Overview of Tau decays

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Indiana University/Jefferson Laboratory

« New Vistas in Low-Energy Precision Physics »

Mainz, April 6, 2016

Outline :

1. Introduction and Motivation

2. Hadronic τ-decays

3. LFV tau decays

4. Conclusion and outlook

NB: several topics not covered: Lepton Universality,
CP violation in tau decays,
g-2
EDM, etc…

➡ see Alberto Lusiani’s talk

1. Introduction and Motivation

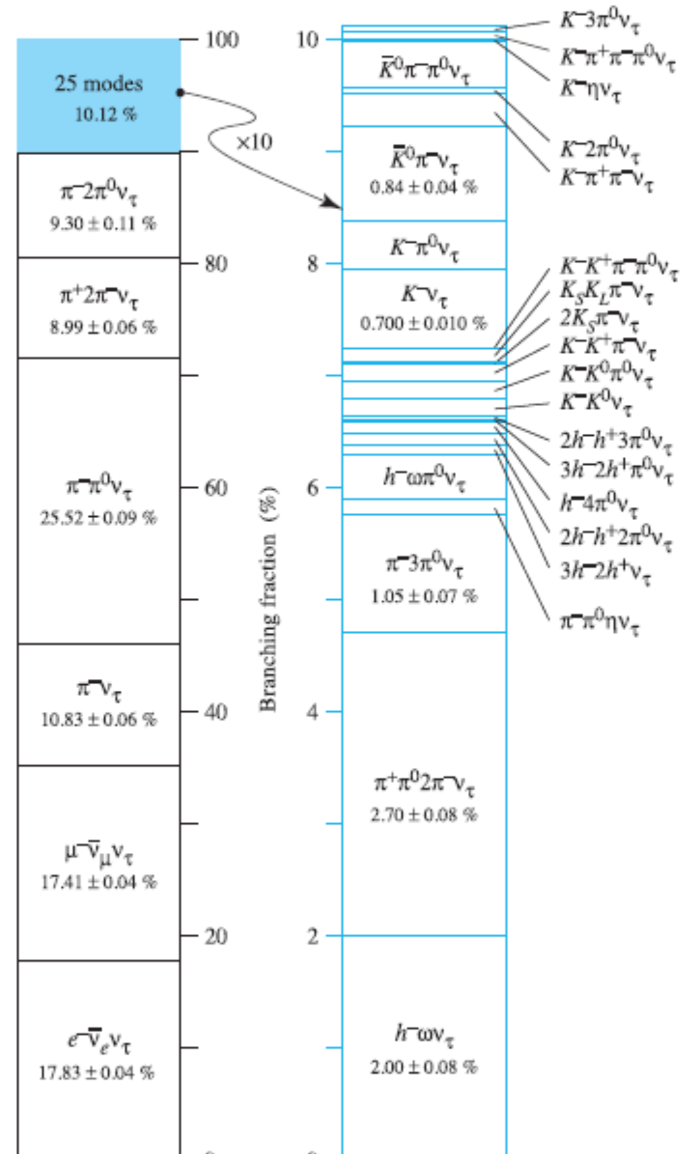
The τ lepton

PDG'14

- τ lepton discovered in 1976 by M. Perl et al. (SLAC-LBL group)
  - Mass: $m_\tau = 1.77682(16)$ GeV
  - Lifetime: $\tau_\tau = 2.096(10)\cdot 10^{-13}$ s

- Enormous progress in tau physics since then (CLEO, LEP, Babar, Belle, BES, VEPP-2M, neutrino experiments,...)
  - Early years: consolidate τ as a standard lepton no invisible decays and standard couplings
  - Better data: determination of fundamental SM parameters and QCD studies

Experiment	Number of τ pairs
LEP	~3x10^5
CLEO	~1x10^7
BaBar	~5x10^8
Belle	~9x10^8



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- Enormous progress in tau physics since then (CLEO, LEP, Babar, Belle, BES, VEPP-2M, neutrino experiments,...)
  - More recently: huge number of tau at the B factories: BaBar, Belle:
    - Tool to search for NP: rare decays, final states in hadron colliders
    - Precision physics: ⇨ α_S, $|V_{us}|$ etc

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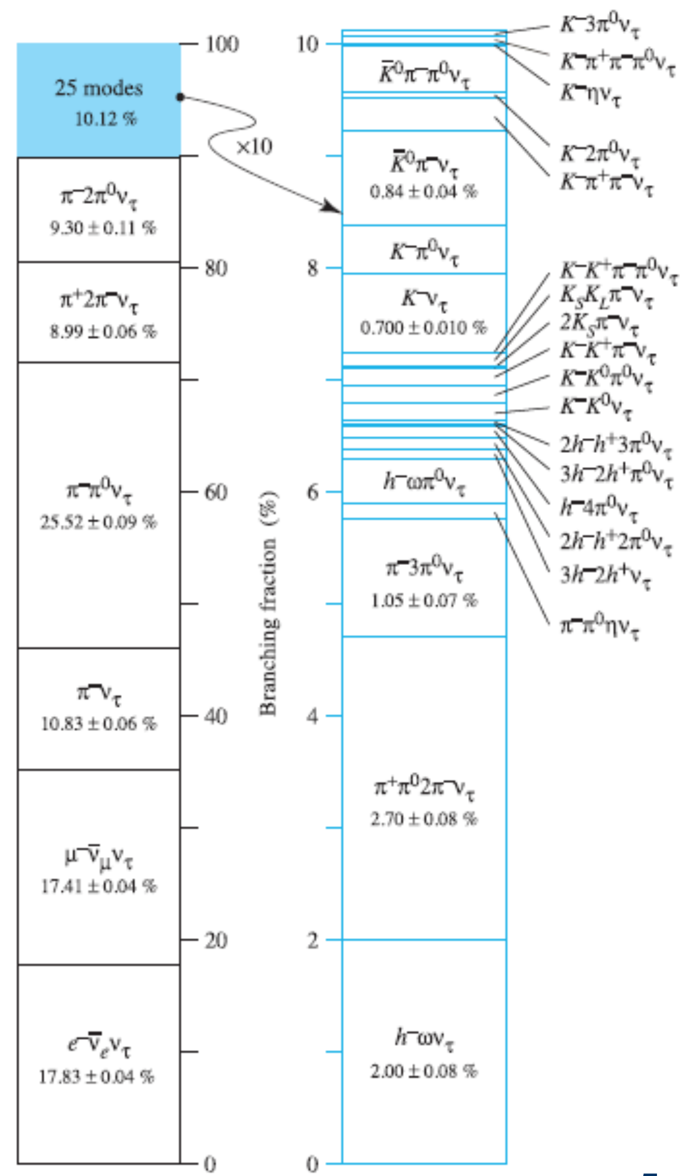
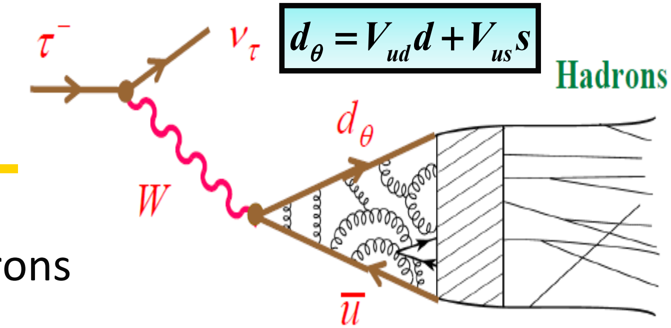


2. Hadronic τ-decays

2.1 Introduction



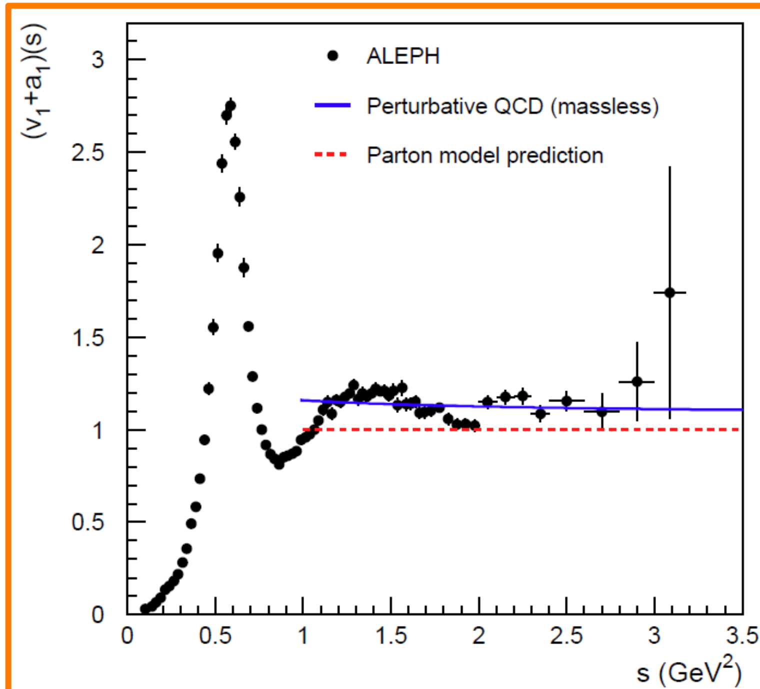
- Tau, the only lepton heavy enough to decay into hadrons

- $\boxed{m_\tau \sim 1.77\,\text{GeV} > \Lambda_{QCD}}$ ⇨ use *perturbative tools: OPE…*

- Inclusive τ decays : $\boxed{\tau \to (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fund. SM parameters $\left(\alpha_S(m_\tau), |V_{us}|, m_s\right)$

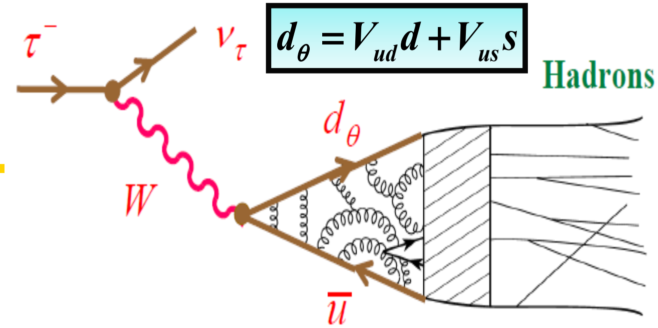
- We consider $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}_{S=0}\right)}$
  $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}_{S\neq 0}\right)}$

- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system ⇨ huge *QCD activity*!



- Observable studied: $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}}$$

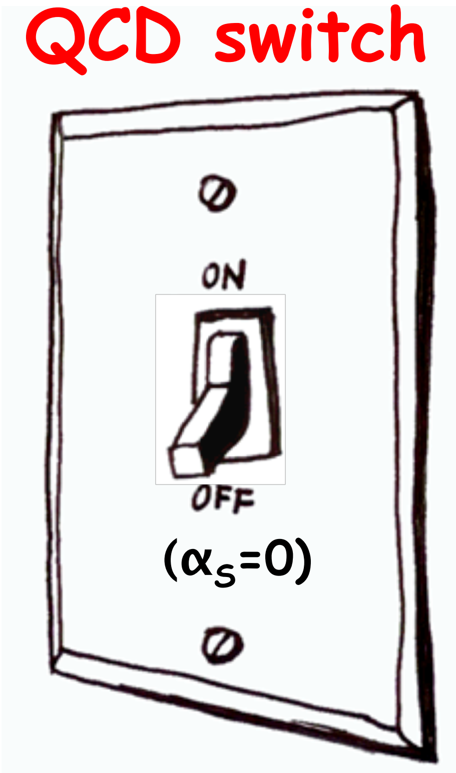
2.2 Theory



- $R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C$ parton model prediction

- $R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$

- $\boxed{\dfrac{|V_{us}|^2}{|V_{ud}|^2} = \dfrac{R_\tau^S}{R_\tau^{NS}}}$ ⇒ $|V_{us}|$



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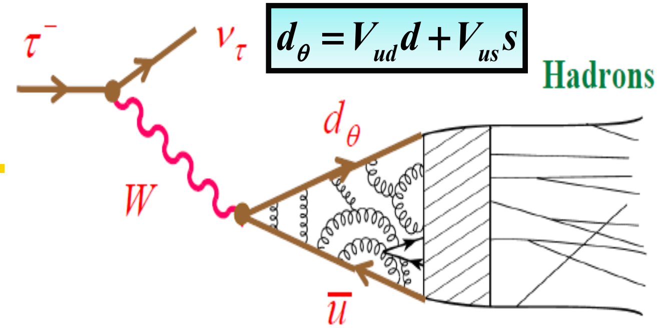
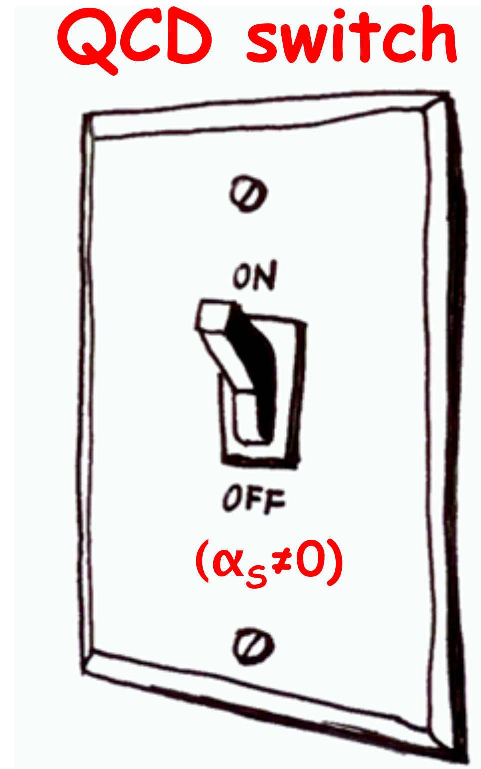


- $$\boxed{R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C}$$ parton model prediction

- $$R_\tau = R_\tau^{NS} + R_\tau^{S} \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$$

- Experimentally: $$\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$$



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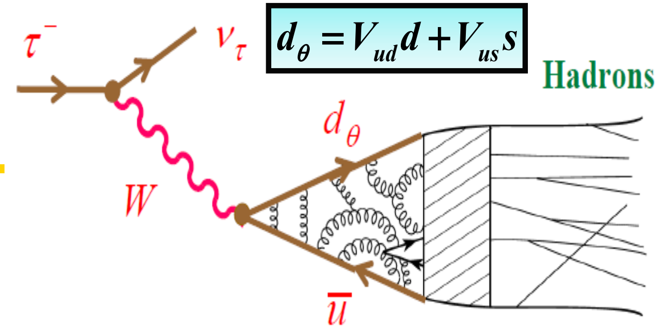

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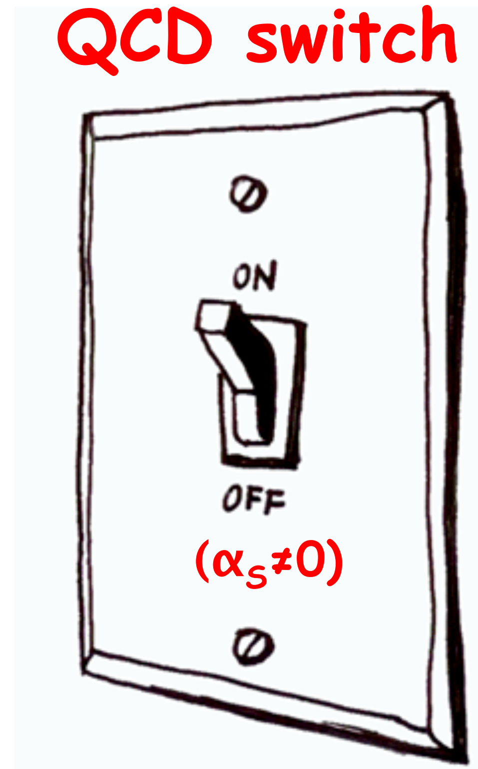
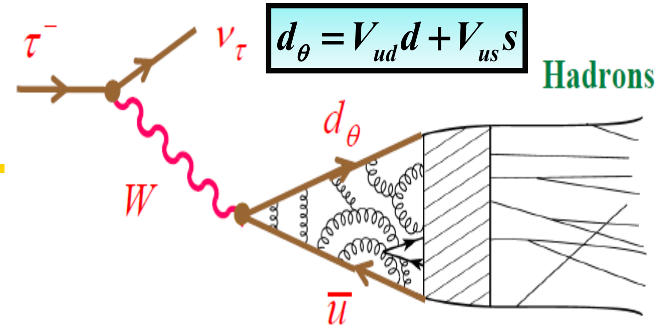
- $$R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

- Experimentally: $$\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$$

- Due to *QCD corrections*: $R_\tau = \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C + O\left(\alpha_s\right)$



2.3 Theory



- From the measurement of the spectral functions, extraction of α_S, $|V_{us}|$

- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C}$$ naïve QCD prediction

- Extraction of the strong coupling constant :

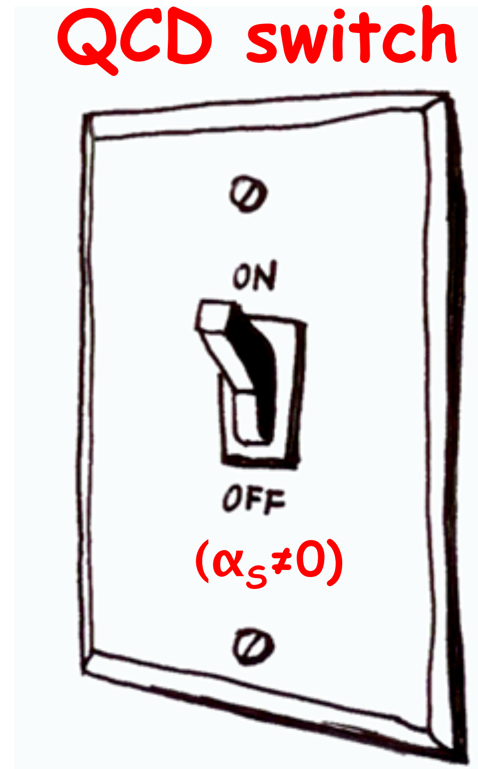
$$R_\tau^{NS} = |V_{ud}|^2 N_C + O(\alpha_s) \Rightarrow \alpha_S$$

measured calculated

- Determination of V_{us} :

$$\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$$

- Aim: compute the QCD corrections with the best accuracy

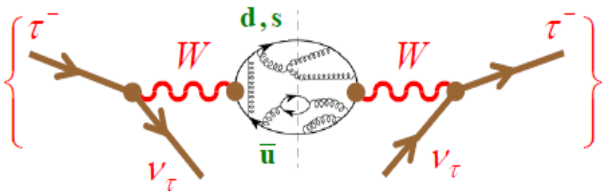


2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}$



$$\mathrm{Im}\Pi^{(1)}_{\bar{u}d,V/A}(s) = \frac{1}{2\pi} v_1/a_1(s)$$

$$\Rightarrow \boxed{R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s+i\varepsilon)+\mathrm{Im}\Pi^{(0)}(s+i\varepsilon)\right]}$$

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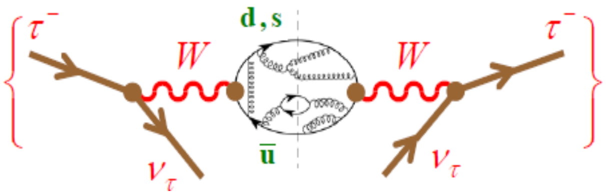
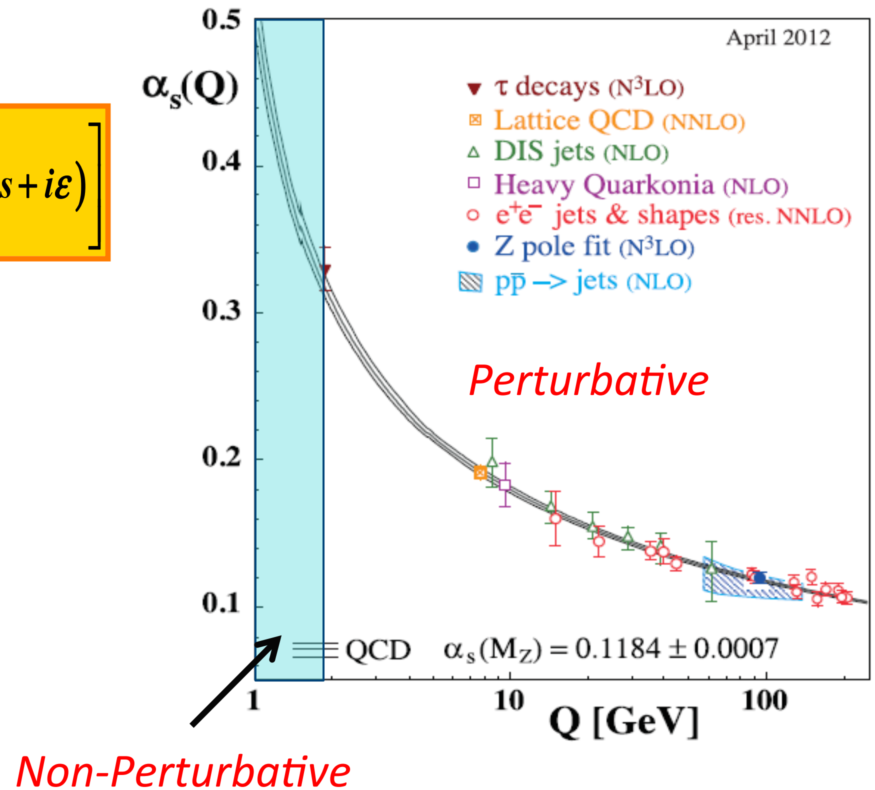


$$R_\tau(m_\tau^2) = 12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon)+\mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

- We are in the *non-perturbative* region: we do not know how to compute!

- Trick: use the analytical properties of Π!



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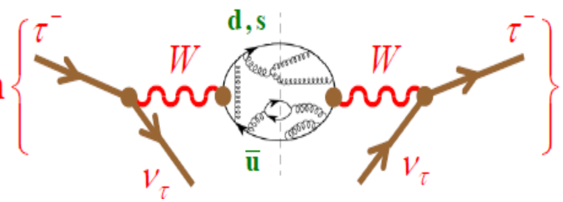


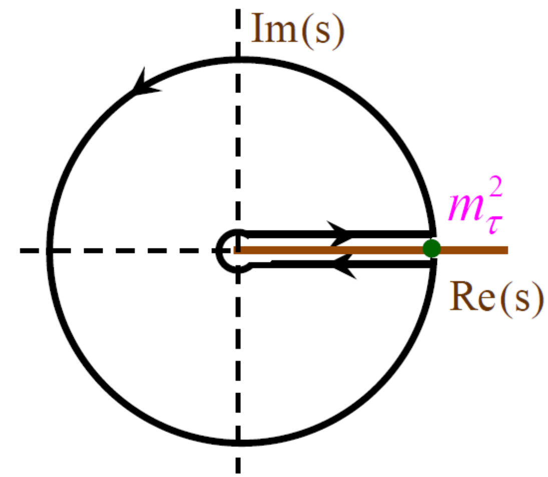
$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇒ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} C^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

2.4 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

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- Perturbative part (D=0): $\boxed{\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%}$ $\quad a_\tau = \dfrac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

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- D=2: quark mass corrections, *neglected* for R_τ^{NS} $\left(\propto m_u, m_d\right)$ but not for R_τ^{S} $\left(\propto m_s\right)$

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- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

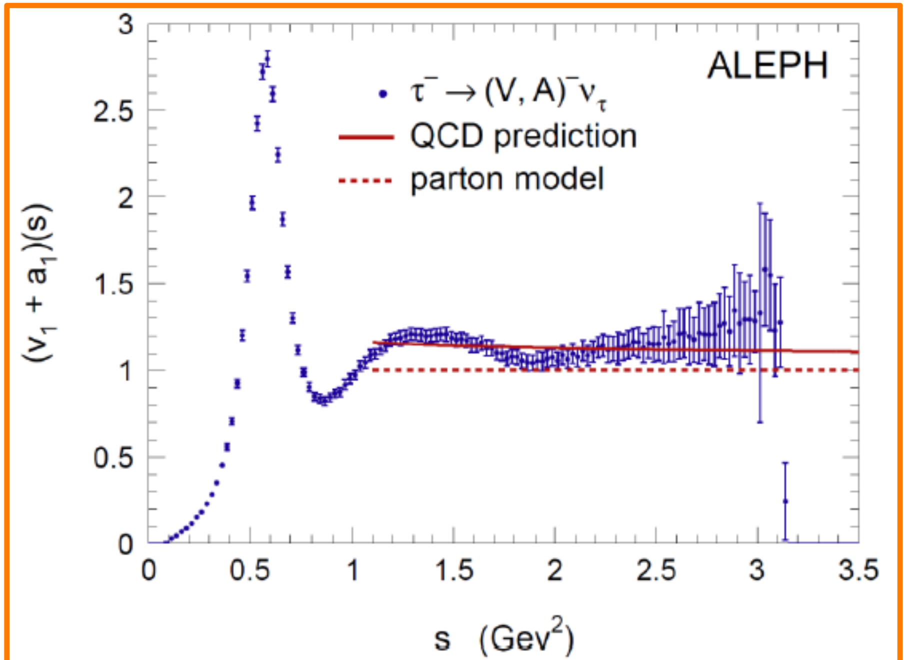
2.4 Calculation of the QCD corrections

Le Diberder & Pich'92

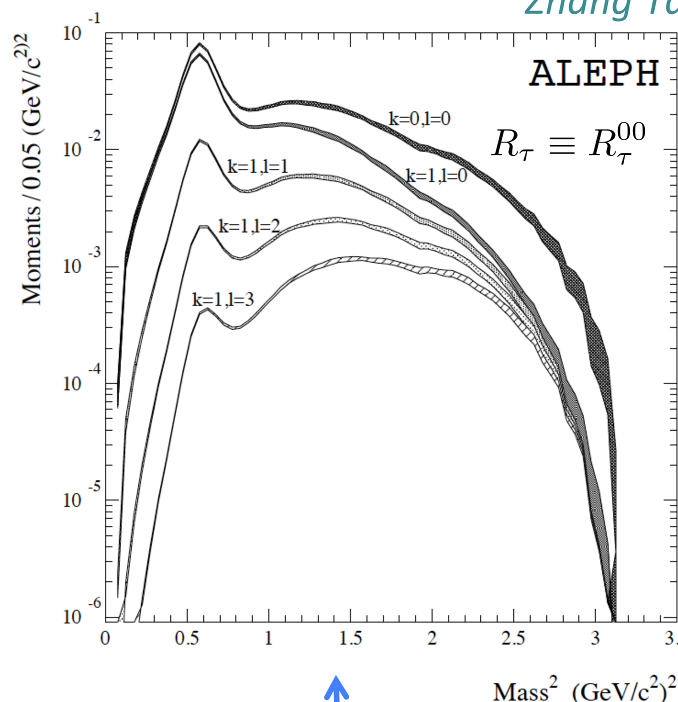
- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R^{k\ell}_{\tau,U}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14



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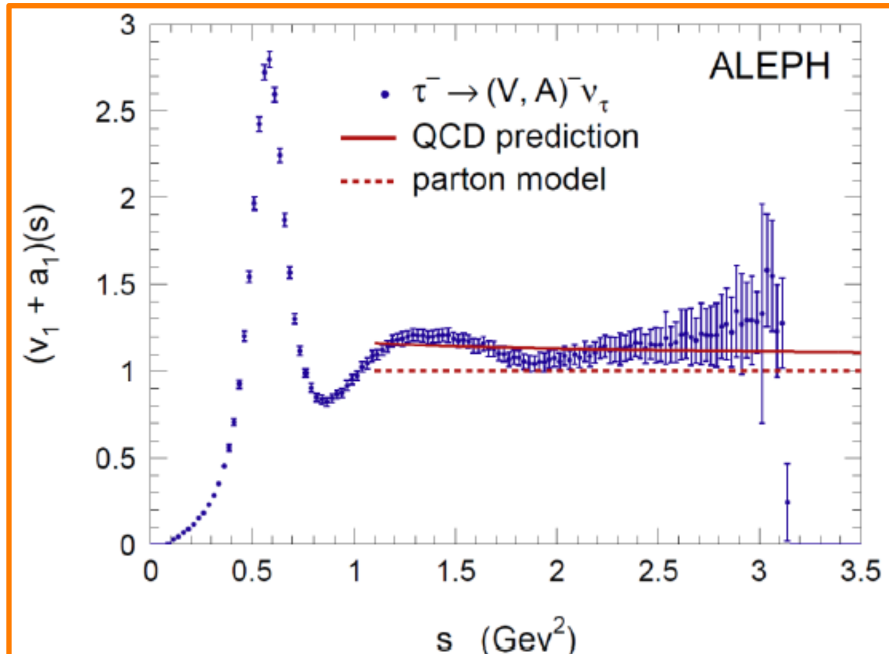
- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$
- Perturbative part (D=0): $\delta_P \approx 20\%$
- D=2: quark mass corrections, *neglected*
- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  Use of weighted distributions

$\Rightarrow$ $\boxed{\delta_{NP} = -0.0064 \pm 0.0013}$

Davier et al'14



- Small unknown NP part $\left(\delta_{NP} : 3\%\ \delta_P\right)$ very precise extraction of α_S !

2.5 Results and determination of α_s

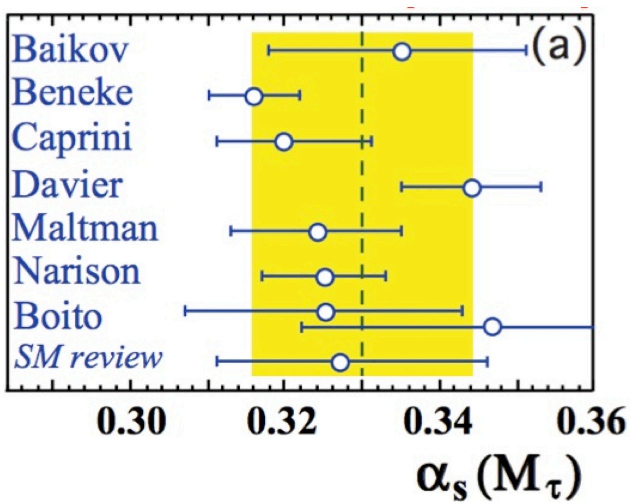
Pich'Tau14

Reference	Method	δ_{NP}	δ_P	$\alpha_s(m_\tau)$	$\alpha_s(m_Z)$
Baikov et al	CIPT, FOPT		0.1998 (43)	0.332 (16)	0.1202 (19)
Davier et al'14	**CIPT, FOPT**	**– 0.0064 (13)**	**–**	**0.332 (12)**	**0.1199 (15)**
Beneke-Jamin	BSR + FOPT	– 0.007 (3)	0.2042 (50)	0.316 (06)	0.1180 (08)
Maltman-Yavin	PWM + CIPT	+ 0.012 (18)	–	0.321 (13)	0.1187 (16)
Menke	CIPT, FOPT		0.2042 (50)	0.342 (11)	0.1213 (12)
Narison	CIPT, FOPT		–	0.324 (08)	0.1192 (10)
Caprini-Fischer	BSR + CIPT		0.2037 (54)	0.322 (16)	–
Abbas et al	IFOPT		0.2037 (54)	0.338 (10)	
Cvetič et al	β_{exp} + CIPT		0.2040 (40)	0.341 (08)	0.1211 (10)
Boito et al	CIPT, DV	– 0.002 (12)	–	0.347 (25)	0.1216 (27)
	FOPT, DV	– 0.004 (12)		0.325 (18)	0.1191 (22)
Pich'14	CIPT	– 0.0064 (13)	0.2014 (31)	0.342 (13)	0.1213 (14)
	FOPT			0.320 (14)	0.1187 (17)
Pich'14	**CIPT, FOPT**	**– 0.0064 (13)**	**0.2014 (31)**	**0.332 (13)**	**0.1202 (15)**

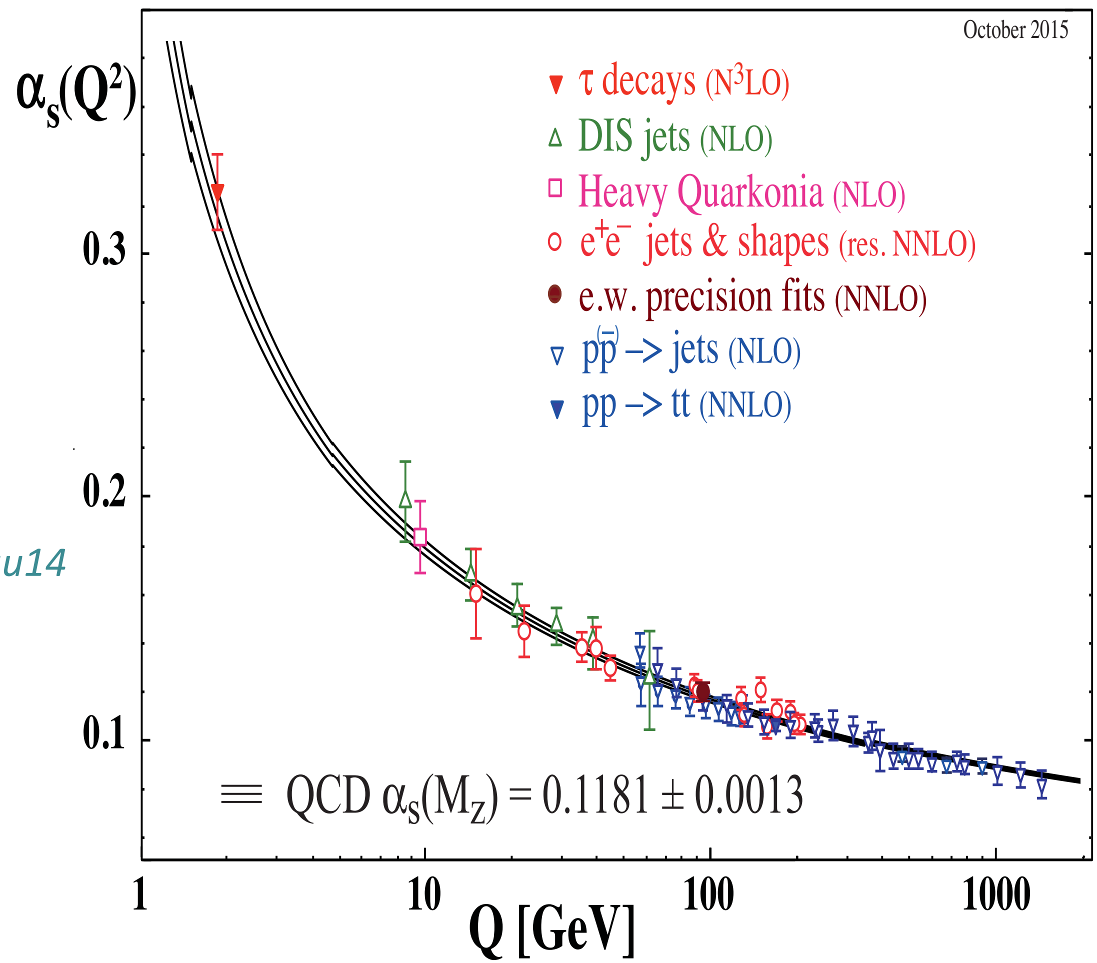
CIPT: Contour-improved perturbation theory
FOPT: Fixed-order perturbation theory
BSR: Borel summation of renormalon series
IFOPT Improved FOPT

β_{exp}: Expansion in derivatives of α_s (β function)
PWM: Pinched-weight moments
CIPTm: Modified CIPT (conformal mapping)
DV: Duality violation (OPAL only)

2.5 Results and determination of α_s



PDG'15



- $$\alpha_S\left(m_\tau^2\right) = 0.332 \pm 0.013$$

Pich'Tau14

$$\alpha_S\left(M_Z^2\right) = 0.1202 \pm 0.0015$$

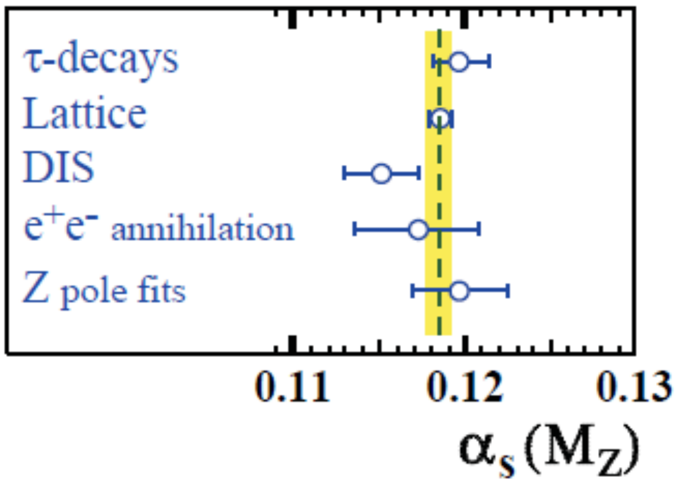
to be compared to

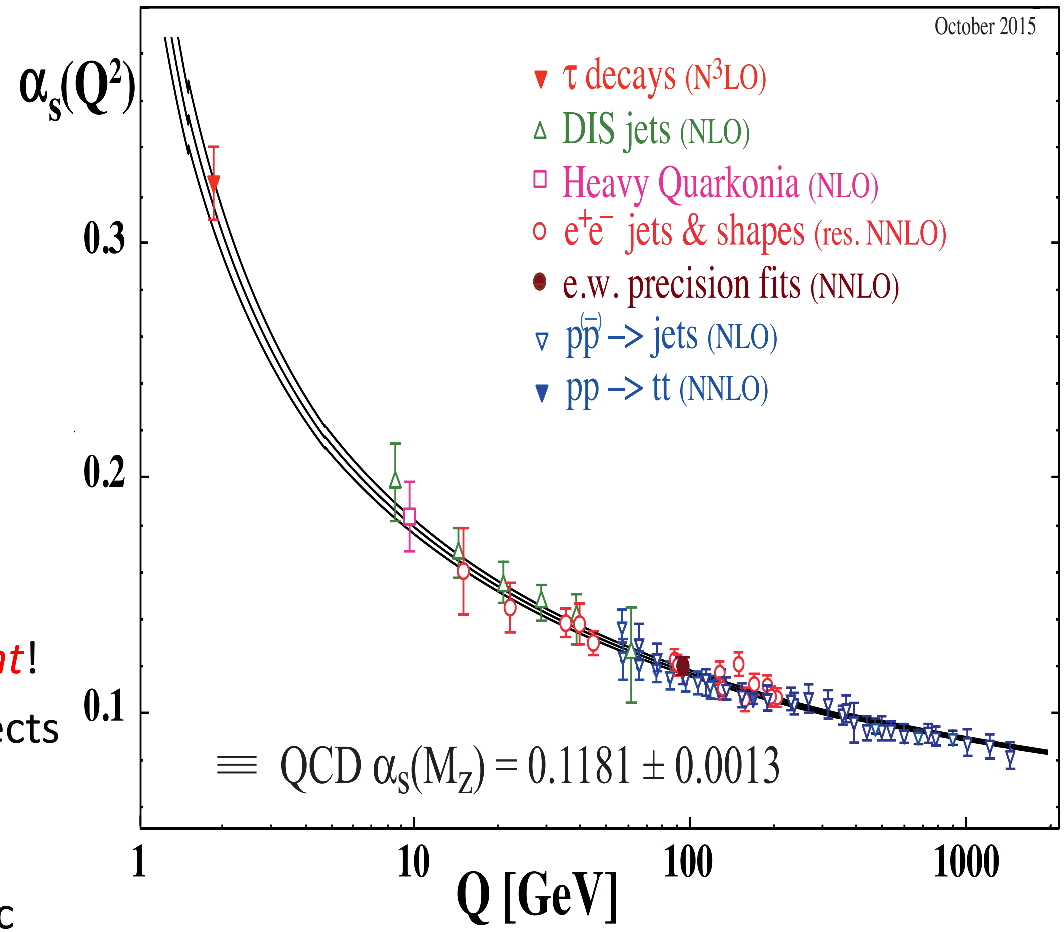
$$\alpha_S\left(M_Z^2\right)_{Z\ \text{width}} = 0.1197 \pm 0.0028$$

- Impressive test of the running of α_S!

2.5 Results and determination of α_s

PDG'15



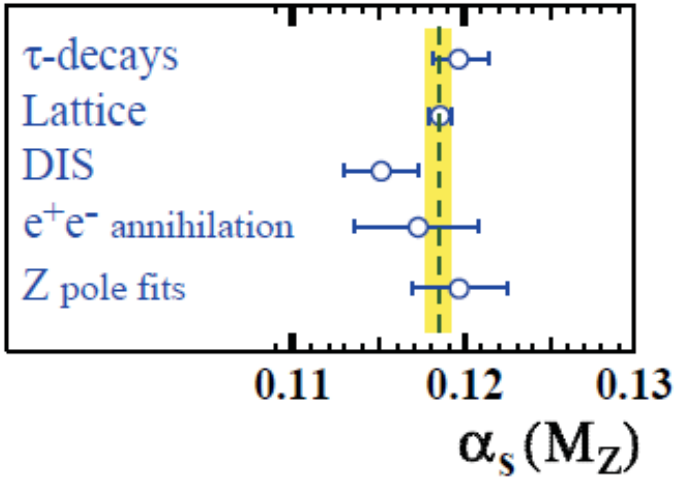


- *Extraction of α_s* from hadronic τ decays very *competitive*!

- If new data room for *improvement*!
  - Study of duality violation effects
  - Improve precision on non-perturbative determination : higher order condensates, etc
  - New physics?

See *MITP workshop* March 7-12, 2016

2.5 Results and determination of α_s



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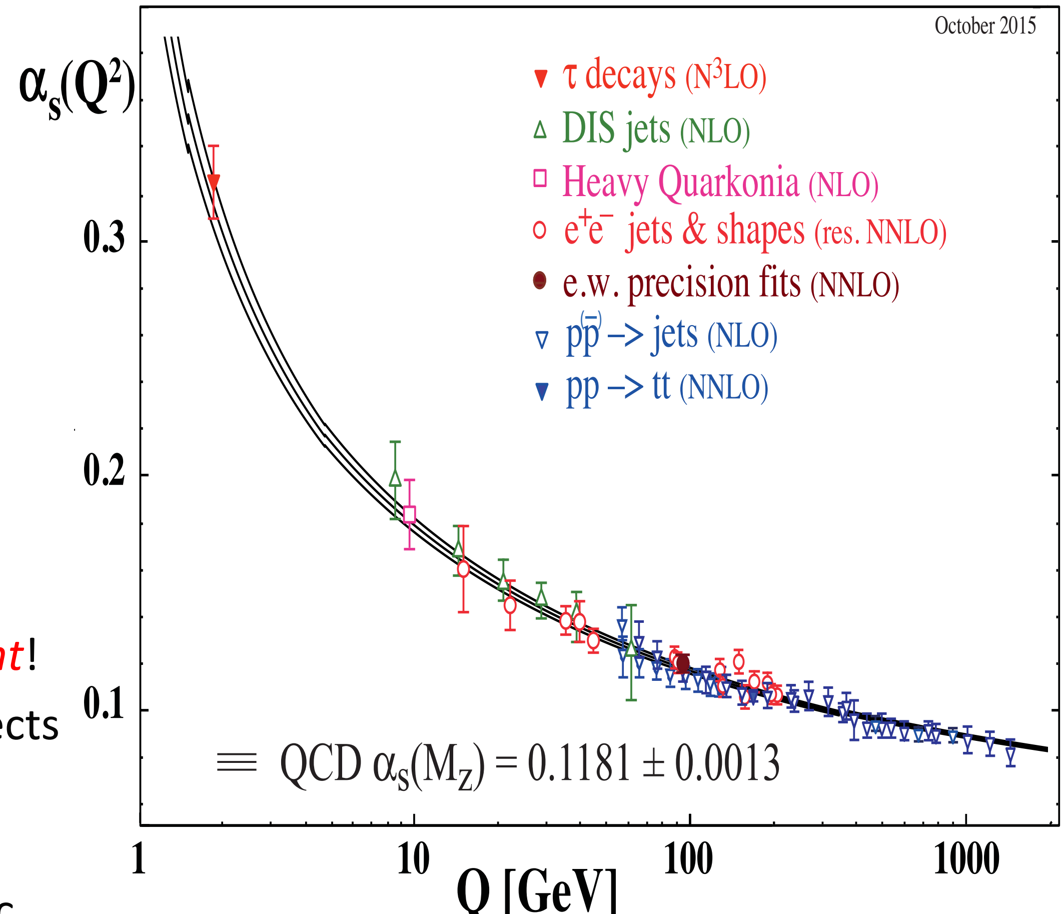

- *Extraction of α_s* from hadronic τ decays very *competitive*!
- If new data room for *improvement*!
  - Study of duality violation effects
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  - New physics?

For V_{us} see talk by *A. Lusiani*

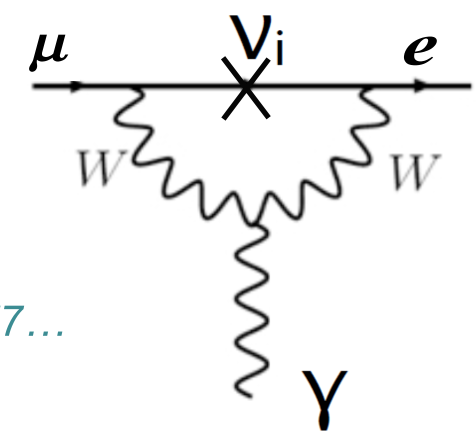
3. Charged Lepton-Flavour Violation

3.1 Introduction and Motivation

- Lepton Flavour Violation is an « accidental » symmetry of the SM (m_ν=0)

- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

  E.g.: $\boxed{\mu \to e\gamma}$



$$\boxed{Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M_W^2}\right|^2 < 10^{-54}}$$

Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$

- Extremely *clean probe of beyond SM physics*

3.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin @ CLFV2013

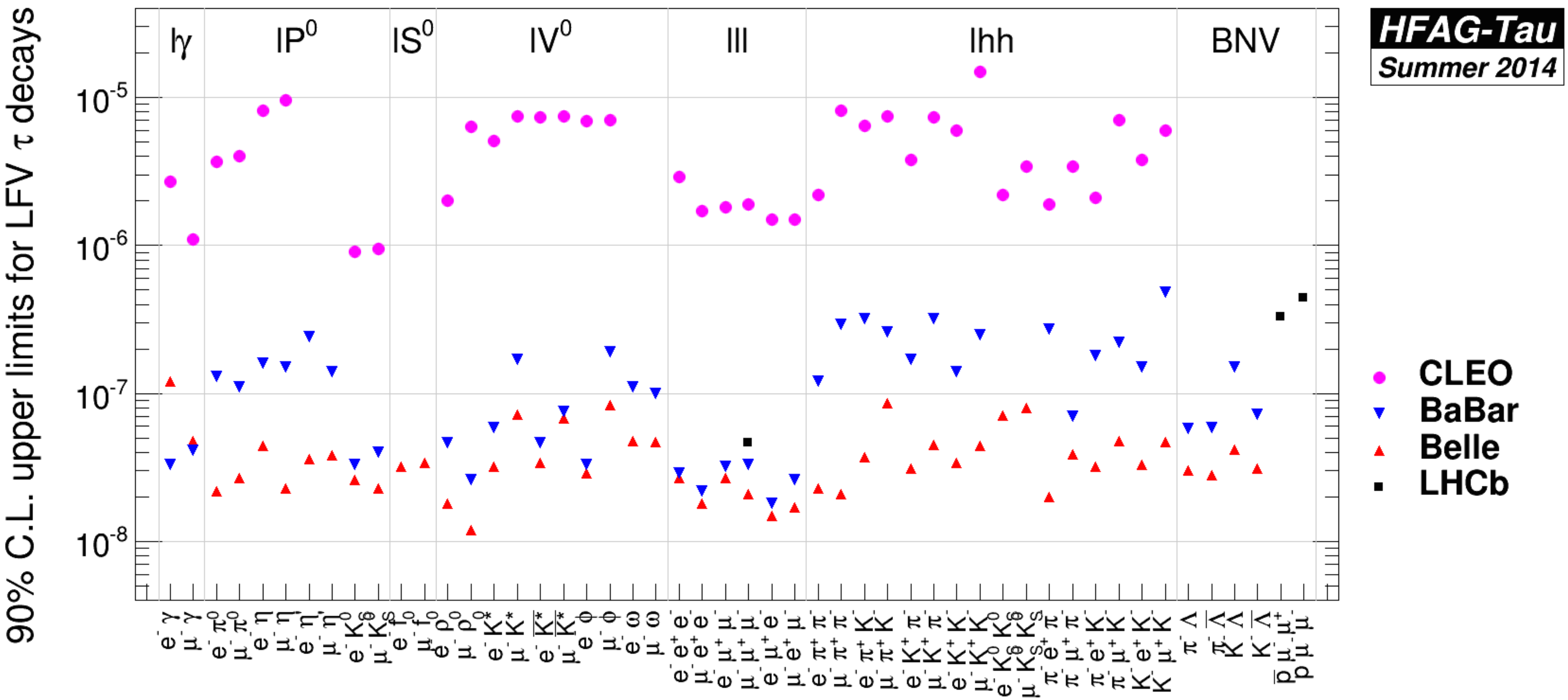
		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent

- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

3.2 CLFV processes: tau decays

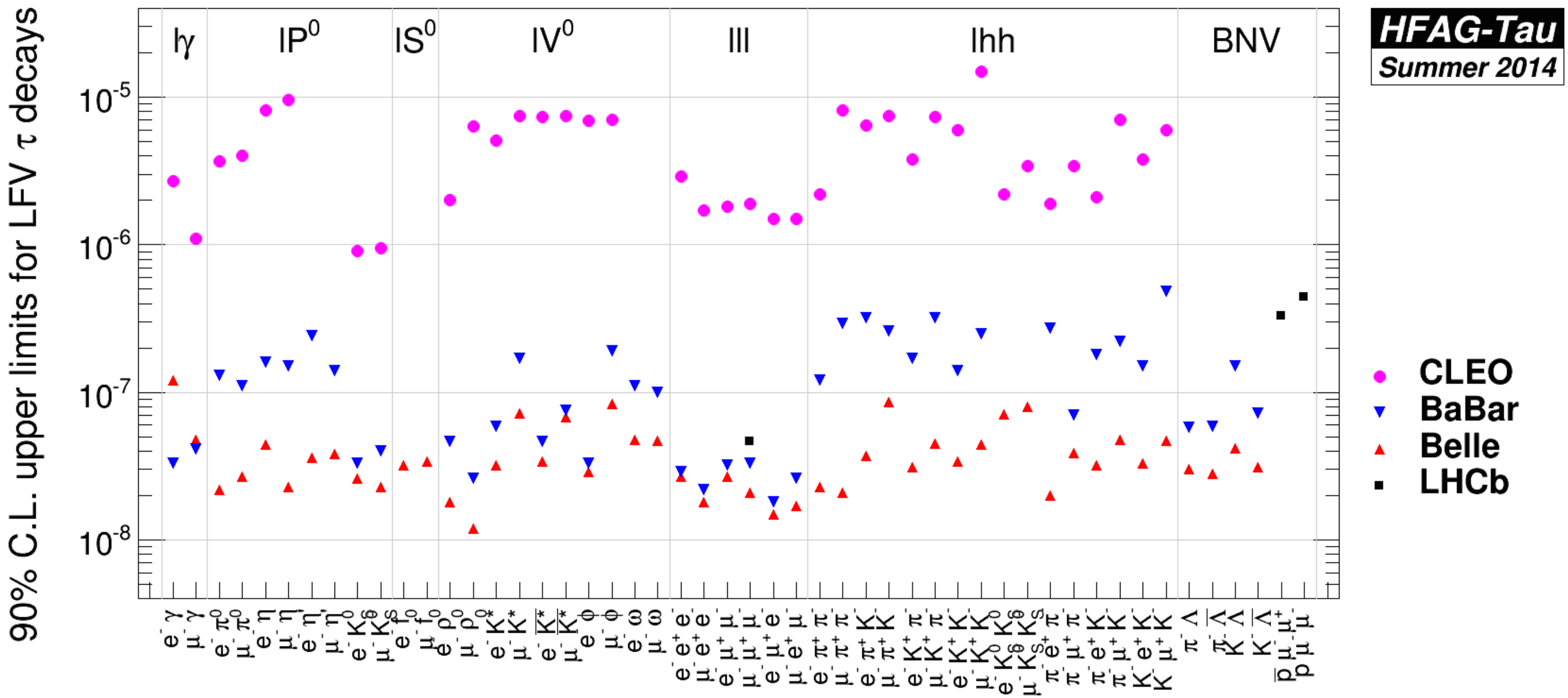
- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ← $P,\ S,\ V,\ P\bar{P}, \ldots$



- 48 LFV modes studied at Belle and BaBar

3.2 CLFV processes: tau decays

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ← $P,\ S,\ V,\ P\bar{P},\ldots$



- Expected sensitivity 10^{-9} or better at *LHCb, Belle II?*

3.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu}\sigma^{\mu\nu} P_{L,R}\tau F_{\mu\nu}}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R}\tau\; \bar{q}\Gamma q}$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R}\tau\; G_{\mu\nu}^{a} G_a^{\mu\nu}}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R}\tau\; \bar{\mu}\, \Gamma P_{L,R}\mu}$

$\boxed{\Gamma \equiv 1\,, \gamma^\mu}$

- Each UV model generates a *specific pattern* of them

3.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

3.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
$\mathrm{O_D}$	✓	✓	✓	✓	–	–
$\mathrm{O_V^q}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O_S^q}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
$\mathrm{O_{GG}}$	–	–	✓	✓	–	–
$\mathrm{O_A^q}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O_P^q}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O_{G\tilde{G}}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part:
form factors $F_i(s)$ and *decay constants* (e.g. f_η, $f_{\eta'}$)

$$\boxed{H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} |0\rangle = \left(Lorentz\ \text{struct.}\right)^i_\mu F_i\left(s\right)} \quad \text{with} \quad s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

4. Charged Lepton-Flavour Violation and Higgs Physics

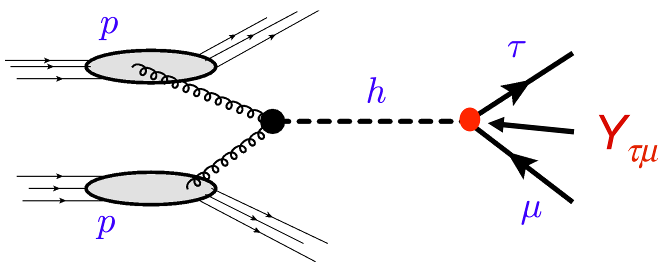
4.1 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H} \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

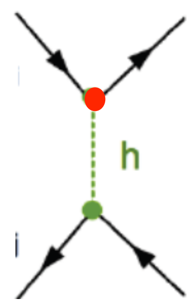
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

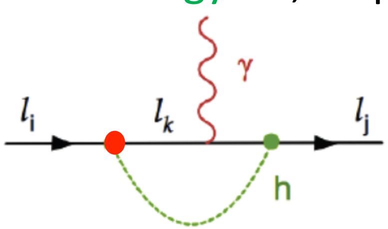
- High energy : LHC

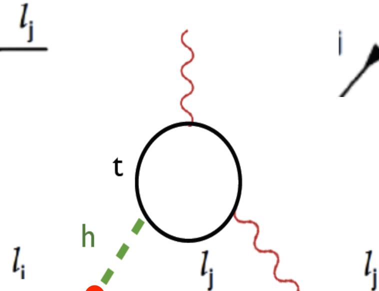


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







4.1 Non standard LFV Higgs coupling

- $$\boxed{\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right)H^\dagger H}$$ $\Rightarrow$ $-Y_{ij}\left(\bar{f}_L^i f_R^j\right)h$

Goudelis, Lebedev, Park'11
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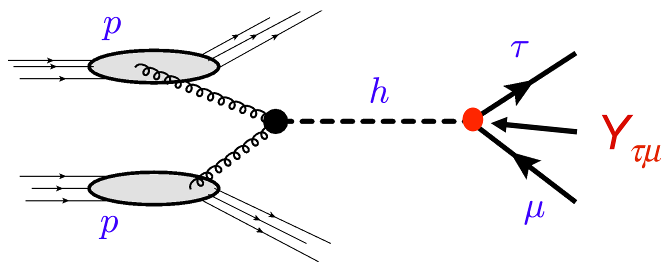
Arhrib, Cheng, Kong'12

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{\mathrm{v}}\delta_{ij}$

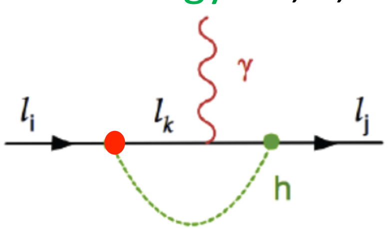
- High energy : LHC

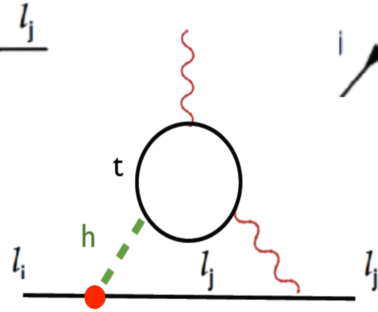


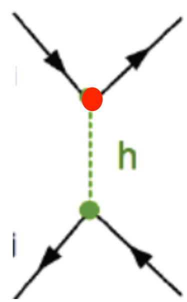
Hadronic part treated with perturbative QCD

Reverse the process

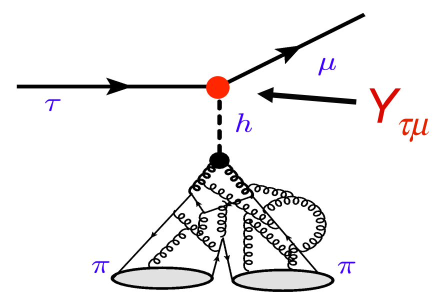
- Low energy : D, S, G operators







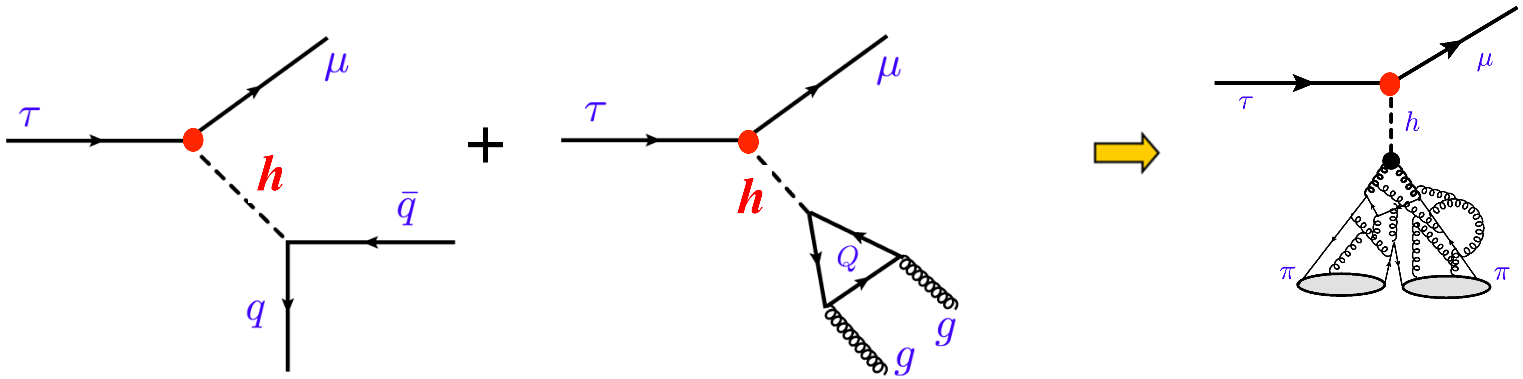
+



Hadronic part treated with non-perturbative QCD

4.2 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



- Problem : Have the hadronic part under control, ChPT not valid at these energies!

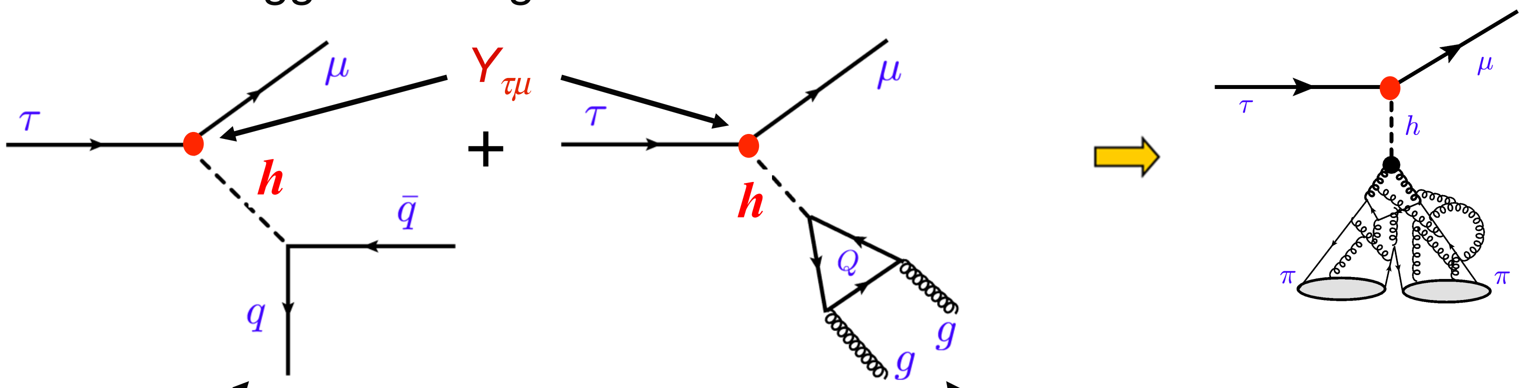
  ⇨ Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

Daub, Dreiner, Hanart, Kubis, Meissner'13

Celis, Cirigliano, E.P.'14

4.2 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



$$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s) \qquad \langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$$

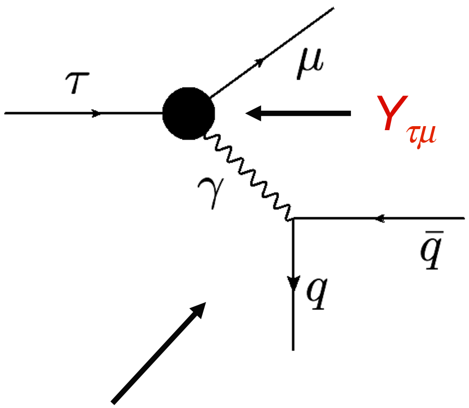
$$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

$$\boxed{\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\Rightarrow \boxed{\Gamma_{\tau\to\mu\pi\pi} \propto \int \left| \Gamma_\pi(s) + \Delta_\pi(s) + \theta_\pi(s) \right|^2 Y^2_{\tau\mu}} \quad \text{with} \quad s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

4.2 Constraints from $\tau \to \mu\pi\pi$

- Contribution from dipole diagrams



$$\Gamma_{\tau\to\mu\pi^+\pi^-} \propto \int \left|F_V(s)\right|^2 Y_{\tau\mu}^2$$

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) \right| \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) \left| 0 \right\rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

- Diagram only there in the case of $\tau^- \to \mu^-\pi^+\pi^-$ absent for $\tau^- \to \mu^-\pi^0\pi^0$
  ⇨ neutral mode more model independent

4.3 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

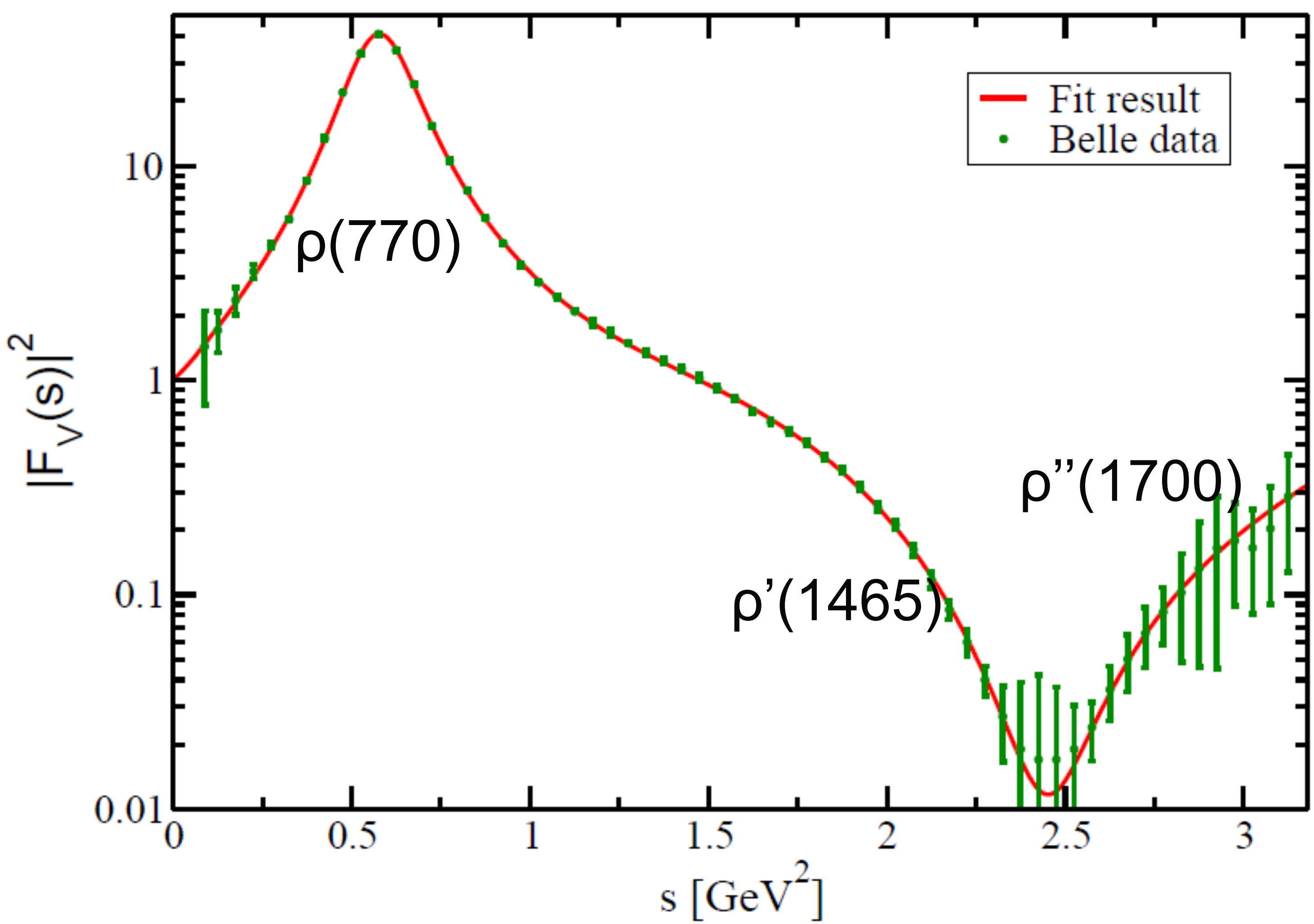
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_V'' - \lambda_V'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{\left(s'-s-i\varepsilon\right)}\right]}$$

Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

4.3 Determination of $F_V(s)$



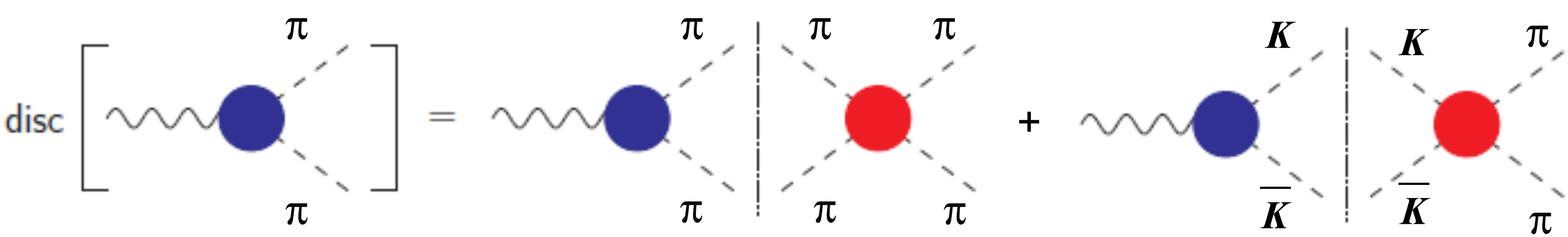
Determination of $F_V(s)$ thanks to precise measurements from Belle!

4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs $\Rightarrow$ *Coupled channel analysis*
  up to $\sqrt{s}\sim 1.4$ GeV
  Inputs: I=0, S-wave $\pi\pi$ and $K\bar{K}$ data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13

- Unitarity:



$$\Rightarrow \quad \boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)} \qquad n = \pi\pi, K\bar{K}$$

4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

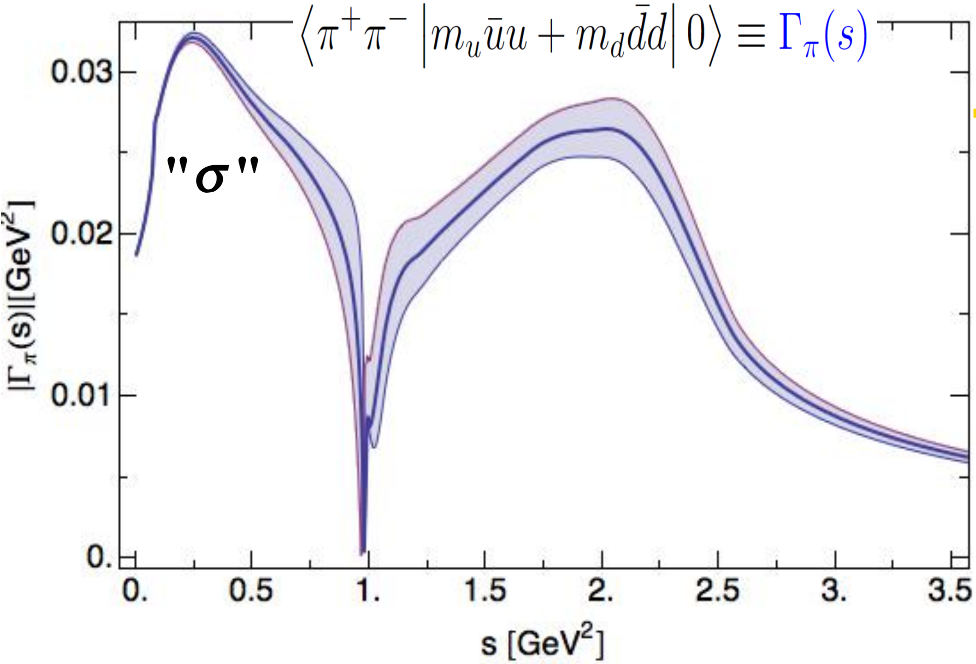
Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

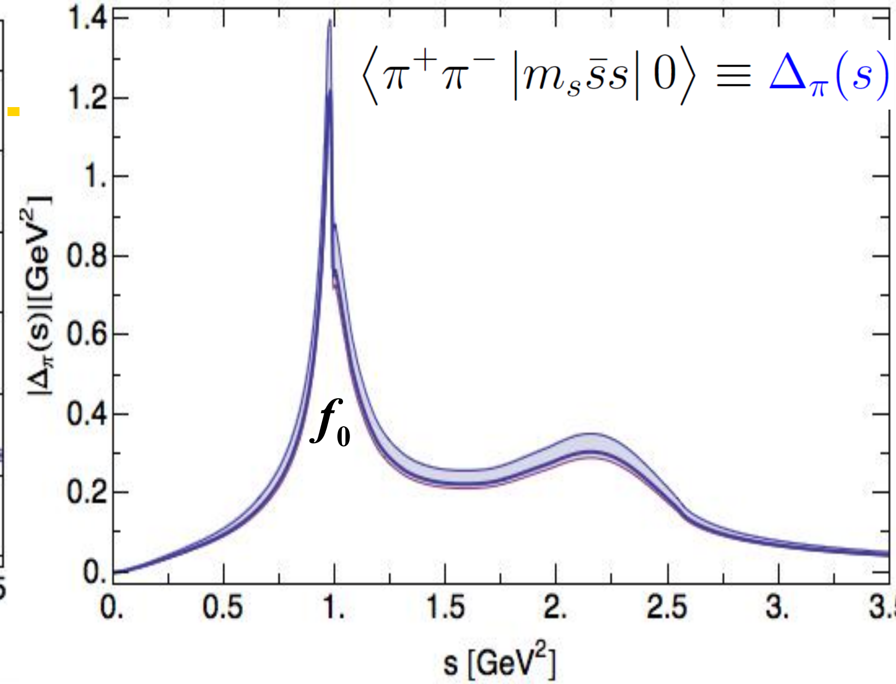
Polynomial determined from a matching to ChPT + lattice

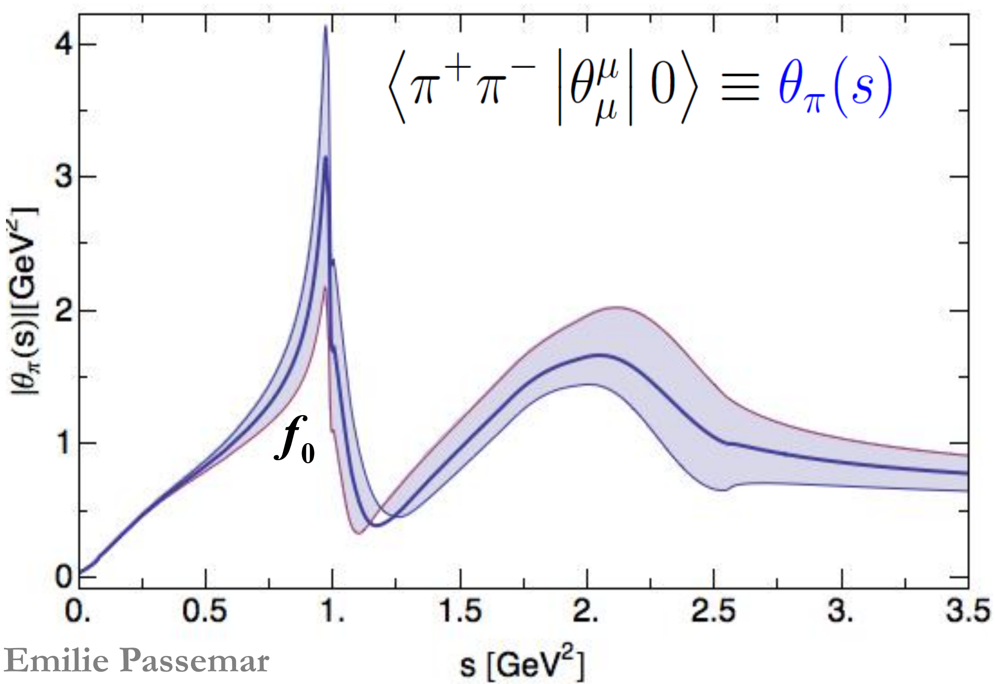
- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

$$\boxed{\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} T_{mn}^* \sigma_m(s) X_m^{(N)}(s)} \longrightarrow \boxed{X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im} X_n^{(N+1)}(s')}{s'-s}}$$

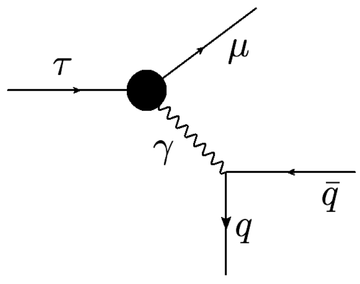

$\langle \pi^+\pi^- | m_u\bar{u}u + m_d\bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$
"σ"
$|\Gamma_\pi(s)|$[GeV^2]
s [GeV^2]

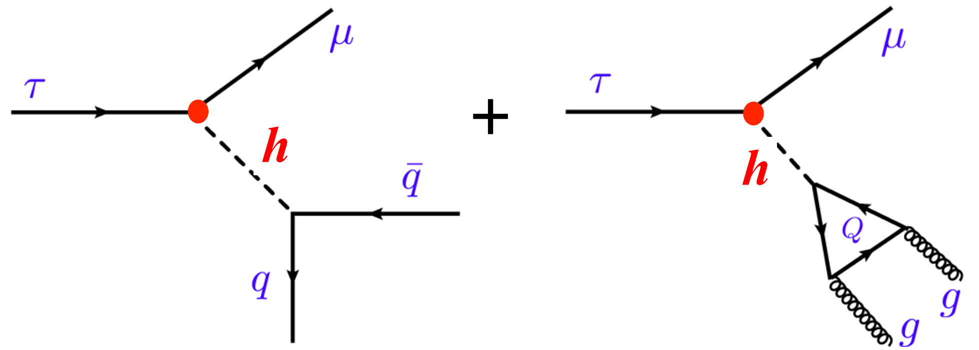

$\langle \pi^+\pi^- | m_s\bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$
f_0
$|\Delta_\pi(s)|$[GeV^2]
s [GeV^2]

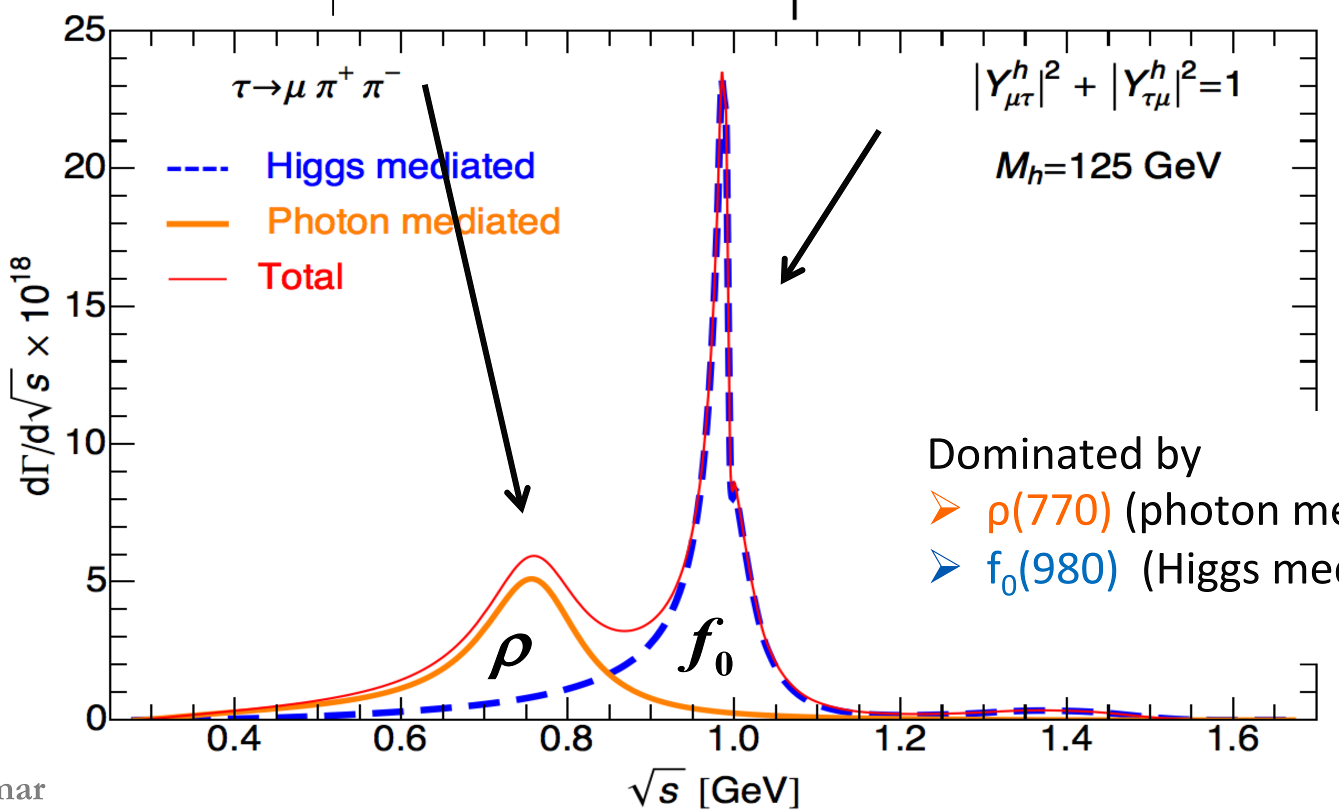

$\langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$
f_0
$|\theta_\pi(s)|$[GeV^2]
s [GeV^2]

4.5 Constraints in the τμ sector

- At low energy
  - $\tau \to \mu\pi\pi$:

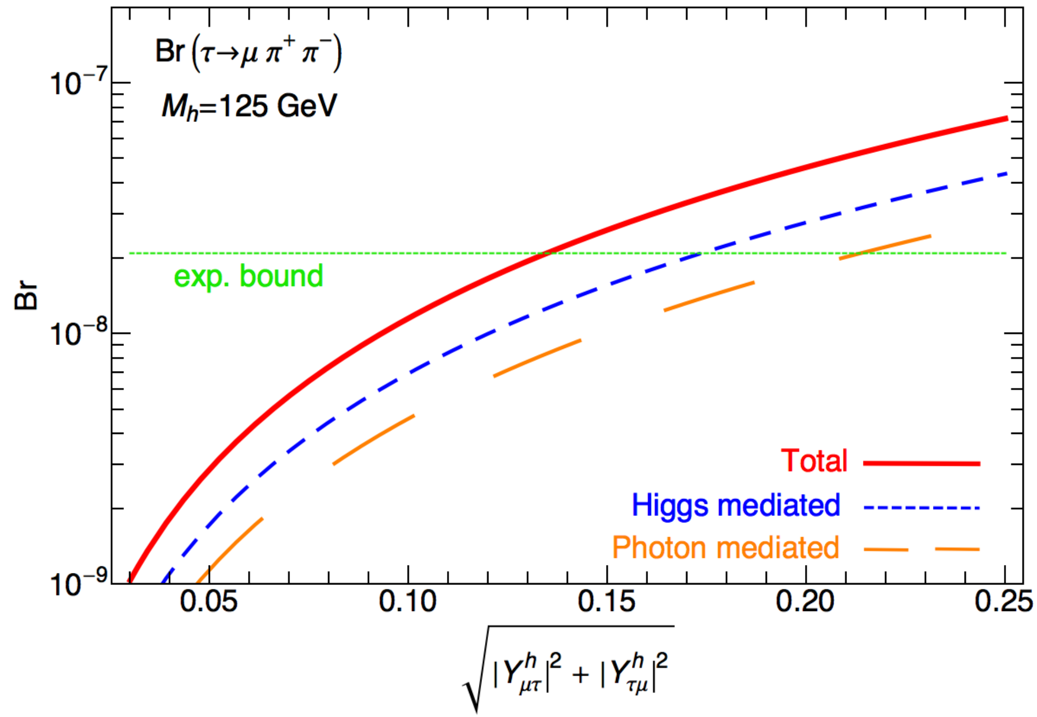






Dominated by
- ρ(770) (photon mediated)
- f_0(980) (Higgs mediated)

4.5 Constraints in the τμ sector



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2+\left|Y^h_{\tau\mu}\right|^2}\le 0.13}$$

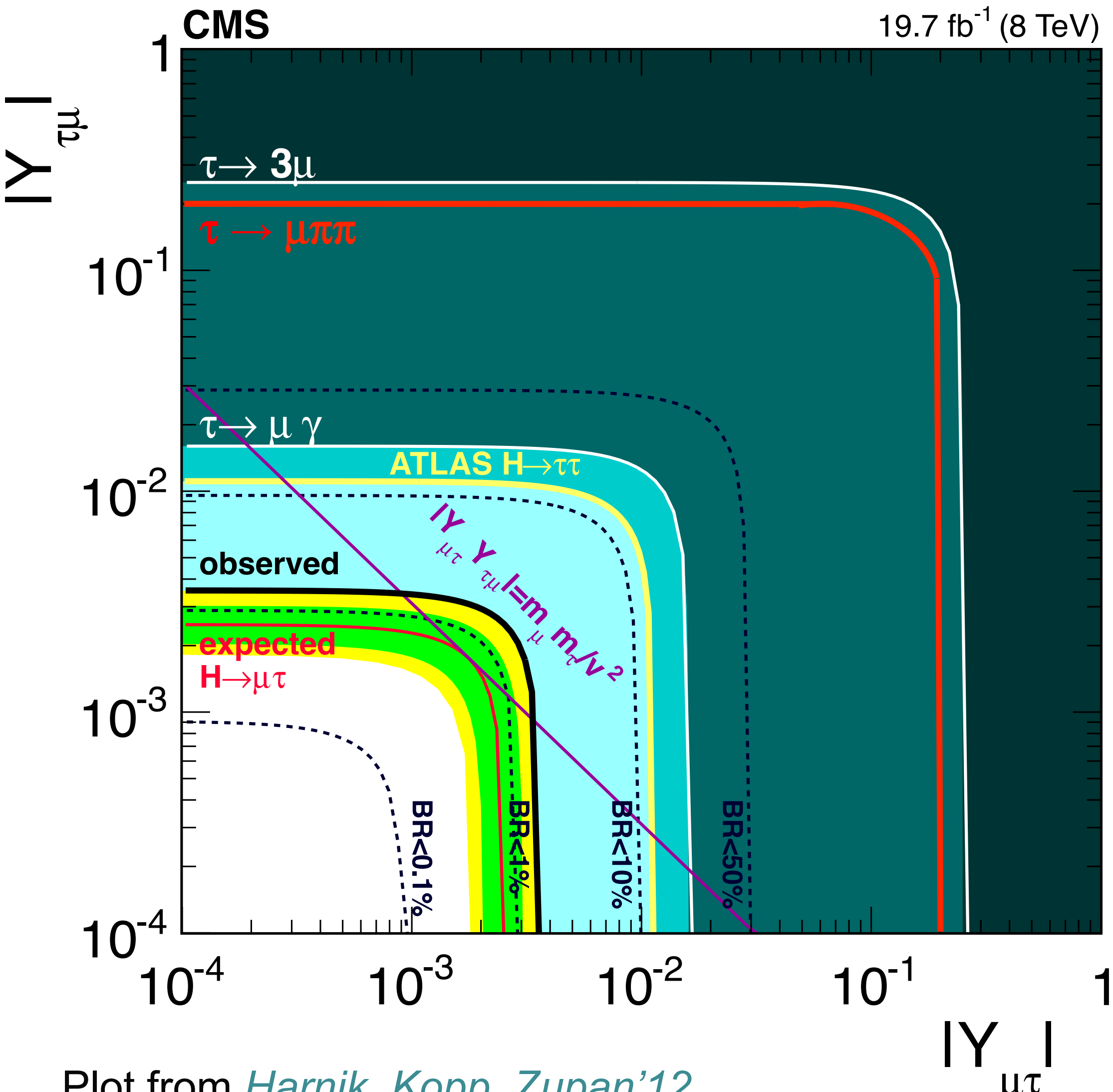
Process	$(\mathrm{BR}\times 10^8)$ 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2+\lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau\to\mu\gamma$	<4.4 [88]	<0.016	Dipole
$\tau\to\mu\mu\mu$	<2.1 [89]	<0.24	Dipole
$\tau\to\mu\pi^+\pi^-$	<2.1 [86]	<0.13	Scalar, Gluon, Dipole
$\tau\to\mu\rho$	<1.2 [85]	<0.13	Scalar, Gluon, Dipole
$\tau\to\mu\pi^0\pi^0$	$<1.4\times10^3$ [87]	<6.3	Scalar, Gluon

Less stringent but more robust handle on LFV Higgs couplings

?

Belle'08'11'12 except last from *CLEO'97*

4.5 Constraints in the τμ sector



Plot from *Harnik, Kopp, Zupan'12* updated *by CMS'15*

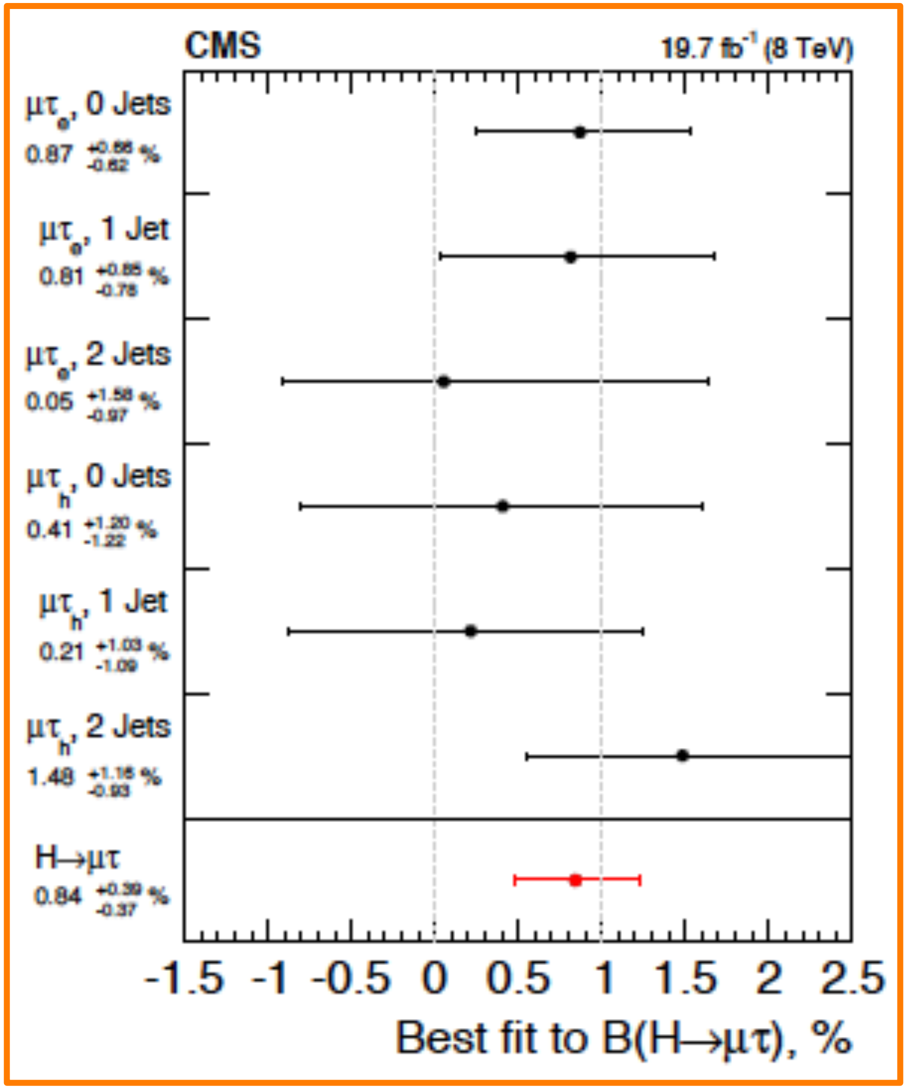
- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level
    ⇨ sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams
    ⇨ robust handle on LFV
- Constraints from HE:
  LHC wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

⇨ $\boxed{BR(\tau \to \mu\gamma) < 2.2\times 10^{-9}}$

$\boxed{BR(\tau \to \mu\pi\pi) < 1.5\times 10^{-11}}$

4.6 Hint of New Physics in $h \rightarrow \tau\mu$?

CMS'15

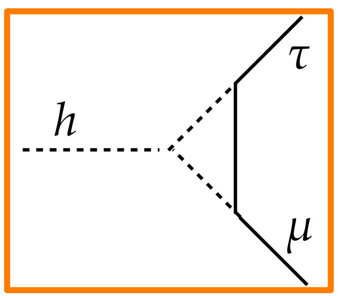


4.7 Interplay between LHC & Low Energy

Dorsner et al.'15

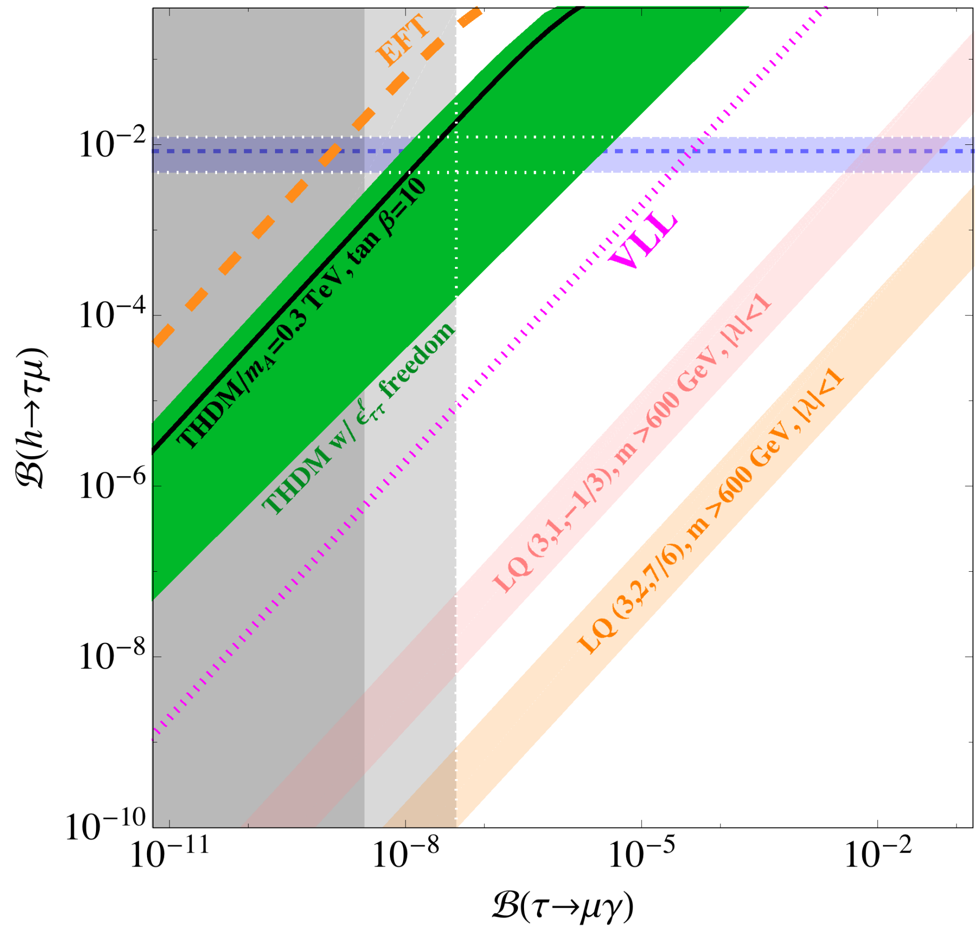
- If real what type of NP?

- If $h \to \tau\,\mu$ due to loop corrections:
  - extra charged particles necessary
  - $\tau \to \mu\gamma$ too large



- $h \to \tau\,\mu$ possible to explain if extra scalar doublet:

  ⇨ *2HDM of type III*



- Constraints from $\tau \to \mu\gamma$ important! ⇨ *Belle II*

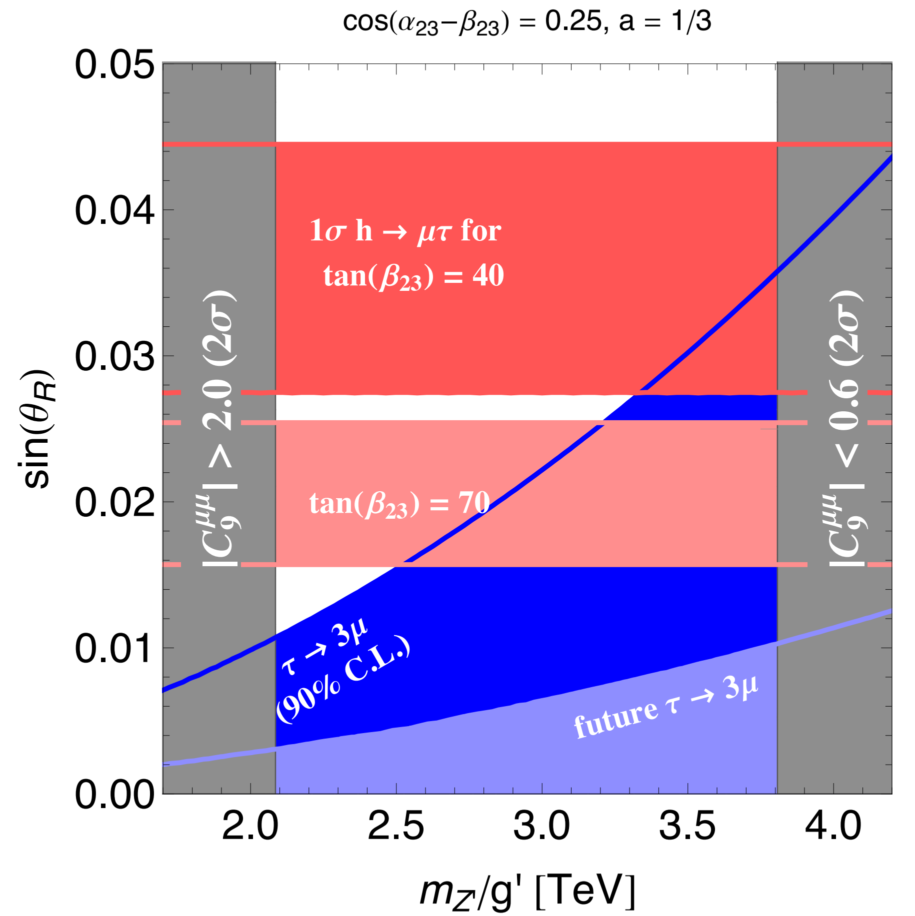
4.8 Interplay between LHC & Low Energy

- *2HDMs* with gauged $L_\mu - L_\tau$
  ⇨ Z', explain anomalies for
  - $h \to \tau\mu$
  - $B \to K^*\mu\mu$
  - $R_K = B \to K\mu\mu / B \to Kee$

- Constraints from $\tau \to 3\mu$ *crucial* ⇨ *Belle II, LHCb*

- See also, e.g.:
  Aristizabal-Sierra & Vicente'14,
  Lima et al'15,
  Omhura, Senaha, Tobe '15
  Altmannshofer et al.'15
  Bauer and Neubert'16, Buschmann et al.'16, etc...

Altmannshofer & Straub'14, Crivellin et al'15
Crivellin, D'Ambrosio, Heeck.'15



5. Conclusion and outlook

5.1 Conclusion

- Hadronic τ-decays very interesting to study
  - Very precise determination of α_s
    But error assignment and treatment of the NP part and new data needed
  - Extraction of V_{us} : ⇨ see Alberto Lusiani's talk

- Charged LFV are a very important probe of new physics
  - Extremely small SM rates
  - Experimental results at low energy are very precise ⇨ very high scale sensitivity
  - Excellent model discriminating tools:
    - BRs
    - Decay distributions

    Hadronic decays such as $\tau \to \mu\pi\pi$ important!

5.1 Conclusion

- Hadronic τ-decays very interesting to study
  - Very precise determination of α_s
    But error assignment and treatment of the NP part and new data needed
  - Extraction of V_{us} : ⇨ see Alberto Lusiani's talk

- Charged LFV are a very important probe of new physics

- Several topics extremely interesting to study that I did not address:
  - CPV asymmetry in $\tau \to K\pi\nu_\tau$
    BaBar result does not agree with SM expectation (2.8σ)
  - Lepton universality tests, Michel parameters…
  - EDM and g-2 of the tau

- A lot of *very interesting physics* remains to be done in the tau sector!

5.2 Prospects: Belle II Theory Interface Platform

- Initiative to coordinate a *joint theory-experiment* effort to study the potential impacts of the Belle II program

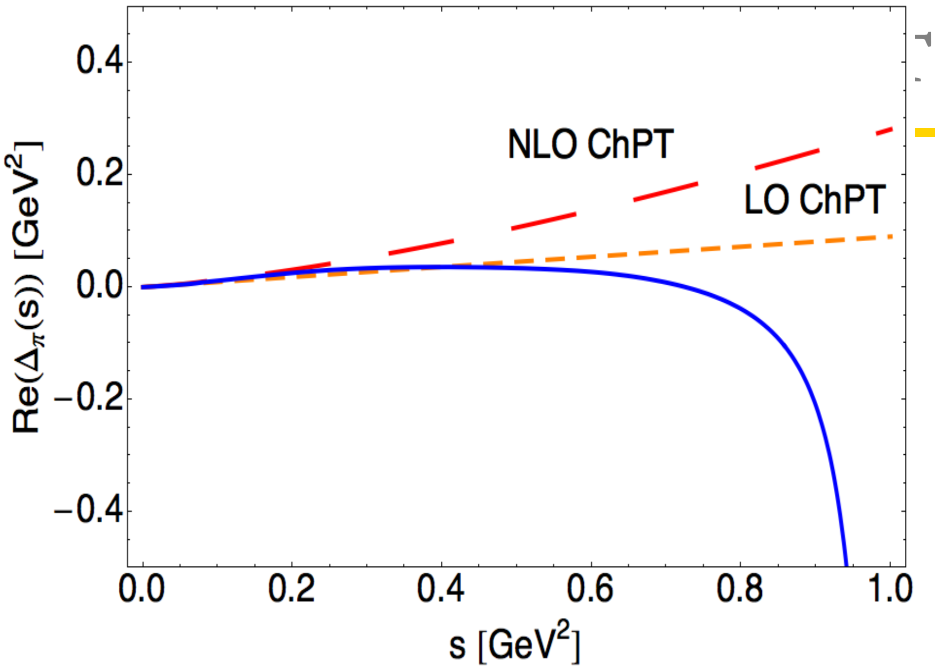
- *Tau, EW and low multiplicity working group*

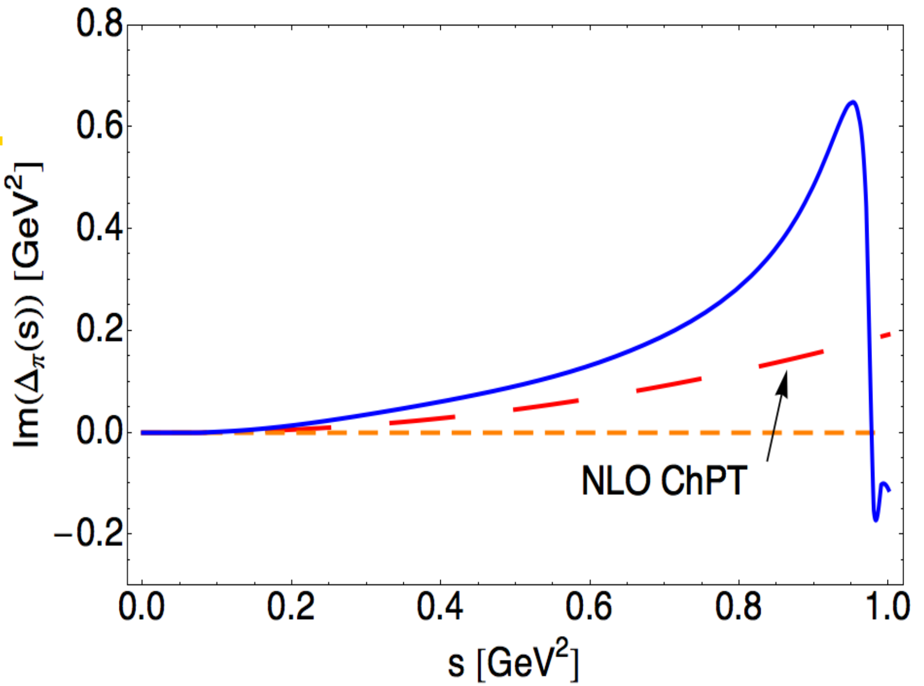
- Meetings twice a year until 2016 gathering theory experts and Belle II members

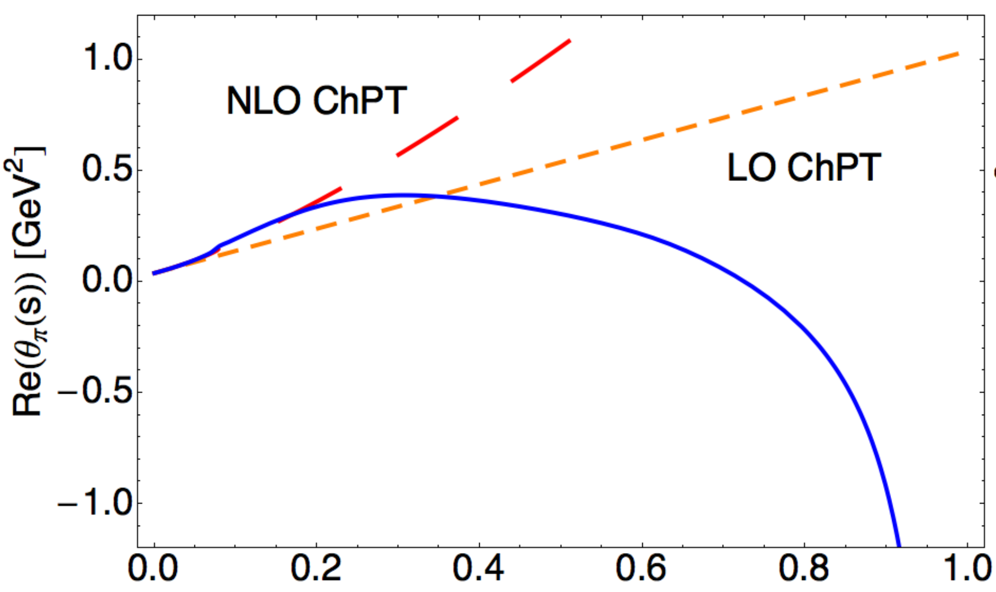
- Next meeting: *May 23 - 25, 2016* @ Pittsburgh

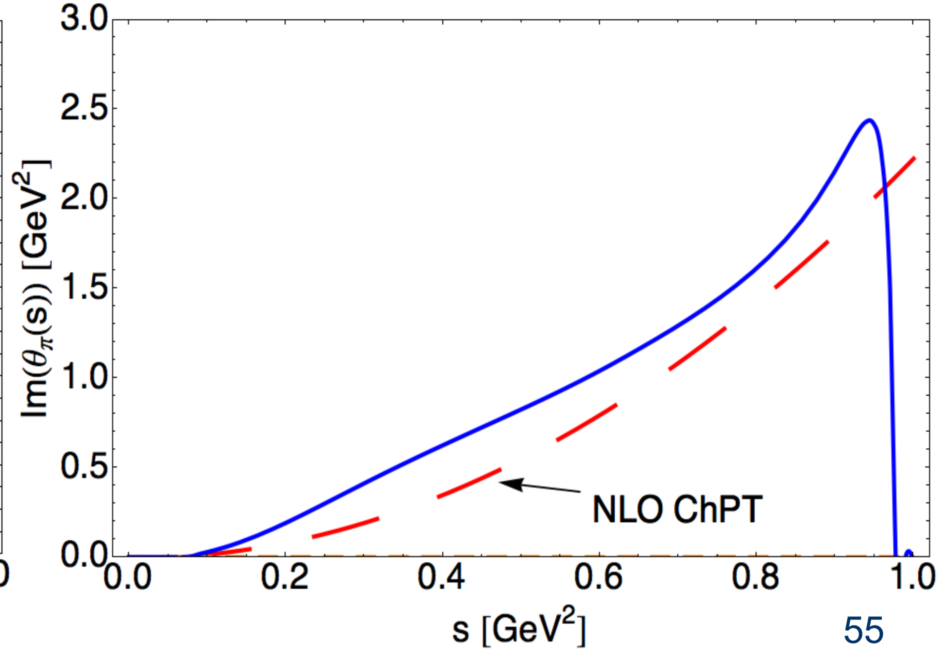
- Visit: *https://belle2.cc.kek.jp/~twiki/bin/view/B2TiP*

6. Back-up


NLO ChPT
LO ChPT
$\mathrm{Re}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]

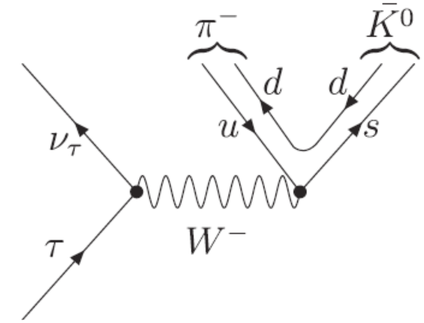

NLO ChPT
$\mathrm{Im}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]


NLO ChPT
LO ChPT
$\mathrm{Re}(\theta_\pi(s))$ [GeV^2]
s [GeV^2]


NLO ChPT
$\mathrm{Im}(\theta_\pi(s))$ [GeV^2]
s [GeV^2]

5. CPV in tau decays

5.1 $\tau \to K\pi\nu_\tau$ CP violating asymmetry



- $$\boxed{A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}}$$

$$= |p|^2 - |q|^2 \; \boxed{\approx (0.36 \pm 0.01)\%}$$ in the SM

Bigi & Sanda'05

Grossman & Nir'11

$$|K_S^0\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$

$$|K_L^0\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

$$\boxed{\langle K_L | K_S \rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}(\varepsilon_K)}$$

- Experimental measurement : *BaBar'11*

$$\boxed{A_{Q\,\mathrm{exp}} = \left(-0.36 \pm 0.23_{\mathrm{stat}} \pm 0.11_{\mathrm{syst}}\right)\%} \Rightarrow \boxed{2.8\sigma}$$ from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

Grossman & Nir'11

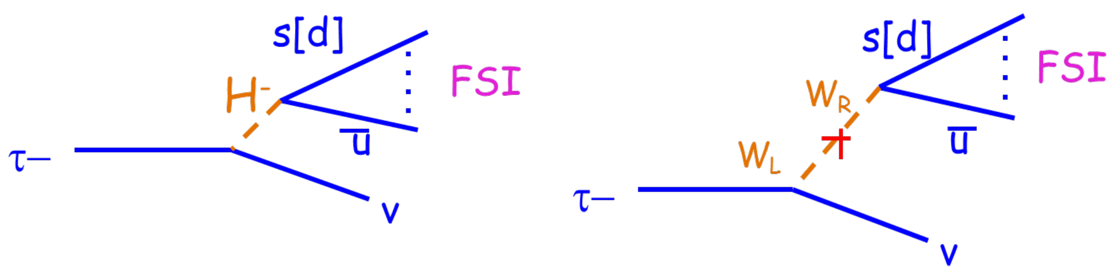
$$\boxed{A_D = \frac{\Gamma\left(D^+ \to \pi^+ K_S^0\right) - \Gamma\left(D^- \to \pi^- K_S^0\right)}{\Gamma\left(D^+ \to \pi^+ K_S^0\right) + \Gamma\left(D^- \to \pi^- K_S^0\right)}} = (-0.54 \pm 0.14)\%$$

Belle, Babar, CLOE, FOCUS

5.1 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, W_L-W_R mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha'14*)?

Bigi'Tau12



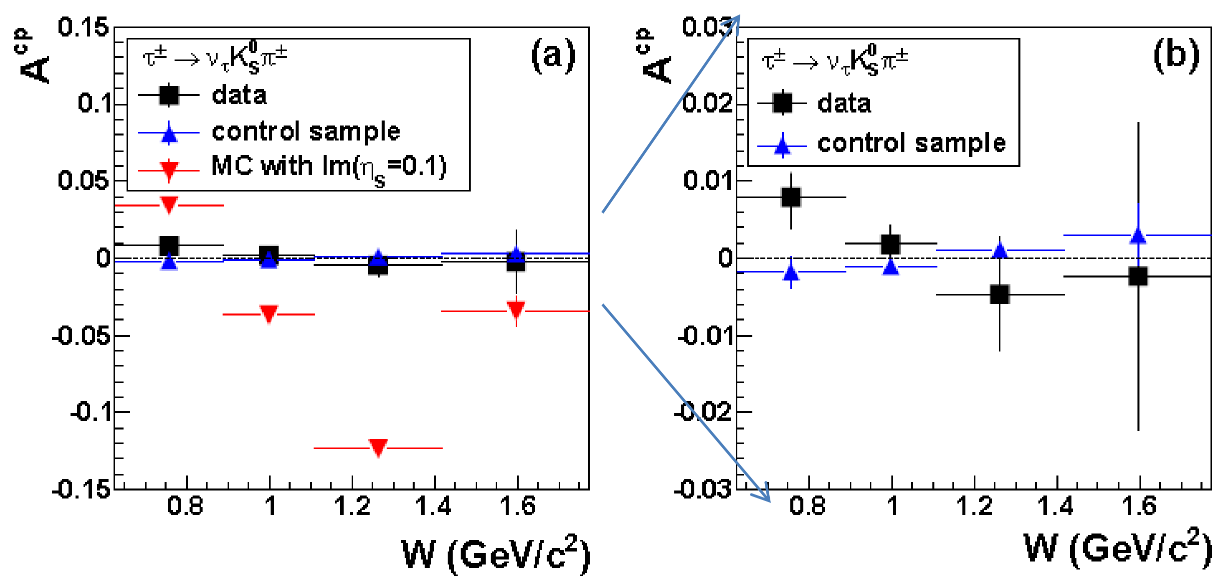
Very difficult to explain!

- Problem with this measurement? ⇨ It would be great to have other experimental measurements from *Belle, BES III or Tau-Charm factory*

Belle'11

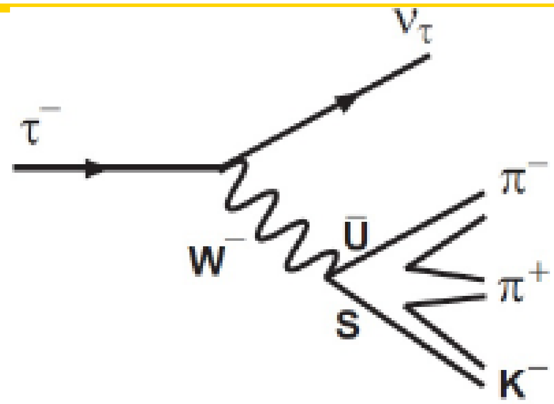
- Measurement of the direct contribution of NP in the angular CP violating asymmetry done by *CLEO* and *Belle*

  ⇨ Belle does not see any asymmetry at the *0.2 - 0.3% level*



5.2 Three body CP asymmetries

- Ex: $\tau \rightarrow K\pi\pi\nu_\tau$



- A variety of CPV observables can be studied :
$\tau \rightarrow K\pi\pi\nu_\tau, \tau \rightarrow \pi\pi\pi\nu_\tau$ rate, angular asymmetries, triple products,….

e.g., Choi, Hagiwara and Tanabashi'98
Kiers, Little, Datta, London et al.,'08
Mileo, Kiers and, Szynkman'14

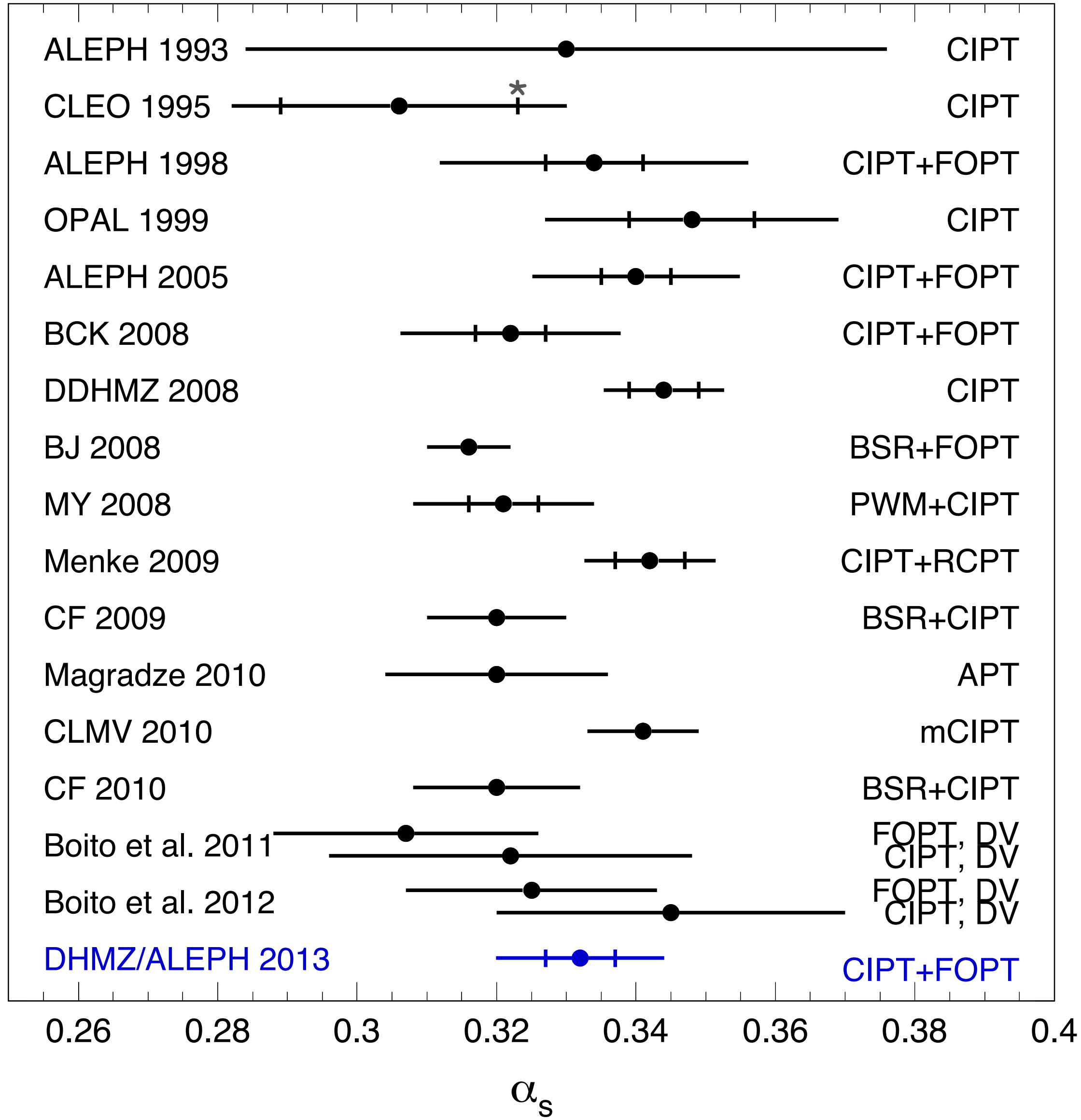
Same principle as in charm

Difficulty : Treatement of the hadronic part
Hadronic final state interactions have to be taken into account!
⇨ Disentangle weak and strong phases

- More form factors, more asymmetries to build but same principles as for 2 bodies

Comparison with Other Determinations

- Baikov, Chetyrkin, Köhn, PRL 101 (2008) 012002, [0801.1821]
- Beneke, Jamin, JHEP 0809 (2008) 044, [0806.3156]
- Maltman, Yavin, PRD78 (2008) 094020, [0807.0650]
- Menke, 0904.1796
- Caprini, Fischer, EPJC64 (2009) 35, [0906.5211]
- Magradze, Few Body Syst. 48 (2010) 143, Erratum-ibid. 53 (2012) 365, [1005.2674]
- Cvetic, Loewe, Martinez, Valenzuela, PRD82 (2010) 093007, [1005.4444]
- Caprini, Fischer, Rom.J.Phys. 55 (2010) 527, [1012.1132]
- Boito et al., PRD84 (2011) 113006, [1110.1127]; PRD85 (2012) 093015, [1203.3146]
- Beneke, Boito, Jamin, JHEP 1301 (2013) 125, [1210.8038]



* experimental uncertainty when available is shown as inner error bar

2.4 Operator Product Expansion

Braaten, Narison, Pich'92

- $$R_{\tau,V+A}(s_0) = N_C\, S_{EW}\, |V_{ud}|^2 \left(1 + \delta_P + \delta_{NP}\right)$$

$$S_{EW} = 1.0201(3)$$ Marciano &Sirlin'88, Braaten & Li'90, Erler'04

- Perturbative part (m_q=0)

$$\boxed{-s\frac{d}{ds}\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_S(\text{-}s)}{\pi}\right)^n}$$

$$K_0 = K_1 = 1,\; K_2 = 1.63982,\; K_3 = 6.37101$$

$$K_4 = 49.07570$$ Baikov, Chetyrkin, Kühn'08

$$\Rightarrow \quad \boxed{\delta_P = \sum_{n=1} K_n A^n(\alpha_S) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots}$$

with $$A^n(\alpha_S) = \frac{1}{2\pi i}\oint_{|s|=m_\tau^2} \frac{ds}{s}\left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8}\right)\left(\frac{\alpha_S(-s)}{\pi}\right)^n$$

$$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

$$\Rightarrow \quad \boxed{\delta_P \approx 20\%}$$

$$\boxed{\left(\delta_P = 0.2066 \pm 0.0070\right)}$$ Davier et al'08

CIPT vs. FOPT

- $$-s\frac{d}{ds}\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_S(-s)}{\pi}\right)^n$$

$$\delta_P = \underbrace{\sum_{n=1} K_n A^n(\alpha_S)}_{\text{CIPT}} = \underbrace{\sum_{n=0} r_n \left(\frac{\alpha_S(-s)}{\pi}\right)^n}_{\text{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^n(\alpha_S) = \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\left(1-2x+2x^3-x^4\right)\left(\frac{\alpha_S(-xm_\tau^2)}{\pi}\right)^n = a_\tau + \dots$$

$$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

n	1	2	3	4	5
K_n	1	1.6398	6.3710	49.0757	
g_n	0	3.5625	19.9949	78.0029	307.78
r_n	1	5.2023	26.3659	127.079	

Pich Tau'10

LeDiberder & Pich''92

⇒ The dominant corrections come from the contour integration
Large running of α_S along the circle $s = m_\tau^2 e^{i\varphi}$ $\varphi \in [0, 2\pi]$

Perturbative $(m_q=0)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

$K_0 = K_1 = 1$, $K_2 = 1.63982$, $K_3 = 6.37101$, $K_4 = 49.07570$ **Baikov-Chetyrkin-Kühn '08**

$$\delta_{\mathrm{P}} = \sum_{n=1} K_n\; A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Power Corrections

$$\Pi^{(0+1)}_{\mathrm{OPE}}(s) \approx \frac{1}{4\pi^2}\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

Braaten-Narison-Pich '92

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{\mathrm{NP}} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;(1-3x^2+2x^3)\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

Suppressed by m_τ^6 **[additional chiral suppression in** $C_6\langle O_6\rangle^{V+A}$ **]**

Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n\; a(-s)^n$$

$$\delta_{\mathrm{P}} = \underbrace{\sum_{n=1} K_n\; A^{(n)}(\alpha_s)}_{\textbf{CIPT}} = \underbrace{\sum_{n=0} r_n\; a_\tau^n}_{\textbf{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a_\tau(-x\,m_\tau^2)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

n	1	2	3	4	5
K_n	1	1.6398	6.3710	49.0757	
g_n	0	3.5625	19.9949	78.0029	307.78
r_n	1	5.2023	26.3659	127.079	

The dominant corrections come from the contour integration

Le Diberder- Pich 1992

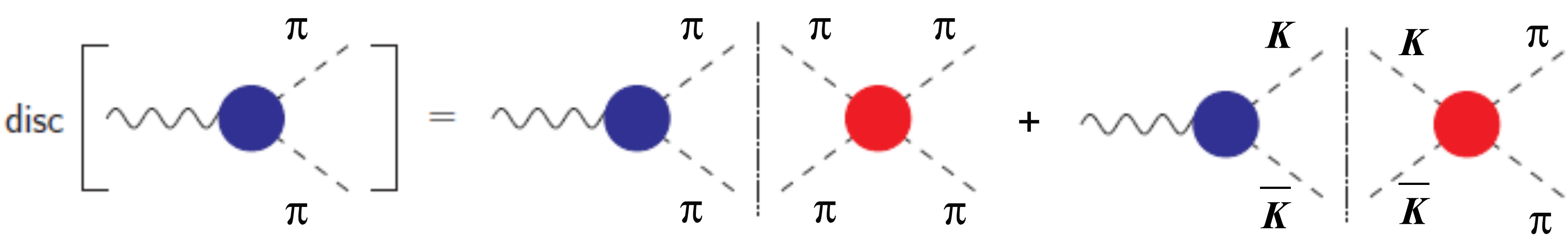
Large running of a_s along the circle $s = m_\tau^2\, e^{i\varphi}$, $\varphi \in [0\,,\,2\pi]$

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs ⇨ *Coupled channel analysis*
  up to $\sqrt{s}\sim1.4$ GeV
  Inputs: I=0, S-wave $\pi\pi$ and $K\bar{K}$ data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13

- Unitarity:

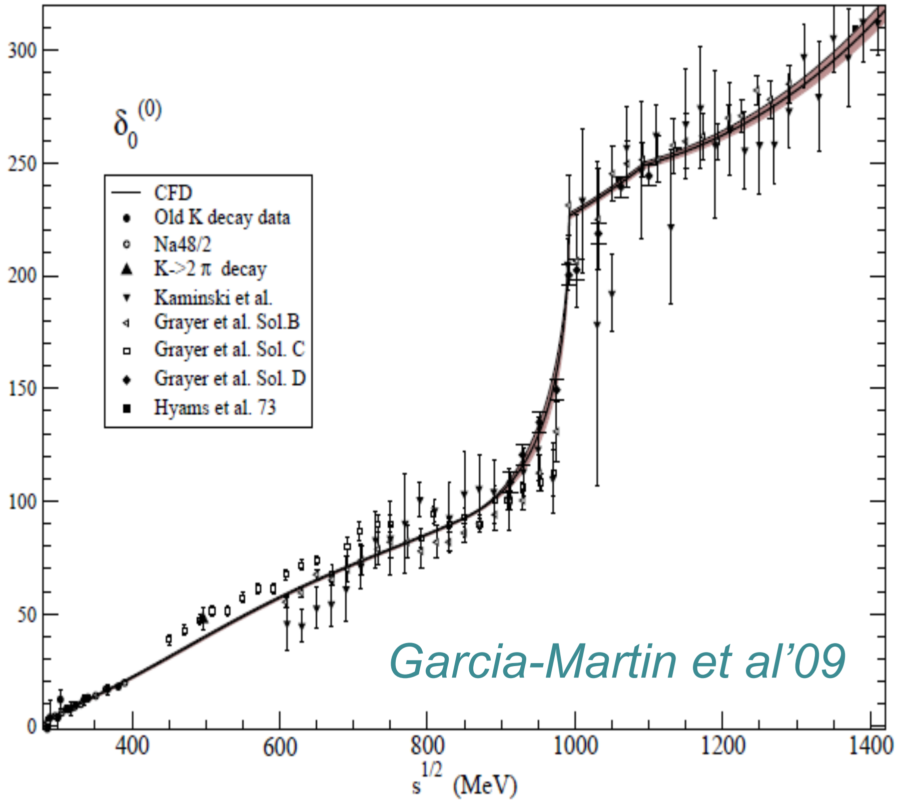


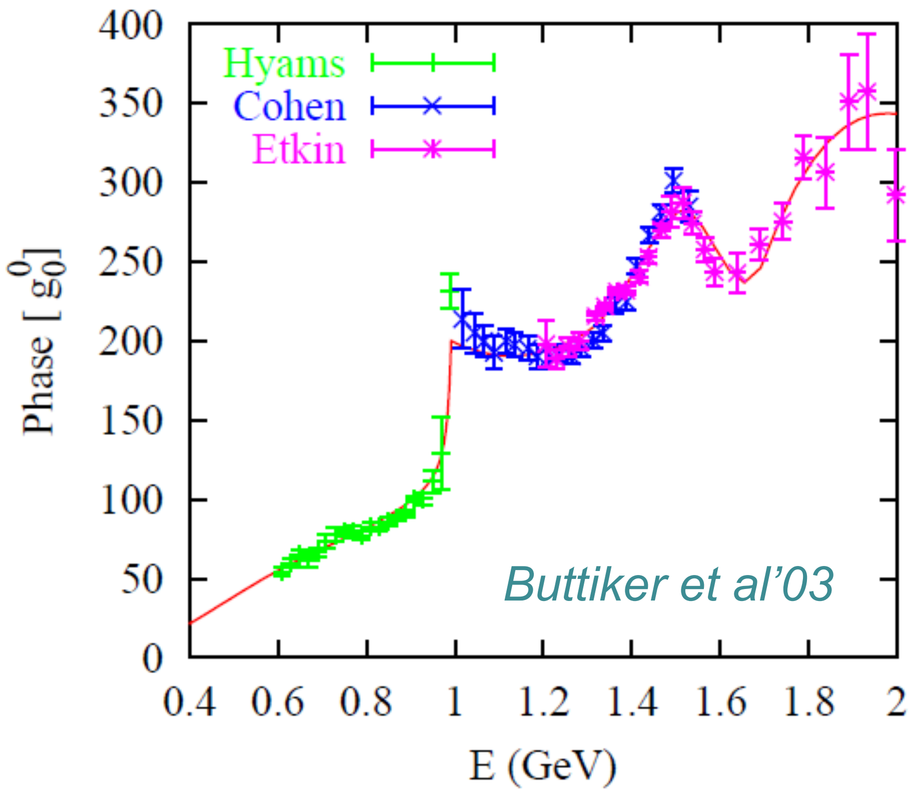
⇨ $$\boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$ $\boldsymbol{n = \pi\pi, K\bar{K}}$

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

Celis, Cirigliano, E.P.'14

- Inputs : $\pi\pi \to \pi\pi, KK$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$\boldsymbol{X(s) = C(s), D(s)}$$

$$\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{T_{nm}^* \sigma_m(s) X_m^{(N)}\right\} \longrightarrow \mathrm{Re} X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \mathrm{Im} X_n^{(N+1)}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \to 1/s$ (*Brodsky & Lepage*) + ChPT:

  Feynman-Hellmann theorem: ⇨

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

⇨

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

$\Rightarrow$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!
  $\Rightarrow$ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\; M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2 M_\pi^2 \right)$$

Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

- General solution

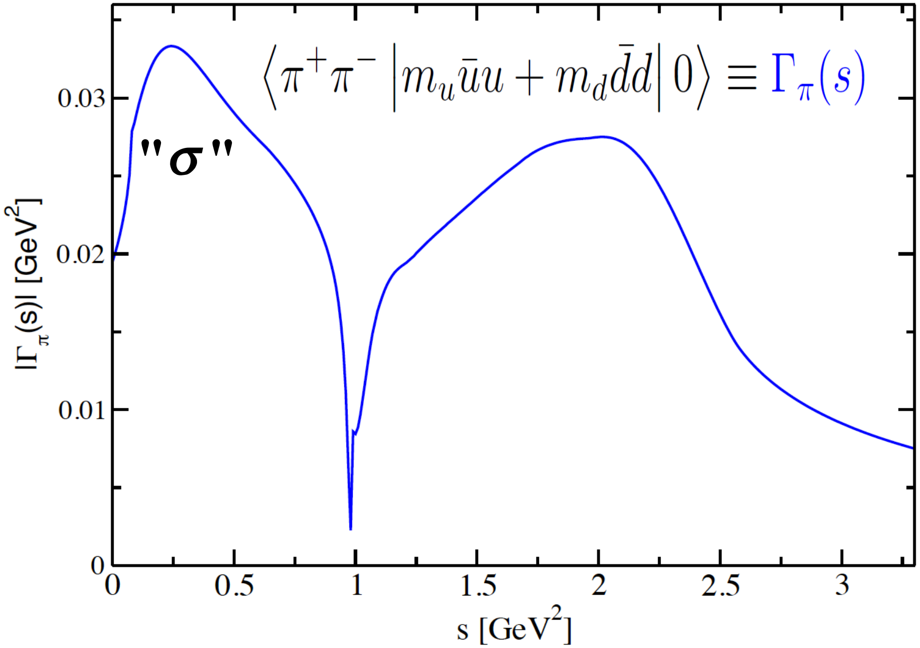
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

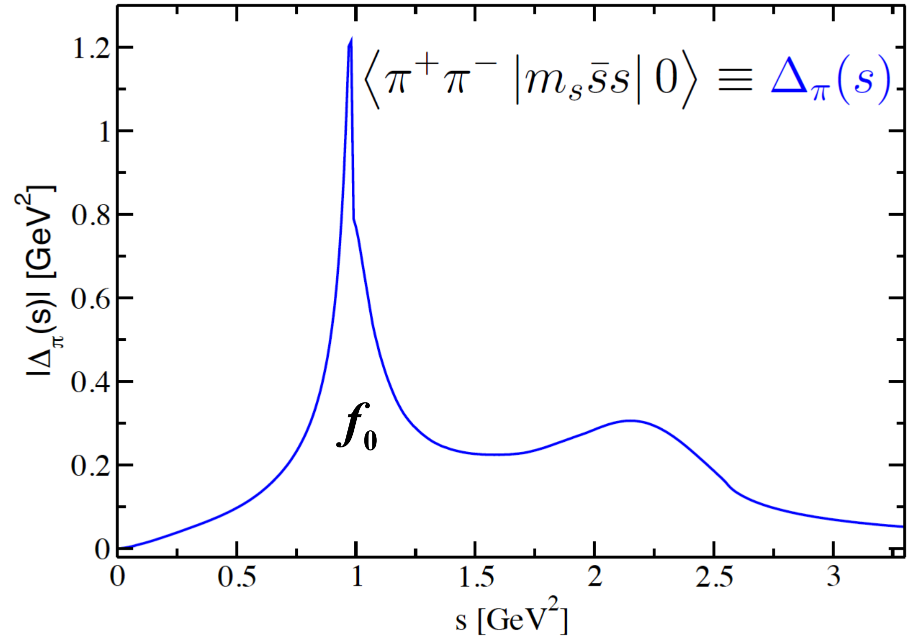
- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

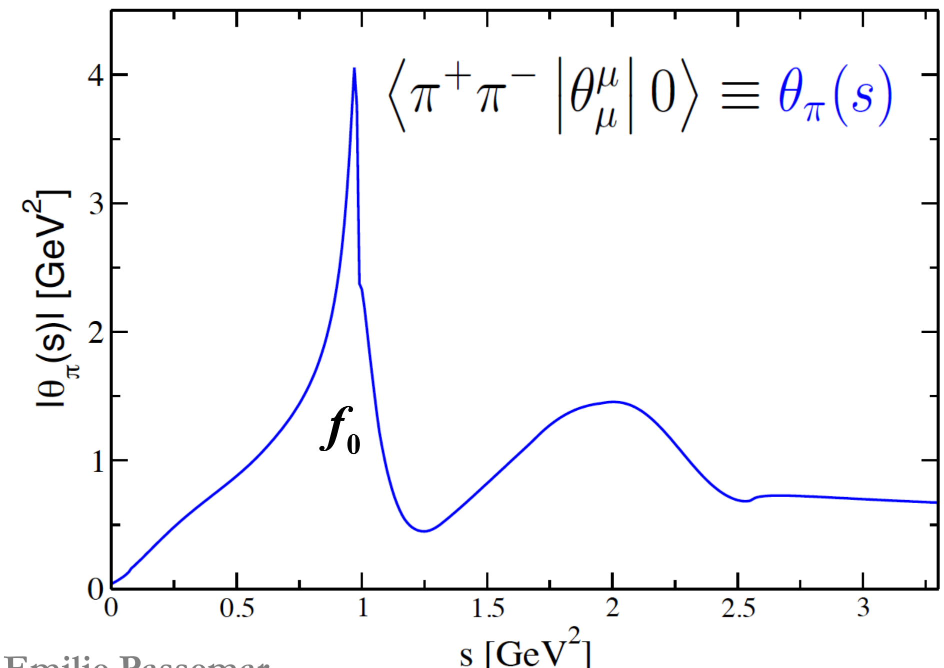
➡ Relax the constraints and match to ChPT

$$P_\theta(s) = 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s$$

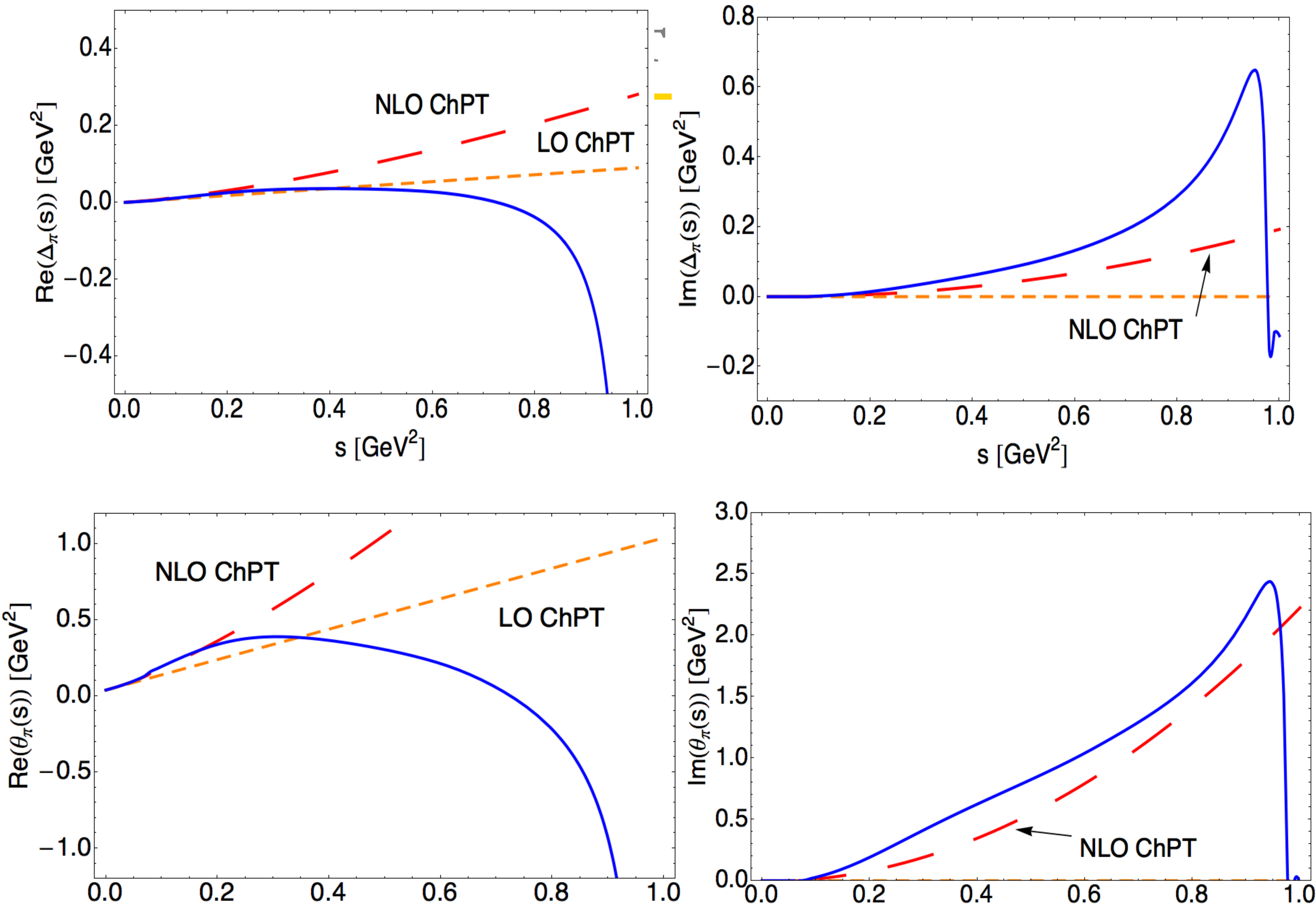
$$Q_\theta(s) = \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s$$







Dispersion relations:
Model-independent method,
based on first principles
that extrapolates ChPT
based on data



NLO ChPT
LO ChPT
$\mathrm{Re}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
$\mathrm{Im}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
LO ChPT
$\mathrm{Re}(\theta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
$\mathrm{Im}(\theta_\pi(s))$ [GeV^2]
s [GeV^2]

3.4.3 Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

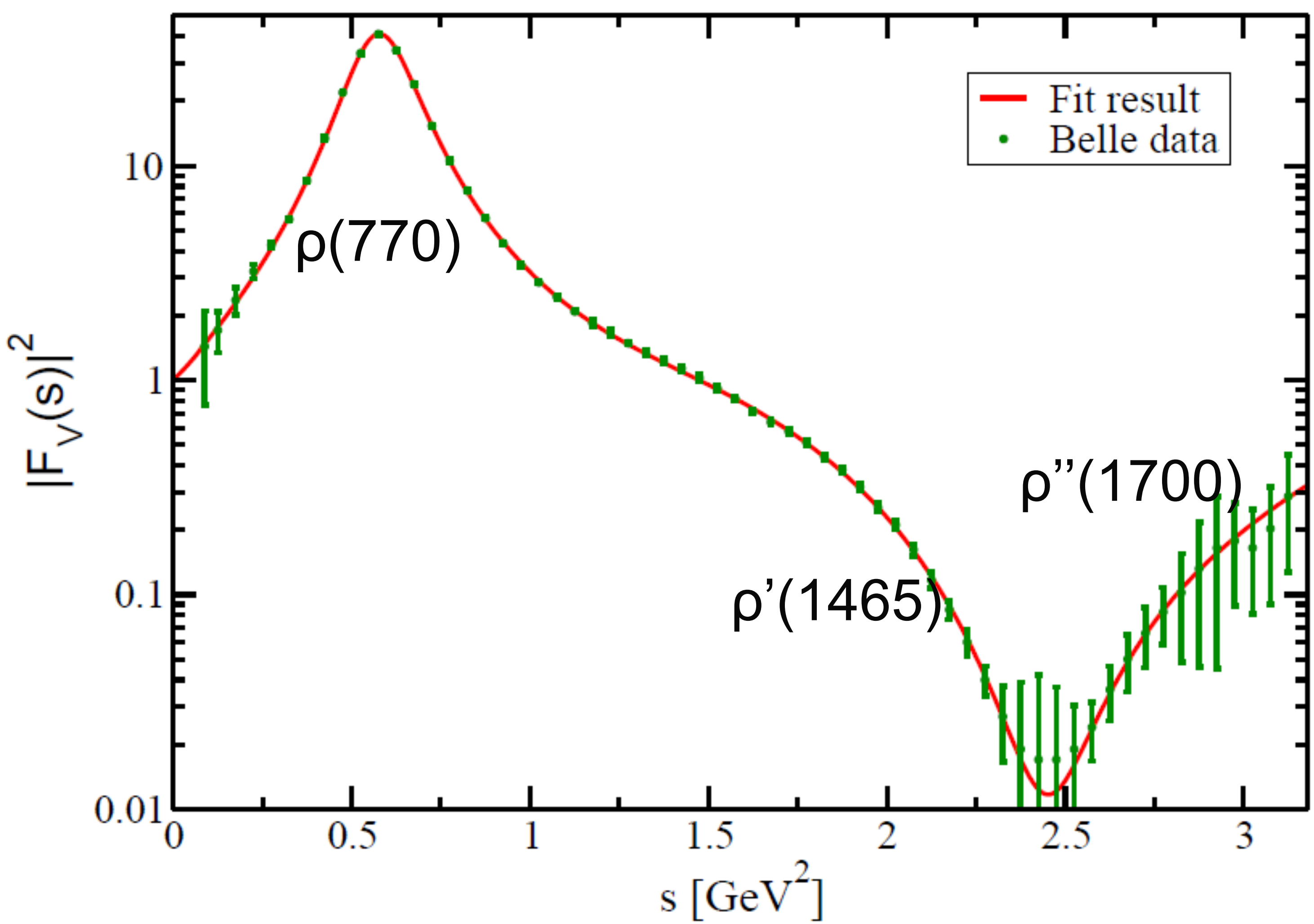
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_V'' - \lambda_V'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3}\frac{\phi_V(s')}{\left(s'-s-i\varepsilon\right)}\right]}$$

Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

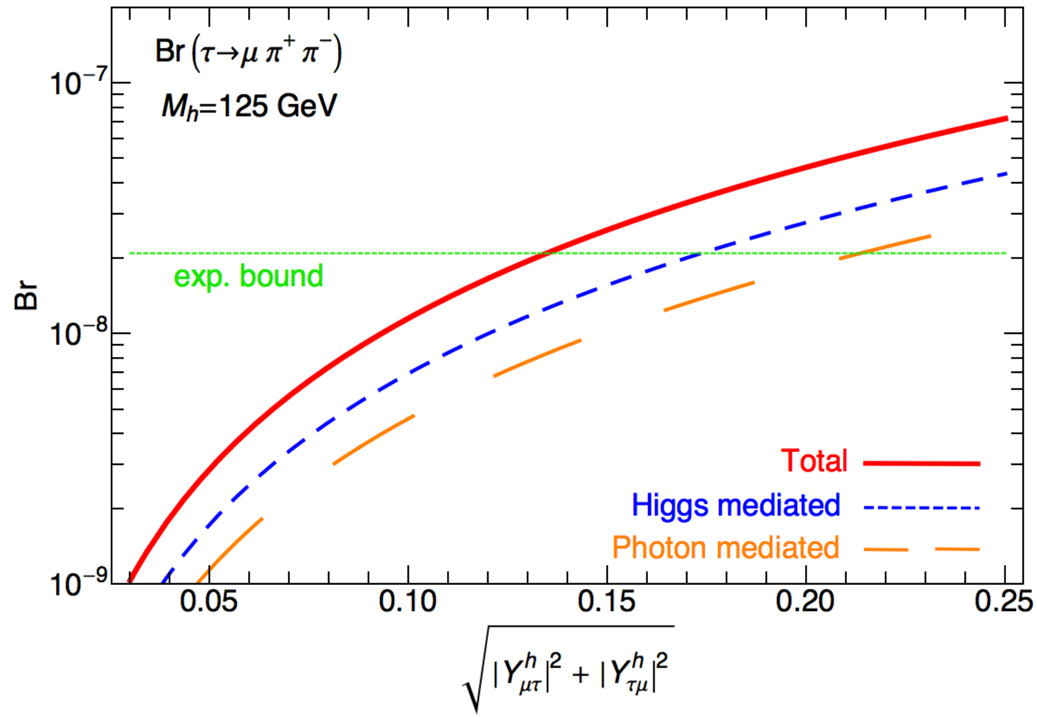
  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

3.4.3 Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.5 Results



Bound:

$$\boxed{\sqrt{\left|Y^h_{\mu\tau}\right|^2+\left|Y^h_{\tau\mu}\right|^2}\leq 0.13}$$

Less stringent but more robust handle on LFV Higgs couplings

Process	$(\mathrm{BR}\times 10^8)$ 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2+\lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau\to\mu\gamma$	<4.4 [88]	<0.016	Dipole
$\tau\to\mu\mu\mu$	<2.1 [89]	<0.24	Dipole
$\tau\to\mu\pi^+\pi^-$	<2.1 [86]	<0.13	Scalar, Gluon, Dipole
$\tau\to\mu\rho$	<1.2 [85]	<0.13	Scalar, Gluon, Dipole
$\tau\to\mu\pi^0\pi^0$	$<1.4\times 10^3$ [87]	<6.3	Scalar, Gluon

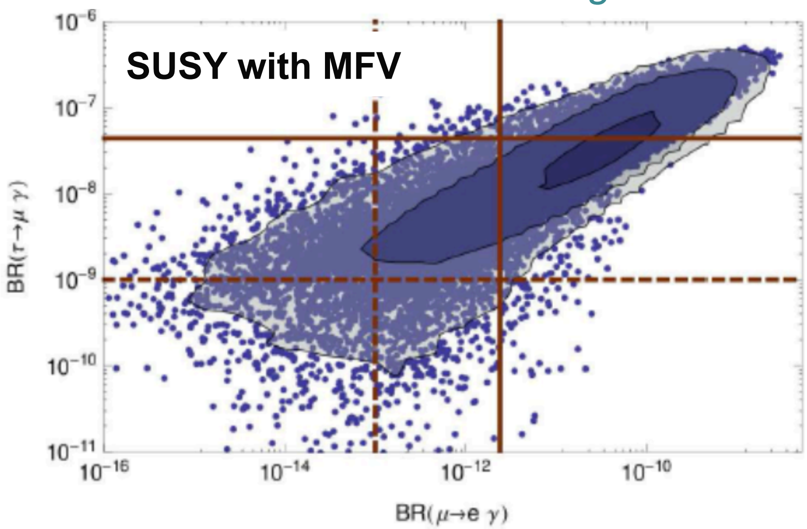
?

Belle'08'11'12 except last from *CLEO'97*

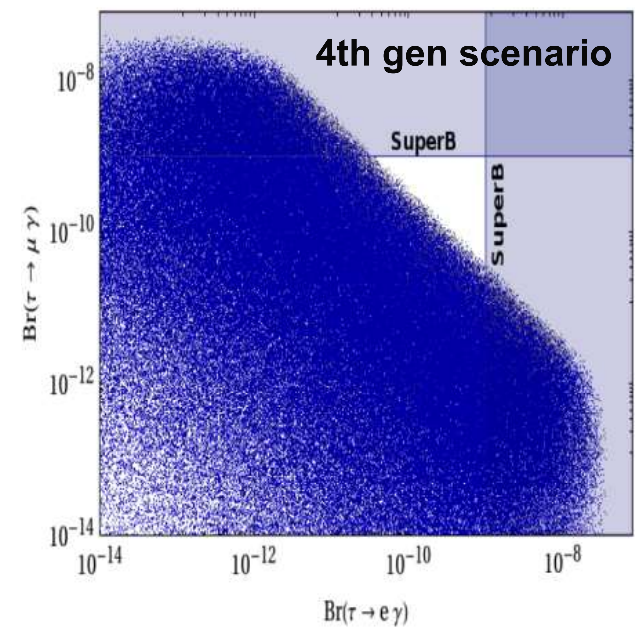
2.5 Model discriminating power of Tau processes

- Depending on the UV model different correlations between the BRs

Blankerburg et al.'12



Buras et al.'10

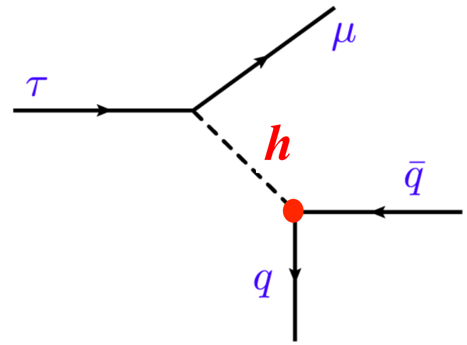


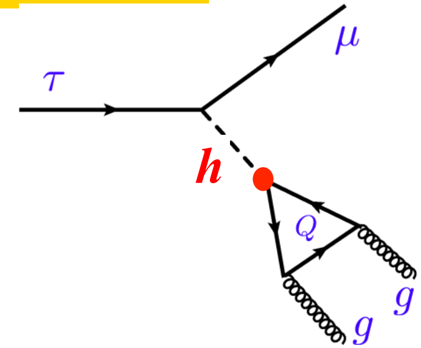
⇨ Interesting to study to determine the underlying dynamics of NP

3.5 What if $\tau \to \mu(e)\pi\pi$ observed?

Reinterpreting *Celis, Cirigliano, E.P'14*

Talk by J. Zupan @ KEK-FF2014FALL





- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!
- $Y_{u,d,s}$ poorly bounded
- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$\boxed{\begin{array}{l} Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \\ Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7} \end{array}}$$

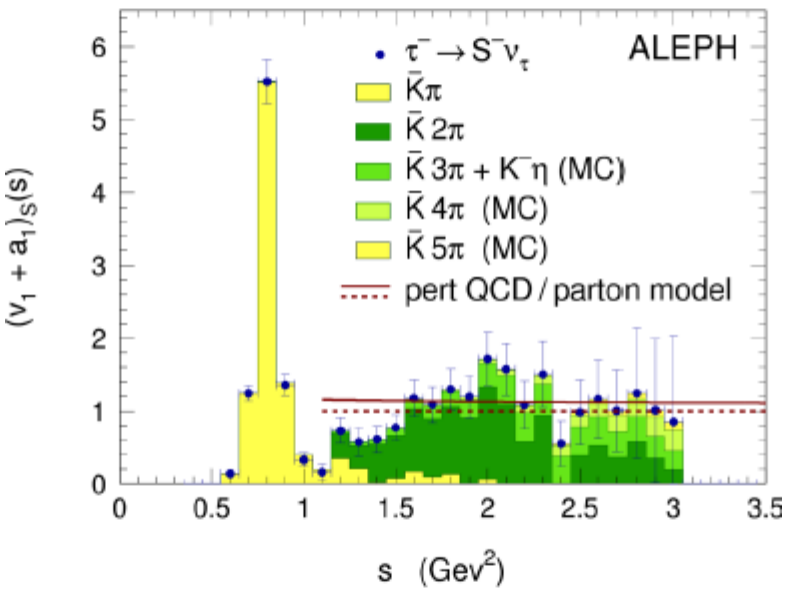
below present experimental limits!

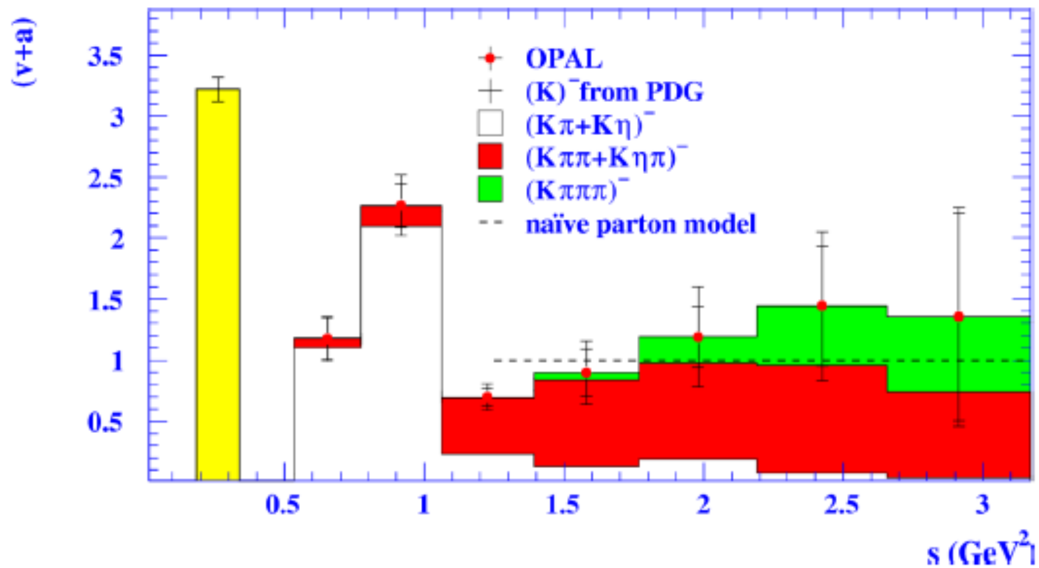
- If discovered ⇨ among other things *upper limit* on $Y_{u,d,s}$!

⇨ Interplay between high-energy and low-energy constraints!

3.6 Prospects : τ strange Brs

- Experimental measurements of the strange spectral functions not very precise





⇨ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

Smaller $\tau \to$ K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller $\boxed{V_{us}}$

$$R_\tau^S\big|_{\text{old}} = 0.1686(47) \quad \Rightarrow \quad R_\tau^S\big|_{\text{new}} = 0.1615(28)$$

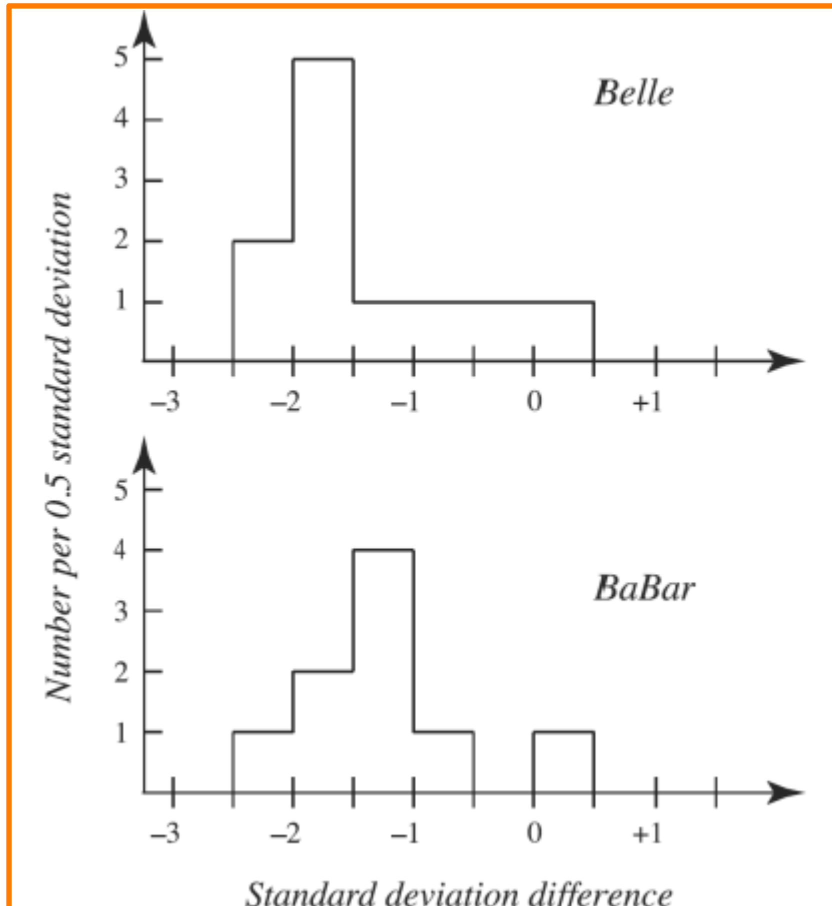
$$\left|V_{us}\right|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}} \quad \Rightarrow \quad \left|V_{us}\right|_{\text{new}} = 0.2176 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}$$

3.6 Prospects : τ strange Brs

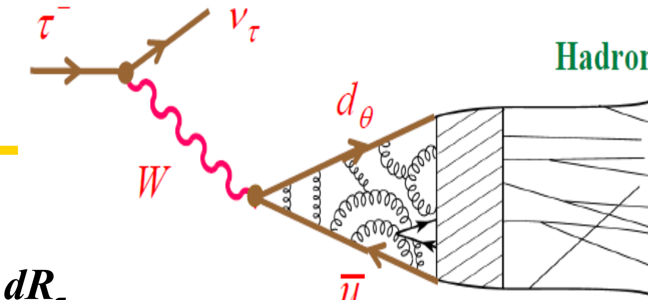
- *PDG 2014*: « Eigtheen of the 20 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.30 » (-1.41 for the 11 Belle measurements and -1.24 for the 9 BaBar measurements)

- Measured modes by the 2 B factories:

Mode	BaBar – Belle Normalized Difference (#σ)
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	+1.4
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	−2.9
$K^-K^+\pi^-\nu_\tau$	−2.9
$K^-K^+K^-\nu_\tau$	−5.4
$\eta\ K^-\nu_\tau$	−1.0
$\phi\ K^-\nu_\tau$	−1.3



2.2 Experimental situation



- Observable studied $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)}}$ and $\dfrac{dR_\tau}{ds}$

- Decomposition as a function of observed and separated final states

$$\boxed{R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}}$$

Zhang'Tau14

$$\upsilon_1 / a_1 \left[\tau^- \to V^- / A^- \nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to V^- / A^- \nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^- \overline{\nu}_e \nu_\tau\right]} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds} \frac{m_\tau^2}{\left(1 - s/m_\tau^2\right)^2 \left(1 + 2s/m_\tau^2\right)}$$

Vector/Axial-vector spectral functions — branching fractions — mass spectrum — kinematic factor

2.6 Inclusive determination of V_{us}

- With QCD on: $$\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$$

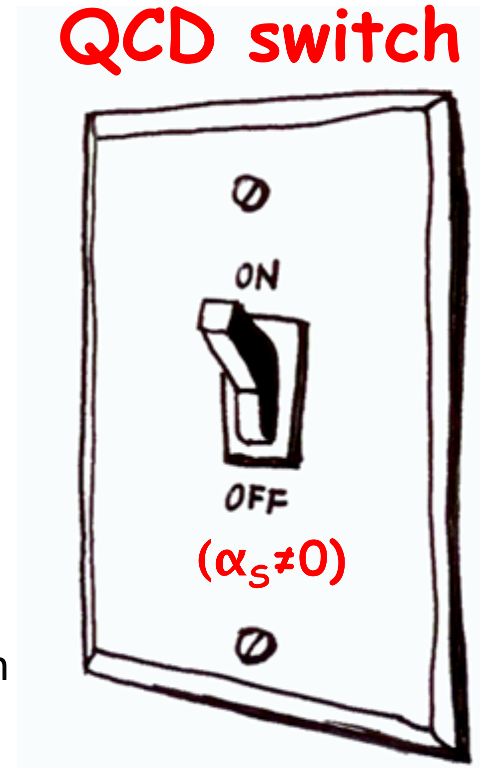
- Use OPE: $$\boxed{R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1+\delta_P+\delta_{NP}^{ud}\right)}$$

$$\boxed{R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1+\delta_P+\delta_{NP}^{us}\right)}$$

- $$\boxed{\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0239(30)$$ *Gamiz et al'07, Maltman'11*



$$\boxed{|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

HFAG'14

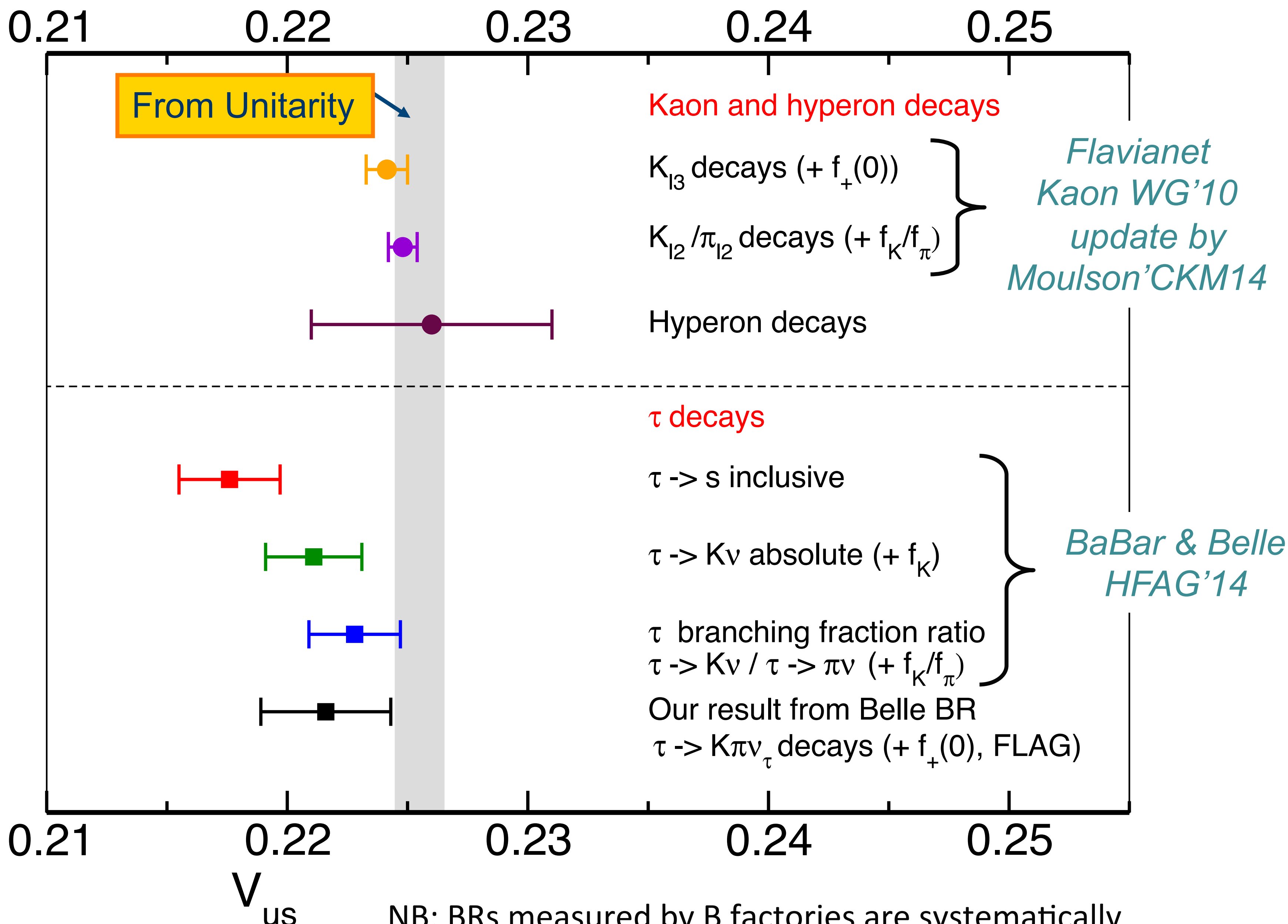
$R_{\tau,S} = 0.1615(28)$

$R_{\tau,NS} = 3.4650(84)$

$|V_{ud}| = 0.97425(22)$

$\Rightarrow$ $$\boxed{|V_{us}| = 0.2176 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}}$$

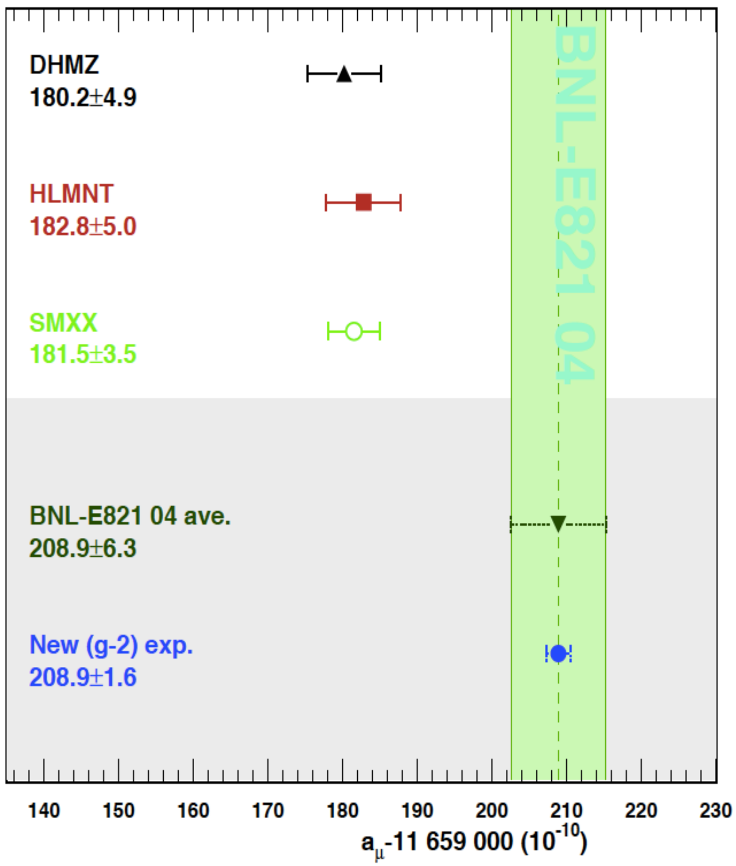
3.4σ away from unitarity!



NB: BRs measured by B factories are systematically smaller than previous measurements

4.3 Confronting measurement and prediction

Blum et al.'13

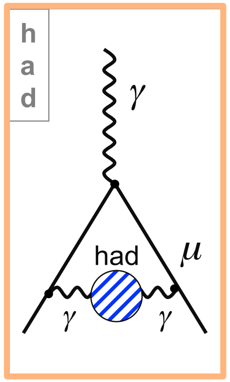


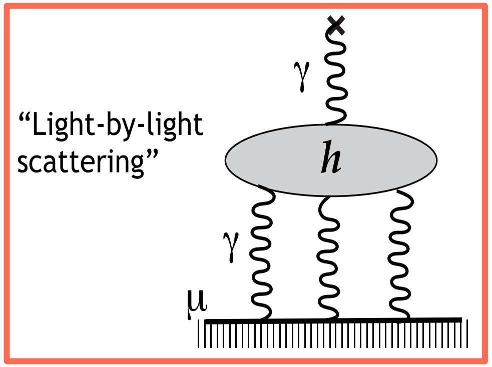
$$a_\mu|_{\rm exp} = 0.001\ 165\ 920\ 80(54)(33)[63]$$
$$a_\mu|_{\rm theo} = 0.001\ 165\ 918\ 40(59)$$
$$a_\mu|_{\rm exp} - a_\mu|_{\rm theo} = 240 \times 10^{-11} \qquad 2.9\sigma \text{ diff.}$$

Theoretical Prediction: *Lafferty, summary talk@Tau2014*

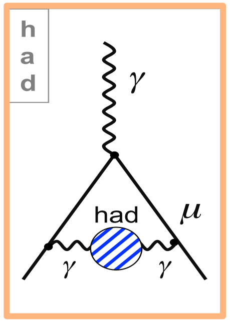
Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	690.8 ± 4.7
HVP(higher order) (*)	-10.0 ± 0.02
HLBL	11.6 ± 4.0
EW	15.4 ± 0.1
Total	11659179.7 ± 6.2

Jegerlehner and Nyffeler '09



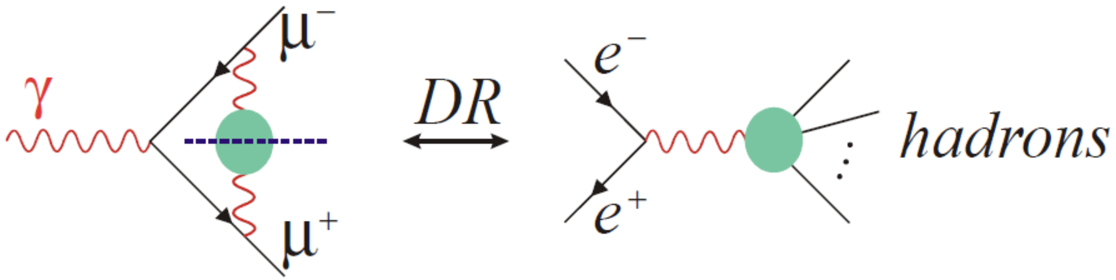


4.4 Towards a model independent determination of HVP



- Hadronic contribution cannot be computed from first principles due to low-energy hadronic effects

- Use analyticity + unitarity ⇨ real part of photon polarisation function from *dispersion relation* over *total hadronic cross section data*



$$R_V(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Leading order hadronic vacuum polarization :

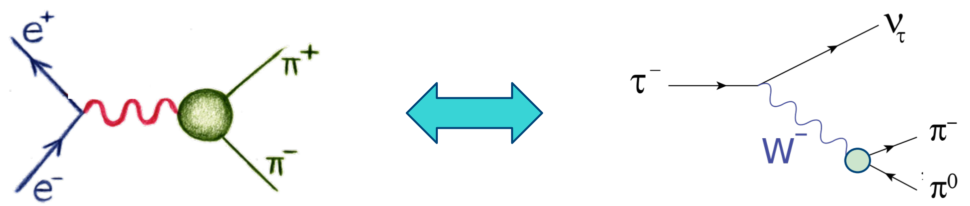
$$a_\mu^{had,LO} = \frac{\alpha^2 m_\mu^2}{(3\pi)^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R_V(s)$$

- Low energy contribution dominates : ~75% comes from s < (1 GeV)^2
  ⇨ ππ contribution extracted from data

4.5 Can precise Tau data help?

Hoecker'11

- Tau data can be used for 2π and 4π channels with isospin rotation



- Tau spectral functions measured by *ALEPH*, *Belle, CLEO*, *OPAL*

- Excellent precision of tau data. Branching ratio (ie, spectral function normalisation) for $\tau \to \pi\pi^0\nu$ known to 0.4%

- Invariant mass spectrum requires unfolding using detector simulation, which is however under good control

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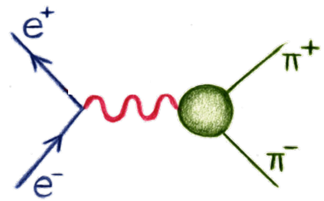
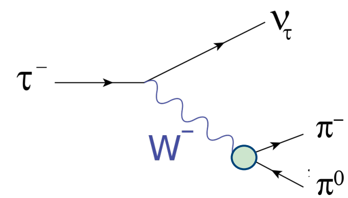




- Tau spectral functions measured by *ALEPH*, *Belle, CLEO*, *OPAL*

- Excellent precision of tau data. Branching ratio (ie, spectral function normalisation) for $\tau \to \pi\pi^0\nu$ known to 0.4%

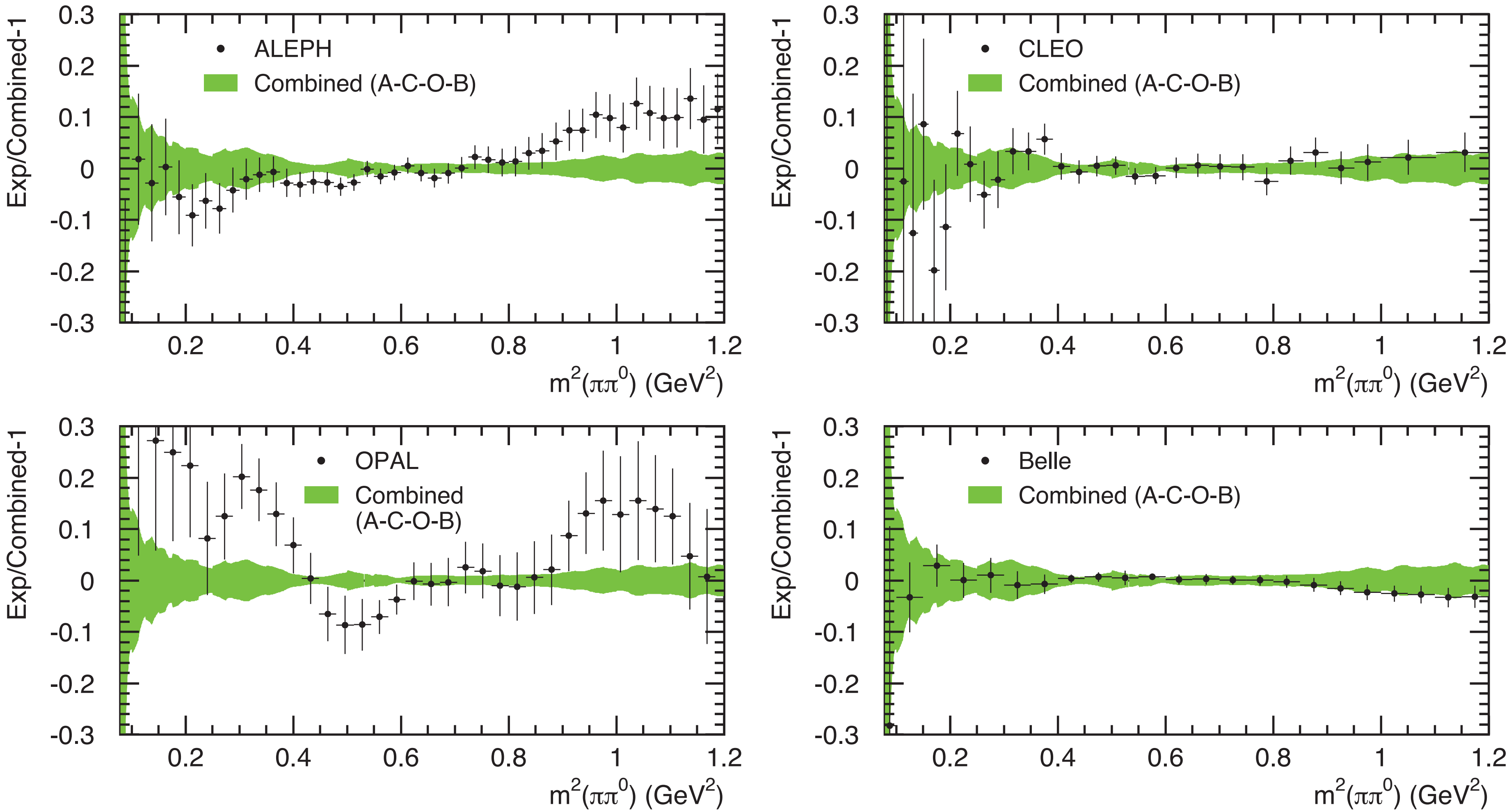
- *Main experimental challenge*: abundance and shape modeling of feed-through from other tau final states

- Main theoretical challenge: *isospin breaking*
Radiative corrections, charged vs. neutral mass splitting and electromagnetic decays: (–3.2 ± 0.4)% correction to a_μ^{had}

4.5 Can precise Tau data help?

Davier et al.'10

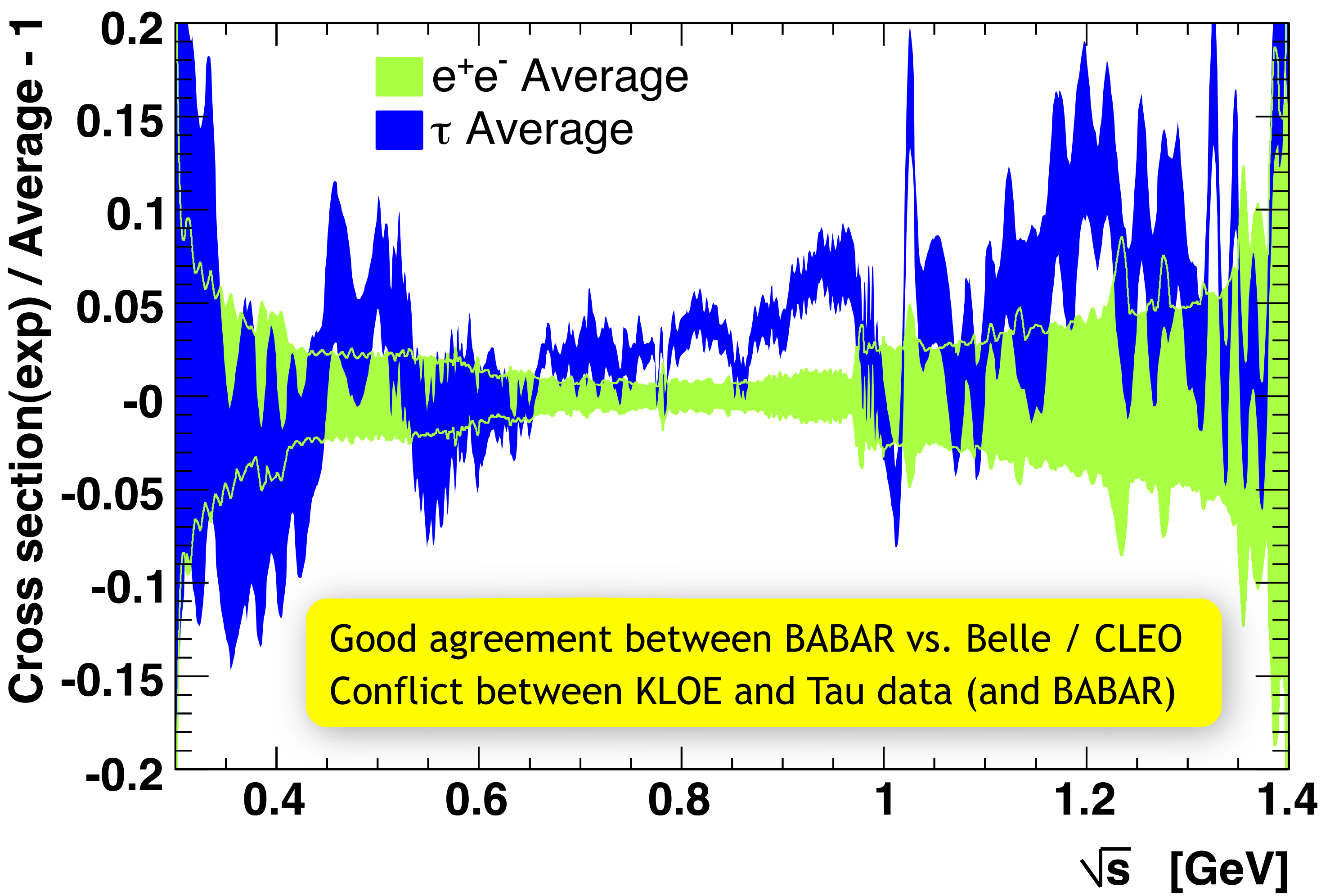
- Situation for the 2 pion channel: τ



4.5 Can precise Tau data help?

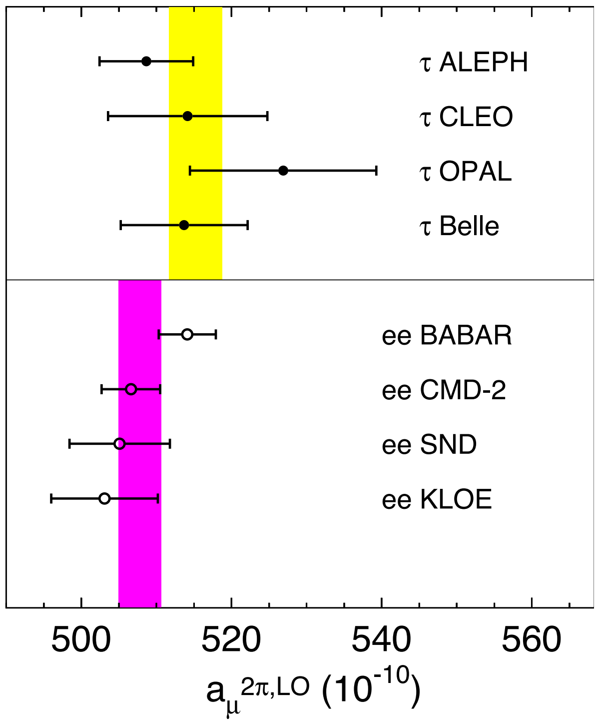
- Situation for the 2 pion channel: e^+e^- vs. τ

Zhang Tau'2012

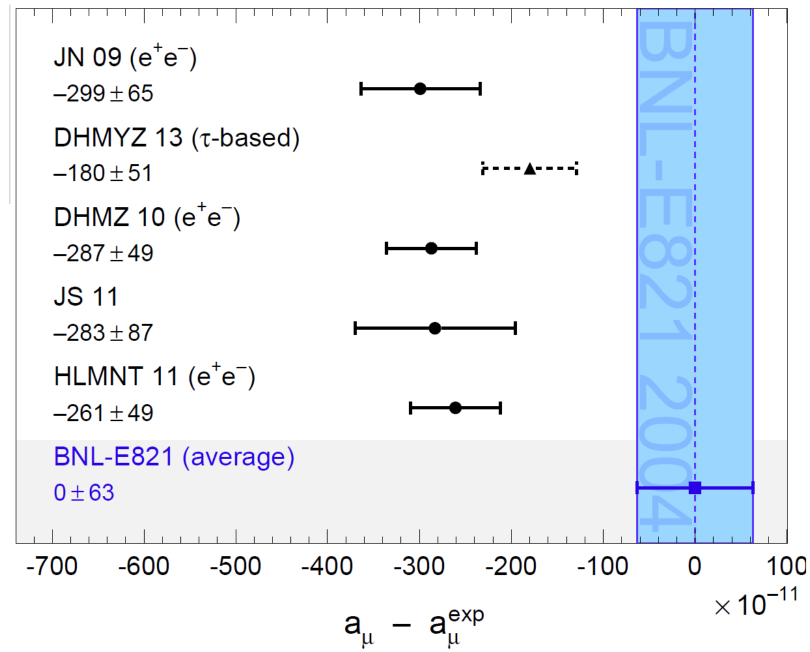


4.5 Can precise Tau data help?

Davier'Tau12



Zhang



- The tau data help to reduce the discrepancy between theory and experiment (1.9 σ) but open issues remain: *isospin breaking*
  Very challenging for theorists!

3.5 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \dfrac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M

  - Spectra for > 2 bodies in the final state:

$$\boxed{\frac{dBR(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}} \quad \text{and} \quad \boxed{dR_{\pi^+\pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu\gamma)} \frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}}}$$

3.6 Model discriminating of BRs

- Studies in specific models

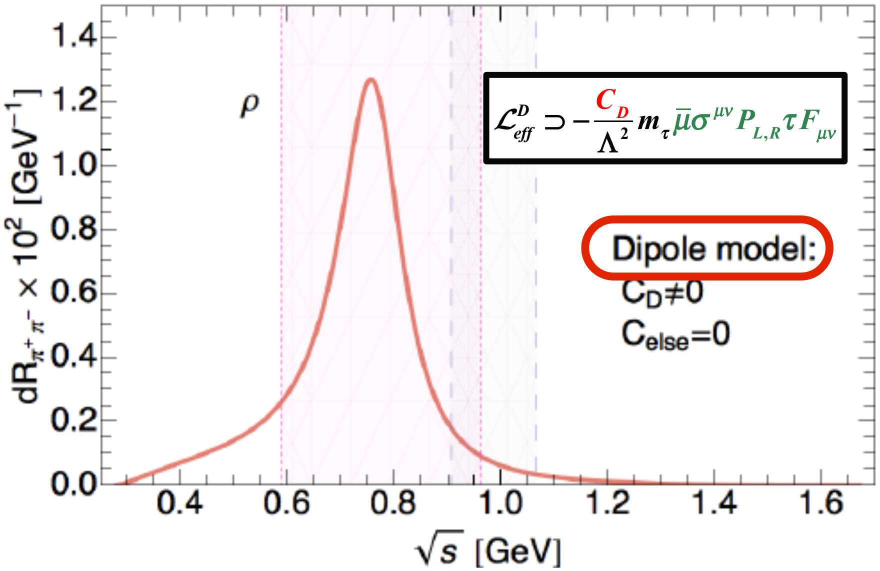
Buras et al.'10

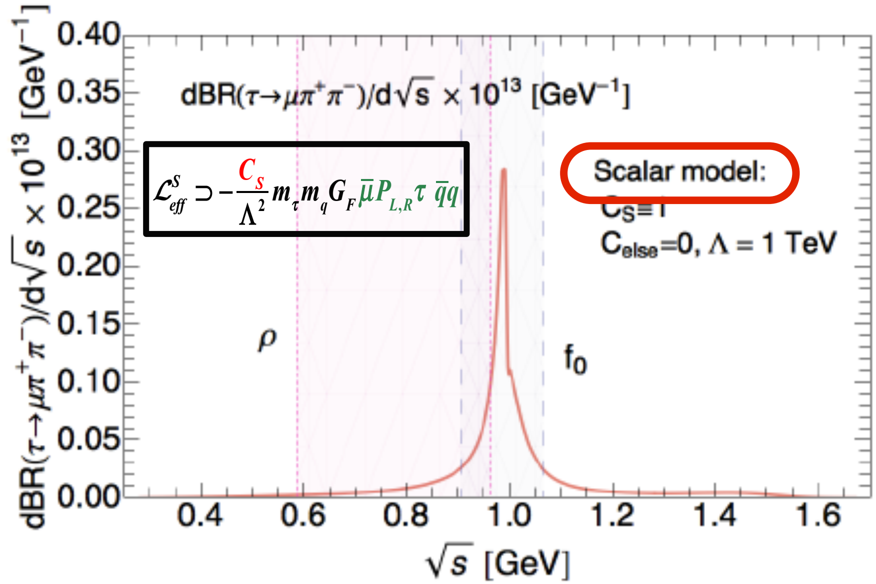
ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^-\to e^-e^+e^-)}{\mathrm{Br}(\mu\to e\gamma)}$	$0.02\ldots1$	$\sim 6\cdot10^{-3}$	$\sim 6\cdot10^{-3}$	$0.06\ldots2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots0.4$	$\sim 1\cdot10^{-2}$	$\sim 1\cdot10^{-2}$	$0.07\ldots2.2$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots0.4$	$\sim 2\cdot10^{-3}$	$0.06\ldots0.1$	$0.06\ldots2.2$
$\frac{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}{\mathrm{Br}(\tau\to e\gamma)}$	$0.04\ldots0.3$	$\sim 2\cdot10^{-3}$	$0.02\ldots0.04$	$0.03\ldots1.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}{\mathrm{Br}(\tau\to \mu\gamma)}$	$0.04\ldots0.3$	$\sim 1\cdot10^{-2}$	$\sim 1\cdot10^{-2}$	$0.04\ldots1.4$
$\frac{\mathrm{Br}(\tau^-\to e^-e^+e^-)}{\mathrm{Br}(\tau^-\to e^-\mu^+\mu^-)}$	$0.8\ldots2$	~ 5	$0.3\ldots0.5$	$1.5\ldots2.3$
$\frac{\mathrm{Br}(\tau^-\to \mu^-\mu^+\mu^-)}{\mathrm{Br}(\tau^-\to \mu^-e^+e^-)}$	$0.7\ldots1.6$	~ 0.2	$5\ldots10$	$1.4\ldots1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti}\to e\mathrm{Ti})}{\mathrm{Br}(\mu\to e\gamma)}$	$10^{-3}\ldots10^{2}$	$\sim 5\cdot10^{-3}$	$0.08\ldots0.15$	$10^{-12}\ldots26$

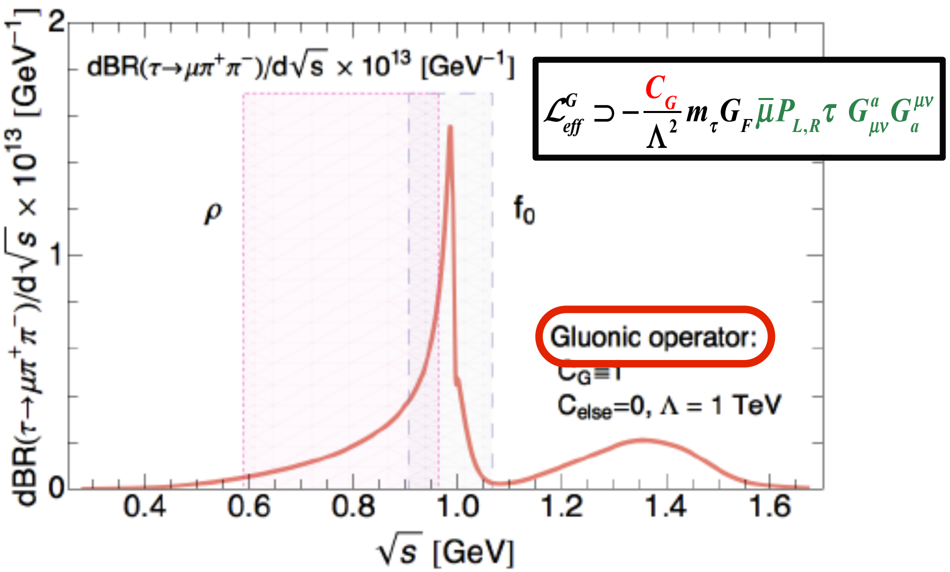
➡ Disentangle the *underlying dynamics* of NP

3.7 Model discriminating of Spectra: $\tau \to \mu\pi\pi$

Celis, Cirigliano, E.P.'14







Very different distributions according to the *final hadronic state!*

NB: See also Dalitz plot analyses for $\tau \to \mu\mu\mu$

Dassinger et al.'07