Tau Decays Measurements

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New Vistas in Low-Energy Precision Physics (LEPP)
4-7 April 2016, Mainz, Germany

Outline

- introduction
- tau mass, tau lifetime, other tau properties
- tau branching fractions and spectral functions
- lepton flavour violation searches
- other measurements
- elaboration of tau results
  - lepton universality
  - $|V_{us}|$

Three phases of tau experimental measurements

Tau discovery phase

- establish evidence for new heavy lepton with:
  $e^+e^- \to \tau^+\tau^-$, $\tau^+ \to e^+\nu_e\bar{\nu}_\tau$, $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$
- MARK I at SPEAR (SLAC), later PLUTO & DASP at DORIS (DESY)

Precision SM tests phase

- lepton universality (with leptonic BFs, tau lifetime, tau mass)
- Z couplings ($\Gamma_{Z\to\tau\tau}$, A^{FB}, A_{pol}, A^{FB}_{pol}, etc.)
- α_s and muon $g{-}2$ hadronic contribution with tau hadronic decays
- $|V_{us}|$ with $\tau \to X_s\nu$ decays
- LEP experiments (∼200k tau pairs each), ARGUS, CLEO (∼14M tau pairs), BES

New Physics search phase

- search for Lepton Flavour Violation (LFV)
- measurement of small BFs whose previous results were statistics-limited
- *B*-factories *BABAR* (∼500M tau pairs), Belle (∼900M tau pairs)

Main differences between recent experiments

around charm-tau threshold (BES, BESIII)

- best for tau mass (beam energy calibration via resonant depolarization)

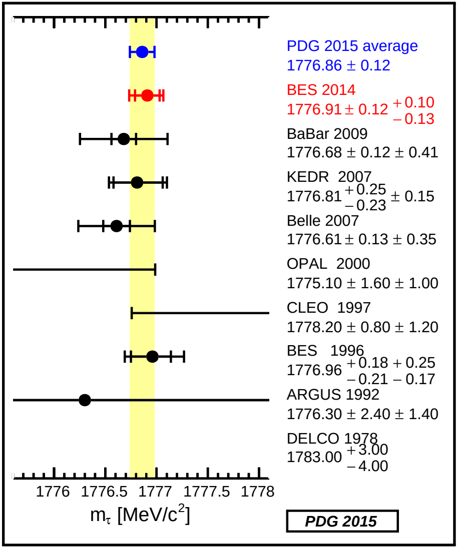
Z^0 peak (LEP 1)

- much smaller samples w.r.t. B-factories
- but several advantages
  - precise absolute luminosity measurements ($\sim$0.5 per-mille)
  - can select tau pairs on just one hemisphere with good efficiency and purity
  - stiff tracks, small amount of multiple scattering
  - large hadron $e^+e^- \to q\bar{q}$ track multiplicity $\Rightarrow$ high rejection of $q\bar{q}$ background
- outstanding analysis contribution by ALEPH

B-factories (CLEO, *BABAR*, Belle)

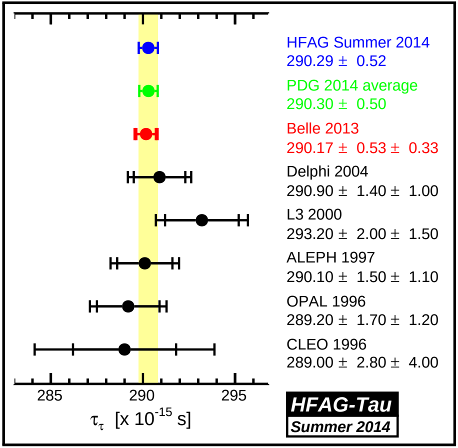
- much larger samples
- cannot select tau pairs on a single hemisphere with decent efficiency and purity
- lowish hadron $e^+e^- \to q\bar{q}$ track multiplicity $\Rightarrow$ difficult rejection of $q\bar{q}$ background
- multiple scattering limits momentum resolution

Tau mass



- most precise measurements by e^+e^- **colliders at** $\tau^+\tau^-$ **threshold**
  - few events but very significant

Tau lifetime



- LEP experiments, many methods
  - impact parameter sum (IPS)
  - momentum dependent impact parameter sum (MIPS
  - 3D impact parameter sum (3DIP)
  - impact parameter difference (IPD)
  - decay length (DL)
- Belle
  - 3-prong vs. 3-prong decay length
  - largest syst. error: alignment

Mass & lifetime difference τ^+ vs. τ^-, dipole moments

$(m_{\tau+} - m_{\tau-})/m_{\text{average}}$ (can signal *CPT* violation)

$< 2.8 \cdot 10^{-4}$	BELOUS 2007	Belle, 414 fb^{-1}
$< 5.5 \cdot 10^{-4}$	AUBERT 2009AK	*BABAR*, 423 fb^{-1}
$< 3.0 \cdot 10^{-3}$	ABBIENDI 2000A	OPAL

$(\tau_{\tau+} - \tau_{\tau-})/\tau_{\text{average}}$ (can signal *CPT* violation)

$< 7.0 \cdot 10^{-3}$	BELOUS 2014	Belle, 711 fb^{-1}

dipole moments (EDM $\neq 0$ can signal *CP*, *T* violation)

$-0.052 < a_\tau < 0.013$ at 95% CL	DELPHI 2004 [$(g-2)_\tau/2$]
$-0.22 < \text{Re}(d_\tau)[10^{-16}\,e\,\text{cm}] < 0.013$ at 95% CL	Belle 2003 [τ EDM]
$-0.25 < \text{Im}(d_\tau)[10^{-16}\,e\,\text{cm}] < 0.008$ at 95% CL	Belle 2003 [τ EDM]
$\text{Re}(a_\tau^W) < 1.1 \cdot 10^{-3}$ at 95% CL	ALEPH 2003 [weak $(g-2)_\tau/2$]
$\text{Im}(a_\tau^W) < 2.7 \cdot 10^{-3}$ at 95% CL	ALEPH 2003 [weak $(g-2)_\tau/2$]
$\text{Re}(d_\tau^W)[10^{-16}\,e\,\text{cm}] < 0.05$ at 95% CL	ALEPH 2003 [weak τ EDM]
$\text{Im}(d_\tau^W)[10^{-16}\,e\,\text{cm}] < 0.11$ at 95% CL	ALEPH 2003 [weak τ EDM]

Branching fractions and spectral functions

Branching fractions

- leptonic BFs $\Rightarrow$ lepton universality tests (SM EW tests)
- leptonic radiative BFs $\Rightarrow$ tau dipole moments (S.Eidelman, M.Passera et.al., arXiv:1601.07987 [hep-ph])

BFs + spectral functions (hadronic invariant mass distributions)

- hadronic final states $\Rightarrow$
  - $\alpha_s(m_\tau)$, running of α_s from m_τ to m_{Z0}
  - alternative way to determine muon $g-2$ hadronic contribution
- "strange" hadronic final states $\Rightarrow$
  - alternative $|V_{us}|$ determination, CKM unitarity test (theory systematics different from lattice QCD systematics on kaon decays)

Branching fraction fit - HFAG 2016 prelim.

- global fit: best way to combine measurements on BFs, BF ratios, inclusive BFs
- since 2010, fit in by-yearly reports by **Heavy Flavour Averaging Group** (**HFAG**)
  - common systematic errors taken into account
  - published results improved using updated values for external parameters
  - no PDG-style automatic error-scaling, exceptions analyzed case-by-case
  - using selection of preliminary results
- less complete and less refined fit in PDG
- **work in progress (A.L.)**: port HFAG fit to **PDG 2016**
  - drop preliminary results
  - investigate all differences in common set of measurements and their relations
- in the following, results labeled **"HFAG 2016 preliminary"**
  (under PDG review for PDG 2016)

Branching fraction fit - HFAG 2016 prelim.

General information

- 171 measurements
  (no new results since HFAG 2014)
- fit 104 quantities
  (BFs or ratios of linear comb. of BFs)
  related by 58 constraints
- $\chi^2/\text{d.o.f.} = 134.9/125, \quad \text{CL} = 25.73\%$
- use unitarity constraint (PDG tradition)
  (in HFAG no unitarity constraint enforced to reduce "pollution" from hadronic to leptonic modes)
- 5.44 error scale factor for inconsistent *BABAR* and Belle $K^-K^-K^+\nu_\tau$
- without unitarity constraint, fitted results sum up to 1 within the statistical uncertainty of ∼1 per mille

Results by experiment

experiment	number of results
ALEPH	40
CLEO	35
BaBar	23
OPAL	19
Belle	15
DELPHI	14
L3	11
CLEO3	6
TPC	3
ARGUS	2
HRS	2
CELLO	1

Unitarity constraint branching fractions - HFAG 2016 prelim.

$B\ (\tau \to \ldots)$	HFAG 2016 prelim
$\mu^- \bar{\nu}_\mu \nu_\tau$	$(17.3951 \pm 0.0385) \cdot 10^{-2}$
$e^- \bar{\nu}_e \nu_\tau$	$(17.8199 \pm 0.0399) \cdot 10^{-2}$
$\pi^- \nu_\tau$	$(10.8194 \pm 0.0513) \cdot 10^{-2}$
$K^- \nu_\tau$	$(0.6965 \pm 0.0097) \cdot 10^{-2}$
$\pi^- \pi^0 \nu_\tau$	$(25.4967 \pm 0.0893) \cdot 10^{-2}$
$K^- \pi^0 \nu_\tau$	$(0.4330 \pm 0.0148) \cdot 10^{-2}$
$\pi^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(9.2638 \pm 0.0964) \cdot 10^{-2}$
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0652 \pm 0.0218) \cdot 10^{-2}$
$\pi^- 3\pi^0 \nu_\tau$ (ex. K^0)	$(1.0436 \pm 0.0707) \cdot 10^{-2}$
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0483 \pm 0.0212) \cdot 10^{-2}$
$h^- 4\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.1118 \pm 0.0391) \cdot 10^{-2}$
$\pi^- \bar{K}^0 \nu_\tau$	$(0.8398 \pm 0.0140) \cdot 10^{-2}$
$K^- K^0 \nu_\tau$	$(0.1479 \pm 0.0053) \cdot 10^{-2}$
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	$(0.3823 \pm 0.0129) \cdot 10^{-2}$
$K^- \pi^0 K^0 \nu_\tau$	$(0.1503 \pm 0.0071) \cdot 10^{-2}$
$\pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	$(0.0272 \pm 0.0226) \cdot 10^{-2}$
$\pi^- K^0_S K^0_S \nu_\tau$	$(0.0233 \pm 0.0007) \cdot 10^{-2}$
$\pi^- K^0_S K^0_L \nu_\tau$	$(0.1091 \pm 0.0241) \cdot 10^{-2}$
$\pi^- \pi^0 K^0_S K^0_S \nu_\tau$	$(0.0018 \pm 0.0002) \cdot 10^{-2}$
$\pi^- \pi^0 K^0_S K^0_L \nu_\tau$	$(0.0327 \pm 0.0119) \cdot 10^{-2}$
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0255 \pm 0.0199) \cdot 10^{-2}$
$\pi^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(8.9911 \pm 0.0511) \cdot 10^{-2}$
$\pi^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω)	$(2.7384 \pm 0.0710) \cdot 10^{-2}$
$h^- h^- h^+ 2\pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0979 \pm 0.0357) \cdot 10^{-2}$

$B\ (\tau \to \ldots)$	HFAG 2016 prelim.
$h^- h^- h^+ 3\pi^0 \nu_\tau$	$(0.0212 \pm 0.0030) \cdot 10^{-2}$
$\pi^- K^- K^+ \nu_\tau$	$(0.1437 \pm 0.0027) \cdot 10^{-2}$
$\pi^- K^- K^+ \pi^0 \nu_\tau$	$(0.0061 \pm 0.0018) \cdot 10^{-2}$
$3h^- 2h^+ \nu_\tau$ (ex. K^0)	$(0.0822 \pm 0.0032) \cdot 10^{-2}$
$3h^- 2h^+ \pi^0 \nu_\tau$ (ex. K^0)	$(0.0164 \pm 0.0011) \cdot 10^{-2}$
$\pi^- \pi^0 \eta \nu_\tau$	$(0.1389 \pm 0.0072) \cdot 10^{-2}$
$K^- \eta \nu_\tau$	$(0.0155 \pm 0.0008) \cdot 10^{-2}$
$K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\pi^- \bar{K}^0 \eta \nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$h^- \pi^0 \omega \nu_\tau$	$(0.4089 \pm 0.0420) \cdot 10^{-2}$
$\pi^- \omega \nu_\tau$	$(1.9499 \pm 0.0645) \cdot 10^{-2}$
$K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2930 \pm 0.0069) \cdot 10^{-2}$
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0395 \pm 0.0142) \cdot 10^{-2}$
$\pi^- K^0_L K^0_L \nu_\tau$	$(0.0233 \pm 0.0007) \cdot 10^{-2}$
$a_1^- (\to \pi^- \gamma) \nu_\tau$	$(0.0401 \pm 0.0145) \cdot 10^{-2}$
$\pi^- \pi^0 K^0_L K^0_L \nu_\tau$	$(0.0018 \pm 0.0002) \cdot 10^{-2}$

- 42 modes (PDG 2015 has 31)
- unitarity is enforced in the fit
- w/o enforcement, $1 - \sum B_i = (0.091 \pm 0.106)\%$

Most recent measurements in HFAG fit, high multiplicity final states

BABAR PRD 86, 092010 (2012)
Study of high-multiplicity 3-prong and 5-prong tau decays at BABAR

$\Gamma_{811} = \pi^- 2\pi^0 \omega \nu_\tau$ (ex. K^0)	$(7.3 \pm 1.2 \pm 1.2) \cdot 10^{-5}$
$\Gamma_{812} = 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)	$(0.1 \pm 0.08 \pm 0.30) \cdot 10^{-4}$
$\Gamma_{821} = 3\pi^- 2\pi^+ \nu_\tau$ (ex. K^0, ω, f_1)	$(7.68 \pm 0.04 \pm 0.40) \cdot 10^{-4}$
$\Gamma_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	$(0.6 \pm 0.5 \pm 1.1) \cdot 10^{-6}$
$\Gamma_{831} = 2\pi^- \pi^+ \omega \nu_\tau$ (ex. K^0)	$(8.4 \pm 0.4 \pm 0.6) \cdot 10^{-5}$
$\Gamma_{832} = 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0, η, ω, f_1)	$(0.36 \pm 0.03 \pm 0.09) \cdot 10^{-4}$
$\Gamma_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	$(1.1 \pm 0.4 \pm 0.4) \cdot 10^{-6}$
$\Gamma_{910} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to 3\pi^0$) (ex. K^0)	$(8.27 \pm 0.88 \pm 0.81) \cdot 10^{-5}$
$\Gamma_{911} = \pi^- 2\pi^0 \eta \nu_\tau$ ($\eta \to \pi^+\pi^-\pi^0$) (ex. K^0)	$(4.57 \pm 0.77 \pm 0.50) \cdot 10^{-5}$
$\Gamma_{920} = \pi^- f_1 \nu_\tau$ ($f_1 \to 2\pi^- 2\pi^+$)	$(5.20 \pm 0.31 \pm 0.37) \cdot 10^{-5}$
$\Gamma_{930} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to \pi^+\pi^-\pi^0$) (ex. K^0)	$(5.39 \pm 0.27 \pm 0.41) \cdot 10^{-5}$
$\Gamma_{944} = 2\pi^- \pi^+ \eta \nu_\tau$ ($\eta \to \gamma\gamma$) (ex. K^0)	$(8.26 \pm 0.35 \pm 0.51) \cdot 10^{-5}$

Most recent measurements in HFAG fit, modes $\tau \to X \geq 1 K^0_S \, \nu_\tau$

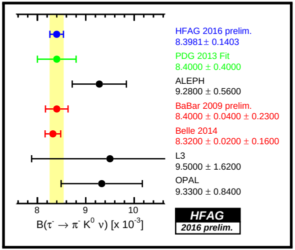
BABAR PRD 86, 092013 (2012)
The branching fraction of $\tau \to \pi^- K^0_S K^0_S (\pi^0) \nu$ decays

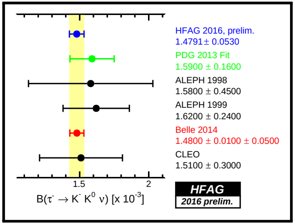
$\Gamma_{47} = \pi^- K^0_S K^0_S \nu_\tau$	$(2.31 \pm 0.04 \pm 0.08) \cdot 10^{-4}$
$\Gamma_{50} = \pi^- \pi^0 K^0_S K^0_S \nu_\tau$	$(1.60 \pm 0.20 \pm 0.22) \cdot 10^{-5}$

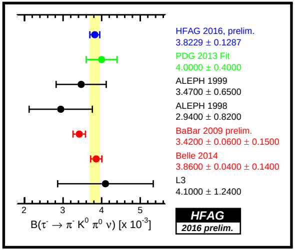
Belle PRD 89, 072009 (2014)
Measurements of Branching Fractions of τ decays with ≥ 1 K^0_S

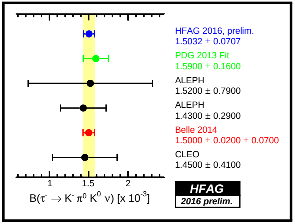
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$8.32 \cdot 10^{-3} \pm 0.3\% \pm 1.8\%$
$\Gamma_{37} = K^- K^0 \nu_\tau$	$14.8 \cdot 10^{-4} \pm 0.9\% \pm 3.7\%$
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$3.86 \cdot 10^{-3} \pm 0.8\% \pm 3.5\%$
$\Gamma_{42} = K^- \pi^0 K^0 \nu_\tau$	$14.96 \cdot 10^{-4} \pm 1.3\% \pm 4.9\%$
$\Gamma_{47} = \pi^- K^0_S K^0_S \nu_\tau$	$2.33 \cdot 10^{-4} \pm 1.4\% \pm 4.0\%$
$\Gamma_{50} = \pi^- \pi^0 K^0_S K^0_S \nu_\tau$	$2.00 \cdot 10^{-5} \pm 10.8\% \pm 10.1\%$

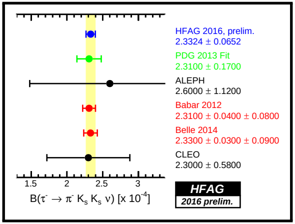
Improvements from recent K_S^0 BFs results

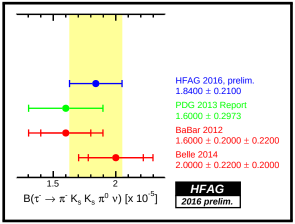






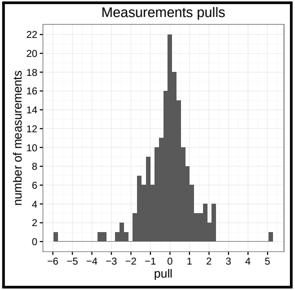


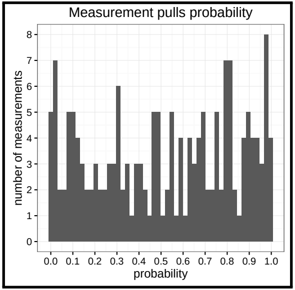


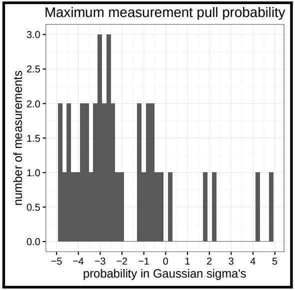


- 2 *BABAR* preliminary results are shown above, but not used in HFAG 2016 prelim. fit

Measurement pulls - HFAG 2016 prelim., no scaling







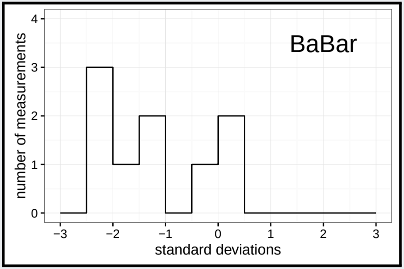
- two outliers: *BABAR* and Belle $B(\tau \to K^- K^- K^+ \nu_\tau)$ results
- (probabilities expressed as n. of Gaussian sigma's)
- rightmost plot: pull probability by measurement, should that pull be the maximum of n.d.o.f. Gaussian pulls: apply scaling for $P_{\text{max}}(\text{pull}_i) > 3\sigma$

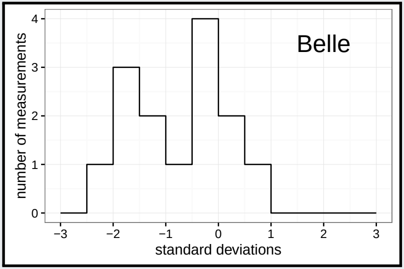
Impact of *BaBar* and Belle *B*-factories - HFAG 2016 prelim.

B-factories improved small BFs, not large BFs

- cannot select tau events with just one hemisphere with good efficiency and purity
- lower hadronic multiplicity $\Rightarrow$ more difficult to discriminate $\tau\tau$ vs. hadrons
- less precise knowledge of the luminosity

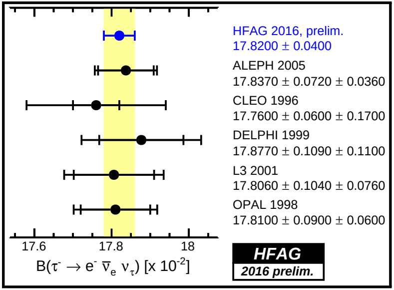
B-factories tend to measure lower BFs

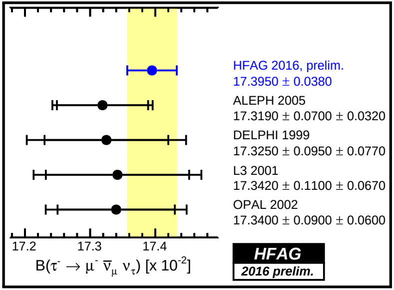


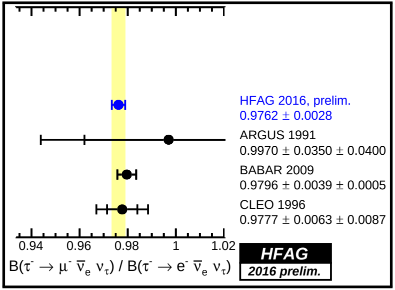


- updated plots of feature mentioned in PDG reviews
- results with no *B*-factories inputs obtained with the HFAG fit techniques (PDG reviews use old enough PDG editions results)

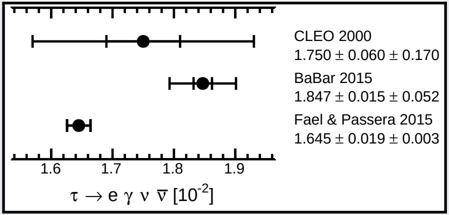
Leptonic branching fractions - HFAG 2016 prelim.

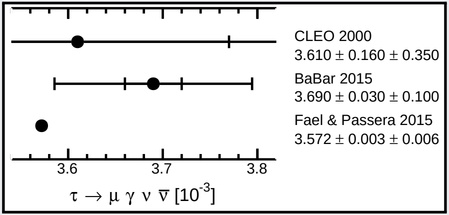






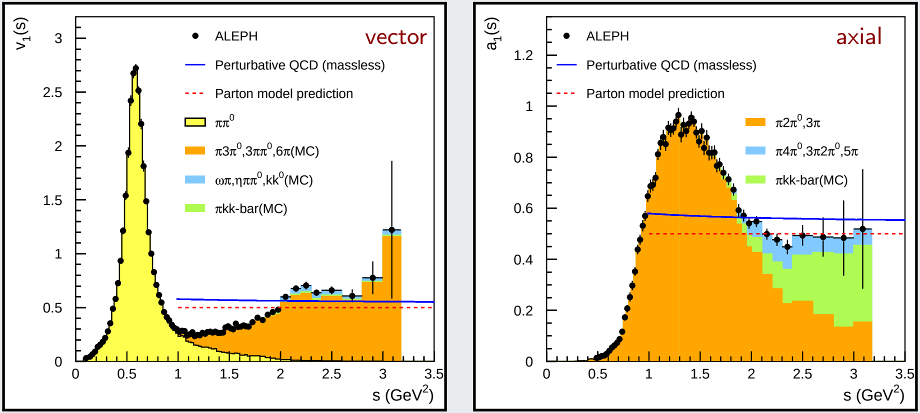
Tau radiative leptonic decays ($E_\gamma > 10\,\mathrm{MeV}$)





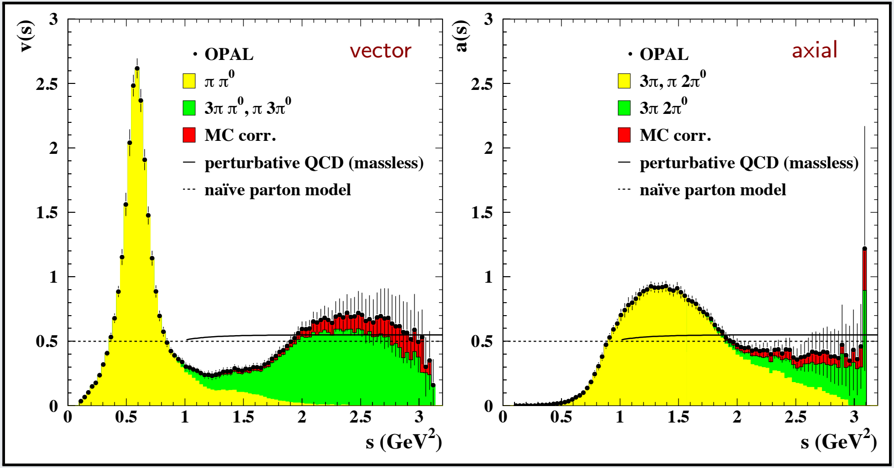
- (see also M.Passera presentation in this workshop)
- CLEO 2000: T. Bergfeld et al., PRL 84 (2000) 830
- *BABAR* 2015: PRD 91, 051103 (2015)
- Fael & Passera 2015: NLO calculation, JHEP 07 (2015) 153, arXiv:1602.00457 [hep-ph]
- 3.5σ discrepancy between *BABAR* 2015 and NLO calculation, to be investigated

ALEPH non-strange spectral functions, 2005, revised in 2014



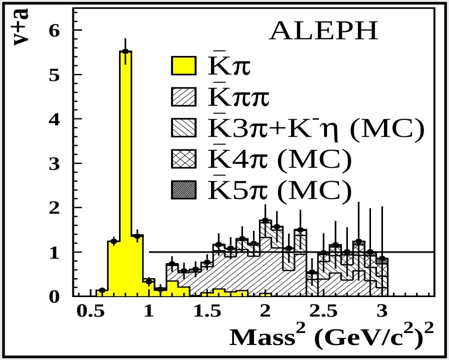
- Davier, Höcker, Malaescu, Yuan, Zhang, EPJC 74 (2014)

OPAL non-strange spectral functions, 1999

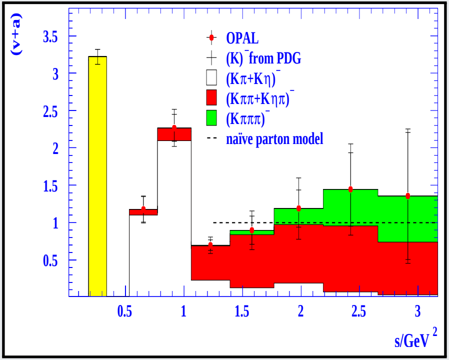


- OPAL coll., EPJC 7 (1999)

ALEPH and OPAL strange $V+A$ spectral functions

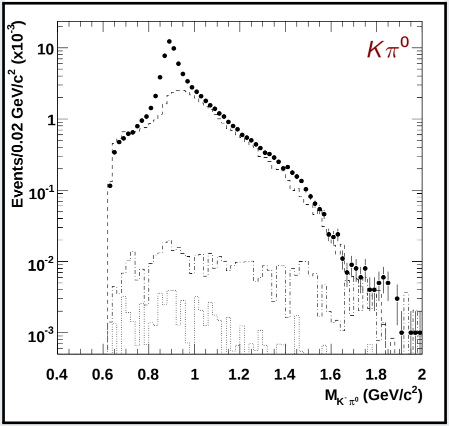


ALEPH, Phys. Rep. 421 (2005) 191

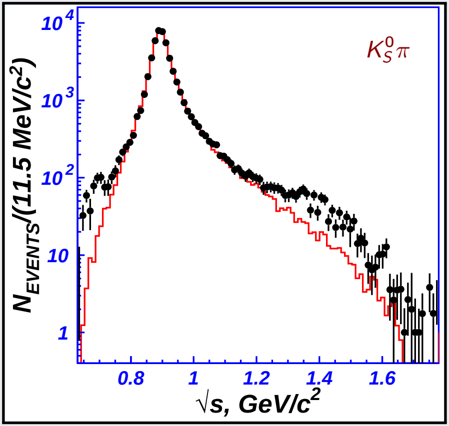


OPAL, EPJC 7 (1999)

B-factories $\tau \to K\pi\nu$ $V{+}A$ spectral functions

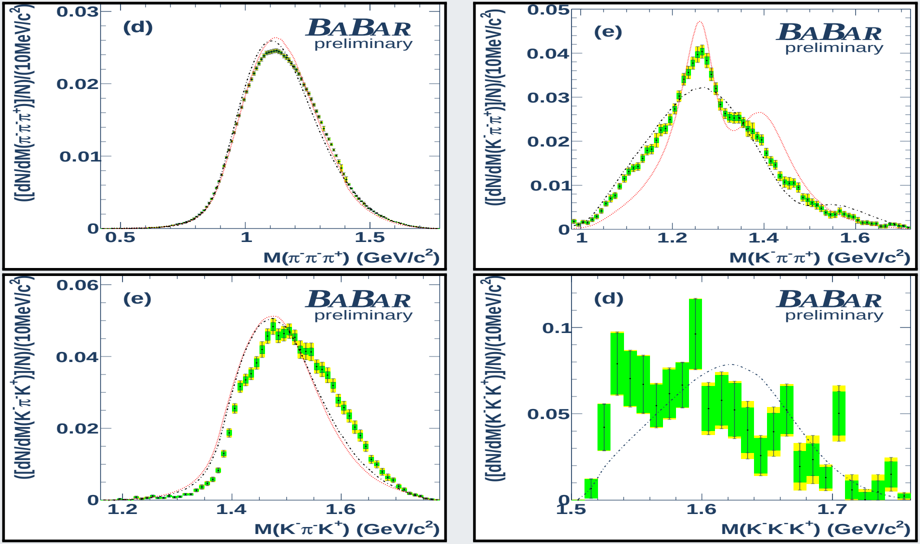


BABAR, PRD 76 (2007) 051104
(not background-subtracted)



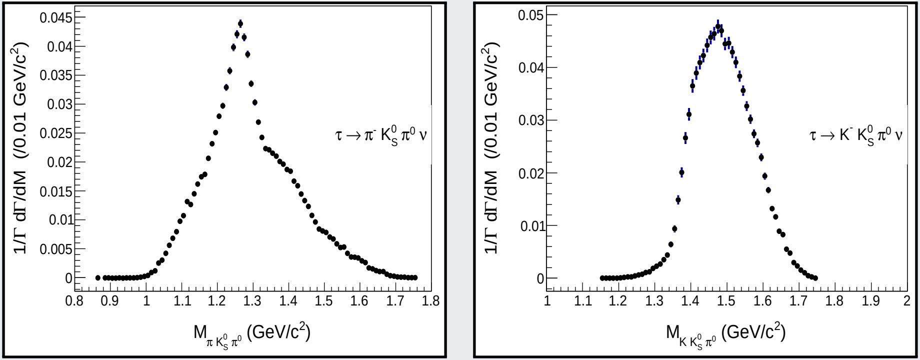
Belle, PLB 654 (2007) 65

BABAR $\tau \to hhh\nu$ spectral functions



Nugent, Nucl.Phys.Proc.Suppl. 253-255 (2014) 38.

Belle $\tau \to h K^0_S \pi^0 \nu$ $V{+}A$ spectral functions



Ryu [Belle] Nucl.Phys.Proc.Suppl. 253-255 (2014) 33

Lepton Flavour Violation results

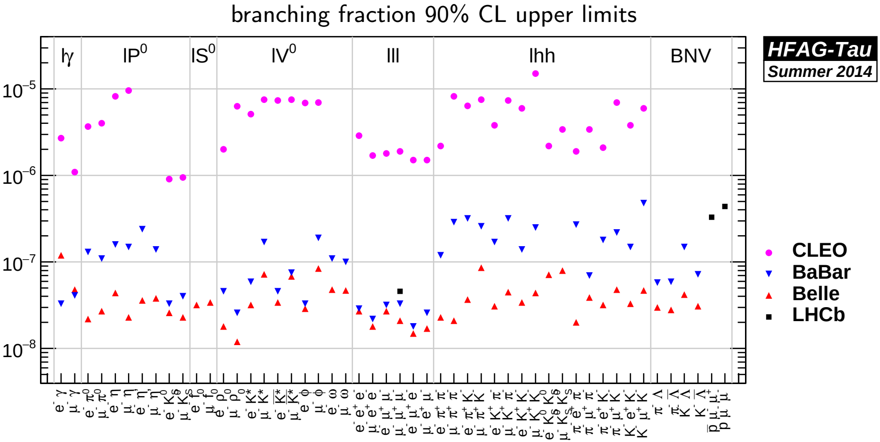
Introduction

- searches for LFV are powerful and clean probe for New Physics effects
- *B*-Factories are best facilities to search for LFV tau decays
- LFV tau decays are easier to detect than SM tau decays
  - typically no undetected neutrinos
  - reconstructed decay products invariant mass peaks at tau mass
  - reconstructed decay products energy in CM-frame peaks at half the event energy

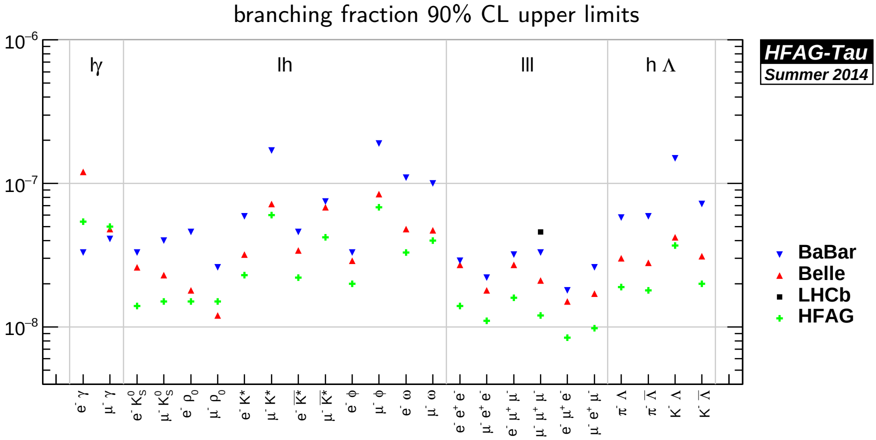
Most recent LFV searches results from the *B*-Factories

- BaBar $\tau \to 3$ leptons, PRD 81 (2010) 111101
- Belle $\tau \to 3$ leptons, PLB 687 (2010) 139
- Belle $\tau \to \ell K_S^0$, $\tau \to \ell K_S^0 K_S^0$, PLB 692 (2010) 4
- Belle $\tau \to \ell V$, PLB 699 (2011) 251
- Belle $\tau \to \ell h h'$, PLB 719 (2013) 346

Tau LFV branching fractions upper limits

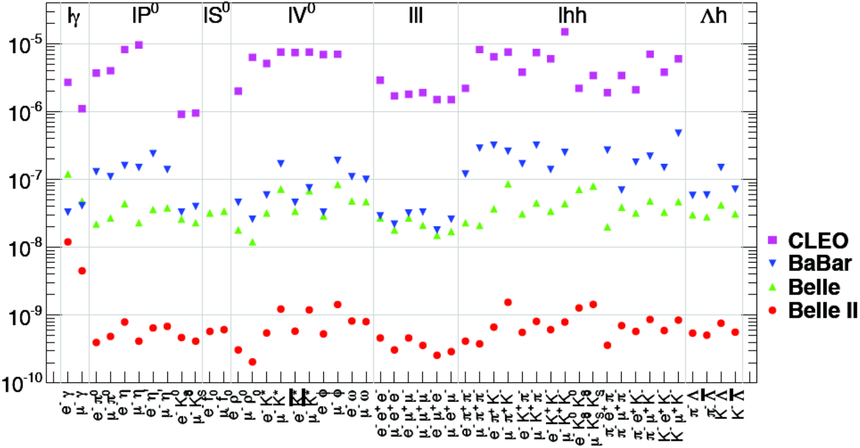


HFAG combined a subset of tau LFV upper limits



Tau LFV expected upper limits for BelleII

branching fraction 90% CL upper limits



(plot from M. Barret, FPCP 2015)

Other measurements

- CP violation in $\tau \to K_S \pi \nu$
- Michel parameters (structure of EW tau decay)
- tau in decay of B, Higgs and other particles
- ...

Elaborations of tau branching fractions and spectral functions

- lepton universality tests
- α_s (see also E.Passemar, this Workshop, and March 2016 Mainz QCD Workshop)
- muon $g-2$ hadronic contribution (see also March 2016 Mainz QCD Workshop)
- $|V_{us}|$ (see also March 2016 Mainz QCD Workshop)
- fits on moments of spectral functions
  - provide uncalculable QCD predictions
  - include non-perturbative terms
  - help estimating truncated terms in OPE

Lepton universality - HFAG 2016 prelim.

Standard Model for leptons $\lambda,\ \rho = e, \mu, \tau$ (Marciano 1988)

$$\Gamma[\lambda \to \nu_\lambda \rho \overline{\nu}_\rho(\gamma)] = \Gamma_{\lambda\rho} = \Gamma_\lambda B_{\lambda\rho} = \frac{B_{\lambda\rho}}{\tau_\lambda} = \frac{G_\lambda G_\rho m_\lambda^5}{192\pi^3} f\left(\frac{m_\rho^2}{m_\lambda^2}\right) r_W^\lambda r_\gamma^\lambda ,$$

where

$$G_\lambda = \frac{g_\lambda^2}{4\sqrt{2}M_W^2} \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \qquad f_{\lambda\rho} = f\left(\frac{m_\rho^2}{m_\lambda^2}\right)$$

$$r_W^\lambda = 1 + \frac{3}{5}\frac{m_\lambda^2}{M_W^2} \qquad r_\gamma^\lambda = 1 + \frac{\alpha(m_\lambda)}{2\pi}\left(\frac{25}{4} - \pi^2\right)$$

Tests of lepton universality from ratios of above partial widths:

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{B_{\tau e}}{B_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} r_W^\mu r_\gamma^\mu}{\tau_\tau m_\tau^5 f_{\tau e} r_W^\tau r_\gamma^\tau}} = 1.0012 \pm 0.0015 = \sqrt{\frac{B_{\tau e}}{B_{\tau e}^{SM}}}$$

$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{B_{\tau \mu}}{B_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} r_W^\mu r_\gamma^\mu}{\tau_\tau m_\tau^5 f_{\tau \mu} r_W^\tau r_\gamma^\tau}} = 1.0030 \pm 0.0014 = \sqrt{\frac{B_{\tau \mu}}{B_{\tau \mu}^{SM}}}$$

$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{B_{\tau \mu}}{B_{\tau e}} \frac{f_{\tau e}}{f_{\tau \mu}}} = 1.0019 \pm 0.0014$$

- precision: **0.20—0.23%** pre-*B*-Factories ⇒ **0.14—0.15%** today
  thanks essentially to the Belle tau lifetime measurement, PRL 112 (2014) 031801
- $r_\gamma^\tau = 1 - 43.2 \cdot 10^{-4}$ and $r_\gamma^\mu = 1 - 42.4 \cdot 10^{-4}$ (Marciano 1988), M_W from PDG 2013

Coupling constants ratios uncertainties contributions - HFAG 2016 prelim.

quantity	uncertainty	contribution
τ_τ	0.18%	0.090%
$B_{\tau\to\mu,e}$	0.23%	0.115%
m_τ	0.009%	0.022%

Universality improved $B(\tau \to e\nu\bar{\nu})$ and $R_{\rm had}$ - HFAG 2016 prelim.

Universality improved $B(\tau \to e\nu\bar{\nu})$

- (M. Davier, 2005): assume SM lepton universality to improve $B_e = B(\tau \to e\bar{\nu}_e\nu_\tau)$ fit B_e using three determinations:
  - $B_e = B_e$
  - $B_e = B_\mu \cdot f(m_e^2/m_\tau^2)/f(m_\mu^2/m_\tau^2)$
  - $B_e = B(\mu \to e\bar{\nu}_e\nu_\mu) \cdot (\tau_\tau/\tau_\mu) \cdot (m_\tau/m_\mu)^5 \cdot f(m_e^2/m_\tau^2)/f(m_e^2/m_\mu^2) \cdot (\delta_\gamma^\tau \delta_W^\tau)/(\delta_\gamma^\mu \delta_W^\mu)$
    [above we have: $B(\mu \to e\bar{\nu}_e\nu_\mu) = 1$]
- $\boldsymbol{B_e^{\rm univ} = (17.818 \pm 0.022)\%}$ HFAG-PDG 2016 prelim. fit

$R_{\rm had} = \Gamma(\tau \to \text{hadrons})/\Gamma_{\rm univ}(\tau \to e\nu\bar{\nu})$

- $$R_{\rm had} = \frac{\Gamma(\tau \to \text{hadrons})}{\Gamma_{\rm univ}(\tau \to e\nu\bar{\nu})} = \frac{B_{\rm hadrons}}{B_e^{\rm univ}} = \frac{1 - B_e^{\rm univ} - f(m_\mu^2/m_\tau^2)/f(m_e^2/m_\tau^2) \cdot B_e^{\rm univ}}{B_e^{\rm univ}}$$
  - two different determinations, second one not “contaminated” by hadronic BFs
- $\boldsymbol{R_{\rm had} = 3.6359 \pm 0.0074}$ HFAG-PDG 2016 prelim. fit
- $\boldsymbol{R_{\rm had}\text{(leptonic BFs only)} = 3.6397 \pm 0.0070}$ HFAG-PDG 2016 prelim. fit

Lepton Universality tests with hadron decays - HFAG 2016 prelim.

Standard Model:

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{B(\tau \to h\nu_\tau)}{B(h \to \mu\bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1+\delta_h) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2 \quad (h = \pi \text{ or } K)$$

rad. corr. $\delta_\pi = (0.16 \pm 0.14)\%$, $\delta_K = (0.90 \pm 0.22)\%$ (Decker 1994)

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9966 \pm 0.0026 \,, \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9865 \pm 0.0071 \,.$$

(electron tests less precise because hadron two body decays to electrons are helicity-suppressed)

Averaging the three g_τ/g_μ ratios:

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0002 \pm 0.0014 \,, \quad \text{(accounting for statistical correlations)}$$

[recent useful contribution from *BABAR* $\frac{K^-\nu_\tau}{e^-\bar{\nu}_e\nu_\tau}$ measurement, PRL 105 (2010) 051602]

Determination of $|V_{us}|$ from experimental data

from kaon decays

- $\Gamma(K \to \pi\ell\bar{\nu}_\ell[\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\rm EW}^K \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\ell \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm SU(2)}^{K\pi}\right)^2$
- $\frac{\Gamma(K^\pm \to \ell^\pm\nu)}{\Gamma(\pi^\pm \to \ell^\pm\nu)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{m_K(1 - m_\ell^2/m_K^2)^2}{m_\pi(1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{\rm EM})$

from tau decays

- $\frac{R(\tau \to X_{\text{strange}})}{|V_{us}|^2} - \frac{R(\tau \to X_{\text{non-strange}})}{|V_{ud}|^2} = \delta R_{\tau,\text{SU3 breaking}}$, **"tau inclusive"**
  $[R(\tau \to X) = \Gamma(\tau \to X)/\Gamma(\tau \to e\nu\bar{\nu})]$
- $\frac{B(\tau^- \to K^-\nu_\tau)}{B(\tau^- \to \pi^-\nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{(1 - m_K^2/m_\tau^2)^2}{(1 - m_\pi^2/m_\tau^2)^2} \frac{r_{\rm LD}(\tau^- \to K^-\nu_\tau)}{r_{\rm LD}(\tau^- \to \pi^-\nu_\tau)}$
- $B(\tau^- \to K^-\nu_\tau) = \frac{G_F^2 f_K^2 |V_{us}|^2 m_\tau^3 \tau_\tau}{16\pi\hbar} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}^{\tau K}$
- $\Gamma(\tau \to \bar{K}\pi\nu_\tau[\gamma]) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{\rm EW}^{\tau K\pi} \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\tau \left(1 + \delta_{\rm EM}^{K\tau} + \tilde{\delta}_{\rm SU(2)}^{K\pi}\right)^2$

"tau inclusive" $|V_{us}|$ determination

- $\frac{R(\tau \to X_{\text{strange}})}{|V_{us}|^2} - \frac{R(\tau \to X_{\text{non-strange}})}{|V_{ud}|^2} = \delta R_{\tau,\text{SU3 breaking}}$
- $\delta R_{\tau,\text{SU3 breaking}}$ can be computed with OPE
  - finite-energy sum rules (FESR) with
    either fixed-order (FOPT) or contour-improved (CIPT) prescriptions
  - strong dependence from m_s
  - problematic convergence requires special treatment
  - non-pert. terms fitted / estimated using tau spectral functions moments
  - assumptions on $D>4$ OPE contributions
- input $|V_{us}|$ and compute m_s, **Pich & Prades, hep-ph/9909244**
- input m_s and compute $|V_{us}|$
  - **Gamiz, Jamin, Pich, Prades, Schwab, hep-ph/0212230, hep-ph/0408044**,
  - **Maltman, 1011.6391 [hep-ph]**
  - **Maltman (Lattice 2015, 1510.06954 [hep-ph], Mainz QCD Workshop in March 2016**
    - fit of $|V_{us}|$ and $D>4$ condensates on moments of tau spectral functions
    - use QCD lattice to quantify OPE truncation error

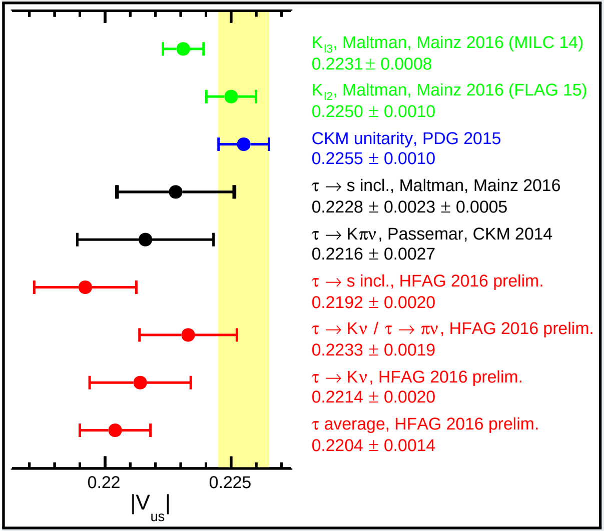
Tau branching fractions to strange final states, HFAG 2016 prelim.

Branching fraction	HFAG-PDG 2016 prelim. fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6965 \pm 0.0097) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4330 \pm 0.0148) \cdot 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0652 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0483 \pm 0.0212) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8398 \pm 0.0140) \cdot 10^{-2}$
$\Gamma_{40} = \pi^- \bar{K}^0 \pi^0 \nu_\tau$	$(0.3823 \pm 0.0129) \cdot 10^{-2}$
$\Gamma_{44} = \pi^- \bar{K}^0 \pi^0 \pi^0 \nu_\tau$ (ex. K^0)	$(0.0272 \pm 0.0226) \cdot 10^{-2}$
$\Gamma_{53} = \bar{K}^0 h^- h^- h^+ \nu_\tau$	$(0.0255 \pm 0.0199) \cdot 10^{-2}$
$\Gamma_{128} = K^- \eta \nu_\tau$	$(0.0155 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \bar{K}^0 \eta \nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^- \omega \nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	$(0.2930 \pm 0.0069) \cdot 10^{-2}$
$\Gamma_{803} = K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	$(0.0395 \pm 0.0142) \cdot 10^{-2}$
$\Gamma_{822} = K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	$(0.0001 \pm 0.0001) \cdot 10^{-2}$
$\Gamma_{833} = K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	$(0.0001 \pm 0.0001) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.9250 \pm 0.0443) \cdot 10^{-2}$

$|V_{us}|$ from $B(\tau \to K\pi\nu)$

- $\boldsymbol{\Gamma(\tau \to \bar{K}\pi\nu_\tau[\gamma])} = \dfrac{G_F^2 m_\tau^5}{96\pi^3}\; C_K^2\; S_{EW}^\tau \left(|\boldsymbol{V_{us}}| f_+^{K\pi}(0)\right)^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2$
- M.Antonelli, V.Cirigliano, A.L., E.Passemar, **arXiv:1304.8134 [hep-ph]**
  - compute the phase space integrals, I_K^ℓ using $K\pi$ form factors
    - from $\tau \to K\pi\nu_\tau$ Belle '08 $K_S^0\pi$ data
    - $K_{\ell 3}$ data may also be used for the low energy end of the integral
  - first estimate of the long-distance electromagnetic corrections ($\delta_{\mathrm{EM}}^{K\tau}$) to $\tau \to K\pi\nu_\tau$
  - isospin breaking corrections ($\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}$) for $\tau \to K^-\pi^0\nu_\tau$ vs. $\tau \to K_S^0\pi\nu_\tau$
  - $f_+^{K\pi}(0)$ from FLAG 2013
  - $f_+^{K\pi}(0)\,|V_{us}| = 0.2141 \pm 0.0014_{IK\tau} \pm 0.0021_{\mathrm{exp}}$
  - $|\boldsymbol{V_{us}}| = \mathbf{0.2216 \pm 0.0027}$ **E. Passemar, CKM 2014**
- V-Bernard, **arXiv:1311.2569 [hep-ph]**
  First determination of $f_+(0)|V_{us}|$ from a combined analysis of $\tau \to K\pi\nu_\tau$ decay and πK scattering with constraints from $K_{\ell 3}$ decays
  - global fit of tau and K data

$|V_{us}|$ results



- Maltman, Mainz 2016 uses
  - HFAG 2014 fit data
  - available spectral functions
  - Adametz thesis on $B(\tau \to K n \pi^0 \nu)$
  - Moulson CKM 2014 for kaon experimental inputs
  - lattice QCD $N_f = 2+1+1$ form factors
    - K_{l3}: FNAL-MILC 2014
    - $K_{\mu 2}$: FLAG 2015
- Passemar CKM 2014 uses
  - HFAG 2014 fit data
- CKM unitarity uses $|V_{ud}|$

Conclusions

- many useful tau measurements are available
- *B*-factories have large samples but
  - relatively unfavorable conditions
  - precision analyses require hard work and their results are just a piece in further elaborations (α_s, $|V_{us}|$, $g-2$ hadronic contribution)
- much more data will be available from BelleII in the near future
- manpower and organization are/will be essential to best exploit the available data

Backup Slides