The muon g-2 recent progress

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New Vistas in Low-Energy Precision Physics
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Theory of the g-2: the beginning

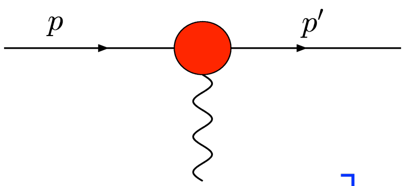
- **Kusch and Foley 1948:**

$$\mu_e^{\exp} = \frac{e\hbar}{2mc}\,(1.00119 \pm 0.00005)$$

- **Schwinger 1948 (triumph of QED!):**

$$\mu_e^{\rm th} = \frac{e\hbar}{2mc}\left(1+\frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- **Keep studying the lepton–γ vertex:**

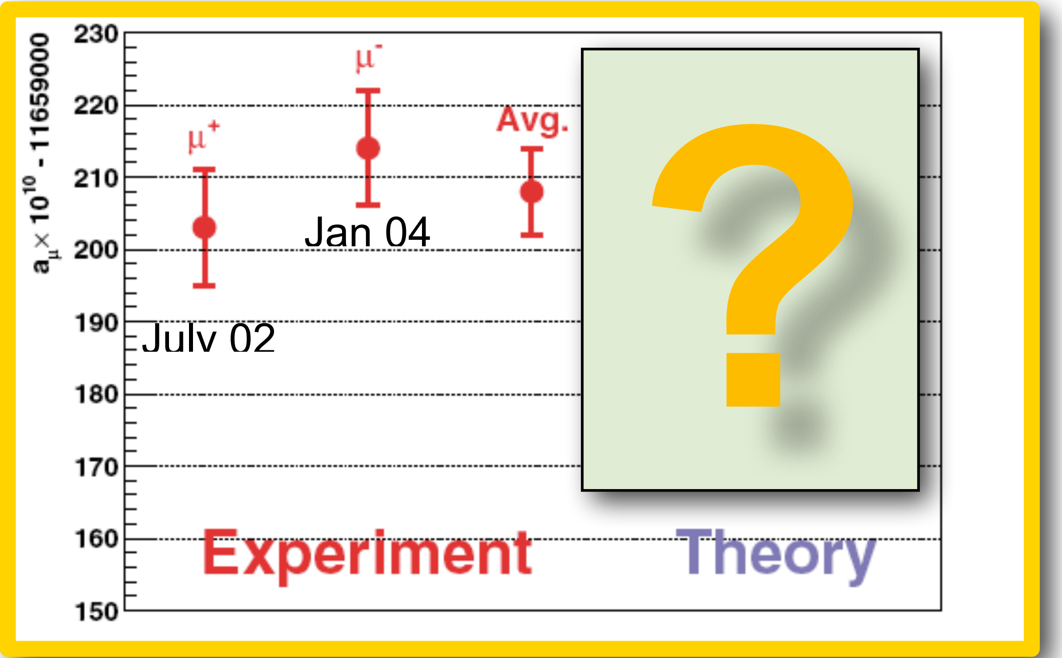


$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$F_1(0) = 1 \qquad F_2(0) = a_l$$

A pure "quantum correction" effect!

The muon g-2: experimental status



- **Today:** $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- **Future:** new muon g-2 experiments at:
  - **Fermilab E989:** aiming at $\pm 16 \times 10^{-11}$, ie 0.14ppm. Beam expected next year. First result expected in 2018 with a precision comparable to that of BNL E821.
  - **J-PARC proposal:** aiming at 2019 Phase 1 start with 0.4ppm.
- **Are theorists ready for this (amazing) precision?** Not yet

The muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+ \; 130.8773\,(61)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Lee, Marquard, Smirnov^2, Steinhauser 2013 (electron loops, analytic),
Kurz, Liu, Marquard, Steinhauser 2013 (τ loops, analytic);
Steinhauser et al. 2015 & 2016 (all electron & τ loops, analytic).

$+ \; 752.85\,(93)\,(\alpha/\pi)^5$ **COMPLETED!**

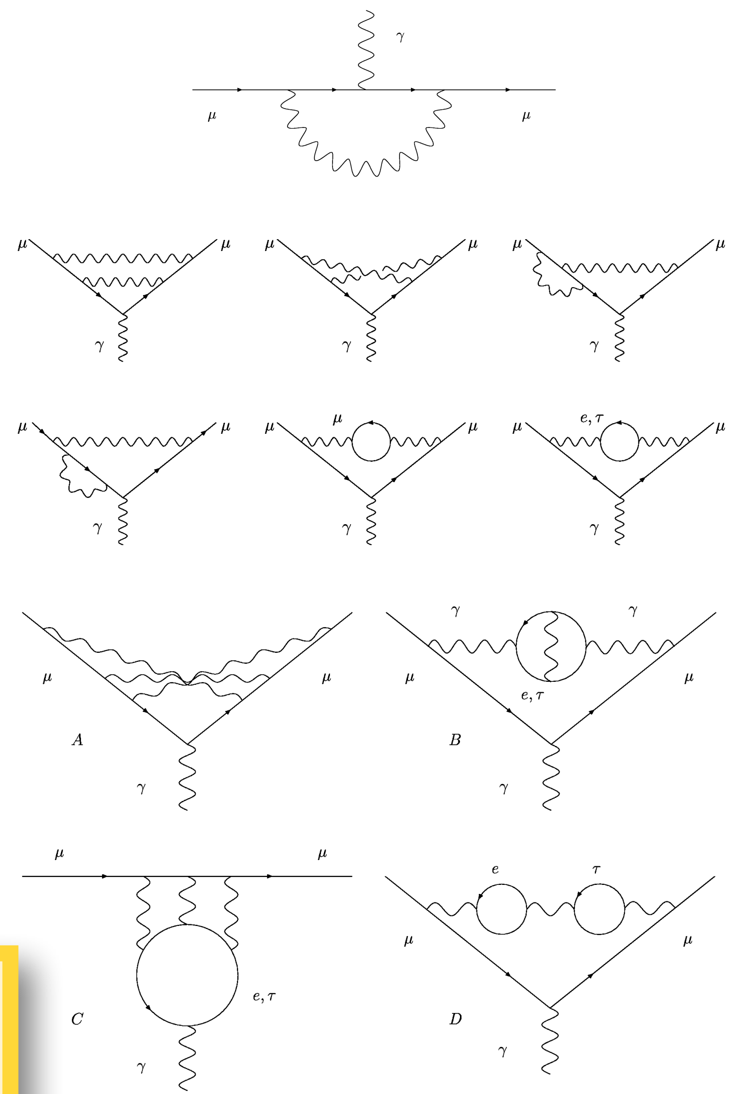
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim,…, Kataev, Kinoshita & Nio '06; Kinoshita et al. 2012 & 2015

Adding up, we get:

$a_\mu^{QED} = 116584718.941\,(21)(77) \times 10^{-11}$

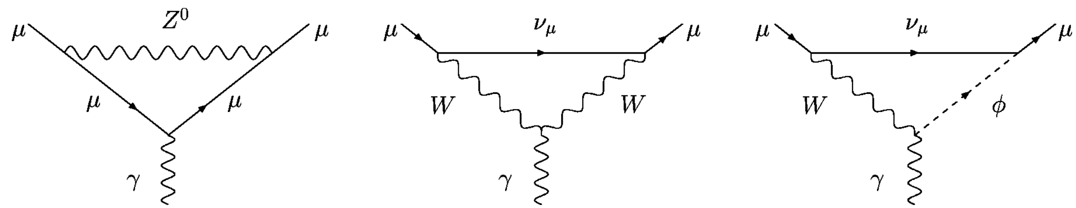
from coeffs, mainly from 4-loop unc ↲ ↳ from $\delta\alpha$(Rb)

with $\alpha = 1/137.035999049(90)$ [0.66 ppb]



The muon g-2: the electroweak contribution

- **One-loop term:**



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

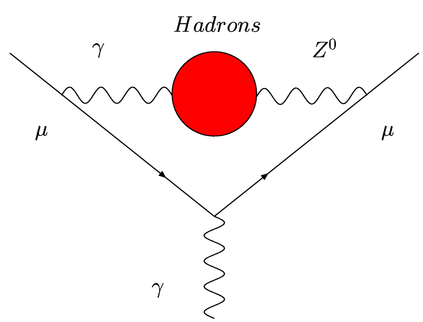
- **One-loop plus higher-order terms:**

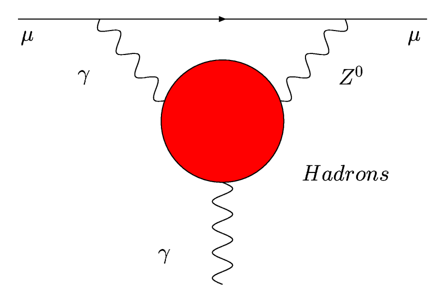
$a_\mu^{\rm EW} = 153.6\ (1) \times 10^{-11}$

with $M_{\rm Higgs}$ = 125.6 (1.5) GeV

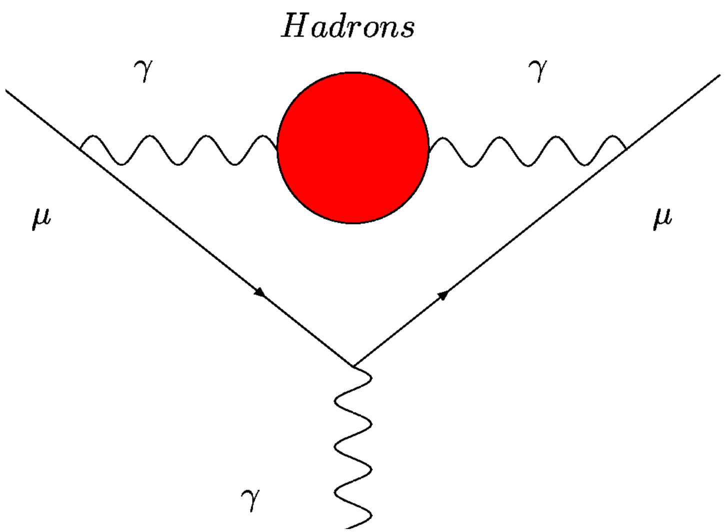
Hadronic loop uncertainties and 3-loop nonleading logs.

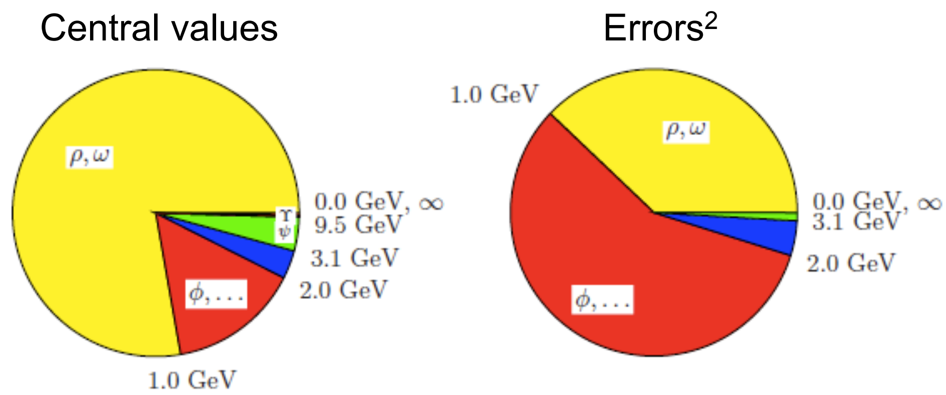
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.





The muon g-2: the hadronic LO contribution (HLO)





F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

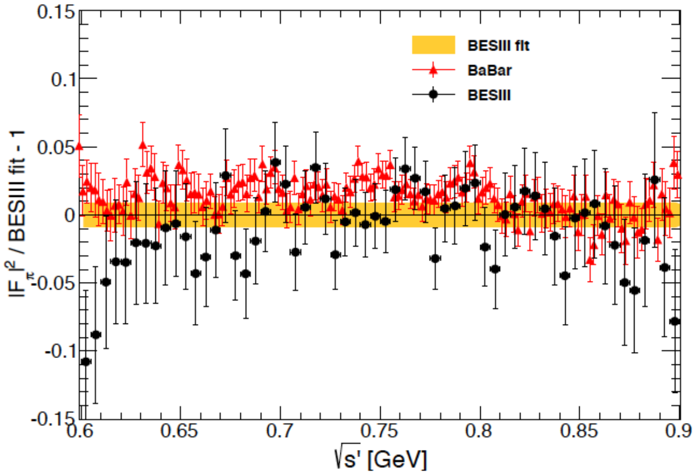
$a_\mu^{\mathrm{HLO}} = 6870\ (42)_{\mathrm{tot}} \times 10^{-11}$ F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2π)

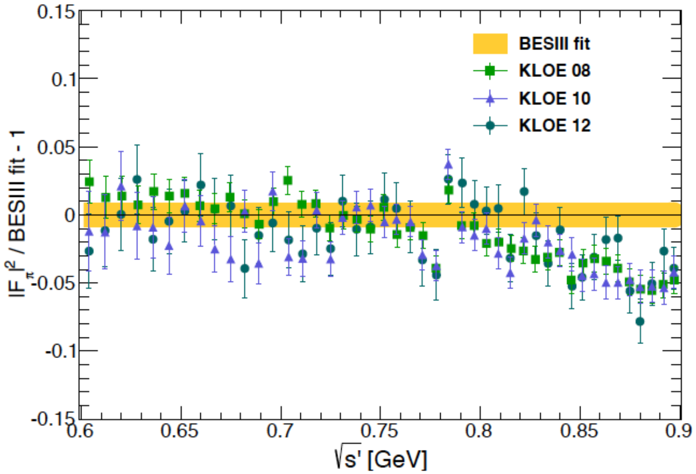
$= 6923\ (42)_{\mathrm{tot}} \times 10^{-11}$ Davier et al, EPJ C71 (2011) 1515

$= 6949\ (37)_{\mathrm{exp}}\ (21)_{\mathrm{rad}} \times 10^{-11}$ Hagiwara et al, JPG 38 (2011) 085003

Radiative Corrections are crucial! S. Actis et al, Eur. Phys. J. C66 (2010) 585

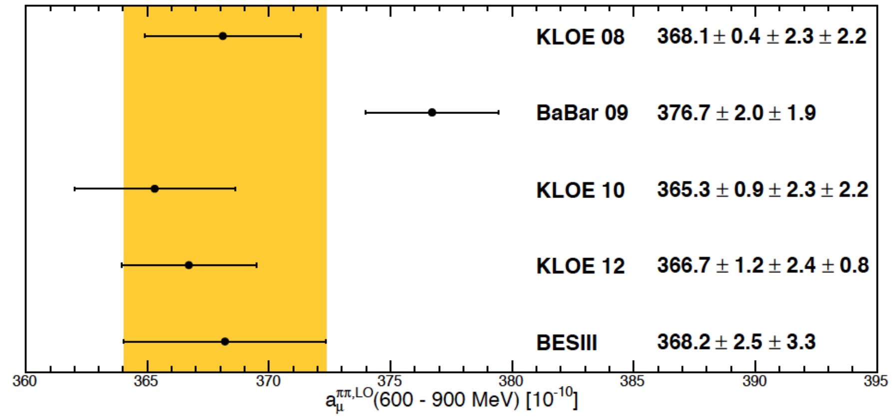
New from BESIII: measurement of the $e^+e^- \to \pi^+ \pi^-$ cross section between 600 & 900 MeV using initial state radiation





BESIII Collaboration, arXiv:1507.08188 (PLB 2016)

See talk of Yuping GUO



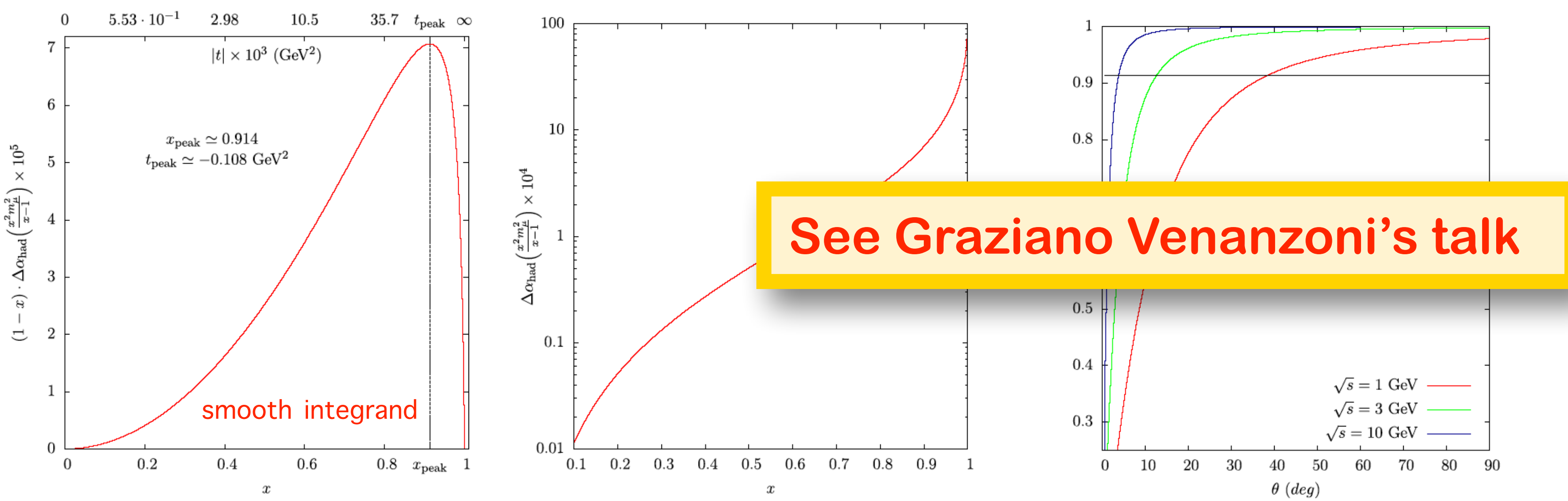
Upcoming $e^+e^- \to \pi^+ \pi^-$ cross section data from VEPP 2000

New independent space-like approach for HLO

- **Alternatively, exchanging the x and s integrations in a_μ^{HLO},**

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)] \qquad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

involving the hadronic contrib. to the running of α in the space-like region, which can be extracted from Bhabha scattering data!



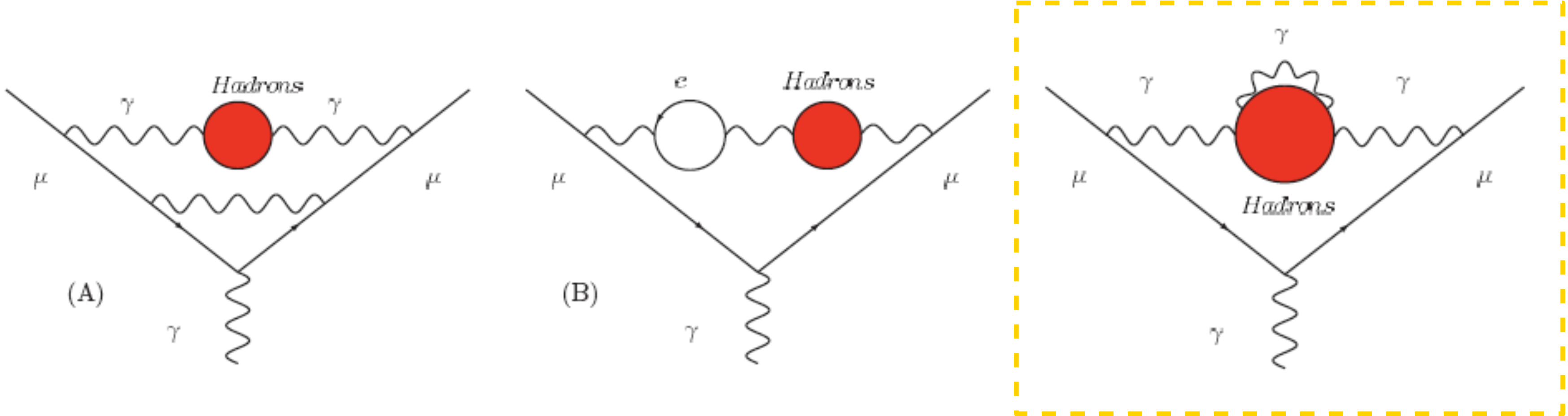
See Graziano Venanzoni's talk

- **Requires Bhabha cross section at small angles at better than 10^{-4}. Challenging: must improve by at least 1 order of magnitude.**
- **A dedicated feasibility study is in progress.**

Carloni Calame, MP, Trentadue, Venanzoni, PLB 746 (2015)

- # HNLO: Vacuum Polarization

Already included in a_μ^{HLO}

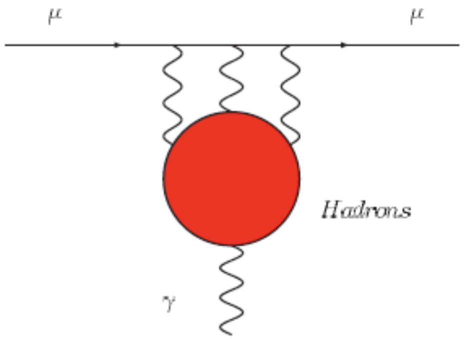


O(α^3) contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{HNLO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

HNLO: Light-by-light contribution



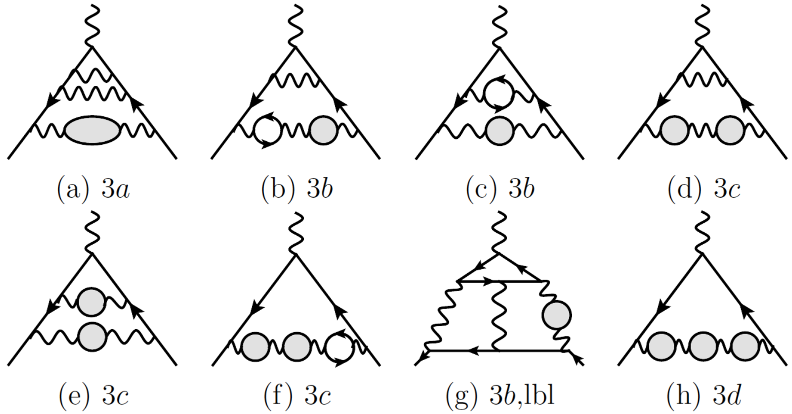
- Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.
- This term had a troubled life! Latest values:

$a_\mu^{HNLO}(lbl) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{HNLO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{HNLO}(lbl) = +105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_\mu^{HNLO}(lbl) = +102\ (39) \times 10^{-11}$	Jegerlehner, arXiv:1511.04473

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- Improvements expected in the π^0 transition form factor A. Nyffeler 1602.0339
- Dispersive approach proposed Colangelo, Hoferichter, Procura, Stoffer, 2014 & 2015 Pauk and Vanderhaeghen 2014.
- Lattice? Very hard but promising Tom Blum et al. 2015

- # HNNLO: Vacuum Polarization



$O(\alpha^4)$ contributions of diagrams containing hadronic vacuum polarization insertions:

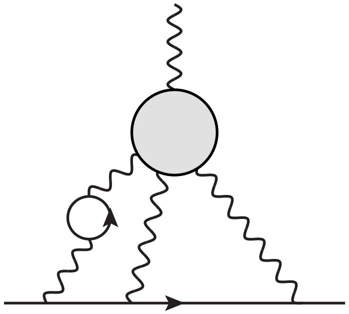
$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

- # HNNLO: Light-by-light

$$a_\mu^{HNNLO}(lbl) = 3\ (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



Comparisons of the SM predictions with the measured g-2 value:

$a_\mu^{EXP} = 116592091\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda=\mu_\mu/\mu_p$ from CODATA'10

$a_\mu^{SM} \times 10^{11}$	$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM}$	σ
116 591 795 (56)	$296\ (86) \times 10^{-11}$	3.5 [1]
116 591 815 (57)	$276\ (85) \times 10^{-11}$	3.2 [2]
116 591 841 (58)	$250\ (86) \times 10^{-11}$	2.9 [3]

with the very recent "conservative" hadronic light-by-light $a_\mu^{HNLO}(lbl) = 102\ (39) \times 10^{-11}$ of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

[1] Jegerlehner, arXiv:1511.04473 (includes BaBar, KLOE10-12 & BESIII 2π)
[2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10 2π)
[3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10 2π)

- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

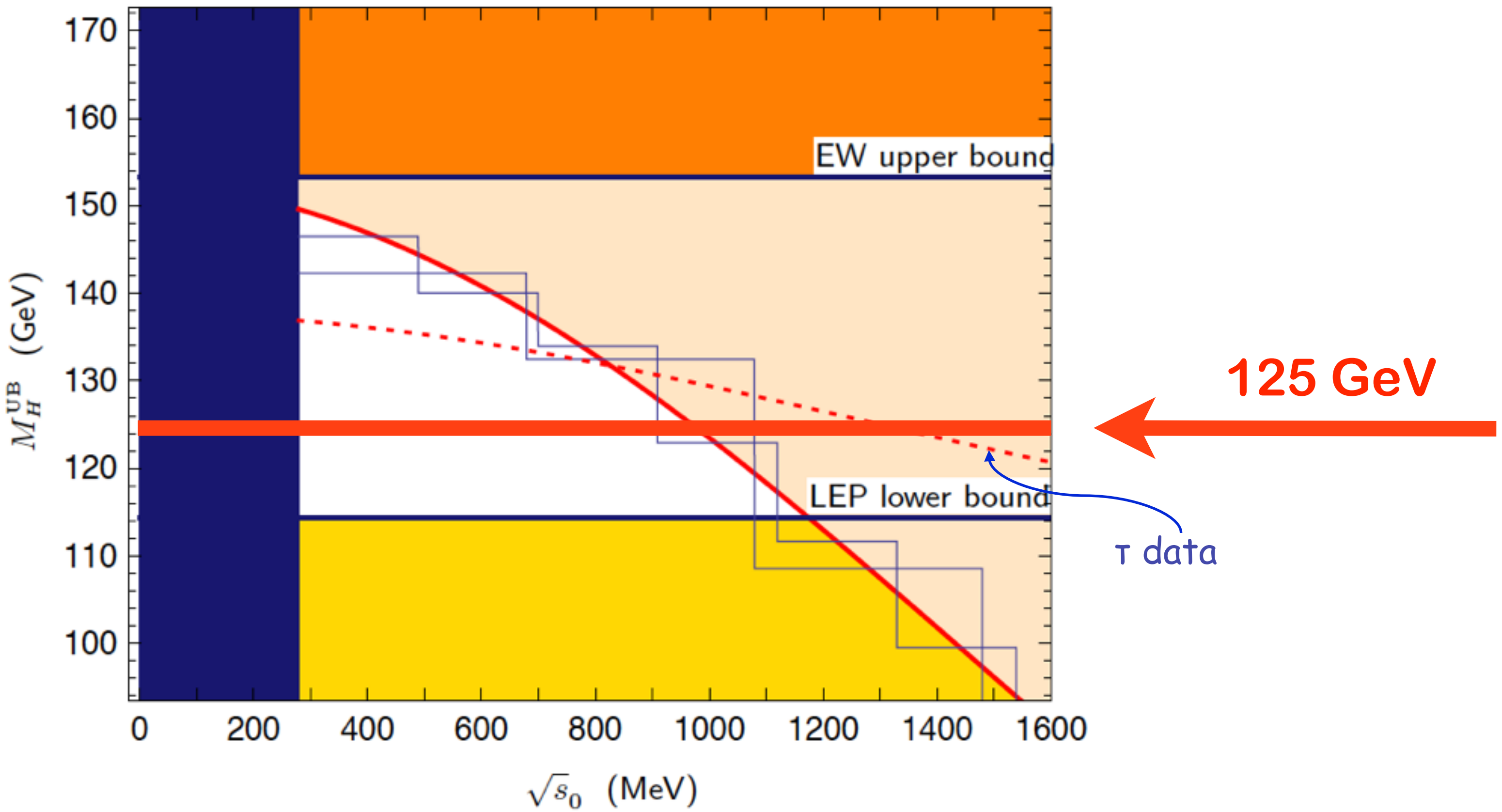
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

(ϵ>0), in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \longrightarrow$$

- **How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?**



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (2)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

- Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.

- Vice versa, assuming we now have a SM Higgs with M_H = 125 GeV, if we bridge the M_H discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The tau g-2: opportunities or fantasies?

The SM prediction of the tau g-2

- The Standard Model prediction of the tau g-2 is:

a_τ^{SM} =	117324 (2)	x 10^{-8}	QED
+	47.4 (0.5)	x 10^{-8}	EW
+	337.5 (3.7)	x 10^{-8}	HLO
+	7.6 (0.2)	x 10^{-8}	HHO (vac)
+	5 (3)	x 10^{-8}	HHO (lbl)

$$a_\tau^{SM} = 117721\ (5) \times 10^{-8}$$

Eidelman & MP 2007

- $(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics, and a "clean" NP test too…

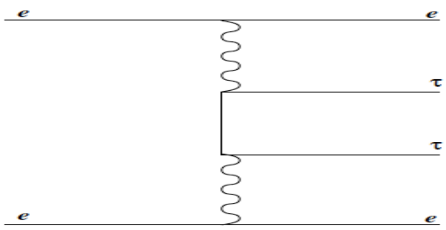
	Muon	Tau
aEW/aH	1/45	1/7
aEW/ δaH	3	10

… if only we could measure it!!

The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine a_τ measuring its spin precession in a magnetic field.

- DELPHI's result, from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):



$$a_\tau = -0.018\ (17)$$

PDG 2014

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

$$-0.007 < a_\tau^{NP} < 0.005 \quad (95\%\ CL)$$

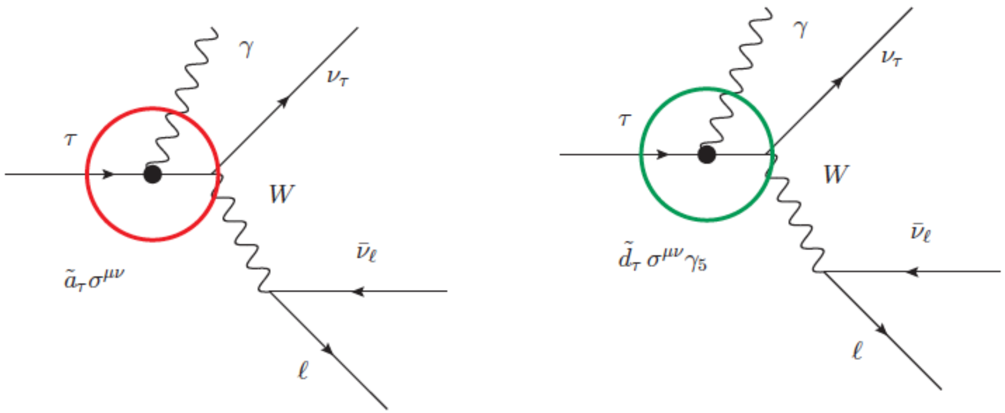
Gonzáles-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of $F_2(q^2{=}M_\Upsilon^2)$ from $e^+e^- \to \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160

A new proposal: the τ g-2 via τ radiative leptonic decays

- **a_τ via the radiative leptonic decays** $\tau \to e\bar{\nu}\nu\gamma, \ \tau \to \mu\bar{\nu}\nu\gamma$ **comparing the theoretical prediction for the differential decay rates with precise data from high-luminosity B factories:**

$$d\Gamma = d\Gamma_0 + \left(\frac{m_\tau}{M_W}\right)^2 d\Gamma_W + \frac{\alpha}{\pi} d\Gamma_{\rm NLO} + \tilde{a}_\tau \, d\Gamma_{\rm a} + \tilde{d}_\tau \, d\Gamma_{\rm d}$$



- **Detailed feasibility study performed in Belle-II conditions: we expect a (modest) improvement of the present PDG bound.**

Eidelman, Epifanov, Fael, Mercolli, MP, arXiv:1601.07987 (JHEP 2016)

Radiative leptonic tau decays: branching ratios

B.R. of radiative τ leptonic decays ($\omega_0 = 10$ MeV)		
	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm LO}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$-1.06\,(1)_n(10)_N \times 10^{-3}$	$-5.8\,(1)_n(2)_N \times 10^{-5}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$-1.89\,(1)_n(19)_N \times 10^{-3}$	$-9.1\,(1)_n(3)_N \times 10^{-5}$
$\mathcal{B}^{\rm Inc}$	$1.728\,(10)_{\rm th}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\rm Exc}$	$1.645\,(19)_{\rm th}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\dagger}_{\rm EXP}$	$1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$	$3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$

(n): numerical errors

(N): uncomputed NNLO corr.

$\sim (\alpha/\pi) \ln r \ln(\omega_0/M) \times \mathcal{B}^{\rm Exc/Inc}_{\rm NLO}$

$(\rm th)$: combined $(n) \oplus (N)$

(τ): experimental error of τ

lifetime: $\tau_\tau = 2.903(5) \times 10^{-13}$ s

† BABAR - PRD 91 (2015) 051103

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\Delta^{\rm Exc}$	$2.02\,(57) \times 10^{-3} \to 3.5\sigma$	$1.2\,(1.0) \times 10^{-4} \to 1.1\sigma$

- **Agreement with MEG's recent μ→eννγ measurement [EPJ C76 (2016) 3, 108]**

Fael, Mercolli and MP, 1506.03416 (JHEP 2015)
Fael and MP, 1602.00457

Conclusions

- The muon g-2 discrepancy is $\Delta a_\mu \sim 3.5\ \sigma$. Is it NP? Or an exp issue? New upcoming g-2 experiment: QED and EW ready for the challenge. How about the hadronic contributions?
- Hadronic VP contribution: new BESIII and upcoming Vepp-2000 time-like data. New space-like data proposal.
- Future of hadronic LBL: dispersive approach and lattice?
- Could Δa_μ be due to mistakes in the hadronic $\sigma(s)$? Given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur below ~ 1GeV: very unlikely.
- The tau g-2 is essentially unknown: new proposal to measure it at Belle II via radiative leptonic tau decays. Modest improvement of the present PDG bound expected.
- BaBar's recent precise measurement of $\mathcal{B}(\tau \to e\bar{\nu}\nu\gamma)$ differs from our SM prediction by $3.5\ \sigma$!

The End