; SELF-COUPLING & COUPLING TO GRAVITY

$$-\frac{1}{2}\int \mathrm{d}^D x \sqrt{-g}\,\mathrm{Tr}\overset{w}{\nabla}_\mu \phi \overset{w}{\nabla}_\nu \phi g^{\mu\nu}$$

$$\int d^D x \sqrt{-g}\left\{\frac{1}{2}\alpha^2 \phi^2 R - \frac{\lambda}{4!}\phi^4\right\}$$

VECTOR & FERMIONIC MATTER

- CONFORMAL WEIGHT w_ϕ OF A CANONICAL SCALAR:

$$\psi \to e^{-\frac{D-1}{2}\theta}\psi$$

$$A_\mu \to e^{-\frac{D-4}{2}\theta}A_\mu$$

$$\Rightarrow w_\psi = -\frac{D-1}{2}, \quad w_A = -\frac{D-4}{2}$$

- CONFORMAL (WEYL) COVARIANT DERIVATIVE:

$$\overset{w}{\nabla}_\mu\psi = \nabla_\mu\psi + \frac{D-1}{2}\mathcal{T}_\mu\psi$$

$$\mathcal{T} \equiv \mathcal{T}_\mu \mathrm{d}x^\mu = \frac{2}{D-1}T^\lambda{}_{\lambda\mu}\mathrm{d}x^\mu$$

- INVARIANT ACTIONS:

FERMIONS: $$\int \mathrm{d}^4x\sqrt{-g}\left[\frac{i}{2}\left(\bar{\psi}\gamma^\mu(\nabla_\mu + eA_\mu)\psi - (\nabla_\mu - eA_\mu)\bar{\psi}\gamma^\mu\psi\right) - g_y\phi\bar{\psi}\psi\right]$$

VECTORS: $$-\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) \qquad \int d^Dx\, f\,\mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

NB1: IN D≠4, TORSION BREAKS GAUGE SYMMETRY!

NB2: TORSION TRACE ACTS AS A GAUGE CONNECTION (no $\acute{\imath}$)!

(CLASSICALLY) CONFORMAL STANDARD MODEL & GRAVITY

- HIGGS SECTOR

$$\int \mathrm{d}^D x\sqrt{-g}\left[-\frac{1}{2}(D_\mu H)^\dagger D^\mu H - \lambda_H (H^\dagger H)^2 + g_{H\Phi} H^\dagger H \Phi^2 - \lambda_\Phi \Phi^4\right]$$

COVARIANT DERIVATIVE: $D_\mu H = \partial_\mu H + \frac{D-2}{2}\mathcal{T}_\mu H - ig\sum_a W^a_\mu \sigma^a \cdot H - ig' Y B_\mu H$

$\Rightarrow$ CAN EXHIBIT DYNAMICAL SYMMETRY BREAKING VIA THE CW MECHANISM

T. G. Steele, Zhi-Wei Wang, arXiv:1310.1960 [hep-ph]

- DILATON ACTION:

$$S[g_{\mu\nu}, \Phi, \mathcal{T}_\mu] = \int \mathrm{d}^4 x\sqrt{-g}\left(\frac{\Phi^2}{2\alpha^2}R - \frac{g^{\mu\nu}}{2}\overset{w}{\nabla}_\mu \Phi \overset{w}{\nabla}_\nu \Phi - V(\Phi)\right) + S^{SM}$$

- ACTION FOR FERMIONS:

$$\int \mathrm{d}^4 x\sqrt{-g}\left[\frac{i}{2}\left(\bar\psi\gamma^\mu(\nabla_\mu + eA_\mu)\psi - (\nabla_\mu - eA_\mu)\bar\psi\gamma^\mu\psi\right) - g_y\phi\bar\psi\psi\right]$$

- GRAVITATIONAL ACTION (LAST TERM IS BOUNDARY [GB] TERM IN D=4):

$$\int \mathrm{d}^D x\sqrt{-g}\left(\xi_1 R^2 + \xi_2 R_{\mu\nu}R^{\mu\nu} + \xi_3 R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}\right)$$

NB: SM+GRAVITY CAN BE MADE WEYL INVARIANT ONLY IN D=4.

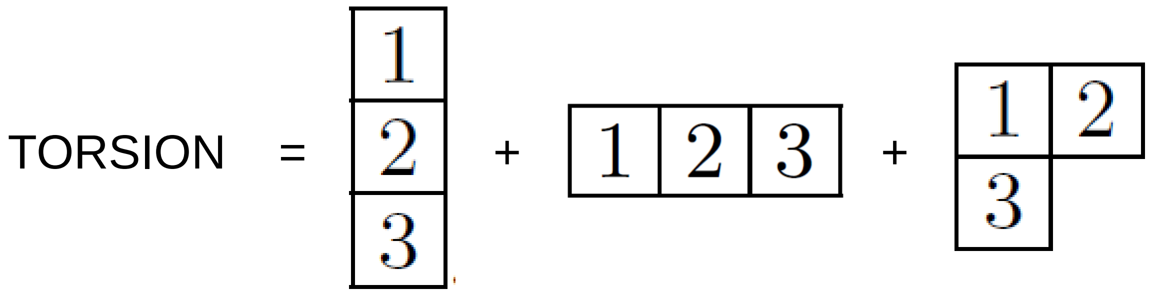
WEYL SYMMETRY IN QUANTUM THEORY AND ITS DYNAMICAL BREAKING

DECOMPOSITION OF TORSION

- TORSION TENSOR CONTAINS DD(D-1)/2=24 COMPONENTS:

- IT CAN BE BROKEN INTO SKEW SYMMETRIC PART (DUAL TO VECTOR), TORSION TRACE (VECTOR) and REMAINDER (16 components: s(12)a(13)).

USING YOUNG DIAGRAMS (THE LAST DIAGRAM IS PROBLEMATIC):



SKEW (1) TRACE (2) MIXED REP (3)

- FORTUNATELY, MATTER SOURCES ONLY TYPE 1 AND 2 TORSION, IMPLYING THAT ONLY THE SKEW SYMMETRIC (1) AND TRACE (2) BECOME DYNAMICAL DUE TO QUANTUM EFFECTS!!

QUANTIZATION

- CANONICAL QUANTIZATION PRESERVES WEYL SYMMETRY.
 INDEED:

$$\phi \to e^{-\frac{D-2}{2}\theta}\phi \qquad \pi_\phi = \sqrt{-g}n_\nu g^{\mu\nu}\bar{\nabla}_\mu\phi\,, \implies \pi_\phi \to \tilde{\pi}_\phi = \Omega^{(D-2)/2}\pi_\phi$$

IMPLIES WEYL INVARIANCE OF THE CANONICAL COMMUTATOR:

$$[\phi,\, \pi_\phi] = [\tilde{\phi},\, \tilde{\pi}_\phi] = \mathbb{1}$$

ALTERNATIVELY, IF CLASSICAL ACTION IS WEYL INVARIANT,
THE PATH INTEGRAL MEASURE IS ALSO INVARIANT, HENCE
GENERATING FUNCTIONAL AND EFFECTIVE ACTION ARE ALSO INVARIANT:

$$\exp\{iW[J_\phi]\} = Z[J_\phi] = \int D\phi D\pi_\phi \exp\{i\int [\pi_\phi\,\dot{\phi} - H(\pi_\phi,\phi) + J_\phi\phi]\}$$

$$\Gamma_{EFF}[\bar{\phi}, g_{\mu\nu}] = J_\phi\bar{\phi} - W[J_\phi], \quad \delta W[J_\phi]/\delta J_\phi = \bar{\phi}$$

- NB: OUR INABILITY TO PROVIDE A REGULAR/RIGOROUS DEFINITION OF THE PATH INTEGRAL MAY BREAK WEYL SYMMETRY.
- INDEED, NO REGULARIZATION SCHEME IS KNOWN (INCLUDING DIM REG) THAT DOES <u>NOT</u> BREAK WEYL SYMMETRY. PROBLEM?
- NB: ANALOGOUS CONSTRUCTION HOLDS FOR FERMIONIC & GAUGE FIELDS

QUANTIZATION 2

- PROBLEM CAN BE RESOLVED BY DEMANDING WEYL INVARIANCE OF THE EFFECTIVE ACTION:

$$\mu \frac{d}{d\mu} \Gamma_{EFF} [\, \bar{\phi}, \; \overline{g_{\mu\nu}} \,] = 0 \qquad (*)$$

- SIMILAR TO RG IMPROVEMENT. DIFFERENT HOWEVER, SINCE THE CLASS OF ALLOWED OPERATORS MUCH SMALLER: ONLY THOSE LOCAL OPERATORS THAT <u>RESPECT</u> WEYL SYMMETRY ARE ALLOWED.

- EQUATION (*) CAN BE INTERPRETED AS THE GENERATOR OF <u>WARD IDENTITITES</u> ASSOCIATED WITH WEYL SYMMETRY.

- WHEN FULLFILLED, THESE WARD IDENTITITES GUARANTEE THAT WEYL SYMMETRY IS (NON-PERTURBATIVELY) REALISED.

<u>**NB**</u>: IT IS REASONABLE TO DEMAND THAT – JUST LIKE GAUGE SYMMETRY – WEYL SYMMETRY CANNOT BE BROKEN.

ONE-LOOP EFFECTIVE ACTION

- CLASSICAL ACTION: $S = \int d^D x \sqrt{-g} \left\{ \frac{1}{2} \alpha^2 \phi^2 R - \frac{1}{2} (\bar{\nabla} \phi)^2 - \frac{\lambda}{4!} \phi^4 \right\}$

- INTEGRATING OUT FLUCTUATIONS IN ϕ $(in\ presence\ of\ a\ bg\ R\ and\ \phi)$ results in divergences of the form:

$$\propto \frac{\mu^{D-4}}{D-4} \delta^D (x - x')$$

- TO RENORMALIZE AWAY THESE ONE ADDS **c.t.**'s THAT REMOVE THE DIVERGENT TERMS. THESE **c.t.**'s BREAK WEYL SYMMETRY!

- RENORMALIZED QUANTUM (1-LOOP) ACTION [F (G) FIELD STRENGTH ASSOCIATED WITH TRACE (SKEW) TORSION]:

$$\Gamma_{1\,LOOP} = -\frac{1}{32\pi^2} \int d^4 x \sqrt{-g} \left\{ \begin{matrix} \left[\left(\alpha^2 - \frac{1}{6} \right) \bar{R} + \alpha^2 \rho + \alpha^2\, \phi^2 \right]^2 \\ -\frac{1}{36} \left[\frac{1}{15}\ \bar{R}_{\alpha)(\beta\gamma)(\delta} \bar{R}^{\alpha\beta\gamma\delta} - \frac{1}{15} \bar{R}_{(\alpha\beta)} \bar{R}^{\alpha\beta} \right] \\ + F_{\alpha\beta} F^{\alpha\beta} + G_{\alpha\beta} G^{\alpha\beta} \end{matrix} \right\} \left\{ \log(\left[\left(\alpha^2 - \frac{1}{6} \right) \bar{R} + \alpha^2 \rho + \alpha^2\, \phi^2 \right] / \mu^2) + C \right\}$$

- WEYL SYMMETRY BREAKING IS "WEAK": IT APPEARS AS μ IN THE LOG(μ).

- THE SYMMETRY CAN BE RESTORED BY IMPOSING WARD IDENTITIES. AT THE LEVEL OF OPERATORS, THEY HAVE THE FORM OF THE CS EQUATION.

- SHOULD REPEAT BY USING GENERAL METHODS [Rieger, Fradkin; Antoniadis et al]

1 LOOP EFFECTIVE ACTION 2

$$\Gamma_{1\,LOOP} = -\frac{1}{32\pi^2}\int d^4x\sqrt{-g}\left\{\begin{matrix}\left[\left(\alpha^2-\frac{1}{6}\right)\bar{R}+\alpha^2\rho+\alpha^2\,\phi^2\right]^2\\ -\frac{1}{36}\left[\frac{1}{15}\;\bar{R}_{\alpha)(\beta\gamma)(\delta}\bar{R}^{\alpha\beta\gamma\delta}-\frac{1}{15}\bar{R}_{(\alpha\beta)}\bar{R}^{\alpha\beta}\right]\\ +F_{\alpha\beta}F^{\alpha\beta}+G_{\alpha\beta}G^{\alpha\beta}\end{matrix}\right\}\left\{\log(\left[\left(\alpha^2-\frac{1}{6}\right)\bar{R}+\alpha^2\rho+\alpha^2\,\phi^2\right]/\mu^2)+C\right\}$$

- WHEN RG IMPROVED, THE OFFENDING TERMS log(μ) ARE ABSORBED BY μ –DEPENDENT COUPLINGS. FORMALLY, $\Gamma_{1LOOP,RG}$ BECOMES μ –INDEPENDENT

- Riemann-SQUARED TERMS CAN BE REMOVED (REWRITTEN AS A GENERALISED GAUSS-BONNET BOUNDARY TERM. THESE ARE INTERESTING AND THEIR RELEVANCE YET TO BE STUDIED.

- SCALAR MATTER MAKE TORSION TRACE DYNAMICAL,
- FERMIONIC MATTER MAKES SKEW SYMMETRIC TORSION DYNAMICAL
- WE DO NOT YET KNOW ABOUT THE GRAVITONS

♦ CLASSICAL ACTION: 1 TENSOR + 1 MASSLESS SCALAR DoF

♦ QUANTUM ACTION: 1 TENSOR + 2 VECTOR + 1 MASSLESS SCALAR DoF

WARD IDENTITIES

- CONSIDER (WEYL INVARIANT) CLASSICAL ACTION:

$$S = \frac{1}{2}\int \mathrm{d}^D x \sqrt{-g}\, g^{\mu\nu}\left(\partial_\mu\phi + \frac{D-2}{2}T_\mu\phi\right)\left(\partial_\nu\phi + \frac{D-2}{2}T_\nu\phi\right)$$

- INFINITESIMAL WEYL TRANSFORMATION ω, $\Omega = 1+\omega$

$$\delta_\omega\phi = -\frac{D-2}{2}\omega\phi$$

- TO LINEAR ORDER IN $\omega(x)$, THE ACTION CHANGES AS:

$$\delta_\omega S = \int \mathrm{d}^D x\sqrt{-g}\left(\frac{D-2}{\sqrt{-g}}\frac{\delta S}{\delta\phi(x)}\omega(x)\phi(x) - \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}(x)}\omega(x)g^{\mu\nu} - \bar{\nabla}_\mu\left(\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta T_\mu(x)}\right)\omega(x)\right)$$

- WHEN THIS IS INSERTED INTO THE GENERATING FUNCTIONAL, AND ONE REQUIRES THAT THE TERM LINEAR IN ω TO VANISH, ONE GETS THE FOLLOWING WARD IDENTITY:

$$\int \mathcal{D}\phi e^{iS}\left(T^\mu_\mu + \bar{\nabla}_\mu\Pi^\mu\right) = 0 \qquad T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad \Pi^\mu = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta T_\mu(x)}$$

♦ HERE TENSOR Π^μ IS THE SOURCE FOR TORSION TRACE VECTOR

WARD IDENTITIES 2

- TO RECAP, WE HAVE DERIVED THE FUNDAMENTAL WARD IDENTITY:

$$\langle T^{\mu}_{\mu} \rangle + \langle \bar{\nabla}_{\mu} \Pi^{\mu} \rangle = 0 \quad (*)$$

- SINCE $\langle T_{\mu}{}^{\mu} \rangle$ IS GENERATED BY THE TRACE ANOMALY, PART OF IT PROPORTIONAL TO THE EULER DENSITY E_4 (GAUSS BONNET), AND Box(R), WHICH ARE TOTAL DERIVATIVE, e.g.

$$\langle T_{\mu}{}^{\mu} \rangle = c_T E_4 = c_T \bar{\nabla}_{\mu} E^{\mu},\ c_T = \text{const.}$$

- THIS MEANS THAT THE ANOMALY (except for Weyl^2) CAN BE REABSORBED INTO THE TORSION TRACE SOURCE :

$$\widetilde{\Pi^{\mu}} \equiv \Pi^{\mu} + c_T E^{\mu} \Rightarrow \nabla_{\mu} \widetilde{\Pi^{\mu}} = 0$$

♦ HENCE, NO MORE (TRACE) ANOMALY!

- THE FUNDAMENTAL WARD IDENTITY (*) CAN BE USED TO GENERATE HIGHER ORDER IDENTITIES (w_O= CONFORMAL WEIGHT OF O):

$$\langle T[(T_{\mu}{}^{\mu} + c_T \bar{\nabla}_{\mu} G^{\mu}) O(\{x_{(i)}\})] \rangle = w_O \sum_j \frac{\delta^D(x - x_{(j)})}{\sqrt{-g}} \langle T[O(\{x_{(i)}\})] \rangle$$

GENERATION OF MASS (SPECULATIVE)

► RECALL:

$$\mu \frac{d}{d\mu} \Gamma_{EFF} [\, \bar{\phi},\ \overline{g_{\mu\nu}}\,] = 0$$

$$\Gamma_{EFF} [\, \bar{\phi},\ \overline{g_{\mu\nu}}\,] = S[\bar{\phi},\ \overline{g_{\mu\nu}}\,] + \text{TADPOLE}$$
$$- \frac{1}{2} \iint d^D x d^D x' \, \bar{\phi}\,(x)\, \mathrm{M}(\mathrm{x};\mathrm{x}')\bar{\phi}\,(x') + ..$$

► CONJECTURE: THERE ARE NO LOCAL (SG) TERMS IN M(x;x') THAT ARE PROPORTIONAL TO μ^2, i.e.

$$M(x;x') \quad = (\alpha R + \beta \phi^2)\, \delta^4(\mathrm{x-x'}) + M(x;x')_{\mathrm{non-local}}$$

COLEMAN WEINBERG MECHANISM

S. Coleman, E. Weinberg (1973)

- MASSLESS SELF-INTERACTING SCALAR GETS A 1-LOOP CORRECTION:

$$L_\phi = -\frac{Z(\phi^2/\mu^2)}{2}(\partial\phi)^2 - \frac{\lambda}{4!}\phi^4 - \frac{\lambda^2}{256\pi^2}\phi^4\left[\ln\left(\frac{\phi^2}{\mu^2}\right) - \frac{25}{6}\right]$$

► QUANTUM EFFECTS BREAK WEYL SYMMETRY:

SCALE μ INTRODUCED BY COUNTER-TERMS NEEDED TO RENORMALIZE

► QUANTUM EFFECTS CAN GENERATE A LOCAL MINIMUM AT $\phi \neq 0$.
THAT IS NOT PERTURBATIVELY RELIABLE, QUANTUM O(ħ) AND CLASSICAL CONTRIBUTION ARE COMPARABLE AT THE MINIMUM.

►CW USED CALLAN-SYMANZIK EQUATION TO RG IMPROVE RESULT.
THE SECOND MINIMUM THEN DISAPPEARS.

►HOPE: IN MULTIFIELD SCALAR MODELS, RELIABLE MINIMA OCCUR SUCH THAT ONE CAN USE CW MECHANISM INSTEAD THE BEH MECHANISM TO EXPLAIN MASS GENERATION. STILL NOT KNOWN WHETHER IT CAN BE MADE CONSISTENT WITH OBSERVATIONS (STABILITY OF POTENTIAL AND NONTRIVIAL MINIMA).

T.P.,da Rocha, Schmidt, Swiezewska (2017), in progress

QUANTUM THEORY CONFRONTS OBSERVATIONS

CONFRONTING OBSERVATIONS

$$\Gamma_{1\,LOOP} \supset -\int d^4x\sqrt{-g}\,\{[\alpha(\mu)\bar{R}^2 + \beta(\mu)\phi^2 R] + \gamma(\mu)F_{\alpha\beta}\,F^{\alpha\beta}\}$$

EARLY COSMOLOGY

- CAN BE TESTED BY STUDYING INFLATIONARY MODELS GENERATED BY CONDENSATION OF SCALARON, DILATON OR LONG TORSION TRACE.
- PRELIMINARY RESULTS: CAN GET (quasi)de SITTER UNIVERSE AND NEARLY SCALE INVARIANT SCALAR SPECTRUM.

LATE COSMOLOGY

- CAN BE TESTED BY STUDYING e.g. DARK ENERGY AND STRUCTURE FORMATION, POSSIBLY DARK MATTER CANDIDATE.
- STILL ON A TO DO LIST ..

CONCLUSIONS AND OUTLOOK

CONCLUSIONS AND OUTLOOK

● CHALLENGE: USE FRG METHODS TO STUDY HOW THIS THEORY DIFFERS FROM THE USUAL GRAVITY,
i.e. WHETHER IT IS ASYMPTOTICALLY SAFE/ADMITS UV COMPLETION.

● CHALLENGE 2: IS ANYTHING DIFFERENT WRT UNITARITY.
NOTE THAT DUE TO ABSENCE OF THE PLANCK SCALE,
THE GHOST PROPAGATOR SHOULD BE MASSLESS (WORSE?)

● CHALLENGE 3: CONFRONT THIS NOVEL THEORY AS MUCH AS POSSIBLE WITH OBSERVATIONS

● CHALLENGE 4: CAN WE GET RID OF (COSMOLOGICAL AND BLACK HOLE) SINGULARITIES?

HINT: RECALL: $(d\tau)_{g.i.} = \exp\left(-\int_{x_0}^{x} T_\mu dx^\mu\right) d\tau :=$ PHYSICAL TIME OF COMOVING OBSERVERS