Electron scattering off few-nucleon systems: theory meets experiment

J.Golak,

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JAGIELLONIAN UNIVERSITY

New Vistas in Low Energy Precision Physics (LEPP)

Mainz, 7 April, 2016

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Introduction

My talk at JGU on 15 June, 2015

Few-nucleon systems with chiral nuclear forces

with the outline:

- Chiral effective field theory up to 2014
- Improved NN potentials from E. Epelbaum *et al.*
- 2N and 3N systems with new chiral potentials
- Selected electromagnetic reactions with 2N and 3N systems
- Muon capture on ^{2}H and ^{3}He
- Conclusions and outlook

Introduction (*cont.*)

Lectures on few-nucleon calculations in momentum space

- 2N bound state
- Nucleon-nucleon scattering (phase shifts, observables)
- Elastic electron-nucleon scattering
- Elastic electron-deuteron scattering
- Inelastic electron-deuteron scattering
- Some aspects of electron scattering off ^{3}He

still available (notes+computer codes)

at http://users.uj.edu.pl/~golak/JGU2015/

Introduction (*cont.*)

A very efficient momentum space framework to deal with nucleon-nucleon scattering, nucleon-deuteron scattering and many electroweak processes has been constructed and tested:
Phys. Rept. 274, 107 (1996); Phys. Rept. 415, 89 (2005);
Eur. Phys. J. A24, 31 (2005)

Limitations: nonrelativistic character and lack of Coulomb force in the 3N continuum

Calculations performed with semi-phenomenological 2N and 3N potentials: Bonn B, AV18, Urbana IX, older chiral potentials from E. Epelbaum *et al.* and recently with the improved chiral potentials from EE *et al.*

Introduction (*cont.*)

Prof. Dr. Hartmuth Arenhövel
has been working for years on many electromagnetic reactions in the few-nucleon systems and it is impossible to compete with him !

There are also other groups: Pisa, Trento, Vilnius, Grenoble, ...

We had to find a new territory and study electromagnetic reactions using the potentials and current operators derived within ChEFT !

Introduction (*cont.*)

New improved chiral NN potentials from E. Epelbaum *et al*. are available

Substantial improvement in the description of many observables in 2N and 3N systems

Matrix elements of all chiral 3NF up to N3LO calculated but have to be adjusted to new NN potentials

LENPIC (Low Energy Nuclear Physics International Collaboration) established to coordinate few-nucleon and many-nucleon calculations

http://www.lenpic.org

``to understand nuclear structure and reactions with chiral forces''

LENPIC

Sven Binder, Kai Hebeler,
Joachim Langhammer, Robert Roth

Evgeny Epelbaum, Hermann Krebs

Pieter Maris, Hugh Potter, James Vary

Richard J. Furnstahl,

Angelo Calci

Andreas Nogga

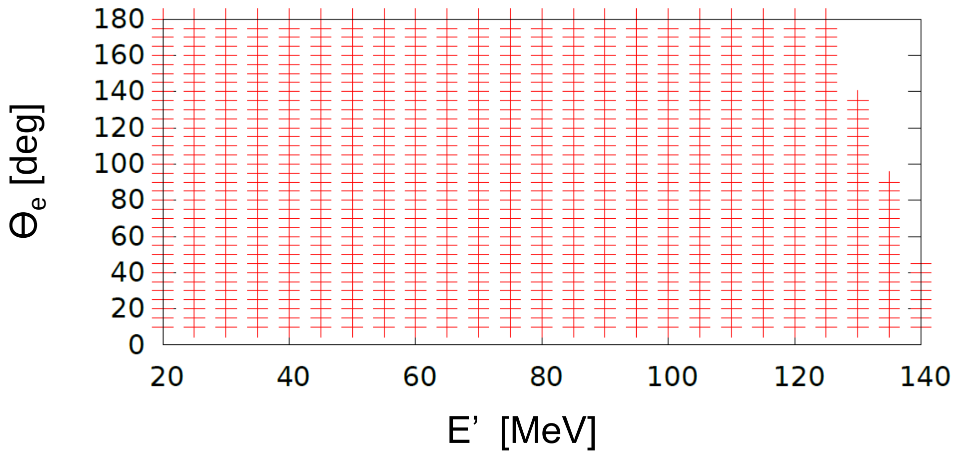
Veronique Bernard

Jacek Golak, Roman Skibiński,
Kacper Topolnicki, Henryk Witała

Hiroyuki Kamada

Ulf-G.Meißner

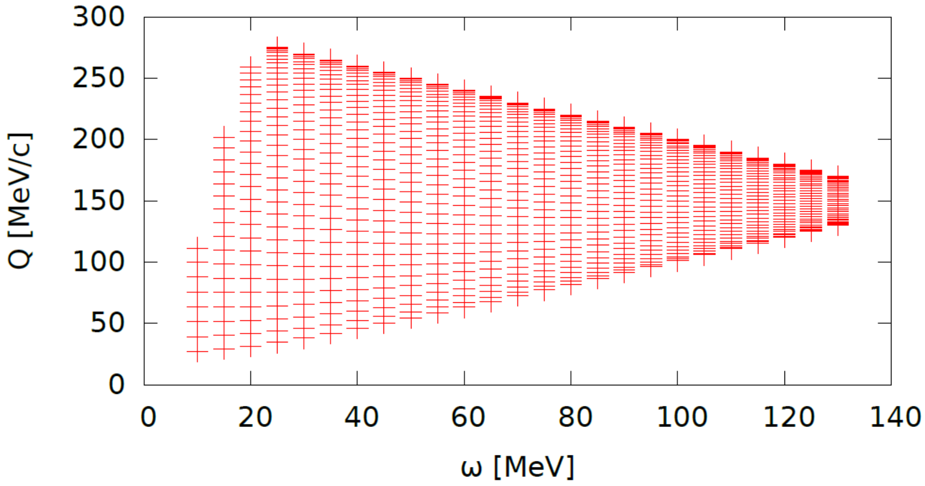
Introduction *(cont.)*



Expected MESA parameters

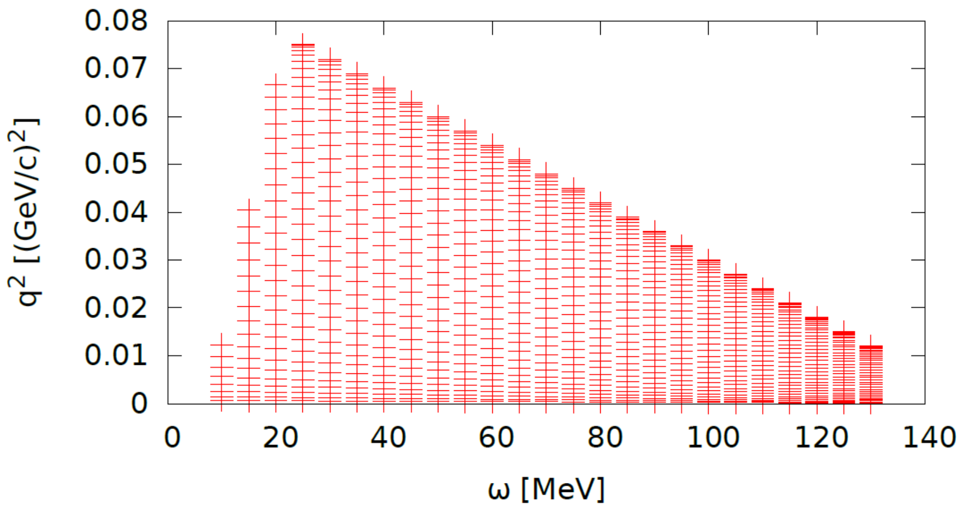
E= 150 MeV
E' > 20 MeV
Θ_e > 10 deg

ideal to study few-nucleon dynamics within the nonrelativistic framework with the input from ChEFT !

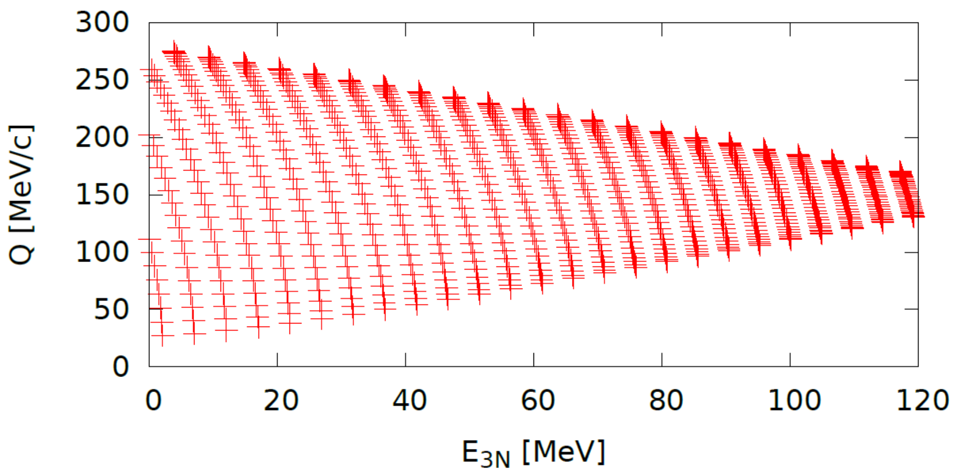


magnitude of three-momentum transfer vs. energy transfer

Introduction *(cont.)*



four-momentum transfer squared vs. energy transfer



magnitude of three-momentum transfer vs. internal energy of 3N system

Input from ChEFT

General strategy in the few-nucleon systems:

- use (**consistent !!!)** dynamical ingredients (2N and 3N potentials, electroweak current operators)
- solve the dynamical equations (Schrödinger equation, Lippmann-Schwinger equation, Faddeev equations)
- give predictions for bound states and reaction observables
- confront results of theoretical calculations with experimental data

Important message from Evgeny's talk:
Improved 2N chiral potentials reveal very welcome properties in 2N and 3N systems

Work on consistent 3N potentials up to N3LO is being finalized

Input from ChEFT (*cont.*)

Work on EM and weak current operators consistent with the improved chiral forces not yet finished

S. Kölling, PhD thesis

Leading two-pion-exchange current operator
S. Kölling *et al*., Phys. Rev. C80, 045502 (2009)

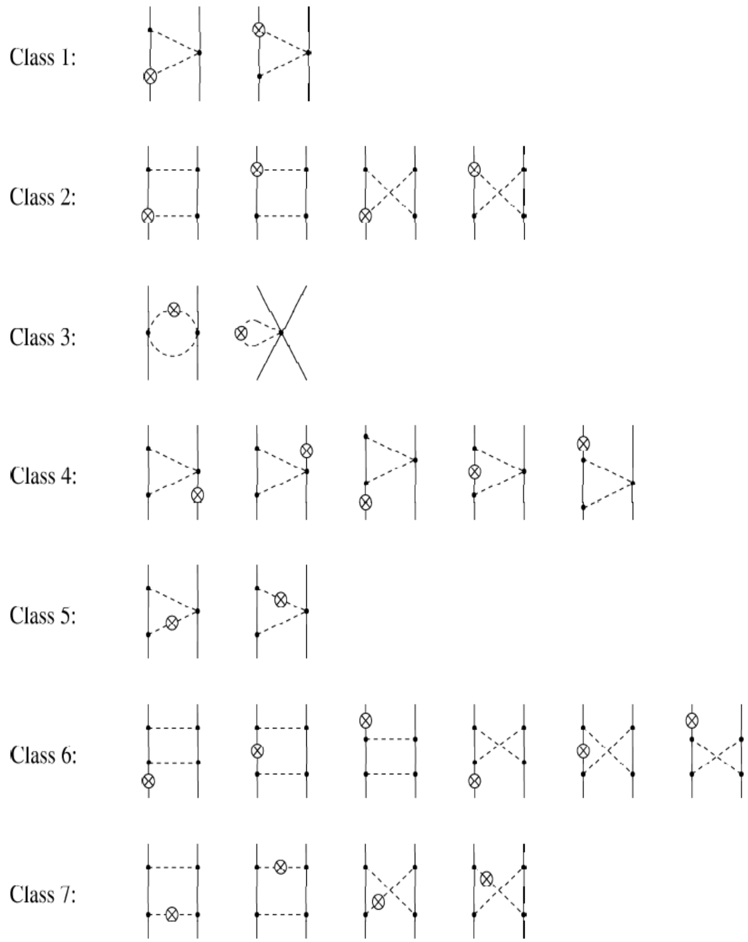
Corrections to one-pion exchange and short-range contributions
S. Kölling *et al*., Phys. Rev. C84, 054008 (2011)

Highly non-trivial task !

Input from ChEFT (*cont.*)

Leading two-pion-exchange current operator

Phys. Rev. C80, 045502 (2009)



$$\vec{J} = \sum_{i=1}^{5} \sum_{j=1}^{24} f_i^j(\vec{q}_1, \vec{q}_2) T_i \vec{O}_j ,$$

Nonvanishing contributions come from

$$f_1^3, f_1^4, f_1^5, f_1^6, f_1^7, f_1^8, f_1^9, f_1^{10},$$
$$f_2^3, f_2^4, f_2^5, f_2^6, f_2^7, f_2^8, f_2^9, f_2^{10}, f_3^1, f_3^2$$

$$J^0 = \sum_{i=1}^{5} \sum_{j=1}^{8} f_i^{jS}(\vec{q}_1, \vec{q}_2) T_i O_j^S ,$$

Nonvanishing contributions come from

$$f_1^{1S}, f_1^{4S}, f_1^{5S}, f_1^{6S}, f_1^{7S}, f_1^{8S},$$
$$f_2^{1S}, f_2^{7S}, f_2^{8S}, f_3^{2S}, f_3^{3S}$$

Input from ChEFT (*cont.*)

Spin operators

$$
\begin{aligned}
\vec{O}_1 &= \vec{q}_1 + \vec{q}_2, \\
\vec{O}_2 &= \vec{q}_1 - \vec{q}_2, \\
\vec{O}_3 &= [\vec{q}_1 \times \vec{\sigma}_2] + [\vec{q}_2 \times \vec{\sigma}_1], \\
\vec{O}_4 &= [\vec{q}_1 \times \vec{\sigma}_2] - [\vec{q}_2 \times \vec{\sigma}_1], \\
\vec{O}_5 &= [\vec{q}_1 \times \vec{\sigma}_1] + [\vec{q}_2 \times \vec{\sigma}_2], \\
\vec{O}_6 &= [\vec{q}_1 \times \vec{\sigma}_1] - [\vec{q}_2 \times \vec{\sigma}_2], \\
\vec{O}_7 &= \vec{q}_1 \left(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2]\right) + \vec{q}_2 \left(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1]\right), \\
\vec{O}_8 &= \vec{q}_1 \left(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2]\right) - \vec{q}_2 \left(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1]\right), \\
\vec{O}_9 &= \vec{q}_2 \left(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2]\right) + \vec{q}_1 \left(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1]\right), \\
\vec{O}_{10} &= \vec{q}_2 \left(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2]\right) - \vec{q}_1 \left(\vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1]\right), \\
\vec{O}_{11} &= (\vec{q}_1 + \vec{q}_2)\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2), \\
\vec{O}_{12} &= (\vec{q}_1 - \vec{q}_2)\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2), \\
\vec{O}_{13} &= \vec{q}_1 \,(\vec{q}_1 \cdot \vec{\sigma}_1)\,(\vec{q}_1 \cdot \vec{\sigma}_2) + \vec{q}_2 \,(\vec{q}_2 \cdot \vec{\sigma}_1)\,(\vec{q}_2 \cdot \vec{\sigma}_2), \\
\vec{O}_{14} &= \vec{q}_1 \,(\vec{q}_1 \cdot \vec{\sigma}_1)\,(\vec{q}_1 \cdot \vec{\sigma}_2) - \vec{q}_2 \,(\vec{q}_2 \cdot \vec{\sigma}_1)\,(\vec{q}_2 \cdot \vec{\sigma}_2),
\end{aligned}
$$

...

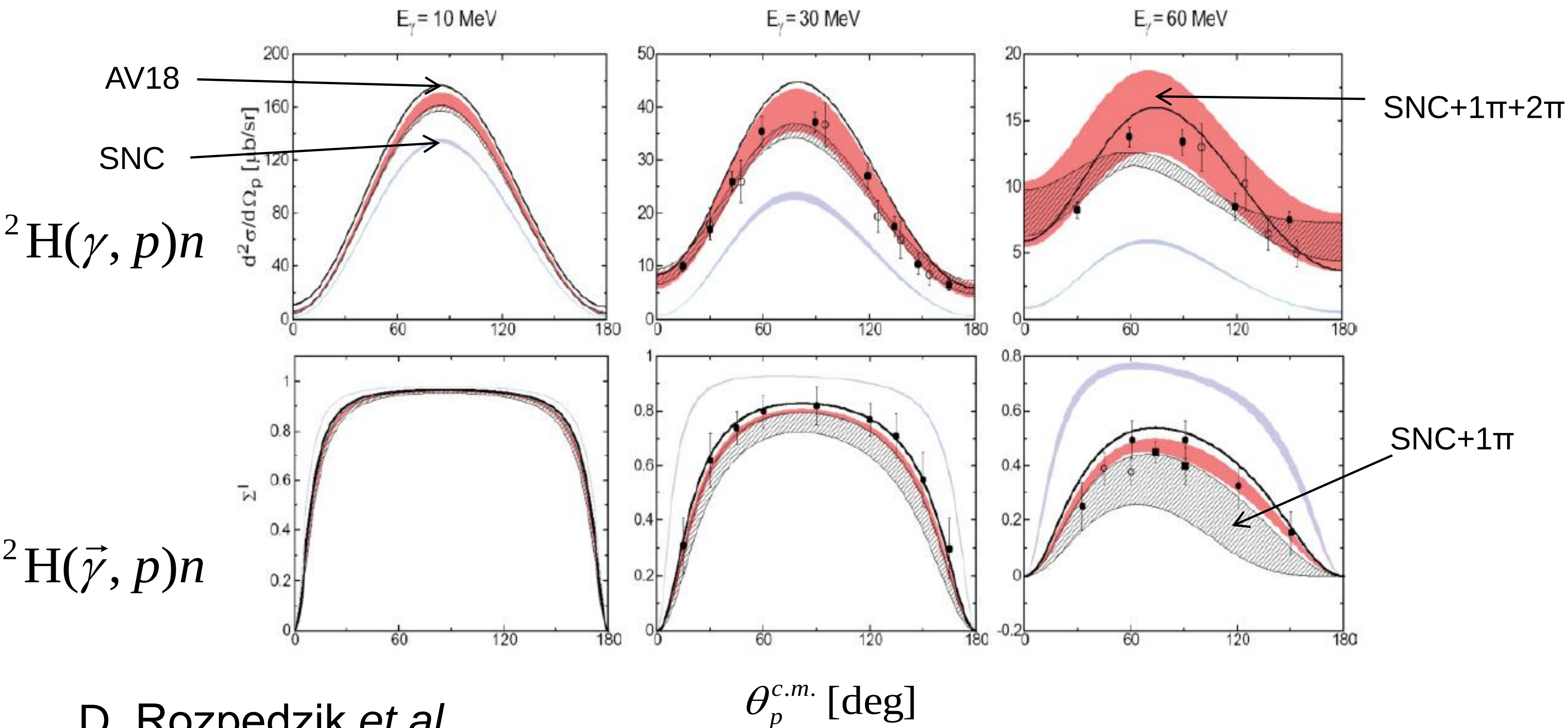
$$
\begin{aligned}
O_1^S &= \mathbb{1}, \\
O_2^S &= \vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2] + \vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1], \\
O_3^S &= \vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_2] - \vec{q}_1 \cdot [\vec{q}_2 \times \vec{\sigma}_1], \\
O_4^S &= \vec{\sigma}_1 \cdot \vec{\sigma}_2, \\
O_5^S &= (\vec{q}_1 \cdot \vec{\sigma}_2)\,(\vec{q}_2 \cdot \vec{\sigma}_1), \\
O_6^S &= (\vec{q}_1 \cdot \vec{\sigma}_1)\,(\vec{q}_2 \cdot \vec{\sigma}_2), \\
O_7^S &= (\vec{q}_2 \cdot \vec{\sigma}_1)\,(\vec{q}_2 \cdot \vec{\sigma}_2) + (\vec{q}_1 \cdot \vec{\sigma}_1)\,(\vec{q}_1 \cdot \vec{\sigma}_2), \\
O_8^S &= (\vec{q}_2 \cdot \vec{\sigma}_1)\,(\vec{q}_2 \cdot \vec{\sigma}_2) - (\vec{q}_1 \cdot \vec{\sigma}_1)\,(\vec{q}_1 \cdot \vec{\sigma}_2).
\end{aligned}
$$

Isospin operators

$$
\begin{aligned}
T_1 &= (\vec{\tau}_1 + \vec{\tau}_2)_3, \\
T_2 &= (\vec{\tau}_1 - \vec{\tau}_2)_3, \\
T_3 &= (\vec{\tau}_1 \times \vec{\tau}_2)_3, \\
T_4 &= \vec{\tau}_1 \cdot \vec{\tau}_2, \\
T_5 &= \mathbb{1},
\end{aligned}
$$

Older chiral forces: EM reactions

Older chiral forces at N2LO with leading one-pion exchange and two-pion exchange currents

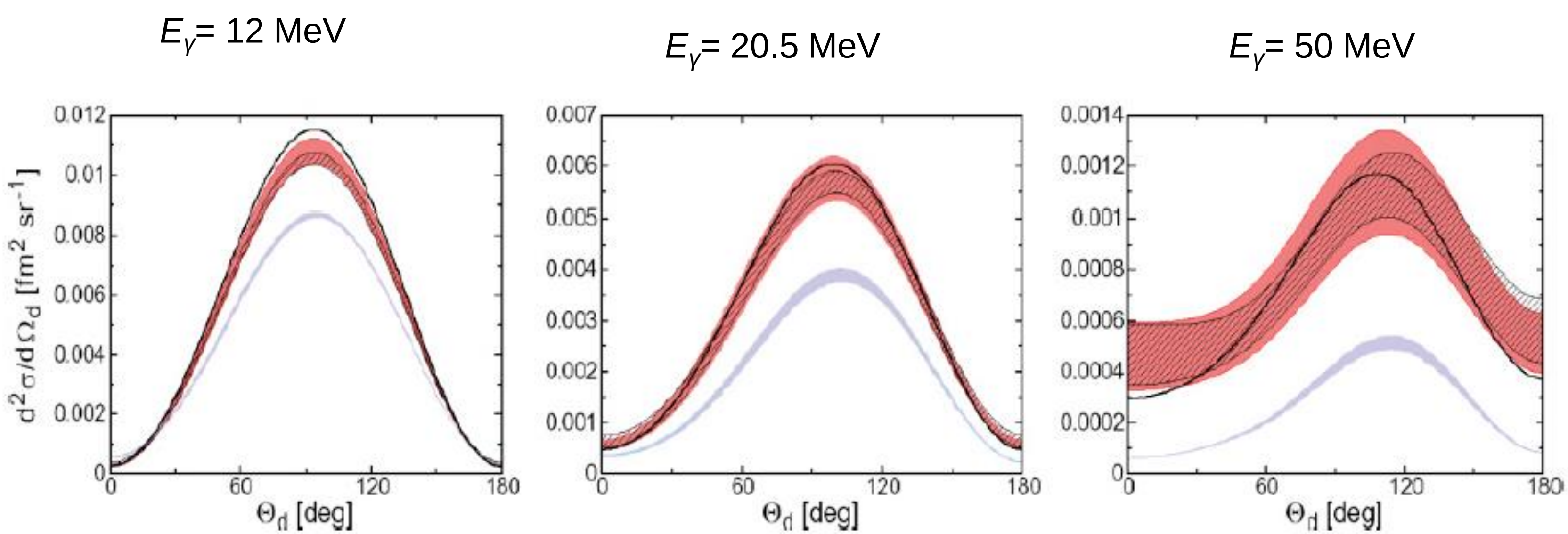


D. Rozpędzik *et al.*

Older chiral forces: EM reactions

Similar behaviour in 3N system !

$$^3He(\gamma, d)p$$

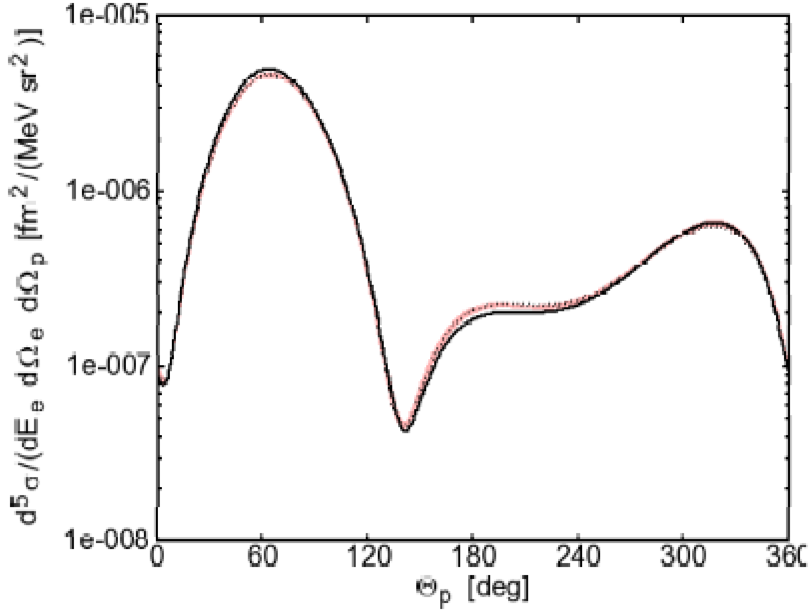


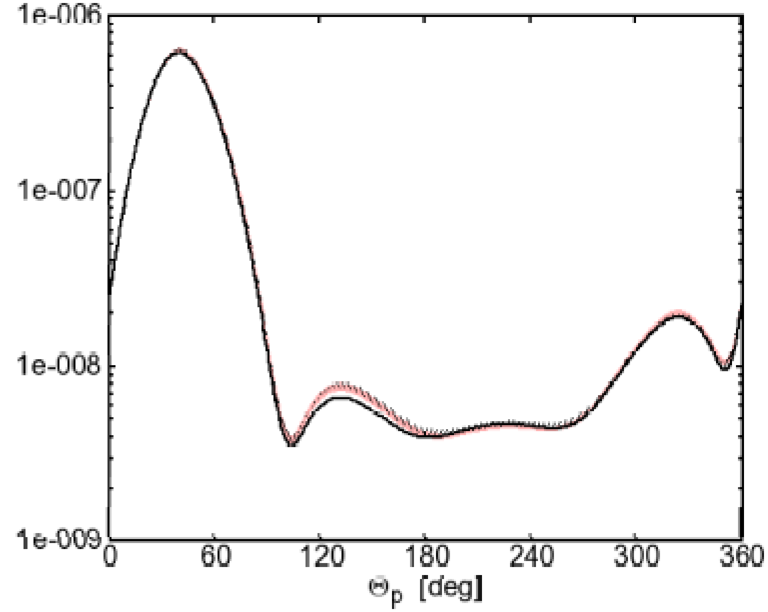
Older chiral forces: EM reactions

$^{3}He(e,e'p)d$ two-body break-up of ^{3}He

$E = 100$ MeV, $\theta_e = 30^o$, $E' = 80$ MeV
$\omega = 20$ MeV, $Q = 50$ MeV/c

$E = 100$ MeV, $\theta_e = 88^o$, $E' = 70$ MeV
$\omega = 30$ MeV, $Q = 120$ MeV/c



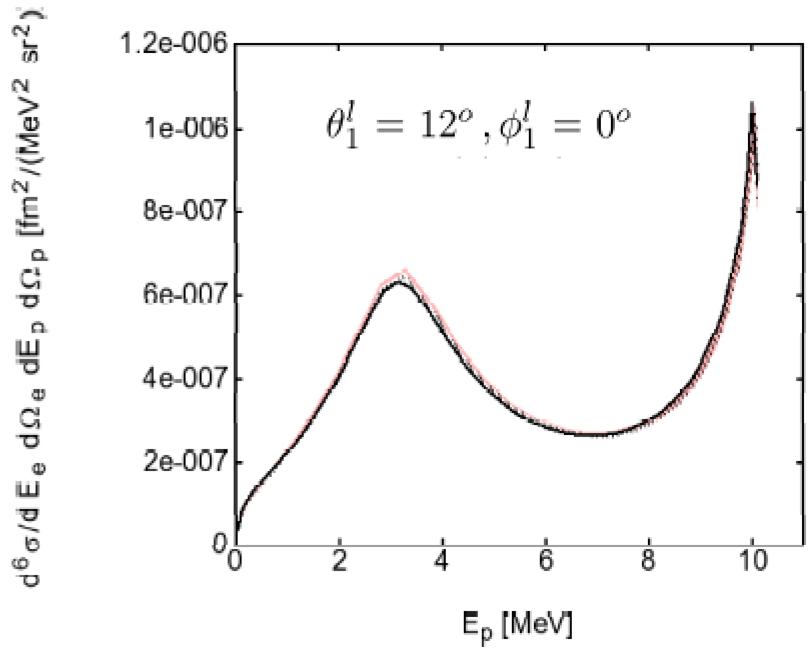


very small sensitivity of unpolarized xs to current operator !

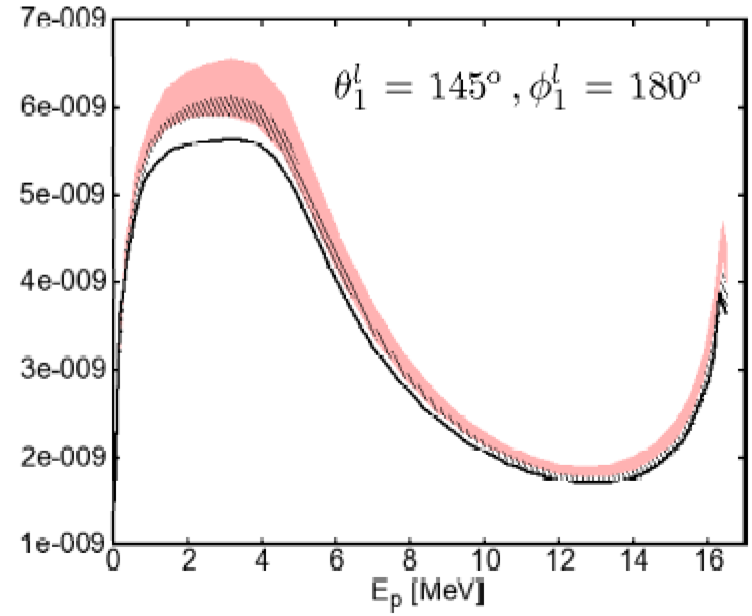
Older chiral forces: EM reactions

$^3He(e,e'\,p)\,pn$ three-body break-up of ^{3}He

$E = 100\ \text{MeV}, \theta_e = 30^o, E' = 80\ \text{MeV}$
$\omega = 20\ \text{MeV}, Q = 50\ \text{MeV}$



$E = 100\ \text{MeV}, \theta_e = 88^o, E' = 70\ \text{MeV}$
$\omega = 30\ \text{MeV}, Q = 120\ \text{MeV}$

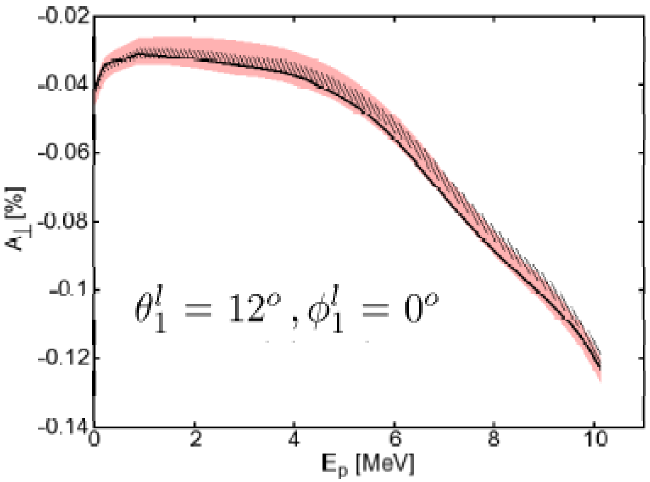


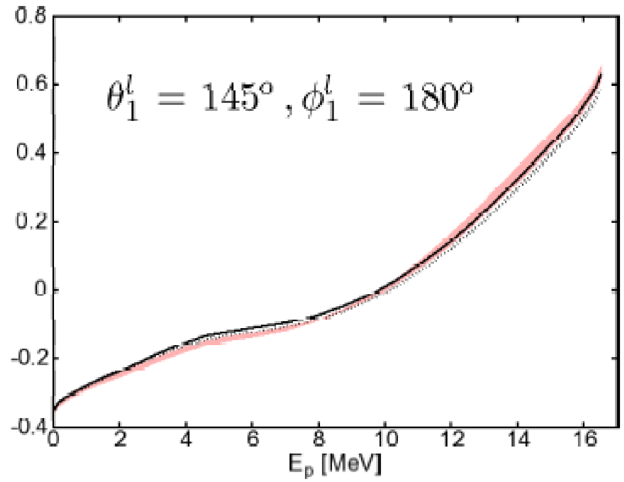
Older chiral forces: EM reactions

$$^{3}He(e,e'p)pn$$

$E = 100$ MeV, $\theta_e = 30^o, E' = 80$ MeV
$\omega = 20$ MeV, $Q = 50$ MeV

$E = 100$ MeV, $\theta_e = 88^o, E' = 70$ MeV
$\omega = 30$ MeV, $Q = 120$ MeV





Improved chiral forces: EM reactions

What can be done without explicit EM current operators ?

Use Siegert theorem to implicitly include many-nucleon contributions:

- Replace a part of a electric multipole by a Coulomb multipole calculated from the single nucleon charge density.

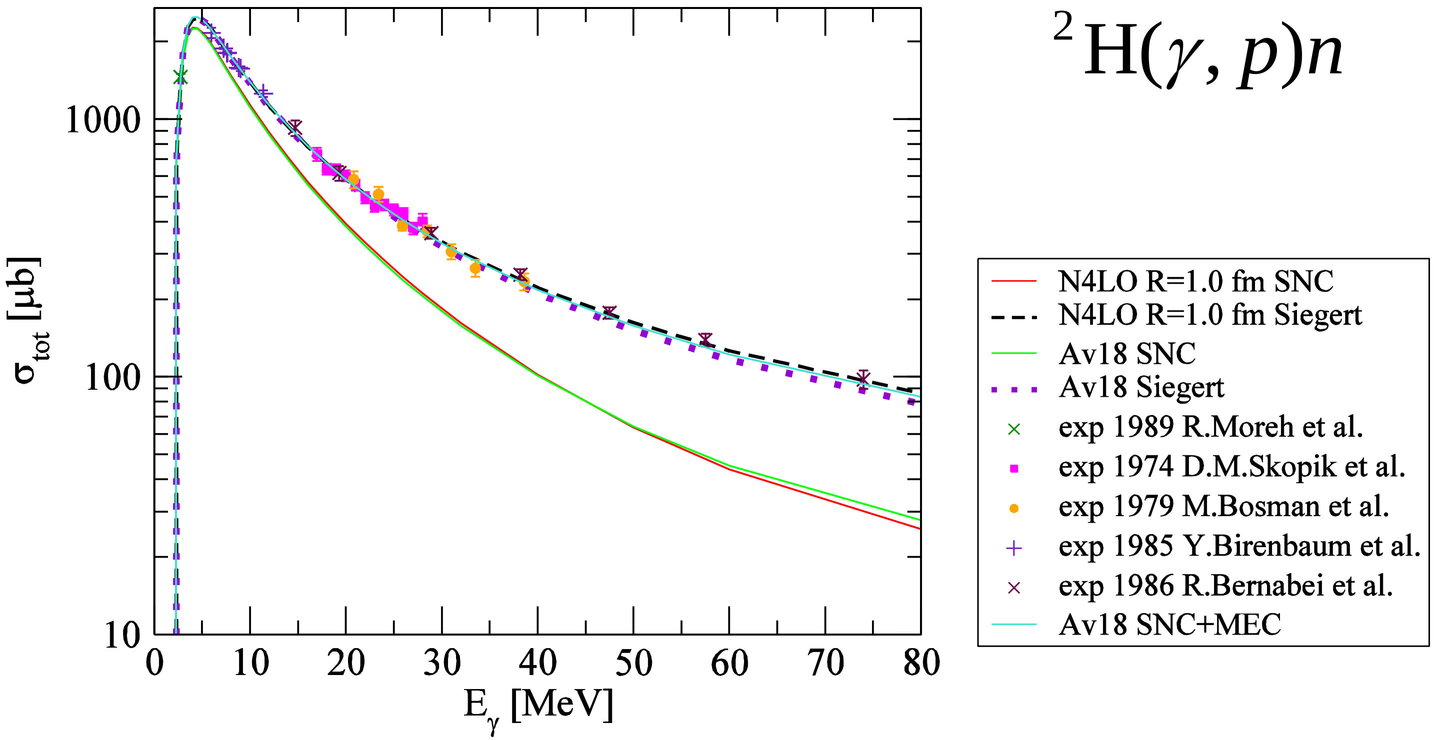
$$\hat{Q} Y_{JM}(\hat{Q}) = -\sqrt{\frac{J+1}{2J+1}}\, \vec{Y}^{M}_{J+1\,1\,J}(\hat{Q}) + \sqrt{\frac{J}{2J+1}}\, \vec{Y}^{M}_{J-1\,1\,J}(\hat{Q})$$

- Calculate magnetic multipoles from the single nucleon current operator

Pretty simple in momentum space !

Improved chiral forces: EM reactions

The total deuteron photodisintegration cross section



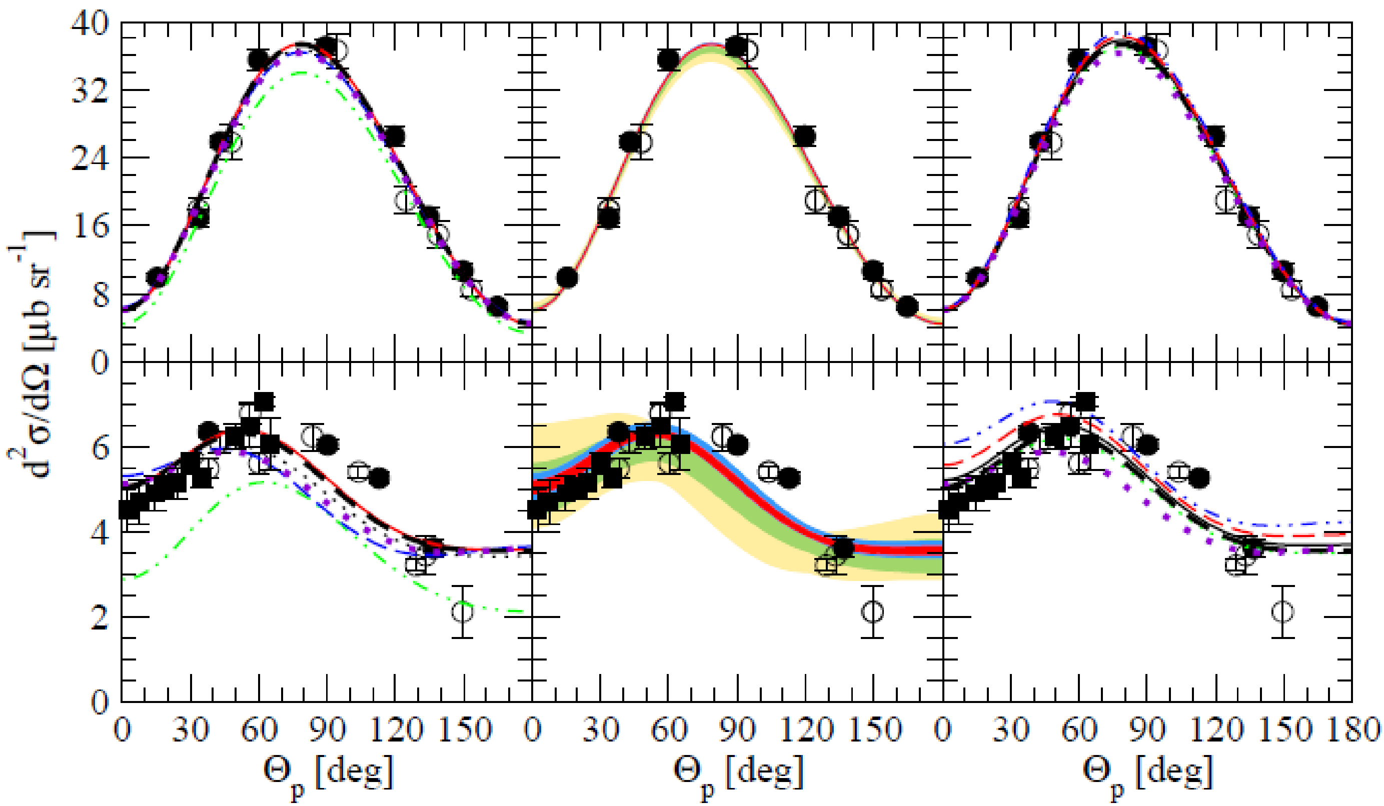
$$^2\mathrm{H}(\gamma, p)n$$

Improved chiral forces: EM reactions

$^2\mathrm{H}(\gamma, p)n$

E_γ= 30 MeV

E_γ= 100 MeV



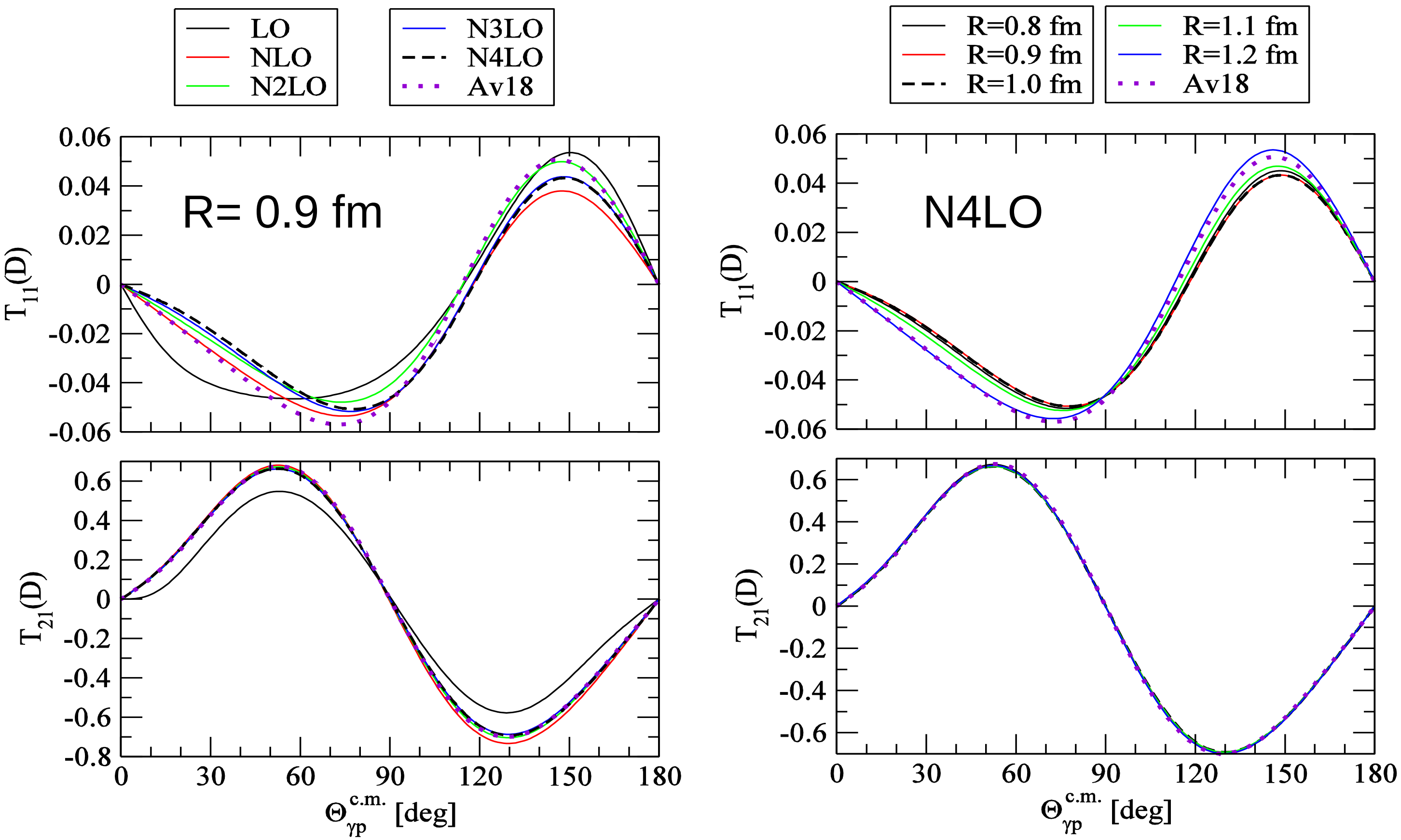
R=0.9 fm
various orders

truncation
errors

N4LO
various R-values

Improved chiral forces: EM reactions

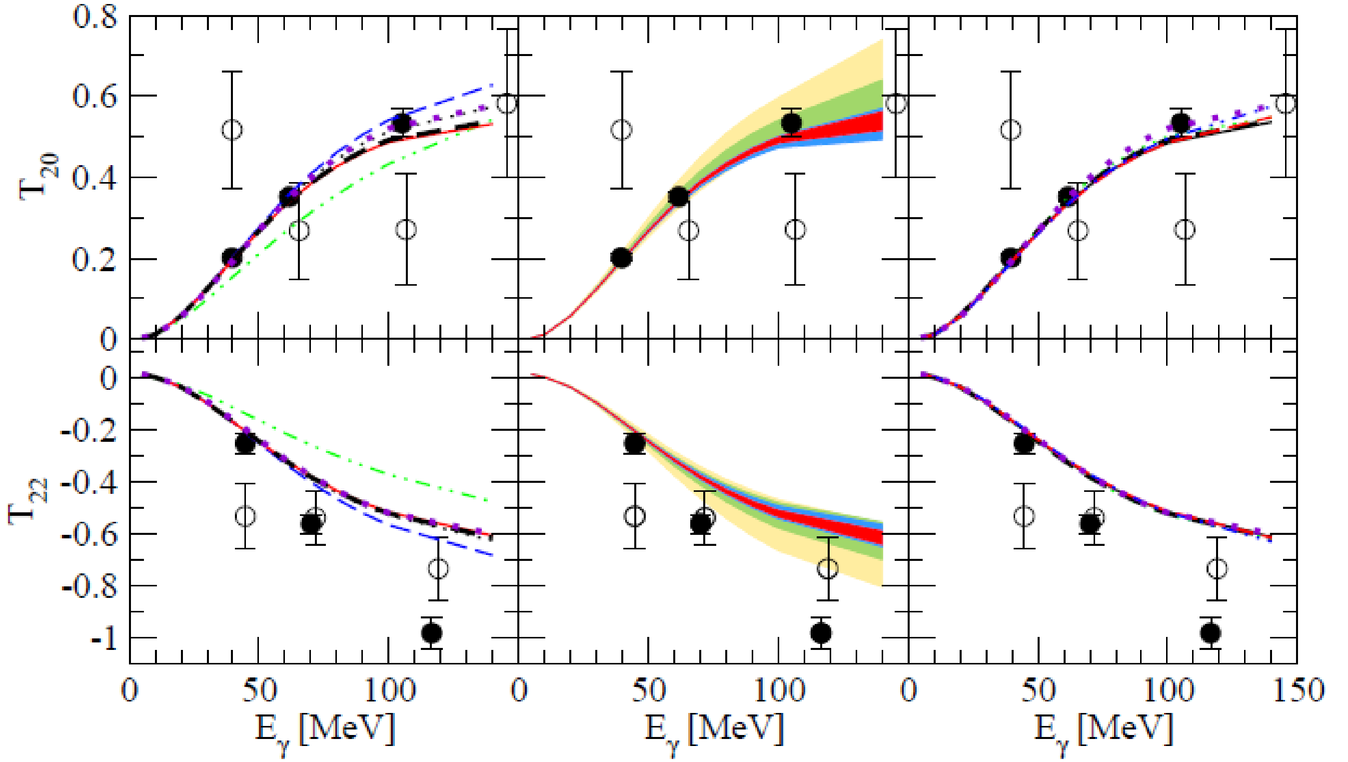
$$^2\overrightarrow{\mathrm{H}}(\gamma, p)n \qquad E_\gamma = 100\ \mathrm{MeV}$$



Improved chiral forces: EM reactions

$$\overrightarrow{{}^{2}\mathrm{H}}(\gamma, p)n$$

$$\theta_p^{c.m.} = 88^o$$



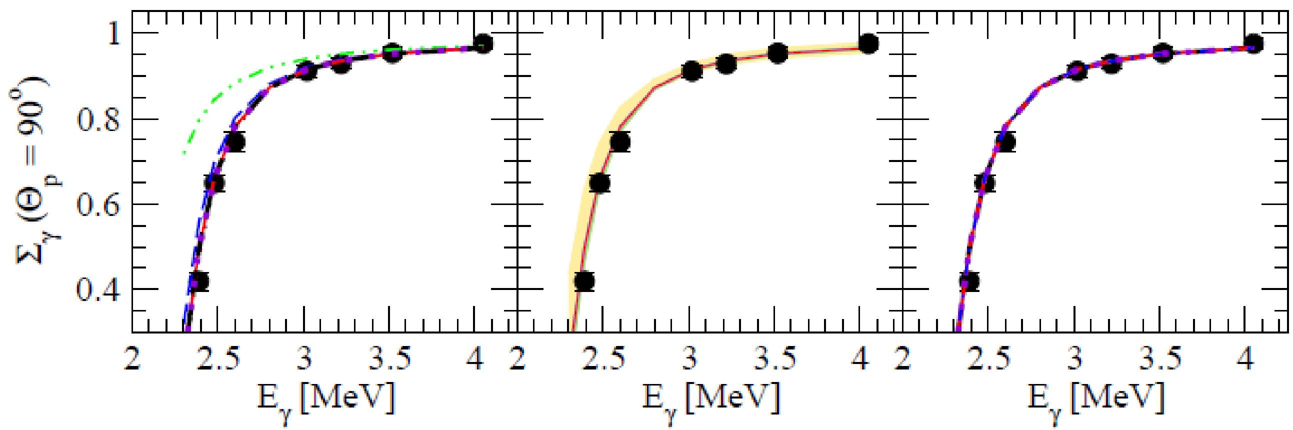
R=0.9 fm
various orders

truncation
errors

N4LO
various R-values

Improved chiral forces: EM reactions

$\theta_p = 90^o$ $\qquad$ ${}^2\mathrm{H}(\vec{\gamma}, p)n$



R=0.9 fm
various orders

truncation
errors

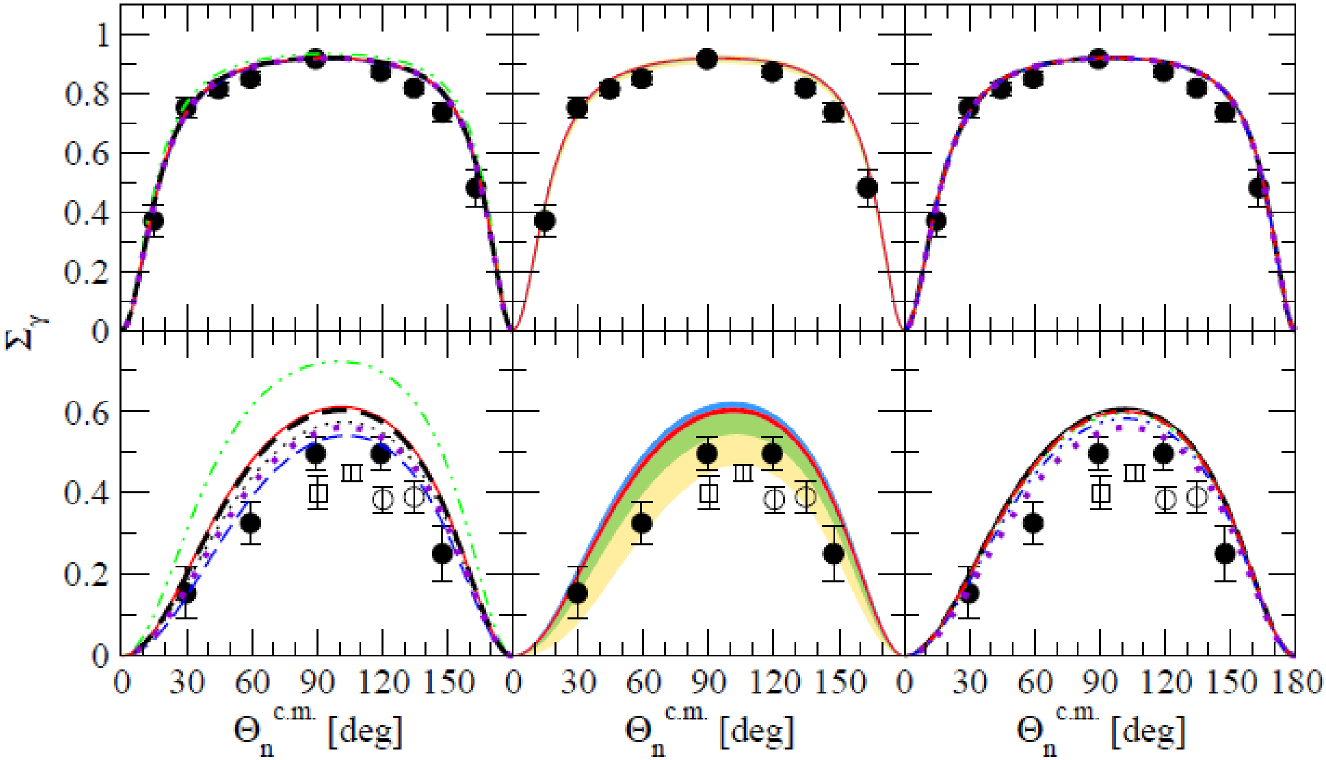
N4LO
various R-values

Improved chiral forces: EM reactions

$$^{2}\mathrm{H}(\vec{\gamma}, n)p$$

E_γ= 19.8 MeV

E_γ= 60.8 MeV



R=0.9 fm
various orders

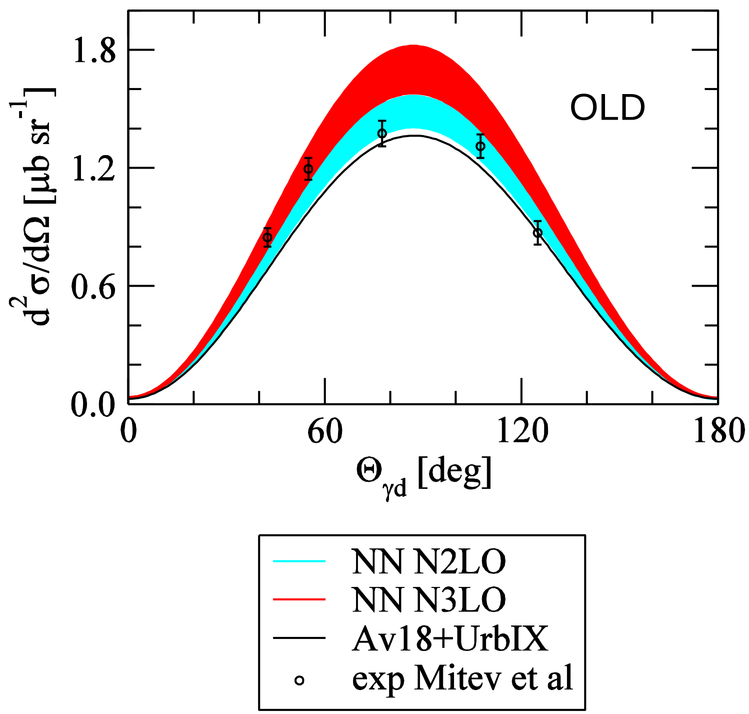
truncation
errors

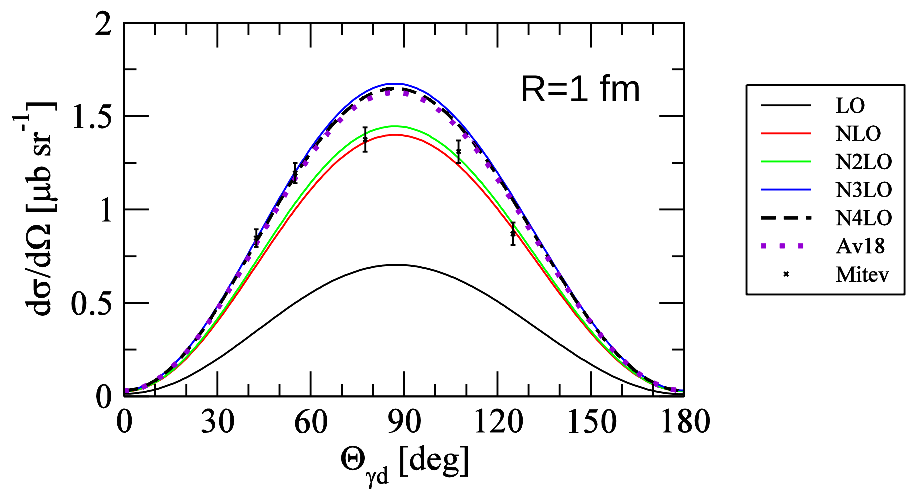
N4LO
various R-values

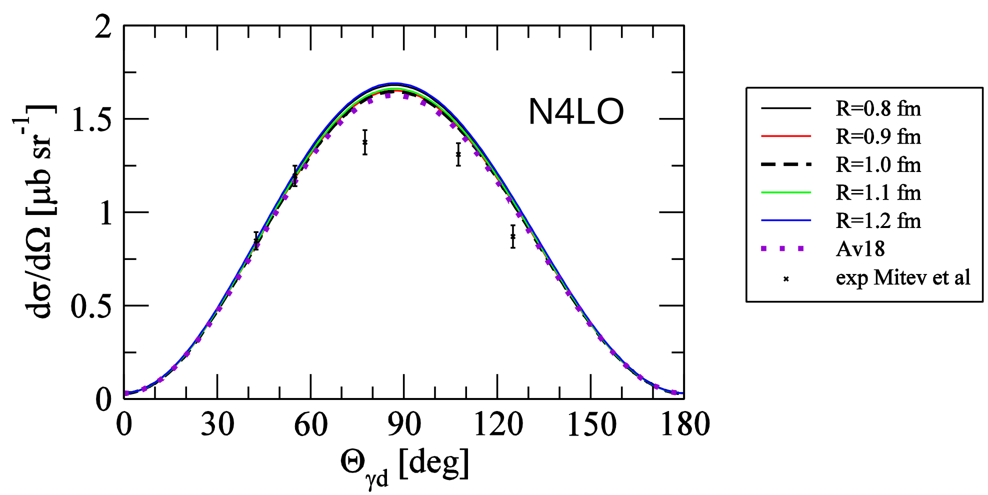
Improved chiral forces: EM reactions

$$n + d \rightarrow \gamma + {}^{3}\mathrm{H}$$

The c.m. neutron-deuteron capture cross section at E_n^{lab} = 9 MeV

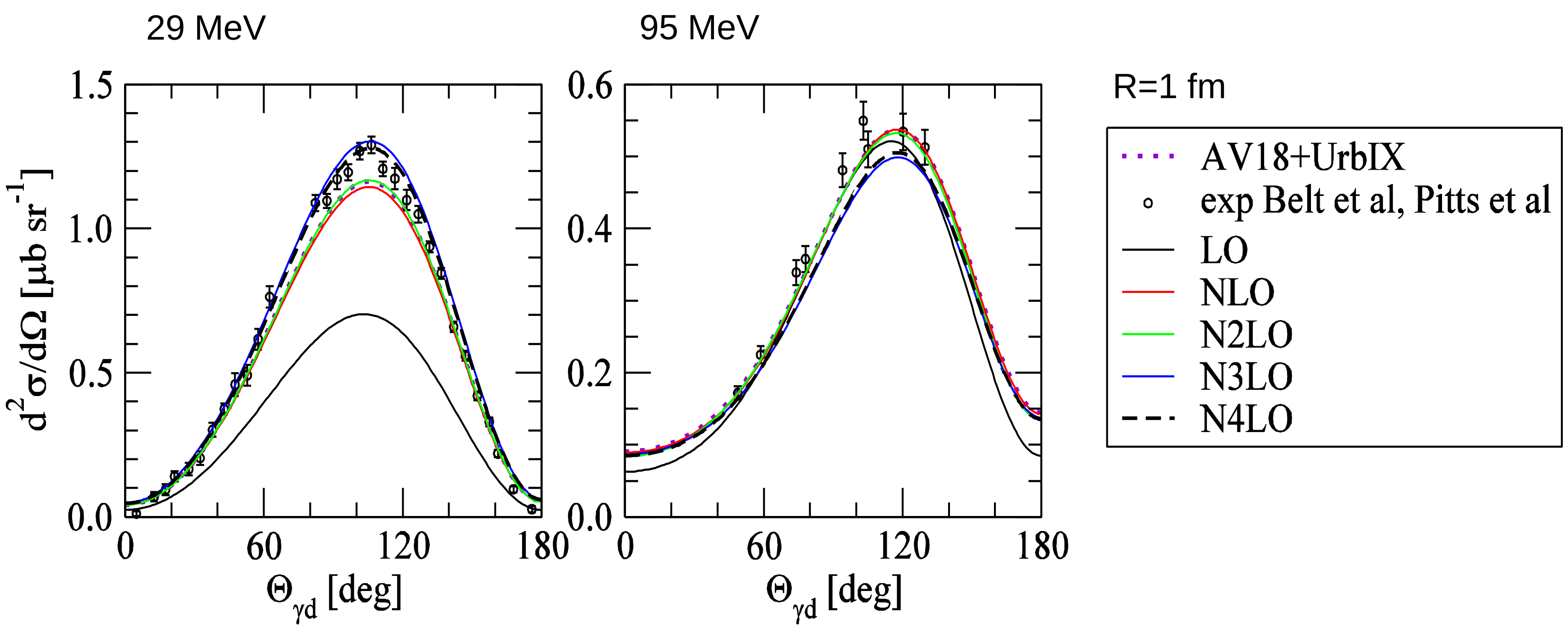




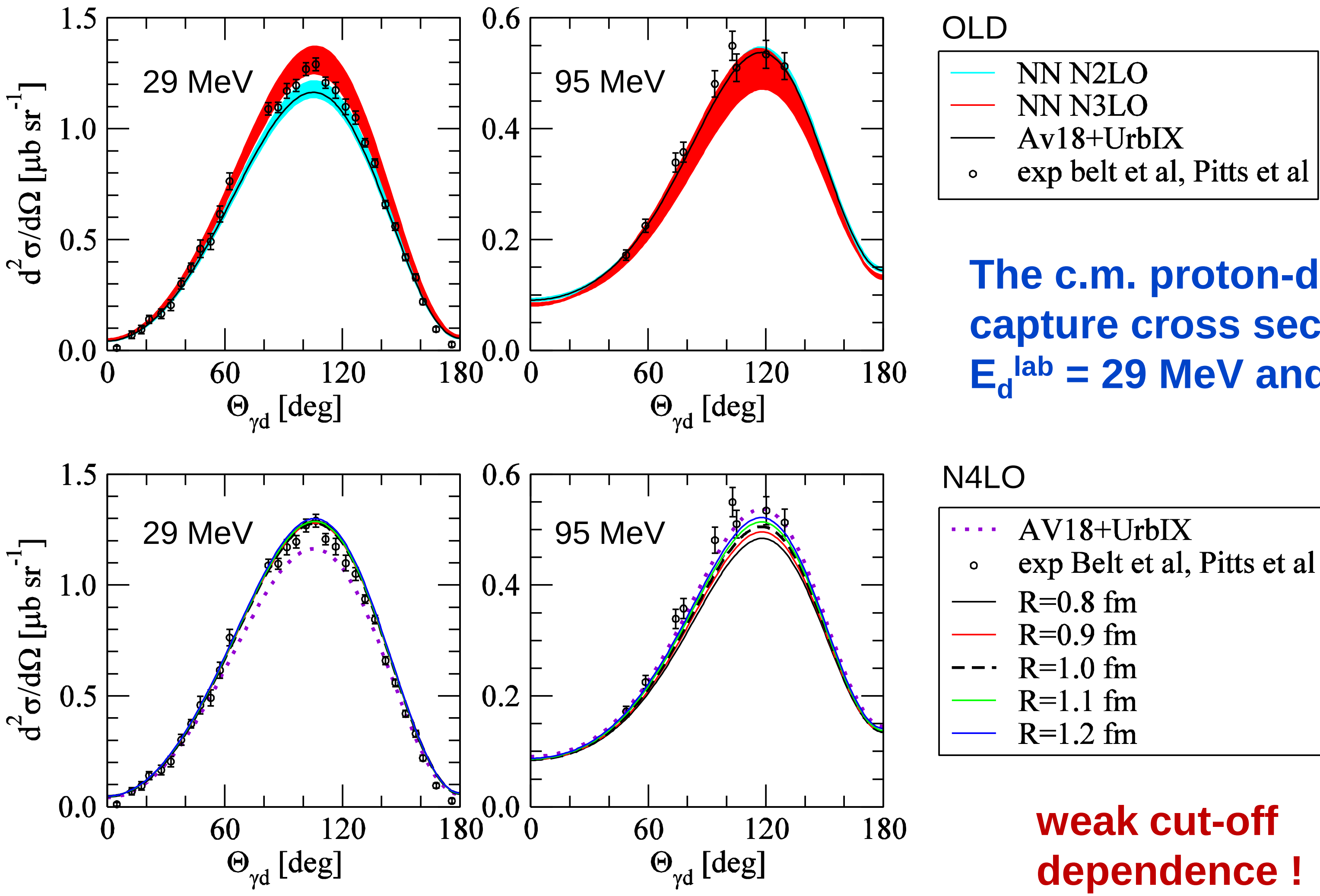


Improved chiral forces: EM reactions

The c.m. proton-deuteron capture cross section at E_d^{lab} = 29 MeV and 95 MeV



Improved chiral forces: EM reactions

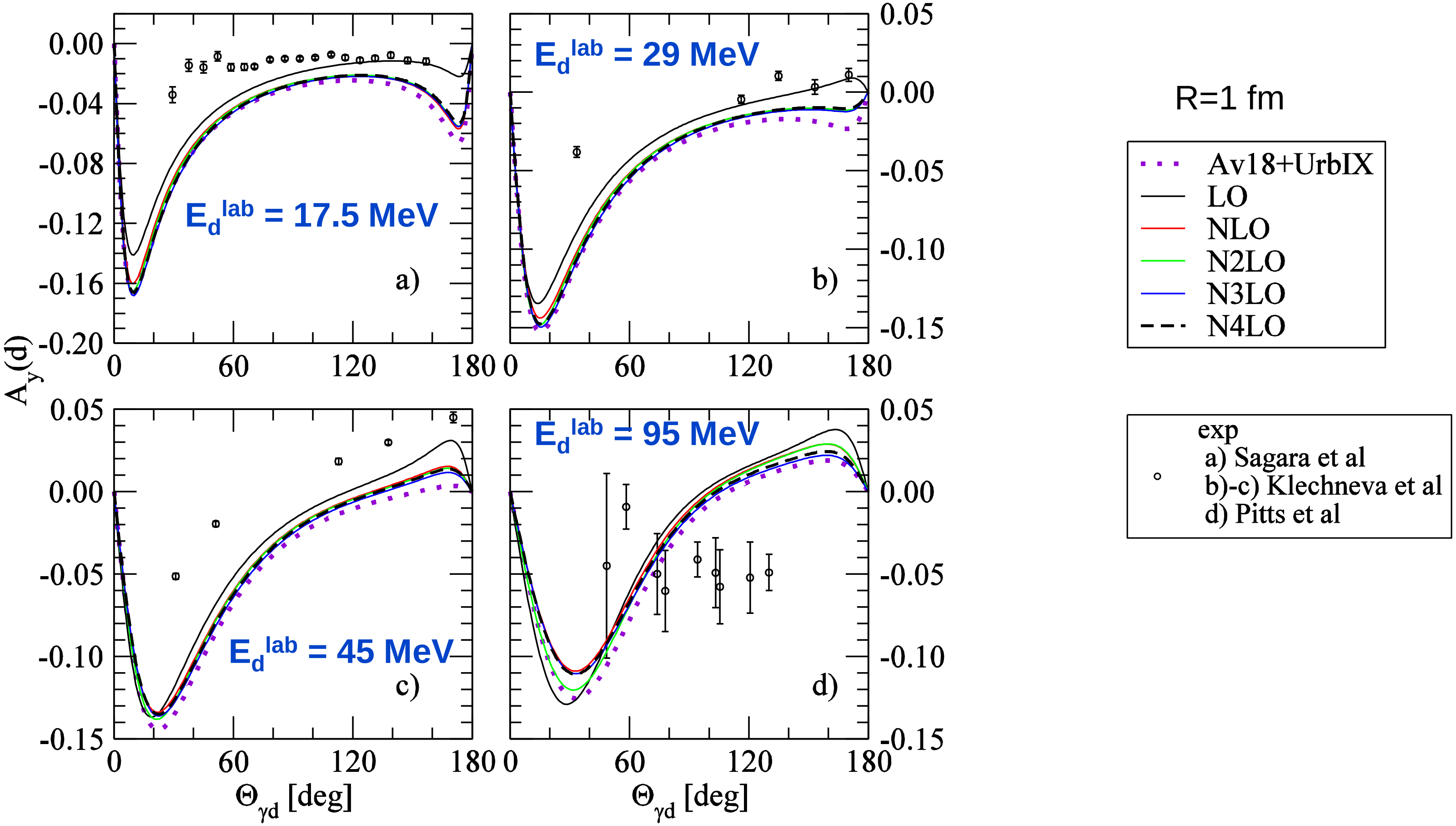


The c.m. proton-deuteron capture cross section at E_d^{lab} = 29 MeV and 95 MeV

weak cut-off dependence !

Improved chiral forces: EM reactions

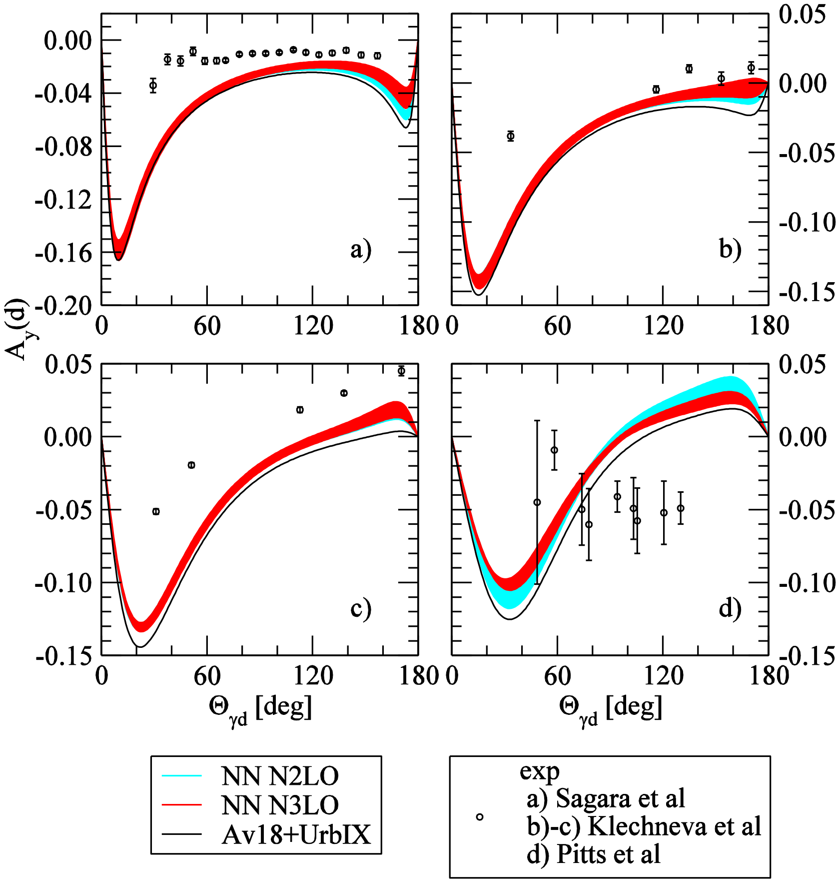
The deuteron vector analyzing power at four energies



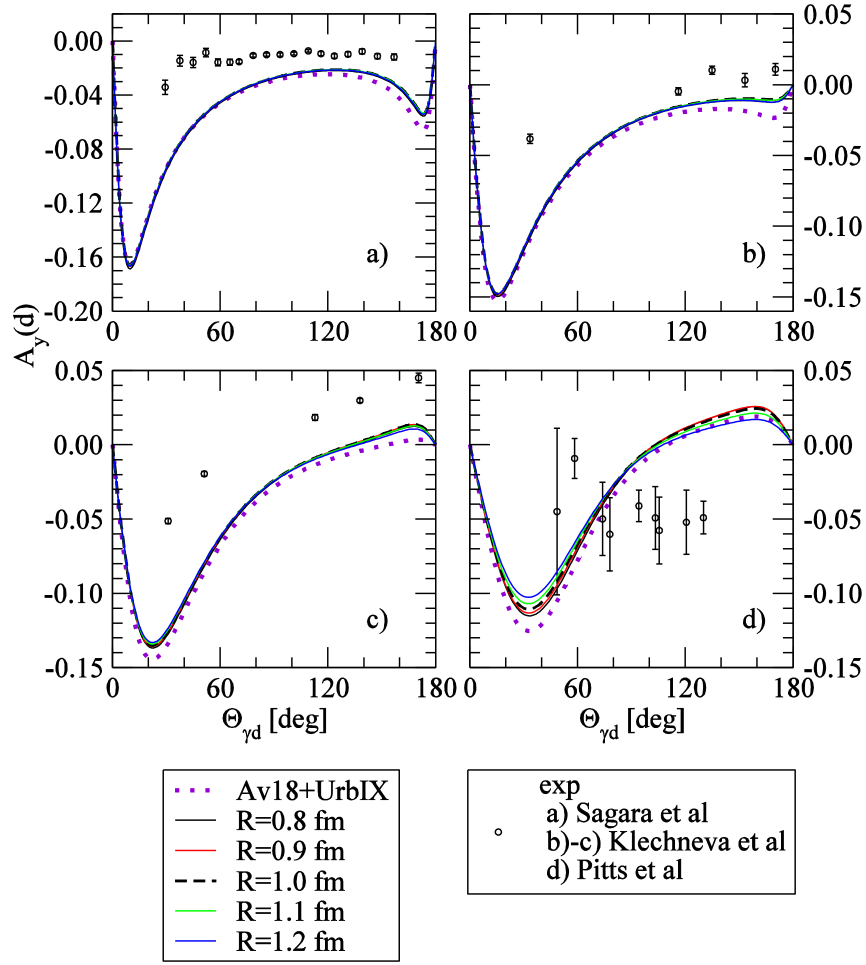
Improved chiral forces: EM reactions

The deuteron vector analyzing power at four energies

OLD



N4LO



Improved chiral forces: EM reactions

$n + d \rightarrow \gamma + {}^3\text{H}$

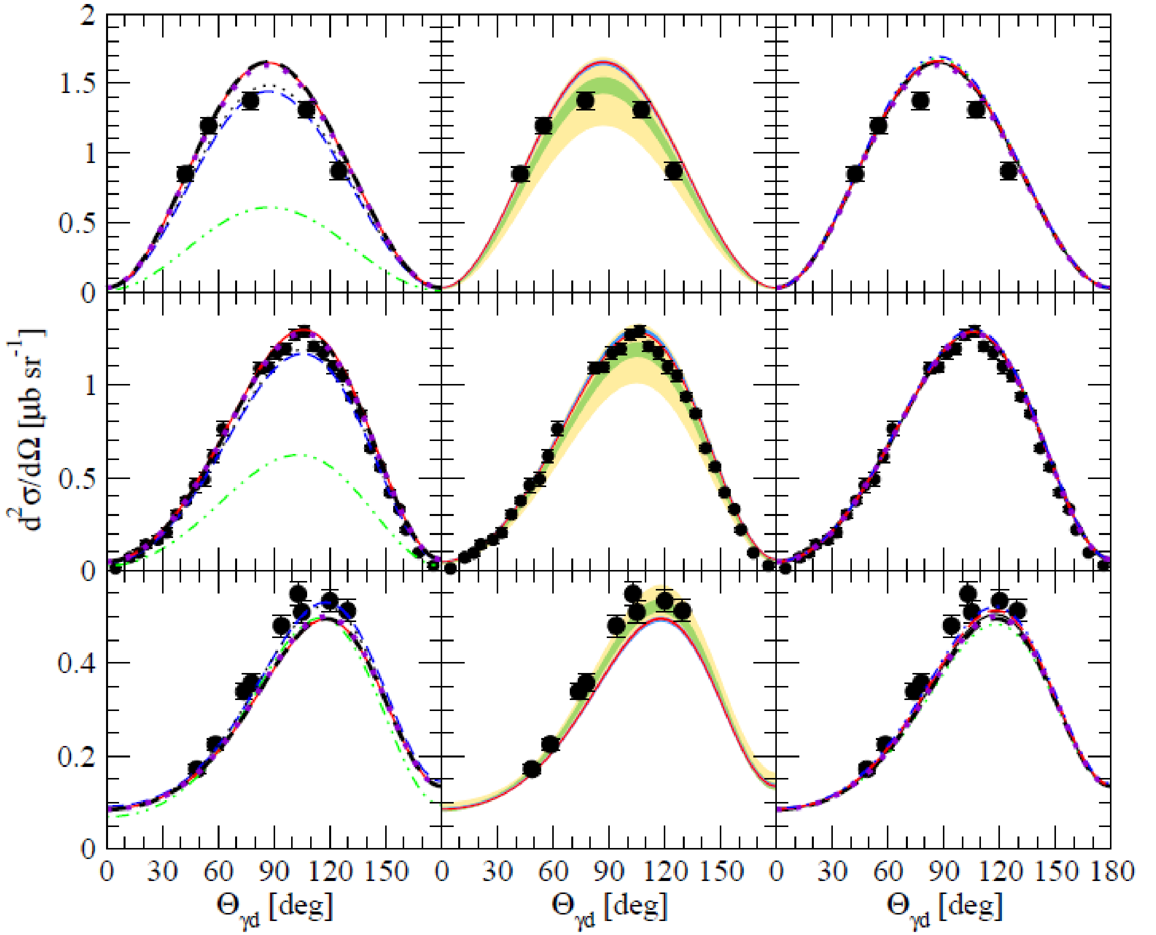
$E_n = 9$ MeV

$p + d \rightarrow \gamma + {}^3\text{He}$

$E_p = 29$ MeV

$p + d \rightarrow \gamma + {}^3\text{He}$

$E_p = 95$ MeV



R=0.9 fm
various orders

truncation
errors

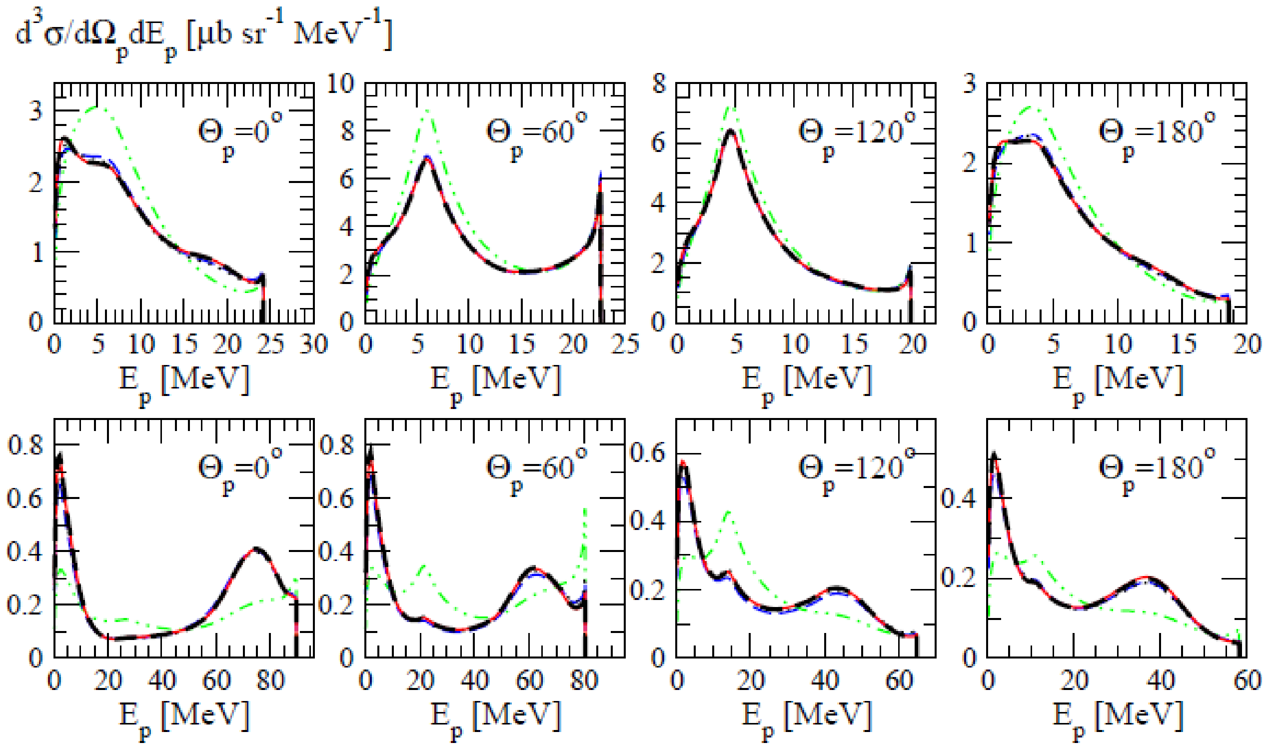
N4LO
various R-values

Improved chiral forces: EM reactions

$$^3\mathrm{He}(\gamma, p)pn$$

E_γ= 40 MeV

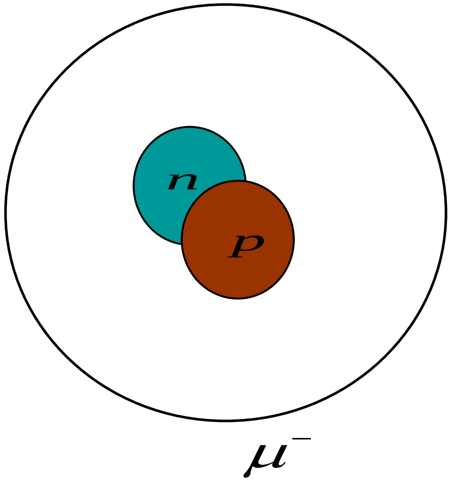
E_γ= 120 MeV



different chiral orders, R=0.9 fm

Muon capture on ^{2}H and ^{3}He

Phys. Rev. C **90**, 024001 (2014)



Muon capture from the lowest K-shell of the muonic atom studied with the single nucleon weak current operator
→ ≈ 10 % error in predictions

$$\psi_K(r) \equiv \psi_{100}(r) = \sqrt{\frac{(Z\,m'\alpha)^3}{\pi}}\, e^{-Z m' \alpha r}$$

$$m' \equiv \frac{m_\mu m_Z}{m_\mu + m_Z}$$

reduced mass

$$E_1 = -\frac{Z^2 \alpha^2 m'}{2}$$

negligible for *Z=1,2* when compared to the muon or nucleon mass

Muon capture on ^{2}H and ^{3}He

Methods developed for electromagnetic reactions can be easily applied to following reactions

$$\mu^- + d \rightarrow \nu_\mu + n + n$$

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + {}^3\mathrm{H}$$

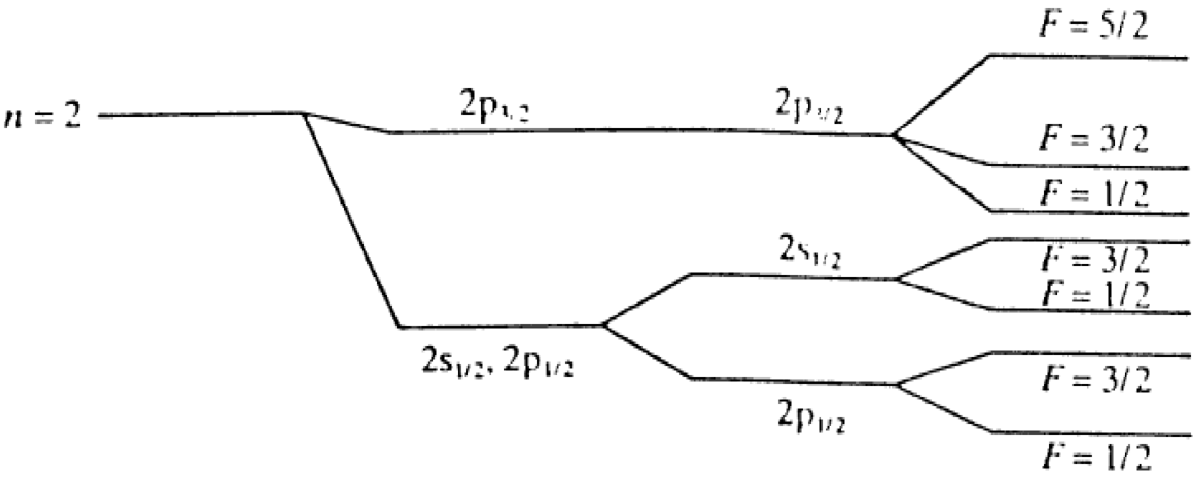
$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + d + n$$

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + p + n + n$$

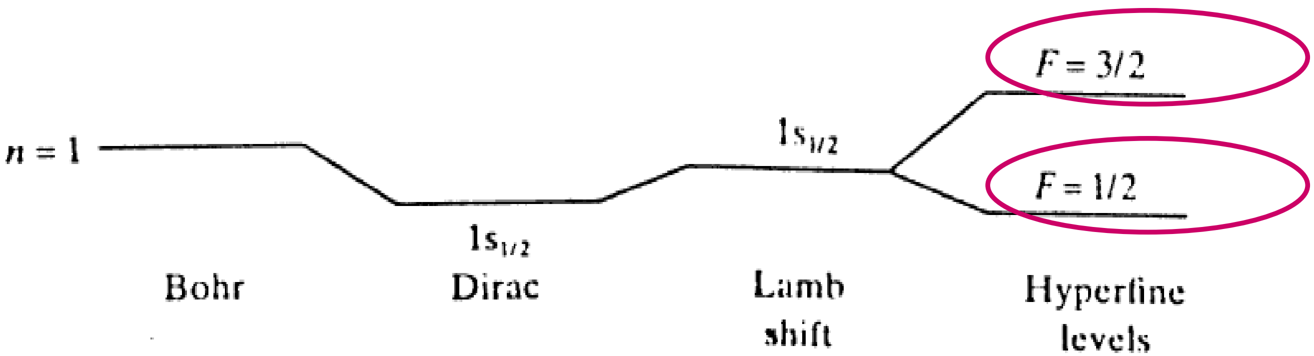
$$\mu^- + {}^3\mathrm{H} \rightarrow \nu_\mu + n + n + n$$

$$\ldots$$

Muon capture on ^{2}H



Hyperfine structure in deuteron



quadruplet states

doublet states

Muon capture on ^2H

$\mu^- + d \rightarrow \nu_\mu + n + n$

Doublet (F=1/2) and quadruplet (F=3/2) capture rates in s^{-1} calculated with the AV18 NN potential (neutron mass is used)

	F=1/2 PW	F=1/2 full	F=3/2 PW	F=3/2 full
SNC	351.8	382.3	9.8	11.4
SNC+MEC	356.9	391.0	10.3	12.1

agrees with results of the Pisa group: L.E. Marcucci *et al.*, Phys. Rev. C83, 014002 (2011)

Muon capture on ^2H

$\mu^- + d \rightarrow \nu_\mu + n + n$

Doublet capture rates (F=½) in s^{-1} calculated with the improved chiral potentials and the single nucleon current operator with RC

Chiral order	R=0.8 fm	R=0.9 fm	R=1 fm	R=1.1fm	R=1.2 fm	Γ_{max} - Γ_{min}
LO	396.0	397.4	398.4	398.9	399.2	3.3
NLO	384.2	385.8	387.2	388.6	389.8	5.7
N2LO	385.0	386.1	387.2	388.3	389.3	4.3
N3LO	386.8	386.4	385.2	384.3	383.2	3.6
N4LO	385.5	386.1	386.3	385.6	384.6	1.7

very weak dependence on the regulator parameter R

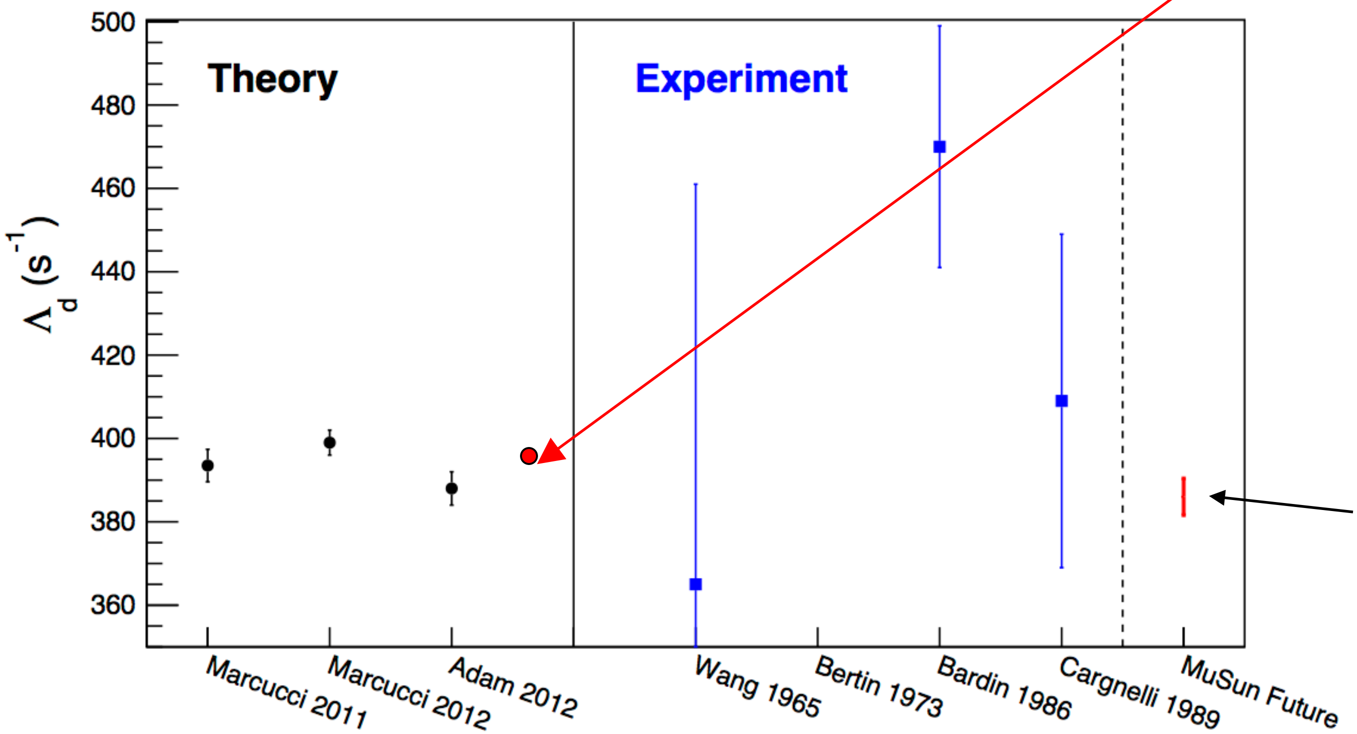
AV18 382.3

Muon capture on ^{2}H

Results from the MuSun experiment will be very important !

http://muon.npl.washington.edu/exp/MuSun/

our present prediction without 2N currents



expected error size in MuSun

Muon capture on ^3He: triton channel

$\mu^- + {}^3He \rightarrow \nu_\mu + {}^3H$

Total capture rates in s^{-1} calculated with the improved chiral potentials and the single nucleon current operator with RC

Chiral order	R=0.8 fm	R=0.9 fm	R=1 fm	R=1.1fm	R=1.2 fm	$\Gamma_{max} - \Gamma_{min}$
LO	1610	1618	1610	1594	1572	46
NLO	1330	1357	1381	1405	1427	97
N2LO	1337	1356	1376	1395	1415	78
N3LO	1314	1304	1289	1278	1266	48
N4LO	1296	1307	1308	1299	1285	23

very weak dependence on the regulator parameter R

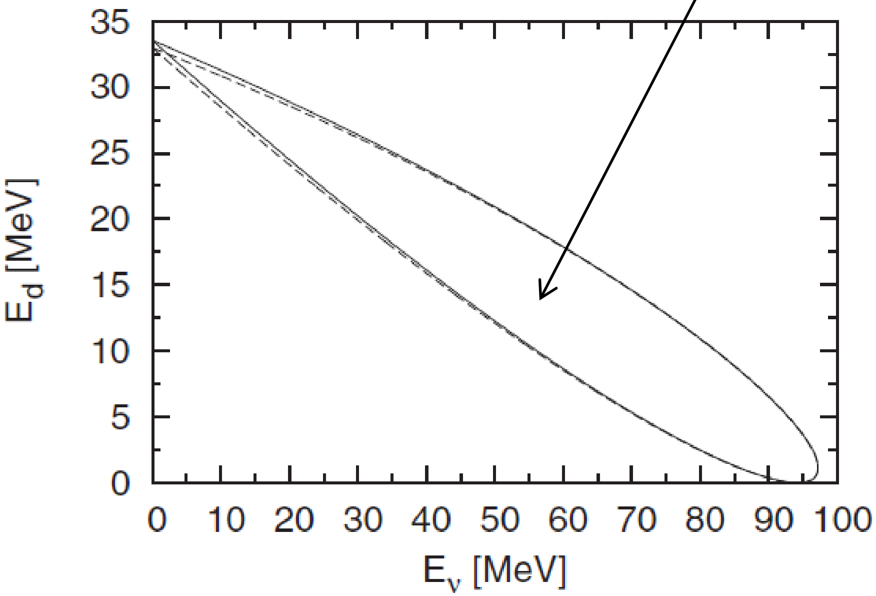
AV18 1353

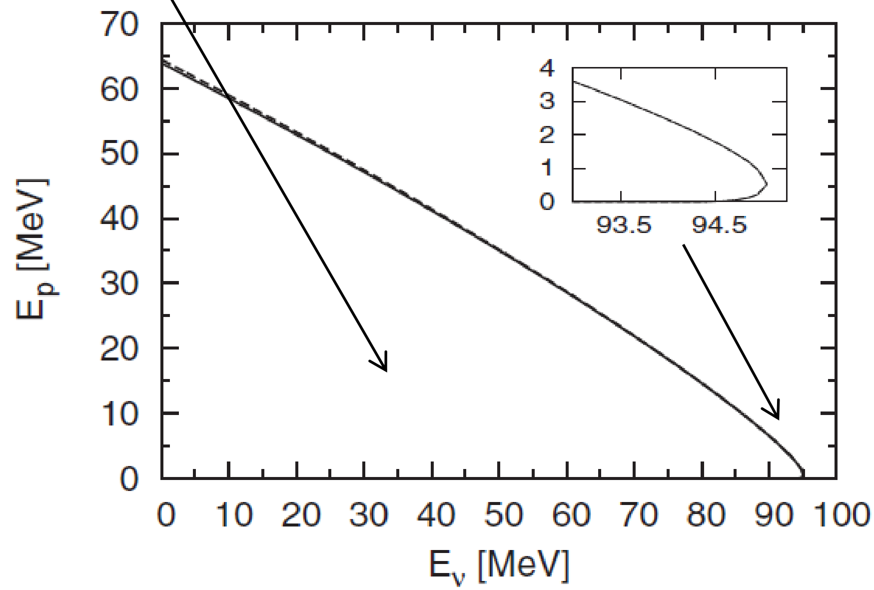
Muon capture on ^{3}He: breakup channels

$$\mu^- + {}^3\text{He} \rightarrow \nu_\mu + d + n$$

$$\mu^- + {}^3\text{He} \rightarrow \nu_\mu + p + n + n$$

kinematically allowed regions





No relativity in the kinematics !

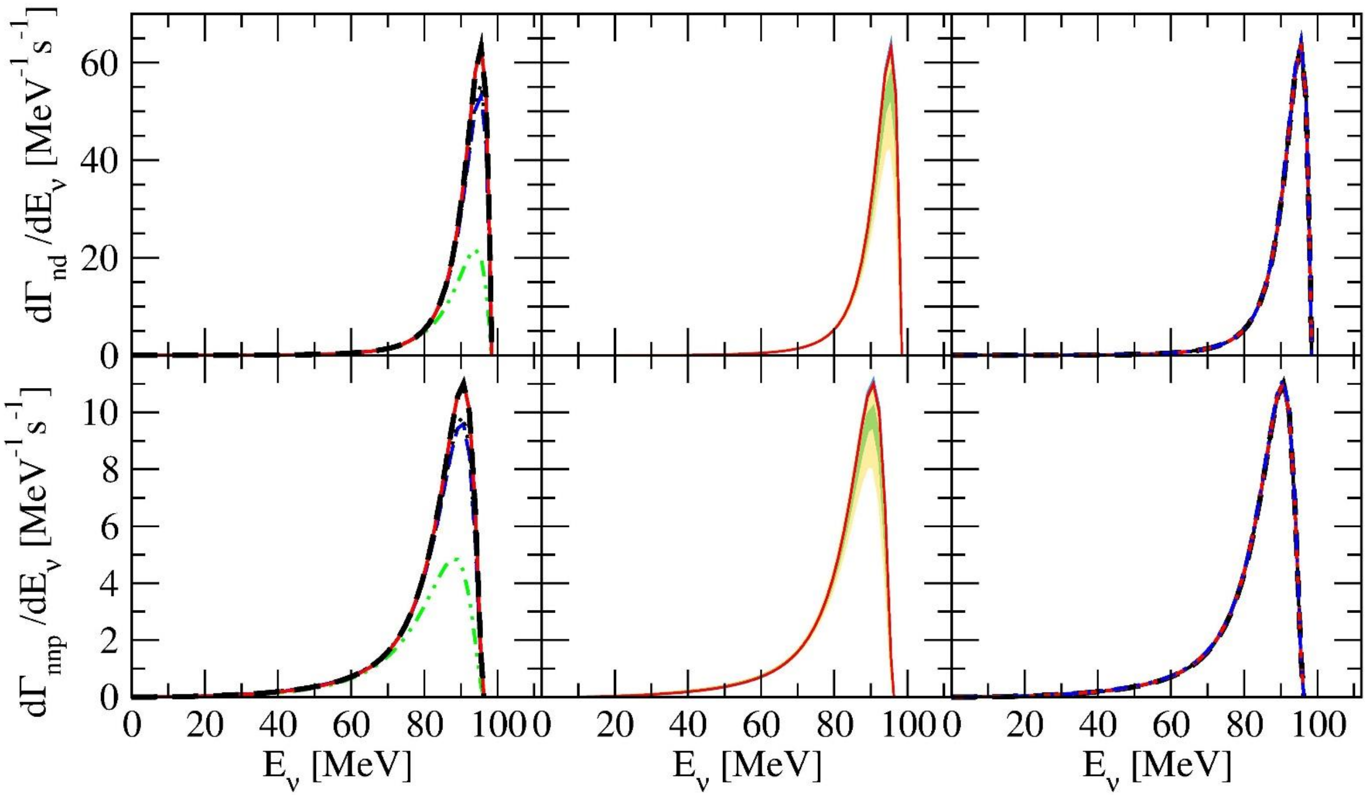
Muon capture on ^{3}He: breakup channels

Total capture rates

	Capture rate Γ in s^{-1}				
	Γ_{nd}			Γ_{nnp}	$\Gamma_{nd} + \Gamma_{nnp}$
	PW	SPW	Full	Full	Full
AV18 ($j_{\max} = 3$)	1917	2046	604	169	773
AV18 ($j_{\max} = 4$)	1917	2046	606	170	776
AV18 + Urbana IX ($j_{\max} = 3$)	1853	1956	544	154	698
Earlier theoretical predictions:					
Yano [34]		510		160	670
Philips *et al.* [35]		414		209	623
Congleton [36]					650
Experimental results:					
Zaĭmidoroga *et al.* [37]					660 ± 160
Auerbach *et al.* [38]					665^{+170}_{-430}
Maev *et al.* [39]					720 ± 70
Bystritsky *et al.* [15]					
Method I		491 ± 125		187 ± 11	678 ± 126
Method II		497 ± 57		190 ± 7	687 ± 60

← best predictions !

Muon capture on ^{3}He: breakup channels



R=0.9 fm
various orders

truncation
errors

N4LO
various R-values

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Muon capture on ^{3}He: two-body break-up

$\mu^- + {}^3He \rightarrow \nu_\mu + n + d$

Total capture rates in s^{-1} calculated with the improved chiral potentials and the single nucleon current operator with RC

Chiral order	R=0.8 fm	R=0.9 fm	R=1 fm	R=1.1fm	R=1.2 fm	$\Gamma_{max} - \Gamma_{min}$
LO	262	282	312	350	392	130
NLO	536	525	515	504	492	44
N2LO	547	539	529	518	507	40
N3LO	584	586	592	596	603	19
N4LO	590	584	583	587	595	12

very weak dependence on the regulator parameter R

AV18 604

Muon capture on ^{3}He: three-body break-up

$\mu^- + {}^3He \rightarrow \nu_\mu + p + n + n$

Total capture rates in s^{-1} calculated with the improved chiral potentials and the single nucleon current operator with RC

Chiral order	R=0.8 fm	R=0.9 fm	R=1 fm	R=1.1fm	R=1.2 fm	Γ_{max} - Γ_{min}
LO	95	99	105	113	120	26
NLO	159	157	154	151	148	11
N2LO	161	159	157	154	151	10
N3LO	169	169	171	172	175	6
N4LO	170	169	169	170	173	4

very weak dependence on the regulator parameter R

AV18 169

Conclusions and outlook

- Very robust momentum space framework to deal many electroweak processes has been constructed and tested (limitations)
- New input: improved chiral 2N and 3N potentials (even 4N potentials) from E. Epelbaum *et al*. are available
- Substantial improvement in description of many observables
- LENPIC (Low Energy Nuclear Physics International Collaboration) to coordinate few-nucleon and many-nucleon Calculations

  ⟶ See Kai Hebeler's talk today !
- Consistent electroweak current operators are needed and are being prepared

- MESA results will be of great importance !

Conclusions and outlook (*cont.*)

BUT BEFORE MESA starts

- Energy ranges and phase-space regions best suited to study the nuclear current operator and three-nucleon force effects should be identified for considered reaction channels
- Achievable accuracy of theoretical predictions for various observables should be estimated
- Consistent chiral potentials and EM current operators are necessary as input to these calculations

Conclusions and outlook (*cont.*)

What should be measured ?

Various observables in deuteron electrodisintegration (polarization might be crucial !)

Two-body break-up of ^{3}He

1. unpolarized proton angular distributions (for a wide range of angles)
2. ^{3}He analyzing power
3. Spin-dependent helicity asymmetries

Three-body break-up of ^{3}He

1. Semi-exclusive cross sections (proton and neutron) at various emission angles with respect to the momentum transfer
2. ^{3}He analyzing power
3. Spin-dependent helicity asymmetries

Thank you !