

Hadronic Contributions to $(g-2)_\mu$

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New Vistas in Low-Energy Precision Physics
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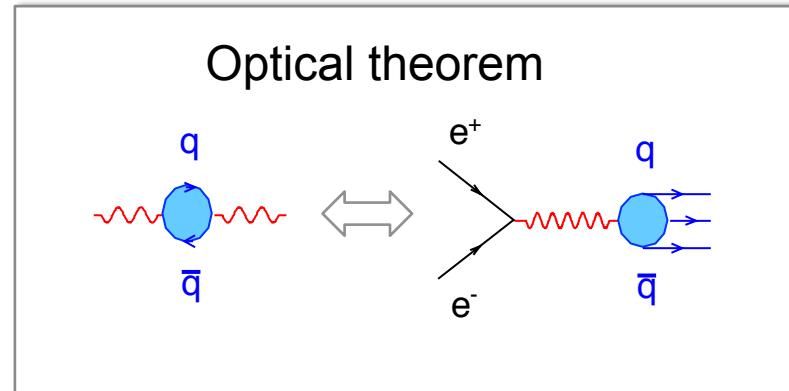
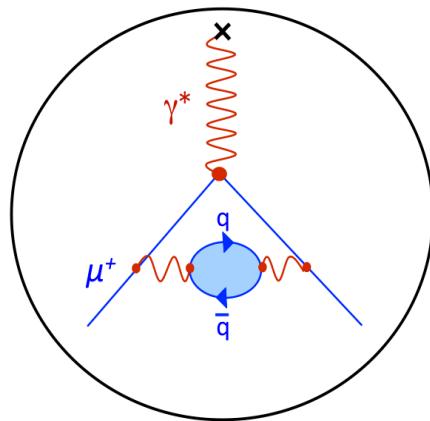
Muon Anomaly: $a_\mu = (g-2)_\mu / 2$

- **Experimental:** $a_{\mu}^{\text{exp}} = 1165\ 920\ 8.9\ (6.3) \times 10^{-10}$ (0.54 ppm)
[BNL-E821: PRD 73 072003]
- **Standard Model prediction:** $a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{weak}} + a_{\mu}^{\text{had}}$

Contribution	in units 10^{-10}	
QED(γ +lepton)	$1165\ 847\ 1.8951 \pm 0.0080$	Kinoshita et.al. (2012)
EW	15.36 ± 0.10	Gnendiger, Stöckinger, Stöckinger-kim (2013)
HVP,LO	692.3 ± 4.2	Davier et. al. (2011)
HVP,NLO	-9.84 ± 0.07	Hagiwara et al. (2009)
HLbL	11.6 ± 4.0	Jegerlehner, Nyffler (2009)
Total	$1165\ 918\ 1.3 \pm 5.8$	

Hadronic Contributions

Hadronic Vacuum Polarization



Dispersion integral

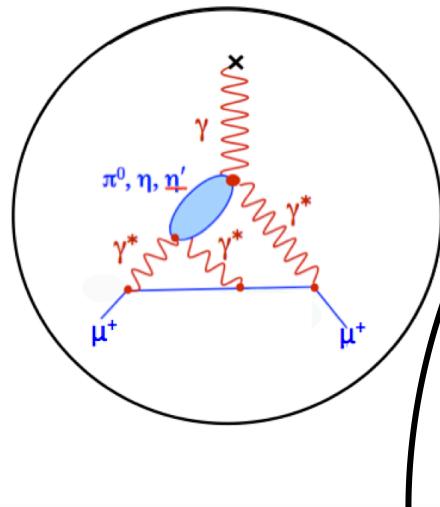
$$a_{\mu,LO}^{\text{HVP}} = \frac{1}{4\pi^3} \int_{m_{\pi^0}}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$

Kernel function $\sim 1/s$

$\sigma_{\text{had}} = \sigma(e^+e^- \rightarrow \text{hadrons})$
 $\sim 1/s \rightarrow$ Low energy contributions important!

Hadronic Contributions

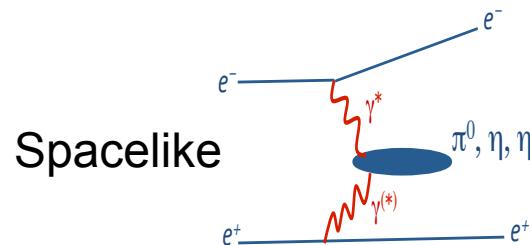
Hadronic Light-by-Light



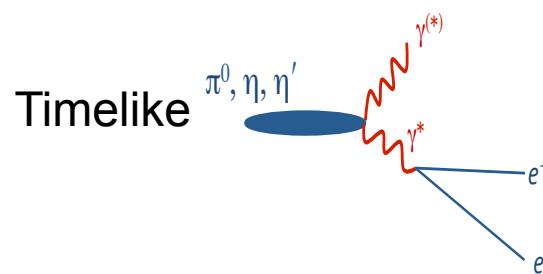
Transition form factors

- Only model calculations so far
- Data-driven approach been developed

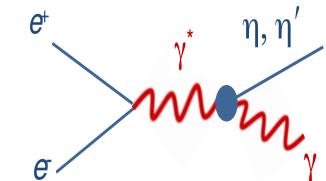
[Colangelo et al '14; Pauk, Vanderhaeghen '14]



Spacelike



Timelike

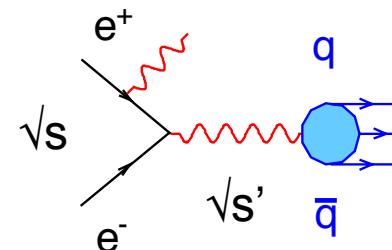
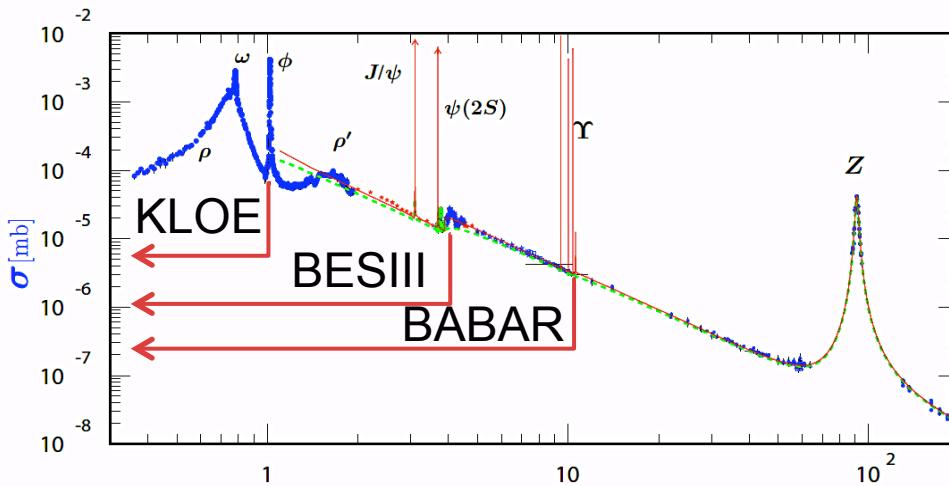


Hadronic Cross Section

$\sigma_{\text{had}} (\text{e}^+ \text{e}^- \rightarrow \text{hadrons})$

Hadronic Cross Section

- Energy Scan:
 - CMD & SND at VEPP-2M & VEPP-2000 in Novosibirsk
 - BESIII at BEPCII in Beijing
- Initial State Radiation:
 - KLOE at DA ϕ NE in Frascati
 - BABAR at PEP-II in Stanford
 - BESIII at BEPCII in Beijing

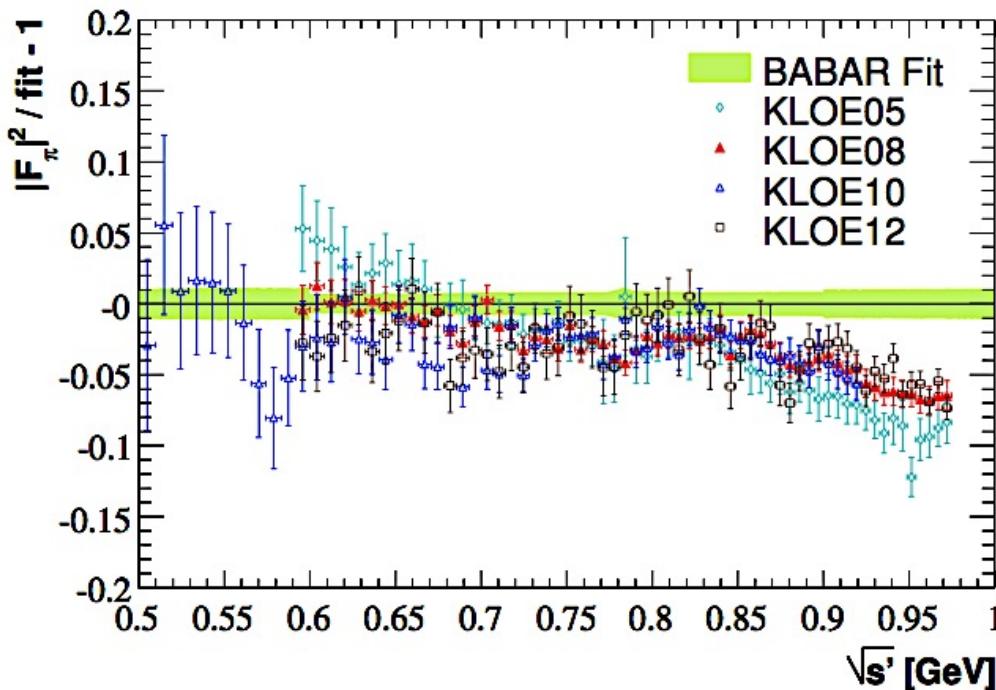


- Needs **no** systematic variation of beam energy
- High statistics thanks to high integrated luminosities

Most Relevant Channel:

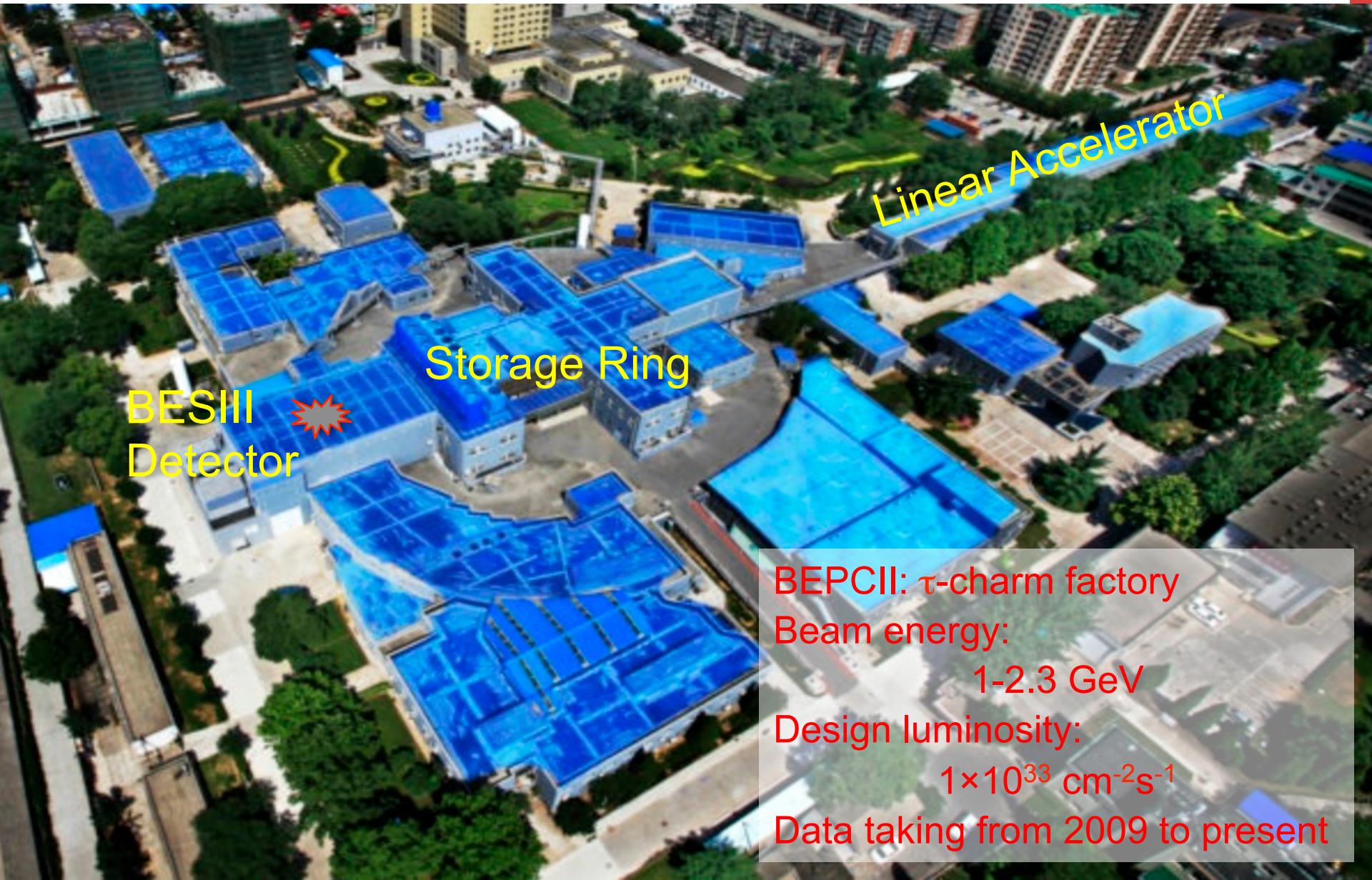
$$e^+ e^- \rightarrow \pi^+ \pi^-$$

- KLOE and BABAR dominate the world average
- Both with uncertainties smaller than 1%
- Relatively large systematic differences, especially above ρ peak
- Knowledge of a_μ^{had} dramatically limited due to this difference

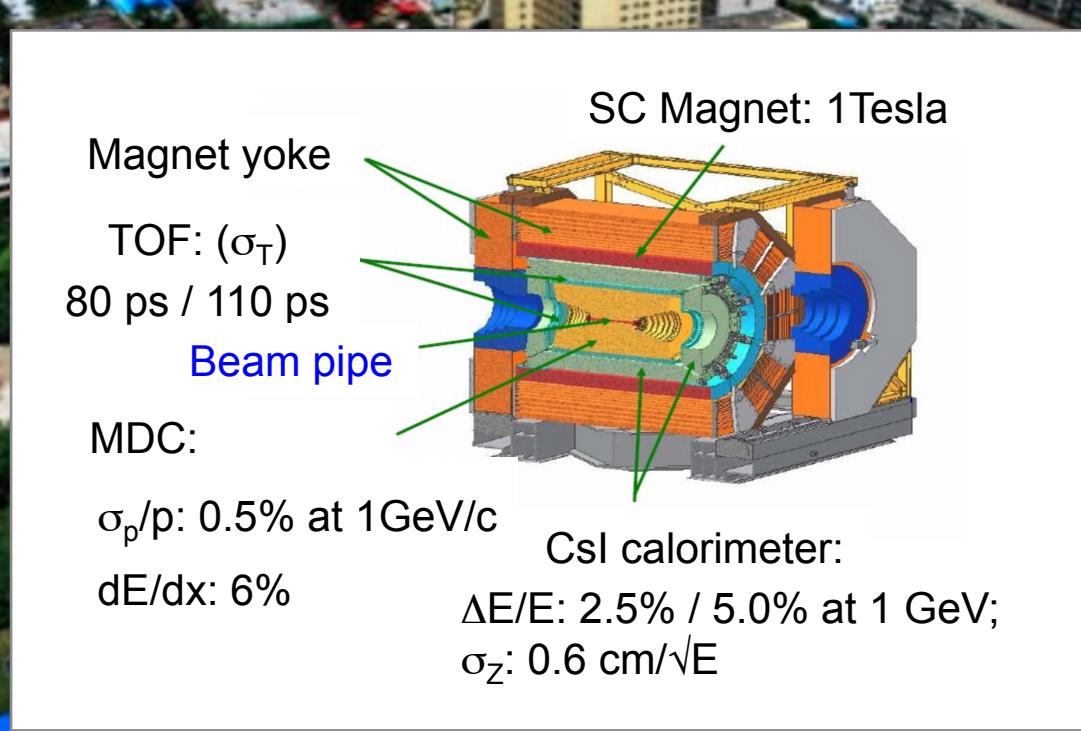


Note: KLOE05 super-
seded by KLOE08

Beijing Electron Positron Collider-II

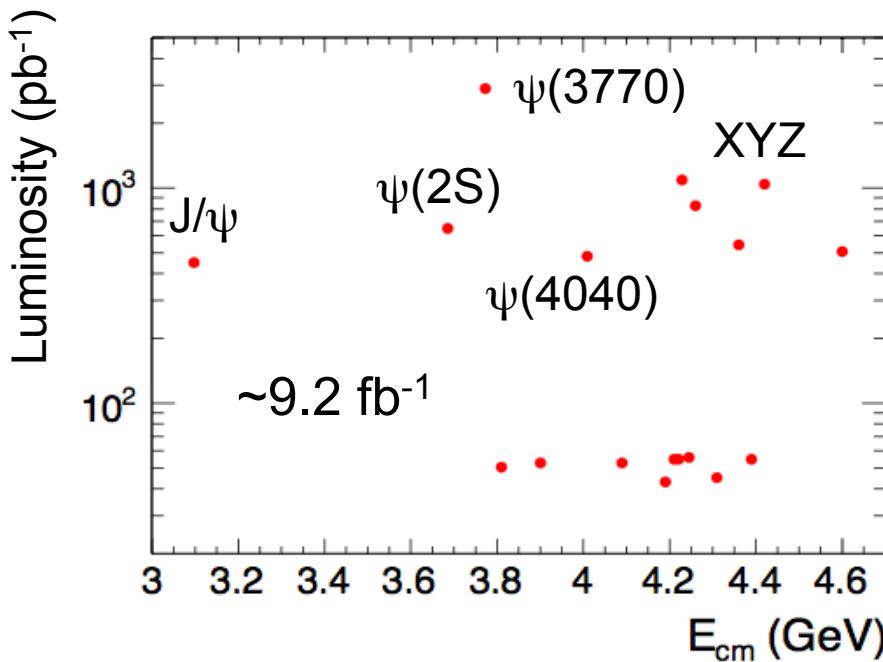


Beijing Electron Positron Collider-II

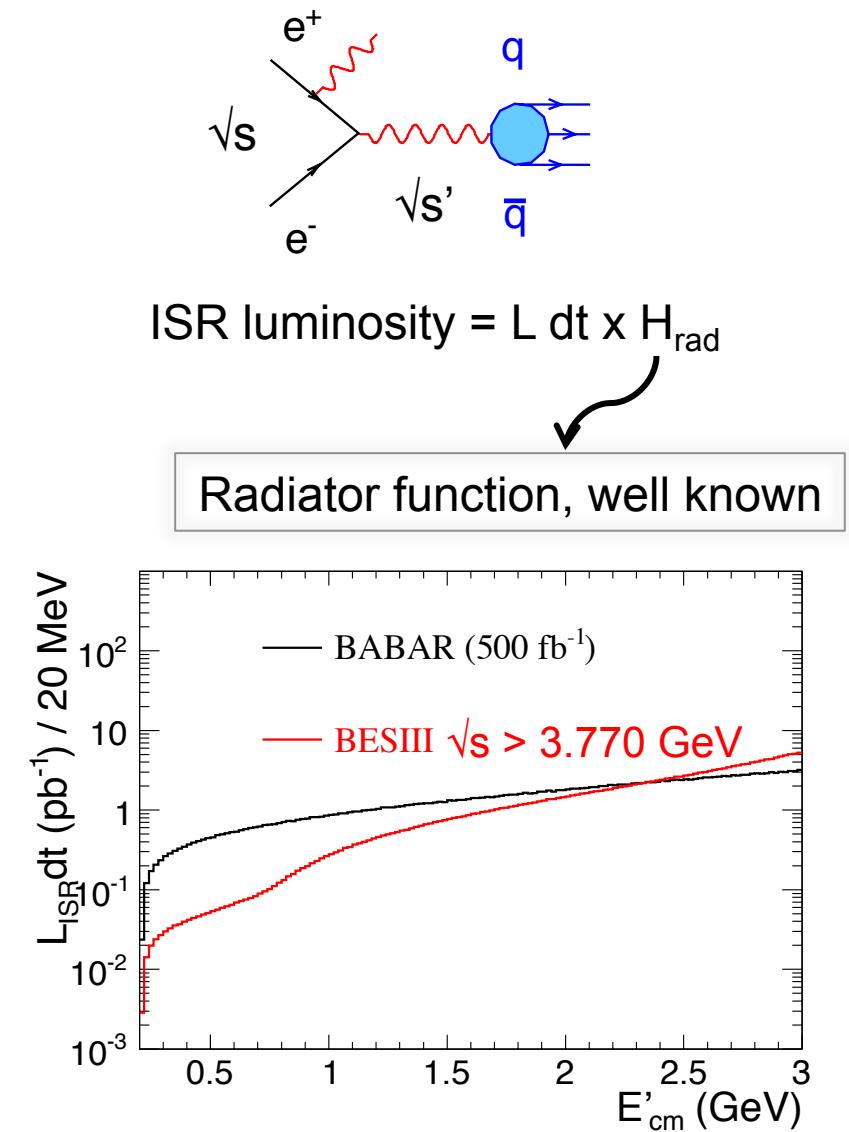


BEPCII: τ -charm factory
Beam energy:
1-2.3 GeV
Design luminosity:
 $1 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$
Data taking from 2009 to present

Data Samples for ISR Study

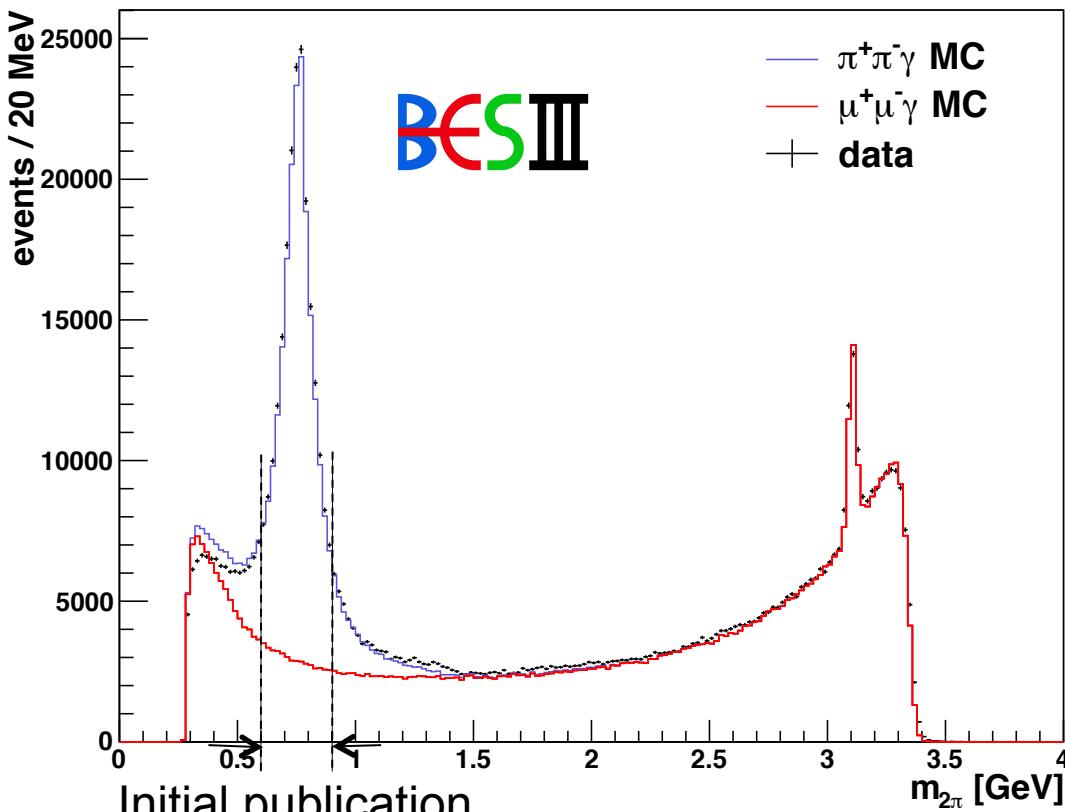


Scan data samples later



$e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^-$ at BESIII

Event yield after preliminary selection

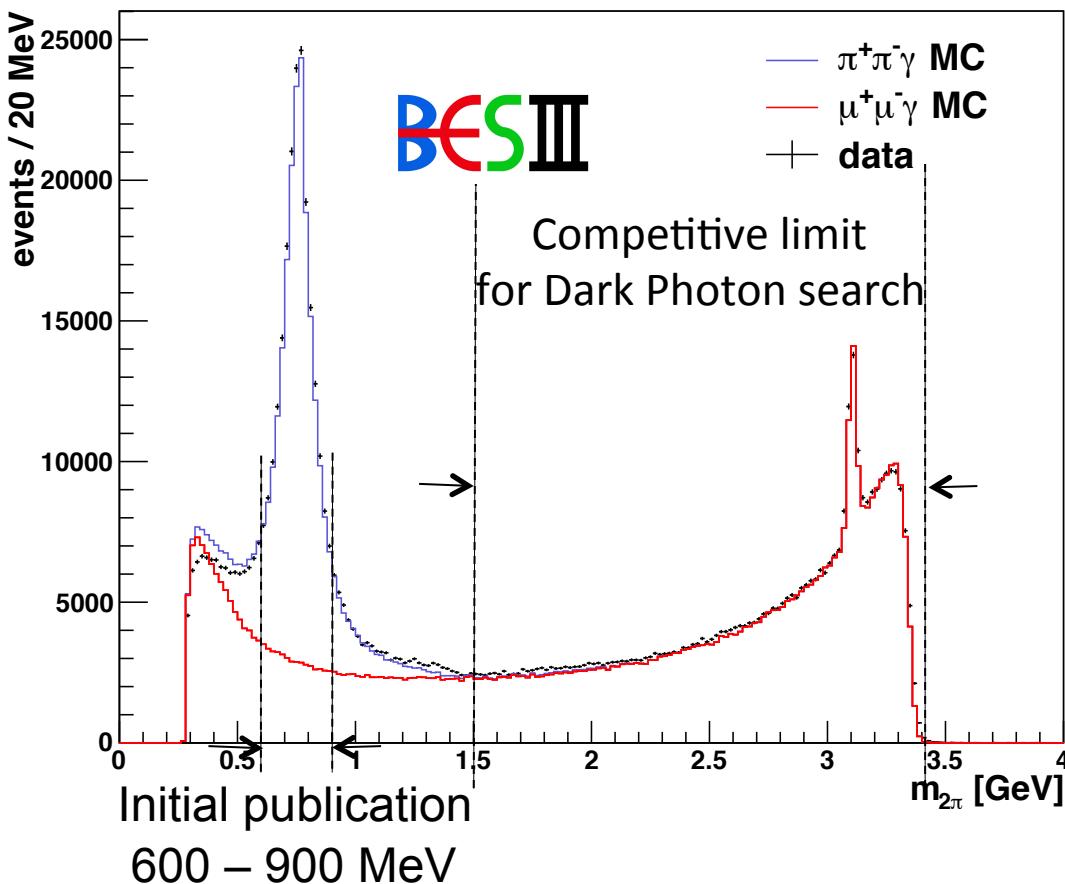


- $\psi(3770)$ data only (2.9 fb^{-1})
- Tag ISR photon
- No dedicated background subtraction
- $e^+e^- \rightarrow \gamma\pi^+\pi^-$: large statistics
- $e^+e^- \rightarrow \gamma\mu^+\mu^-$: dominate background
- Data - MC differences visible

[Phys. Lett. B753 (2016) 629]

$e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^-$ at BESIII

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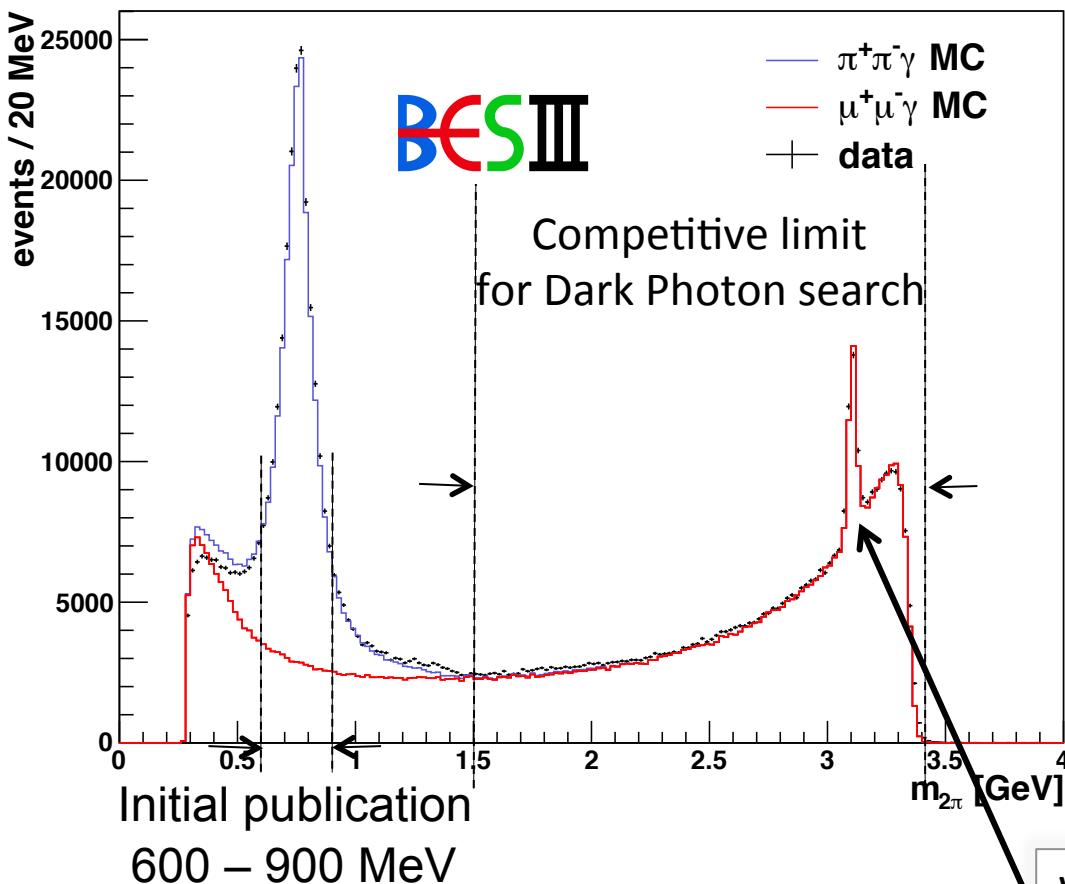


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[Phys. Lett. B753 (2016) 629]

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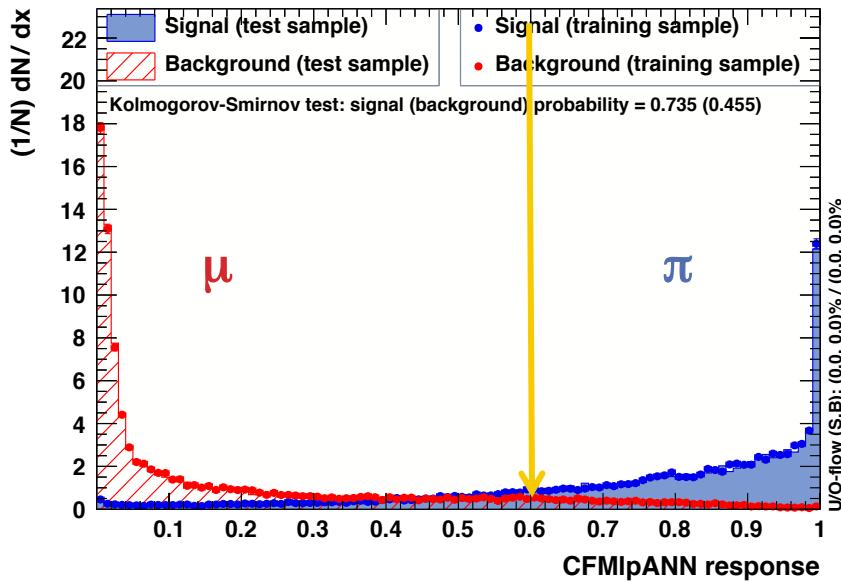
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World's best measurement
of Γ_{ee} of J/ψ

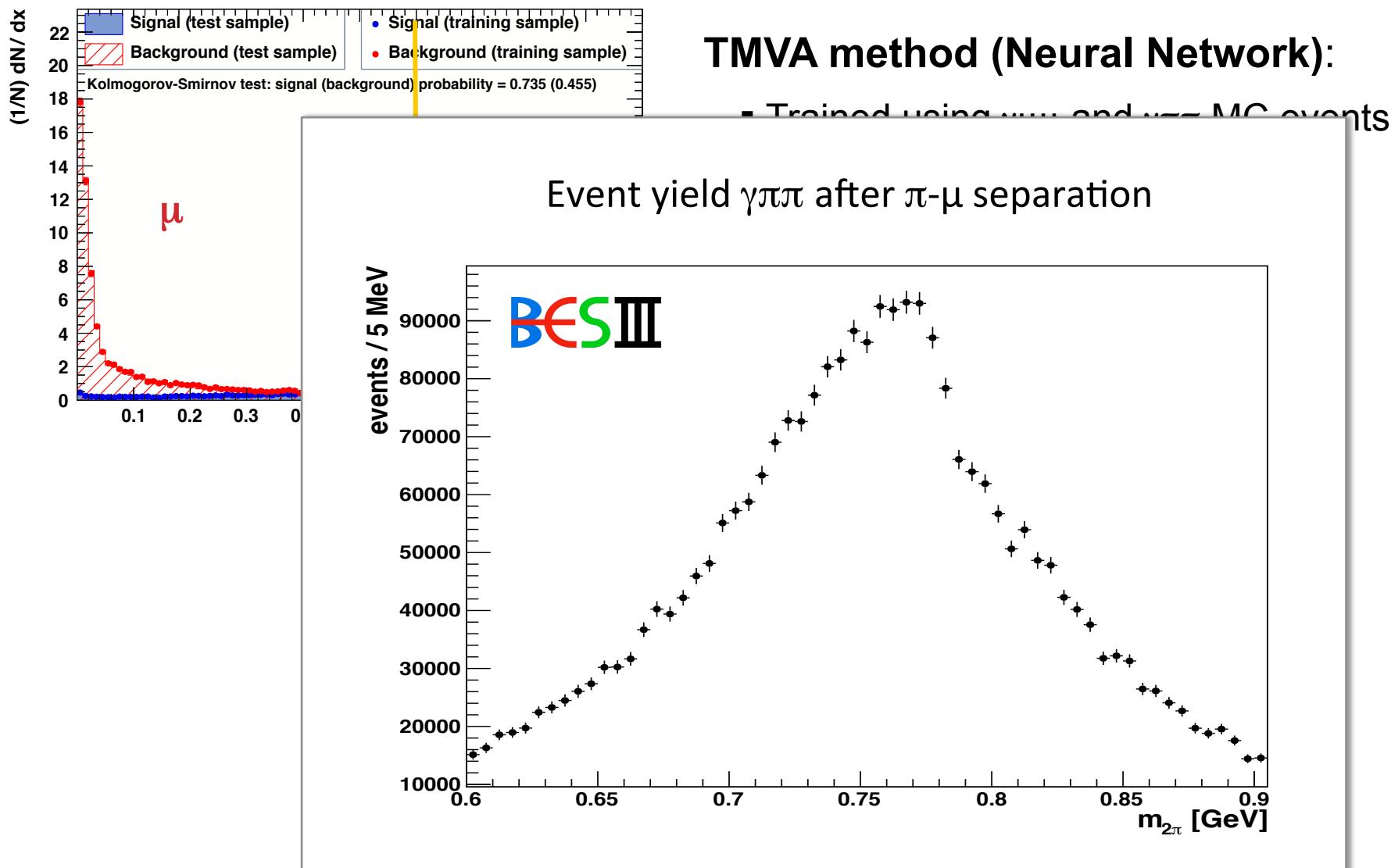
$e^+e^- \rightarrow \gamma_{ISR} \pi^+\pi^-$: π - μ separation



TMVA method (Neural Network):

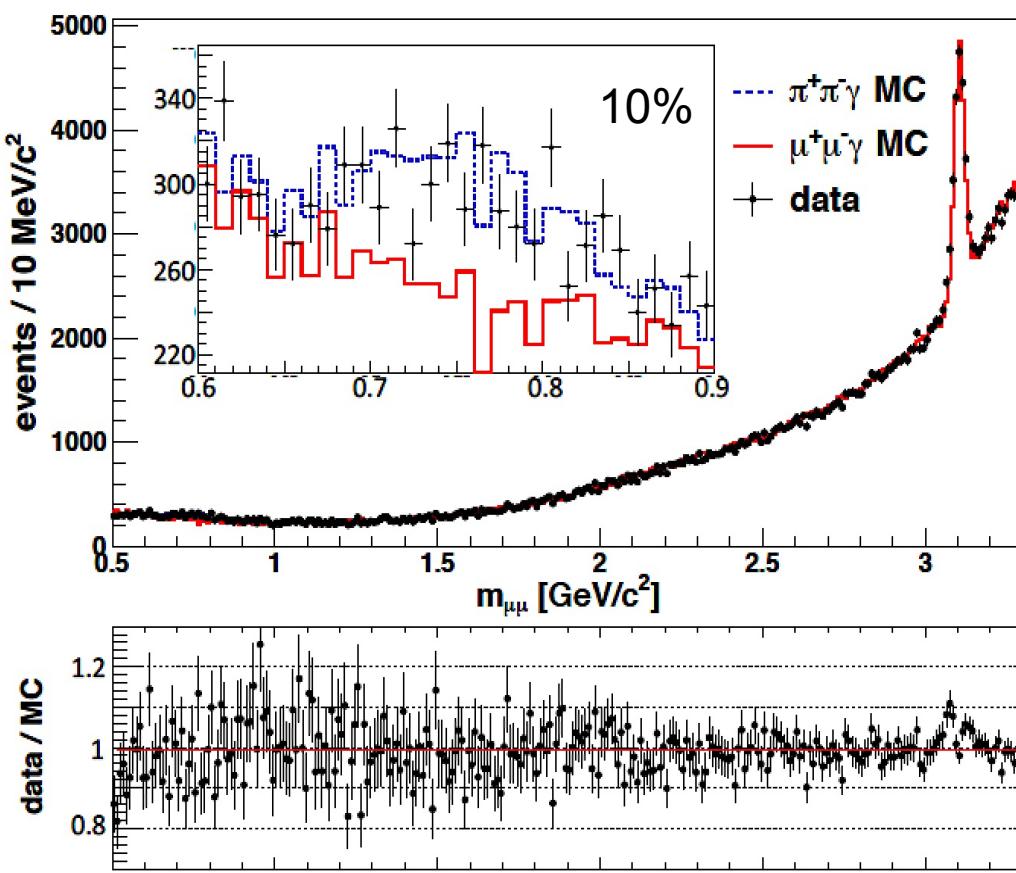
- Trained using $\gamma\mu\mu$ and $\gamma\pi\pi$ MC events
- Information based on track level
- Efficiency matrix (p, θ) for data, MC
- Correct for data - MC differences
- Cross checked for different TMVA methods

$e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^-$: $\pi\text{-}\mu$ separation



QED Test: $e^+e^- \rightarrow \gamma\mu^+\mu^-$

Event yield $\gamma\mu\mu$ after $\pi\text{-}\mu$ separation and all efficiency corrections



- Background from $\gamma\pi\pi$ small
- PHOKHARA uncertainty < 0.5%
- Luminosity measurement based on Bhabha events, 0.5% accuracy

$$\Delta(\text{MC/QED-data}) - 1 = (1.0 \pm 0.3_{\text{stat}} \pm 0.9_{\text{syst}}) \%$$

- Excellent agreement with QED
- Accuracy on 1% level as needed to be competitive !

$e^+e^- \rightarrow \pi^+\pi^-$ Cross section

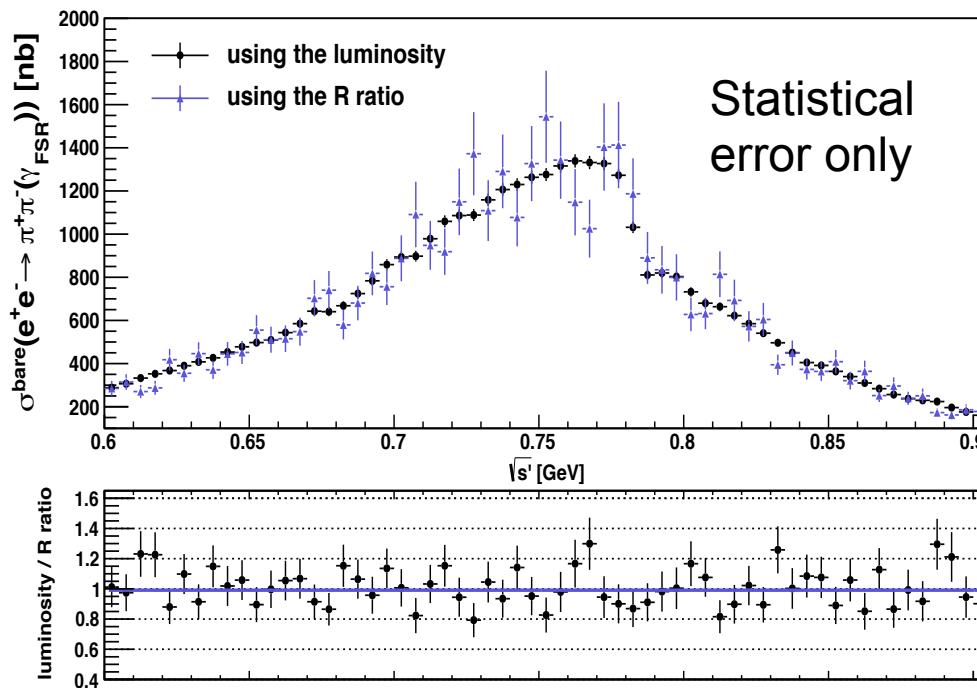
2 normalization methods:

- Normalization to L_{int} (obtained from Bhabha events)

$$\sigma_{\pi\pi(\gamma)}^{\text{bare}} = \frac{N_{\pi\pi\gamma} \cdot (1 + \delta_{\text{FSR}}^{\pi\pi})}{\mathcal{L}_{int} \cdot \epsilon_{\text{global}}^{\pi\pi\gamma} \cdot H(s) \cdot \delta_{vac}}$$

- Normalization to $\gamma\mu\mu$ events, i.e. R ratio ($\gamma\pi\pi/\gamma\mu\mu$)

$L_{int}, H_{rad}, \delta_{vac}$ cancel in ratio



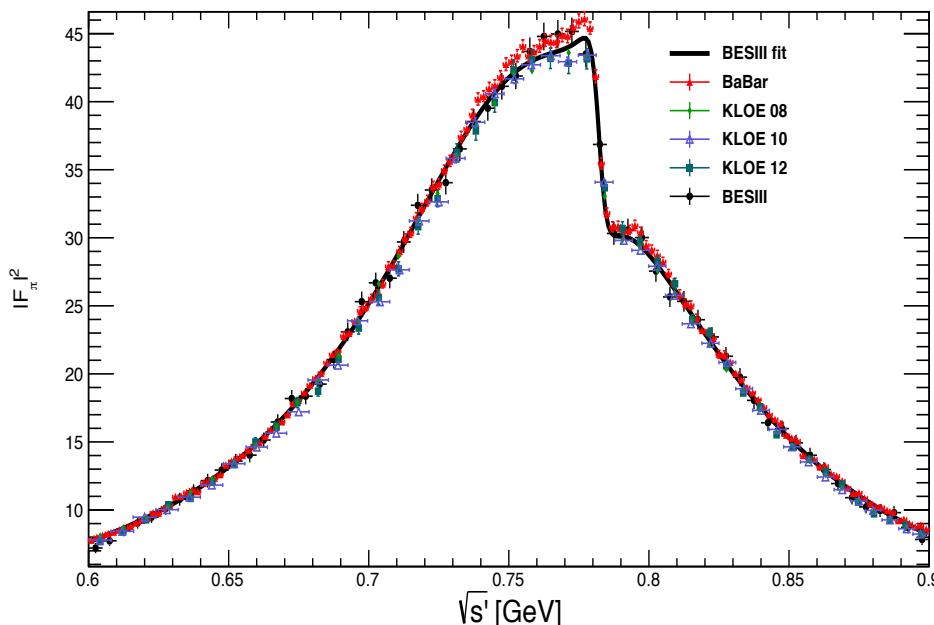
Good agreement between
two methods

Luminosity / R ratio -1
= $(0.85 \pm 1.68) \%$

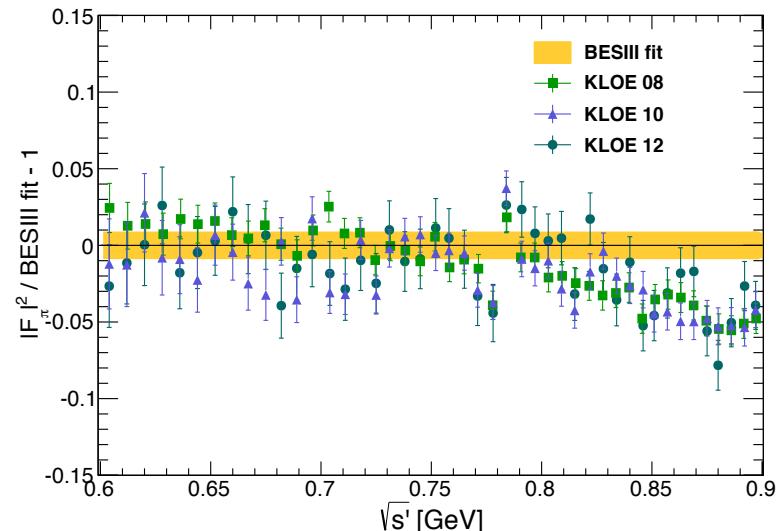
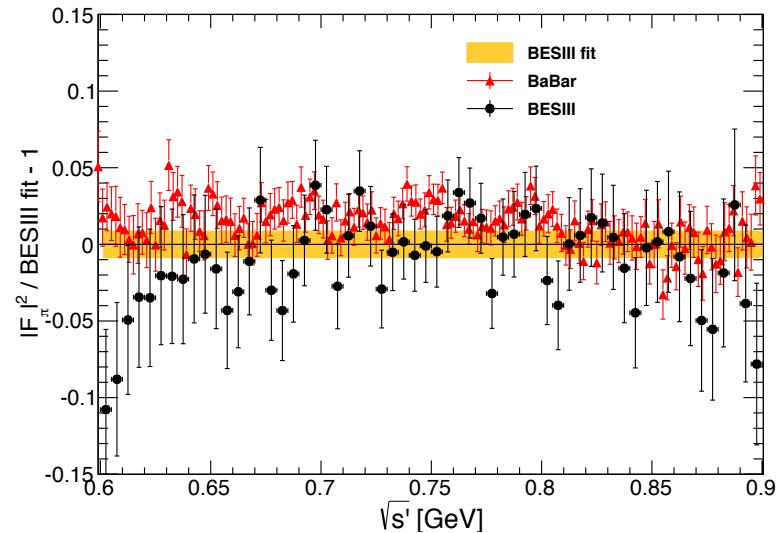
limited by low $\gamma\mu\mu$ statistics

Compare with Existing Data

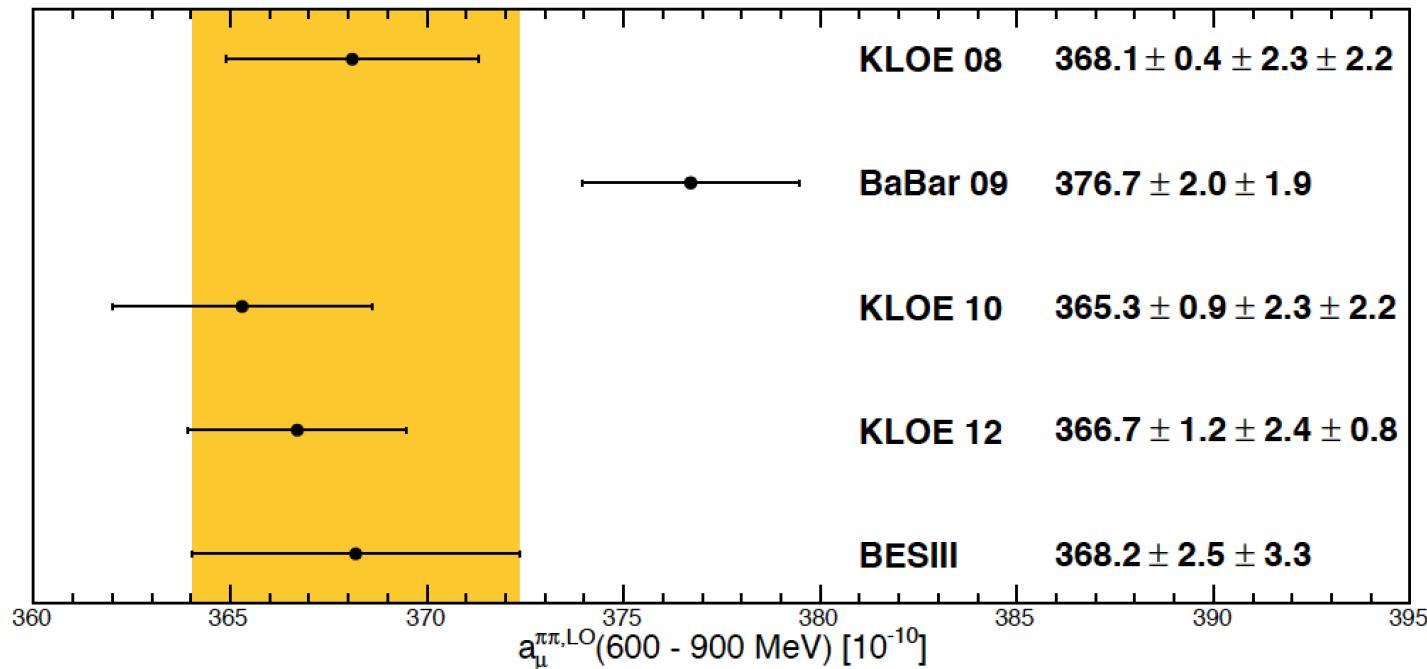
Pion Form Factor F_π



- 0.9 % accuracy (dominated by theory)
- Normalization to luminosity \times radiator function
- Gounaris and Sakurai parameterization

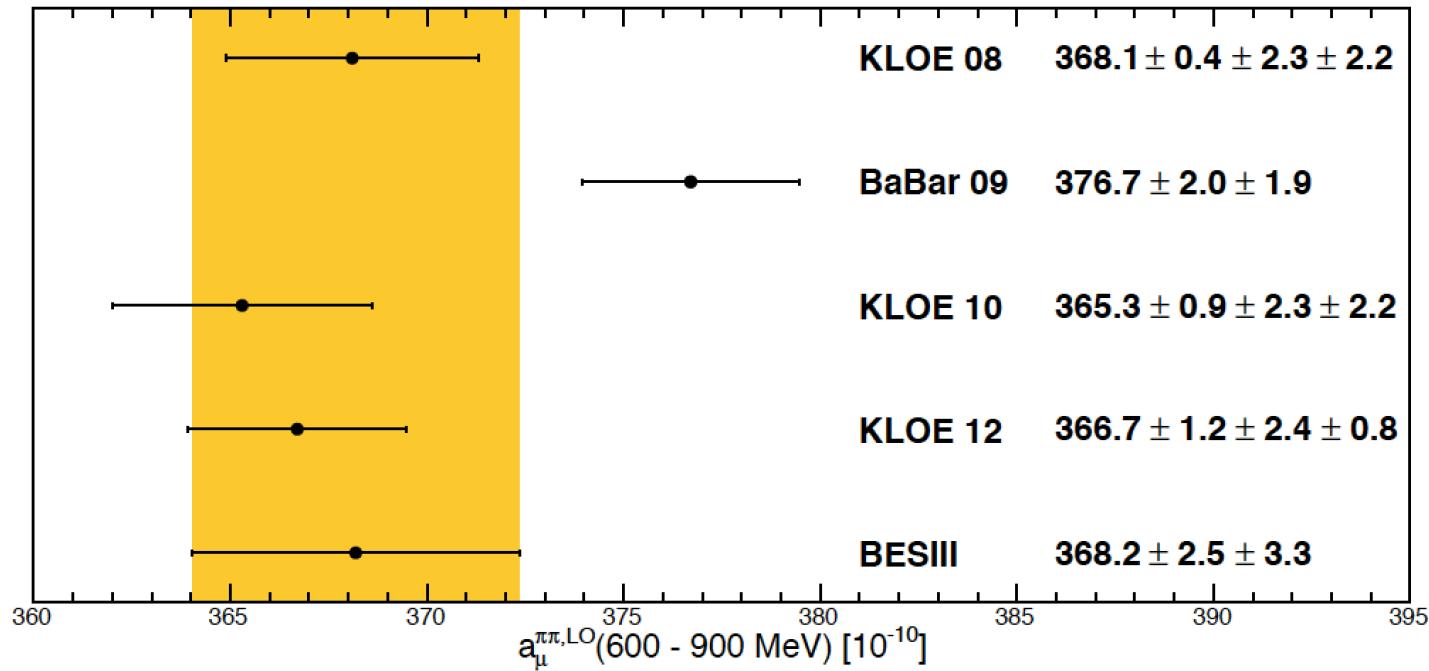


Impact on a_{μ}^{HVP}

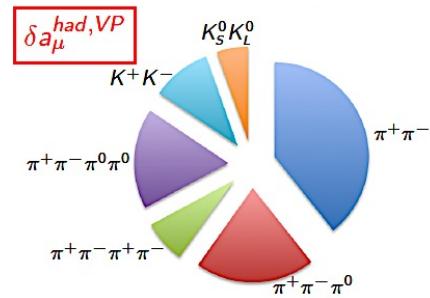


Deviation on $(g-2)_{\mu}$ between experimental and SM has been confirmed

Impact on a_μ^{HVP}



Deviation on $(g-2)_\mu$ between experimental and SM has been confirmed



Study of $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\pi^0\pi^0$ processes undergoing at BESIII

Energy Scan from 2.0 to 4.6 GeV

World's best measurement from BES/BESII with 5% ~ 8% total uncertainty (statistical uncertainty: 3% ~ 5%)

BESIII: aim at systematic uncertainty: 3.0%

151 energy points $>10^5$ hadronic events each → statistical error negligible

Energy region	Energy points	Note
2.400~3.400	4	Mini-scan
3.800~4.590	104	Fine-scan heavy charm resonant
2.000~3.080	21	R&QCD-scan
3.050~3.120	16	J/ ψ -scan
3.542~3.600	5	τ -scan
3.650,3.671	2	$\psi(3686)$ -scan

Reducing the uncertainty of $\alpha_{\text{em}}(M_Z^2)$ by a factor of 2

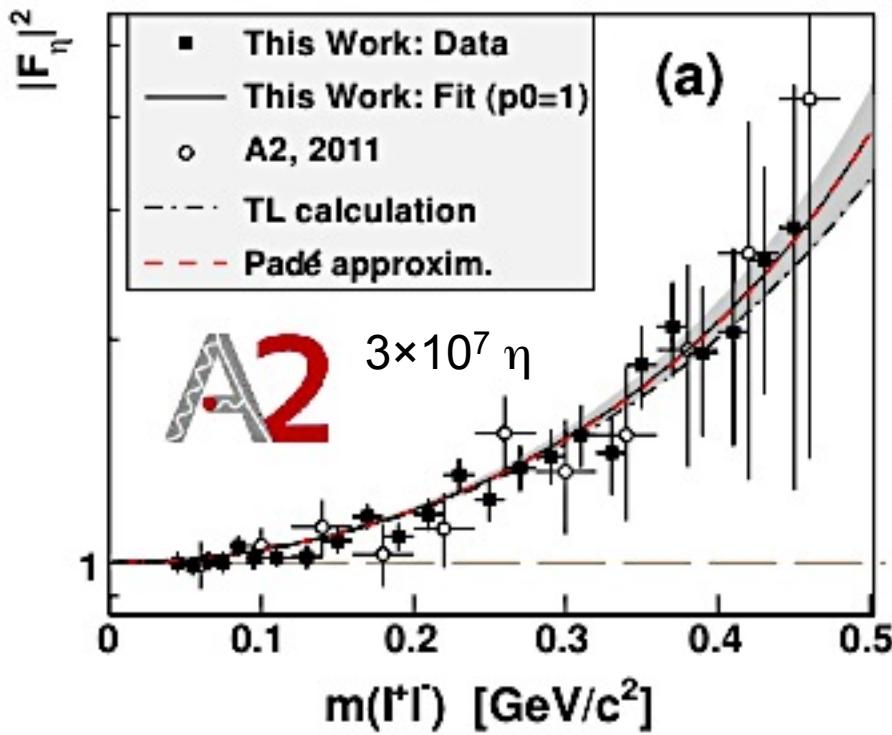
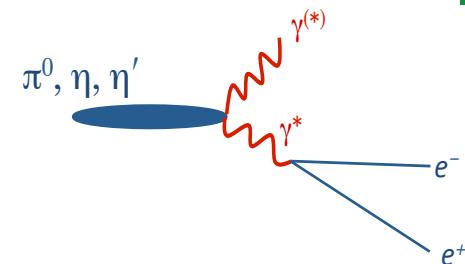
→ A new quantity of electroweak precision fits

Meson Transition Form Factor $|F(Q^2)|$

Timelike Transition FFs



$$\frac{d\Gamma(\eta \rightarrow l^+ l^- \gamma)}{dm_{ll} \Gamma(\eta \rightarrow \gamma\gamma)} = [QED] \cdot |F_\eta(m_{ll})|^2$$



[Phys. Rev. C89 (2014) 044608]

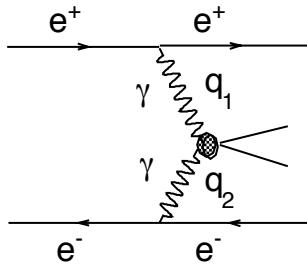
Vector Meson Dominance Model:
single-pole:

$$F(q) = (1-q^2/\Lambda^2)^{-1}$$

$$\Lambda^{-2} = (1.95 \pm 0.15 \pm 0.10) \text{ GeV}^{-2}$$

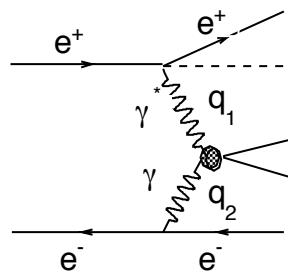
In good agreement with previous measurements!

Spacelike Transition FFs



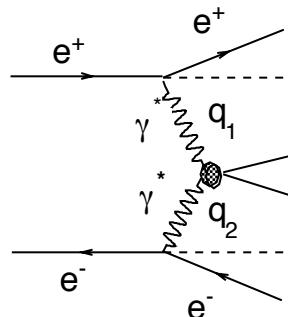
Untag:

- Only tag the hadron products, P_t -balance
- $Q_i^2 \sim 0 \text{ GeV}^2$, quasi-real photon



Single tag:

- Tag the hadron products
- Tag only one lepton, missing momentum direction
- $Q_1^2 \sim 0 \text{ GeV}^2$, $Q_1^2 = -q_2^2 \text{ GeV}^2$; highly virtual photon



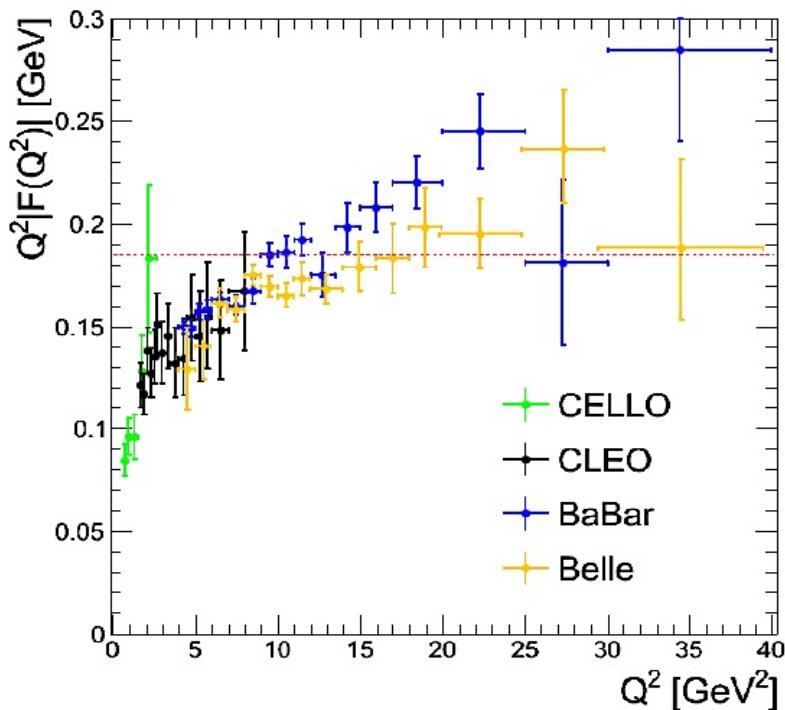
Double tag:

- Tag the hadron products
- Tag both leptons
- Both photons are virtual

Input for data-driven approach

Existing Data on Spacelike TFFs

$e^+e^- \rightarrow e^+e^- \pi^0$



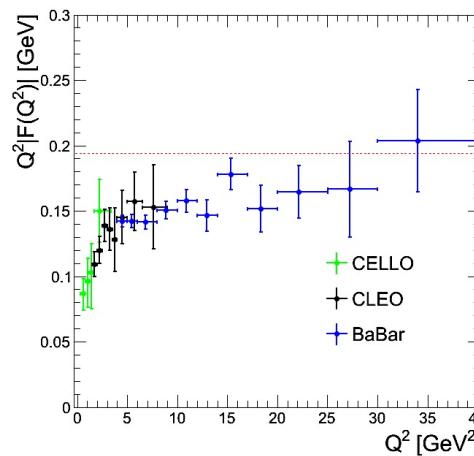
CELLO: Z. Phys. C 49 401 (1991)

CLEO: Phys. Rev. D57 33 (1998)

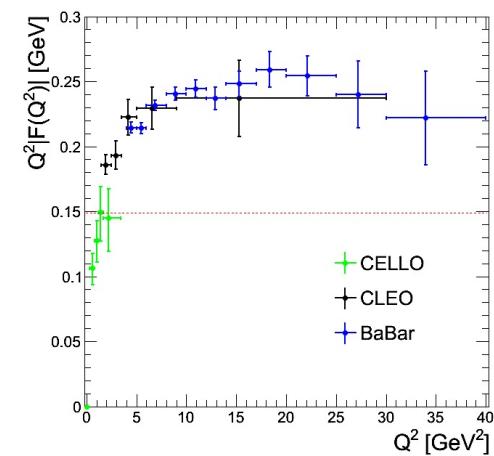
BaBar: Phys. Rev. D80 052002 (2009)

Belle: Phys. Rev. D86 092007 (2012)

$e^+e^- \rightarrow e^+e^- \eta$



$e^+e^- \rightarrow e^+e^- \eta'$

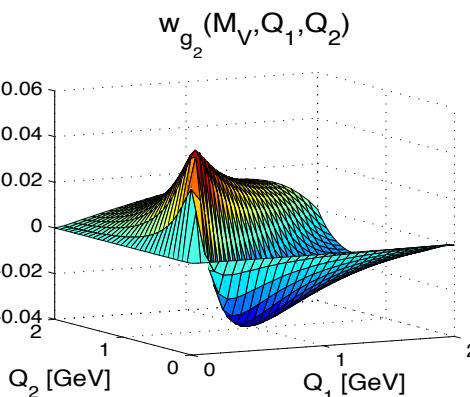
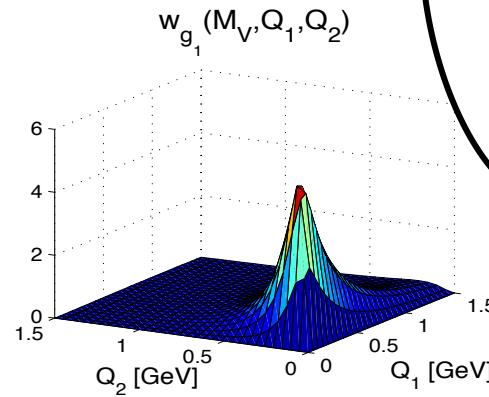
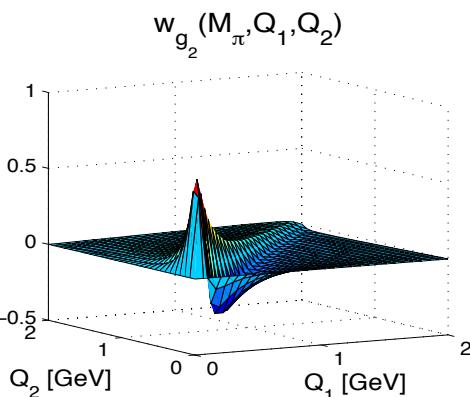
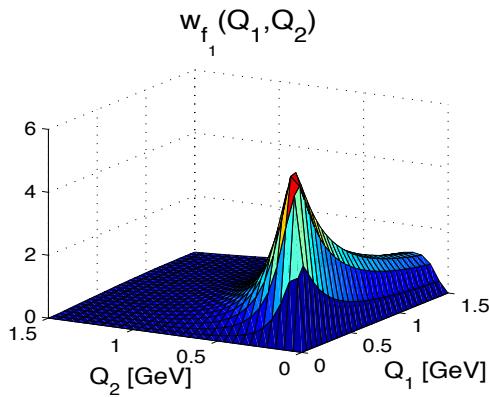


- Recent results from BABAR and BELLE:
 $Q^2 > 4 \text{ GeV}^2$
- CLEO: $Q^2 > 1.5 \text{ GeV}^2$
- CELLO: $Q^2 < 1.5 \text{ GeV}^2$, very poor accuracy

Low Q^2 range not covered/precise

Relevant Q^2 Region

$$a_{\mu}^{\text{HLBL};\pi^0} = \int_0^{\infty} dQ_1 \int_0^{\infty} dQ_2 \sum_i w_i(Q_1, Q_2) f_i(Q_1, Q_2)$$



Form factor dependent

Universal weight functions

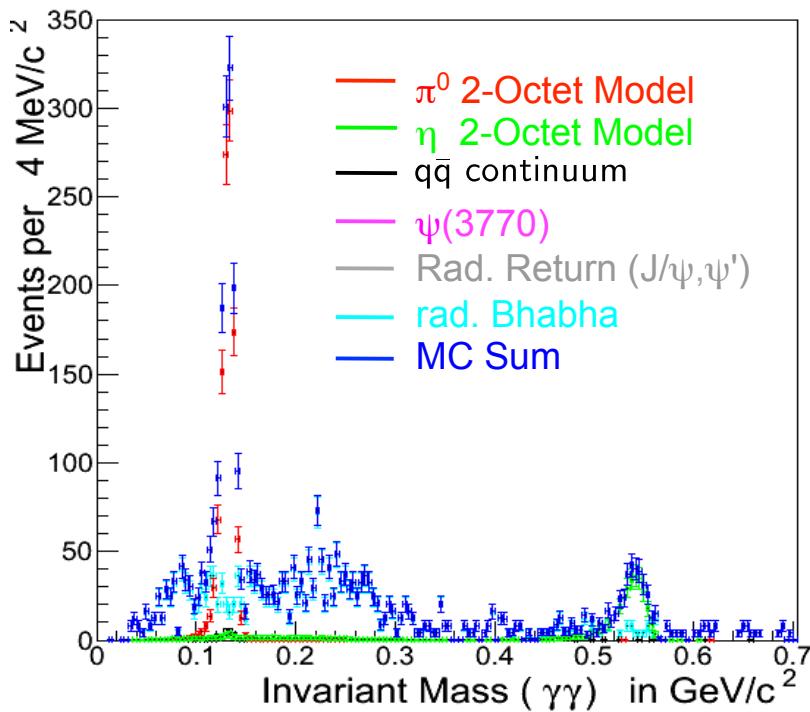
Relevant Q^2 region:
 $< 1.5 \text{ GeV}^2$

[M. Knecht and A. Nyffeler: Phys. Rev. D 65, 073034 (2002)]

$e^+e^- \rightarrow e^+e^- \pi^0$ at BESIII

L_{int} : 927 pb $^{-1}$ Tagged lepton: e^-

MC only, part of full statistics



Event Selection:

- Exactly one lepton candidate
- At least two, max four photons
- Helicity angle $\cos \theta_H > 0.8$
- Kinematic cuts to reject ISR background
- Cut on angle of missing momentum

Strategy:

Count
 π^0 yield in
bins of Q^2

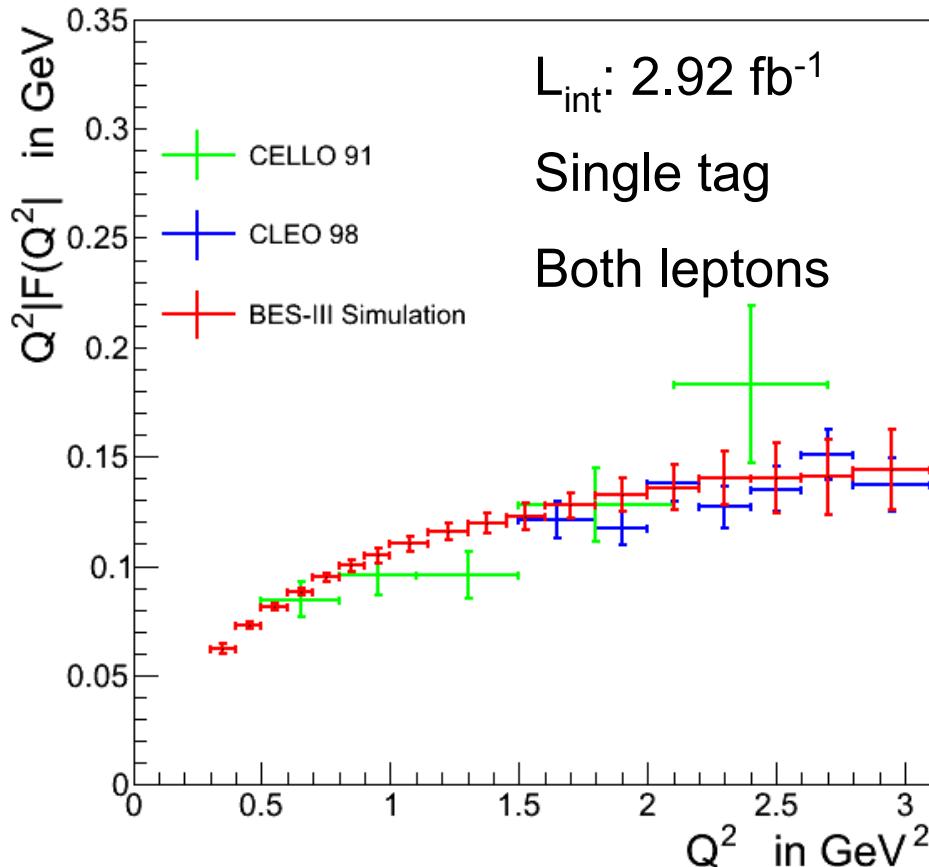


$d\sigma/dQ^2$



Form factor
 $F(Q^2)$

Spacelike transition FFs: π^0



[CELLO: Z. Phys. C 49 401 (1991)]
[CLEO: Phys. Rev. D57 33 (1998)]

MC only, red error bars
corresponding to BESIII statistics

Extract TFF for:

$$0.3 \leq Q^2[\text{GeV}^2] \leq 3.1$$

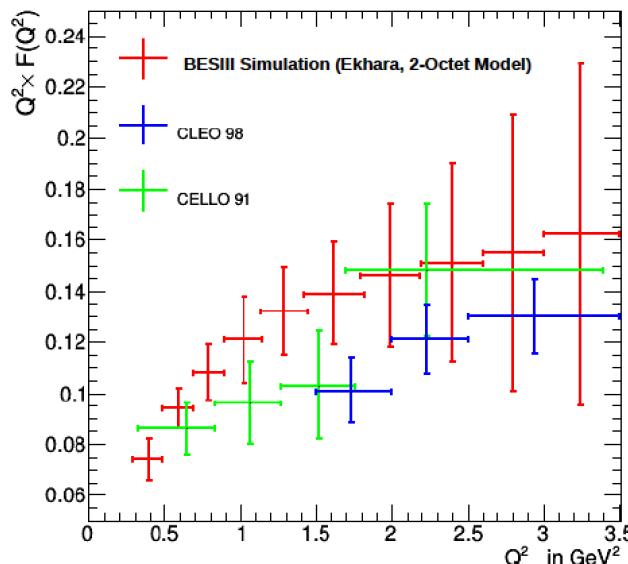
Significantly improves and
extends data set below $Q^2 =$
 1.5 GeV^2

Input for $(g-2)_\mu$!

Spacelike transition FFs: η / η'

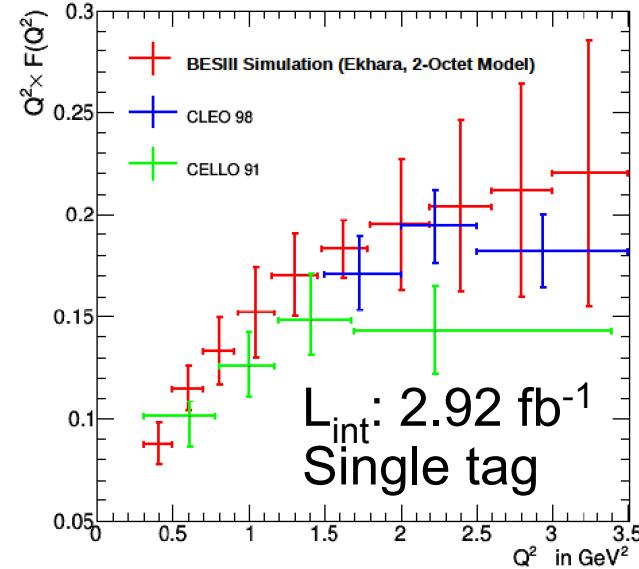
MC only, red error bars corresponding to BESIII statistics

$$F_{\eta, \gamma, \gamma^*}(Q^2)$$



$$\eta \rightarrow \pi^+ \pi^- \pi^0 \rightarrow \pi^+ \pi^- \gamma \gamma$$

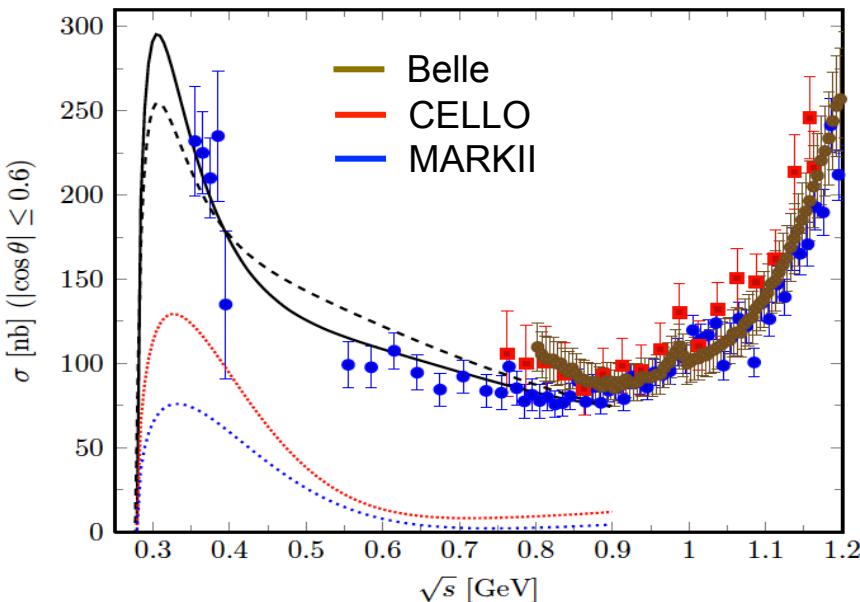
$$F_{\eta', \gamma, \gamma^*}(Q^2)$$



$$\eta' \rightarrow \pi^+ \pi^- \eta \rightarrow \pi^+ \pi^- \gamma \gamma$$

- Results competitive to previous measurement
- More data and more decay modes → order of magnitude improvement

$\gamma \gamma^* \rightarrow \pi^+ \pi^-$ at BESIII



- All in two real photon case:
 $\gamma\gamma \rightarrow \pi^+ \pi^-$
- In low mass region, only measurement come from MarkII

[MARKII: Phys. Rev. D42, 5, 1990]

[CELLO: Z. Phys. C56, 381, 1992]

[Belle: Phys. Rev. D75, 051101(R), 2007]

[N. Asmussen, P. Masjuan, M. Vanderhaeghen]

Access to:

Q^2 region: $0.2 \sim 2.0 \text{ GeV}^2$

$M_{\pi\pi}$: threshold $\sim 2.0 \text{ GeV}$

Conclusion

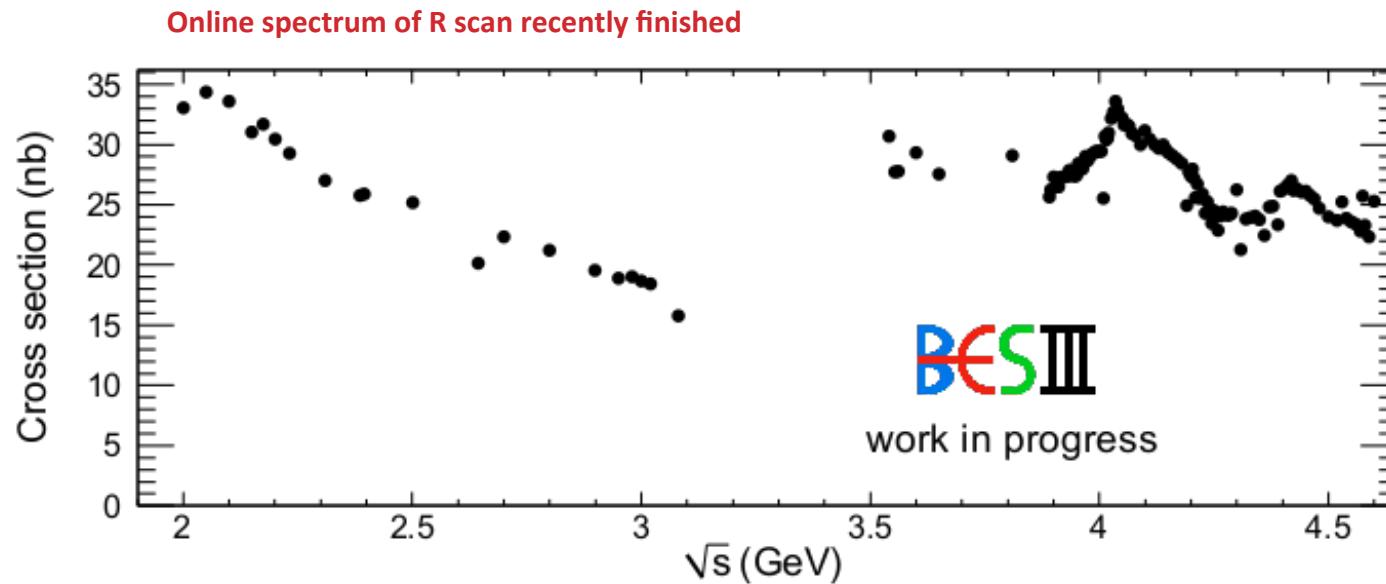
- Important results (to be expected) from BESIII for SM prediction of $(g-2)_\mu$
 - HVP: precision inclusive and exclusive measurements
 - HLbL: spacelike form factors measurement in relevant region
- Competing experiments: Frascati, Novosibirsk, Belle-II(?)
- Reduction of factor of 2 of the uncertainty of a_μ^{had} in reach

THANK YOU FOR YOUR ATTENTION!

Backup slides

Energy Scan from 2.0 to 4.6 GeV

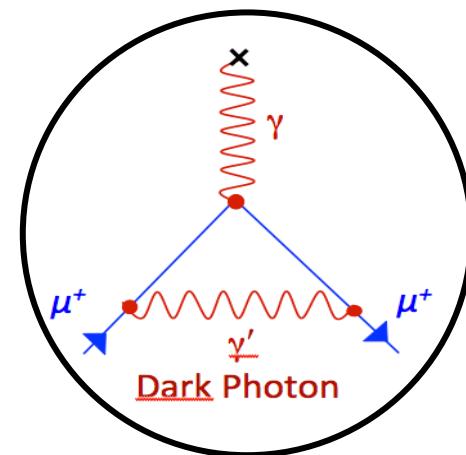
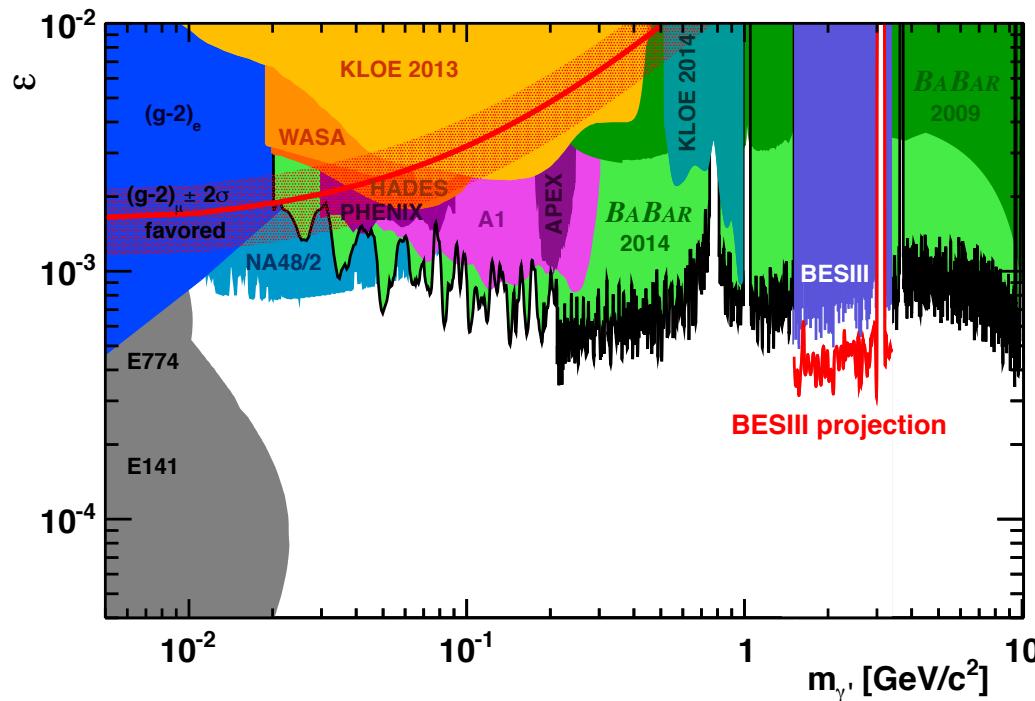
Reducing the uncertainty of $\alpha_{\text{em}}(M_Z^2)$ by a factor of 2
→ A new quantity of electroweak precision fits



$R = \sigma_{\text{had}}/\sigma_{\mu\mu}$ with targeted systematic accuracy: 3.0%

- 125 scan points with $>10^5$ hadronic events each → statistical error negligible
- World's best measurement so far from BES /BESII with 5 ... 8 % total error (with 3 ... 5% statistical error)

Dark photon and $(g-2)_\mu$



BESIII (Mass 1.5 -3.4 GeV/c 2)

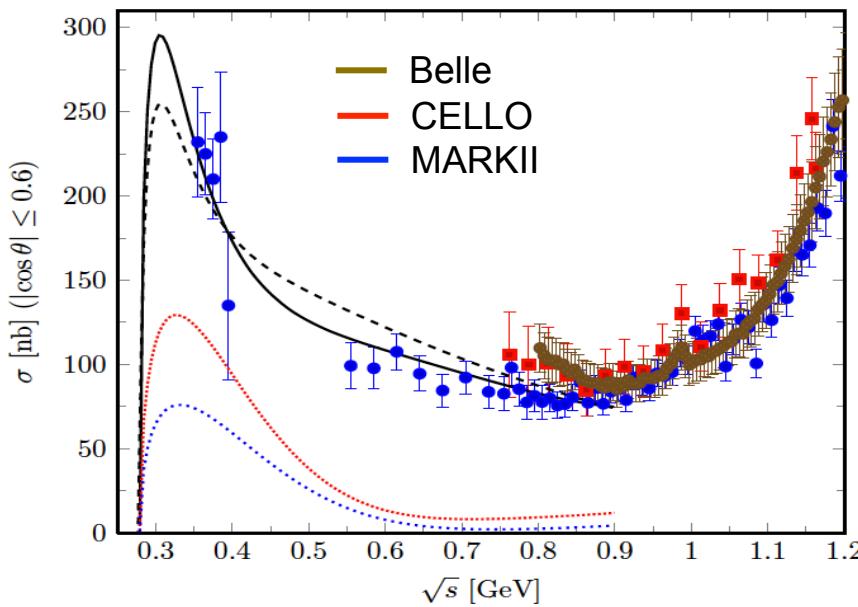
- Spin-off of pion FF analysis ($e^+e^- \rightarrow \gamma\mu^+\mu^- / \gamma e^+e^-$)

A1 / MAMI (Mass 30 – 855 MeV/c 2)

- Search in Bethe-Heitler process (eZ scattering)

$\gamma\gamma^* \rightarrow \pi^+\pi^-$ at BESIII

- Hadron form factor
- Resonance parameters for $0^{\pm+}$, 2^{++} states
- Pion polarizability, probe the structure of pion
- Re-scattering effect study at low mass region



- All in two real photon case:
 $\gamma\gamma \rightarrow \pi^+\pi^-$
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Access to:

Q^2 region: $0.2 \sim 2.0 \text{ GeV}^2$
 $M_{\pi\pi}$: threshold $\sim 2.0 \text{ GeV}$

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