Moduli of heterotic G2 compactifications

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Women at the Intersection of Mathematics and High Energy Physics
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X. de la Ossa, ML, E. Svanes (1607.03473 & work in progress)

Motivation and summary

Heterotic string compactifications have a long history

- Minkowski 4D $\mathcal{N} = 1$ vacua:

  Calabi–Yau manifolds X with vector bundles V

  Strominger–Hull systems (X, V, H) with flux and bundles.

- 4D $\mathcal{N} = 1/2$ domain wall vacua:

  SU(3) structure manifolds with flux and bundles.

  (may "uplift" non-perturbatively to non-SUSY *AdS* solutions.)

Geometry and topology of X, V determine the low-energy 4D physics.
In particular, *moduli* give

- Couplings between matter fields.
- Cosmological constant, possibly inflation.
- Massless scalar fields unless stabilised: 5th forces etc.

What are the moduli in heterotic compactifications? Moduli stabilisation?

Motivation and summary

This talk: heterotic compactifications on G_2 structure manifolds

- Heterotic 4D $\mathcal{N} = 1/2$ domain wall solutions

  SU(3) structure manifolds with flux and bundles.

  (may "uplift" to max. symmetric (non-SUSY) 4D vacua.)
- Moduli captured by heterotic systems with G_2 holonomy/structure
- Moduli of heterotic system captured by Atiyah-like bundles.

  cf. talk by Xenia de la Ossa
- Flow of SU(3) structures and interconnected moduli spaces (*time permitting*).

Outline

Heterotic supersymmetric vacua

Heterotic string to $\mathcal{O}(\alpha')$

- Bosonic fields: Metric G, B-field B, dilaton ϕ, gauge field A
- Fermionic fields: Gravitino Ψ_M, dilatino λ, gaugino χ

Compactifications

- $\mathcal{M}_{10} = \mathcal{M}_E \times X$: SUSY $\iff$ nowhere vanishing spinor η on X
- Killing spinor equations

$$\nabla_H \eta = \left(\nabla_M + \frac{1}{8} \not{H}_M \right) \eta = 0$$
$$\left(\not{\nabla} \hat{\phi} + \frac{1}{12} \not{H} \right) \eta = 0$$
$$\not{F} \eta = 0$$

  where $\not{\nabla} = \gamma^M \nabla_M$, etc.

- Bianchi identity $\mathrm{d}H = \frac{\alpha'}{4}(\mathrm{tr} R \wedge R - \mathrm{tr} F \wedge F)$

4D Heterotic $\mathcal{N} = 1$ Minkowski vacua *cf.* Xenia de la Ossa's talk

6D Geometry:

- $\nabla_H \eta = 0 \iff X$ has conformally balanced $SU(3)$ structure
  $\iff \mathrm{d}(e^{-2\phi}\Psi) = 0 = \mathrm{d}(e^{-2\phi}\omega \wedge \omega)$.
- No H-flux $\iff X$ is Calabi–Yau $\quad \mathrm{d}\Psi = 0 = \mathrm{d}\omega$.

Candelas, et.al.:85, Hull:86; Strominger:86, Ivanov, Papadopoulos:00; Gauntlett, et.al.:03,...

Gauge fields $\to$ vector bundle V

- $\not{F}\eta = 0 \implies$
  - $F^{(0,2)} = F^{(2,0)} = 0 \rightsquigarrow$ holomorphic V
  - HYM equation $F \lrcorner \omega = 0 \rightsquigarrow$ polystable holomorphic V

Candelas, et.al.:85,Donaldson:85, Uhlenbeck, Yau:86, Li, Yau:87,...

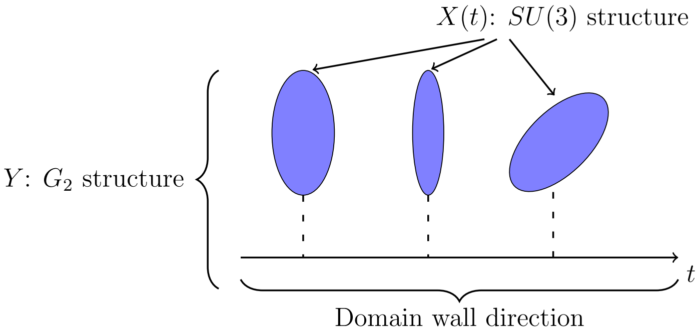
4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Micu et al:04-08, Chatzistavrakidis et al:06,09, Nolle et al:10,Held et al:10...

Lukas et al:10-15; Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14,16,...

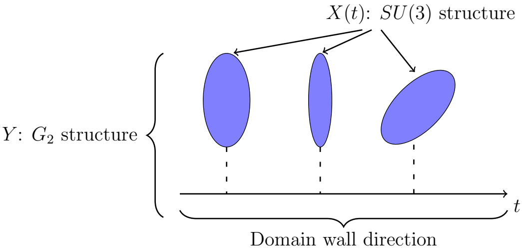
7D Geometry

4D domain wall vacuum: $\mathcal{M}_{10} = \mathcal{M}_4 \times_W X(r) \equiv \mathcal{M}_3 \times Y$
$\mathcal{M}_4 = \mathcal{M}_3 \times \mathbb{R}$, $\mathcal{M}_3$ AdS or Minkowski



4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Lukas et al:10-15; Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14,16,...



Uplift to max. symmetric 4D solution

- Want nearly-Minkowski vacuum *after all* moduli-stabilising effects added in.
- In fine-tuned solutions, non-perturbative effects can balance a *weak* perturbative running in DW direction.
- Analogous to KKLT or LARGE volume vacua in type IIB.
- Example: non-SUSY AdS vacuum on half-flat manifold of [*Lukas et al:15*]

4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Lukas et al:10-15; Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14,16

7D Geometry:

- $\nabla_H \eta = 0 \iff Y$ has integrable G_2 structure

$$\iff \boxed{\mathrm{d}(e^{-2\phi}\psi) = 0 \ , \ \mathrm{d}\varphi = \tau_0\,\psi + \tfrac{3}{2}\,\mathrm{d}\phi \wedge \varphi + *\tau_3.}$$

for positive 3-form φ and $\psi = *\varphi$.

- No H-flux $\iff Y$ has G_2 holonomy

Embed $SU(3)$ in G_2: $\varphi = \mathrm{d}r \wedge \omega(r) + \mathrm{Re}(\Psi(r))$.

G_2 torsion classes: *Fernandez–Gray:82, Chiossi–Salamon:02*

Gauge fields $\rightarrow$ vector bundle V

- $\not{F}\eta = 0 \implies$ instanton bundle: $\boxed{F \wedge \psi = 0.}$

Infinitesimal Moduli of Heterotic SUSY Vacua

- 4D $\mathcal{N} = 1$ vacua: deformations of (X, V, H)
  - X: conformally balanced complex 3-fold
  - V: holomorphic polystable gauge bundle
  - H: α' corrected BI
- "Atiyah class stabilization" *cf. Xenia de la Ossa's talk*

Infinitesimal moduli

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Deformations of holomorphic structure on extension bundle of Atiyah type.

Atiyah:57, Kodaira, Spencer:58,60, Candelas, de la Ossa:91, Becker,et.al:05,06, Anderson,et.al:10,11,13, Fu, Yau:11,Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez,et.al:13,15,...

- 4D $\mathcal{N} = 1/2$ vacua: deformations of (Y, V, H)
  - Y integrable G_2 structure manifold
  - V: instanton gauge bundle
  - H: α' corrected BI

de la Ossa, ML, Svanes:16 + in progress, Clarke,et.al:16

Infinitesimal Moduli: Heterotic G_2 systems

Naive moduli space

Y: integrable G_2 structure manifold ($H = 0$: G_2 holonomy)
V: instanton gauge bundle

- $\partial_t \psi$,$\partial_t \varphi$: geometric moduli
- $\partial_t A$: Vector bundle moduli $H^1(Y, \mathrm{End}(V))$
- $\partial_t B$: deformations of B-field, $H = \mathrm{d}B + \alpha'(...)$

Infinitesimal Moduli: Heterotic G_2 systems

Naive moduli space

Y: integrable G_2 structure manifold $\boxed{H = 0\text{: } G_2 \text{ holonomy}}$

V: instanton gauge bundle

- $\partial_t\psi$,$\partial_t\varphi$: geometric moduli

  Joyce:96, Dai–Wang–Wei:03, de Boer–Naqvi–Shomer:05,...

- $\partial_t A$: Vector bundle moduli $H^1(Y, \mathrm{End}(V))$
- $\partial_t B$: deformations of B-field, $H = \mathrm{d}B + \alpha'(...)$

Infinitesimal Moduli: Heterotic G_2 holonomy system 1

Geometric moduli

Vary $\psi \to \psi + \partial_t \psi$, $\varphi \to \varphi + \partial_t \varphi$

- Preserve $\mathrm{d}\psi = 0 = \mathrm{d}\varphi$:
$$\boxed{\mathrm{d}\partial_t \psi = 0 = \mathrm{d}\partial_t \varphi.}$$
- Trivial deformations
$$\boxed{\partial_{\mathrm{triv}} \psi = \mathcal{L}_V \psi = d(i_V \psi) + i_V(d\psi) = d(i_V \psi)}$$
$$\boxed{\partial_{\mathrm{triv}} \varphi = \mathcal{L}_V \varphi = d(i_V \varphi) + i_V(d\varphi) = d(i_V \varphi)}$$
- Moduli space $\sim$ closed but not exact 3-forms/4-forms.
$$\boxed{\mathcal{T}\mathcal{M}_Y \cong H^3_{\mathrm{d}}(Y)}$$

Simple, but less useful when analysing bundle moduli.
Need to find analogue of Dolbeault cohomology.

Manifolds with G_2 structure

Fernandez–Gray:82, Chiossi–Salamon:02

Decomposition of forms

$\Lambda^k(Y)$ decomposes into $\Lambda^k_p(Y)$, p denotes G_2 irrep. Find these using φ:

Example: $\Lambda^1 = \Lambda^1_7 = T^*Y \cong TY$
$\implies$ *any* $\beta \in \Lambda^2$ decomposes as $\beta = \alpha \lrcorner \varphi + \gamma$, where $\alpha \in \Lambda^1$ and $\gamma \lrcorner \varphi = 0$

$$\Lambda^0 = \Lambda^0_1 \ ,$$
$$\Lambda^1 = \Lambda^1_7 = T^*Y \cong TY \ ,$$
$$\Lambda^2 = \Lambda^2_7 \oplus \Lambda^2_{14} \ ,$$
$$\Lambda^3 = \Lambda^3_1 \oplus \Lambda^3_7 \oplus \Lambda^3_{27} \ .$$

Canonical G_2 cohomology

Decomposition of de Rham cohomology

Reyes-Carrion:93,Fernandez–Ugarte:98

Analogue of Dolbeault operator on a complex manifold: project d onto G_2 irreps.

- The differential operator $\check{\mathrm{d}}$ is defined by

$$\check{\mathrm{d}}_0 = \mathrm{d}\ , \quad \check{\mathrm{d}}_1 = \pi_7 \circ \mathrm{d}\ , \quad \check{\mathrm{d}}_2 = \pi_1 \circ \mathrm{d}\ .$$

- $\tau_2 = 0 \iff \check{\mathrm{d}}^2 = 0$, so can construct differential, elliptic complex

$$0 \to \Lambda^0(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^1(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^2_7(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^3_1(Y) \to 0$$

- $H^*_{\check{\mathrm{d}}}(Y)$ is "canonical G_2-cohomology of Y".

This generalizes to TY-valued forms: Elliptic complex

$$0 \to \Lambda^0(TY) \xrightarrow{\check{\mathrm{d}}_\theta} \Lambda^1(TY) \xrightarrow{\check{\mathrm{d}}_\theta} \Lambda^2_7(TY) \xrightarrow{\check{\mathrm{d}}_\theta} \Lambda^3_1(TY) \to 0$$

with finite-dim cohomology groups $H^p_{\check{\mathrm{d}}_\theta}(Y, TY)$, if $R(\theta) \wedge \psi = 0$

Infinitesimal Moduli: Heterotic G_2 holonomy system 2

Geometric moduli as "G_2 Dolbeault cohomology"

$$\partial_t\psi = \frac{1}{3!}\, M_t^a \wedge \psi_{bcda}\,\mathrm{d}x^{bcd} \;, \qquad M_t^a = M_{t\,b}{}^a\,\mathrm{d}x^b \in \Lambda^1(Y, TY)$$
$$\partial_t\varphi = -\frac{1}{2}\, M_t{}^a \wedge \varphi_{bca}\,\mathrm{d}x^{bc}$$

- Diffeomorphisms: $\boxed{\mathcal{L}_V\psi = -\frac{1}{3!}\,(\mathrm{d}_\theta V^a)\wedge\psi_{bcda}\,\mathrm{d}x^{bcd}}$

  where d_θ is a connection for TY-valued forms.

- Preserve $\mathrm{d}\psi = 0 = \mathrm{d}\varphi$: constraints

$$\mathrm{d}_\theta\Delta_t^a \wedge \psi_{bcda}\,\mathrm{d}x^{bcd} = 0 \;,$$
$$\mathrm{d}_\theta\Delta_t^a \wedge \varphi_{bca}\,\mathrm{d}x^{bc} = 0 \;.$$

  where $\Delta_{t\,b}{}^a = M_{t\,b}{}^a - \frac{1}{7}\,(\mathrm{tr} M_t)$

- Compact G_2 manifold: $\boxed{\mathcal{TM}_Y \cong H^3_{\mathrm{d}}(Y) \subset H^1_{\check{\mathrm{d}}_\theta}(Y, TY)}$

Infinitesimal Moduli: Heterotic G_2 holonomy system

G_2 "Atiyah" class stabilization

Instanton condition $F \wedge \psi = 0$: couples bundle and geometric moduli

$$\boxed{\check{\mathrm{d}}_A(\partial_t A) = -\check{\mathcal{F}}(\Delta_t)\ .}$$

$\Delta_t \in \Lambda^1(Y, TY)$, $\check{\mathrm{d}}_A$, $\check{\mathcal{F}}$: project to G_2 **7** irrep

- "Atiyah" map

$$\begin{array}{cccc} \mathcal{F}: & \Lambda^p(Y, TY) & \longrightarrow & \Lambda^{p+1}(Y, \mathrm{End}(V)) \\ & \Delta & \mapsto & \mathcal{F}(\Delta) = -F_{ab}\, \mathrm{d}x^b \wedge \Delta^a\ . \end{array}$$

Bianchi identity $\mathrm{d}_A F = 0 \implies \check{\mathcal{F}}$ is a map in cohomology

Corrected moduli space for bundle and geometric moduli:

$$\boxed{\mathcal{TM}_{(Y,V)} \subset H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})}$$

Infinitesimal Moduli: Heterotic G_2 holonomy system

Infinitesimal moduli space for bundle and geometry:

$$\boxed{\mathcal{TM}_{(Y,V)} \subset H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})}$$

Remark 1: B-field deformations

- Infinitesimal moduli of G_2-holonomy metrics only spans part of $H^1_{\check{\mathrm{d}}_\theta}(Y, TY))$:
  $H^1_{\check{\mathrm{d}}_\theta}(Y, TY)) \cong \check{\mathcal{H}}^1(Y, TY) = \boxed{\check{\mathcal{S}}^1(Y, TY)} \oplus \check{\mathcal{A}}^1(Y, TY)$
- $\check{\mathcal{A}}^1(Y, TY)$ is spanned by $\partial_t B$
- All $\partial_t B$ are in the kernel of $\check{\mathcal{F}}$.
- Thus easily incorporate B-field deformations in the infinitesimal moduli space.

Infinitesimal Moduli: Heterotic G_2 holonomy system

Infinitesimal moduli space for bundle, geometry and B-field:

$$\boxed{\mathcal{TM}_{(Y,V,B)} \cong H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})}$$

Remark 2: Extension bundle

- Use the G_2 Atiyah map $\check{\mathcal{F}}$ to define a new bundle

$$0 \longrightarrow \mathrm{End}(V) \longrightarrow E \longrightarrow TY \longrightarrow 0 \ ,$$

- E has connection $\mathcal{D}_E$:

$$\mathcal{D}_E = \begin{pmatrix} \check{\mathrm{d}}_A & \check{\mathcal{F}} \\ 0 & \check{\mathrm{d}}_\theta \end{pmatrix} \ .$$

- $\mathcal{D}_E^2 = 0 \iff \check{\mathcal{F}}(\check{\mathrm{d}}_\theta(\Delta)) + \check{\mathrm{d}}_A(\check{\mathcal{F}}(\Delta)) = 0$ (*cf* Atiyah algebroid for $\mathcal{N} = 1$).
- $H^1_{\mathcal{D}_E}(Y, E)$ is the moduli space:

$$0 \to H^1_{\check{\mathrm{d}}_A}(Y, \mathrm{End}(V)) \to H^1_{\mathcal{D}_E}(E) \to H^1_{\check{\mathrm{d}}_\theta}(Y, TY)) \xrightarrow{\check{\mathcal{F}}} H^2_{\check{\mathrm{d}}_A}(Y, \mathrm{End}(V)) \to \dots .$$

Infinitesimal Moduli: Heterotic integrable G_2 system

Geometric moduli for integrable G_2 structure

- Diffeomorphisms: $$\boxed{\mathcal{L}_V \psi = -\frac{1}{3!}\left(\mathrm{d}_\theta V^a\right) \wedge \psi_{bcda}\, \mathrm{d}x^{bcd}}$$

  where d_θ is a connection for TY-valued forms.
- Preserve $\tau_2 = 0$: $$\boxed{(\check{\mathrm{d}}_\theta \Delta_t^a) \wedge \psi_{bcda}\, \mathrm{d}x^{bcd} = 0}$$
- $\partial_t \varphi \implies$ Variational constraints on torsion

$\check{\mathcal{F}}$, $\check{\mathcal{R}}$ maps

SUSY + BI $\implies$ EOM if θ is an instanton connection: $R(\theta) \wedge \psi = 0$.

- $\mathcal{R}$ map: completely analogous to $\mathcal{F}$.
- Extra moduli for connection variations.
  Related to field redefinitions as for $\mathcal{N} = 1$ system?
- $\check{\mathcal{F}}$, $\check{\mathcal{R}}$ in fact map **all** geometric moduli to $\check{\mathrm{d}}_A$,$\check{\mathrm{d}}_\theta$-closed forms

Infinitesimal Moduli: Heterotic integrable G_2 system

Corrected moduli space for bundle, instanton and geometric moduli:

$$\boxed{H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus H^1_{\check{\mathrm{d}}_\theta}(\mathrm{End}(TY)) \oplus \ker(\check{\mathcal{F}} + \check{\mathcal{R}})}$$

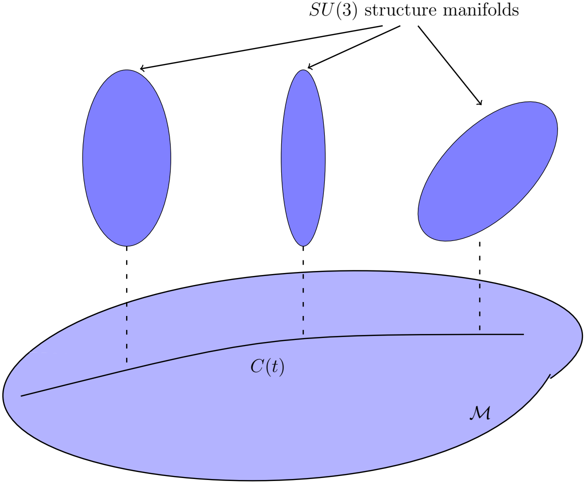
Last equation to bring in: Bianchi identity $\mathrm{d}H = \frac{\alpha'}{4}(\mathrm{tr}R \wedge R - \mathrm{tr}F \wedge F)$

- Recall from $\mathcal{N} = 1$: *Anderson,et.al:14, de la Ossa, Svanes:14*
  Vary BI together with $H = \mathrm{d}^c\omega \rightsquigarrow$ map $\mathcal{H} : \Lambda^p(X, E) \longrightarrow \Lambda^{p+1}(X, T^*X)$
  - $\mathcal{H}$ well-defined in cohomology
  - finite-dim moduli space
- $\mathcal{N} = 1/2$ *de la Ossa, ML, Svanes:17XX*
  Vary BI together with $H = \frac{1}{6}\tau_0\,\varphi - \tau_1 \lrcorner \psi - \tau_3 \rightsquigarrow$ map $\mathcal{H}$
  - $\mathcal{H}$ well-defined in cohomology ✓
  - finite-dim moduli space ✓
  - moduli $\sim$ deformations of Atiyah-like bundle ✓

Compare heterotic generalised geometry. *Clarke, Garcia-Fernandez, Tipler:16*

Flow between different geometries

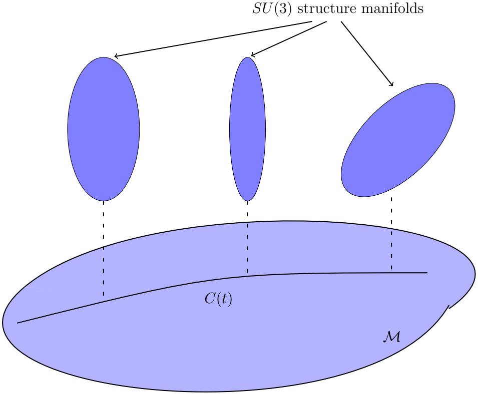
de la Ossa, ML, Svanes:14



t parametrizes a curve in the moduli space of SU(3) structures

Flow between different geometries

de la Ossa, ML, Svanes:14



Two options:

- Fix torsion classes of SU(3) structure.
- Flow between different types of SU(3) structure.

Remark:

- Ignore gauge bundle
- $\mathcal{O}(\alpha'^0)$ analysis of KSE+BI.

Flow between different geometries: Hitchin flow

Hitchin:00

Assume G_2 holonomy: $\tau_a = 0$, $a = 0, .., 4$ and embed $\varphi = \mathrm{d}t \wedge \omega + \mathrm{Re}(\Psi)$

$$\text{SUSY} \implies \text{No flux and constant dilaton}$$
$$\implies \text{Half-flat SU(3) structure}$$

$$\begin{aligned} \mathrm{d}(\omega \wedge \omega) &= 0 \ , \\ \mathrm{dRe}(\Psi) &= 0 \ , \\ \mathrm{dIm}(\Psi) &= \mathrm{Im}(W_0)\, \omega \wedge \omega + \mathrm{Im}(W_2) \wedge \omega \ . \end{aligned}$$

Hitchin flow:

$$\begin{aligned} \partial_t(\omega \wedge \omega) &= 2\mathrm{dIm}(\Psi) \\ \partial_t \mathrm{Re}(\Psi) &= \mathrm{d}\omega \ . \end{aligned}$$

The presence of flux/G_2 torsion allows to find generalisations of Hitchin flow.

Flow between different geometries: Example

de la Ossa, ML, Svanes:14

Calabi–Yau with flux

- Assume X has $W_i = 0$ for $t = 0$.
- Embed $\varphi = N \wedge \omega + \mathrm{Re}(\Psi)$
- 7D flux (determined by SUSY): $H = c_1 \mathrm{d}t \wedge \omega + c_2 \mathrm{Re}(\Psi) + c_3 \mathrm{Im}(\Psi) + J(\gamma)$.

Torsion classes preserved by flow $\iff$ γ $SU(3)$-harmonic.

Non-harmonic γ: flow from CY to *non-complex* SU(3) structure.
Integrability of non-CY flow: to be studied.

Conclusions and outlook

Conclusions

- 4D heterotic $\mathcal{N} = 1/2$ DW solutions
  - Y Integrable G_2 structure $\supset$ G_2 holonomy
  - X Conformally balanced (non-complex) SU(3) structure
- Infinitesimal moduli of G_2 holonomy manifold Y w. instanton bundle V:

$$H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})$$

- **Infinitesimal moduli captured by $H^1_{\mathcal{D}_E}(Y, E)$ of extension bundle**

$$0 \longrightarrow \mathrm{End}(V) \longrightarrow E \longrightarrow TY \longrightarrow 0 \ ,$$

- Similar structure for moduli of heterotic int. G_2 system (Y, V, H).
- Flow of X along DW direction generalize Hitchin flow.

Conclusions and outlook

Conclusions

Atiyah-like bundle captures deformations of (real) integrable G_2 structure manifolds with instanton bundles.

Outlook

- Relation of $SU(3)$ and G_2 structure moduli spaces for domain wall solutions.
- Metric on moduli space.
- 3D/4D perspective: superpotential.
- Relevance for deformations of M-theory and type II string compactifications.

Thank You