String compactifications on string-size tori from double field theory

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In collaboration with

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Motivation

Bosonic closed string

 ℓ_s

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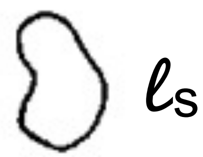
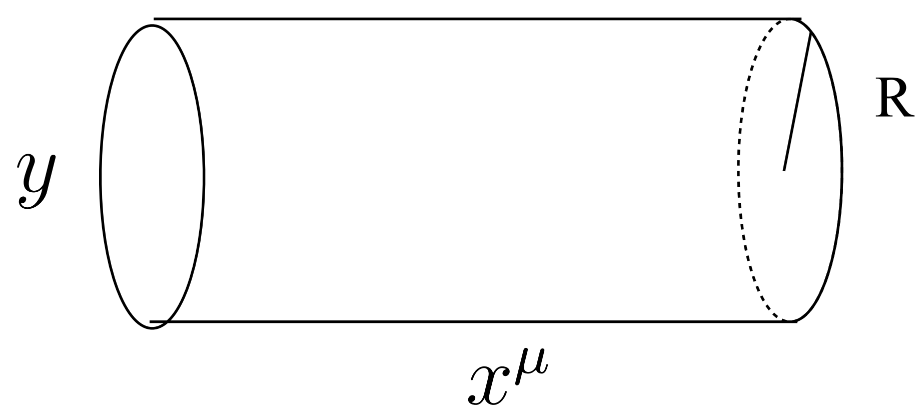
Massless states:

g, B

Motivation

Bosonic closed string





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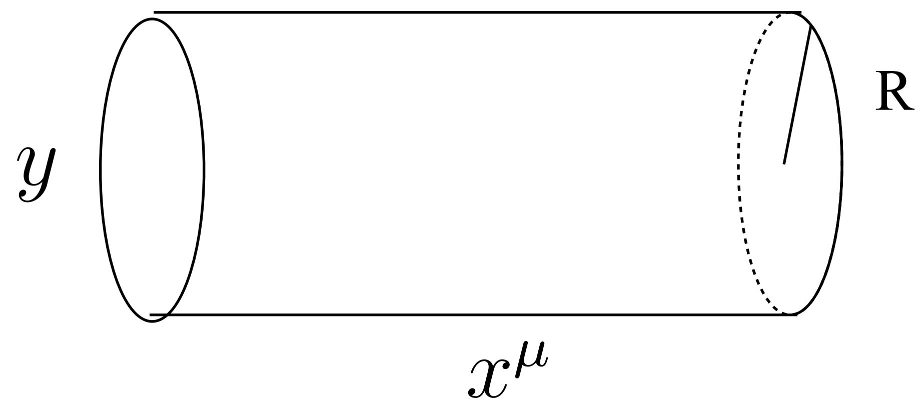

<u>Massless states</u>:

g	$\rightarrow$	$g_{\mu\nu}$	
		$g_{\mu y}$	vector
		g_{yy}	scalar
B	$\rightarrow$	$B_{\mu\nu}$	
		$B_{\mu y}$	vector

Motivation

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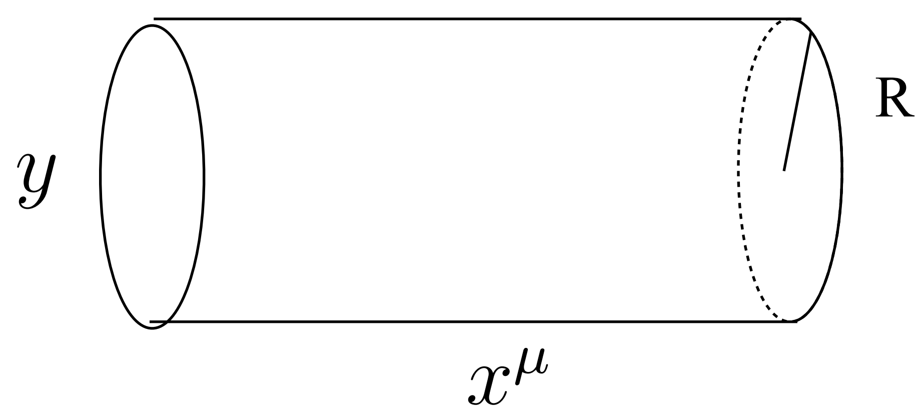

Massless states:

$g \rightarrow g_{\mu\nu}$

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1 scalar $U(1) \times U(1)$

Motivation

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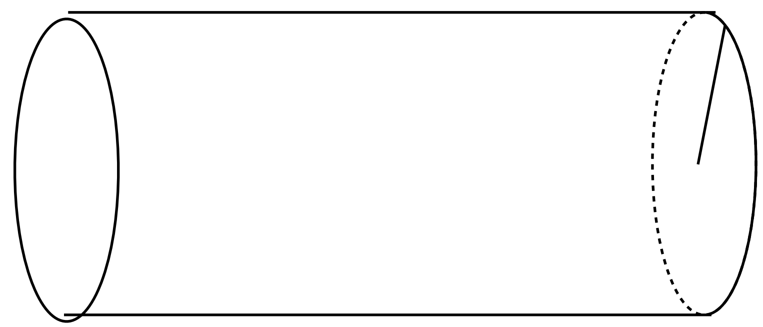

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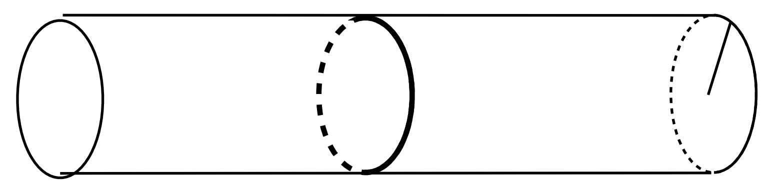

$= \ell_s = \sqrt{\alpha'}$

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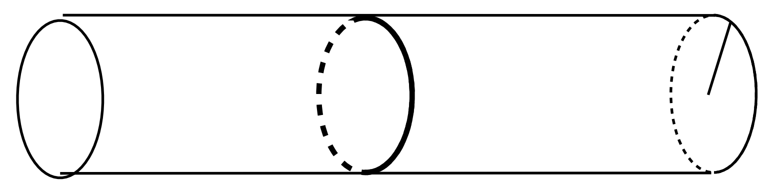
momentum #   winding #

$$M^2 = \frac{2}{\alpha'}(N + \bar{N} - 2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2} \qquad \tilde{R} = \frac{\alpha'}{R}$$

y



$\mathrm{R} = \ell_s = \sqrt{\alpha'}$

x^μ

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momentum # winding #

Bosonic closed string

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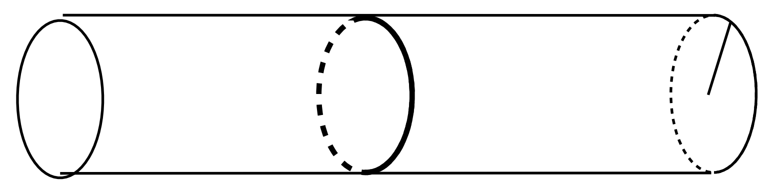

$R = \ell_s = \sqrt{\alpha'}$

Massless states:

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+

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Motivation

Level-matching condition $\bar{N} - N = p\tilde{p}$

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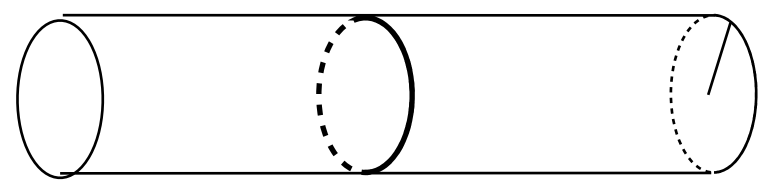

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+

$N = 0 \quad \bar{N} = 1 \quad p = \tilde{p} = \pm 1$

2 vectors 2 scalars

$N = 1 \quad \bar{N} = 0 \quad p = -\tilde{p} = \pm 1$

2 vectors 2 scalars

$N = \bar{N} = 0 \qquad p = \pm 2$ 2 scalars

$\tilde{p} = \pm 2$ 2 scalars

$SU(2) \times SU(2)$ 9 scalars

Motivation

Level-matching condition $\bar{N} - N = p\tilde{p}$

momentum # winding #

$$M^2 = \frac{2}{\alpha'}(N + \bar{N} - 2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2} \qquad \tilde{R} = \frac{\alpha'}{R}$$

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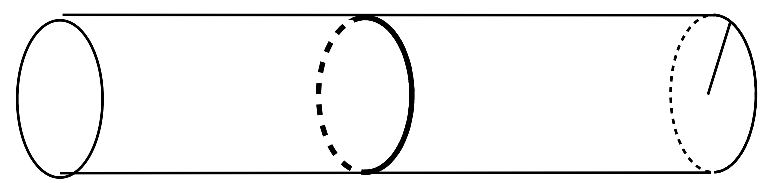

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$R \leftrightarrow \tilde{R}$

$p \leftrightarrow \tilde{p}$

T-duality

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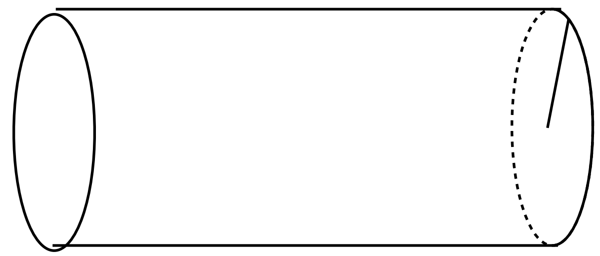
Closed string



effective theory from
reduction of 10d sugra: U(1) x U(1) + 1 massless scalar

valid at $E << \frac{1}{R} << \frac{1}{\sqrt{\alpha'}}$

Closed string

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Can we get an effective description valid at $E << \frac{1}{R} \sim \frac{1}{\sqrt{\alpha'}}$?

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Cf. Taormina's talk

$R >> l_s$	$R \sim l_s$
geometric tools	CFT

Closed string

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$\downarrow$

Double Field Theory

Cf. Taormina's talk

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Closed string

ℓ_s

R $= \ell_s = \sqrt{\alpha'} = 1$

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↓

Double Field Theory

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geometric tools

R~l_s
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here: want geometric tools to describe this

$\boxed{\alpha' = 1}$

Outline

- Double field theory (DFT)
- Compactifications of bosonic string on string size T^d
- Effective action from DFT

Double Field Theory inspired by **generalized complex geometry**

Hitchin 02
Gualtieri 04

- A geometry for geometry + B-field $\quad g + B$

Double Field Theory inspired by **generalized complex geometry**

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Gualtieri 04

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symmetries

diffeomorphisms (vectors)

gauge transf. of B (one-forms)

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diffeomorphisms (vectors) $\delta g = \mathcal{L}_v g \quad \delta B = \mathcal{L}_v B$

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$$V = \begin{pmatrix} v \\ \lambda \end{pmatrix}$$

Fund of O(d,d)

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Fund of O(d,d)

O(d,d) invariant
$$\langle V, V \rangle = V^t \eta V \qquad \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$= 2\iota_v \lambda$$

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- Generated by generalized Lie derivative

V=v+λ
U=u+ξ

$$\delta_V U = L_V U = \mathcal{L}_v u + \mathcal{L}_v \xi - \iota_u d\lambda$$

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Algebra : Courant bracket $[[U,V]] = \frac{1}{2}(L_U V - L_V U)$

Double Field Theory inspired by **generalized complex geometry** for $\mathcal{M}_d$

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- A geometry for geometry + B-field   g + B

symmetries

diffeomorphisms (vectors) $\delta g = \mathcal{L}_v g \quad \delta B = \mathcal{L}_v B$ $V = \begin{pmatrix} v \\ \lambda \end{pmatrix}$ **Fund of O(d,d)**

gauge transf. of B (one-forms) $\delta B = \, \mathrm{d}\lambda$

O(d,d) invariant $\langle V, V \rangle = V^t \, \eta \, V \quad \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

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Algebra : Courant bracket $[[U, V]] = \frac{1}{2}(L_U V - L_V U)$

LvU can be written in an O(d,d) covariant way by defining $\partial_M = (\partial_m, 0) \qquad M = 1, ..., 2d$

$$L_V U^M = \underbrace{V^P \partial_P U^M - U^P \partial_P V^M}_{\mathcal{L}_V U^M} + U^P \partial^M V_P$$

$V \in \Gamma(T\mathcal{M} \oplus T^*\mathcal{M})$ Fund of O(d,d)

$$V \in \Gamma(T\mathcal{M} \oplus T^*\mathcal{M})$$

Fund of O(d,d)

Generalized metric

$$\mathcal{H} = \qquad \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix}$$

$V \in \Gamma(T\mathcal{M} \oplus T^*\mathcal{M})$

Fund of O(d,d)

Generalized metric

$$\mathcal{H} = \begin{pmatrix} \mathbb{1} & -B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \underbrace{\begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix}}_{\substack{\text{B-transform} \\ \text{(adj O(d,d))}}} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

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$$\in \frac{O(d,d,\mathbb{R})}{O(d,\mathbb{R}) \times O(d,\mathbb{R})}$$

O(d)xO(d) structure

$V \in \Gamma(T\mathcal{M} \oplus T^*\mathcal{M})$

Fund of O(d,d)

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$$\mathcal{H} \to O\mathcal{H}O^t \qquad O \in O(d,d)$$

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O(d)xO(d) structure

$$\mathcal{H} \to O\mathcal{H}O^t \qquad O \in O(d,d)$$

Take $O = \eta \qquad \mathcal{H} \to \eta\mathcal{H}\eta^t$

$V \in \Gamma(T\mathcal{M} \oplus T^*\mathcal{M})$

Fund of O(d,d)

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T-duality

$V \in \Gamma(T\mathcal{M} \oplus T^*\mathcal{M})$ **Fund of O(d,d)**

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Frame on $T\mathcal{M} \oplus T^*\mathcal{M}$

frame e_a dual frame e^a

$$E_A = \begin{pmatrix} e_a - \iota_{e_a}B \\ e^a \end{pmatrix} \qquad \mathcal{H} = E^t E$$

Double field theory

Hull & Zwiebach 06

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In GCG, we defined $\partial_M = (\partial_m, 0)$ $M = 1, ..., 2d$

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In DFT, we define $\partial_M = (\partial_m, \partial^m)$

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Was ist das ????

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Nobody really knows, but here is what we know, and why we want to do that

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$d = 1$ In GCG on S^1

$$TS^1 \oplus T^*S^1$$

$$\partial_y + dy$$

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$$\partial_y + dy$$

$$\begin{array}{c} \partial_y \quad + \quad dy \\ \quad\quad\ \wr| \\ \quad\quad\ \partial_{\tilde{y}} \end{array}$$

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$d = 1$ In DFT on S^1 $\qquad < \partial_y, \partial_{\tilde{y}} >= 1 \qquad \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$TS^1 \oplus T\tilde{S}^1$ $\qquad$ O(1,1) pairing

$$\partial_y + dy$$
$$\wr|$$
$$\partial_{\tilde{y}}$$

Double field theory

Hull & Zwiebach 06

In GCG, we defined $\partial_M = (\partial_m, 0)$ $M = 1, ..., 2d$

In DFT, we define $\partial_M = (\partial_m, \partial^m)$

Was ist das ????

Nobody really knows, but here is what we know, and why we want to do that

$d = 1$ In DFT on S^1

$< \partial_y, \partial_{\tilde{y}} >= 1$

$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$TS^1 \oplus T\tilde{S}^1$

O(1,1) pairing

$\partial_y + dy$

$\wr|$

$\partial_{\tilde{y}}$

$\mathcal{M}_D \times S^1$

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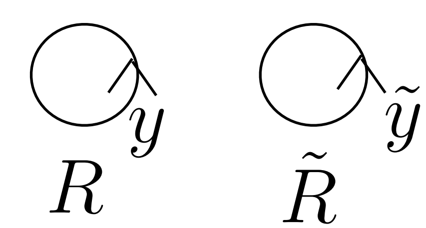

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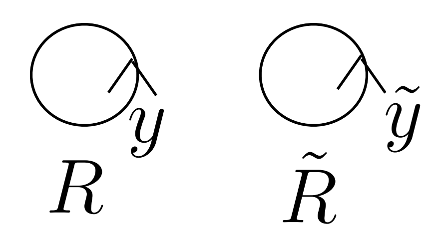

Why, why tilde?

Why, why tilde?

momentum p

winding $\tilde{p}$

Recall mass formula strings on S^1

$$M^2 = \frac{2}{\alpha'}(N + \bar{N} - 2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2}$$

$$R \leftrightarrow \tilde{R} = \tfrac{\alpha'}{R}$$

$$p \leftrightarrow \tilde{p}$$

T-duality

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momentum $p \longrightarrow y$

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However, it requires constraints for closure of algebra of symmetries

$\partial_y \qquad \partial_{\tilde{y}}$

Level matching condition $\quad \underbrace{\bar{N} - N}_{=0 \text{ in usual massless states}} = \underbrace{p\tilde{p}}_{=0} \quad \Rightarrow \quad \partial_y \partial_{\tilde{y}}(\quad) = 0$

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$1, \dots, 2D$

weak constraint

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strong constraint
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so, we're back to square zero...

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Include winding modes here , violating weak constraint

(though satisfying level matching condition)

Bosonic string on T^d

Narain 86

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Massless states:

$g_{\mu m}, B_{\mu m}$ 2d vectors: $U(1)^d \times U(1)^d$

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lots of extra vectors & scalars
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2dx2d

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form a lattice Lorentzian (d,d), even, self-dual

lattice of enhanced gauge group

$$G \times G$$

rank d d

Symmetry enhancement

$$M^2 = 2(N + \bar{N} - 2) + Z^t \underbrace{\mathcal{H}}_{E^T E} Z$$

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$$\mathcal{M}_{D-d} \times T^d$$

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D-dim Vectors

$$\bar{N}_x = 1$$

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D-dim Vectors

$N_y = 1$

$N_y = 0$

$\bar{N}_x = 1$

Symmetry enhancement

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D-dim Vectors

$$N_y = 1 \qquad N_y = 0$$

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LMC $\quad p_L^2 - p_R^2 = 0$

$M^2=0$ $\quad p_L^2 + p_R^2 = 0$

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M²=0 $\quad p_L^2 + p_R^2 = 0$

$$p_L = p_R = 0$$

no mom or winding

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$$0 = 2(N - \bar{N}) + \left(p_L^2 - p_R^2\right)$$

D-dim Vectors

$$N_y = 1 \qquad\qquad N_y = 0$$

$$\bar{N}_x = 1$$

LMC $\quad p_L^2 - p_R^2 = 0$

M²=0 $\quad p_L^2 + p_R^2 = 0$

$$p_L = p_R = 0$$

no mom or winding

$$A^m$$

Symmetry enhancement

$$\mathcal{M}_{D-d} \times T^d$$

$$x \;,\; y$$

$$M^2 = 2(N + \bar{N} - 2) + \left(p_L^2 + p_R^2\right) \qquad p = EZ$$

$$\begin{pmatrix} p_L \\ p_R \end{pmatrix} = \begin{pmatrix} e_a^m \left[p_m + (g_{mn} + B_{mn})\tilde{p}^n\right] \\ e_a^m \left[p_m - (g_{mn} - B_{mn})\tilde{p}^n\right] \end{pmatrix}$$

$$0 = 2(N - \bar{N}) + \left(p_L^2 - p_R^2\right)$$

D-dim Vectors

$$\bar{N}_x = 1$$

	$N_y = 1$	$N_y = 0$
LMC	$p_L^2 - p_R^2 = 0$	$p_L^2 - p_R^2 = 2$
M²=0	$p_L^2 + p_R^2 = 0$	$p_L^2 + p_R^2 = 2$
	$p_L = p_R = 0$	
	no mom or winding	

$$A^m$$

Symmetry enhancement

$$\mathcal{M}_{D-d} \times T^d$$

$$x \ , \ y$$

$$M^2 = 2(N + \bar{N} - 2) + \left(p_L^2 + p_R^2\right) \qquad p = EZ$$

$$0 = 2(N - \bar{N}) + \left(p_L^2 - p_R^2\right)$$

$$\begin{pmatrix} p_L \\ p_R \end{pmatrix} = \begin{pmatrix} e_a^m \left[p_m + (g_{mn} + B_{mn})\tilde{p}^n\right] \\ e_a^m \left[p_m - (g_{mn} - B_{mn})\tilde{p}^n\right] \end{pmatrix}$$

D-dim Vectors

$$N_y = 1 \qquad N_y = 0$$

$\bar{N}_x = 1$

		$N_y = 1$	$N_y = 0$
	LMC	$p_L^2 - p_R^2 = 0$	$p_L^2 - p_R^2 = 2$
	M²=0	$p_L^2 + p_R^2 = 0$	$p_L^2 + p_R^2 = 2$
		$p_L = p_R = 0$	$p_R = 0 \quad p_L^2 = 2 = \lvert\alpha\rvert^2$
		no mom or winding	mom and/or winding

$$A^m$$

Symmetry enhancement

$$\mathcal{M}_{D-d} \times T^d$$

$$x \ , \ y$$

$$M^2 = 2(N + \bar{N} - 2) + \left(p_L^2 + p_R^2\right) \qquad p = EZ$$

$$0 = 2(N - \bar{N}) + \left(p_L^2 - p_R^2\right)$$

$$\begin{pmatrix} p_L \\ p_R \end{pmatrix} = \begin{pmatrix} e_a^m \left[p_m + (g_{mn} + B_{mn})\tilde{p}^n\right] \\ e_a^m \left[p_m - (g_{mn} - B_{mn})\tilde{p}^n\right] \end{pmatrix}$$

D-dim Vectors

		$N_y = 1$	$N_y = 0$
$\bar{N}_x = 1$	LMC	$p_L^2 - p_R^2 = 0$	$p_L^2 - p_R^2 = 2$
	$M^2=0$	$p_L^2 + p_R^2 = 0$	$p_L^2 + p_R^2 = 2$
		$p_L = p_R = 0$	$p_R = 0 \quad p_L^2 = 2 = \lvert\alpha\rvert^2$
		no mom or winding	mom and/or winding
		A^m	A^α

root of a simply-laced Lie algebra rank d

G

Symmetry enhancement

$$\mathcal{M}_{D-d} \times T^d$$

$$x \ , \ y$$

$$M^2 = 2(N + \bar{N} - 2) + \left(p_L^2 + p_R^2\right) \qquad p = EZ$$

$$0 = 2(N - \bar{N}) + \left(p_L^2 - p_R^2\right)$$

$$\begin{pmatrix} p_L \\ p_R \end{pmatrix} = \begin{pmatrix} e_a^m \left[p_m + (g_{mn} + B_{mn})\tilde{p}^n\right] \\ e_a^m \left[p_m - (g_{mn} - B_{mn})\tilde{p}^n\right] \end{pmatrix}$$

D-dim Vectors

$$\bar{N}_x = 1$$

	$N_y = 1$	$N_y = 0$
LMC	$p_L^2 - p_R^2 = 0$	$p_L^2 - p_R^2 = 2$
$M^2=0$	$p_L^2 + p_R^2 = 0$	$p_L^2 + p_R^2 = 2$
	$p_L = p_R = 0$	$p_R = 0 \quad p_L^2 = 2 = \lvert\alpha\rvert^2$
	no mom or winding	mom and/or winding

Cartans of G

A^m

root of a simply-laced
Lie algebra
rank d

A^α

G

Symmetry enhancement

$$\mathcal{M}_{D-d} \times T^d$$

$$x \ , \ y$$

$$M^2 = 2(N + \bar{N} - 2) + \left(p_L^2 + p_R^2\right) \qquad p = EZ$$

$$0 = 2(N - \bar{N}) + \left(p_L^2 - p_R^2\right)$$

$$\begin{pmatrix} p_L \\ p_R \end{pmatrix} = \begin{pmatrix} e_a^m \left[p_m + (g_{mn} + B_{mn})\tilde{p}^n\right] \\ e_a^m \left[p_m - (g_{mn} - B_{mn})\tilde{p}^n\right] \end{pmatrix}$$

D-dim Vectors

$\bar{N}_x = 1$

		$N_y = 1$	$N_y = 0$
	LMC	$p_L^2 - p_R^2 = 0$	$p_L^2 - p_R^2 = 2$
	$M^2=0$	$p_L^2 + p_R^2 = 0$	$p_L^2 + p_R^2 = 2$
		$p_L = p_R = 0$	$p_R = 0 \quad p_L^2 = 2 = \lvert\alpha\rvert^2$
		no mom or winding	mom and/or winding

Cartans of G

root of a simply-laced Lie algebra rank d

A^m $\qquad$ A^α $\qquad$ G

D-dim Vectors

A^m $\qquad$ A^α $\qquad$ G

$N = 1$

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

maximal enhancement

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

maximal enhancement

d=1 ● ● $SU(2) \times SU(2)$

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

maximal enhancement

d=1 ● ● $SU(2) \times SU(2)$ $A_1 \times A_1$

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

maximal enhancement

d=1  $SU(2) \times SU(2)$ $\quad A_1 \times A_1$

d=2  $SU(2)^2 \times SU(2)^2$

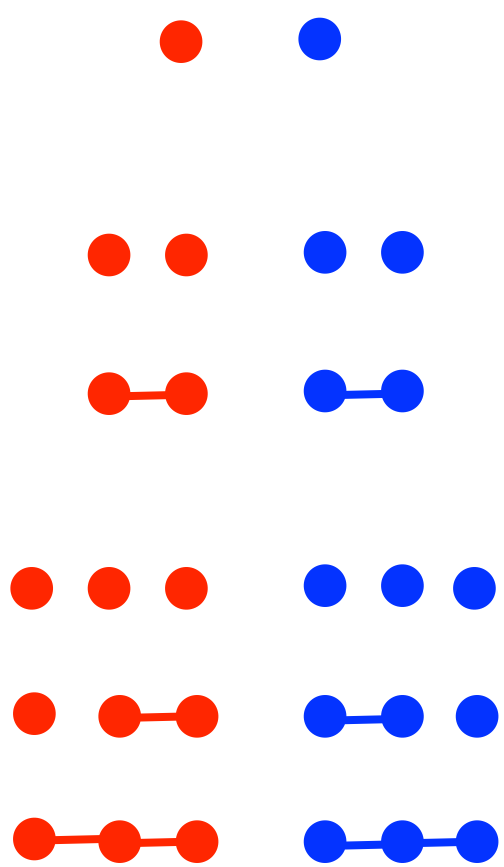
 $SU(3) \times SU(3)$ $\quad A_2 \times A_2$

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

maximal enhancement



d=1

$SU(2) \times SU(2)$ $\quad A_1 \times A_1$

d=2

$SU(2)^2 \times SU(2)^2$

$SU(3) \times SU(3)$ $\quad A_2 \times A_2$

d=3

$SU(2)^3 \times SU(2)^3$

$SU(2) \times SU(3) \times SU(3) \times SU(2)$

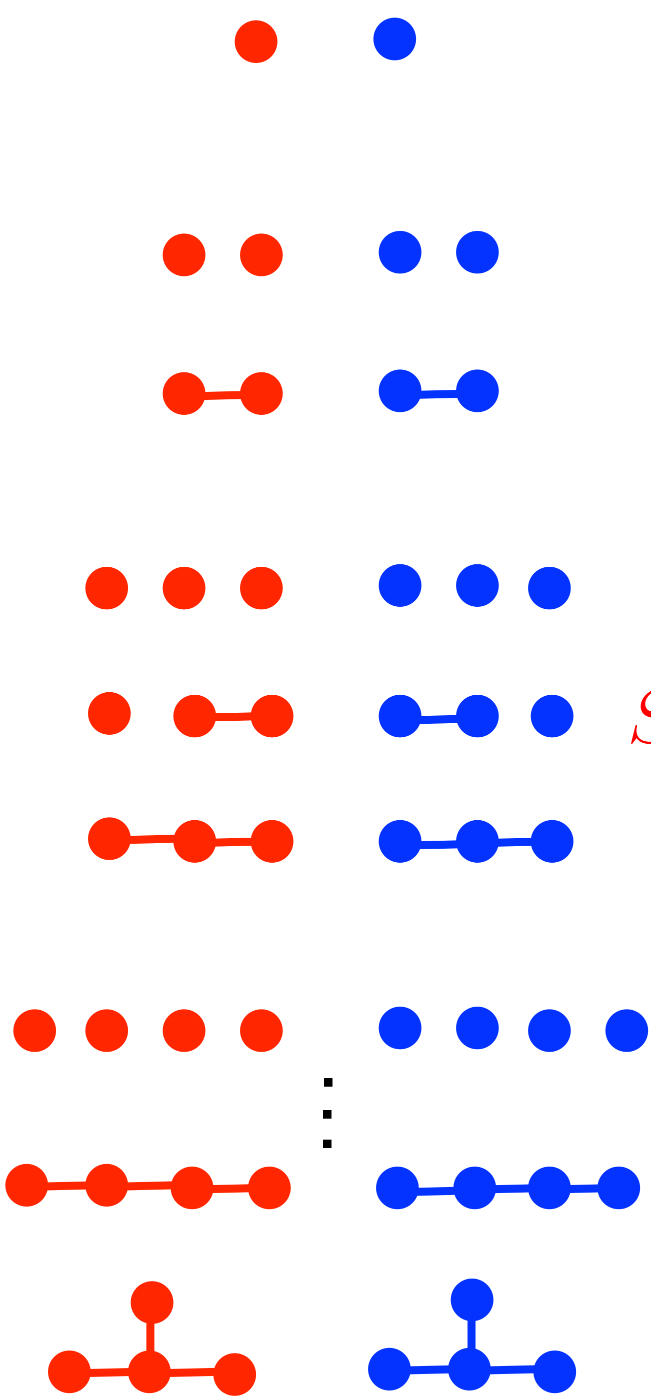
$SU(4) \times SU(4)$ $\quad A_3 \times A_3$

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

maximal enhancement



d=1 $SU(2) \times SU(2)$ $A_1 \times A_1$

d=2 $SU(2)^2 \times SU(2)^2$

$SU(3) \times SU(3)$ $A_2 \times A_2$

d=3 $SU(2)^3 \times SU(2)^3$

$SU(2) \times SU(3) \times SU(3) \times SU(2)$

$SU(4) \times SU(4)$ $A_3 \times A_3$

d=4 $SU(2)^4 \times SU(2)^4$

$\vdots$

$SU(5) \times SU(5)$ $A_4 \times A_4$

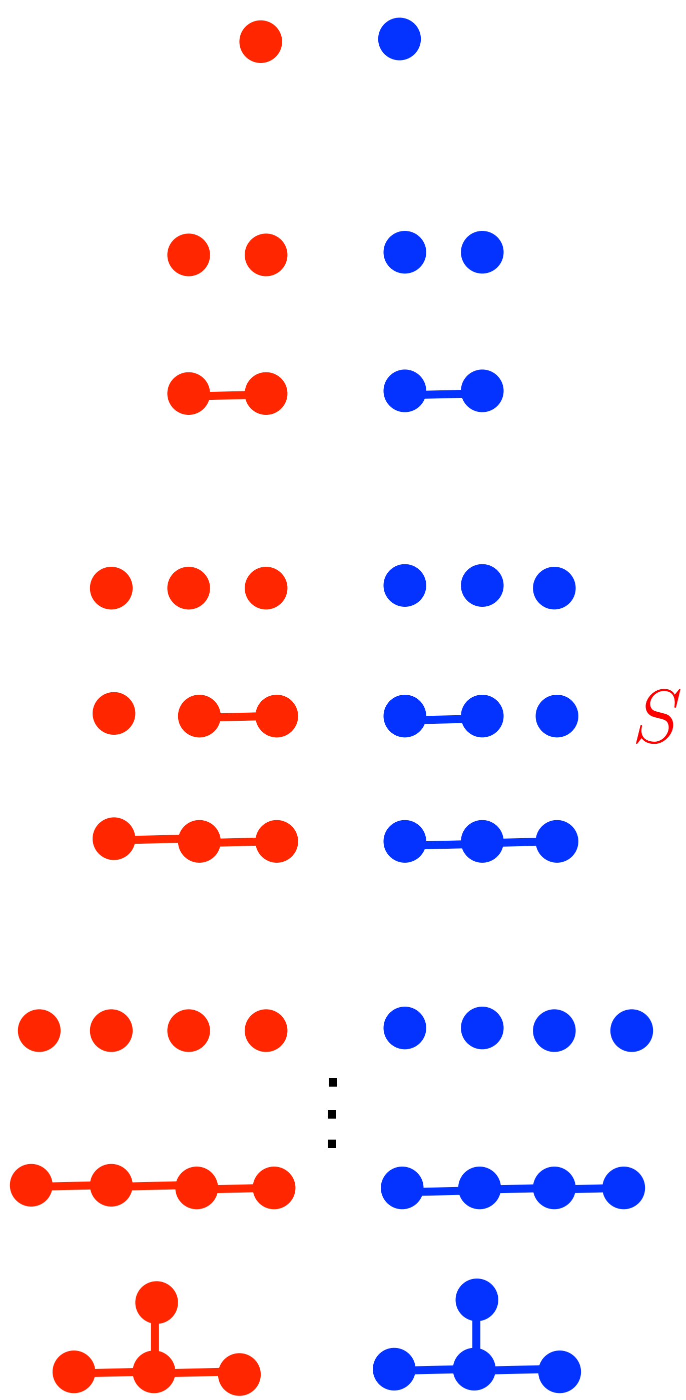
$SO(8) \times SO(8)$ $D_4 \times D_4$

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

maximal enhancement



d=1 $\quad SU(2) \times SU(2) \quad A_1 \times A_1$

d=2 $\quad SU(2)^2 \times SU(2)^2$

$SU(3) \times SU(3) \quad A_2 \times A_2$

d=3 $\quad SU(2)^3 \times SU(2)^3$

$SU(2) \times SU(3) \times SU(3) \times SU(2)$

$SU(4) \times SU(4) \quad A_3 \times A_3$

d=4 $\quad SU(2)^4 \times SU(2)^4$

$\vdots$

$SU(5) \times SU(5) \quad A_4 \times A_4$

$SO(8) \times SO(8) \quad D_4 \times D_4$

ADE series

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $\quad 0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$$p = EZ$$

Vectors $\quad N = 0,\ \bar{N} = 1$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \quad A^m, A^\alpha, A^{-\alpha}$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

2d vectors

2n vectors

G G

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

positive root

negative root

$A^m \quad \bar{A}^m$ $\qquad$ $A^m, A^\alpha, A^{-\alpha} \quad A^m, A^\alpha, A^{-\alpha}$

2d vectors $\qquad$ 2n vectors

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

LMC $\quad 0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$$p = EZ$$

Vectors $\quad N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

G $\quad$ G

Scalars $\quad \bar{N}_y = N_y = 1$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$$p = EZ$$

Vectors $N = 0,\ \bar{N} = 1$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \quad A^m, A^\alpha, A^{-\alpha}$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

2d vectors

2n vectors

G G

M^{mn}

d² scalars

Scalars $\bar{N}_y = N_y = 1$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N+\bar{N}-2) + \left(p_L^2 + p_R^2\right)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $\quad 0 = 2(N-\bar{N}) + \left(p_L^2 - p_R^2\right)$

$$p = EZ$$

Vectors $\quad N=0,\ \bar{N}=1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

positive root

negative root

$A^m \quad \bar{A}^m$

2d vectors

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

2n vectors

G G

M^{mn}

d² scalars

Scalars $\quad \bar{N}_y = N_y = 1$

$$N = \bar{N} = 0$$

$$p_L^2 - p_R^2 = 2$$

$$p_L^2 + p_R^2 = 4$$

$$p_L^2 = p_R^2 = 2$$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$$p = EZ$$

positive root

negative root

A^m $\bar{A}^m$

2d vectors

$A^m, A^{\alpha}, A^{-\alpha}$ $A^m, A^{\alpha}, A^{-\alpha}$

2n vectors

M^{mn}

d² scalars

Vectors $N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

G G

Scalars $\bar{N}_y = N_y = 1$

$$N = \bar{N} = 0$$

$$p_L^2 - p_R^2 = 2$$

$$p_L^2 + p_R^2 = 4$$

$$p_L^2 = p_R^2 = 2$$

$M^{\alpha\beta}$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$$p = EZ$$

Vectors $N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

positive root

negative root

$A^m \quad \bar{A}^m$

2d vectors

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

2n vectors

G G

M^{mn}

d² scalars

$$\underbrace{M^{\alpha\beta} \quad M^{mn} \quad M^{\alpha n} \quad M^{m\beta}}_{M^{ab}}$$

$a = 1, \ldots, n$

n² scalars

Scalars $\bar{N}_y = N_y = 1$

$N = \bar{N} = 0$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2 \quad M^{\alpha\beta}$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

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Vectors $N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

positive root

negative root

$A^m \quad \bar{A}^m$

2d vectors

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

2n vectors

G G

M^{mn}

d² scalars

$g_{mn} + B_{mn}$

$$\underbrace{M^{\alpha\beta} \quad M^{mn} \quad M^{\alpha n} \quad M^{m\beta}}_{M^{ab}}$$

$a = 1, \ldots, n$

n² scalars

Scalars $\bar{N}_y = N_y = 1$

$N = \bar{N} = 0$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2 \quad M^{\alpha\beta}$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$$p = EZ$$

Vectors $N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

positive root

negative root

$A^m \quad \bar{A}^m$

2d vectors

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

2n vectors

G G

M^{mn}

d² scalars

$g_{mn} + B_{mn}$

$$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}}$$

$$\underbrace{M^{\alpha\beta} \quad M^{mn} \quad M^{\alpha n} \quad M^{m\beta}}_{M^{ab}}$$

$a = 1, \ldots, n$

n² scalars

Scalars $\bar{N}_y = N_y = 1$

$N = \bar{N} = 0$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2 \quad M^{\alpha\beta}$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

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$$p = EZ$$

Vectors $N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

$A^m \quad \bar{A}^m$

2d vectors

positive root

negative root

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

2n vectors

G G

M^{mn}

d² scalars

$g_{mn} + B_{mn}$

$$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}}$$

$$\underbrace{M^{\alpha\beta} \quad M^{mn} \quad M^{\alpha n} \quad M^{m\beta}}$$

M^{ab} $a = 1, ..., n$

n² scalars

$$\boxed{\mathcal{H} \in \frac{O(n,n)}{O(n)\times O(n)}}$$

Scalars $\bar{N}_y = N_y = 1$

$N = \bar{N} = 0$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2$ $M^{\alpha\beta}$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$p = EZ$

Vectors $N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

2d vectors

2n vectors

2dD dof

$g_{\mu m} \pm B_{\mu m}$

G G

M^{mn}

$$\underbrace{M^{\alpha\beta} \quad M^{mn} \quad M^{\alpha n} \quad M^{m\beta}}$$

d² scalars

$g_{mn} + B_{mn}$

M^{ab}

n² scalars

$a = 1, \ldots, n$

Scalars $\bar{N}_y = N_y = 1$

$N = \bar{N} = 0$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2 \quad M^{\alpha\beta}$

$$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d) \times O(d)}} \qquad \boxed{\mathcal{H} \in \frac{O(n,n)}{O(n) \times O(n)}}$$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $\quad 0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$p = EZ$

D^2 dof

tensors

$g_{\mu\nu}, B_{\mu\nu}$

Vectors $\quad N = 0,\ \bar{N} = 1$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ $M^2=0$

2dD dof

2d vectors

$g_{\mu m} \pm B_{\mu m}$

2n vectors

G G

M^{mn}

$$\underbrace{M^{\alpha\beta} \quad M^{mn} \quad M^{\alpha n} \quad M^{m\beta}}$$

d² scalars

$g_{mn} + B_{mn}$

M^{ab}

n² scalars

$a = 1, \ldots, n$

Scalars $\quad \bar{N}_y = N_y = 1$

$N = \bar{N} = 0$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2 \quad M^{\alpha\beta}$

$$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}} \qquad \boxed{\mathcal{H} \in \frac{O(n,n)}{O(n)\times O(n)}}$$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$p = EZ$

D² dof

tensors $g_{\mu\nu}, B_{\mu\nu}$

tensors $g_{\mu\nu}, B_{\mu\nu}$

Vectors $N = 0,\ \bar{N} = 1$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

2d vectors

2dD dof

$g_{\mu m} \pm B_{\mu m}$

2n vectors

G G

M^{mn}

$$\underbrace{M^{\alpha\beta} \quad M^{mn} \quad M^{\alpha n} \quad M^{m\beta}}$$

d² scalars

$g_{mn} + B_{mn}$

M^{ab}

n² scalars

$a = 1, \ldots, n$

$$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d) \times O(d)}} \qquad \boxed{\mathcal{H} \in \frac{O(n,n)}{O(n) \times O(n)}}$$

Scalars $\bar{N}_y = N_y = 1$

$N = \bar{N} = 0$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2 \quad M^{\alpha\beta}$

Symmetry enhancement $\mathcal{M}_D \times T^d$

$$M^2 = 2(N+\bar{N}-2) + (p_L^2 + p_R^2)$$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N-\bar{N}) + (p_L^2 - p_R^2)$

D² dof

tensors

$g_{\mu\nu}, B_{\mu\nu}$

tensors

$g_{\mu\nu}, B_{\mu\nu}$

$p = EZ$

Vectors $N=0,\ \bar{N}=1$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

2dD dof

2d vectors

$g_{\mu m} \pm B_{\mu m}$

2n vectors

G G

M^{mn}

$$\underbrace{M^{\alpha\beta} \quad M^{mn} \quad M^{\alpha n} \quad M^{m\beta}}$$

Scalars $\bar{N}_y = N_y = 1$

$N = \bar{N} = 0$

d² scalars

$g_{mn} + B_{mn}$

M^{ab}

n² scalars

$a = 1, ..., n$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2 \quad M^{\alpha\beta}$

$$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}} \qquad \boxed{\mathcal{H} \in \frac{O(n,n)}{O(n)\times O(n)}}$$

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DFT

Effective action from string theory

Computing 3-point functions <VVV> at a point of enhancement we read off

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a} + \frac{1}{4} \bar{F}^{a}_{\mu\nu} \bar{F}^{\mu\nu}_{a}$$

$$+\frac{1}{4} M_{aa'} F^{a}_{\mu\nu} \bar{F}^{a'\mu\nu} + D_{\mu} M_{aa'} D^{\mu} M^{aa'} - \frac{1}{12} f_{abc} \bar{f}_{a'b'c'} M^{aa'} M^{bb'} M^{cc'}$$

$$H = dB + A^{a} \wedge F_{a} + f_{abc} A^{a} \wedge A^{b} \wedge A^{c}$$

$$- \bar{A}^{a} \wedge \bar{F}_{a} - \bar{f}_{abc} \bar{A}^{a} \wedge \bar{A}^{b} \wedge \bar{A}^{c}$$

$$F^{a} = dA^{a} + f^{a}_{bc} A^{b} \wedge A^{c}$$

$$D_{\mu} M^{aa'} = \partial_{\mu} M^{aa'} + f^{a}_{bc} A^{b}_{\mu} M^{ca'} + f^{a'}_{b'c'} \bar{A}^{b'}_{\mu} M^{ac'}$$

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Higgs mechanism

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Higgs mechanism

$$M^{mn} = \underbrace{v^{mn}}_{\substack{\text{deviation from} \\ \text{point of enhancement}}} + M'^{mn}$$

$$\delta(g + B)$$

Effective action from string theory

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A^α

$\bar{A}^\alpha$ acquire mass² $\sim v v^t$

$$H = dB + A^a \wedge F_a + f_{abc} A^a \wedge A^b \wedge A^c$$

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$M^{\alpha\beta}$ acquire mass² $\sim v$

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G x G → $U^d(1)$ x $U^d(1)$

$$D_\mu M^{aa'} = \partial_\mu M^{aa'} + f^a_{bc} A^b_\mu M^{ca'} + f^{a'}_{b'c'} \bar{A}^{b'}_\mu M^{ac'}$$

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$M^{\alpha\beta}$

acquire mass² $\sim v$

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$\delta(g+B)$

Can we get this action from DFT ??

DFT action

DFT action

DFT O(D,D) action

Hull & Zweibach 09

$$S = \int dX \left(-\partial_{MN}\mathcal{H}^{MN} + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \right)$$

DFT action

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Equivalent to

$$S = \int dX\, \mathbb{R}$$

generalized Ricci scalar

Coimbra, Strickland-Constable, Waldram 09

DFT action

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Generalized Scherk-Schwarz reduction of DFT action

$$\mathcal{M}_{D-d} \times \mathcal{M}^d$$

Aldazabal, Baron, Marques, Nuñez 11
Geissbuhler 11

$$\text{O(D,D)} \longrightarrow \underset{\text{external}}{\text{O(D-d,D-d)}} \times \underset{\text{internal}}{\text{O(d,d)}}$$

DFT action

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Hull & Zweibach 09

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Generalized Scherk-Schwarz reduction of DFT action

$$\mathcal{M}_{D-d} \times \mathcal{M}^d$$

$x \quad , \quad y$

generalized parallelizable

$$\mathcal{H}^{MN} = \delta^{AB} E_A{}^M E_B{}^N \qquad E_A(x,y) = U_A{}^{A'}(x)\, E'_{A'}(y)$$

Aldazabal, Baron, Marques, Nuñez 11
Geissbuhler 11

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DFT action

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Hull & Zweibach 09

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$$\text{O(D,D)} \longrightarrow \underbrace{\text{O(D-d,D-d)}}_{\text{external}} \times \underbrace{\text{O(d,d)}}_{\text{internal}}$$

$$\partial_M = (\underbrace{\partial_\mu,}_{\text{D-d}} \underbrace{\partial_i, \partial_{\bar{i}},}_{\substack{\text{2d} \\ I}} \cancel{\partial^\mu})$$

From now on discuss simplest case, d=1 $\mathcal{M}_{D-1} \times S^1$

Frame on $T\mathcal{M}_D \oplus T^*\mathcal{M}_D$

frame e_a dual frame e^a

$$T\mathcal{M}_D = T\mathcal{M}_{D-1} \oplus TS^1$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

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$$y \sim y + 2\pi$$

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$e^{\hat{a}}$

$\phi\,(dy + V_1)$ $g_{\mu y}$

$\sqrt{g_{yy}} = R$

From now on discuss simplest case, d=1 $\mathcal{M}_{D-1} \times S^1$

Frame on $T\mathcal{M}_D \oplus T^*\mathcal{M}_D$

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Frame on $TM_D \oplus T^*\mathcal{M}_D$

frame e_a dual frame e^a

$$T\mathcal{M}_D = T\mathcal{M}_{D-1} \oplus TS^1$$

$$y \sim y + 2\pi$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

#

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$\phi\,(dy + V_1)$ $\quad g_{\mu y}$

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From now on discuss simplest case, d=1 $\qquad \mathcal{M}_{D-1} \times S^1$

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frame $\; e_a \qquad$ dual frame $\; e^a$

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$B_{\mu y}$

#

$\phi^{-1}(\partial_y + B_1)$

$e^{\hat{a}}$

$\phi\,(dy + V_1)$ $\; g_{\mu y}$

$\sqrt{g_{yy}} = R$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$B_{\mu y}$

#

$$\boxed{\begin{array}{c} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{array}}$$

$g_{\mu y}$

$e^{\hat{a}}$

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$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix}$$

$B_{\mu y}$

$\#$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix} \qquad \boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}} \quad g_{\mu y}$$

$e^{\hat{a}}$

$\sqrt{g_{yy}} = R$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} U^+ & U^- \\ U^- & U^+ \end{pmatrix} \quad \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$U^\pm = \frac{1}{2}(\phi^{-1} \pm \phi) \qquad \begin{aligned} A &= V_1 + B_1 & J &= \partial_y + dy \\ \bar{A} &= V_1 - B_1 & \bar{J} &= \partial_y - dy \end{aligned}$$

$B_{\mu y}$

$\#$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix} \qquad \boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}} \qquad g_{\mu y}$$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}\underbrace{\langle M^{11}\rangle}_{v^{11} = \epsilon} + \ldots$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow{LR} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} U^+ & U^- \\ U^- & U^+ \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$U^\pm = \frac{1}{2}(\phi^{-1} \pm \phi)$$

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$B_{\mu y}$

\#

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix} \qquad \boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}} \qquad g_{\mu y}$$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}\underbrace{\langle M^{11}\rangle}_{v^{11} = \epsilon} + \ldots$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} U^+ & U^- \\ U^- & U^+ \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$U^+ \approx 1 \qquad U^- \approx \tfrac{1}{2}M$$

$$U^{\pm} = \frac{1}{2}(\phi^{-1} \pm \phi)$$

$$A = V_1 + B_1 \qquad J = \partial_y + dy$$

$$\bar{A} = V_1 - B_1 \qquad \bar{J} = \partial_y - dy$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$B_{\mu y}$

#

$$\boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}}$$

$g_{\mu y}$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}\underbrace{\langle M^{11}\rangle}_{v^{11} = \epsilon} + \ldots$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \tfrac{1}{2}M \\ \tfrac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$U^+ \approx 1 \qquad U^\pm = \frac{1}{2}(\phi^{-1} \pm \phi) \qquad A = V_1 + B_1 \qquad J = \partial_y + dy$$

$$U^- \approx \tfrac{1}{2}M \qquad \bar{A} = V_1 - B_1 \qquad \bar{J} = \partial_y - dy$$

Effective action valid at energies $E \sim \frac{1}{\sqrt{\alpha'}}\,\epsilon << \frac{1}{\sqrt{\alpha'}}$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$B_{\mu y}$

$\#$

$$\boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}}$$

$g_{\mu y}$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}\underbrace{\langle M^{11}\rangle}_{v^{11} = \epsilon} + \dots$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

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Effective action valid at energies $E \sim \frac{1}{\sqrt{\alpha'}}\,\epsilon << \frac{1}{\sqrt{\alpha'}}$

So far, no enhancement of symmetry

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$\#$

$B_{\mu y}$

$$\boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}} \quad g_{\mu y}$$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}\underbrace{\langle M^{11}\rangle}_{v^{11} = \epsilon} + \dots$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow{LR} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$U^+ \approx 1 \qquad U^{\pm} = \frac{1}{2}(\phi^{-1} \pm \phi) \qquad A = V_1 + B_1 \qquad J = \partial_y + dy$$

$$U^- \approx \tfrac{1}{2}M \qquad \bar{A} = V_1 - B_1 \qquad \bar{J} = \partial_y - dy$$

Effective action valid at energies $E \sim \frac{1}{\sqrt{\alpha'}}\,\epsilon << \frac{1}{\sqrt{\alpha'}}$

So far, no enhancement of symmetry , no double field theory

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \;\longrightarrow\; T\mathcal{M}_d \oplus TS^1 \oplus T^*S^1 \oplus T^*\mathcal{M}_d$$

$$J = \partial_y + dy$$

$$\bar{J} = \partial_y - dy$$

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$dy \eqsim \partial_{\tilde{y}}$$

$$J = \partial_y + dy$$

$$\bar{J} = \partial_y - dy$$

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$dy \approx \partial_{\tilde{y}}$$

$$J = \partial_y + dy = \partial_y + \partial_{\tilde{y}} = \partial_{y^L}$$

$$\bar{J} = \partial_y - dy = \partial_y - \partial_{\tilde{y}} = \partial_{y^R}$$

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$dy \approx \partial_{\tilde{y}}$$

$$J = \partial_y + dy = \partial_y + \partial_{\tilde{y}} = \partial_{y^L}$$

$$\bar{J} = \partial_y - dy = \partial_y - \partial_{\tilde{y}} = \partial_{y^R}$$

Still, this is formal. No dependence on y or $\tilde{y}$

DFT & Enhancement of symmetry

$$TM \oplus T^*M \longrightarrow TM_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*M_d$$

$$dy \approx \partial_{\tilde{y}}$$

$$J = \partial_y + dy = \partial_y + \partial_{\tilde{y}} = \partial_{y^L}$$

$$\bar{J} = \partial_y - dy = \partial_y - \partial_{\tilde{y}} = \partial_{y^R}$$

Still, this is formal. No dependence on y or $\tilde{y}$

Of course, we have not included momentum/winding modes $\sim e^{2iy}/e^{2i\tilde{y}}$

To include winding modes we need dependence on $S^1, \tilde{S}^1$

To account for the enhancement of symmetry, we need to enlarge the generalized tangent space

Enhancement of symmetry

$$T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$\begin{pmatrix} E^{L} \\ E^{R} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

Enhancement of symmetry

$$\begin{pmatrix} E^{L} \\ E^{R} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$\downarrow$$

$$T\mathcal{M}_d \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\mathrm{O}(3,3)} \oplus T^*\mathcal{M}_d$$

Enhancement of symmetry

$$TM_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*M_d$$

$$\downarrow$$

$$TM_d \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\text{O(3,3)}} \oplus T^*M_d$$

$$\begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} E^i \\ E^{\bar{\imath}} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M^{i\bar{\jmath}} \\ \frac{1}{2}M^{\bar{\imath}j} & 1 \end{pmatrix} \begin{pmatrix} J^j + A^j \\ \bar{J}^{\bar{\jmath}} - \bar{A}^{\bar{\jmath}} \end{pmatrix}$$

$M^{i\bar{\jmath}}(x)$

9 scalar fields

$A^i(x)$

$\bar{A}^{\bar{\imath}}(x)$

6 vector fields

Enhancement of symmetry

$$T\mathcal{M}_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_d$$

$$\downarrow$$

$$T\mathcal{M}_d \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\mathrm{O(3,3)}} \oplus T^*\mathcal{M}_d$$

$$\begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} E^i \\ E^{\bar{\imath}} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M^{i\bar{\jmath}} \\ \frac{1}{2}M^{\bar{\imath}j} & 1 \end{pmatrix} \begin{pmatrix} J^j + A^j \\ \bar{J}^{\bar{\jmath}} - \bar{A}^{\bar{\jmath}} \end{pmatrix}$$

$M^{i\bar{\jmath}}(x)$

9 scalar fields

$A^i(x)$

$\bar{A}^{\bar{\imath}}(x)$

6 vector fields

$J^i(y, \tilde{y})$

$\bar{J}^{\bar{\imath}}(y, \tilde{y})$

Enhancement of symmetry

$$TM_d \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*M_d$$

$$\downarrow$$

$$TM_d \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\text{O(3,3)}} \oplus T^*M_d$$

$$\begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} E^i \\ E^{\bar{\imath}} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M^{i\bar{\jmath}} \\ \frac{1}{2}M^{\bar{\imath}j} & 1 \end{pmatrix} \begin{pmatrix} J^j + A^j \\ \bar{J}^{\bar{\jmath}} - \bar{A}^{\bar{\jmath}} \end{pmatrix}$$

$M^{i\bar{\jmath}}(x)$

9 scalar fields

$A^i(x)$

$\bar{A}^{\bar{\imath}}(x)$

6 vector fields

Should satisfy SU(2)$_L$ algebra $J^i(y, \tilde{y})$

Should satisfy SU(2)$_R$ algebra $\bar{J}^{\bar{\imath}}(y, \tilde{y})$

under C-bracket

Effective action

Generalized Sherk-Schwarz compactification of DFT action

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$
\begin{aligned}
\mathcal{L} \quad = \quad & R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ} \\
& - \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)
\end{aligned}
$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$-\frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$\mathcal{L} \;=\; R-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}+\frac{1}{4}\mathcal{H}_{IJ}F^{I\mu\nu}F^{J}_{\mu\nu}+(D_\mu\mathcal{H})_{IJ}(D^\mu\mathcal{H})^{IJ}$$

$$-\frac{1}{12}f_{IJK}f_{LMN}\left(\mathcal{H}^{IL}\mathcal{H}^{JM}\mathcal{H}^{KN}-3\,\mathcal{H}^{IL}\eta^{JM}\eta^{KN}+2\,\eta^{IL}\eta^{JM}\eta^{KN}\right)$$

$$\begin{pmatrix} E_a \\ E^{L} \\ E^{R} \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a}A & \iota_{e_a}\bar{A} & \iota_{e_a}B \\ 0 & 1 & \frac{1}{2}M & M\bar{A} \\ 0 & \frac{1}{2}M^t & 1 & M^tA \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x,y,\tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y,\tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$-\frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$\begin{pmatrix} E_a \\ E^{L} \\ E^{R} \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\square^2$$

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$\begin{pmatrix} E_a \\ E^{L} \\ E^{R} \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\boxed{}^{\,2} \approx \begin{pmatrix} 1 & M \\ M^t & 1 \end{pmatrix}$$

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$\begin{pmatrix} E_a \\ E^{L} \\ E^{R} \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\boxed{}^{\,2} \approx \begin{pmatrix} 1 & M \\ M^t & 1 \end{pmatrix}$$

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$F^I \;=\; dA^I + \boxed{{f^I}_{JK}}\, A^J \wedge A^K$$

$$\boxed{[E'_J, E'_K]_C = f^I_{JK} E'_I}$$

$$\begin{pmatrix} E_a \\ E^L \\ E^R \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad {U_A}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\boxed{\phantom{\mathcal{H}}}^{\,2} \approx \begin{pmatrix} 1 & M \\ M^t & 1 \end{pmatrix}$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$F^I = dA^I + \boxed{f^I{}_{JK}}\, A^J \wedge A^K$$

$$\boxed{[E'_J, E'_K]_C = f^I_{JK} E'_I}$$

$J, \bar{J}$ $\qquad$ $\epsilon^{ijk}, \epsilon^{\overline{ijk}}$

$$\begin{pmatrix} E_a \\ E^L \\ E^R \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\boxed{}^{\,2} \approx \begin{pmatrix} 1 & M \\ M^t & 1 \end{pmatrix}$$

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$H \;=\; dB + F^I \wedge A_I$$

$$F^I \;=\; dA^I + \boxed{f^I{}_{JK}}\, A^J \wedge A^K$$

$$\boxed{[E'_J, E'_K]_C = f^I_{JK} E'_I}$$

J, $\bar{J}$ $\qquad$ ϵ^{ijk}, $\epsilon^{\overline{ijk}}$

$$\begin{pmatrix} E_a \\ E^L \\ E^R \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^{i}_{\mu\nu} F^{i\mu\nu} + \frac{1}{4} \bar{F}^{i}_{\mu\nu} \bar{F}^{i\mu\nu} + \frac{1}{4} M^{ij} F^{i}_{\mu\nu} \bar{F}^{j\mu\nu} + D_\mu M^{ij} D^\mu M^{ij}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$H = dB + F^I \wedge A_I$$

$$F^I = dA^I + \boxed{f^I{}_{JK}} A^J \wedge A^K$$

$$\boxed{[E'_J, E'_K]_C = f^I_{JK} E'_I}$$

J, $\bar{J}$ $\qquad$ ϵ^{ijk}, $\epsilon^{\overline{ijk}}$

$$\begin{pmatrix} E_a \\ E^L \\ E^R \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^{i}_{\mu\nu} F^{i\mu\nu} + \frac{1}{4} \bar{F}^{i}_{\mu\nu} \bar{F}^{i\mu\nu} + \frac{1}{4} M^{ij} F^{i}_{\mu\nu} \bar{F}^{j\mu\nu} + D_\mu M^{ij} D^\mu M^{ij}$$

$$+ f_{ijk}\, f_{\bar{\imath}\bar{\jmath}\bar{k}}\, M^{i\bar{\imath}} M^{j\bar{\jmath}} M^{k\bar{k}}$$

$$H \;=\; dB + F^I \wedge A_I$$

$$F^I \;=\; dA^I + \boxed{f^I{}_{JK}} A^J \wedge A^K$$

$$\boxed{[\underset{\uparrow}{E'_J}, \underset{\uparrow}{E'_K}]_C = \underset{\uparrow}{f^I_{JK}} E'_I}$$

$J, \bar{J}$ $\quad\quad$ $\epsilon^{ijk}, \epsilon^{\overline{ijk}}$

$$\begin{pmatrix} E_a \\ E^L \\ E^R \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \quad U_A{}^{A'}(x) \quad E'_{A'}(y, \tilde{y})$$

Effective action

Generalized Sherk-Schwarz compactification of DFT action

$$I = i, \bar{\imath}$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^{i}_{\mu\nu} F^{i\mu\nu} + \frac{1}{4} \bar{F}^{i}_{\mu\nu} \bar{F}^{i\mu\nu} + \frac{1}{4} M^{ij} F^{i}_{\mu\nu} \bar{F}^{j\mu\nu} + D_\mu M^{ij} D^\mu M^{ij}$$

$$+ f_{ijk}\, f_{\bar{\imath}\bar{\jmath}\bar{k}}\, M^{i\bar{\imath}} M^{j\bar{\jmath}} M^{k\bar{k}}$$

Exactly string theory action!

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SU(2) x SU(2) algebra

C-bracket

$$[V_1, V_2]_C = \frac{1}{2}(\mathcal{L}_{V_1} V_2 - \mathcal{L}_{V_2} V_1)$$

$$(\mathcal{L}_{V_1} V_2)^I = V_1^J \partial_J V_2^I + (\partial^I V_{1J} - \partial_J V_1^I) V_2^J$$

generalized Lie derivative

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$$\boxed{V_2 + TS^1 + T\tilde{S}^1 + V_2^* .}$$

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The following J and $\bar{J}$ do the job

$$J = \begin{pmatrix} \cos 2y^L & \sin 2y^L & 0 \\ -\sin 2y^L & \cos 2y^L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \partial_{w_1^L} \\ \partial_{w_2^L} \\ \partial_{y^L} \end{pmatrix} \qquad \bar{J} = \begin{pmatrix} \cos 2y^R & \sin 2y^R & 0 \\ -\sin 2y^R & \cos 2y^R & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \partial_{w_1^R} \\ \partial_{w_2^R} \\ \partial_{y^R} \end{pmatrix}$$

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- A setup where we know was ist das and it does tell us about string theory!