Probes of Cosmic Acceleration

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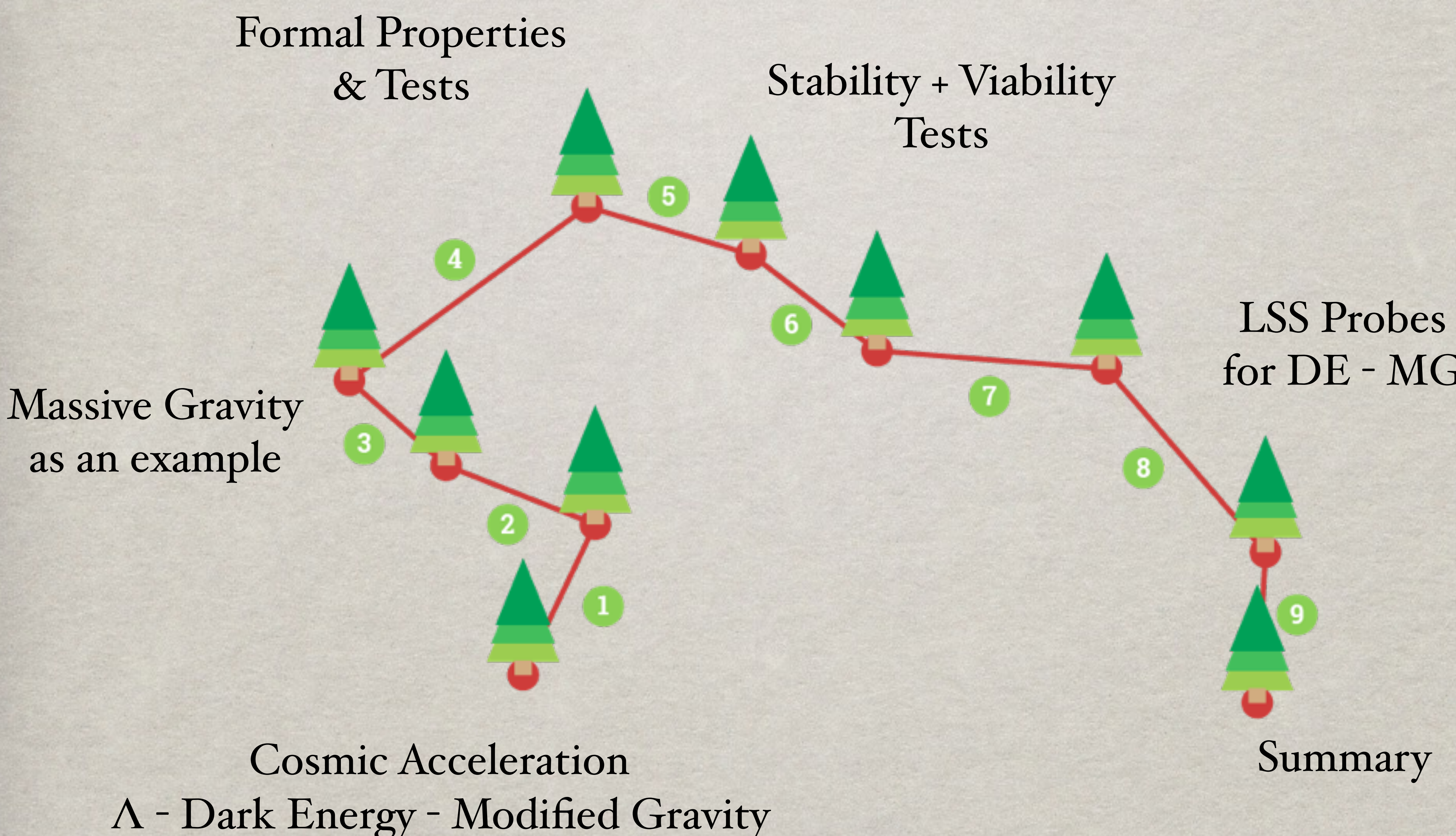
based on a series of works with C. de Rham + A. Tolley and with Z. Vlah

March 21st, 2017, Mainz Institute for Theoretical Physics

Outline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Motivations

c.c. problem

Old: "understand why the vacuum energy is so small"

Weinberg

New: "why it is comparable to the present mass density"

Accept Λ

"..wait for a better solution, go anthropic etc..."

Dark Energy/Modified Gravity

Uniqueness of GR

Lorentz-invariant
massless
spin-2

L-breaking theories

massive gravity

directly add e.g.
scalars, DE

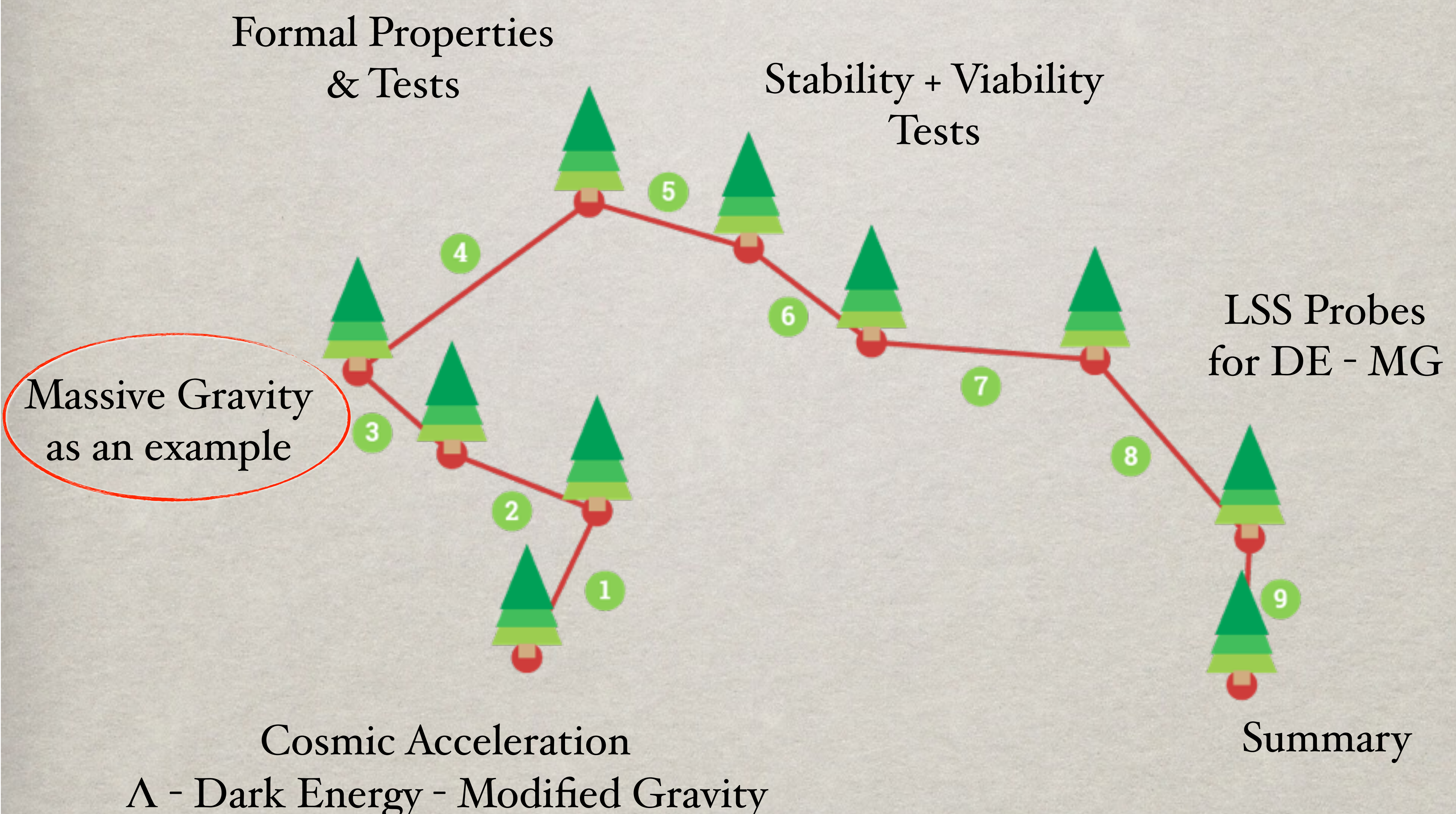
Massive Gravity

Why massive gravity ? Technically natural mechanism for cosmic acceleration

Non-linear massive gravity (dRGT) + extensions

$\Uparrow$

Ghost-free, Lorentz-invariant 4-d theory of a ~~massless~~ massive spin 2 field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Linearly

Fierz and Pauli (1939)

$$-\underbrace{\frac{1}{4}h^{\mu\nu}\hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}}_{\text{E-H}} \quad -\underbrace{\frac{1}{8}m^2\left(h^2_{\mu\nu}-h^2\right)}_{\text{F-P}}$$

; 5 dof of healthy massive spin 2

breaks diff invariance: $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

vDVZ discontinuity

$$\text{Theory}_{m\to 0} \neq \text{Theory}_{m=0}$$

At odds w/ observations: angle for the bending of light at impact parameter b off by *25%* w.r.t. GR

Non-linearities better play a crucial role,  they do

Non Linearly

Vainshtein effect: non-linearities screen helicity-0 mode in the presence of matter

$$r_V = \left(\frac{M}{M_P^2 m^2}\right)^{1/3}$$

Most of what is verified analytically is static and spherical

screening in an area within r_V, where GR is recovered ✓ linearized theory good outside

Not easy

$$\mathcal{L}^{n-l}_{\mathrm{F-P}} = -m^2 M_{\mathrm{Pl}}^2 \sqrt{-g}\left([(\mathbb{I}-\mathbb{X})^2] - [\mathbb{I}-\mathbb{X}]^2\right)$$

where $\mathbb{X}^\mu_\nu = g^{\mu\alpha}\tilde{f}_{\alpha\nu}$

helicity-0 mode π

$$\mathbb{X}^\mu_\nu = \delta^\mu_\nu - \frac{2}{M_{\mathrm{Pl}} m^2}\partial^\mu\partial_\nu\pi + \frac{1}{M_{\mathrm{Pl}}^2 m^4}\partial^\mu\partial_\alpha\pi\partial^\alpha\partial_\nu\pi$$

generic non-linear interaction will carry an Ostrogradski ghost

Non Linear with Special Structure, *dRGT*

de Rham, Gabadadze (2010)
de Rham, Gabadadze, Tolley (2010)

$$S_{\rm mGR} = \frac{M_{\rm Pl}}{2}\int d^4x\sqrt{-g}\left(R[g] + \frac{m^2}{2}\sum_{n=0}^{4}\alpha_n\mathcal{L}_n[\mathcal{K}[g,f]]\right)$$

where

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$$

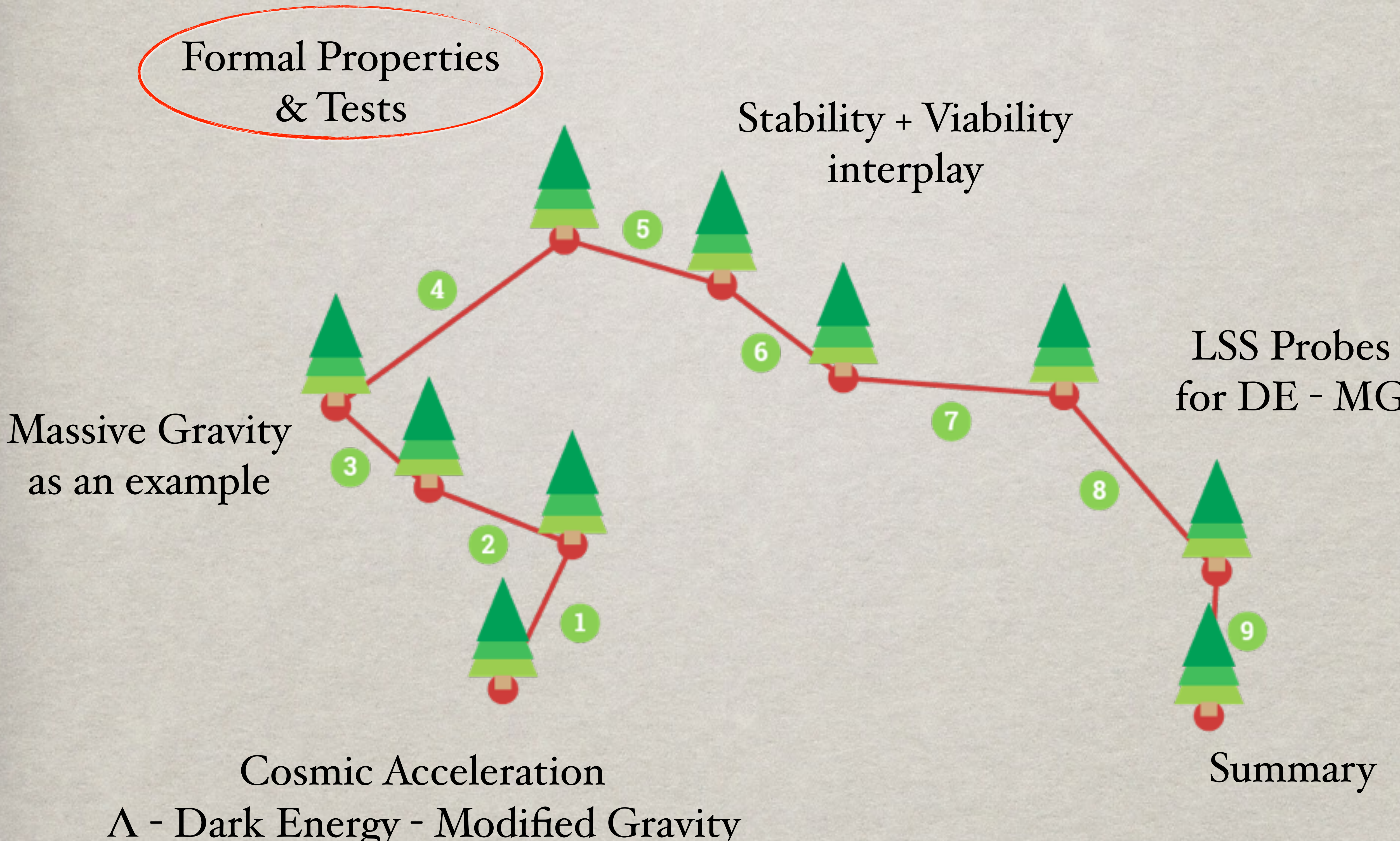
and

$$\begin{aligned}
\mathcal{L}_0[\mathcal{K}] &= 4!\\
\mathcal{L}_1[\mathcal{K}] &= 3!\,[\mathcal{K}]\\
\mathcal{L}_2[\mathcal{K}] &= 2!([\mathcal{K}]^2 - [\mathcal{K}^2])\\
\mathcal{L}_3[\mathcal{K}] &= ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3])\\
\mathcal{L}_4[\mathcal{K}] &= ([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4])
\end{aligned}$$

Absence of ghosts verified in countless ways at full non-linear level

Hassan, Rosen
(2011,2011)

Extensions, e.g. bigravity = massive gravity + H-E for metric *f*; 7 healthy dof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Robustness under Quantum Corrections

Nicolis, Rattazzi (2012)
de Rham at al (2012)
de Rham, Heisenberg, Ribeiro (2015)

(I) Preserve ghost-free structure

(II) Small renormalization of the graviton mass

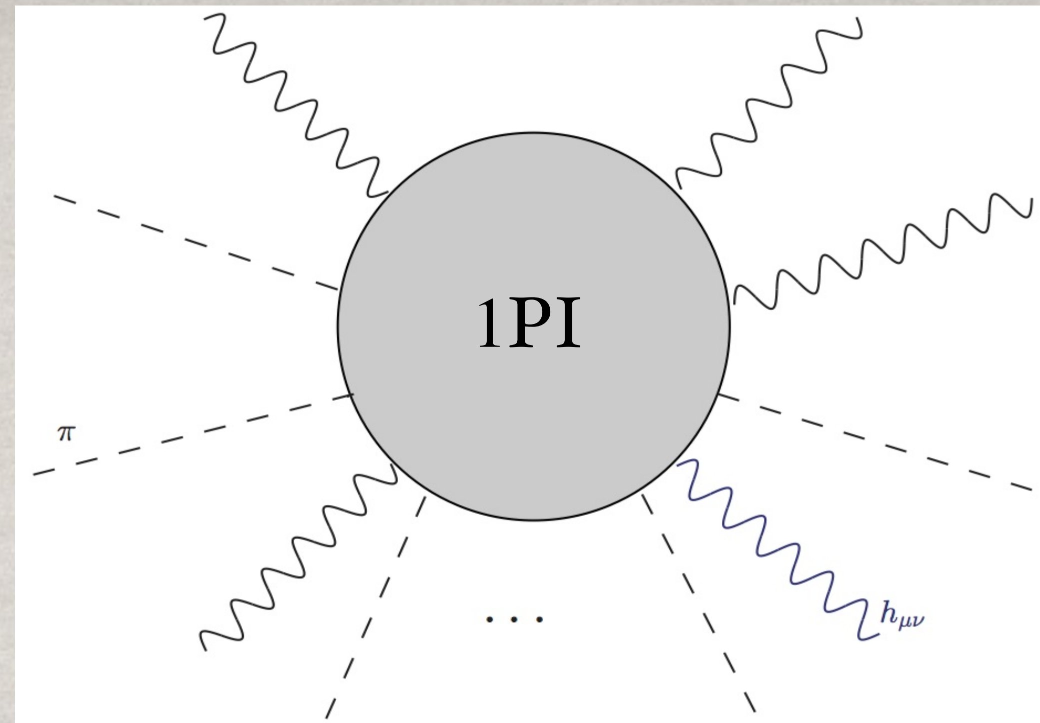
Exact non-renormalization theorem exists in Decoupling (scaling limit)

$$m \to 0, \quad M_P \to \infty, \quad (m^2 M_P)^{1/3} \equiv \Lambda_3 \to \text{fixed}$$

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \frac{3}{2}\pi \Box \pi + (h^{\mu\nu} + \pi \eta^{\mu\nu}) \sum_{n=2}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}(\Pi)$$

$$X^{(1)}_{\mu\nu}(\Pi) = \epsilon_\mu^{\ \alpha\rho\sigma} \epsilon_\nu^{\ \beta}{}_{\rho\sigma} \Pi_{\alpha\beta} \, ,$$
$$\Pi_{\alpha\beta} \equiv \partial_\alpha \partial_\beta \pi$$

scalar d.o.f., Galileon,
vectors turned off



Special structure (antisymmetry) of Galileon interaction

==>

external particles always at least two derivatives acting on it in the 1PI action

==>

Galileons not renormalized in DL

In the full theory

(II) $\delta m^2 \lesssim m^2 \left(\frac{m}{M_P}\right)^{2/3}$ ✓ technically natural small mass

(I) Ghost-free structure detuned only at Planck scale

Unitarity and Analyticity of Scattering Amplitudes

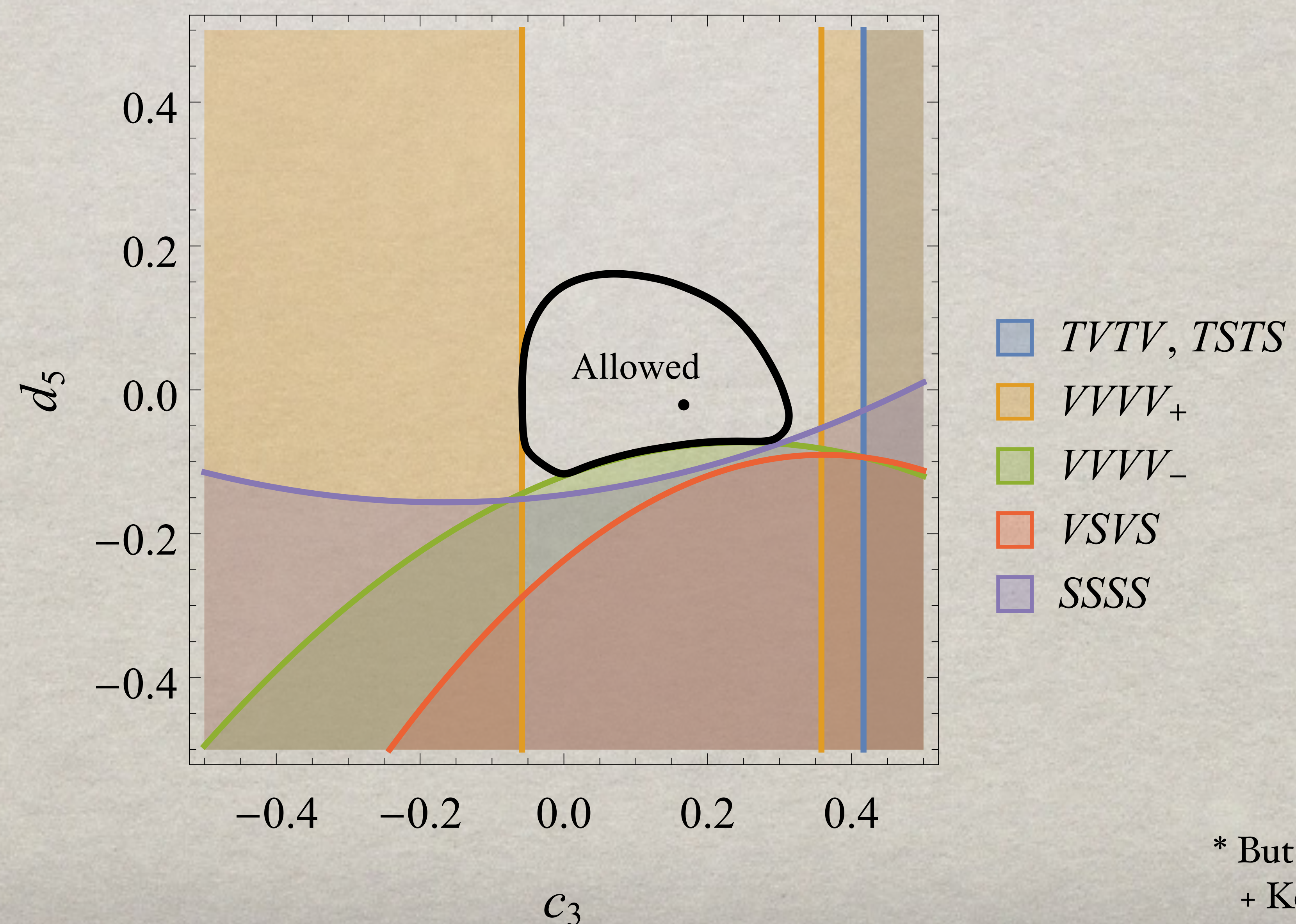
Cheung, Remmen (2016)

analyticity + unitarity *

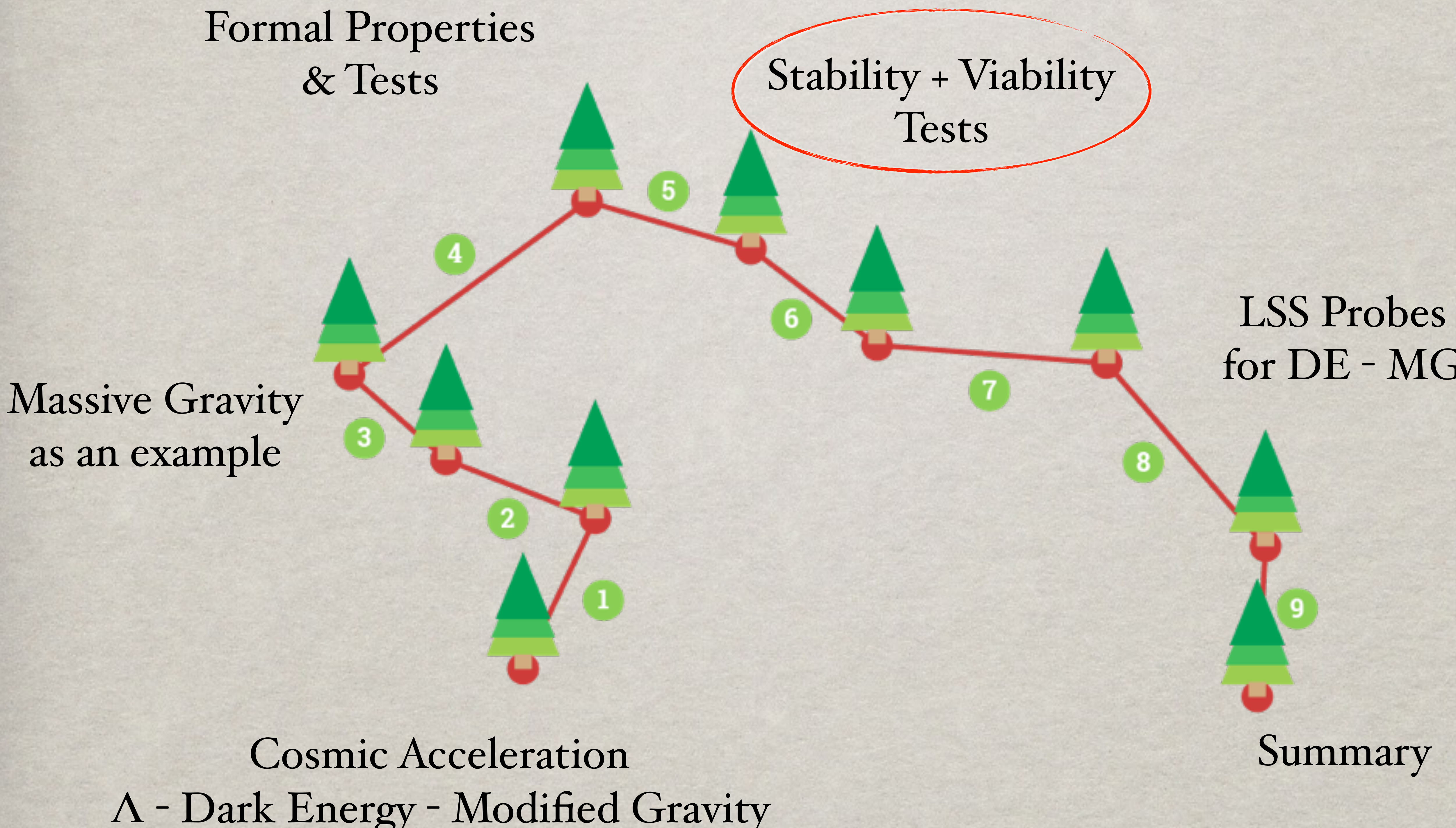
==>

positivity constraints on coefficients in forward amplitude

$$c_3, d_5 \sim a_n \sim \alpha_n$$



* But see Dvali et al
+ Keltner, Tolley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Is it empty?

Is is stable?

Is is observationally viable?

No FRW solutions in dRGT if "f" Minkowski*

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, Tolley

Yes, we can live with inhomogeneities

Vainshtein guarantees inhomogeneities unobservable before late times

Inhomogenities only appear on scale set by inverse graviton mass

Volkov; Koyama; Gumrukcuoglu et al; Gratia, Hu, Wyman; Kobayashi et al; DeFelice et al; Tasinato et al; Not updated, many many more!!

*natural for interacting massive spin-2 representation of Poincare` group

Solutions with metric "f" as dS or FRW

MF, Tolley

Add matter content to gauge model independence

$$\mathcal{L}_M \sim \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\partial\phi)^2 + V(\phi)\Big]$$

Simple algorithm

Existence of the solution

Check stability of the theory

Early + late-time dynamics from Friedman equation

CHANGE F

Stability bound

$$H = \alpha\, p^2 + \beta\, q^2 + \gamma(\nabla q)^2 + \ldots$$

coefficient of kinetic term > 0

tachyon inst.

gradient inst.

Quickest route to the Higuchi/unitarity bound in dS:

"In the the linear (massive) theory there exist a unitary spin 2 representation of the dS group iff:"

$$m^2 = 0$$ G.R.

$$m^2 = 2H^2$$ Partially massless theory

Higuchi bound in massive gravity → $$m^2 > 2H^2$$ Massive

CHANGE F

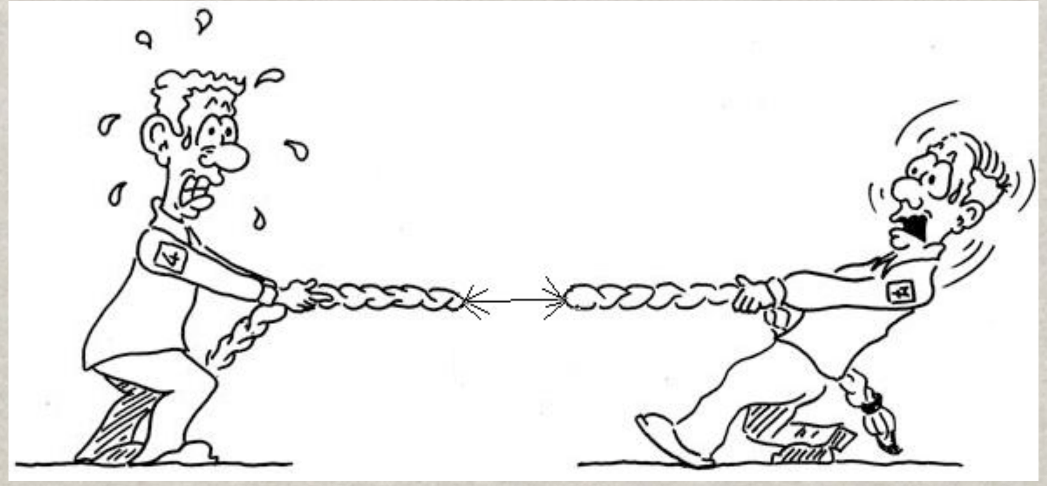
Bound from Observations

Before Dark Energy epoch sets in, G.R. good description:

$$3H^2 = \Lambda + 3m^2 \times \Theta(1) + \ldots$$

$$\boxed{m^2 \lesssim H^2}$$

combining Stability and Observations then:



want our theory to be stable

GR over many cosmo epochs

$$m^2 > 2H^2$$

$$m^2 \lesssim H^2$$

CHANGE F

Generalized Higuchi:

$$\tilde{m}^2(H) = m^2 \frac{H}{H_f}\left((3+3\alpha_3+\alpha_4) - 2(1+2\alpha_3+\alpha_4)\frac{H}{H_f} + (\alpha_3+\alpha_4)\frac{H^2}{H_f^2}\right) \geq 2H^2$$

N.B. independent on precise form of matter

Friedman Side:

$$m^2\left(\frac{2}{3}(-6-4\alpha_3-\alpha_4) + 2\left(\frac{H}{H_f}\right)(3+3\alpha_3+\alpha_4) - 2\left(\frac{H}{H_f}\right)^2(1+2\alpha_3+\alpha_4) + \frac{2}{3}\left(\frac{H}{H_f}\right)^3(\alpha_3+\alpha_4)\right) \ll 2H^2$$

Combined:

$$\frac{\mathrm{poly}_1^{(\mathrm{k})}(\mathrm{z})}{\mathrm{poly}_2^{(\mathrm{k})}(\mathrm{z})} \gg 1$$

Hard to satisfy even using α_3, α_4

Impossible when we account $H = H(t)$

CHANGE F

What now? Go bigravity !

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left[M_P^2\, R(g) - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n\left(g^{-1}f\right)\right] + \frac{1}{2}\sqrt{-f}M_f^2\, R(f) + \mathcal{L}_M$$

$$g_{\mu\nu} \leftrightarrow f_{\mu\nu},\ M_P \leftrightarrow M_f,\ \beta_n \leftrightarrow \beta_{4-n} \quad {}^*$$

This fact must be reflected on the bound itself

Crucial for Galileon Duality

*Matter breaks this

Stability Bound in Bigravity

MF, Tolley

Soon in a more symmetric form

Stability bound

$$\tilde{m}^2 \left[1 + \left(\frac{H_f / M_f}{H / M_P} \right)^2 \right] \geq 2H^2$$

- Minisuperspace action ✓
- Decoupling limit ✓
- PM check ✓
- Symmetric ✓

Recover Massive gravity bound in the limit $M_f \to \infty, \;\; M_P, H_f$ finite.

Not possible before: $\frac{H_f}{M_f} \gg \frac{H}{M_P}$ not directly invoking m

Friedman side

$$H^2 = \frac{1}{3M_P^2}\left[\rho(a) + \sum_{n=0}^{3} \frac{3m^2\beta_n}{(3-n)!n!}\left(\frac{H}{H_f}\right)^n\right] \quad ; \quad H_f^2 = \frac{1}{3M_f^2}\left[\sum_{n=0}^{3} \frac{3\beta_{n+1}}{(3-n)!n!}\left(\frac{H}{H_f}\right)^{(n-3)}\right]$$

$m^2 \times \Theta(1) \ll H^2$ it's the only direct requirement on m, but now:

In the $\frac{H_f}{M_f} \gg \frac{H}{M_P}$ region with $\beta_1 \neq 0$ solve for $\tilde{m}^2$; Hf, bound reads:

$$3H^2 > 2H^2 \checkmark$$

The (most pressing) stability vs observations tension is resolved in bigravity !

Stable Self-accelerating Solution

Akrami, Koivisto, Sandstad (2012,2013)

Set: $\beta_2 = 0 = \beta_3;\ \beta_1 = 2M_P^2$

$$H^2 = \frac{1}{6M_P^2}\left(\rho(a) + \sqrt{\rho(a)^2 + \frac{12\,m^4 M_P^6}{M_f^2}}\right)$$

Model	B_0	B_1	B_2	B_3	B_4	Ω_m	χ^2_{min}	p-value	log-evidence
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
(B_1, Ω_m^0)	0	free	0	0	0	free	551.60	0.8355	-281.73

Observationally viable (?) ! Small part of the whole table

Stability bound? It reduces to

$$\left(\frac{1}{M_P^2} + \frac{12M_f^2}{m^4\beta_1^2}H^4\right) > 0 \quad \checkmark$$

Stable as well.

Further work on Cosmo Solutions

Our focus has been on unitarity (Higuchi), but ought to check gradient instability

$$H = \alpha p^2 + \beta q^2 + \gamma(\nabla q)^2$$

Comelli et al (2012); Konnig et al; Comelli et al #2

Non-linearities via Vainshtein? (in progress...)

Viable (in a reduced parameter space region) bigravity model put forward by

especially useful in the "low energy regime"

De Felice et al (2014)
De Felice et al (2013)

$$h_{ij}T^{ij} = \left(H^+_{ij} + C_{(\mathrm{r})}H^-_{ij}\right)T^{ij}$$

massive(massless) tensors decouple and simplify analysis of e.g. gravitational waves

MF, Ribeiro

Generalized Massive Gravity

De Rham, MF, Tolley (2014)

Let the $\alpha_n(\beta_n)$ now depend on the Stueckelbergs fields as

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\left[R[g] + \frac{m^2}{2}\sum_{n=0}^{4}\tilde{\alpha}_n(\tilde{\phi}^a\tilde{\phi}_a)\mathcal{U}_n[\mathcal{K}]\right] + \mathcal{L}_{\rm matter}[g,\psi^{(i)}]$$

In pure mGR, isometry group of the reference metric is Poincare'

$$f_{\mu\nu} = \partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}$$

Now:

- keep — Lorentz ✓
- give up — Translations ✗

Gen. mGR theory still has 5 d.o.f. proven ghost-free

↓

De Rham, Keltner, Tolley (2014)

①

New Solutions

$$m^2 H\left[\frac{3}{2}\beta_1 a^2+\beta_2 a+\frac{1}{4}\beta_3\right]=\frac{m^2}{2a}\left[4\beta_0' a^3+3\beta_1' a^2+\beta_2' a+\frac{1}{6}\beta_3'\right]$$

zero in pure massive gravity, hence lack of solution for f Minkowski

②

Hints of self-acceleration

$$3M_{\rm PL}^2H^2=\rho+\frac{m^3M_{\rm PL}^2}{2H}\left[\frac{\left(4\bar\beta_{0,1}+3\bar\beta_{1,1}a^{-1}\right)^2}{\bar\beta_{1,1}}\right]-2m^2M_{\rm PL}^2\frac{\bar\beta_{0,1}\bar\beta_2}{a\bar\beta_{1,1}}$$

$(\bar\beta_{0,1}\,,\ \bar\beta_{1,1}\,,\ \bar\beta_2\neq 0=\forall_{\rm else})$

③

Stability

- Higuchi bound
- Gradient instability
- ✓ Coupling with matter
- Vector sector
- Tensor sector as usual

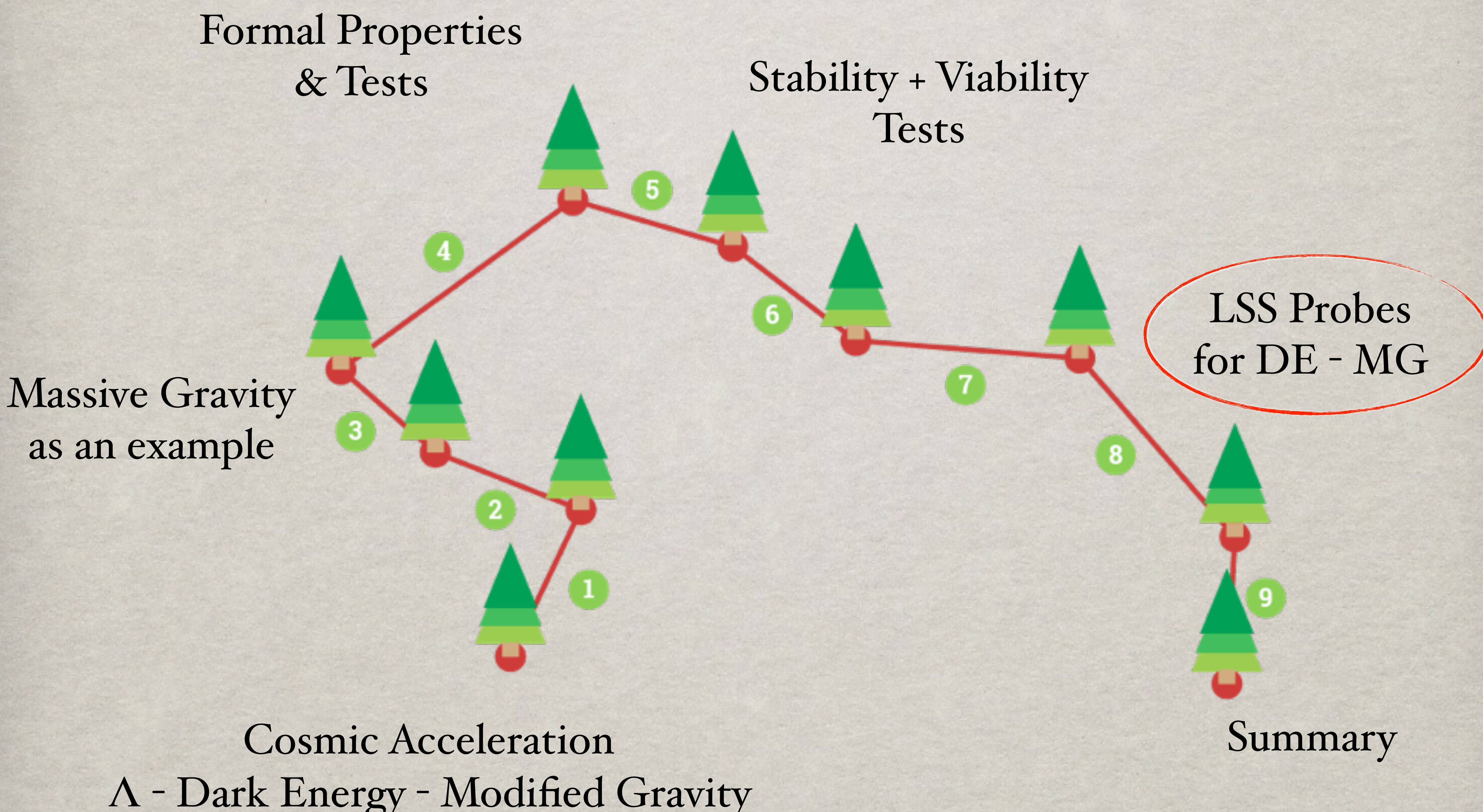
Full-fledged analysis

Small Recap

No FRW solutions in dRGT if "f" Minkowski

CHOICE

KEEP
CALM
and
do
COSMOLOGICAL
PERTURBATION
THEORY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Extra Scalar

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}(\phi, \partial\phi, \partial^2\phi, ..)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T$$

Screening where GR extremely well-tested, e.g. Solar system

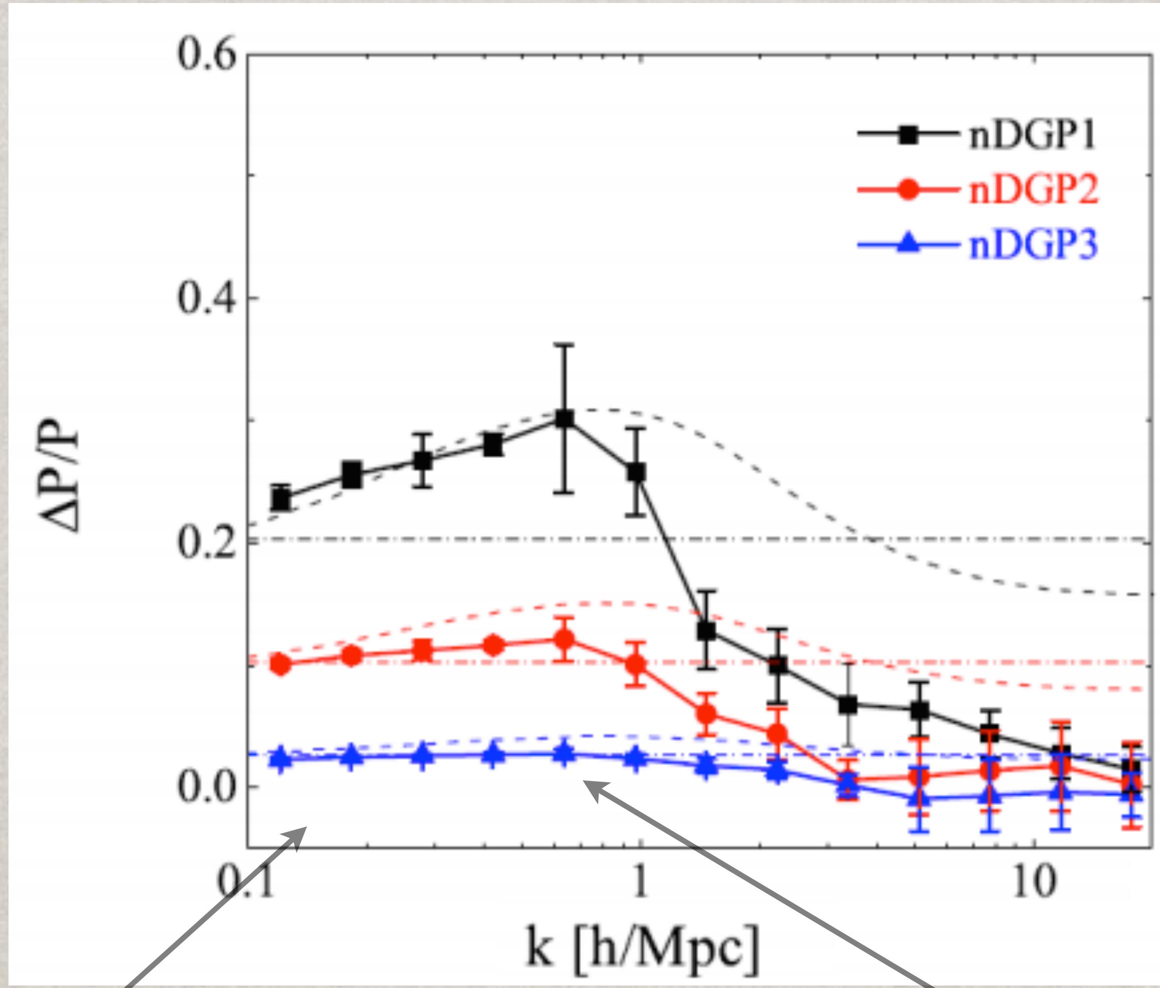
$$V(r) \sim -\frac{g^2(\phi)}{Z(\phi)} \frac{e^{-\frac{m(\phi)}{\sqrt{Z(\phi)}} r}}{4\pi r} \mathcal{M}$$

Symmetron

Vainshtein

Chameleon

N-body

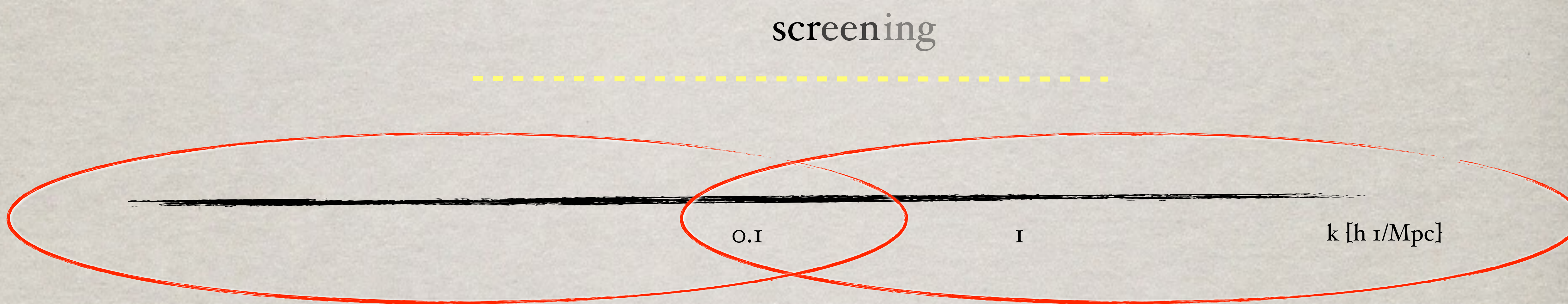


Falck, Koyama, Zhao (2015)

Linear scales

Onset Vainshtein Screening

Perturbation Theory



full perturbative control

bulk of the information on growth

A lot going on to conquer the quasi-linear scales

RPT (Crocce, Scoccimarro)
TRG (Matarrese, Pietroni; Pietroni)
TSPT(Blas, Garny, Ivanov, Sibiryakov)
....
Lagrangian approach
(Matsubara; Porto et al; Vlah et al)
...

EFT of LSS

Why Perturbation Theory?

Underlying physical principles

More clear cut map with DE/MG model space

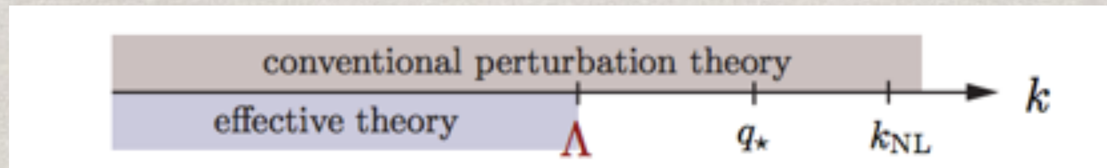
Symmetries (e.g. their role in consistency conditions)

Easy to include additional layers (e.g. non-Gaussianity, baryons..)

Perturbative approaches to LSS

conquering quasi-linear scales

EFT of LSS



Baumann et al (2010)

For dark matter (or more), use fluid description

$$\dot{\theta}_\ell + \mathcal{H}\theta_\ell + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_\ell = -\frac{1}{\rho_\ell}\nabla_i \nabla_j \langle \tau_{ij} \rangle$$

the "EFTness" of the approach is in the fact one describes long-wavelength dynamic

informed by a few UV inputs

$$\begin{aligned} \langle \tau_{ij} \rangle \;&=\; \rho \left[c_1 \left(\frac{\langle v_s^2 \rangle \partial^2}{\mathcal{H}^2} \right)_{ij} + c_2 \left(\frac{\langle v_s^2 \rangle \partial^2}{\mathcal{H}^2} \right)^2_{ij} + \cdots \right] \phi_\ell \;+ \\ &+\; \rho \left[\left(d_1^{(n)} \left(\frac{\partial^2}{\Lambda^2} \right) + d_2^{(n)} \left(\frac{\partial^2}{\Lambda^2} \right)^2 + \cdots \right) \left\{ v_\ell^2 \,,\, \delta_\ell \phi_\ell \,,\, \cdots \right\} \right]_{ij} \end{aligned}$$

Example

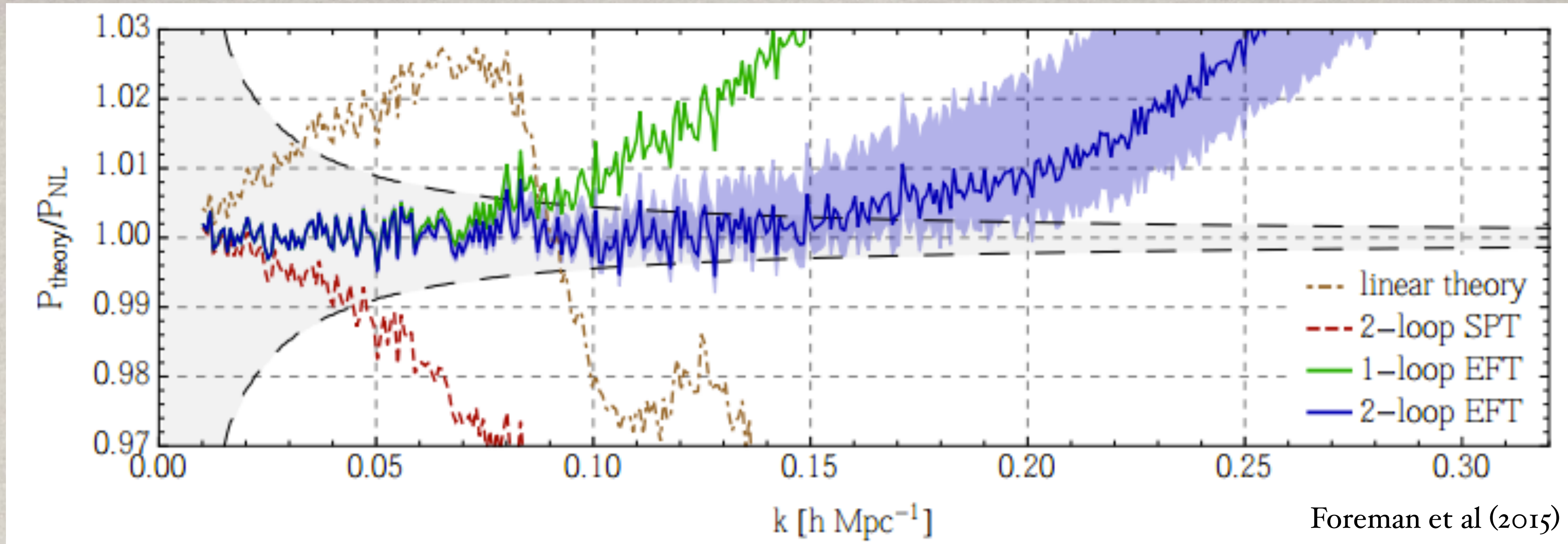
$$P_{\mathrm{EFT-1-loop}}(k,z) = [D_1(z)]^2 P_{11}(k) + [D_1(z)]^4 P_{1-\mathrm{loop}}(k) - 2(2\pi) c^2_{s(1)}(z)[D_1(z)]^2 \frac{k^2}{k^2_{NL}} P_{11}(k)$$

linear

1-L, usual
$P_{22} + P_{13}$

New: counterterm

Locality (analyticity)
and rotational invariance



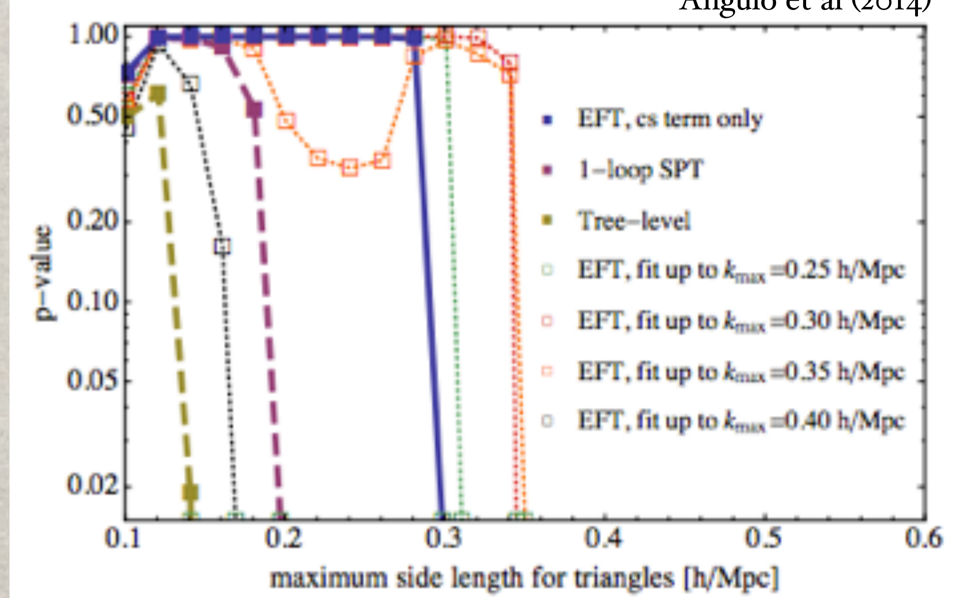
Foreman et al (2015)

fit it

$$P_{\rm EFT-1-loop}(k,z) = D_1(z)^2\, P_{11}(k) + D_1(z)^4\, P^{\rm usual}_{1-\rm loop}(k) - 2(2\pi) c^2_{s(1)}(z) D_1(z)^2 \frac{k^2}{k^2_{\rm NL}} P_{11}(k)$$

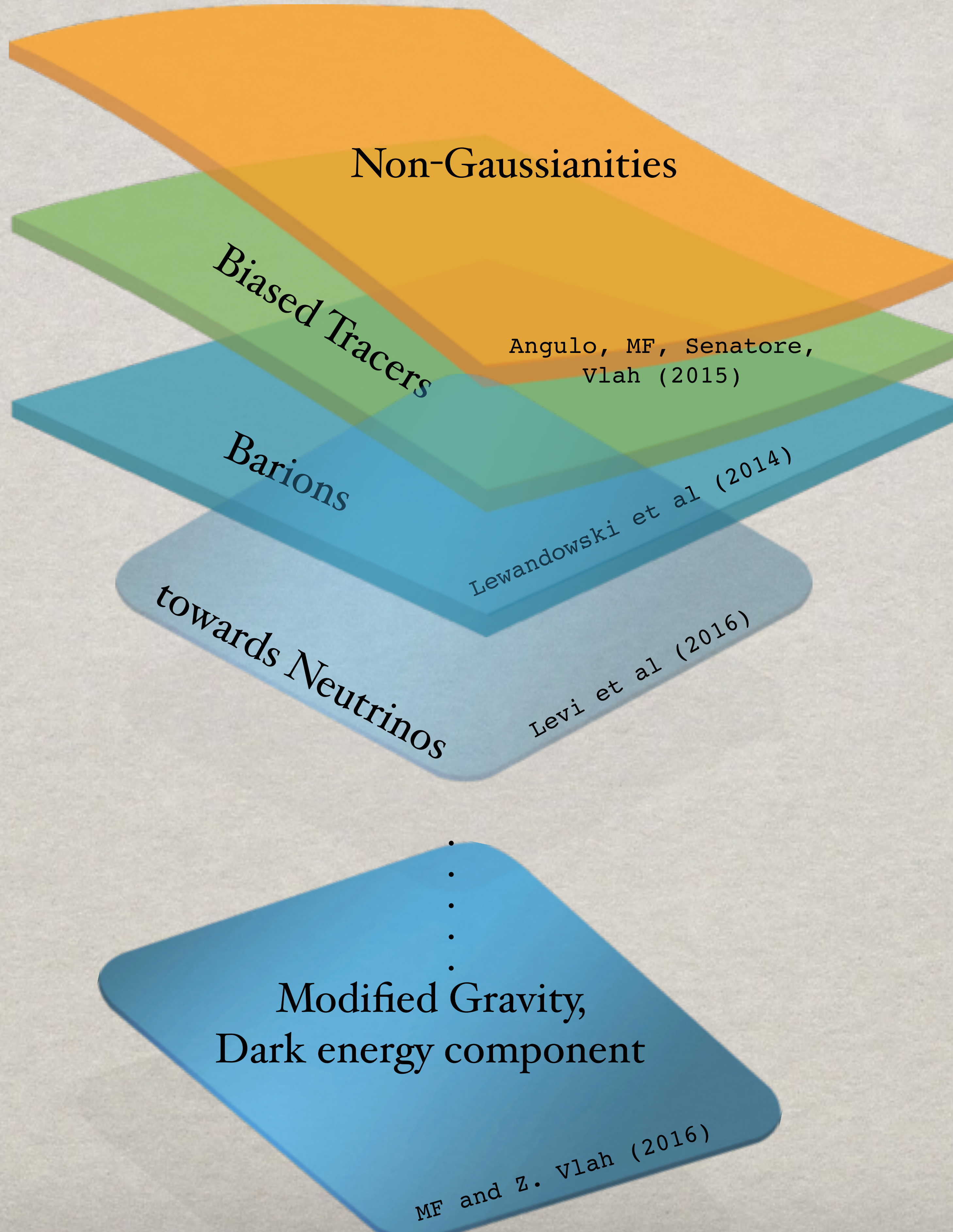
Once "Calibrate" $c_{s(1)}$ on the power spectrum

Angulo et al (2014)



Move on to other observables

Layers of physics



Adding a MG or DDE component

Creminelli et al (2009);
Sefusatti, Vernizzi (2011);
Anselmi et al (2011);
D'Amico, Sefusatti (2011);

$$
\begin{cases}
\dfrac{\partial \delta_m}{\partial \tau} + \vec{\nabla} \cdot [(1+\delta_m)\vec{v}] = 0 \\
\dfrac{\partial \delta_Q}{\partial \tau} - 3\omega \mathcal{H} \delta_Q + \vec{\nabla} \cdot [(1+\omega+\delta_Q)\vec{v}] = 0 \\
\dfrac{\partial v}{\partial \tau} + \mathcal{H}\vec{v} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\nabla \Phi
\end{cases}
$$

clustering quintessence, $c_s=0$

$$
\nabla^2 \Phi = \frac{3}{2}\mathcal{H}^2 \Omega_m \underbrace{(\delta_m + \delta_Q \frac{\Omega_Q}{\Omega_m})}_{\delta_T}
$$

known exactly only up to quadratic order new variables $\delta = \delta_T$; $\Theta = -\frac{C}{\mathcal{H} f_+}\theta$

$$\begin{cases} \frac{\partial \delta_{\mathbf{k}}}{\partial \eta} - \Theta_{\mathbf{k}} = \frac{\alpha(\mathbf{q_1}, \mathbf{q_2})}{C} \Theta_{\mathbf{q_1}} \delta_{\mathbf{q_2}} \, , \\ \frac{\partial \Theta_{\mathbf{k}}}{\partial \eta} - \Theta_{\mathbf{k}} - \frac{f_-}{f_+^2}(\Theta_{\mathbf{k}} - \delta_{\mathbf{k}}) = \frac{\beta(\mathbf{q_1}, \mathbf{q_2})}{C} \Theta_{\mathbf{q_1}} \Theta_{\mathbf{q_2}} \, , \end{cases}$$

$$C(\eta) = 1 + (1+w)\frac{\Omega_Q}{\Omega_m}$$

reduced to familiar system,
C features in the non-linear

All orders, integral & differential solutions

MF, Vlah (2016)

$$\delta_{\mathbf{k}}(\eta) \quad = \sum_{n=1}^{\infty} F_n^s(\mathbf{q}_1..\mathbf{q}_n,\eta) D_+^n(\eta) \delta^{\rm in}_{\mathbf{q_1}}..\delta^{\rm in}_{\mathbf{q_n}}$$

$$\Theta_{\mathbf{k}}(\eta) \quad = \sum_{n=1}^{\infty} G_n^s(\mathbf{q}_1..\mathbf{q}_n,\eta) D_+^n(\eta) \delta^{\rm in}_{\mathbf{q_1}}..\delta^{\rm in}_{\mathbf{q_n}}$$

$$F_n(\eta) = \int_{-\infty}^{\eta} \frac{d\tilde\eta}{C(\tilde\eta)} \left\{ e^{(n-1)(\tilde\eta-\eta)} \frac{\tilde f_+}{\tilde f_+ - \tilde f_-} \left[\left(\tilde h_\beta^{(n)} - \frac{\tilde f_-}{\tilde f_+} \tilde h_\alpha^{(n)} \right) + e^{\tilde\eta-\eta} \frac{D_-(\eta)}{\tilde D_-(\eta)} \left(\tilde h_\alpha^{(n)} - \tilde h_\beta^{(n)} \right) \right] \right\}$$

$$G_n(\eta) = \int_{-\infty}^{\eta} \frac{d\tilde\eta}{C(\tilde\eta)} \left\{ e^{(n-1)(\tilde\eta-\eta)} \frac{\tilde f_+}{\tilde f_+ - \tilde f_-} \left[\left(\tilde h_\beta^{(n)} - \frac{\tilde f_-}{\tilde f_+} \tilde h_\alpha^{(n)} \right) + e^{\tilde\eta-\eta} \frac{f_-}{f_+} \frac{D_-(\eta)}{\tilde D_-(\eta)} \left(\tilde h_\alpha^{(n)} - \tilde h_\beta^{(n)} \right) \right] \right\}$$

$$C = 1 + (1+\omega) \frac{\Omega_Q(\eta)}{\Omega_m(\eta)}$$

iteratively derived, first recursion are usual

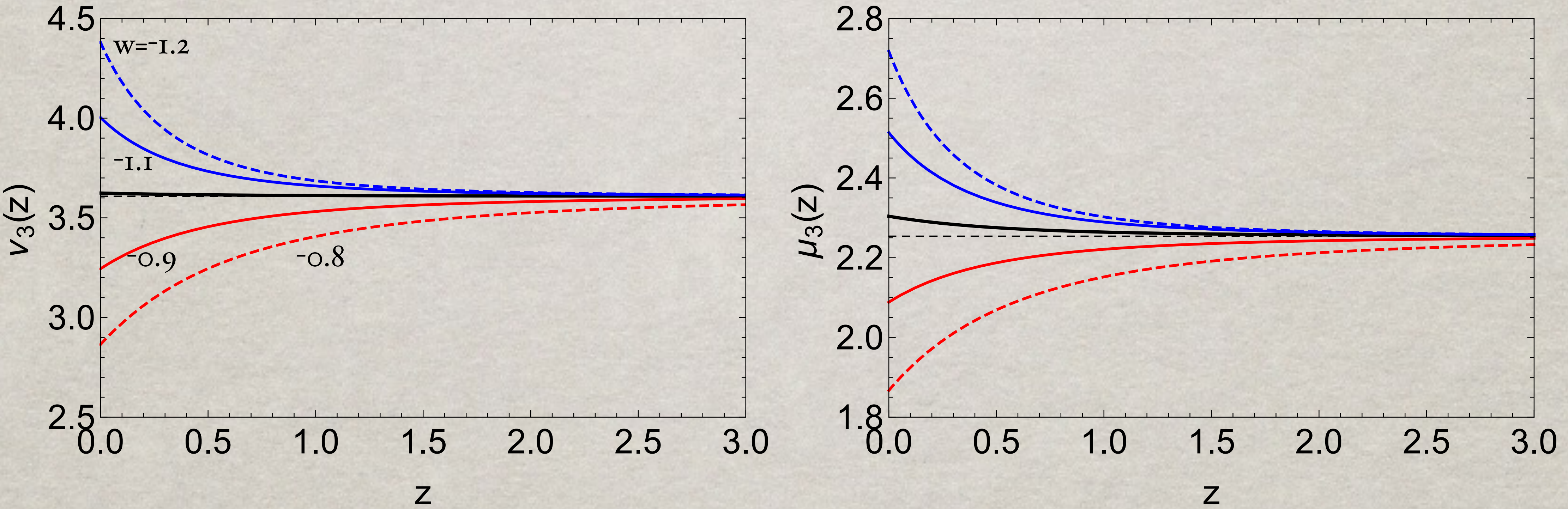
$\alpha(\mathbf{q}_1, \mathbf{q}_2), \beta(\mathbf{q}_1, \mathbf{q}_2)$

related to

$\propto$ linear growth rate

$$F_3 \quad = (1-\epsilon^{(2)})\mathcal{F}_3^{\epsilon} + \nu_3\mathcal{F}_3^{\nu_3} + (1-\epsilon^{(1)})\nu_2\mathcal{F}_3^{\nu_2} + \lambda_1\mathcal{F}_3^{\lambda_1} + \lambda_2\mathcal{F}_3^{\lambda_2}$$

$$G_3 \quad = (1-\epsilon^{(2)})\mathcal{G}_3^{\epsilon} + \mu_3\mathcal{G}_3^{\mu_3} + (1-\epsilon^{(1)})\mu_2\mathcal{G}_3^{\mu_2} + \kappa_1\mathcal{G}_3^{\kappa_1} + \kappa_2\mathcal{G}_3^{\kappa_2}$$

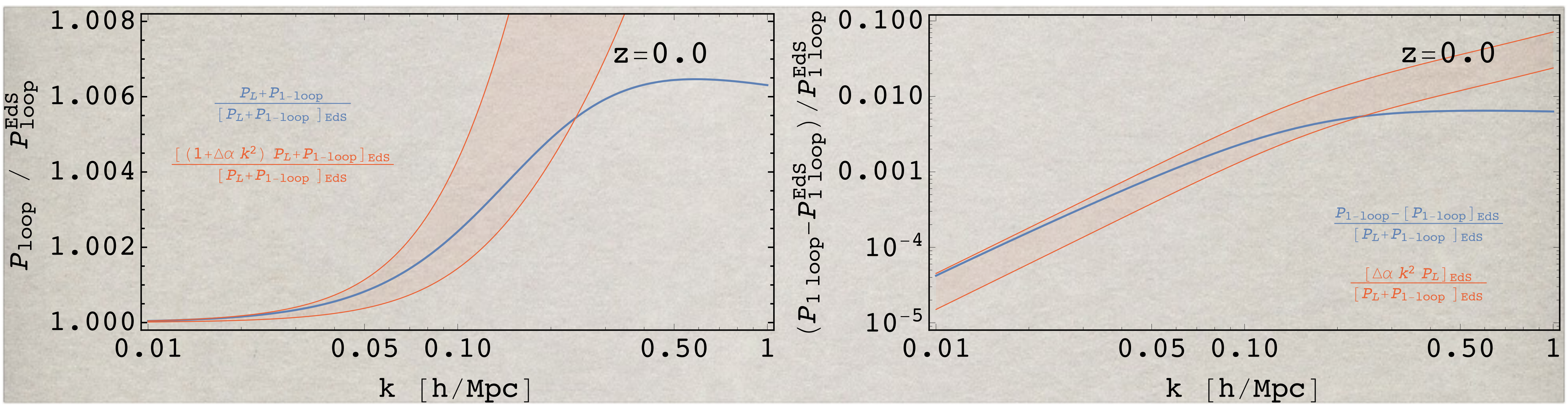


similarly for $\lambda_1, \lambda_2, \kappa_1, \kappa_2(z)$ while $\mathcal{F}_3 = \mathcal{F}_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$

Observables

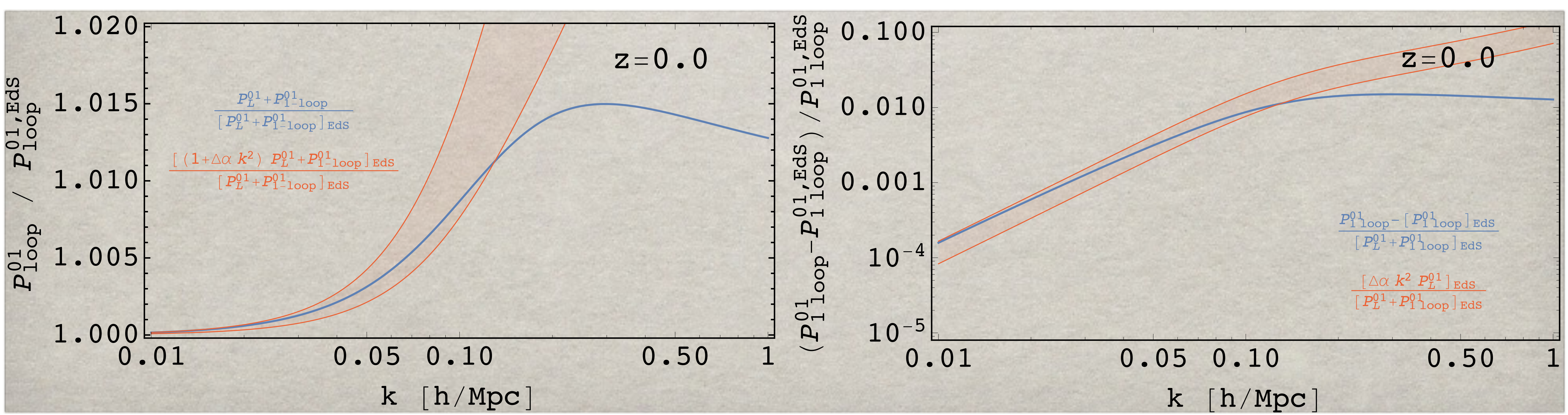
MF, Vlah (2016)

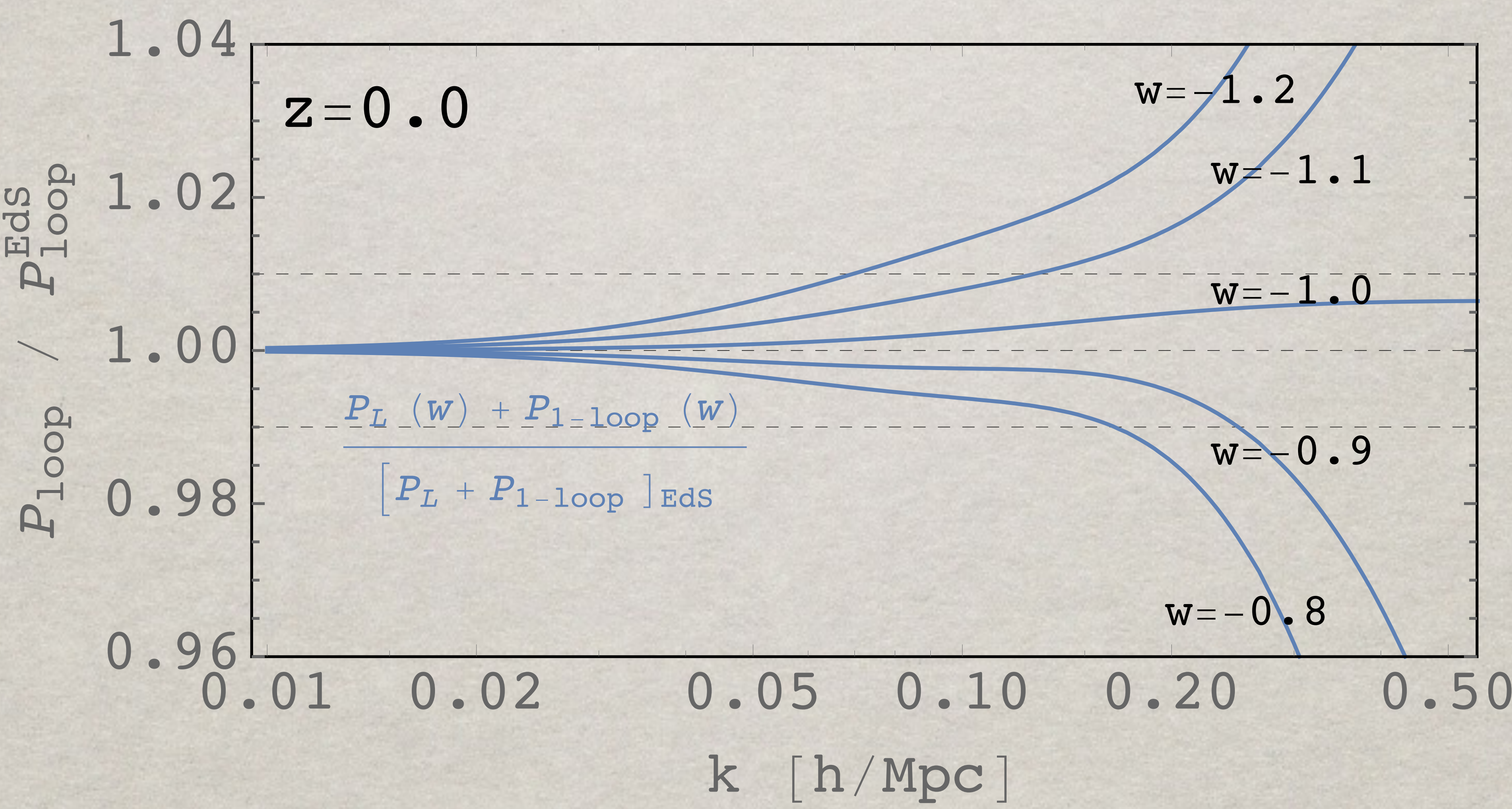
$$P_{1-\text{loop}}(k,a) = P_{\rm L}(k,a) + P_{22}(k,a) + 2P_{13}(k,a) + P_{\rm c.t.}(k,a)$$



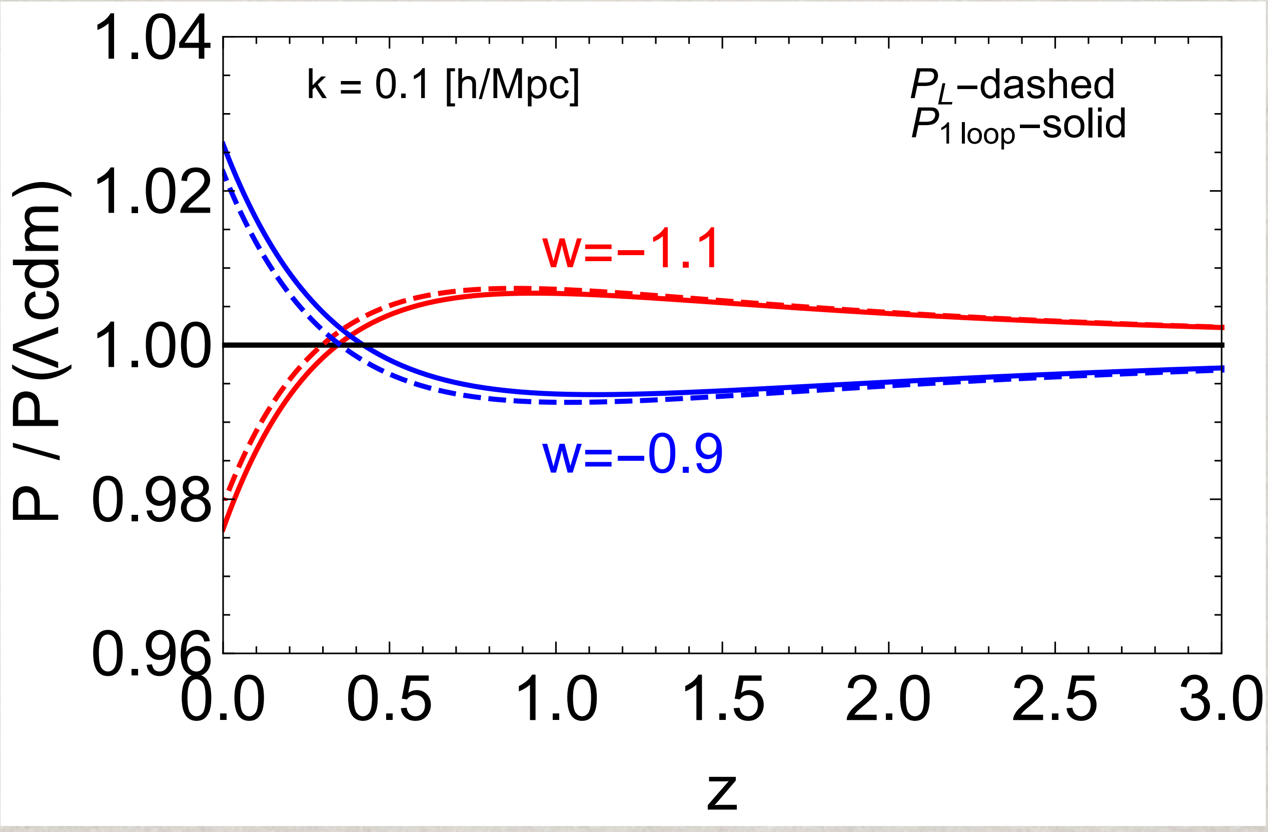
$C(\eta) = 1$

test with ΛCDM




z=0.0
w=-1.2
w=-1.1
w=-1.0
w=-0.9
w=-0.8
$\frac{P_L(w) + P_{1\text{-}loop}(w)}{[P_L + P_{1\text{-}loop}]_{EdS}}$
1.04
1.02
1.00
0.98
0.96
0.01
0.02
0.05
0.10
0.20
0.50
k [h/Mpc]
$P_{loop} / P_{loop}^{EdS}$

MF, Vlah



All the way to Biased tracers

$$\delta_h(\vec{x},t) \simeq \int^t H(t') \left[c_{\delta_T}(t')\, \frac{\delta_T(\vec{x}_{\rm fl},t')}{H(t')^2} + c_{\delta_{\rm d.e.}}(t')\, \delta_{\rm d.e.}(\vec{x}_{\rm fl}) + c_{\partial v_c}(t')\, \frac{\partial_i v_c^i(\vec{x}_{\rm fl},t')}{H(t')} + c_{\partial v_{\rm d.e.}}(t')\, \frac{\partial_i v_{\rm d.e.}^i(\vec{x}_{\rm fl},t')}{H(t')} + \right.$$

$$\left. + c_{\epsilon_c}(t')\epsilon_c(\vec{x}_{\rm fl},t') + c_{\epsilon_{\rm d.e.}}(t')\, \epsilon_{\rm d.e.}(\vec{x}_{\rm fl},t') + c_{\partial^2\delta_T}(t')\, \frac{\partial^2_{x_{\rm fl}}}{k_{\rm M}^2} \frac{\delta_T(\vec{x}_{\rm fl},t')}{H(t')^2} + \dots \right] .$$

Consistency Conditions

MF, Vlah

CDM
$$\begin{cases} \delta_T' + \partial_i[(1+\delta_T)v^i] = 0, \quad \nabla^2\Phi = \dfrac{3}{2}\mathcal{H}^2\delta_T \\ \dfrac{\partial v^i}{\partial \tau} + \mathcal{H}v^i + v^j\partial_j v^i = -\nabla^i\Phi\,; \end{cases}$$

Ccs checklist:

- residual gauge symmetry ✓
$$\begin{cases} \tau \to \tilde{\tau} = \tau\,;\; x^i \to \tilde{x}^i = x^i + n^i(\tau)\,;\; v^i \to \tilde{v}^i = v^i + n^{i'} \\ \delta_m \to \tilde{\delta}_m = \delta_m\,;\; \Phi \to \tilde{\Phi} = \Phi - x^i(\mathcal{H}n^{i\prime} + n^{i\prime\prime})\,, \end{cases}$$
- initial conditions: Gaussian ✓
- adiabaticity ✓

$$\langle \delta_L^m\, \delta_S^m\, \delta_S^m \rangle \sim \frac{\partial}{\partial k}\langle \delta_S^m\, \delta_S^m \rangle$$

Consistency Conditions

MF, Vlah

Full system, now $c_s \neq 0$

$$\begin{cases} \dfrac{\partial \delta_m}{\partial \tau} + \partial_i[(1+\delta_m)v^i] = .. \,, \\ \delta_Q' - 3(w - c_s^2)\mathcal{H}\delta_Q + \partial_i[(1+\omega)v^i] = 3\Psi'(1+w), \\ \dfrac{\partial v_m^i}{\partial \tau} + \mathcal{H}v_m^i + v_m^j \partial_j v_m^i = -\nabla^i \Phi \,, \\ \dfrac{\partial v_Q^i}{\partial \tau} + \mathcal{H}(1-3w)v_Q^i = -\partial_i \Phi - \dfrac{c_s^2\, \partial_i \delta_Q}{1+w} \\ \nabla^2 \Phi = \dfrac{3}{2}\mathcal{H}^2 \Omega_m \left(\delta_m + \dfrac{\Omega_q}{\Omega_m}\delta_Q\right) \equiv \dfrac{3}{2}\mathcal{H}^2\Omega_m \delta_T \end{cases}$$

c_s ,w proportional breaking of Ccs

$$\langle \delta_L^m\, \delta_S^m\, \delta_S^m \rangle|_{\text{usual}} + c_s^2\, \langle \delta_L^m\, \delta_S^m\, \delta_S^m \rangle|^{\perp}_{\text{mediated by DE}} \sim \frac{\partial}{\partial k}\langle \delta_S\, \delta_S \rangle$$

Action of long mode cannot be reabsorbed by gauge transformation

==>

Enhanced squeezed signal, e.g. bispectrum

What's next

MF, Z. Vlah &..

Richer dynamics

small deformations within CQ, $c_s \neq 0$

DGP, Galileons (proxy for mGR)
+
more

Thank you!