Evgeny Epelbaum, RUB

New Vistas in Low Energy Precision Physics (LEPP),
Mainz, April 4-7, 2016

Theoretical overview of nuclear chiral EFT

Bundesministerium für Bildung und Forschung

DFG Deutsche Forschungsgemeinschaft

erc European Research Council

The landscape of computational nuclear physics

Ultimate goal: predictive & systematically improvable QCD-based theory for nuclei/ nuclear reactions/nuclear matter with quantified uncertainties

The method: chiral EFT for nuclear forces/currents + ab-initio „few"-body approaches
[Faddeev-Yakubovski, No Core Shell Model, Quantum Monte Carlo, Lorentz Integral Transform, Coupled Cluster, Lattice, self-consistent Gorkov-Green's functions,…]

Open questions:

— quantitative understanding of Nd scattering and light nuclei (3NF problem)
— systematic underbinding of heavier nuclei (A ~ 40): too soft interactions?
— is it possible to describe heavy nuclei without additional fine tuning?
— nuclei on the edge of stability, exotics (e.g. tetra-neutron?)
— interface with lattice QCD

Some strategies:

— high orders, no fine tuning in LECs, no tuning to heavy nuclei, error analysis
EE, Krebs, Meißner; Low Energy Nuclear Physics International Collaboration (LENPIC)

— allow for some fine tuning in LECs and fit to heavy nuclei, error analysis
The Oak Ridge group: Ekström, Carlsson, Wendt, Papenbrock, Hagen, …

— interactions optimized for specific few-body methods, e.g. local forces & QMC
Gezerlis, Tews, EE, Gandolfi, Hebeler, Nogga, Schwenk, Piarulli, Girlanda, Schiavilla, Navarro Perez, …

Chiral Effective Field Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of Q,

Weinberg, Gasser, Leutwyler, Meißner, ...

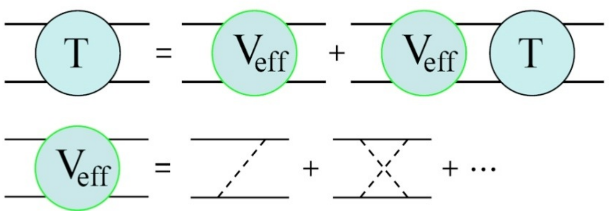
$$\boxed{Q = \frac{\text{momenta of external particles or } M_\pi \sim 140 \text{ MeV}}{\text{breakdown scale } \Lambda_b}}$$

Write down $L_{\text{eff}}\, [\pi, N, \ldots]$,
identify relevant diagrams at a given order,
do Feynman calculus,
fit LECs to exp data,
make predictions…

Chiral EFT for nuclear systems: expansion for nuclear forces + resummation (Schrödinger eq.)

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt, Krebs, ...

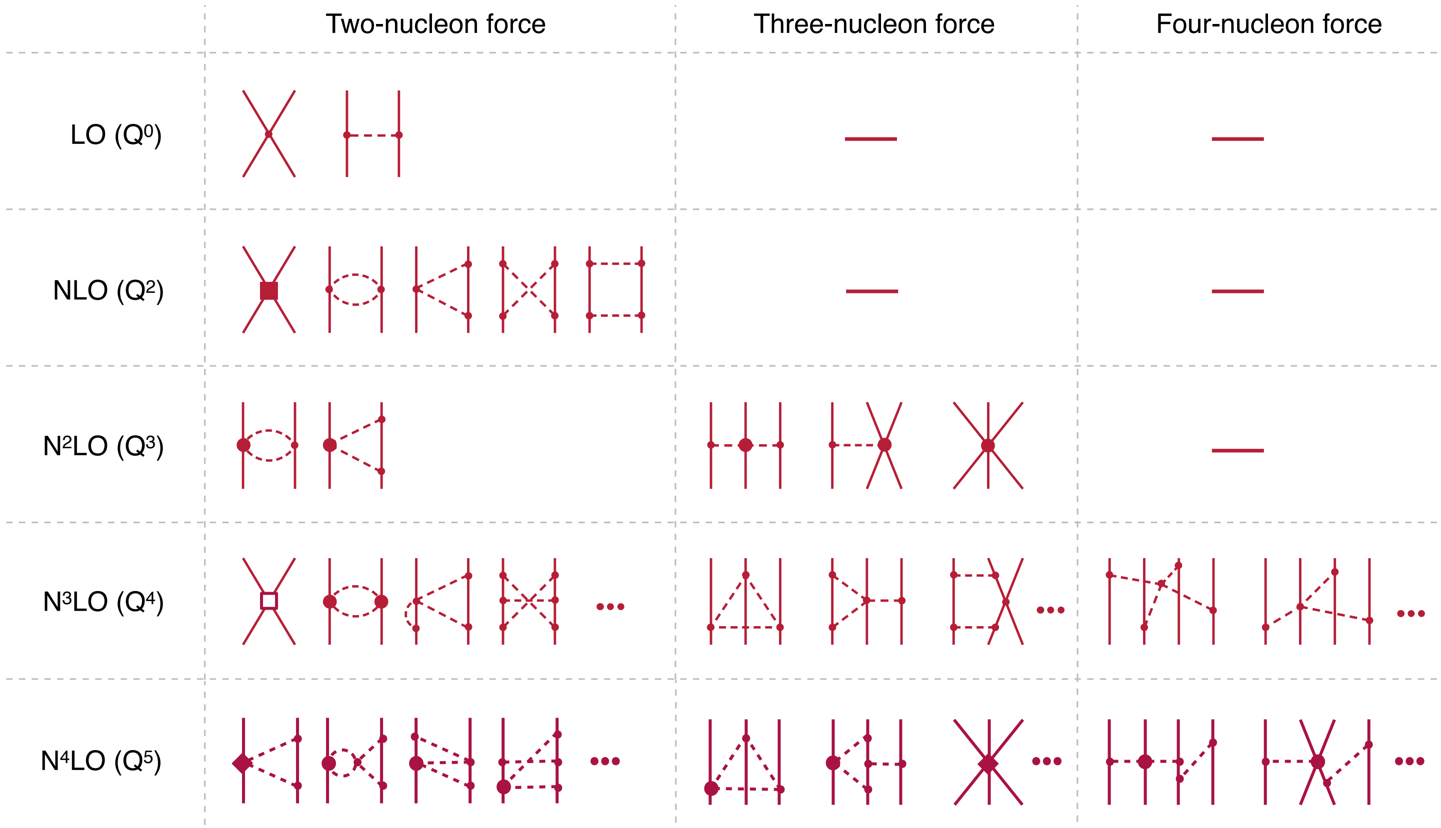
$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}} \right] |\Psi\rangle = E|\Psi\rangle$$



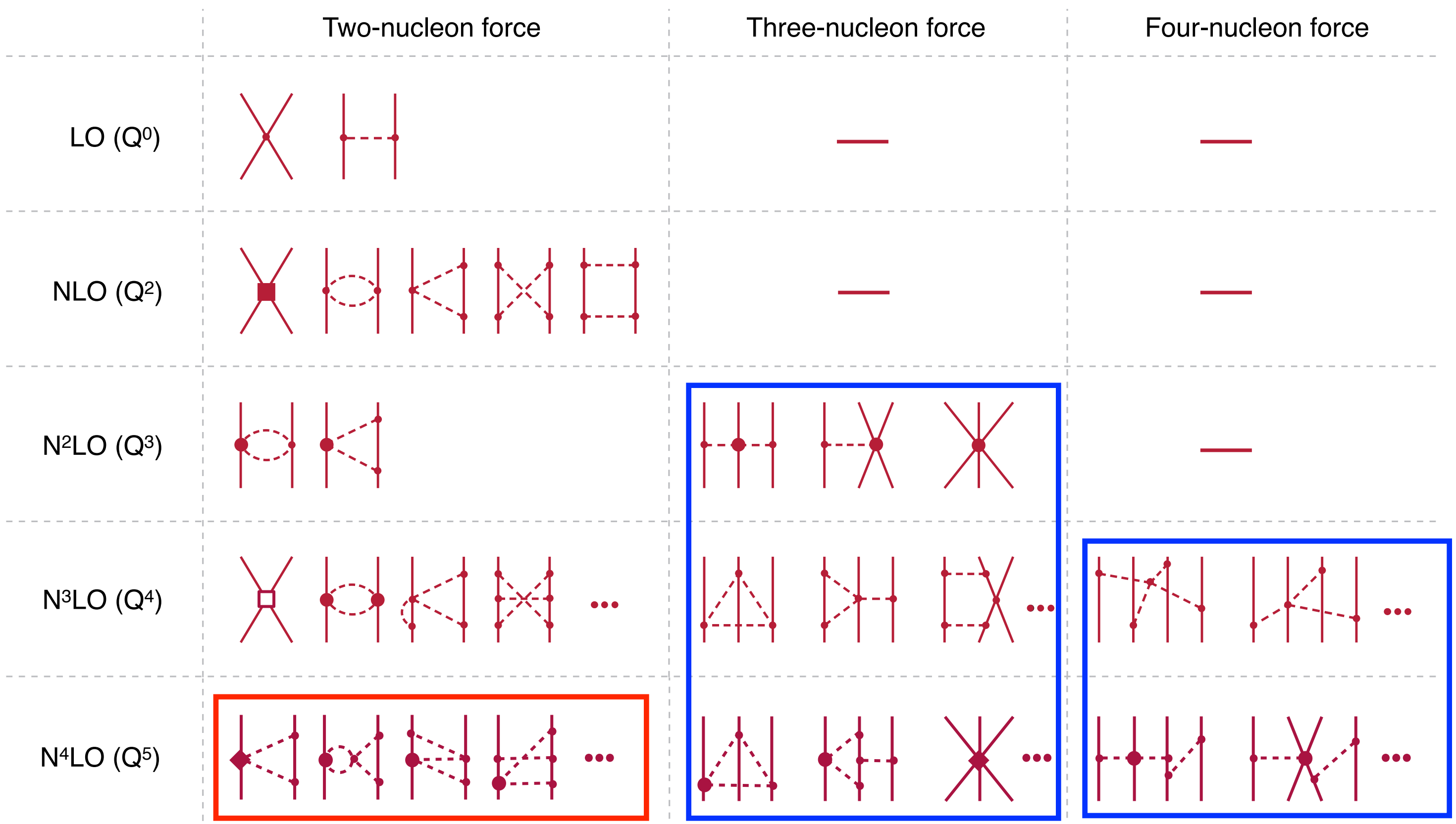
- systematically improvable
- unified approach for $\pi\pi$, πN, NN
- consistent many-body forces and currents **[talk by Jacek Golak]**
- error estimations

Notice: nonperturbative treatment of chiral nuclear forces in the Schrödinger eq. requires the introduction of a finite cutoff **[Alternatively, use semi-relativistic approach, EE, Gegelia, et al. '12…'15]**

Chiral expansion of the nuclear forces [NDA]



Chiral expansion of the nuclear forces [NDA]



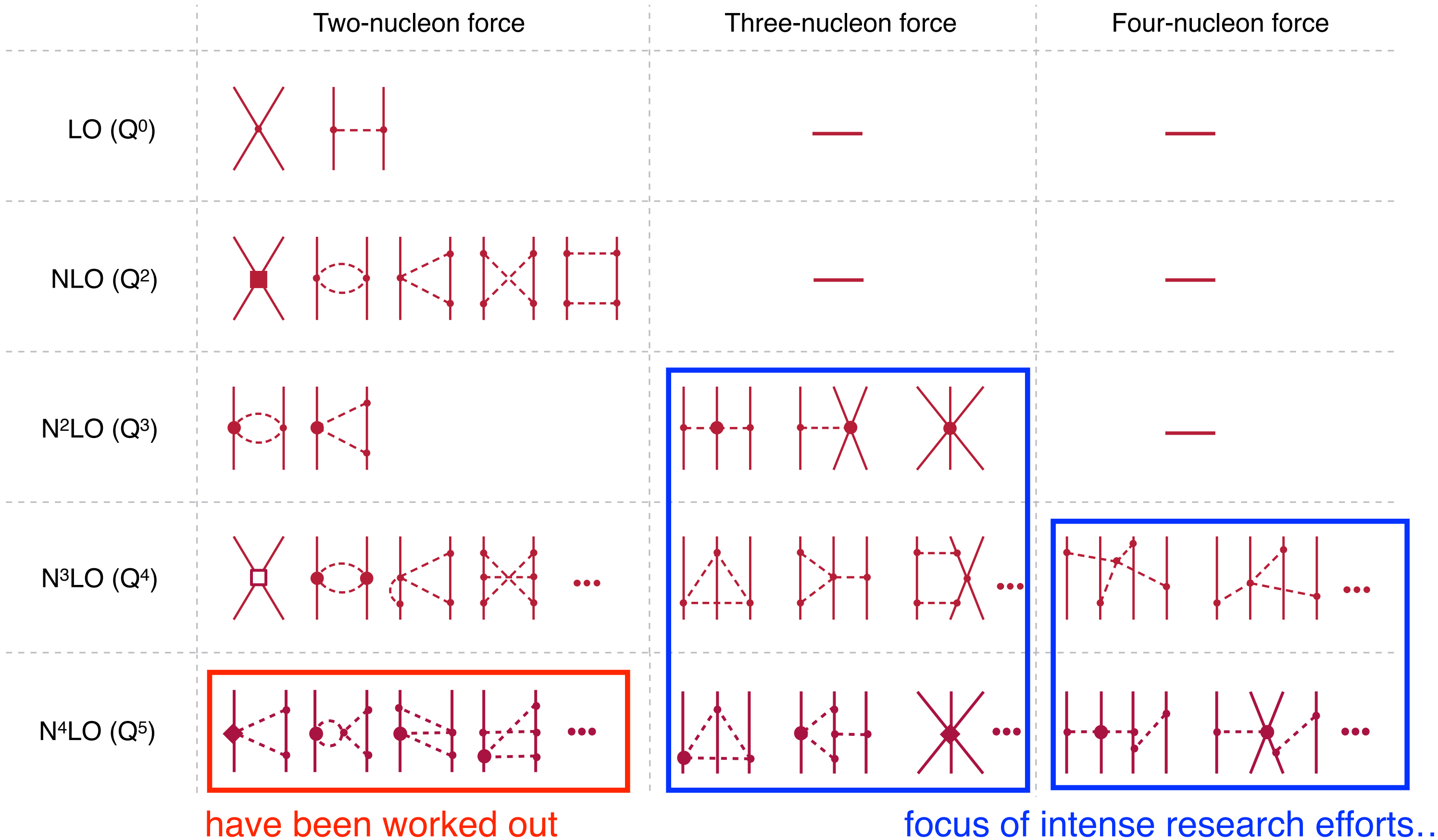
have been worked out

focus of intense research efforts…

Entem, Kaiser, Machleidt, Nosyk, PRC 91 (2015) 014002

EE, Krebs, Meißner, PRL 115 (2015) 122301

Chiral expansion of the nuclear forces [NDA]



Entem, Kaiser, Machleidt, Nosyk, PRC 91 (2015) 014002
EE, Krebs, Meißner, PRL 115 (2015) 122301

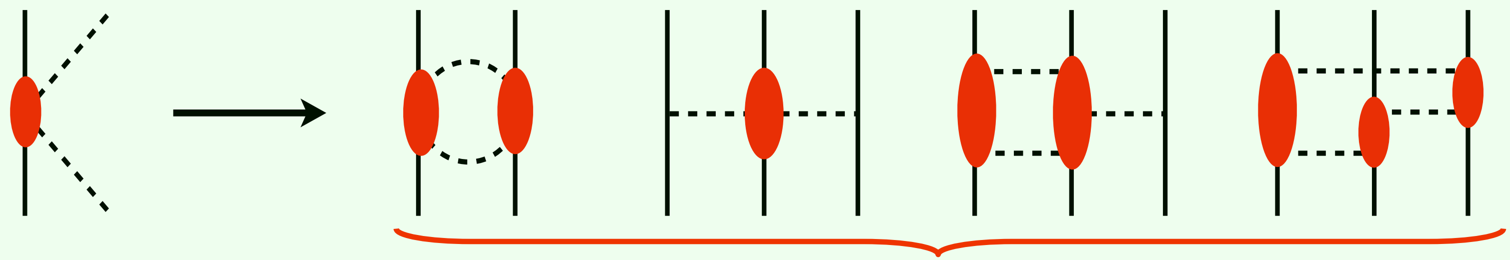
Why go to fifth order (N^{4}LO) in the chiral expansion?

— no additional parameters in the NN force (except for 1 IB term) ➙ testing the theory

— there is evidence that χ-expansion for the 3NF is not yet converged at Q^4

The long-range part of the nuclear forces

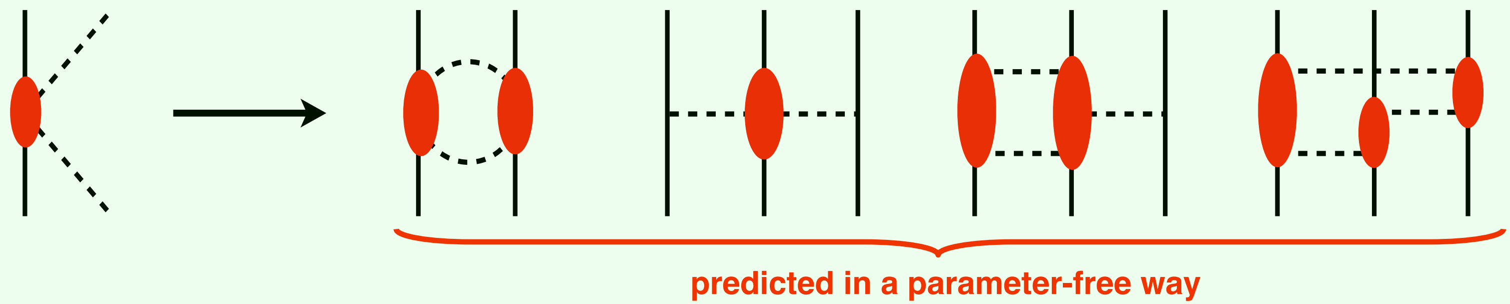
Long-range nuclear forces are completely determined by the
chiral symmetry of QCD + experimental information on πN scattering



predicted in a parameter-free way

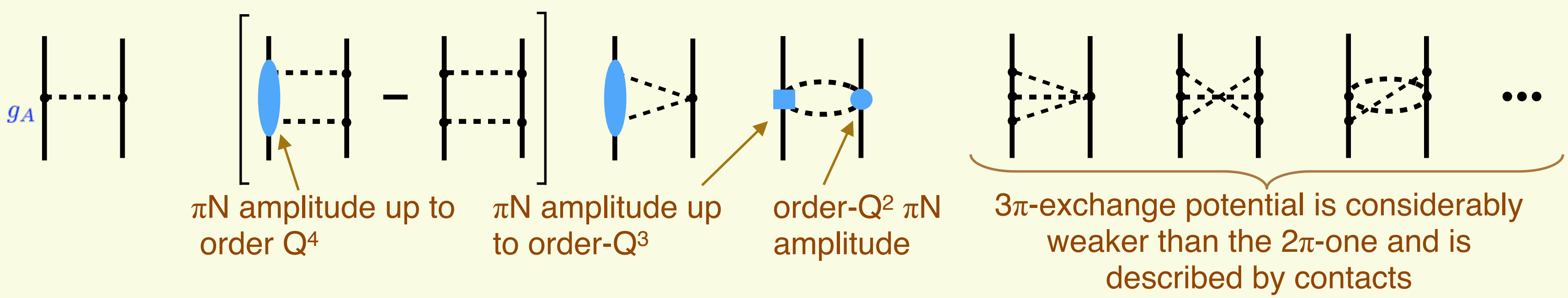
The long-range part of the nuclear forces

Long-range nuclear forces are completely determined by the
chiral symmetry of QCD + experimental information on πN scattering



The long-range NN force up to N^{4}LO [Q^5]

Ordonez et al.; Kaiser; EE, Krebs, Meißner; Entem, Machleidt; …



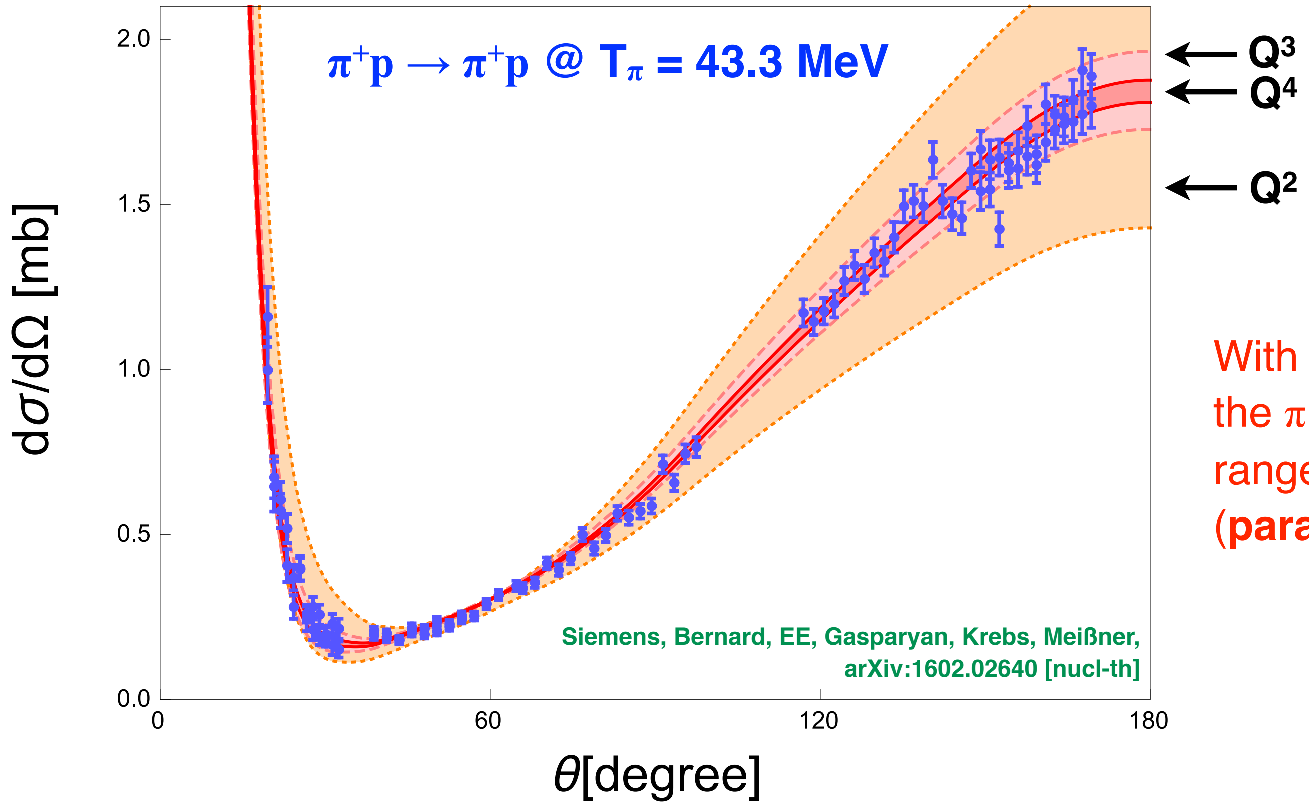
The TPE potential can be derived by taking the phase-space integral of the πN amplitudes computed in ChPT (Lorentz-transformed to the proper kinematics…) Kaiser ’00

Determination of πN LECs

— determination of πN LECs from πN PWA Fettes, Meißner, Alarcon, Camalich, Gasparyan, EE, Krebs, Deliang, …

— LECs from Roy-Steiner-eq. analysis of πN-scattering Hoferichter, Ruiz de Elvira, Kubis, Meißner, Yao, Gegelia, …

— determination of LECs from πN data Wendt, Ekström, Siemens, Bernard, EE, Gasparyan, Krebs, Meißner, …



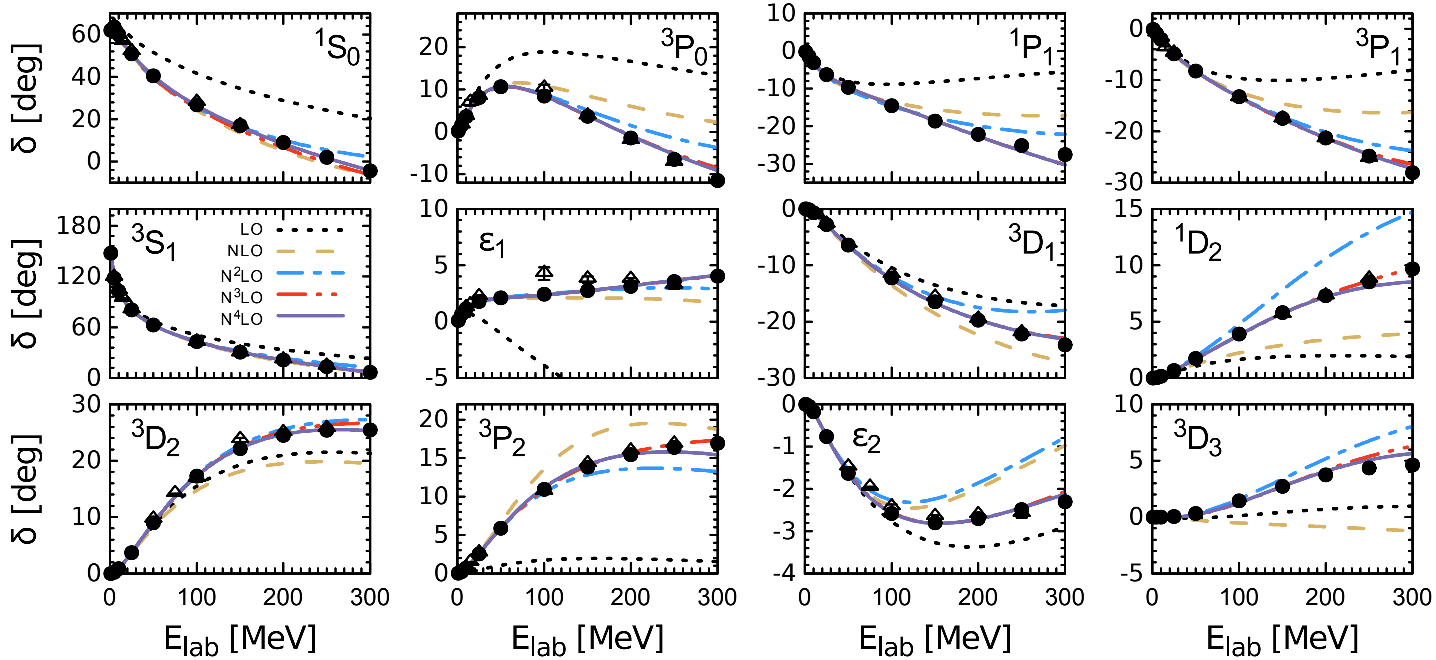
With the LECs taken from the πN system, the long-range NN force is fixed (**parameter-free**)!

- Used **local regularization** for long-range forces to reduce the amount of finite cutoff artifacts
- Short-range contact interactions are fitted to Nijmegen PWA

NN phase shifts order by order

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; PRL 115 (2015) 122301

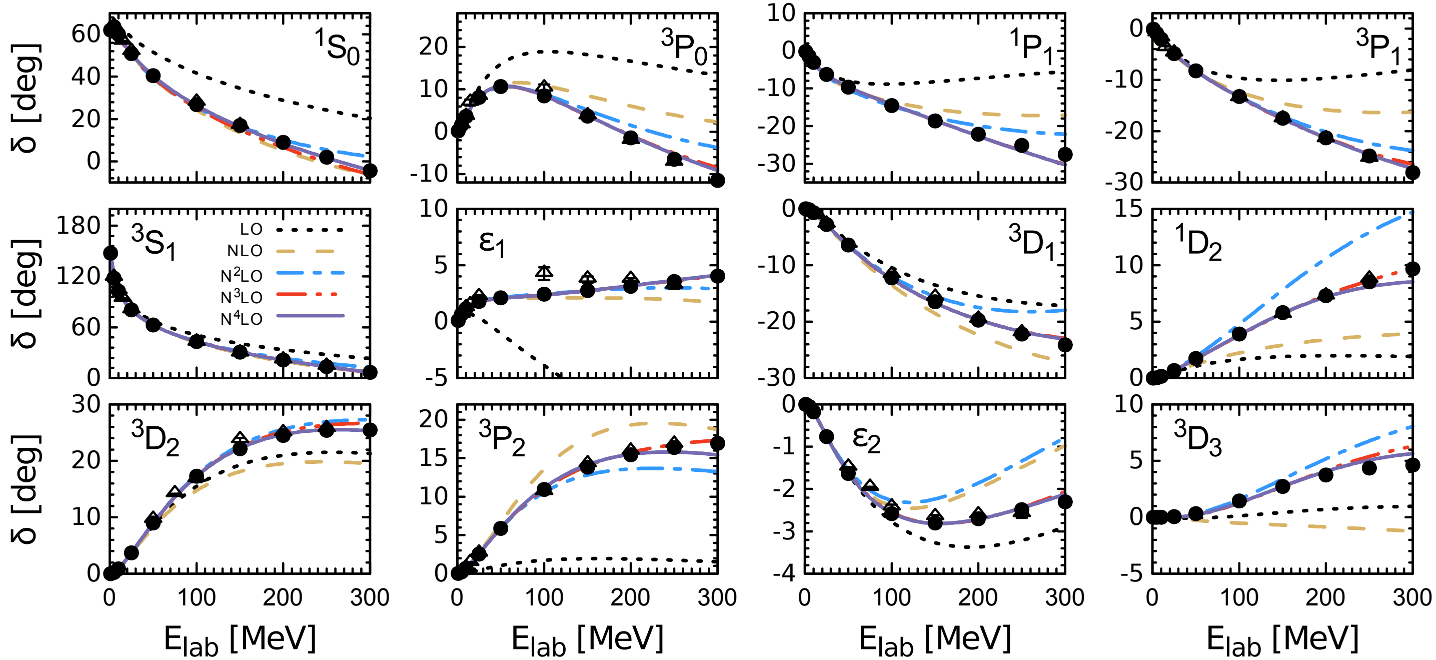
Convergence of the chiral expansion for neutron-proton phase shifts [using R = 0.9 fm]



NN phase shifts order by order

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; PRL 115 (2015) 122301

Convergence of the chiral expansion for neutron-proton phase shifts [using R = 0.9 fm]



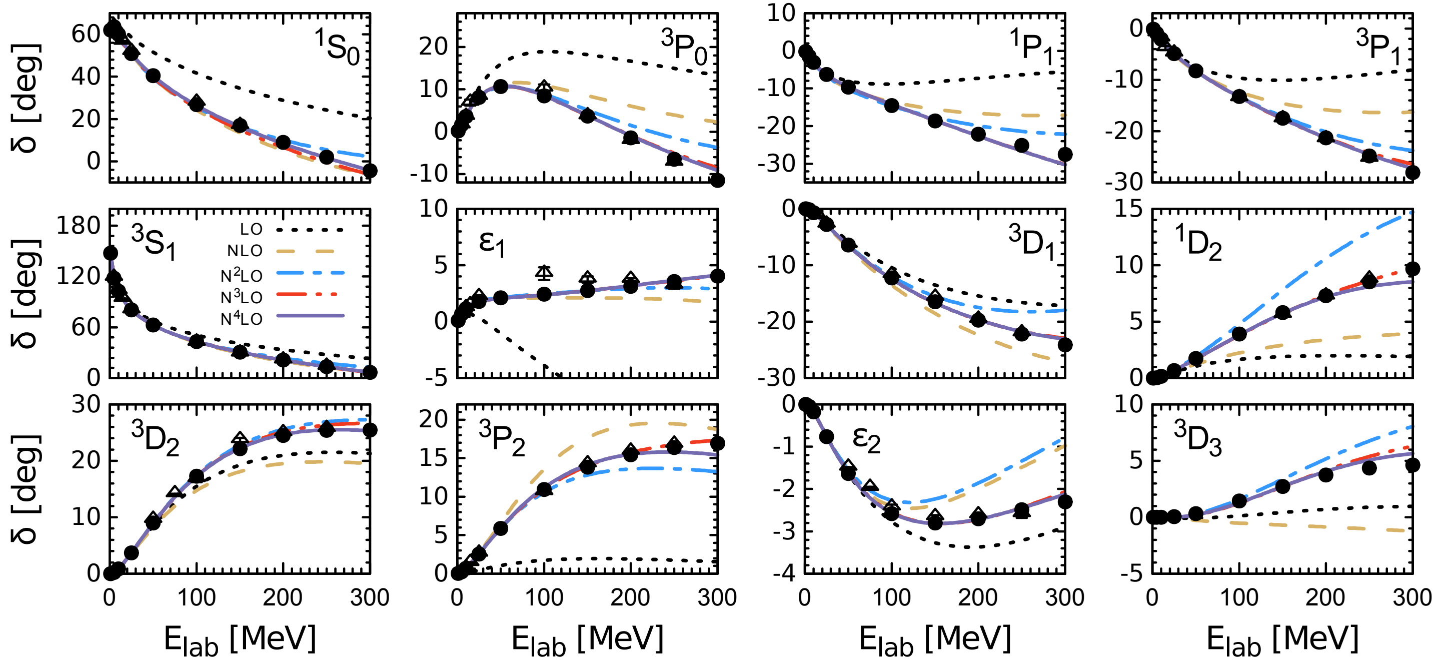
Quality of the reproduction of the Nijmegen PWA („χ^2_{datum}“) [using R = 0.9 fm]

$E_{\rm lab}$ bin	LO [Q^0]	NLO [Q^2]	N^{2}LO [Q^3]	N^{3}LO [Q^4]	N^{4}LO [Q^5]
neutron-proton phase shifts					
0–100	360	31	4.5	0.7	0.3
0–200	480	63	21	0.7	0.3
proton-proton phase shifts					
0–100	5750	102	15	0.8	0.3
0–200	9150	560	130	0.7	0.6
	2 LECs	+ 7 LECs + 2 IB LECs		+ 15 LECs	+ 1 IB LEC

NN phase shifts order by order

EE, Krebs, Meißner, EPJ A51 (2015) 5, 53; PRL 115 (2015) 122301

Convergence of the chiral expansion for neutron-proton phase shifts [using R = 0.9 fm]



Quality of the reproduction of the Nijmegen PWA („χ^2_{datum}“) [using R = 0.9 fm]

$E_{\rm lab}$ bin	LO [Q⁰]	NLO [Q²]		N²LO [Q³]	N³LO [Q⁴]		N⁴LO [Q⁵]
neutron-proton phase shifts							
0–100	360	31	no new LECs →	4.5	0.7	1 LEC ($^1S_0$) →	0.3
0–200	480	63		21	0.7		0.3
proton-proton phase shifts							
0–100	5750	102		15	0.8	no new LECs →	0.3
0–200	9150	560		130	0.7		0.6
	2 LECs	+ 7 LECs + 2 IB LECs			+ 15 LECs		+ 1 IB LEC

Quantification of
Theoretical Uncertainties

PHYSICAL REVIEW A
atomic, molecular, and optical physics

Highlights Recent Accepted Authors Referees Search About

Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities. […]

Phys. Rev. A 83, 040001 (2011)

Uncertainty quantification

Sources of uncertainty:

- Uncertainty in NN PWA used as input to fix contact interactions (probably small)
- Uncertainty in the values of πN LECs (might be significant, **Carlsson et al.'15**)
- Statistical uncertainty for NN LECs (small, **Ekström et al.'14**)
- **Uncertainty due to truncation of the chiral expansion at a given order**

Uncertainty quantification

Sources of uncertainty:

- Uncertainty in NN PWA used as input to fix contact interactions (probably small)
- Uncertainty in the values of πN LECs (might be significant, Carlsson et al.'15)
- Statistical uncertainty for NN LECs (small, Ekström et al.'14)
- **Uncertainty due to truncation of the chiral expansion at a given order**

A simple algorithm [EE, Krebs, Meißner, EPJA 51 (2015) 53]

Let $X^{(i)}(p)$ be an observable of interest calculated in chiral EFT up to the order Q^i:

$$X^{(i)}(p) = X^{(0)} + \underbrace{\Delta X^{(2)}}_{\sim Q^2 X^{(0)}} + \ldots + \underbrace{\Delta X^{(i)}}_{\sim Q^i X^{(0)}} \quad \text{with} \quad Q = \max(p/\Lambda_b,\, M_\pi/\Lambda_b)$$

estimated from the error plots $\Lambda_b \sim 600$ MeV

Use the explicitly calculated $\Delta X^{(i)}$ to estimate the uncertainty $\delta X^{(i)}$ at order Q^i:

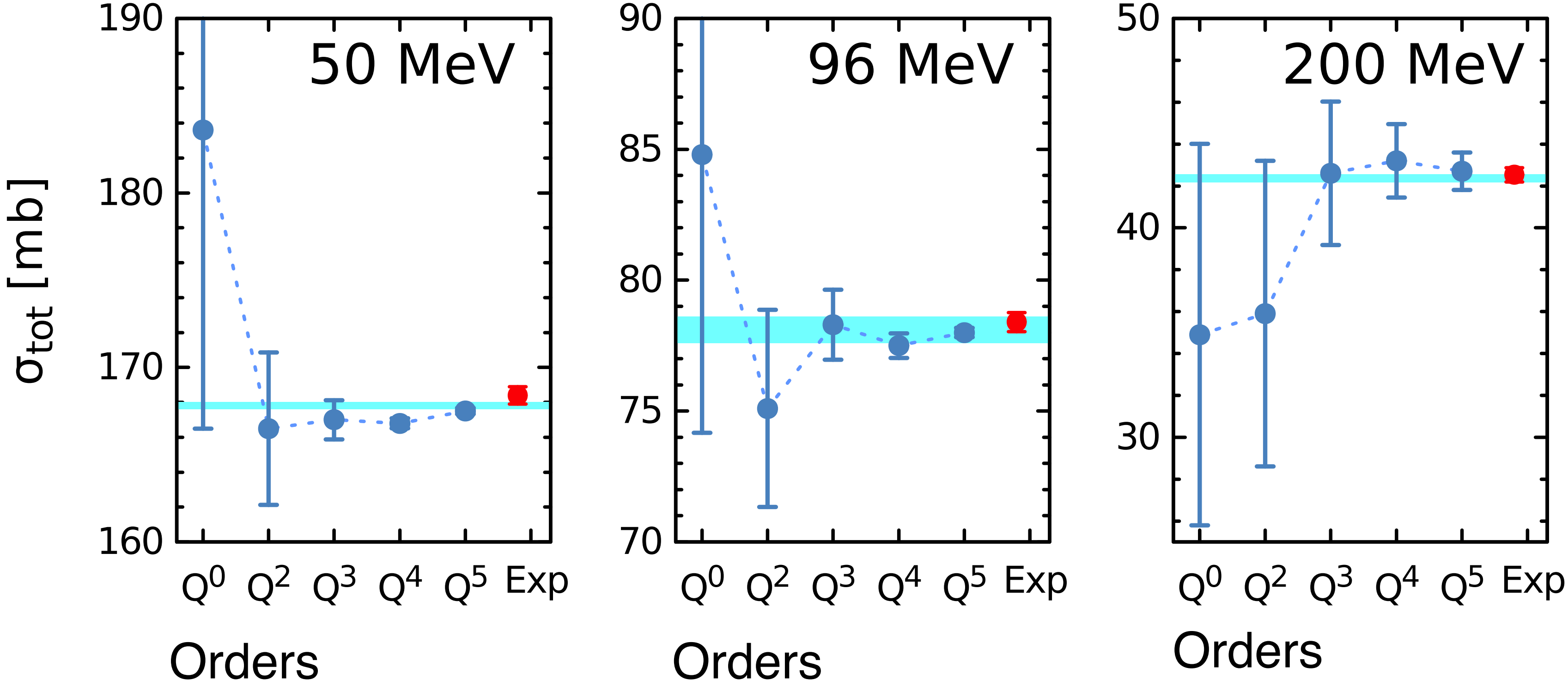
$$\delta X^{(0)} = Q^2 |X^{(0)}|, \quad \delta X^{(i)} = \max_{2 \le j \le i} \left(Q^{i+1} |X^{(0)}|,\, Q^{i+1-j} |\Delta X^{(j)}| \right)$$

subject to the additional constraint $\delta X^{(i)} \ge \max_{j,k} \left(|X^{(j \ge i)} - X^{(k \ge i)}| \right)$.

- no reliance on cutoff variation (not reliable)
- easily applicable to any observable of interest (scattering, bound states, 3N, …)
- error bars consistent with 68% degree-of-belief intervals Furnstahl et al., PRC 92 (2015) 024005

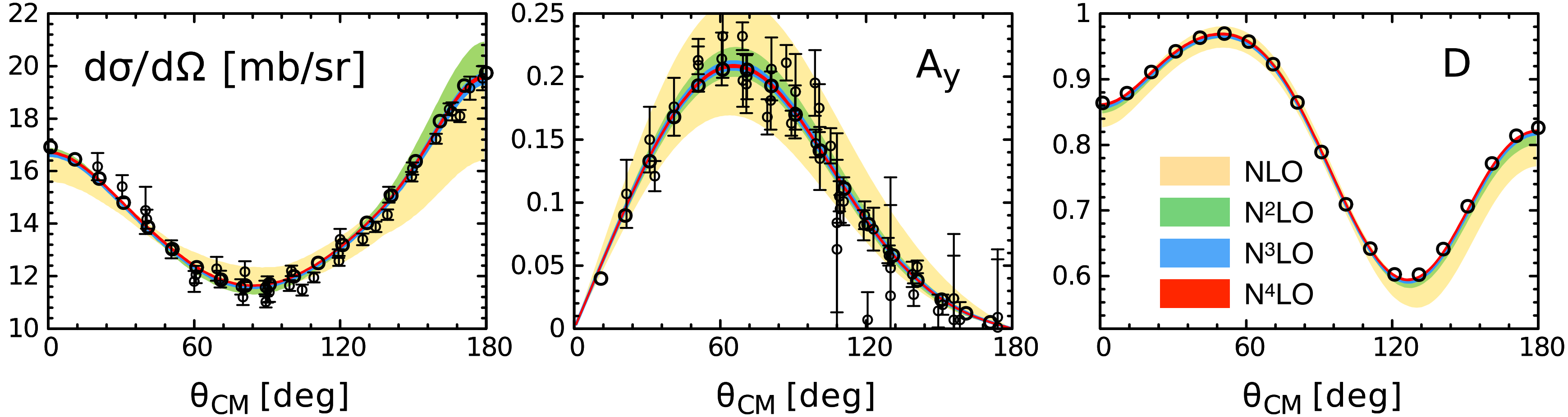
Uncertainty quantification

Total cross section in np scattering [R = 0.9 fm]

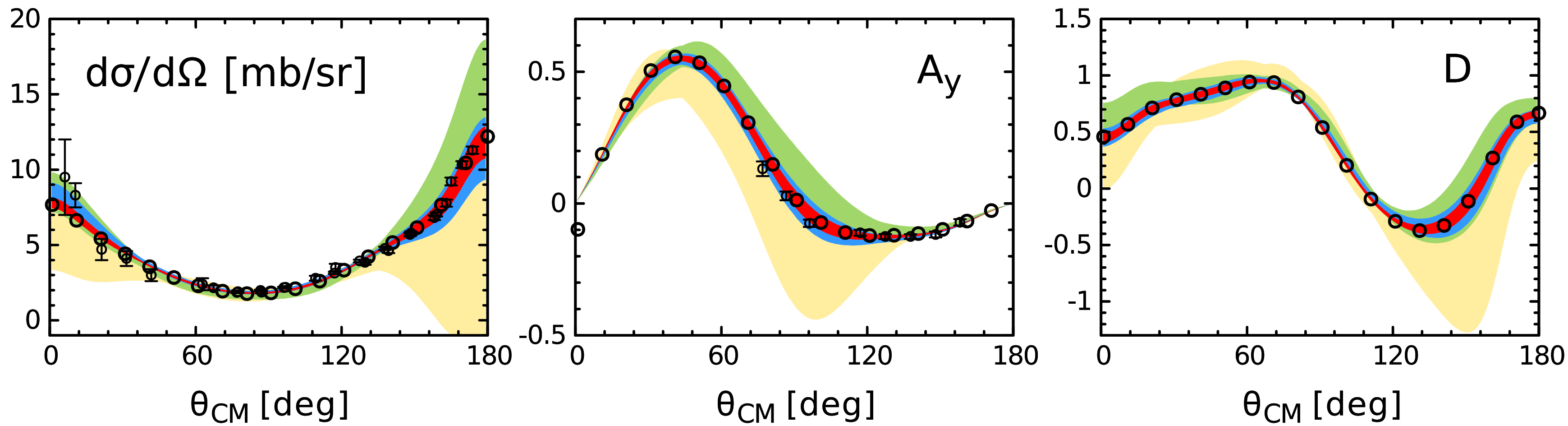


Uncertainty quantification

Proton-neutron scattering at E_{lab} = 50 MeV [R = 0.9 fm]



Proton-neutron scattering at E_{lab} = 200 MeV [R = 0.9 fm]



Work in progress:

Testing the chiral 3NF

Having developed these tools, namely

— accurate and precise NN potentials up to N^4LO

— reliable approach for quantifying theoretical uncertainties,

we are well equipped to address the three-nucleon force problem.

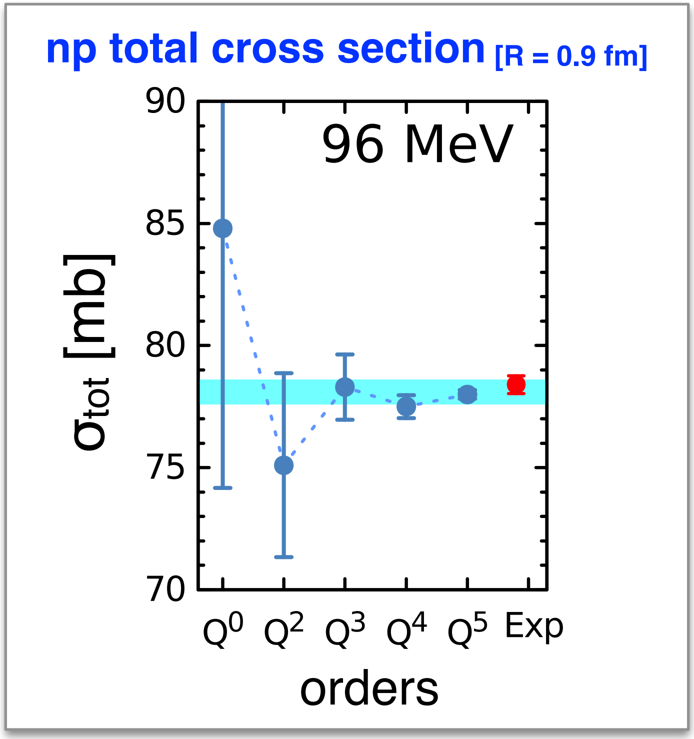
Notice: none of the existing 3NF models is able to reproduce 3N scattering observables [Kalantar-Naestanaki, EE, Messchendorp, Nogga, Rept. Prog. Phys. 75 (12) 016301]

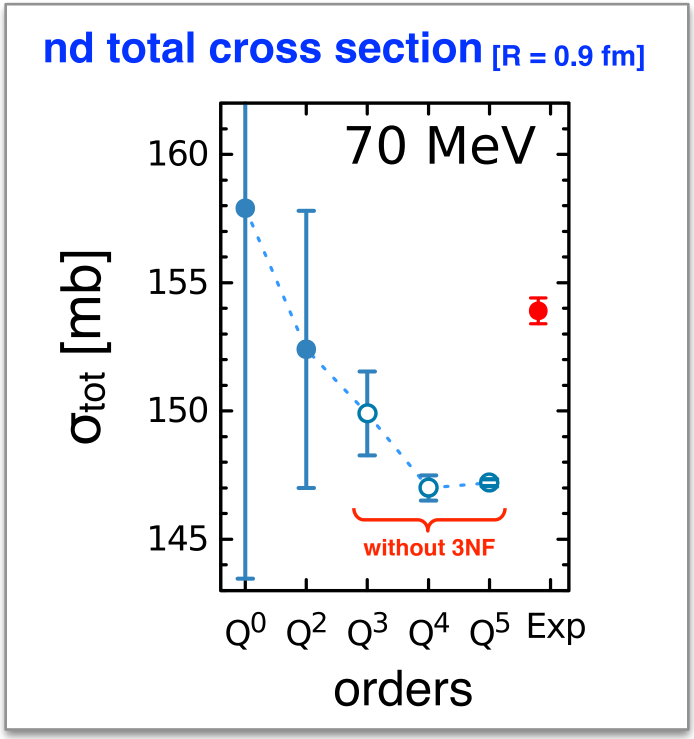
LENPIC: Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration

3N force: first insights

LENPIC, arXiv:1505.07218[nucl-th], to appear in PRC

Is there evidence for missing 3N forces effects? Yes!





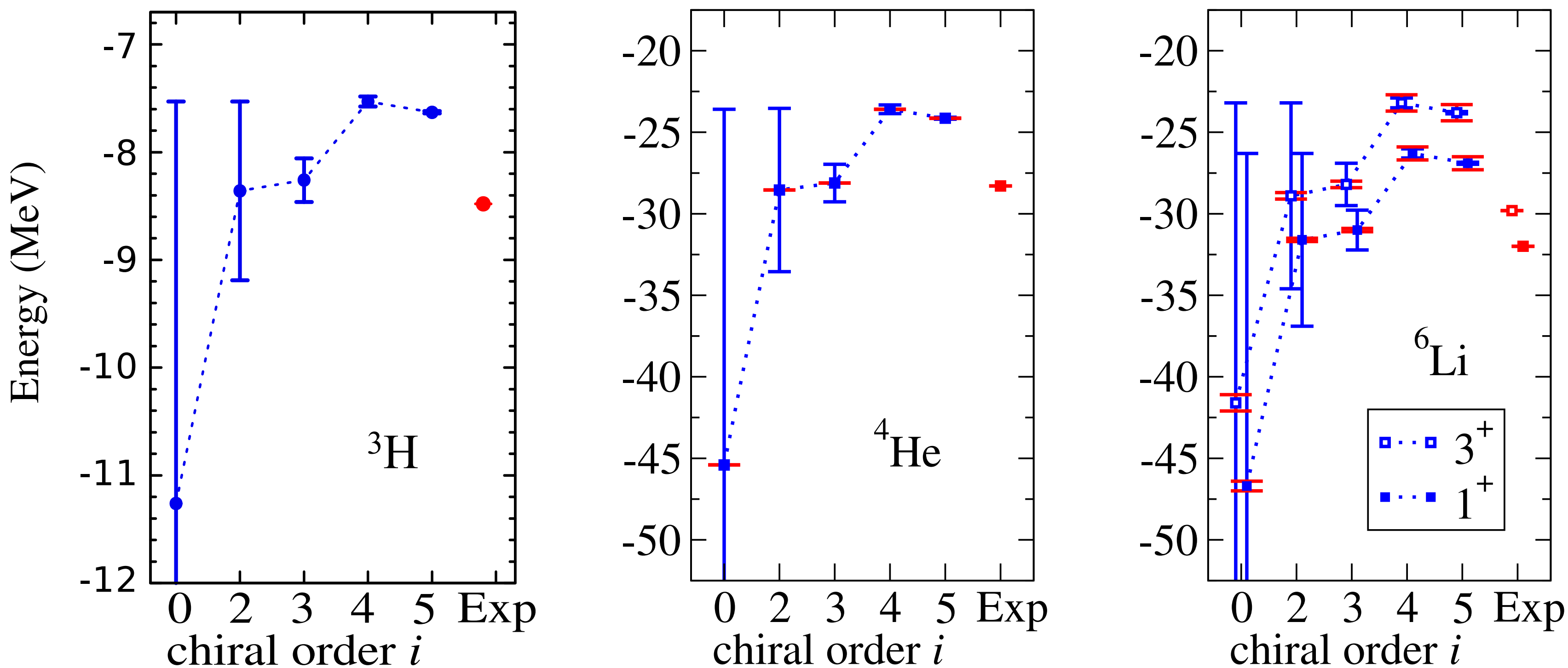
- Discrepancies between theory and data well outside the range of quantified uncertainties
  → **clear evidence for missing 3NF effects**
- Magnitude of the required 3NF contributions matches well the estimated size of N^2LO terms
  → **consistent with the chiral power counting**

LENPIC: Low Energy Nuclear Physics International Collaboration

3N force: first insights

LENPIC, arXiv:1505.07218[nucl-th], to appear in PRC

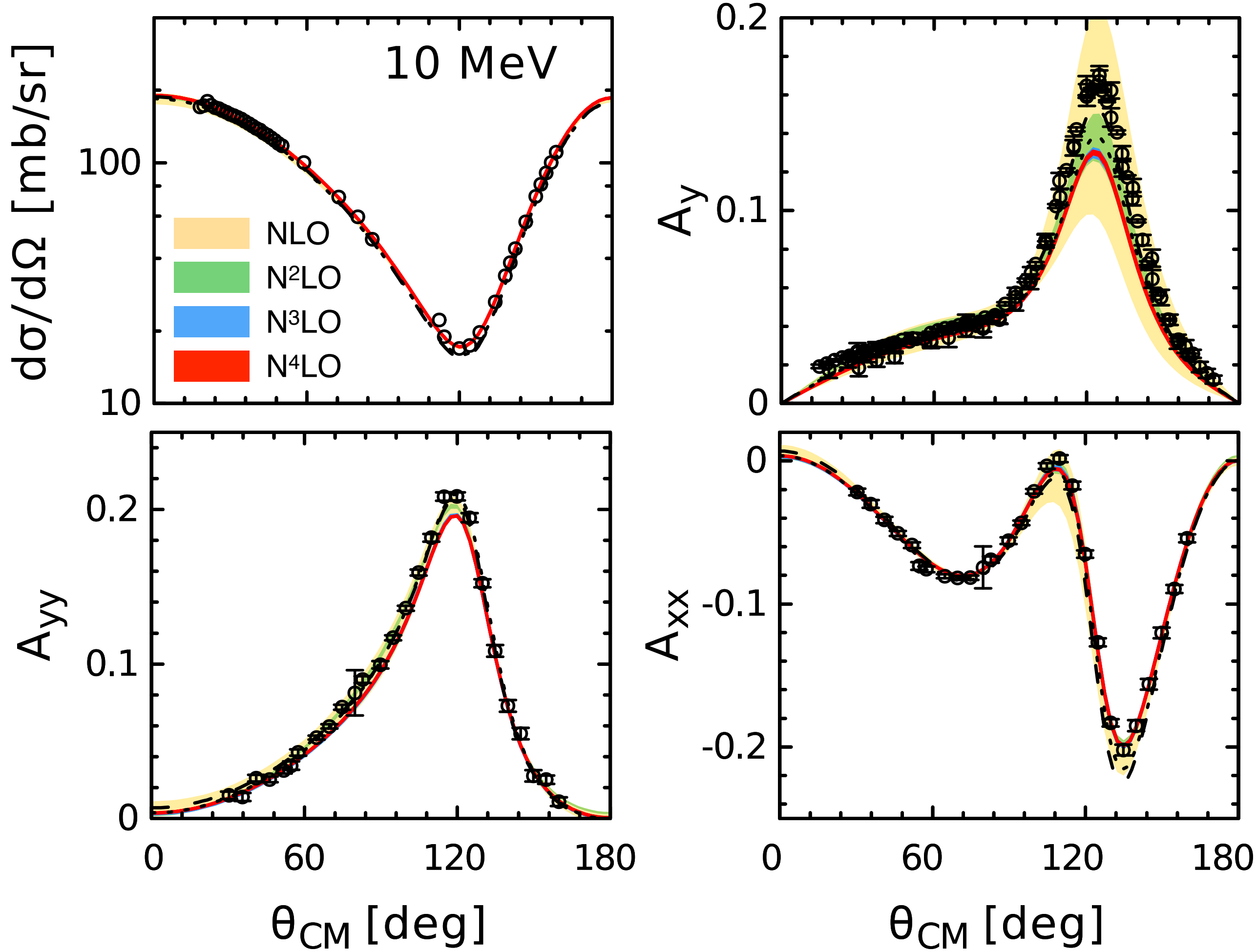
Similar pattern is observed for the properties of the light nuclei:



LENPIC: Low Energy Nuclear Physics International Collaboration

Nd elastic scattering at 10 MeV [r = 0.9 fm]

LENPIC, arXiv:1505.07218[nucl-th], to appear in PRC

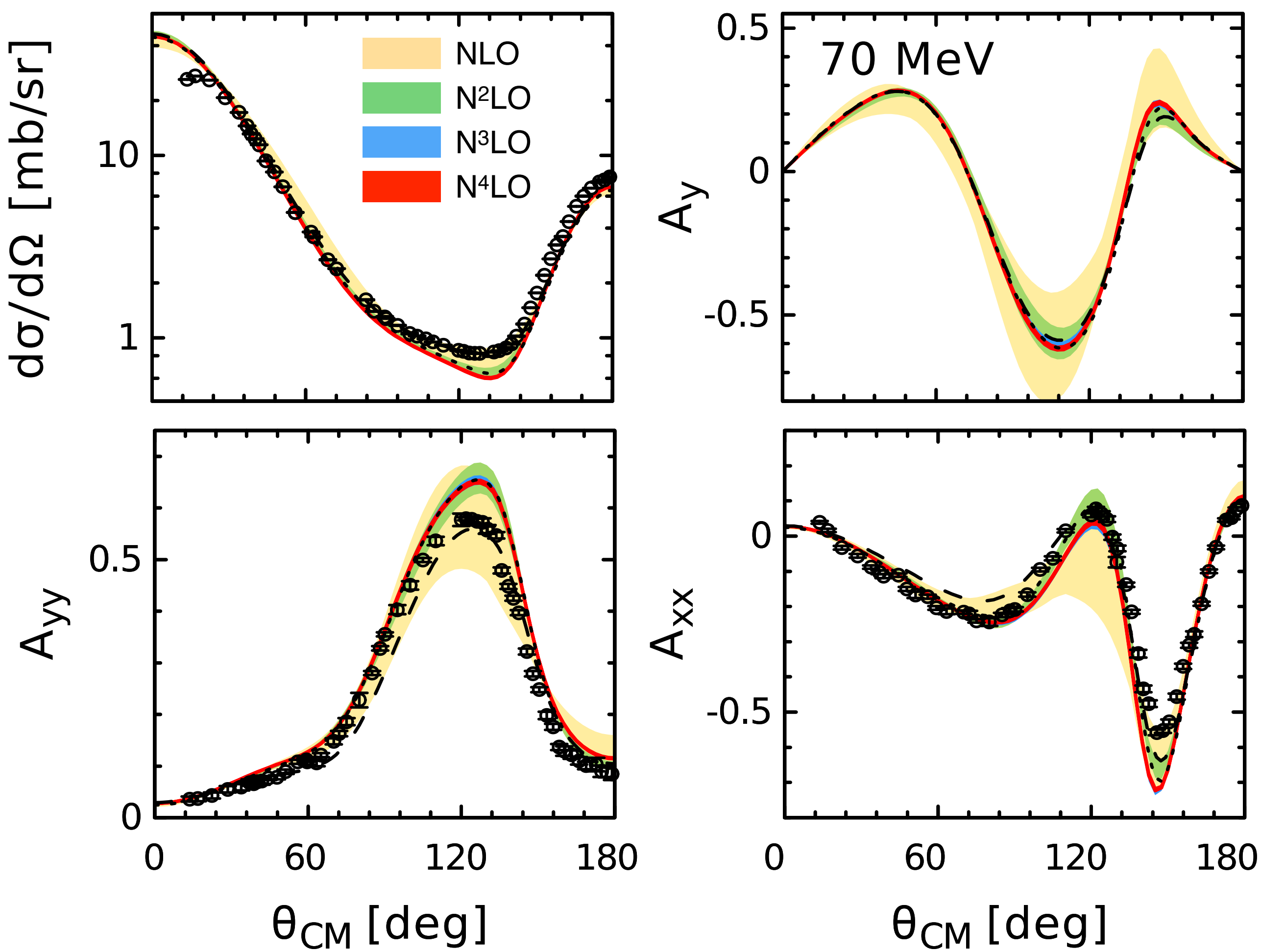


LENPIC: Low Energy Nuclear Physics International Collaboration

RUB · universität bonn · Technische Universität Darmstadt · Jagiellonian University in Krakow · Iowa State · Jülich Forschungszentrum · Kyutech · The Ohio State University · IPN Orsay · TRIUMF · Oak Ridge National Laboratory

Nd elastic scattering at 70 MeV [r = 0.9 fm]

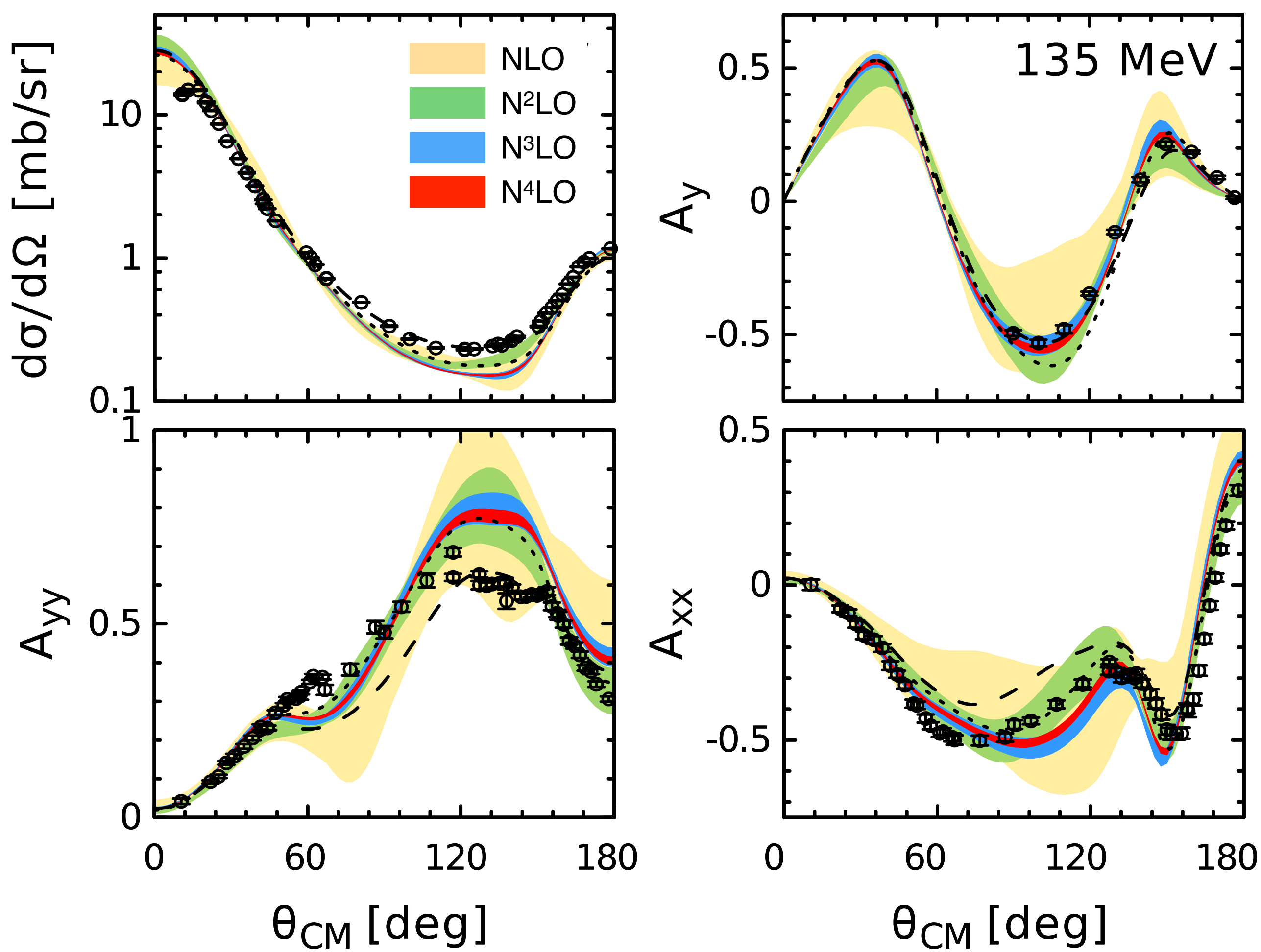
LENPIC, arXiv:1505.07218[nucl-th], to appear in PRC



LENPIC: Low Energy Nuclear Physics International Collaboration

Nd elastic scattering at 135 MeV [r = 0.9 fm]

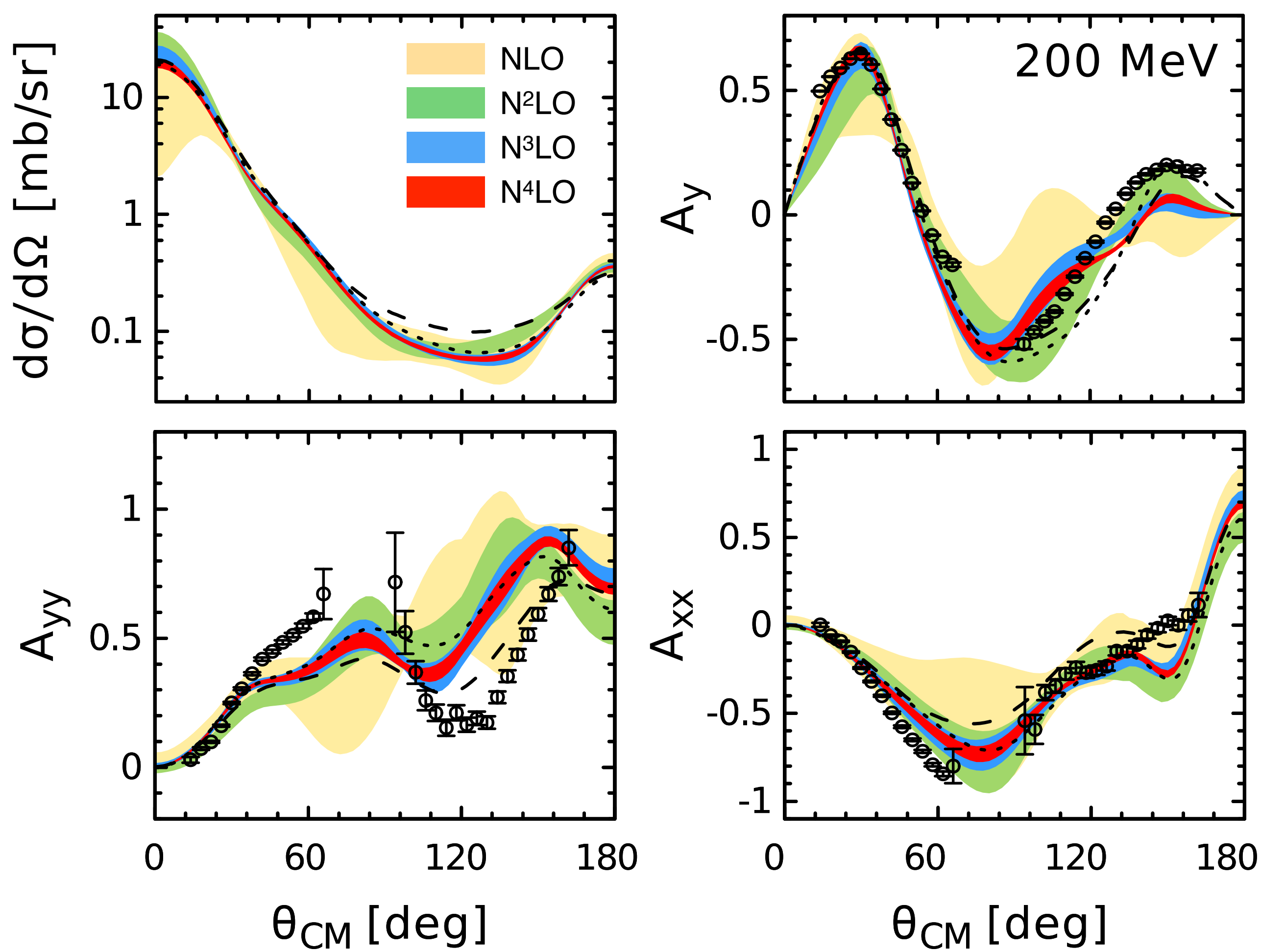
LENPIC, arXiv:1505.07218[nucl-th], to appear in PRC



LENPIC: Low Energy Nuclear Physics International Collaboration

Nd elastic scattering at 200 MeV [r = 0.9 fm]

LENPIC, arXiv:1505.07218[nucl-th], to appear in PRC



LENPIC: Low Energy Nuclear Physics International Collaboration

Nuclear lattice simulations:

A new ab initio approach to nuclei and nuclear reactions

D. Lee, EE, H. Krebs, T. Lähde, T. Luu, U.-G. Meißner, G. Rupak, …

Some recent highlights:

Ab initio calculation of the Hoyle state

EE, H. Krebs, D. Lee, U.-G. Meißner, PRL 106 (11) 192501;
EE, H. Krebs, T.A.Lähde, D. Lee, U.-G. Meißner, PRL 109 (12) 252501

Ab initio calculation of the spectrum and structure of ^{16}O

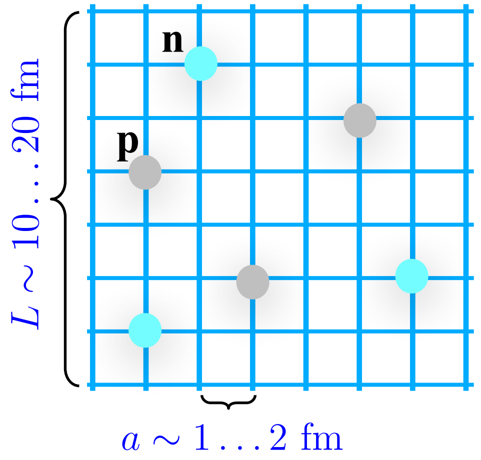
EE, H. Krebs, T. A. Lähde, D. Lee, U.-G. Meißner, G. Rupak, PRL 112 (14) 102501

Lattice EFT for medium-mass nuclei

T. A. Lähde, EE, H. Krebs, D. Lee, U.-G. Meißner, G. Rupak, PLB 732 (14) 110

Symmetry-sign extrapolations

T.A. Lähde, T. Luu, D. Lee, U.-G. Meißner, EE, H. Krebs, G. Rupak, EPJ A51 (15) 92



Ab initio alpha-alpha scattering

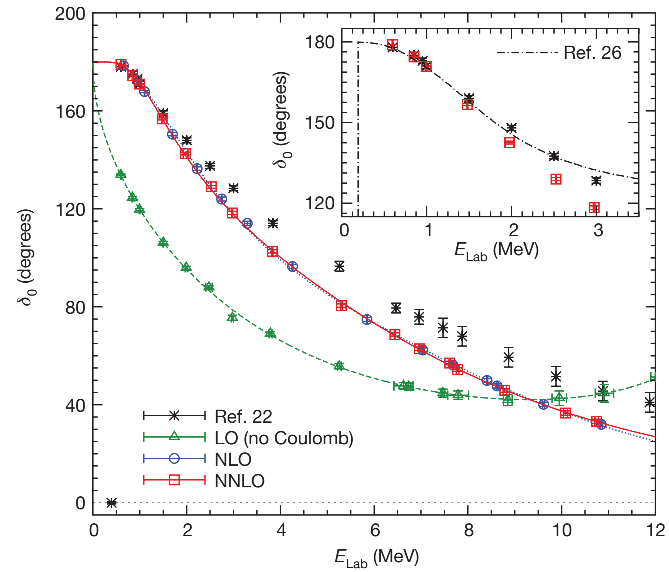
Elhatisari, Lee, Rupak, EE, Krebs, Lähde, Luu, Meißner, Nature 528 (2015) 111

nature International weekly journal of science

Ab initio alpha–alpha scattering

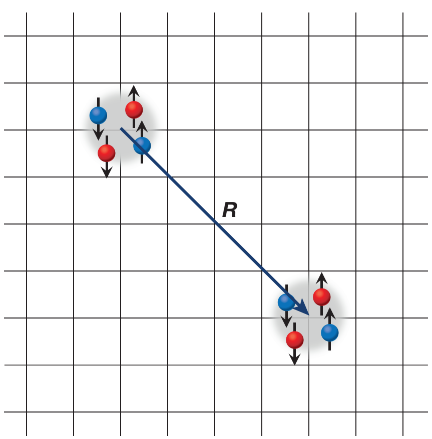
Serdar Elhatisari[1], Dean Lee[2], Gautam Rupak[3], Evgeny Epelbaum[4], Hermann Krebs[4], Timo A. Lähde[5], Thomas Luu[1,5] & Ulf-G. Meißner[1,5,6]

Nature **528**, 111–114 (03 December 2015) | doi:10.1038/nature16067
Received 12 June 2015 | Accepted 30 September 2015 | Published online 02 December 2015



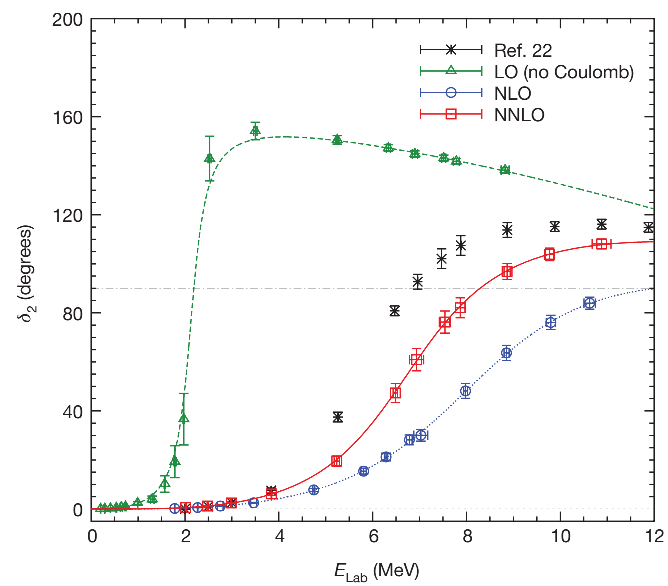
First ab initio calculation of alpha-alpha scattering!

Used lattice EFT to extract the effective Hamiltonian for two interacting α-clusters (adiabatic projection method [A. Rokash et al., PRC 92 (15) 054612])



Phase shifts obtained employing a hard spherical wall boundary at asymptotically large distances

Promising scaling with respect to the number of particles as $\sim (A_1 + A_2)^2$

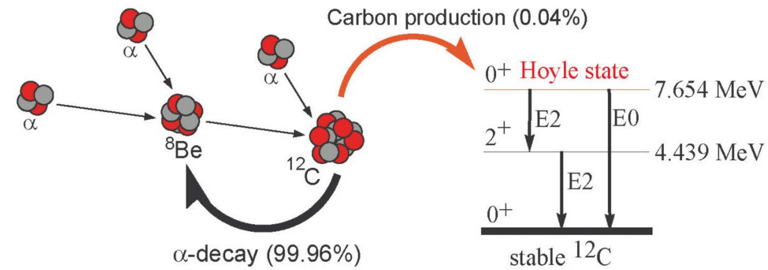


Dependence of the 3α reaction on m_q

EE, Krebs, Lähde, Lee, Meißner, PRL 110 (2013) 112502; EPJA 49 (2013)

Production of ^{12}C in stars depends sensitively on $\varepsilon \equiv E_{12}^{\star} - 3E_4 = 379.47(18)\ \mathrm{keV}$

$$r_{3\alpha} \simeq 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_{\rm B} T}\right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_{\rm B}T}\right)$$



Changing ε by ~100 keV destroys production of either ^{12}C or ^{16}O [Livio et al.'89]:

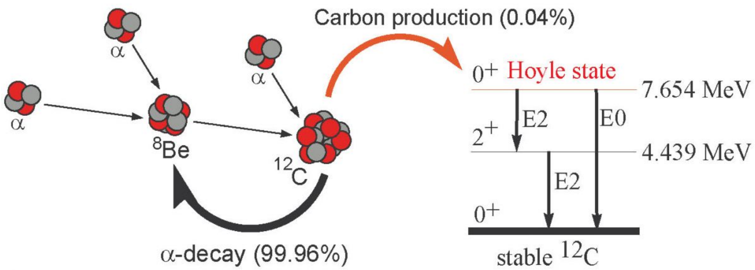
A good candidate for the anthropic principle?

Dependence of the 3α reaction on m_q

EE, Krebs, Lähde, Lee, Meißner, PRL 110 (2013) 112502; EPJA 49 (2013)

Production of ^{12}C in stars depends sensitively on $\varepsilon \equiv E_{12}^{\star} - 3E_4 = 379.47(18)\ \mathrm{keV}$

$$r_{3\alpha} \simeq 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_{\rm B} T}\right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_{\rm B}T}\right)$$

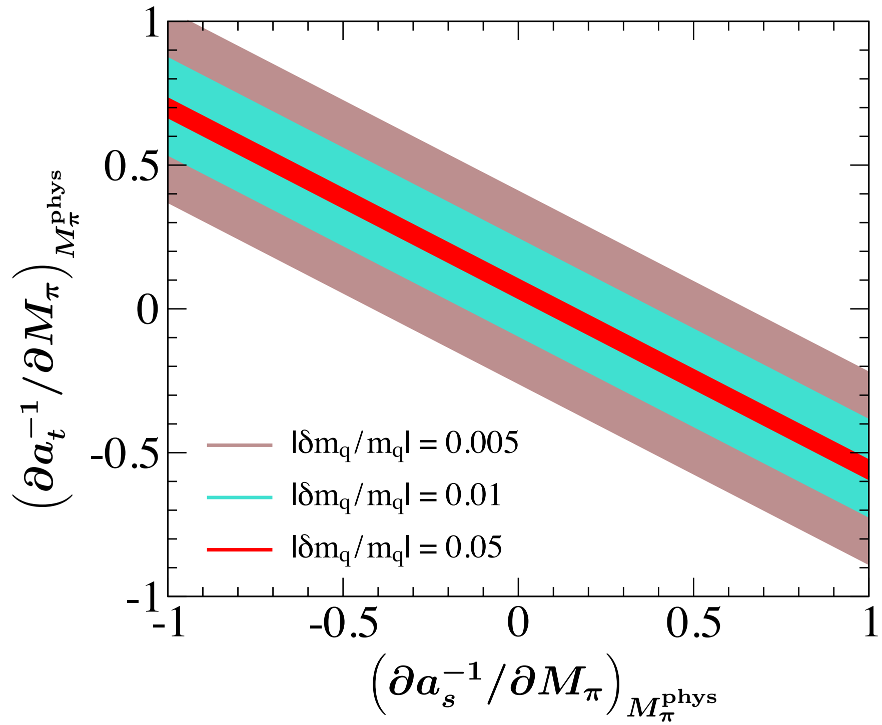


Changing ε by ~100 keV destroys production of either ^{12}C or ^{16}O [Livio et al.'89]:
A good candidate for the anthropic principle?

Input: the slopes of the inverse S-wave scattering lengths:

$$\left(\partial a_s^{-1}/\partial M_\pi\right)_{M_\pi^{\rm phys}} \qquad \left(\partial a_t^{-1}/\partial M_\pi\right)_{M_\pi^{\rm phys}}$$

Output: „Survivability bands“ ⟶

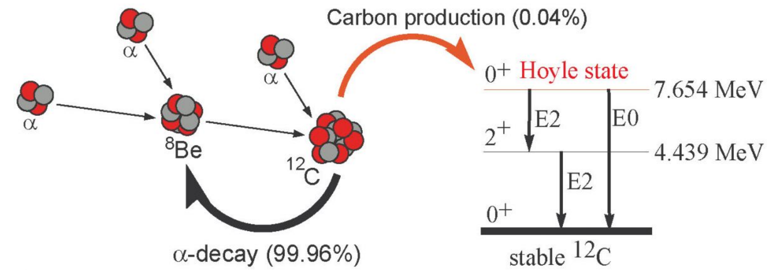


Dependence of the 3α reaction on m_q

EE, Krebs, Lähde, Lee, Meißner, PRL 110 (2013) 112502; EPJA 49 (2013)

Production of ^{12}C in stars depends sensitively on $\varepsilon \equiv E_{12}^{\star} - 3E_4 = 379.47(18)\ \mathrm{keV}$

$$r_{3\alpha} \simeq 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_\mathrm{B} T}\right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_\mathrm{B} T}\right)$$

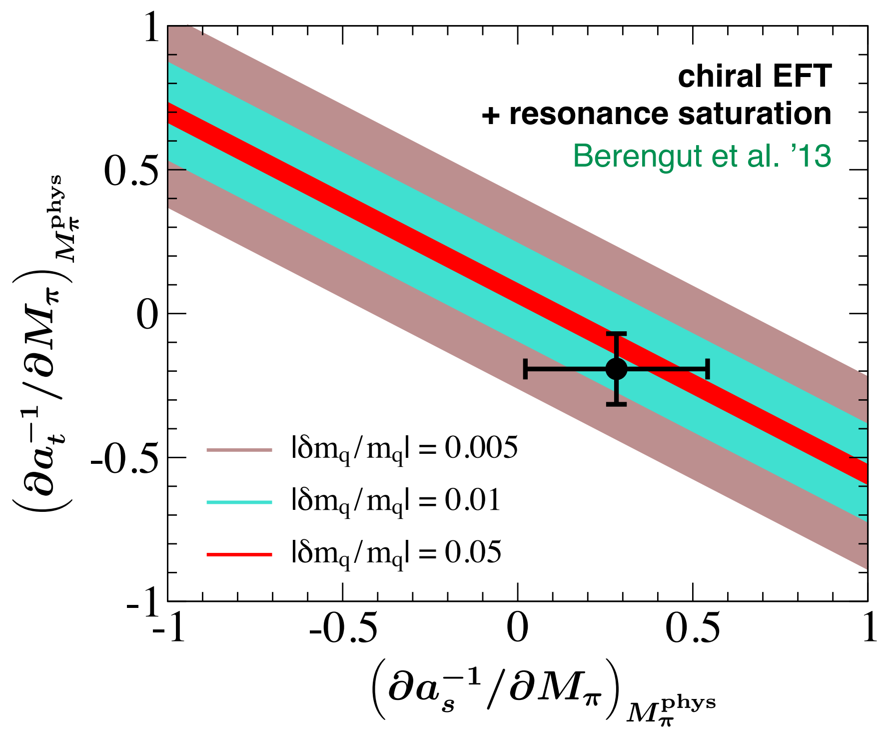


Changing ε by ~100 keV destroys production of either ^{12}C or ^{16}O [Livio et al.'89]:
A good candidate for the anthropic principle?

Input: the slopes of the inverse S-wave scattering lengths:

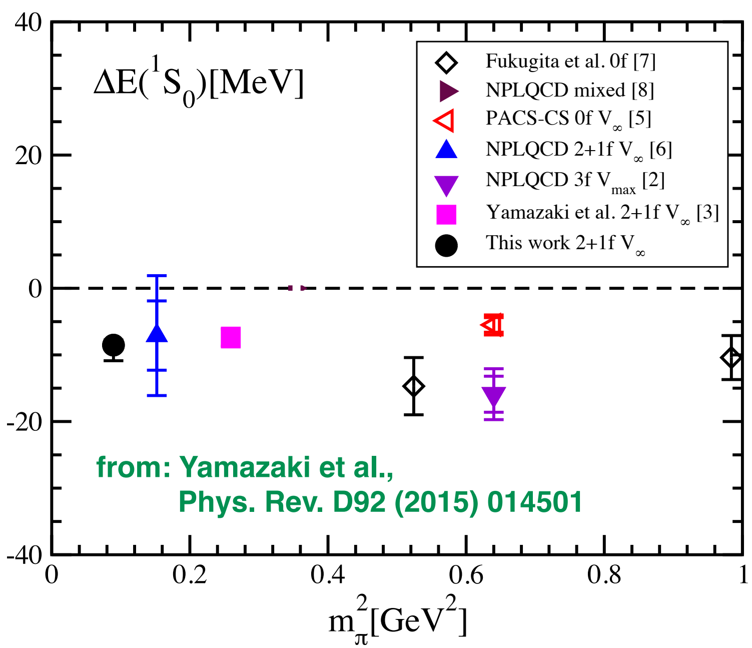
$$\left(\partial a_s^{-1}/\partial M_\pi\right)_{M_\pi^\mathrm{phys}} \qquad \left(\partial a_t^{-1}/\partial M_\pi\right)_{M_\pi^\mathrm{phys}}$$

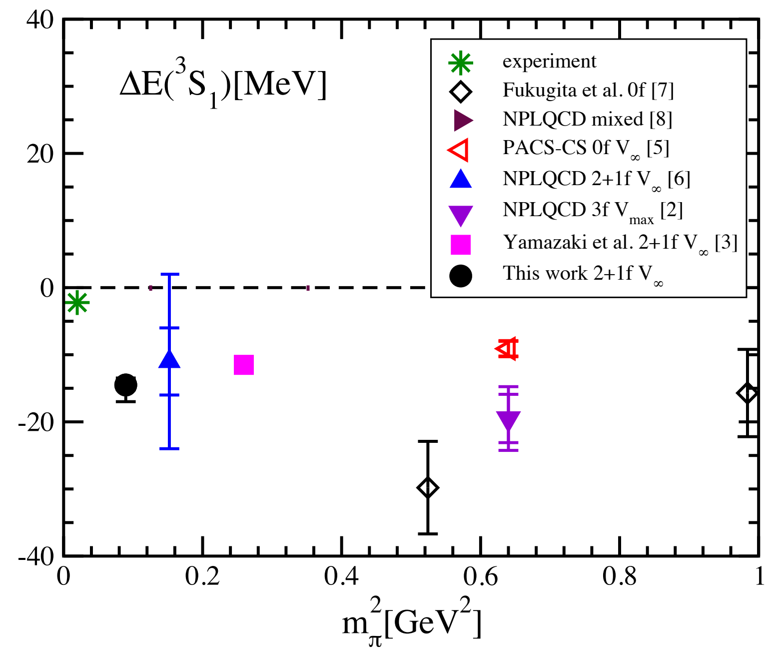
Output: „Survivability bands" ⟶



For more decisive conclusions, need lattice-QCD calculations of the quark mass dependence of the scattering lengths!

Lattice QCD for the NN system



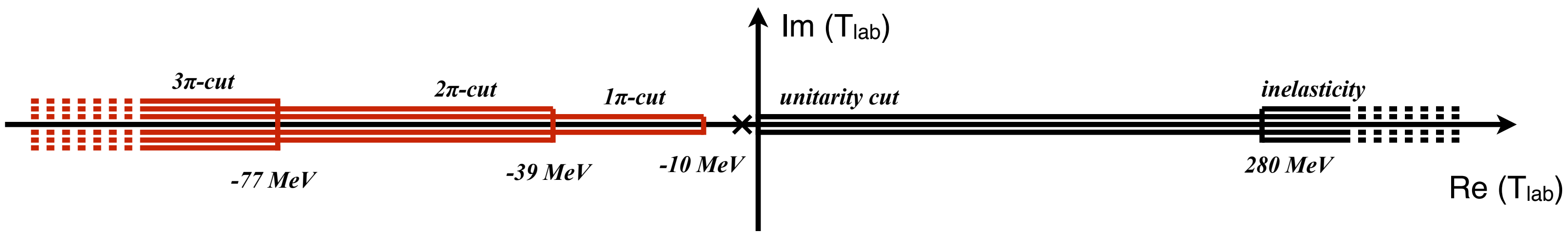


Further, the HAL QCD Collaboration finds no bound states for $M_\pi > 411$ MeV

Tools to extrapolate/check consistency of lattice QCD results

- **Pionless EFT:** extrapolate in the number of nucleons [large M_π] Barnea, Kirscher, van Kolck, …
- **Chiral EFT:** extrapolate in M_π (and the number of nucleons) [small M_π]
  Beane, Savage, EE, Glöckle, Meißner, Gegelia, Soto, Chen, …
- **Low-Energy Theorems (LETs) for the NN system:** extrapolate in energy at fixed M_π
  Baru, EE, Filin, Gegelia, PRC 92 (2015) 014001

The long-range interaction (1π) governs the low-energy behavior of the amplitude and implies correlations between coefficients in the ERE which may be regarded as Low Energy Theorems

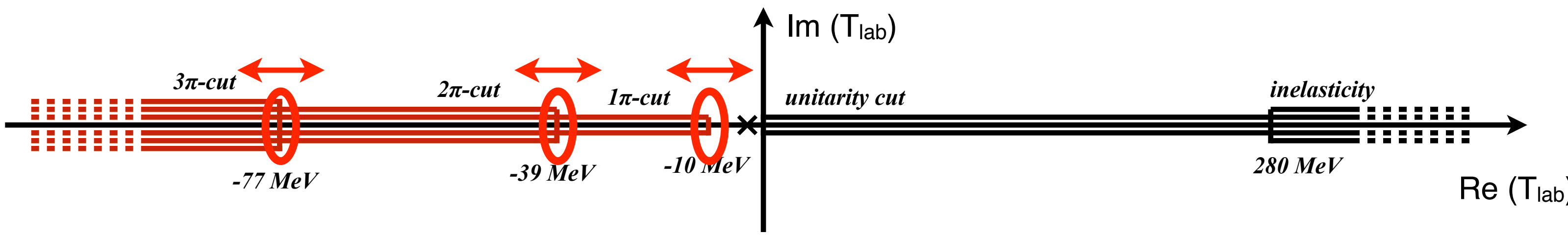


— very good / fair predictive power in the $^3S_1$ / $^1S_0$ channel at physical M_π

Tools to extrapolate/check consistency of lattice QCD results

- **Pionless EFT:** extrapolate in the number of nucleons [large M_π] Barnea, Kirscher, van Kolck, …
- **Chiral EFT:** extrapolate in M_π (and the number of nucleons) [small M_π]
  Beane, Savage, EE, Glöckle, Meißner, Gegelia, Soto, Chen, …
- **Low-Energy Theorems (LETs) for the NN system:** extrapolate in energy at fixed M_π
  Baru, EE, Filin, Gegelia, PRC 92 (2015) 014001

The long-range interaction (1π) governs the low-energy behavior of the amplitude and implies correlations between coefficients in the ERE which may be regarded as Low Energy Theorems



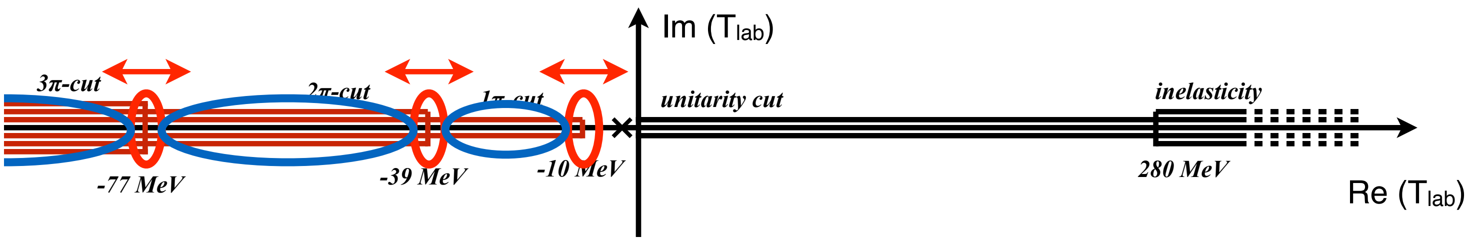
— very good / fair predictive power in the $^3S_1$ / $^1S_0$ channel at physical M_π

— when going to unphysical pion masses, the main change in the left-hand singularities is due to threshold shifts (explicit M_π-dependence)

Tools to extrapolate/check consistency of lattice QCD results

- **Pionless EFT:** extrapolate in the number of nucleons [large M_π] Barnea, Kirscher, van Kolck, …
- **Chiral EFT:** extrapolate in M_π (and the number of nucleons) [small M_π]
  Beane, Savage, EE, Glöckle, Meißner, Gegelia, Soto, Chen, …
- **Low-Energy Theorems (LETs) for the NN system:** extrapolate in energy at fixed M_π
  Baru, EE, Filin, Gegelia, PRC 92 (2015) 014001

The long-range interaction (1π) governs the low-energy behavior of the amplitude and implies correlations between coefficients in the ERE which may be regarded as Low Energy Theorems



— very good / fair predictive power in the $^3S_1$ / $^1S_0$ channel at physical M_π

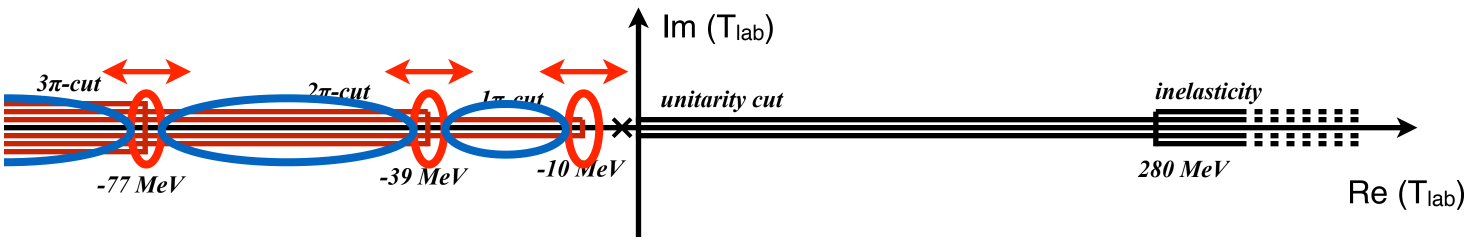
— when going to unphysical pion masses, the main change in the left-hand singularities is due to threshold shifts (explicit M_π-dependence)

— changes in discontinuity across the left-hand cuts (M_π-dependence of g_A, F_π, m_N) are known from lattice QCD

Tools to extrapolate/check consistency of lattice QCD results

- **Pionless EFT:** extrapolate in the number of nucleons [large M_π] Barnea, Kirscher, van Kolck, …
- **Chiral EFT:** extrapolate in M_π (and the number of nucleons) [small M_π]
  Beane, Savage, EE, Glöckle, Meißner, Gegelia, Soto, Chen, …
- **Low-Energy Theorems (LETs) for the NN system:** extrapolate in energy at fixed M_π
  Baru, EE, Filin, Gegelia, PRC 92 (2015) 014001

  The long-range interaction (1π) governs the low-energy behavior of the amplitude and implies correlations between coefficients in the ERE which may be regarded as Low Energy Theorems



— very good / fair predictive power in the $^3S_1$ / $^1S_0$ channel at physical M_π

— when going to unphysical pion masses, the main change in the left-hand singularities is due to threshold shifts (explicit M_π-dependence)

— changes in discontinuity across the left-hand cuts (M_π-dependence of g_A, F_π, m_N) are known from lattice QCD

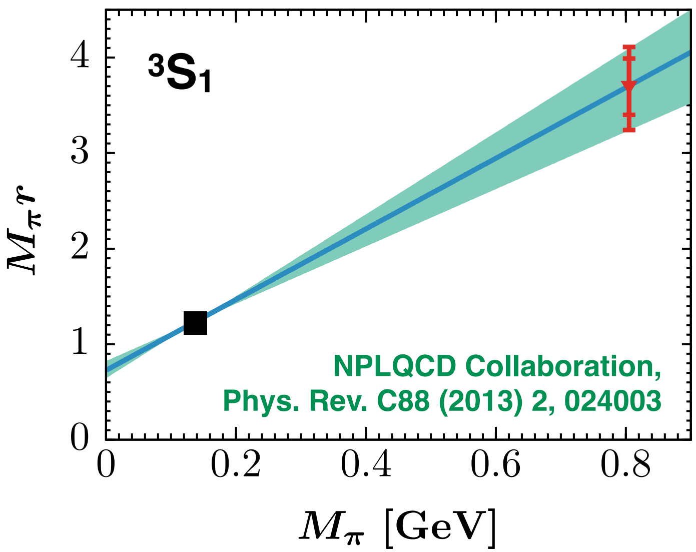
— need a single lattice data (e.g. BE) as input at a given M_π to reconstruct the amplitude…

Notice: No reliance on the chiral expansion, $M_\pi \to \infty$ limit well defined!

Low-energy theorems for NN scattering

- Is the conjectured linear M_π-behavior of $M_\pi\, r^{(3S1)}$ consistent with the trend in BEs?

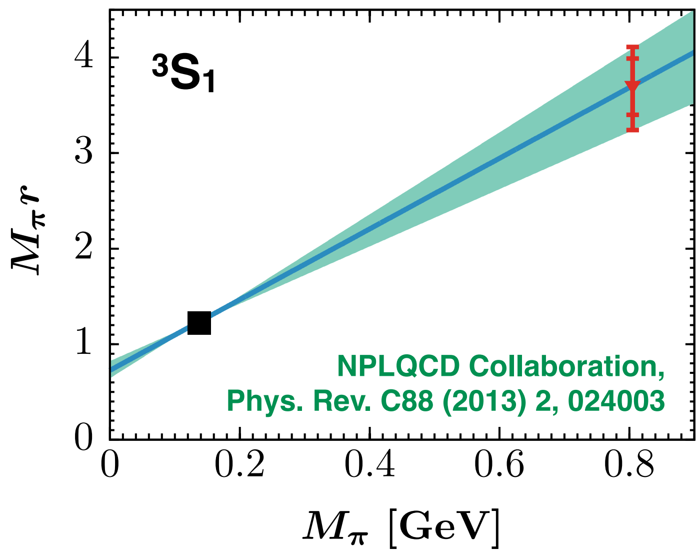
  Baru, EE, Filin, Gegelia '15

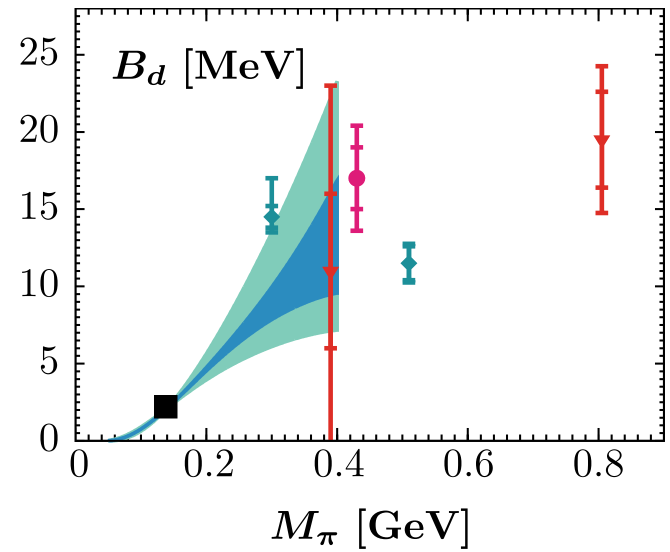


Low-energy theorems for NN scattering

- Is the conjectured linear M_π-behavior of $M_\pi\, r^{(3S1)}$ consistent with the trend in BEs?

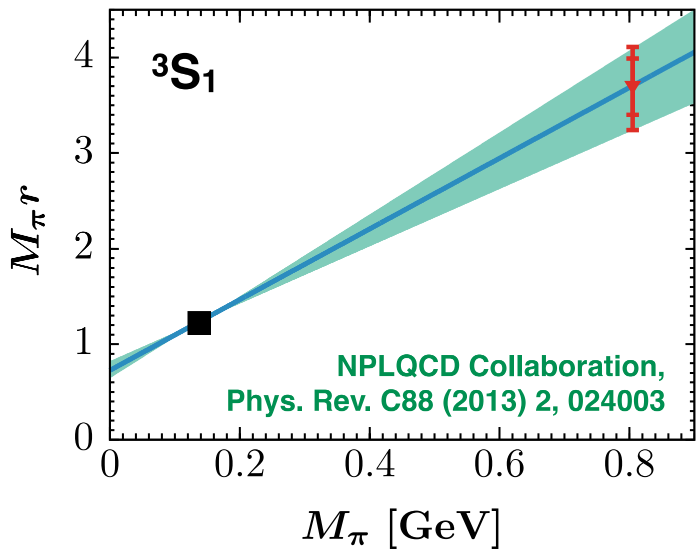
Baru, EE, Filin, Gegelia '15



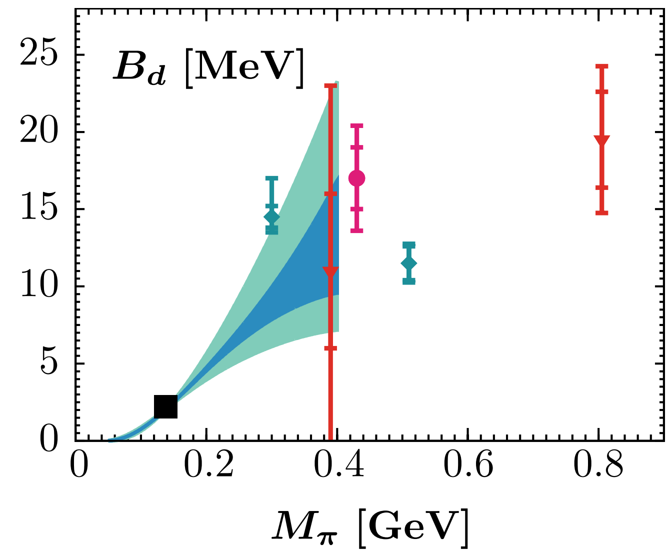


Low-energy theorems for NN scattering

- Is the conjectured linear M_π-behavior of $M_\pi\, r^{(3S1)}$ consistent with the trend in BEs?
  Baru, EE, Filin, Gegelia '15



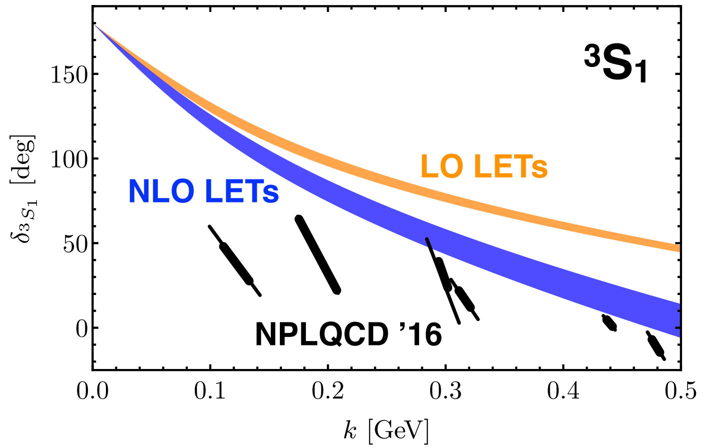
LETs →



- Are the NPLQCD results for BE & phase shifts @ M_π=450 MeV consistent?
  Baru, EE, Filin, to appear

Use $B_d = 14.4\left(^{+3.2}_{-2.6}\right)$ MeV [Beane et al.'16]
as input to predict phase shifts via LETs

NPLQCD results for phase shifts at the two lowest energies are incompatible with their results for B_d: Underestimated systematics??



Summary

A new generation of chiral NN potentials up to N^{4}LO

— excellent description of NN data

— good convergence of the chiral expansion

A simple approach to estimate theoretical uncertainty at a given order

— applicable to any observable and for a particular choice of the regulator

— results in the NN system at all orders and for all cutoffs are consistent with each other and with experimental data (within uncertainties)

Application to few-N systems

— clear evidence for missing 3NF effects within our scheme

— Nd scattering at 50…150 MeV: a golden window to test the 3NF (in progress)

Nuclear lattice simulations

— promising novel approach to nuclei & reactions (first ab initio αα scattering!)

Lattice QCD & nuclear physics

— EFT, LETs: useful tools to extend/test lattice-QCD results

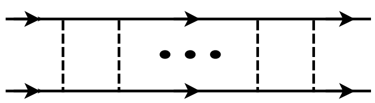
Good progress towards reliable ab initio description of nuclear systems based on chiral EFT with quantified theoretical uncertainties!

spares…

Regularization, renormalization and all that...

$$T(\vec{p}\,', \vec{p}) = V_{2\mathrm{N}}(\vec{p}\,', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2\mathrm{N}}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon}$$ with $$V_{2\mathrm{N}} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

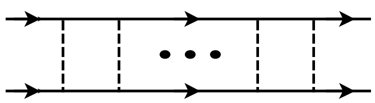
→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



Regularization, renormalization and all that...

$$T(\vec{p}\,', \vec{p}) = V_{2N}(\vec{p}\,', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



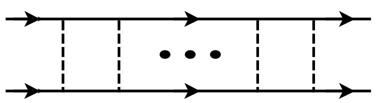
Possible approaches:

- Use a manifestly Lorentz-invariant approach (3D-eqs. fulfilling relativistic unitarity)
  EE, Gegelia'12,'13; EE, Gasparyan, Gegelia, Krebs, Schindler '14,'15
  - — integral eq. is renormalizable at LO (only log-divergences), Λ can be removed!
  - — Caveat: calculations are complicated, hard to go beyond the NN system…

Regularization, renormalization and all that...

$$T(\vec{p}\,', \vec{p}) = V_{2N}(\vec{p}\,', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



Possible approaches:

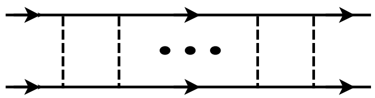
- Use a manifestly Lorentz-invariant approach (3D-eqs. fulfilling relativistic unitarity)
  EE, Gegelia'12,'13; EE, Gasparyan, Gegelia, Krebs, Schindler '14,'15
  - — integral eq. is renormalizable at LO (only log-divergences), Λ can be removed!
  - — Caveat: calculations are complicated, hard to go beyond the NN system…

- Use a finite UV cutoff (most frequently used) Lepage '97
  - — simple, well suited for few- and many-body calculations
  - — it is not legitimate to take Λ well above Λ_b (unless all UV div. can be subtracted)
    Lepage '97; EE, Gegelia '09
  - — Caveat: finite-cutoff artifacts…

Regularization, renormalization and all that...

$$T(\vec{p}\,', \vec{p}) = V_{2N}(\vec{p}\,', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



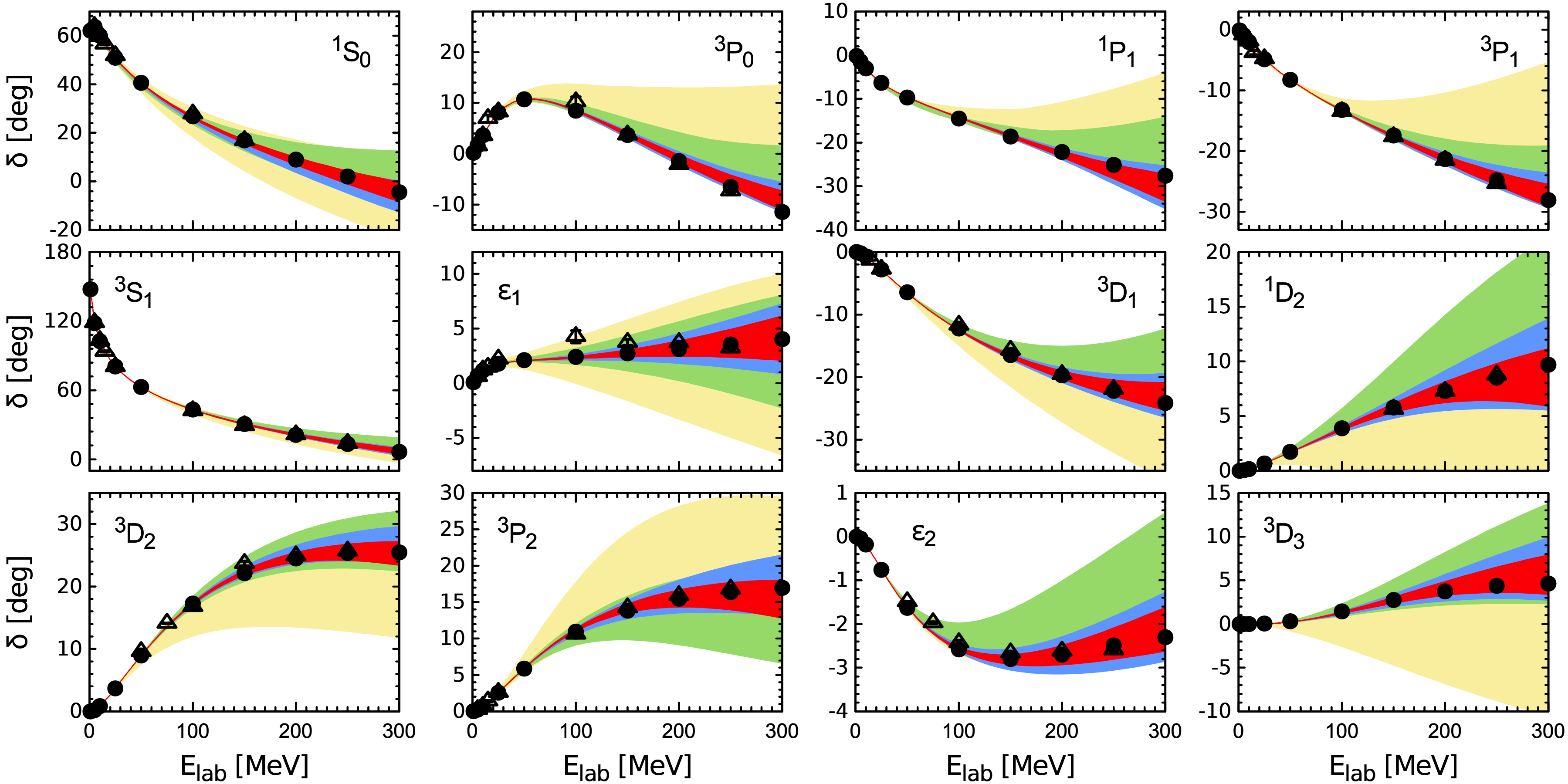
Possible approaches:

- Use a manifestly Lorentz-invariant approach (3D-eqs. fulfilling relativistic unitarity)
  EE, Gegelia'12,'13; EE, Gasparyan, Gegelia, Krebs, Schindler '14,'15
  - — integral eq. is renormalizable at LO (only log-divergences), Λ can be removed!
  - — Caveat: calculations are complicated, hard to go beyond the NN system…

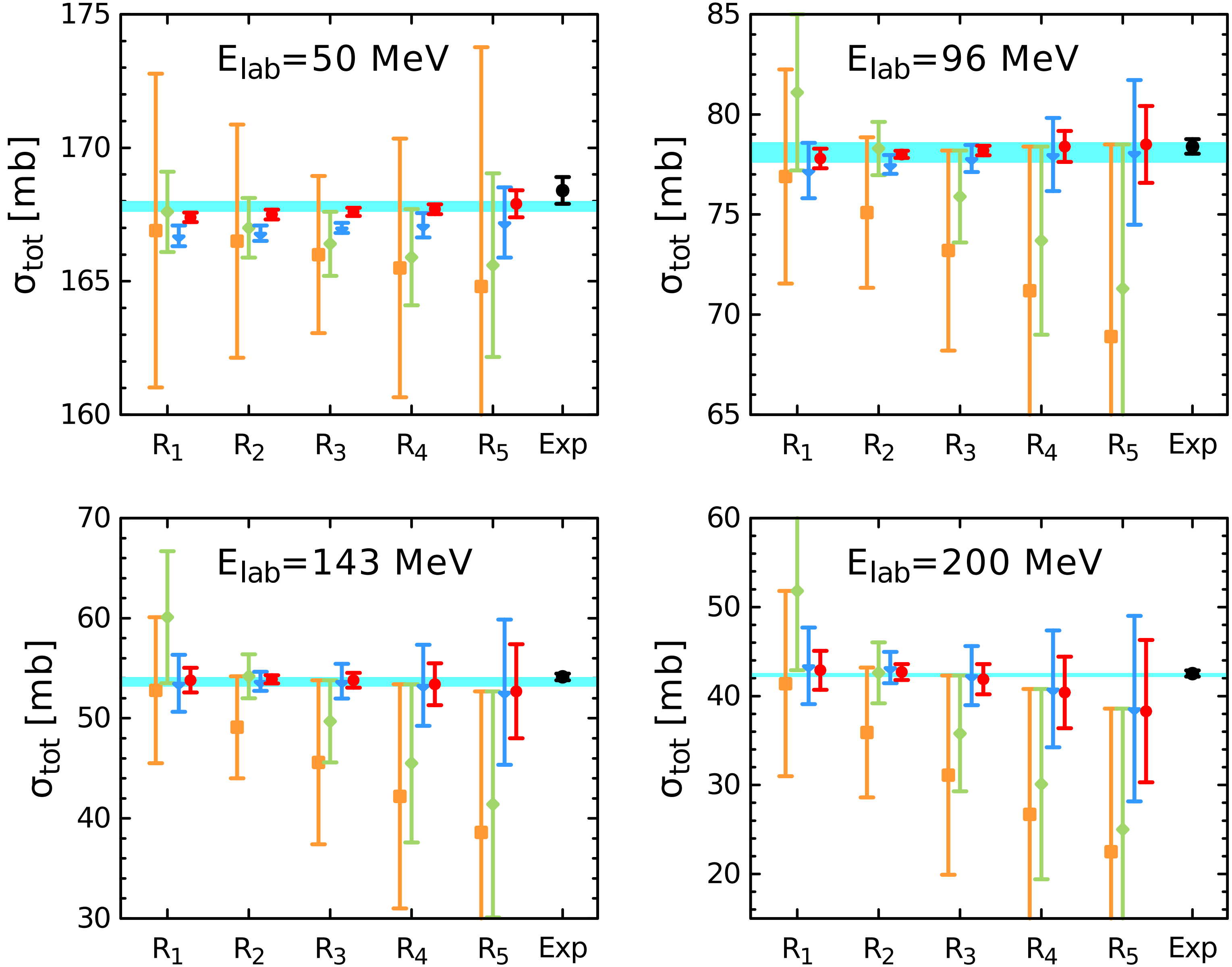
- Use a finite UV cutoff (most frequently used) Lepage '97
  - — simple, well suited for few- and many-body calculations
  - — it is not legitimate to take Λ well above Λ_b (unless all UV div. can be subtracted)
    Lepage '97; EE, Gegelia '09
  - — Caveat: finite-cutoff artifacts…

(Implicit) renormalization: express bare LECs in terms of observables (phase shifts)
Notice: LECs **have to be** refitted at each chiral order

Neutron-proton phase shifts



Neutron-proton total cross section



Regulator (in)dependence

How do our results depend on the specific form of the regulator $f\left(\frac{r}{R}\right) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n$

and/or additional spectral function regularization $V_C(q) = \frac{2}{\pi}\int_{2M_\pi}^{\Lambda_{\rm SFR}} d\mu\,\mu\,\frac{\rho_C(\mu)}{\mu^2+q^2}$

Selected phase shifts (in deg.) for different values of $\Lambda_{\rm SFR}$ and n at $N^3LO_{[R = 0.9\ {\rm fm}]}$

Lab. energy	NPWA	our result	DR, $n=5$	DR, $n=7$	SFR, 1.0 GeV	SFR, 1.5 GeV	SFR, 2.0 GeV
proton-proton $^1S_0$ phase shift							
10 MeV	55.23	55.22 ± 0.08	55.22	55.22	55.22	55.22	55.22
100 MeV	24.99	24.98 ± 0.60	24.98	24.98	24.98	24.98	24.98
200 MeV	6.55	6.56 ± 2.2	6.55	6.56	6.56	6.56	6.57
neutron-proton $^3S_1$ phase shift							
10 MeV	102.61	102.61 ± 0.07	102.61	102.61	102.61	102.61	102.61
100 MeV	43.23	43.22 ± 0.30	43.28	43.20	43.17	43.21	43.22
200 MeV	21.22	21.2 ± 1.4	21.2	21.2	21.2	21.2	21.2
proton-proton $^3P_0$ phase shift							
10 MeV	3.73	3.75 ± 0.04	3.75	3.75	3.75	3.75	3.75
100 MeV	9.45	9.17 ± 0.30	9.15	9.18	9.18	9.17	9.17
200 MeV	-0.37	-0.1 ± 2.3	-0.1	-0.1	-0.1	-0.1	-0.1
proton-proton $^3P_1$ phase shift							
10 MeV	-2.06	-2.04 ± 0.01	-2.04	-2.04	-2.04	-2.04	-2.04
100 MeV	-13.26	-13.42 ± 0.17	-13.43	-13.41	-13.41	-13.42	-13.42
200 MeV	-21.25	-21.2 ± 1.6	-21.2	-21.2	-21.2	-21.2	-21.2
proton-proton $^3P_2$ phase shift							
10 MeV	0.65	0.65 ± 0.01	0.66	0.65	0.65	0.65	0.65
100 MeV	11.01	11.03 ± 0.50	10.97	11.06	11.07	11.05	11.04
200 MeV	15.63	15.6 ± 1.9	15.6	15.5	15.5	15.5	15.6

→ negligible regulator dependence (compared to the estimated theor. accuracy)

Deuteron properties R=0.9 fm

EE, Krebs, Meißner, arXiv:1412.0142 [nucl-th], arXiv:1412.4623 [nucl-th]

	LO	NLO	N	N	N	empirical
B	2.0235	2.1987	2.2311	2.2246*	2.2246*	2.224575(9)
A	0.8333	0.8772	0.8865	0.8845	0.8844	0.8846(9)
η	0.0212	0.0256	0.0256	0.0255	0.0255	0.0256(4)
r_d	1.990	1.968	1.966	1.972	1.972	1.97535(85)
Q [fm	0.230	0.273	0.270	0.271	0.271	0.2859(3)
P_D	2.54	4.73	4.50	4.19	4.29	

— fast convergence of the chiral expansion (P_D is not observable)

— error estimation (assuming $Q=M_\pi/\Lambda_b$)

A_S: LO: 0.83(5) → NLO: 0.878(13) → N²LO: 0.887(3) → N³LO: 0.8845(8) → N⁴LO: 0.8844(2)

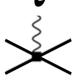
η: LO: 0.021(5) → NLO: 0.026(1) → N²LO: 0.0256(3) → N³LO: 0.0255(1) → N⁴LO: 0.0255

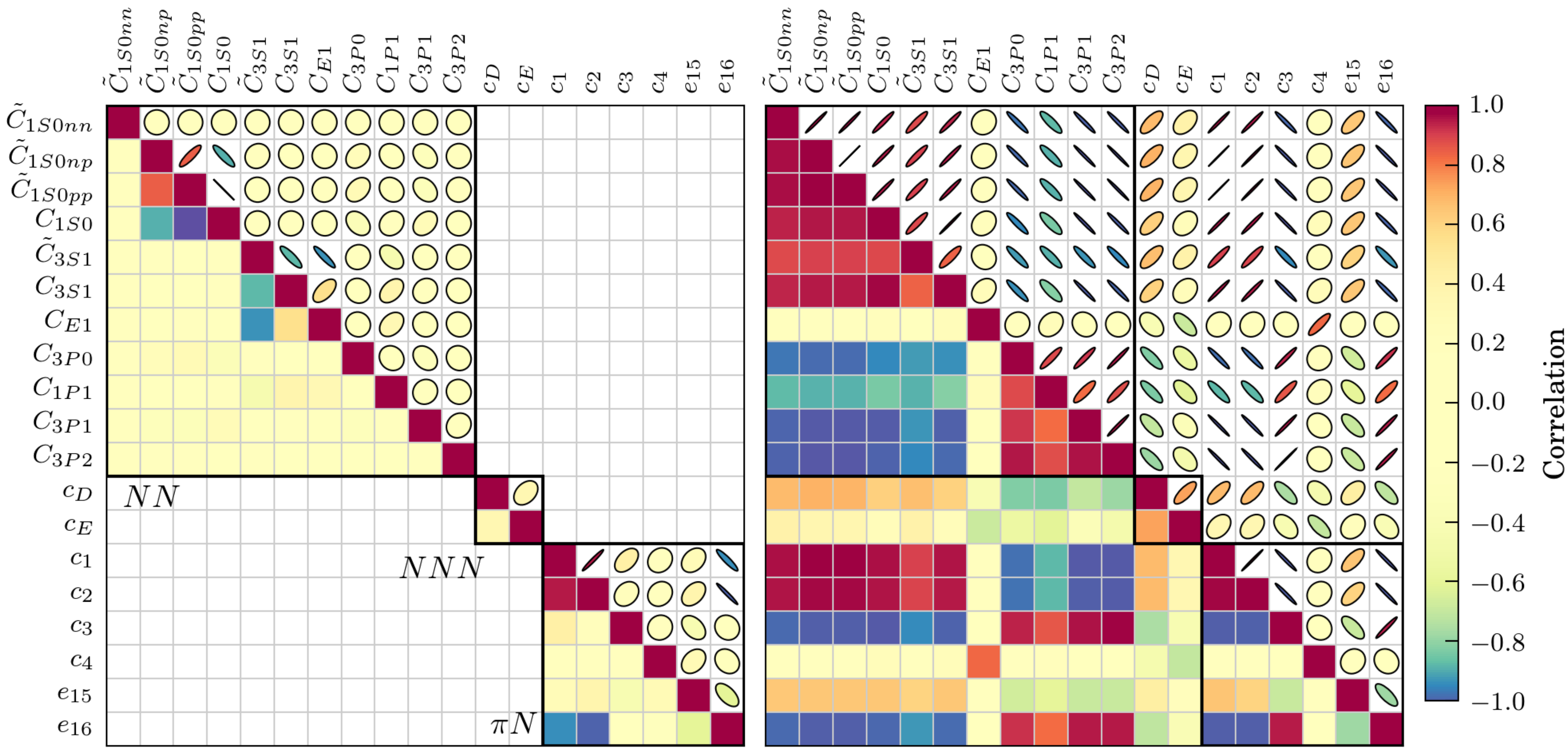
→ theoretical results for A_S,η at N⁴LO are more accurate than empirical numbers

— results for r_d and Q do not take into account MECs and relativistic corrections:

r_d: $|\Delta r_d| \simeq 0.004$ fm [Kohno '83] → predictions in agreement with the data

Q: rel. corrections + 1π-exchange MEC: $\Delta Q \simeq +0.008\ \mathrm{fm}^2$ [Phillips '07] → $Q \simeq 0.279\ \mathrm{fm}^2$

the remaining deviation of 0.007 fm² agrees with the expected size of  [Phillips '07]



No justification for making combined πN and NN fits!

Table VIII. NNLOsep and NNLOsim ($\Lambda = 500, T_{\max} = 290$)

LEC	NNLOsep	NNLOsim
$\tilde{C}^{(np)}_{^1S_0}$	$-0.15387(10)$	$-0.1474(20)$
$\tilde{C}^{(pp)}_{^1S_0}$	$-0.152935(72)$	$-0.1465(20)$
$\tilde{C}^{(nn)}_{^1S_0}$	$-0.15354(43)$	$-0.1471(20)$
$C_{^1S_0}$	$+2.7442(19)$	$+2.548(47)$
$\tilde{C}_{^3S_1}$	$-0.1671(10)$	$-0.1687(21)$
$C_{^3S_1}$	$+0.8738(64)$	$+0.705(47)$
C_{E_1}	$+0.6899(67)$	$+0.597(11)$
$C_{^3P_0}$	$+1.2782(66)$	$+1.161(31)$
$C_{^1P_1}$	$+0.521(12)$	$+0.520(33)$
$C_{^3P_1}$	$-0.9378(69)$	$-0.955(31)$
$C_{^3P_2}$	$-0.68645(76)$	$-0.658(30)$
c_D	$-0.581(28)$	$-0.325(51)$
c_E	$-0.6666(99)$	$-0.521(17)$
c_1	$-0.69(50)$	$+0.22(30)$
c_2	$+3.0(14)$	$+5.1(10)$
c_3	$-4.12(32)$	$-3.56(13)$
c_4	$+5.35(81)$	$+3.933(85)$
d_1+d_2	$+6.22(44)$	$+5.320(94)$
d_3	$-5.31(30)$	$-4.83(22)$
d_5	$-0.46(18)$	$-0.24(14)$
$d_{14}-d_{15}$	$-11.00(42)$	$-10.23(27)$
e_{14}	$-0.63(95)$	$-0.26(89)$
e_{15}	$-7.7(26)$	$-9.3(24)$
e_{16}	$+5.9(49)$	$-0.0(41)$
e_{17}	$+2.1(18)$	$+1.5(18)$
e_{18}	$-8.1(42)$	$-1.2(16)$

?

TABLE I: Numerical values of the πN LECs that result from the optimization with respect to explrimental observables. The resulting values are grouped from left to right in the order they appear in the Lagrangian.

$\mathcal{O}(Q^1)$ LECs [GeV^{-1}]		$\mathcal{O}(Q^2)$ LECs [GeV^{-2}]		$\mathcal{O}(Q^3)$ LECs [GeV^{-3}]	
c_1	-1.40 ± 0.12	$\bar{d}_1 + \bar{d}_2$	$+5.80 \pm 0.14$	$\bar{e}_{14}$	$+1.53 \pm 0.31$
c_2	$+1.71 \pm 0.33$	$\bar{d}_3$	-5.66 ± 0.08	$\bar{e}_{15}$	-11.91 ± 0.87
c_3	-4.56 ± 0.11	$\bar{d}_5$	$+0.03 \pm 0.06$	$\bar{e}_{16}$	$+11.43 \pm 1.23$
c_4	$+3.72 \pm 0.27$	$\bar{d}_{14} - \bar{d}_{15}$	-11.50 ± 0.12	$\bar{e}_{17}$	$+0.73 \pm 0.51$
				$\bar{e}_{18}$	$+0.57 \pm 1.36$

Also, does it make sense to make combined πN and NN fits with πN amplitude taken at different chiral orders?

LECs (in GeV^{-n}) extracted from πN scattering

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$
$[Q^4]_{\text{HB, NN}}$, GW PWA	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-0.58
$[Q^4]_{\text{HB, NN}}$, KH PWA	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-0.37
$[Q^4]_{\text{covariant}}$, data	-0.82	3.56	-4.59	3.44	5.43	-4.58	-0.40	-9.94	-0.63	-0.90