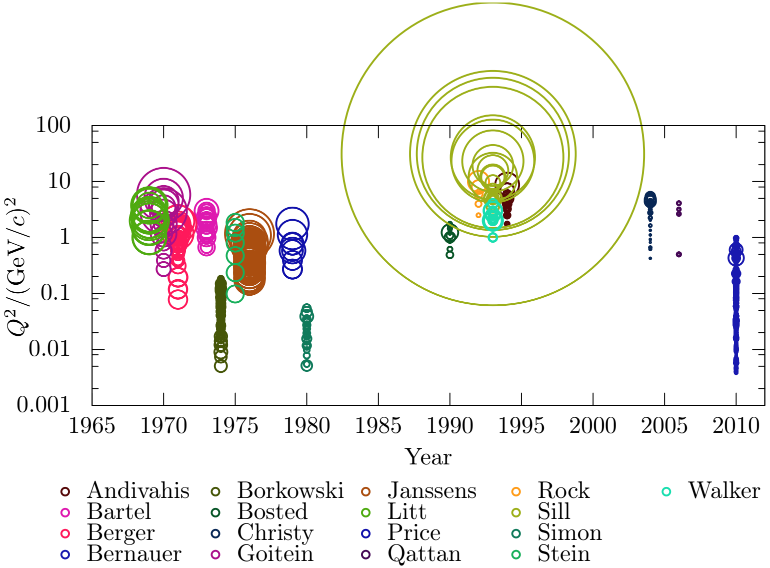
Precision Form Factor Measurements in Electron-Nucleon Scattering: Assorted Topics

Jan C. Bernauer

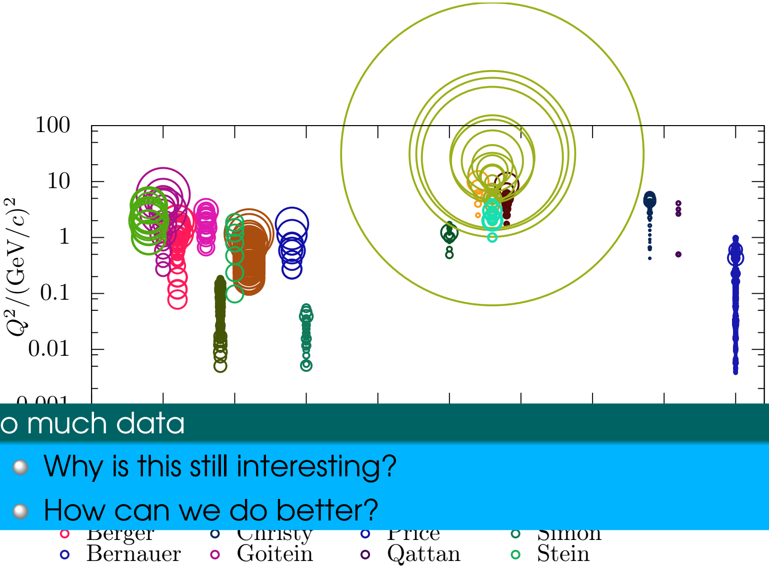
New Vistas in Low-Energy Precision Physics (LEPP) 2016

Massachusetts Institute of Technology

History of unpolarized electron-proton scattering



History of unpolarized electron-proton scattering



So much data

- Why is this still interesting?
- How can we do better?

Cross section and form factors for elastic lepton-proton scattering

The cross section:

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}} = \frac{1}{\varepsilon(1+\tau)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

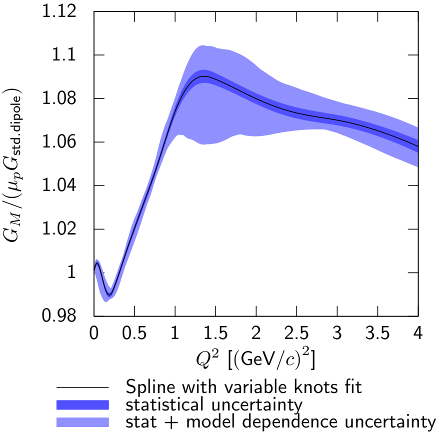
with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

Fourier-transform of $G_E,\ G_M \longrightarrow$ spatial distribution (Breit frame)

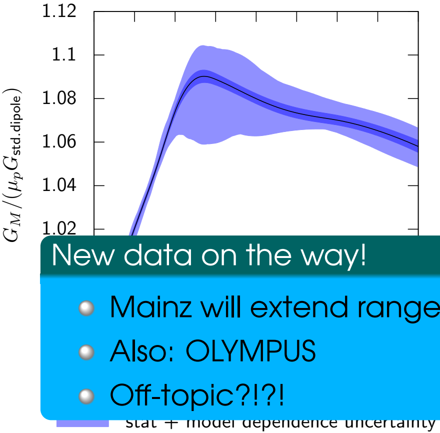
$$\left\langle r_E^2 \right\rangle = -6\hbar^2 \left.\frac{\mathrm{d}G_E}{\mathrm{d}Q^2}\right|_{Q^2=0} \qquad \left\langle r_M^2 \right\rangle = -6\hbar^2 \left.\frac{\mathrm{d}\left(G_M/\mu_p\right)}{\mathrm{d}Q^2}\right|_{Q^2=0}$$

Magnetic form factor: High-Q



- Cusp between 1 and 1.5 (GeV/c)^2
- Seen in older fits
- Mainz data $<$ 1 (GeV/c)^2
- Should be visible in Lattice QCD

Magnetic form factor: High-Q

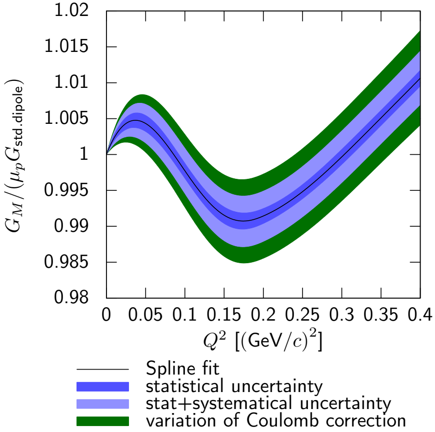


- Cusp between 1 and 1.5 $(\text{GeV}/c)^2$
- Seen in older fits
- Mainz data < 1 $(\text{GeV}/c)^2$

New data on the way!

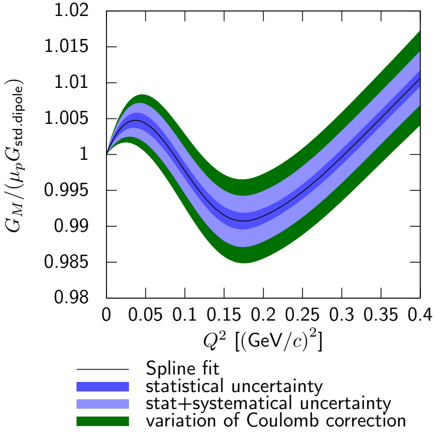
- Mainz will extend range to about 2 $(\text{GeV}/c)^2$
- Also: OLYMPUS
- Off-topic?!?!

Magnetic form factor: Low-Q



- Up-Down-Up structure
- Not seen in older fits
  - They approach from below
  - Lack of data
- Gives rise to small r_m
- Sensitive to radiative corrections

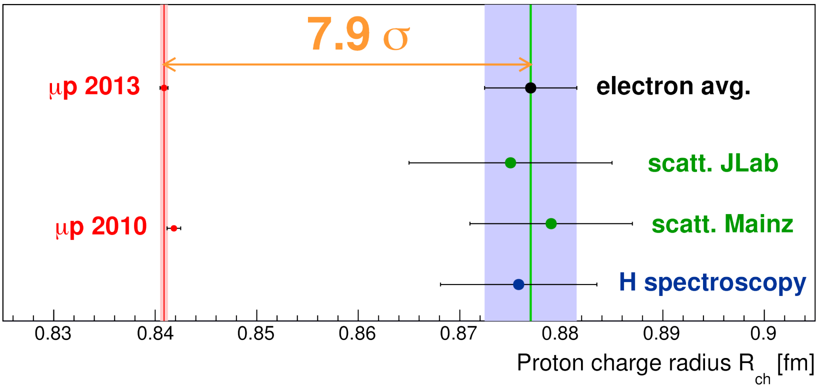
Magnetic form factor: Low-Q



- Up-Down-Up structure
- Not seen in older fits
  - They approach from below
  - Lack of data
- Gives rise to small r_m
- Sensitive to radiative corrections

- Measure backward angles so $\tau > \varepsilon$
- Lowest possible beam energy!
- Measurements like this are planned at MAGIX

Proton electric radius



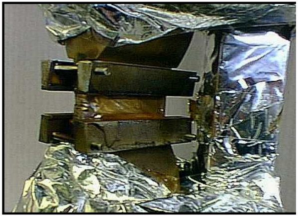
$$\left\langle r_E^2 \right\rangle = -6\hbar^2 \left. \frac{\mathrm{d}G_E}{\mathrm{d}Q^2} \right|_{Q^2=0} \qquad \left\langle r_M^2 \right\rangle = -6\hbar^2 \left. \frac{\mathrm{d}\left(G_M/\mu_p\right)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

Current state

- Current best measurement:
  systematic errors dominate

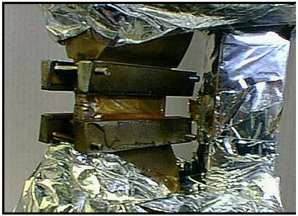
Current state

- Current best measurement: systematic errors dominate
  - Background from target walls
  - Acceptance correction for extended target



Current state $\Longrightarrow$ Next gen.

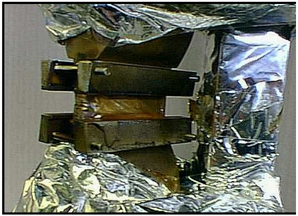
- Current best measurement: systematic errors dominate
  - Background from target walls
  - Acceptance correction for extended target
- Eliminate with jet target
  - point-like
  - no walls



$\Downarrow$

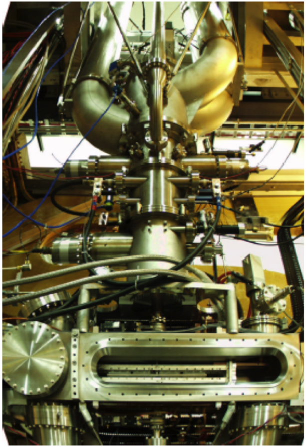
Current state $\Longrightarrow$ Next gen.

- Current best measurement: systematic errors dominate
  - Background from target walls
  - Acceptance correction for extended target
- Eliminate with jet target
  - point-like
  - no walls
- Rinse, repeat with D,^{3}He,^{4}He, ...



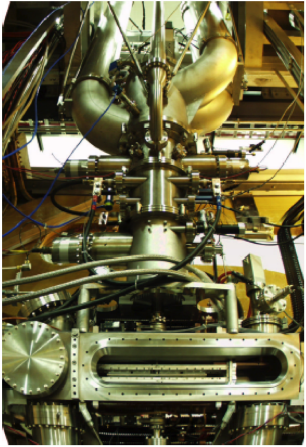
$\Downarrow$

"Complete" form factor experiment



- Replace target with polarized hydrogen gas (jet or storage cell)
- Polarized beam
- Measure G_E, G_M via Rosenbluth and G_E/G_M via polarization

"Complete" form factor experiment

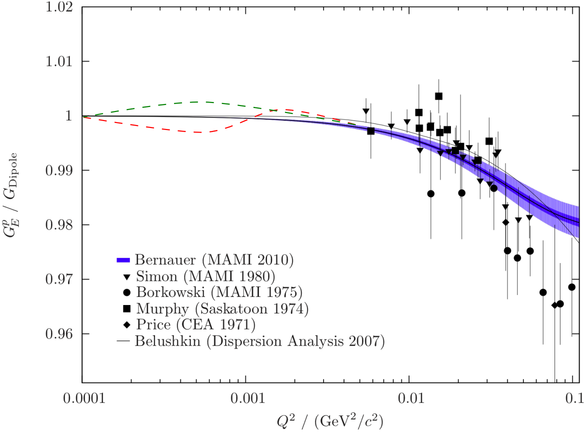


- Replace target with polarized hydrogen gas (jet or storage cell)
- Polarized beam
- Measure G_E, G_M via Rosenbluth and G_E/G_M via polarization

Impact

G_E/G_M: Rosenbluth $\Longleftrightarrow$ polarization
$\Longrightarrow$ Test of radiative corrections

Extrapolation problematic? Structures at low Q^2?



Smaller is better

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon\left(1+\tau\right)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

$$\tau = \frac{Q^2}{4m_p^2}, \quad Q^2 = 4EE' \sin^2\frac{\theta}{2}$$

- For smaller Q^2 :
  - smaller θ (PRad)
  - smaller E (Rosenbluth / ISR)

Smaller is better

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon\left(1+\tau\right)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

$$\tau = \frac{Q^2}{4m_p^2}, \quad Q^2 = 4EE'\sin^2\frac{\theta}{2}$$

- For smaller Q^2 :
  - smaller θ (PRad)
  - smaller E (Rosenbluth / ISR)
- At small Q^2, small $\theta \Longrightarrow \varepsilon >> \tau$, probably OK to use model for G_M

Some further thoughts

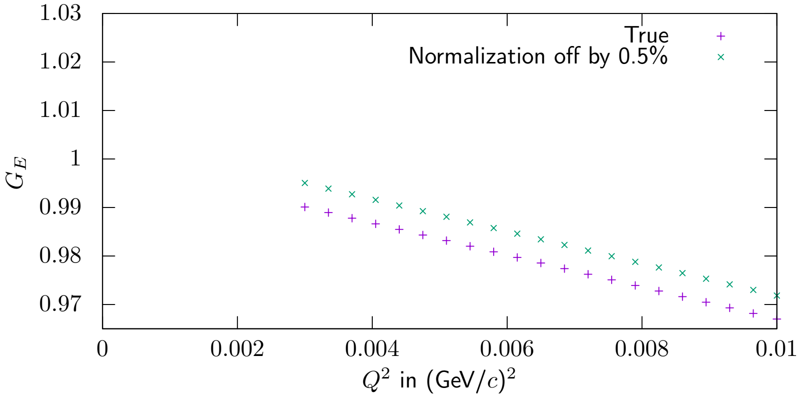
A.K.A.: The rant section.

Does good absolute normalization help?

- Assume: Form factor has this form:
  $G_E(Q^2) = 1 + a \cdot Q^2 + b \cdot Q^4$ (Q^2 in GeV^2)
- (For Mainz fit: $a \approx -3.3$, $b \approx 13$, for $\mu P : a \approx -3.08$)
- Here: $b = 0$

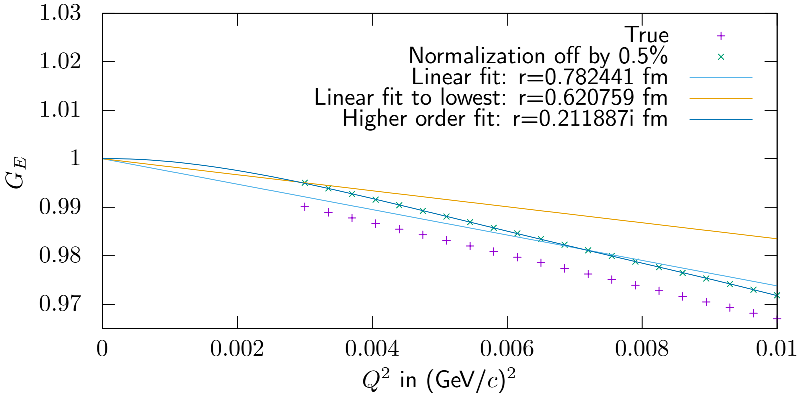
Does good absolute normalization help?

- Assume: Form factor has this form:
  $G_E(Q^2) = 1 + a \cdot Q^2 + b \cdot Q^4$ (Q^2 in GeV^2)
- (For Mainz fit: $a \approx -3.3$, $b \approx 13$, for μP : $a \approx -3.08$)
- Here: $b = 0$



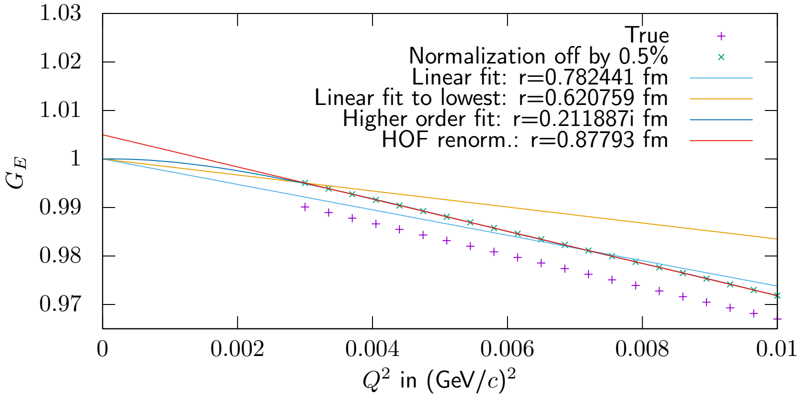
Does good absolute normalization help?

- Assume: Form factor has this form: $G_E(Q^2) = 1 + a \cdot Q^2 + b \cdot Q^4$ (Q^2 in GeV^2)
- (For Mainz fit: $a \approx -3.3$, $b \approx 13$, for $\mu P : a \approx -3.08$)
- Here: $b = 0$



Does good absolute normalization help?

- Assume: Form factor has this form: $G_E(Q^2) = 1 + a \cdot Q^2 + b \cdot Q^4$ (Q^2 in GeV^2)
- (For Mainz fit: $a \approx -3.3$, $b \approx 13$, for μP : $a \approx -3.08$)
- Here: $b = 0$



Does good absolute normalization help?

- Assume: Form factor has this form: $G_E(Q^2) = 1 + a \cdot Q^2 + b \cdot Q^4$ (Q^2 in GeV^2)
- (For Mainz fit: $a \approx -3.3$, $b \approx 13$, for μP : $a \approx -3.08$)
- Here: $b = 0$

Normalization does not help (much)

- Full finite-size effect $\approx 3\%$ at 0.01 in G_E, 6% in cross section.
- Effect of radius difference $\ll$
- If you believe extrapolation, renormalization is probably better:
  - Difference between b=0 and b=13 only 0.3% at 0.01. Quadratic in Q^2!
- If you don't, can't say anything.

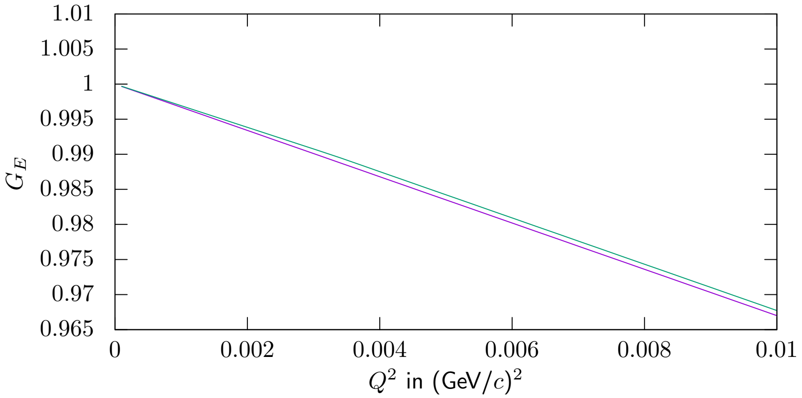
Q^2 in (GeV/c)^2

Will we (be able to) see structures?

- Simplest model: Kink, i.e. linear-kink-linear
- Assume 30 data points between 0.0001 and 0.01 (GeV/c)^2, 0.05% precision
- Kink just below the available data

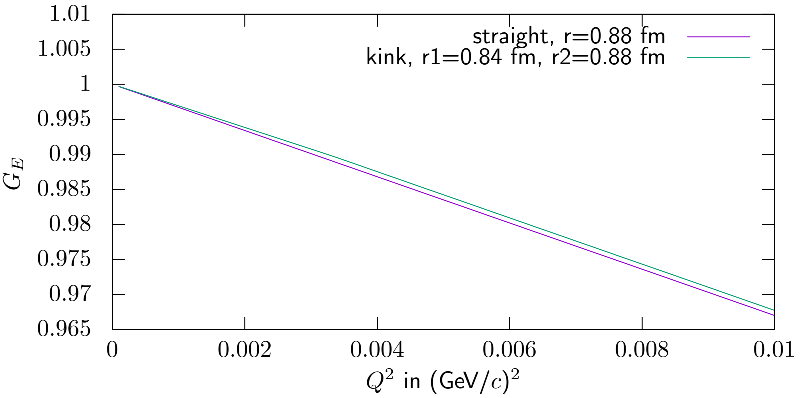
Will we (be able to) see structures?

- Simplest model: Kink, i.e. linear-kink-linear
- Assume 30 data points between 0.0001 and 0.01 (GeV/c)^2, 0.05% precision
- Kink just below the available data



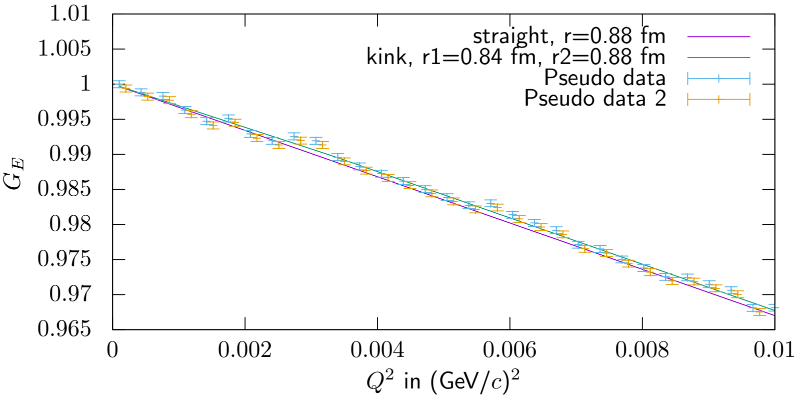
Will we (be able to) see structures?

- Simplest model: Kink, i.e. linear-kink-linear
- Assume 30 data points between 0.0001 and 0.01 (GeV/c)^2, 0.05% precision
- Kink just below the available data



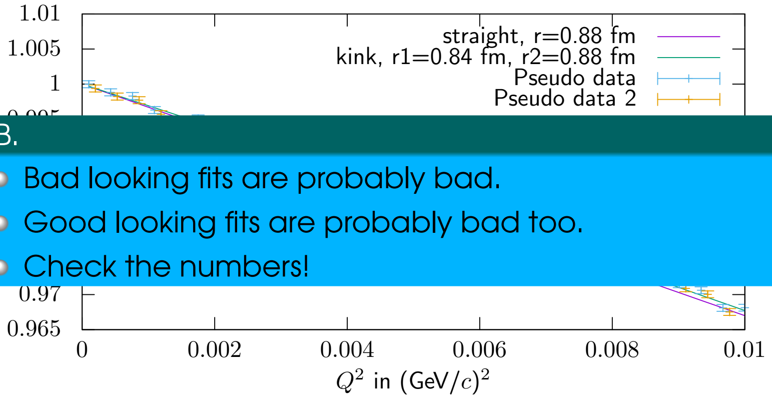
Will we (be able to) see structures?

- Simplest model: Kink, i.e. linear-kink-linear
- Assume 30 data points between 0.0001 and 0.01 (GeV/c)^2, 0.05% precision
- Kink just below the available data



Will we (be able to) see structures?

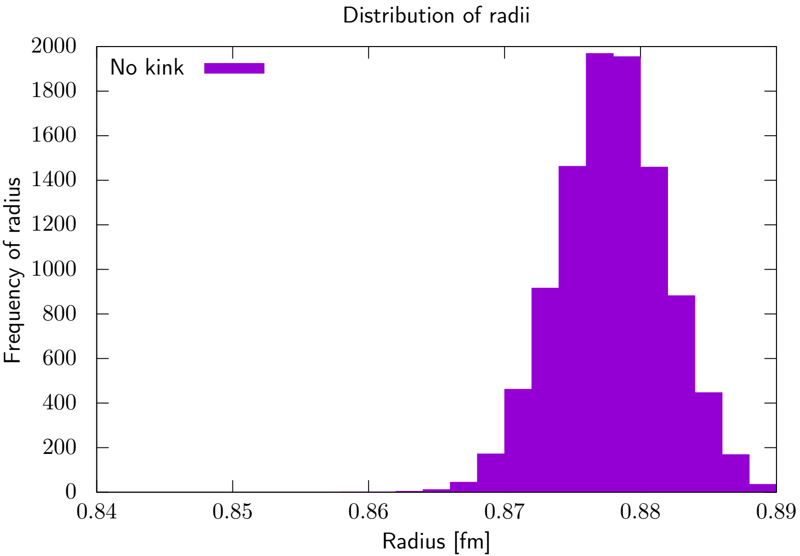
- Simplest model: Kink, i.e. linear-kink-linear
- Assume 30 data points between 0.0001 and 0.01 (GeV/c)^2, 0.05% precision
- Kink just below the available data



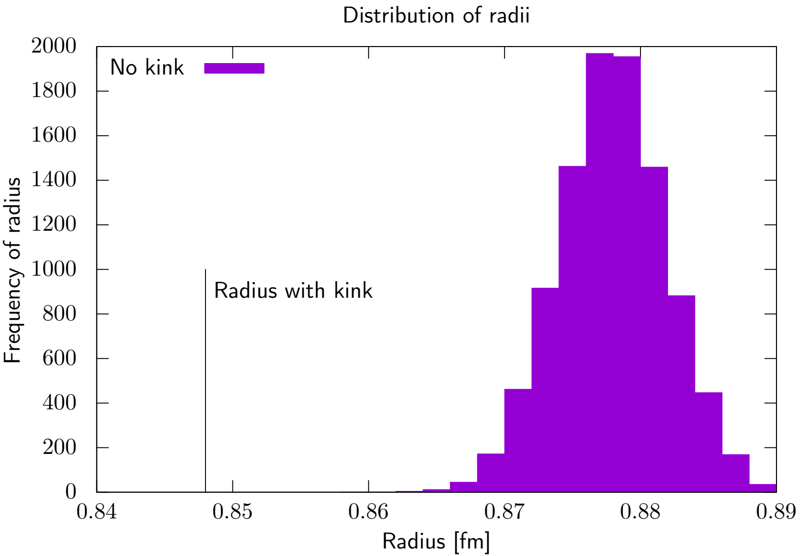
N.B.

- Bad looking fits are probably bad.
- Good looking fits are probably bad too.
- Check the numbers!

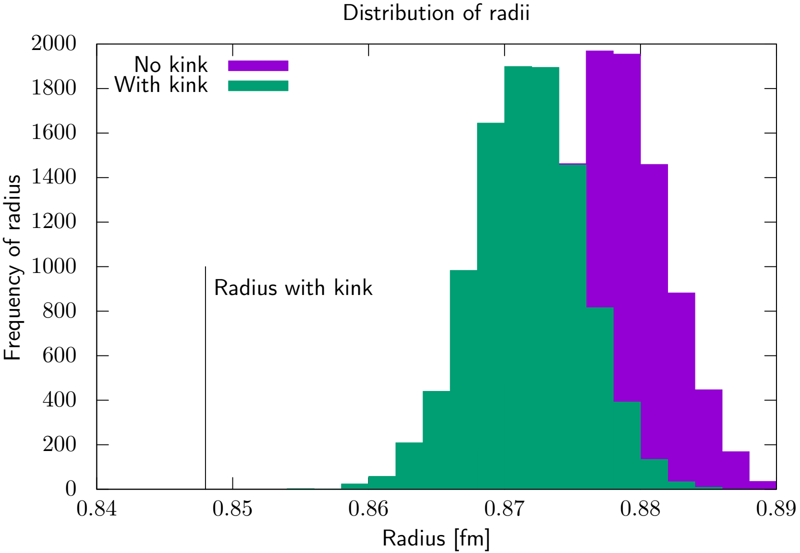
Structures, in numbers

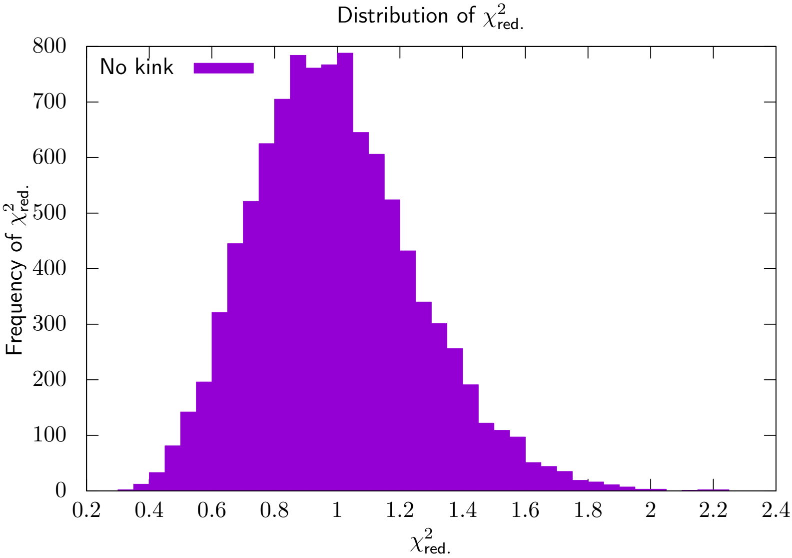


Structures, in numbers

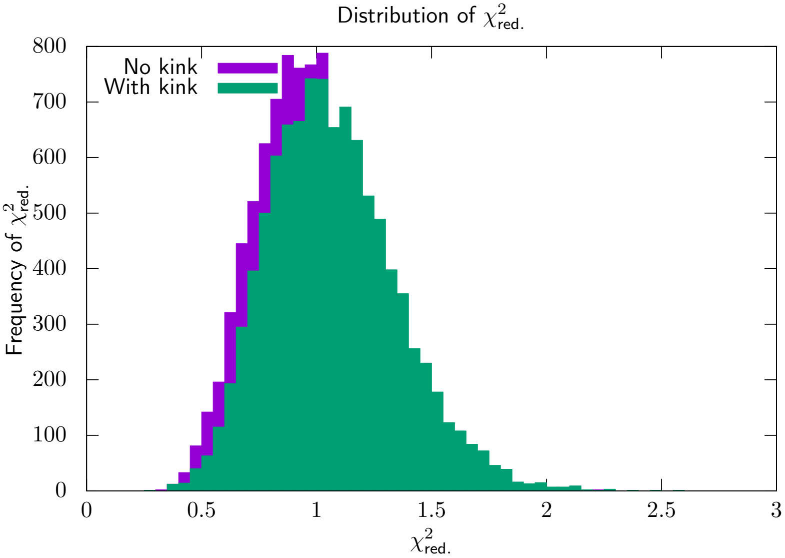


Structures, in numbers

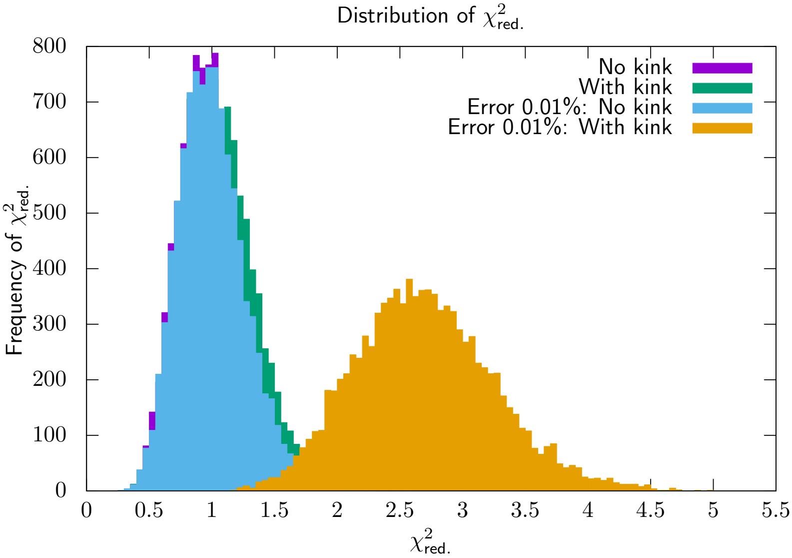



Distribution of $\chi^2_{\text{red.}}$
No kink
Frequency of $\chi^2_{\text{red.}}$
800
700
600
500
400
300
200
100
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
$\chi^2_{\text{red.}}$

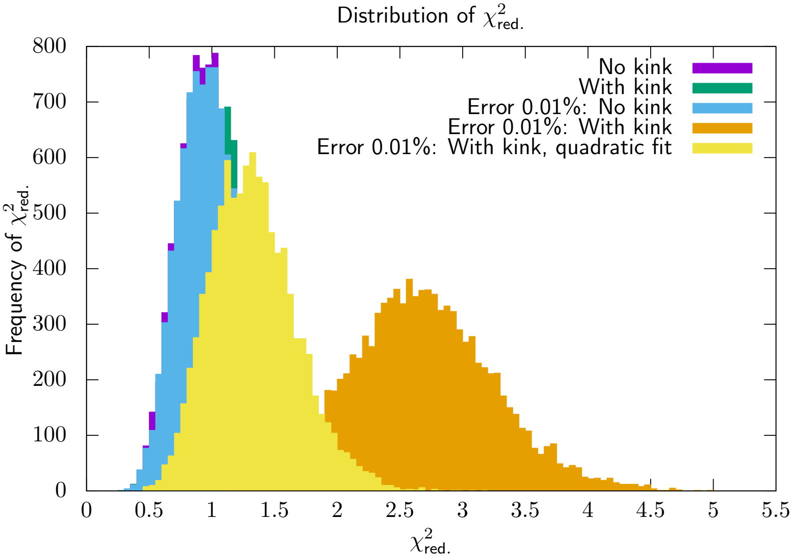
Structures, in numbers



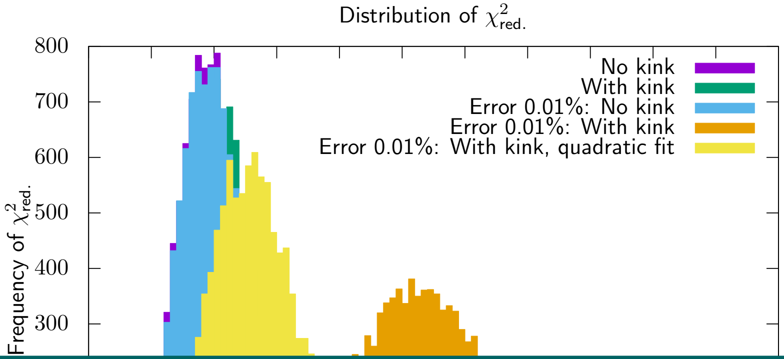
Structures, in numbers



Structures, in numbers



Structures, in numbers



PV like precision may be needed

- Do we actually control radiative corrections on the 10^{-4} level?
- Detector efficiency?

Polynomial fits and Taylor expansions

$$poly(x, \vec{p}) = p_0 + p_1 \cdot x + p_2 \cdot x^2 + ...$$

- This sure looks like a Taylor expansion around zero!

Polynomial fits and Taylor expansions

$$poly(x, \vec{p}) = p_0 + p_1 \cdot x + p_2 \cdot x^2 + ...$$

- This sure looks like a Taylor expansion around zero!
- But:

$$\begin{aligned} b_0 + b_1 \cdot (x - x_0) + b_2(x - x_0)^2 + ... = & \; b_0 + b_1 x_0 + b_2 x_0^2 + ... \\ & + (b_1 - 2b_2 x_0 + ...) \cdot x + \\ & ... \end{aligned}$$

- I.e., every (truncated) Taylor expansion can be transformed to look like a Taylor expansion around 0.

Polynomial fits and Taylor expansions

- Given zero errors, does the fit give a Taylor expansion at all?

Polynomial fits and Taylor expansions

- Given zero errors, does the fit give a Taylor expansion at all?
- In general, no!
- The fit has a different convergence behavior.
  - No convergence radius!

Polynomial fits and Taylor expansions

- Given zero errors, does the fit give a Taylor expansion at all?
- In general, no!
- The fit has a different convergence behavior.
  - No convergence radius!
- Need Weierstrass approximation theorem
  - For the derivative!
  - Can we optimize fitting for this?

Linear fits?

- How bad is it to neglect higher terms?
- Fitting $a \cdot x + o$ to $b_{\text{real}} \cdot x^2 + a_{\text{real}} \cdot x + o_{\text{real}}$.

Linear fits?

- How bad is it to neglect higher terms?
- Fitting $a \cdot x + o$ to $b_{\mathrm{real}} \cdot x^2 + a_{\mathrm{real}} \cdot x + o_{\mathrm{real}}$.
- "Taylor around average Q^2". i.e. $a = a_{\mathrm{real}} + 2b_{\mathrm{real}} Q^2_{\mathrm{avg}}$
- Error in normalization only $b_{\mathrm{real}} \left(Q^2_{\mathrm{avg.}}\right)^2$
- If FF fits are true, $\Delta a = -26\, c^2/\mathrm{GeV}^2$.
- So for 0.01 $(\mathrm{GeV}/c)^2$, $\approx 0.88 \longrightarrow 0.84$

Thank you!

Repeat after me: χ^2 tests data, not models. It only has a meaning if the model is assumed correct. A good χ^2 does NOT mean the model is correct.