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Continuous and Discrete Gauge Symmetries in F-theory

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Outline (Summary): F-theory Compactification

I. Key ingredients:
   brief overview; non-Abelian gauge symmetries

II. Abelian gauge symmetries
   rational sections and Mordell-Weil group
   Highlight insights into Heterotic duality

III. (Abelian) Discrete gauge symmetries
   multi-sections and Tate-Shafarevich group
   Highlight Heterotic duality and Mirror symmetry

IV. Non-Abelian Discrete gauge symmetries
   hard in F-theory → weak coupling limit Type IIB

Time permitting

Emphasize geometric perspective

Apologies: Upenn-centric

Abelian and discrete symmetries in Heterotic/F-theory

M.C., A.Grassi, D.Klevers, M.Poretschkin and P.Song, ``Origin of Abelian Gauge Symmetries in Heterotic/F-theory Duality,'' arXiv:1511.08208 [hep-th]

M.C., A.Grassi and M.Poretschkin, ``Discrete Symmetries in Heterotic/F-theory Duality and Mirror Symmetry,'' arXiv:1607.03176 [hep-th]

Non-Abelian discrete symmetries in Type IIB string

V. Braun, M.C., R. Donagi and M.Poretschkin, ``Type II String Theory on Calabi-Yau Manifolds with Torsion and Non-Abelian Discrete Gauge Symmetries,'' arXiv:1702.08071 [hep-th]

Type IIB perspective

F-THEORY BASIC INGREDIENTS

F-theory?

F-theory	=	Type IIB String
• coupling g_s part of geometry (12dim)		• back-reacted (10dim) (p,q) 7-branes
• elliptically fibered Calabi-Yau manifold		• regions with large g_s on non-CY space

g_s –string coupling

F-theory?

M-theory (11dim SG)

↑ on S^1

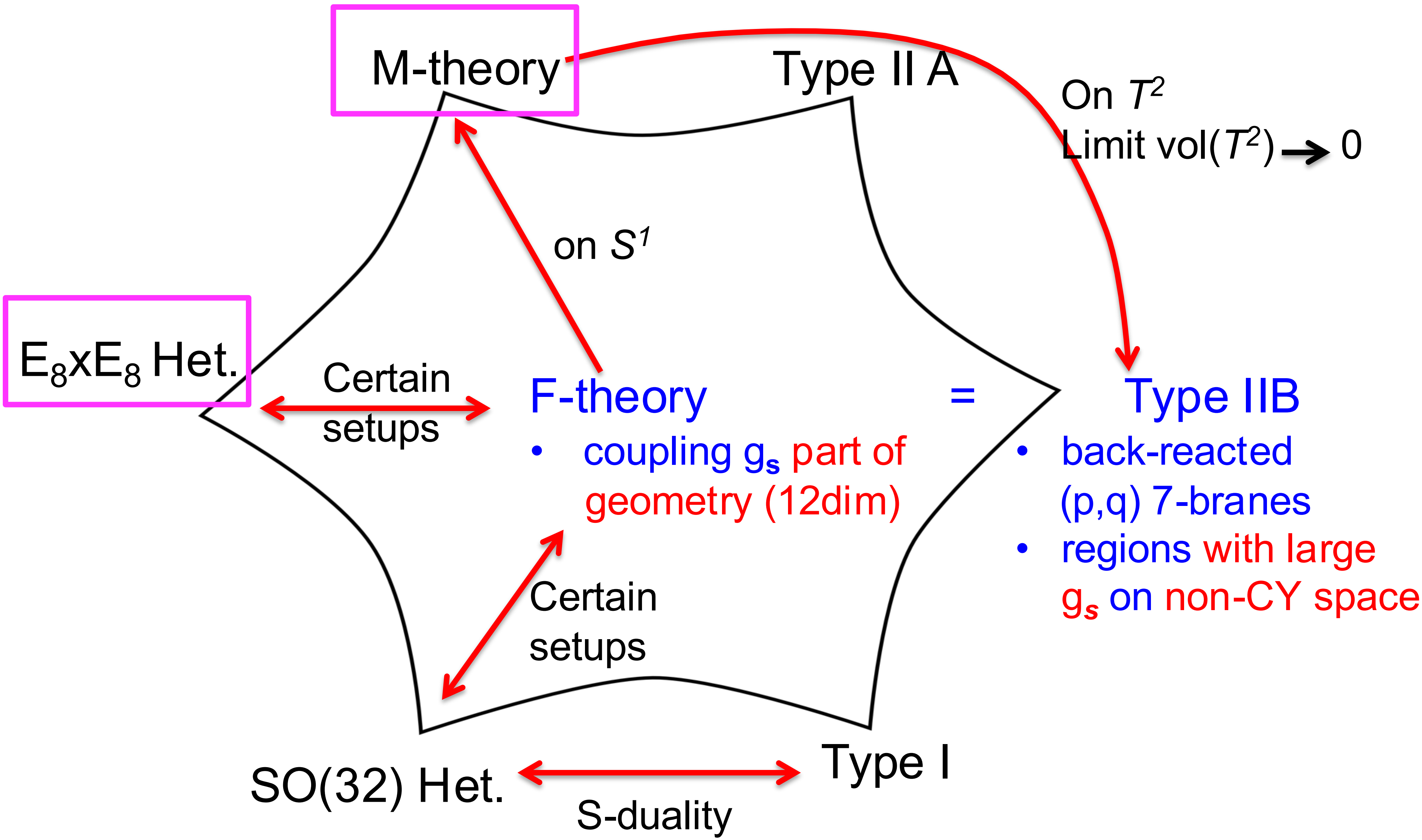
F-theory = Type IIB

F-theory:
- coupling g_s part of geometry (12dim)

Type IIB:
- back-reacted (p,q) 7-branes
- regions with large g_s on non-CY space

g_s –string coupling

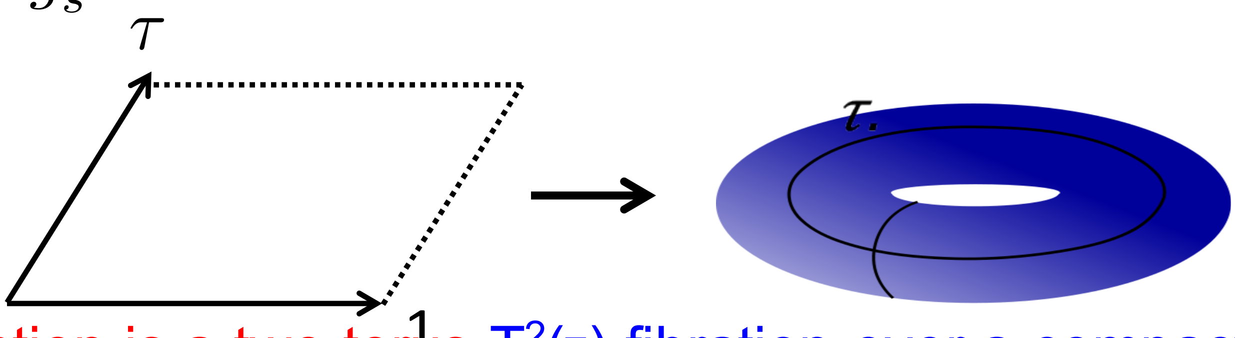
F-theory?


M-theory
Type II A
On T^2
Limit vol(T^2) → 0
on S^1
E_8xE_8 Het.
Certain setups
F-theory
• coupling g_s part of geometry (12dim)
=
Type IIB
• back-reacted (p,q) 7-branes
• regions with large g_s on non-CY space
Certain setups
SO(32) Het.
Type I
S-duality

F-theory Compactification: Basic Ingredients

F-theory geometrizes the (Type IIB) string coupling (axio-dilaton)

$\tau \equiv C_0 + i g_s^{-1}$ as a modular parameter of two-torus $T^2(\tau)$ w/SL(2,Z)



[Vafa]

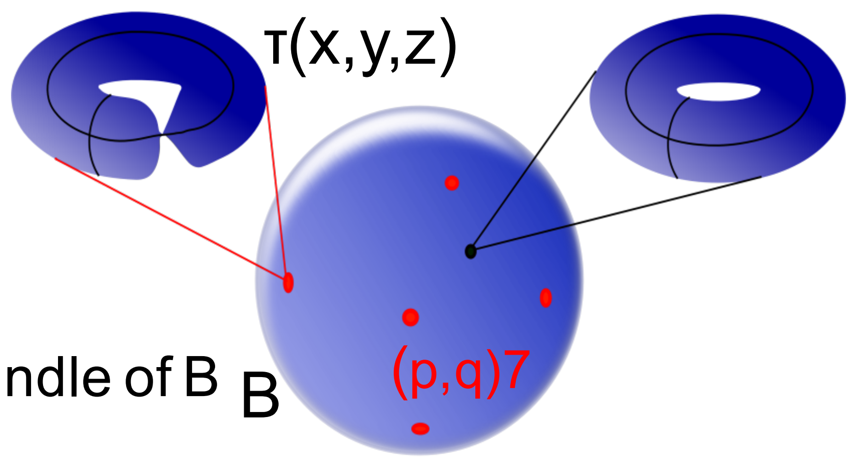
Compactifcation is a two-torus $T^2(\tau)$-fibration over a compact base space B:

Weierstrass normal form:

$$y^2 = x^3 + f x z^4 + g z^6$$

[z:x:y] homog. coords on $\mathbf{P}^2(1,2,3)$

Fibration: f, g, x, y - sections of anti-canonical bundle of B
(holomorphic functions of B)



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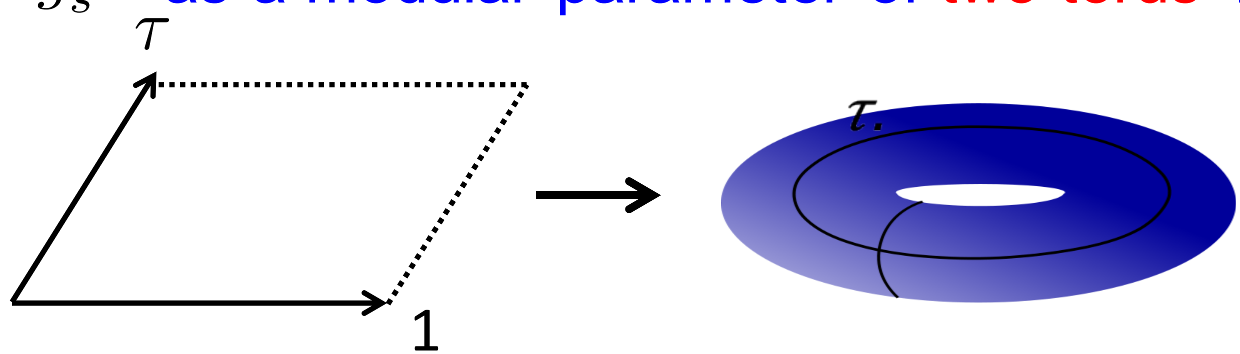

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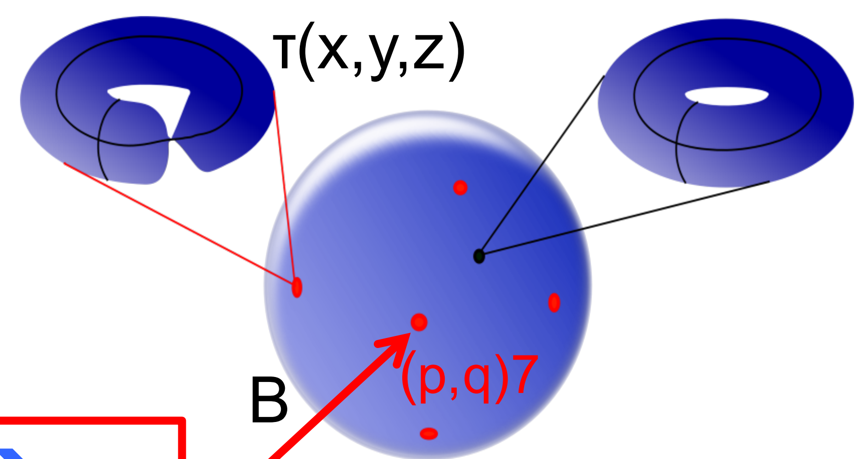


singular $T^2(\tau)$-fibr. → $g_s \to \infty$
location of (p,q) 7-branes

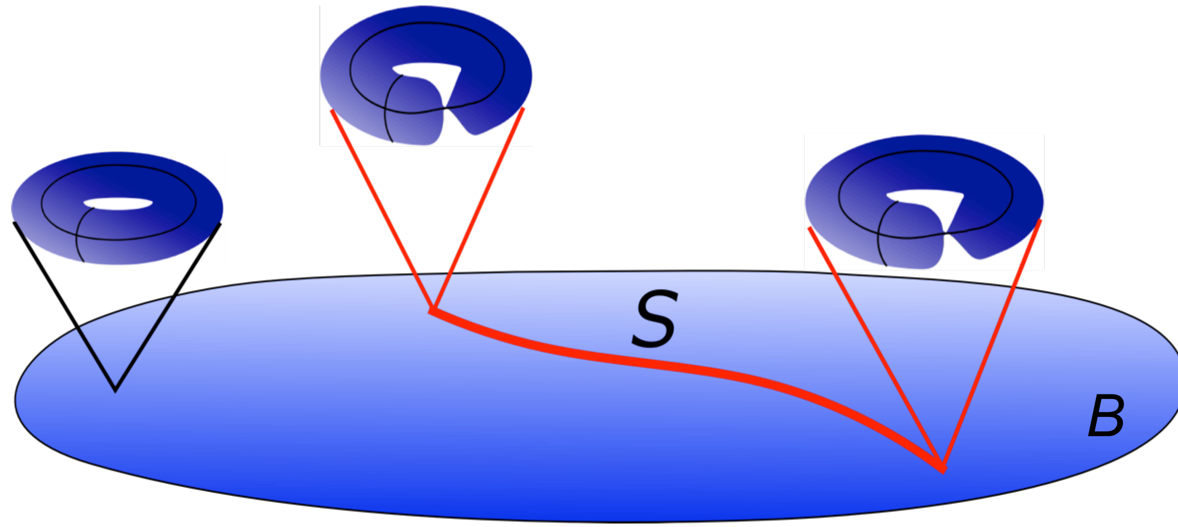
F-theory compactification

[Vafa'96], [Morrison,Vafa'96],...

Singular elliptically fibered Calabi-Yau manifold X

Modular parameter of two-torus
(elliptic curve)

$$\tau \equiv C_0 + i g_s^{-1}$$



Weierstrass normal form for elliptic fibration of *X*

$$y^2 = x^3 + f x z^4 + g z^6$$

[z:x:y] - homogeneous coordinates on $\mathbf{P}^2(1,2,3)$

f, g - sections on (holomorphic functions of) B

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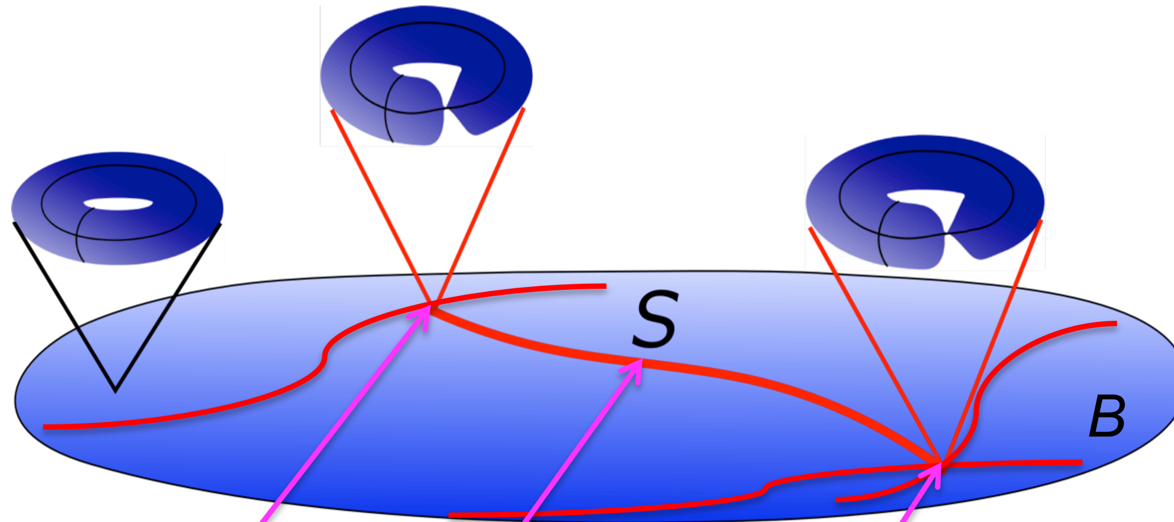

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Matter
(co-dim 2; chirality- G_4-flux)

Yukawa couplings
(co-dim 3)

singular elliptic-fibration, $g_s \rightarrow \infty$
location of (p,q) 7-branes

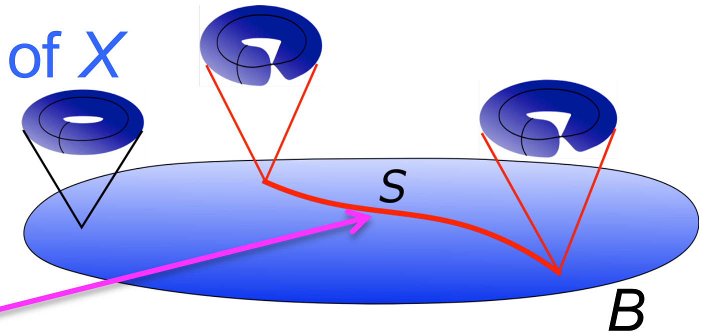
non-Abelian gauge symmetry
(co-dim 1)

Non-Abelian Gauge Symmetry

[Kodaira],[Tate], [Vafa], [Morrison,Vafa],...[Esole,Yau],
[Hayashi,Lawrie,Schäfer-Nameki],[Morrison], ...

- Weierstrass normal form for elliptic fibration of X

$$y^2 = x^3 + fxz^4 + gz^6$$

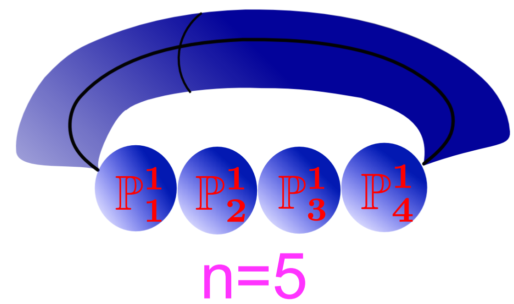


- Severity of singularity along divisor S in B

specified by $[ord_S(f), ord_S(g), ord_S(\Delta)]$

- Resolution: structure of a tree of $\mathbb{P}^1$'s over S

Resolved I_n-singularity ←→ SU(n) Dynkin diagram



Cartan gauge bosons: supported by (1,1) form $\omega_i \leftrightarrow \mathbb{P}^1_i$ on resolved X

(via M-theory Kaluza-Klein reduction of C_3 potential $C_3 \supset A^i \omega_i$)

Deformation: [Grassi, Halverson, Shaneson'14-'15]

II. U(1)-Symmetries in F-Theory

Abelian Gauge Symmetries

Different: (1,1) forms ω_m, supporting U(1) gauge bosons, isolated & associated with I_1-fibers, only

[Morrison,Vafa'96]

(1,1) - form ω_m ⟺ rational section of elliptic fibration

Abelian Gauge Symmetry & Mordell-Weil Group

rational sections of elliptic fibr. ⇔ rational points of elliptic curve

Abelian Gauge Symmetry & Mordell-Weil Group

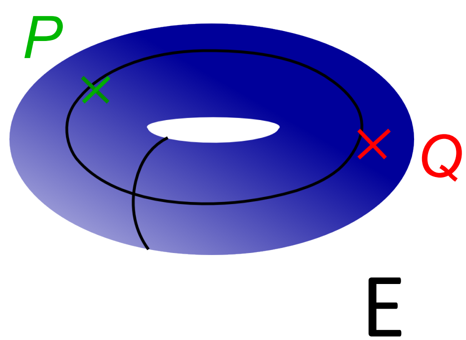
rational sections of elliptic fibr. ⟺ rational points of elliptic curve

Rational point Q on elliptic curve E with zero point P

- is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

- Rational points form group (addition) on E



Abelian Gauge Symmetry & Mordell-Weil Group

rational sections of elliptic fibr. ⟺ rational points of elliptic curve

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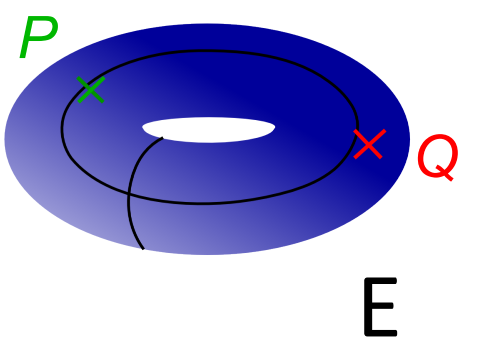
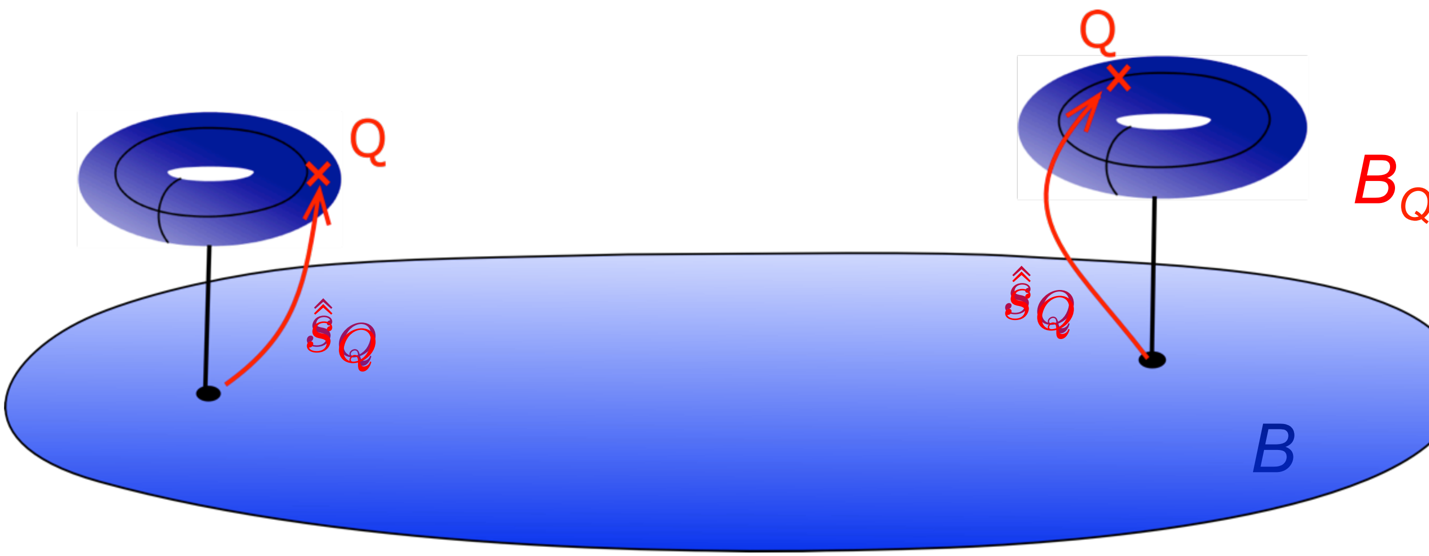


➡ Mordell-Weil group of rational points

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point Q induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

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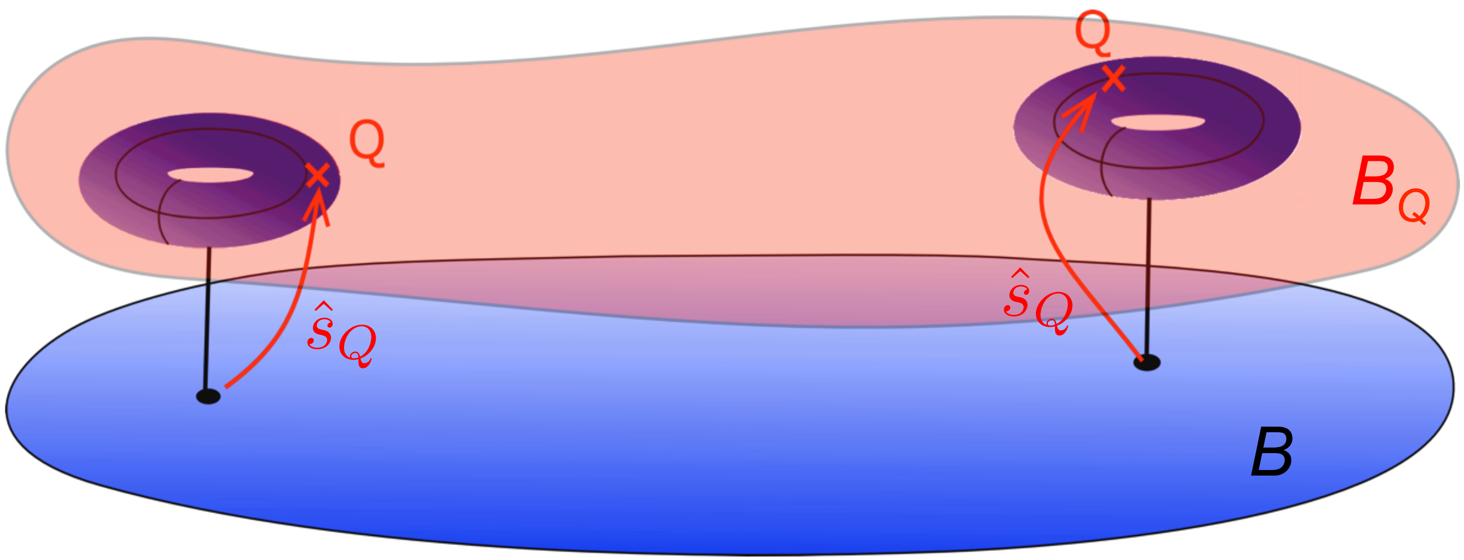

➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

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➡ (1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⬌ rational section

Earlier work: [Grimm,Weigand 1006.0226]...[Grassi,Perduca 1201.0930]
[M.C.,Grimm,Klevers 1210.6034]...

Explicit Examples: (n+1)-rational sections – U(1)^n

[Deligne]

[via line bundle constr. on elliptic curve E- *CY in (blow-up) of* $W\mathbb{P}^m$]

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park 1208.2695]...

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n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park 1208.2695]...

n=2: with *P, Q, R* - specific example: generic CY in dP_2

[Borchmann,Mayerhofer,Palti,Weigand 1303.54054,1307.2902]
[M.C.,Klevers,Piragua 1303.6970,1307.6425]
[M.C.,Grassi,Klevers,Piragua 1306.0236]

generalization to nongeneric cubic in $\mathbb{P}^2[u:v:w]$

[M.C.,Klevers,Piragua,Taylor 1507.05954]

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generalization to nongeneric cubic in $\mathbb{P}^2[u:v:w]$

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n=3: with *P, Q, R, S* - CICY in $\mathrm{Bl}_3\mathbb{P}^3$

n=4: determinantal variety in $\mathbb{P}^4$ [M.C.,Klevers,Piragua,Song 1310.0463]

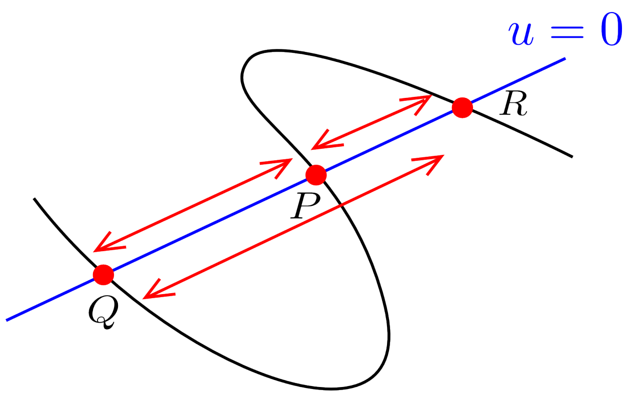
...

higher *n,* not clear…

U(1)xU(1): Further Developments

[M.C., Klevers, Piragua, Taylor 1507.05954]

General U(1)xU(1) construction:



non-generic cubic curve in $\mathbb{P}^2[u:v:w]$:

$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

$f_2(u,v,w)$ degree two polynomial in $\mathbb{P}^2[u:v:w]$

Study of non-Abelian enhancement (unHiggsing) by merging rational points P, Q, R [first symmetric representation of SU(3)]

higher index representations [Klevers, Taylor 1604.01030]

[Morrison, Park 1606.0744]

non-local horizontal divisors (Abelian) turn into local vertical ones (non-Abelian) →

both in geometry (w/ global resolutions) & field theory (Higgsing matter)

U(1)'s in Heterotic/F-theory Duality

[M.C., Grassi, Klevers, Poretschkin, Song 1511.08208]

[Morrison,Vafa '96], [Friedman,Morgan,Witten '97]

Basic Duality (8D):

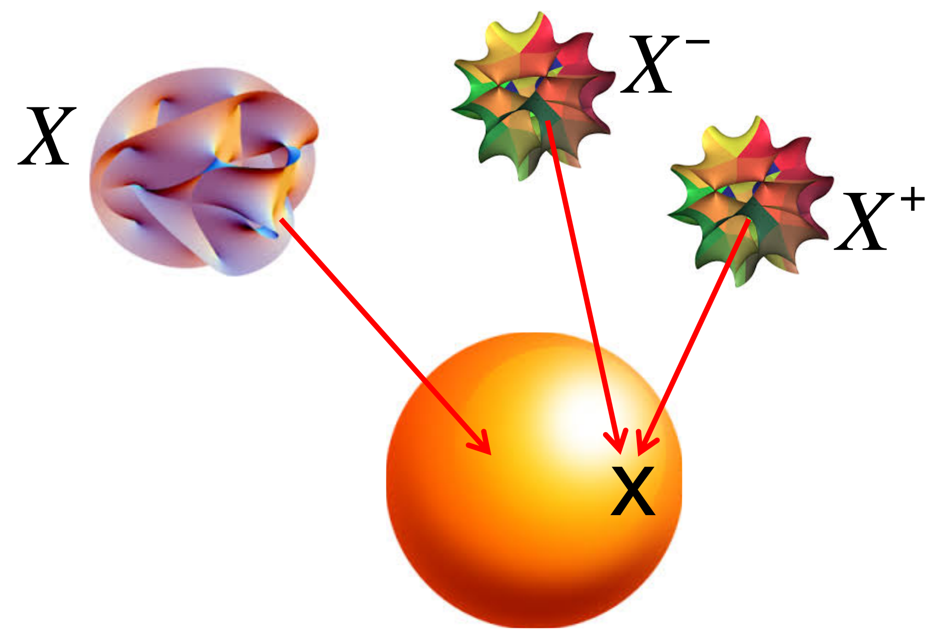
Heterotic E_8 x E_8 String on T^2

⬍ dual to

F-Theory on elliptically fibered K3 surface X

Manifest in stable degeneration limit:

K3 surface X splits into two half-K3 surfaces X^+ and X^-



K3-fibration over P^1 (moduli)

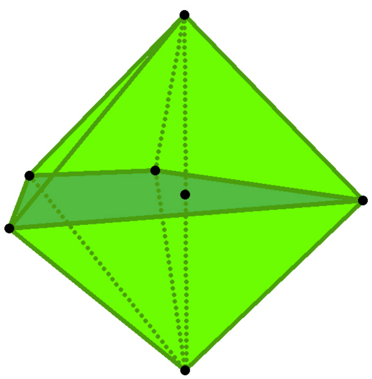
Dictionary:

- X^+ and X^- → background bundles V_1 and V_2
- Heterotic gauge group $G = G_1 \times G_2$ $\quad G_i = [E_8, V_i]$
- The Heterotic geometry T^2: at intersection of X^+ and X^-

U(1)'s in Heterotic/F-theory Duality

Employ toric geometry techniques in 8D/6D to study stable degeneration limit of F-theory models with one U(1)
[elliptic curve: CY hypersurface in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$]

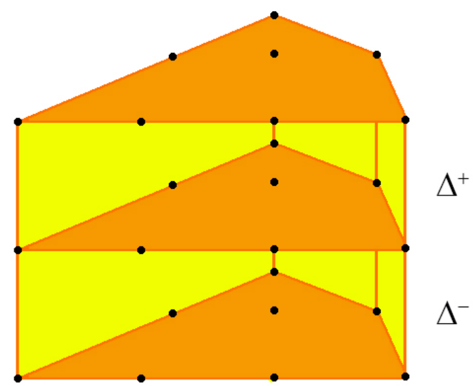
Toric polytope:



specifies the ambient space $P^1 x Bl_1 P^{(1,1,2)}$

Dual polytope:

Newton polytope



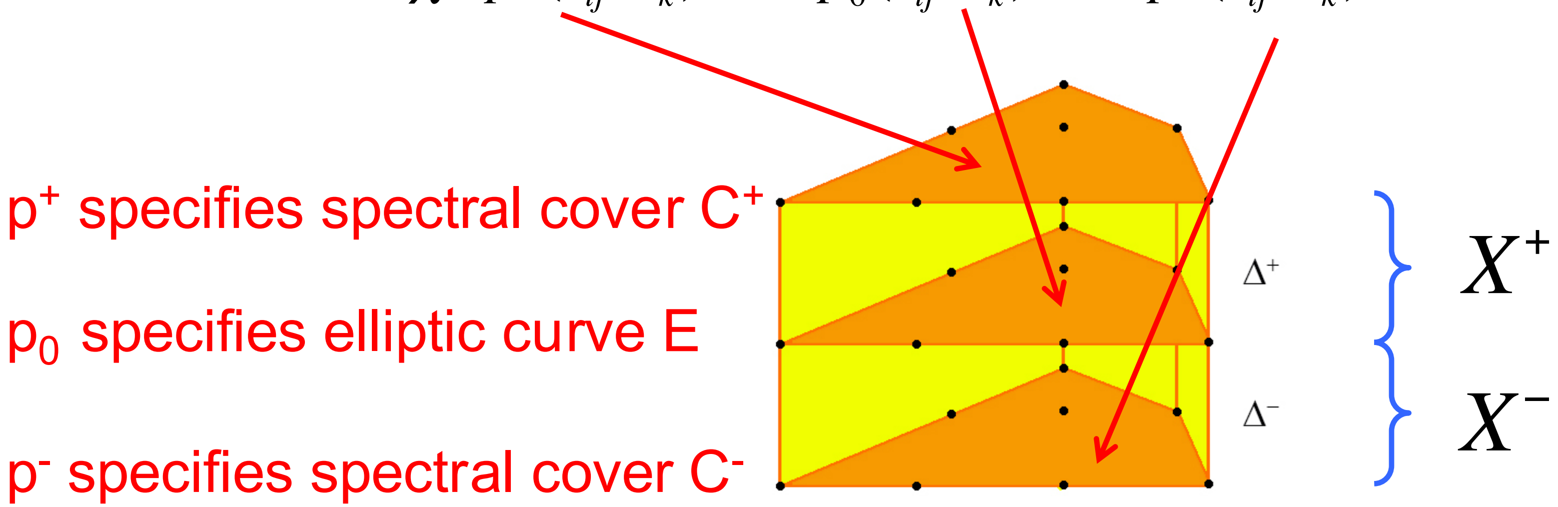
specifies the elements of O($-K_{P^1 \times Bl_1 P^{(1,1,2)}}$) monomials in the ambient space

6D: fiber this construction over another P^1

Decomposing the F-Theory Geometry

[Morrison, Vafa'96], [Berglund, Mayr '98]

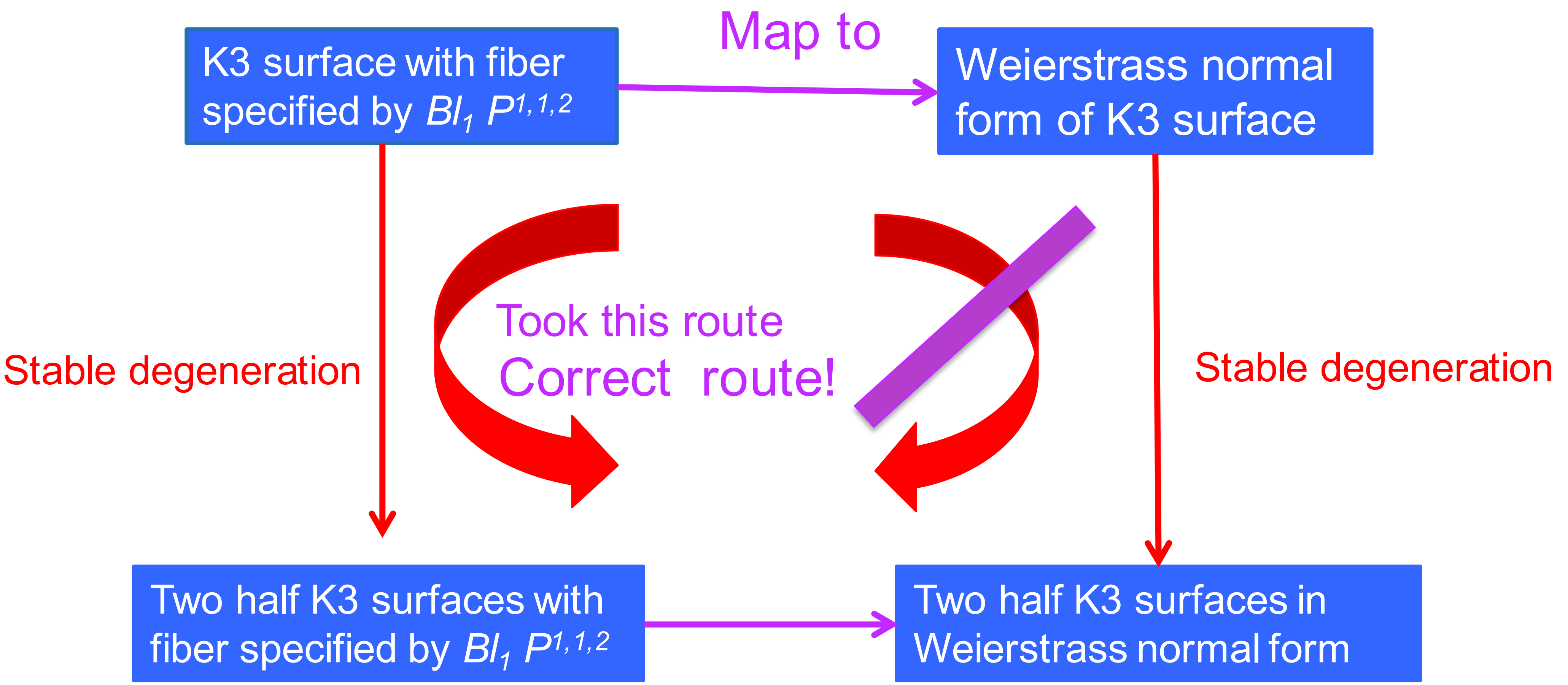
$$\chi : p^{+}(s_{ij}, x_k)U^2 + p_0(s_{ij}, x_k)U^2 + p^{-}(s_{ij}, x_k)V^2$$



- Spectral cover defines a SU(N) vector bundle on E
- Specialize to large gauge groups to keep spectral cover under control

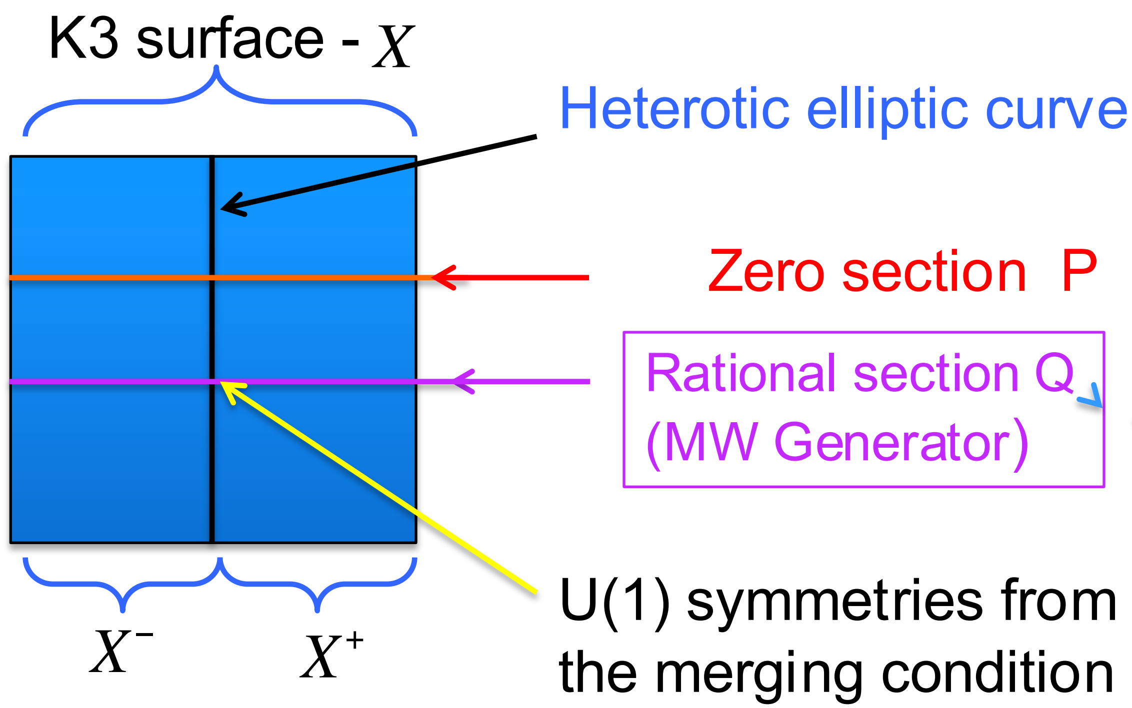
Developed for toric geometry with U(1)

Weierstrass form and stable degeneration with MW



do not commute!

Tracing U(1)s through duality



Case I

Split vector bundle-symmetric:

S(U(N-1) x U(1)) ^{2}

Example N=2: $(E_7 \times U(1))^2$ gauge symmetry

6D: $U(1)^2$-massive (U(1)-background bundle) [Witten]

→ only symmetric comb. U(1)-massless

Vector bundle with torsion:

S(U(N-1) x Z_2)

$E_8 \times E_6 \times U(1)$

6D: U(1)-massless

Case III

SU(N)xSU(M) bundle

6D: U(1)-massless

III. Discrete Symmetries in F-Theory

Abelian Discrete Symmetries in F-theory

Calabi-Yau geometries with genus-one fibrations

These geometries do not admit a section, but a multi-section

Earlier work: [Witten; deBoer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi;…]

Recent extensive efforts'14-'16: [Braun, Morrison; Morrison, Taylor; Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter; Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand; M.C., Donagi, Klevers, Piragua, Poretschkin; Grimm, Pugh, Regalado; M.C., Grassi, Poretschkin;…]

Key features:

Higgsing models w/U(1), charge-n $\langle\Phi\rangle \neq 0$ – conifold transition

Geometries with n-section ⟺ Tate-Shafarevich Group Z_n

Z_2 [Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand'14]

Z_3 [M.C.,Donagi,Klevers,Piragua,Poretschkin 1502.06953]

Abelian & Discrete Gauge Symmetry in F-theory

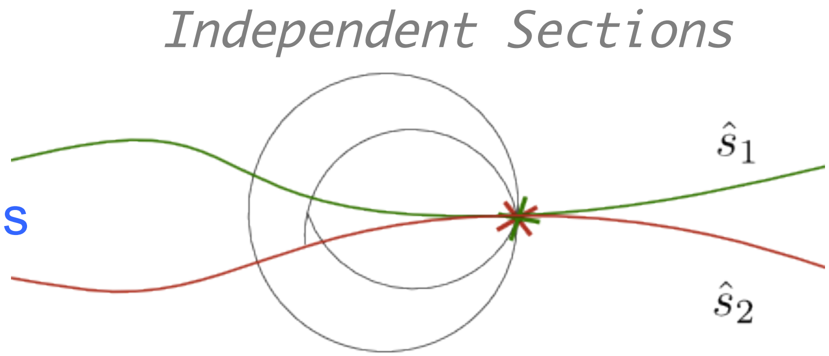
F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example

Singular codim-2 locus

I_2 -fiber

[Morrison,Taylor]
[Anderson,García-Etxebarria, Grimm,Keitel]
[Mayrhofer,Palti,Till,Weigand]



blow-down
(P^1 in the geometry with multiple sections collapses)

deformation [Candelas, de la Ossa]

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

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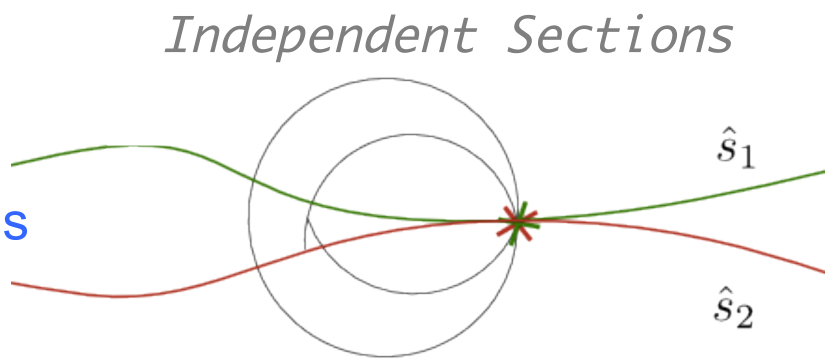
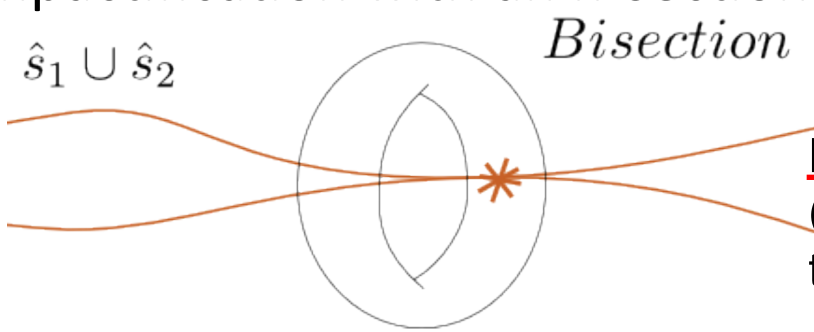


blow-down
(P^1 in the geometry with multiple sections collapses)

appearance of massless field
ϕ with charge 2

F-theory compactification with an n-section (discrete Z_n symmetry)



Deformation
(S^3 glues several sections to a multi-section)

massless field acquires VEV
$\langle\phi\rangle \neq 0$
$U(1) \rightarrow Z_2$

Conifold transition - Effective theory

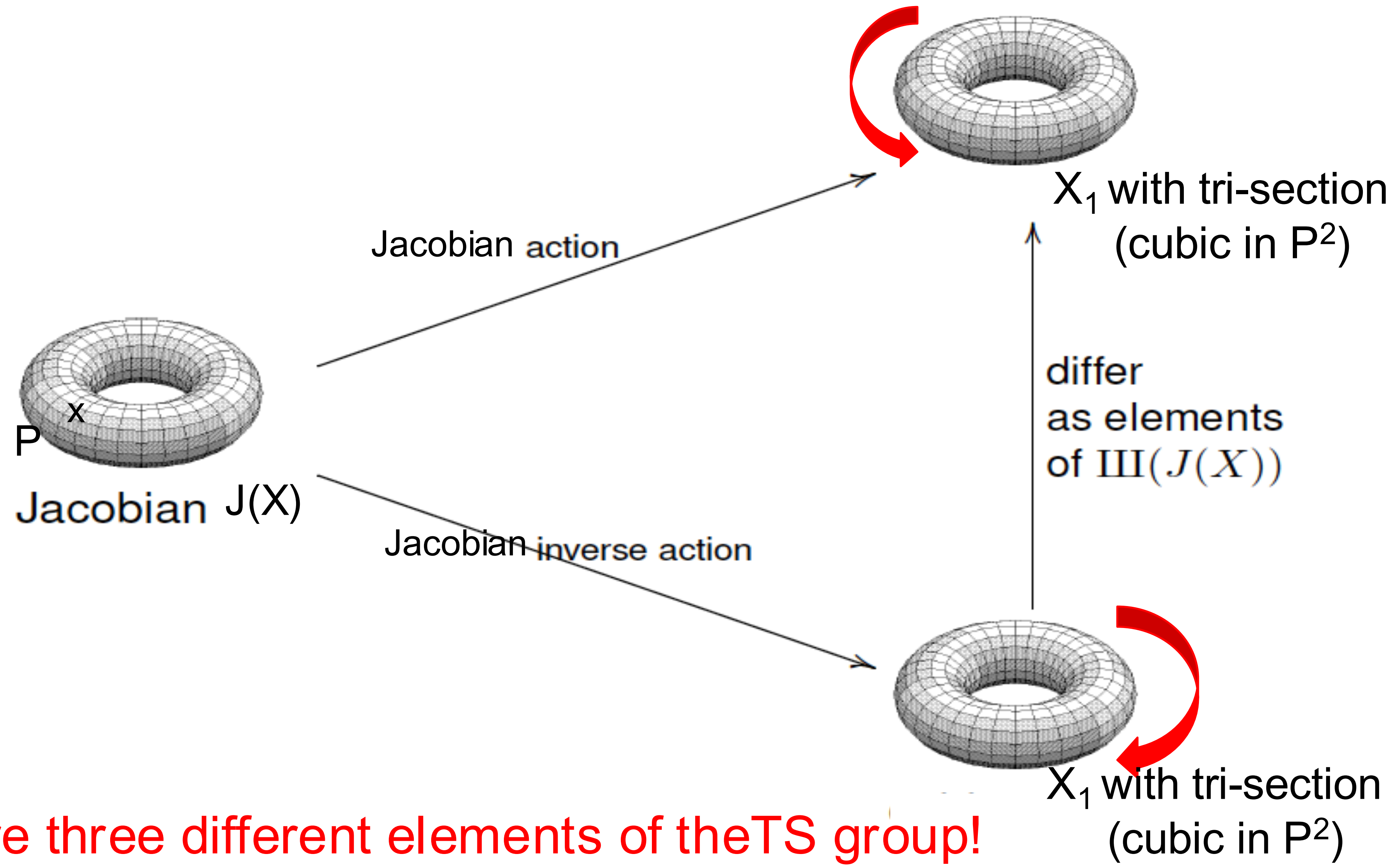
Explicit construction of U(1) geometries with matter charge n → Z_n

n=3: [Klevers, Peña, Piragua, Oehlmann, Reuter 1408.4808]
[M.C., Donagi, Klevers, Piragua, Poretschkin 1502.06953]...

Tate-Shafarevich group and Z_3

[M.C., Donagi, Klevers, Piragua, Poretschkin 1502.06953]

Only two geometries: X_1 w/ trisection and Jacobian $J(X_1)$



There are three different elements of theTS group!

Shown to be in one-to-one correspondence with three M-theory vacua.

Discrete Symmetry in Heterotic/F-theory Duality

[M.C., Grassi, Poretschkin 1607.03176]

Goal: Trace the origin of discrete symmetry D

- Conjecture for $P^2(1,2,3)$ fibration [Berglund, Mayr '98]

  X_2 elliptically fibered, toric K3 with singularities (gauge groups) of type G_1 in X^+ and G_2 in X^-

  ⇕

  its mirror dual Y_2 with singularities (gauge groups) of type H_1 in X^+ and H_2 in X^- with $H_i=[E_8, G_i]$

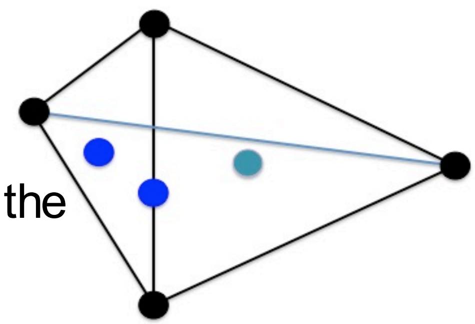
- Employ the conjecture to construct background bundles with structure group G where $D=[E_8, G]$ beyond $P^2(1,2,3)$

- Explore ``symmetric'' stable degeneration with $G_1=G_2$

  → symmetric appearance of discrete symmetry D

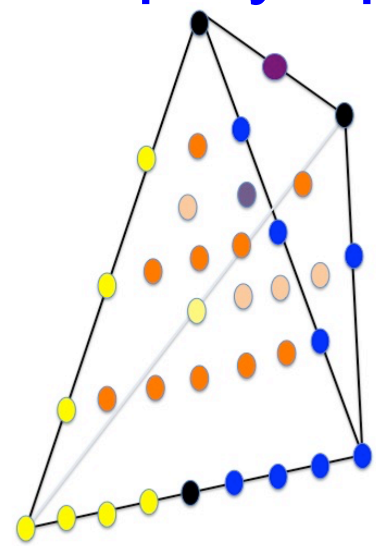
Example with Z_2 symmetry

Polytope:



(monominals of the ambient space)

Dual polytope:



8D: $((E_7 \times SU(2))/\mathbb{Z}_2)^2$ - gauge symmetry
$\mathbb{Z}_2^2$ - vector bundle

${\mathbb{Z}_2}^2$ - gauge symmetry
$((E_7 \times SU(2))/\mathbb{Z}_2)^2$ - vector bundle

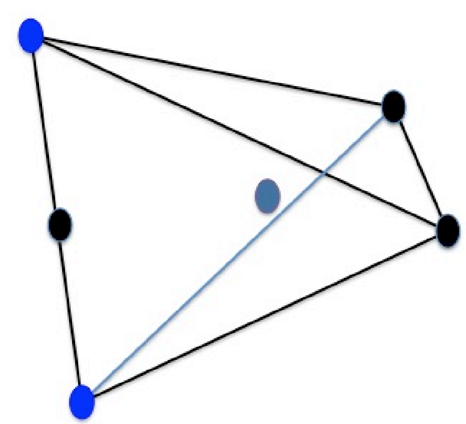
6D: $(E_7 \times SU(2))/\mathbb{Z}_2$ - gauge symmetry

$\mathbb{Z}_2$ - gauge symmetry

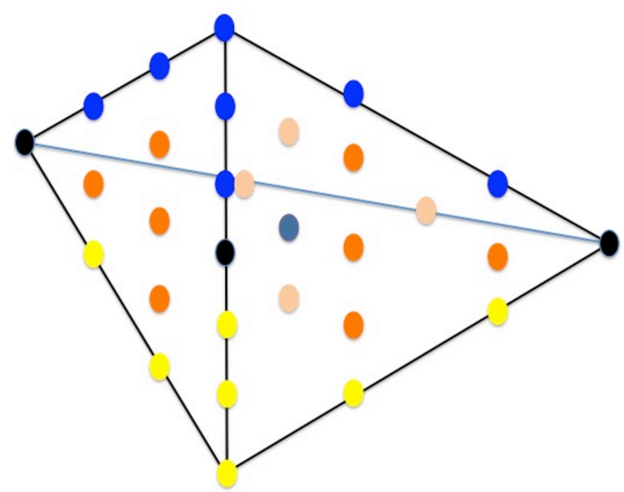
Field theory: Higgsing symmetric U(1) model:
only one (symm. comb.) U(1)-massless
→ only one Z_2 -``massless"

Example with Z_3 symmetry

Polytope:



Dual polytope:



6D: $(E_6 \times E_6 \times SU(3))/\mathbb{Z}_3$ - gauge symmetry

$\mathbb{Z}_3$ - gauge symmetry

These examples demonstrate:

toric CY's with MW torsion of order-n,

⇕ via Heterotic duality related to

mirror dual toric CY's with n-section.

Related: [Klevers, Peña, Piragua, Oehlmann, Reuter '14]

IV. Non-Abelian Discrete Symmetries

Type IIB analysis

[V. Braun, M.C., R. Donagi, M.Poretschkin, arXiv:1702.08071]

no-time

Summary and Outlook

- Key ingredients of F-theory compactification

  Geometric perspective - discrete data

  gauge symmetry, matter, Yukawa couplings

- Recent developments

  Abelian & Discrete symmetries (related to MW & TS group)

  Highlight insights into Heterotic duality.

  → non-Abelian discrete symmetries

- Particle physics models

  SU(5) GUT's & three family Standard Model & with R-parity

  (tip of the iceberg)

→ Future: non-Abelian discrete symmetries in F-theory...

Thank you

Thank you

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