Compton scattering from Protons and Light Nuclei: pinning down the nucleon polarizabilities

Judith McGovern

University of Manchester

Work done in collaboration with Harald Grießhammer, Daniel Phillips,
Vadim Lensky, Vladimir Pascalutsa, Mike Birse, Jerry Feldman, Luke Myers *et al.*

Prog. Nucl. Part. Phys. **67** 841 (2012) Eur. Phys. J. A **49** 12(2013)
Phys. Rev. Lett. **113,** 262506 (2014) Eur. Phys. J. C **75** 604 (2015);
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(1) Compton Scattering and polarisabilities

(2) Quick review of EFT calculations

(3) State of current calculations and fits and future directions

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from careful analysis of proton data set, and new deuteron data, we can now report the most precise value of nucleon scalar polarisabilities to date:

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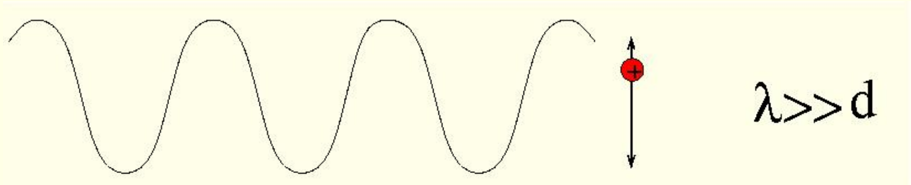
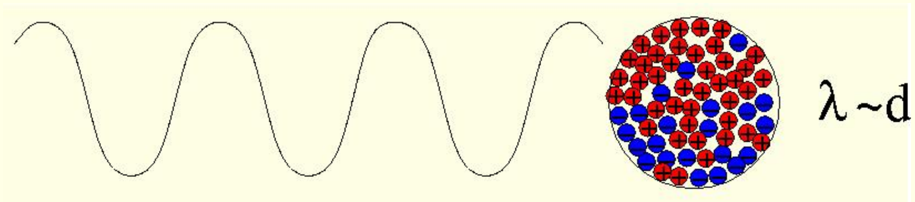
Rest of talk: background to these, and future directions.

Note: Heavy Baryon framework used unless stated to the contrary.

For large wavelengths, only sensitive to overall charge: Thomson scattering



But for smaller wavelengths, the target is polarised by the electric and magnetic fields



Compton Scattering

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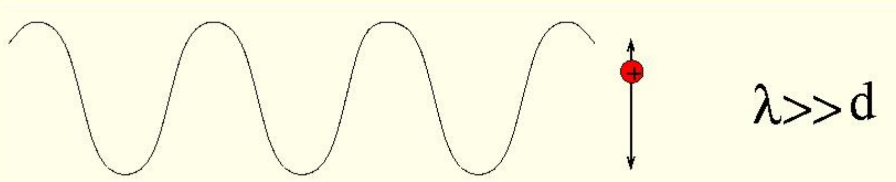

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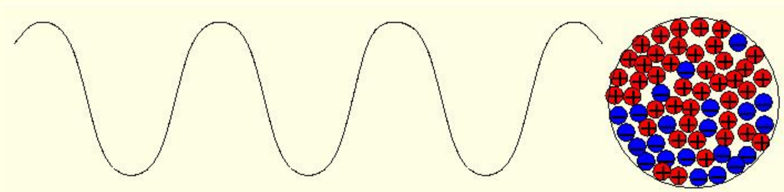
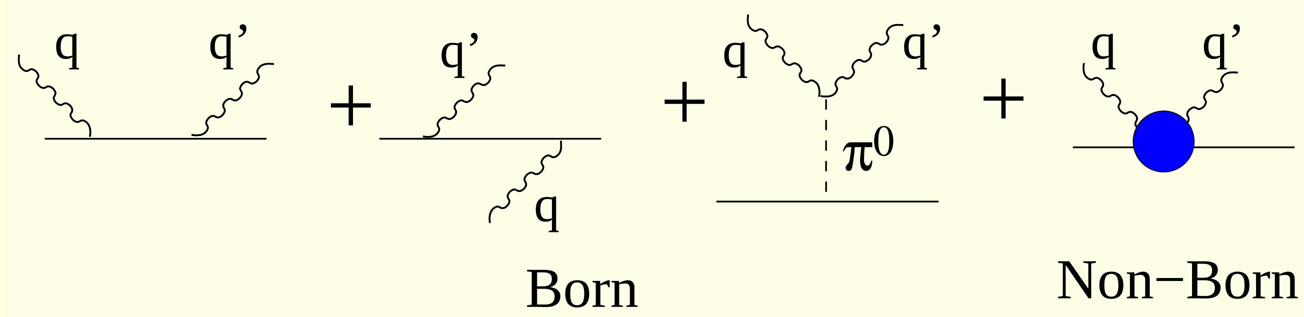


To leading order

$$
\begin{aligned}
H_{eff} &= \frac{(\mathbf{p}-Q\mathbf{A})^2}{2m} + Q\phi - \frac{1}{2}4\pi\left(\alpha\vec{E}^2 + \beta\vec{H}^2\right.\\
&+ \left.\gamma_{E1E1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1M1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} - 2\gamma_{M1E2}E_{ij}\sigma_i H_j + 2\gamma_{E1M2}H_{ij}\sigma_i E_j\right)
\end{aligned}
$$

where $E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$ and $H_{ij} = \frac{1}{2}(\nabla_i H_j + \nabla_j H_i)$

Compton Scattering from the nucleon



The scattering amplitude has Born and non-Born pieces. The latter probe the structure of the nucleon; polarisabilities are leading signs of non-pointlike nucleons as we increase the photon energy.

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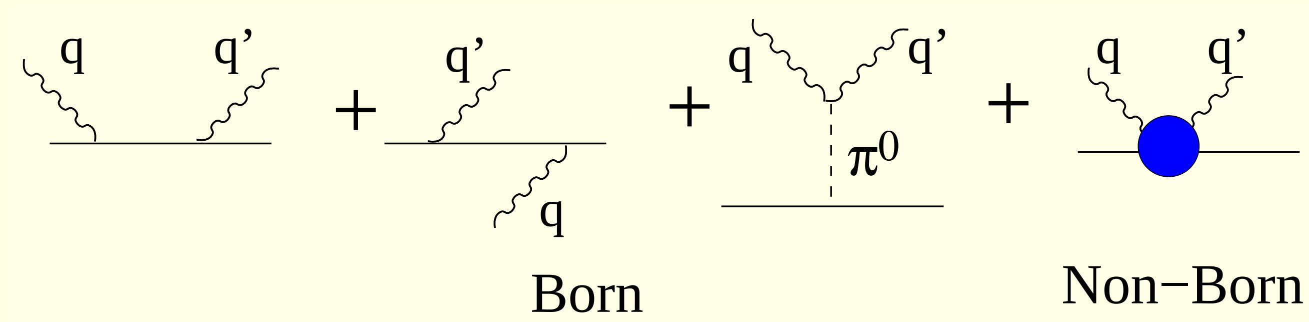

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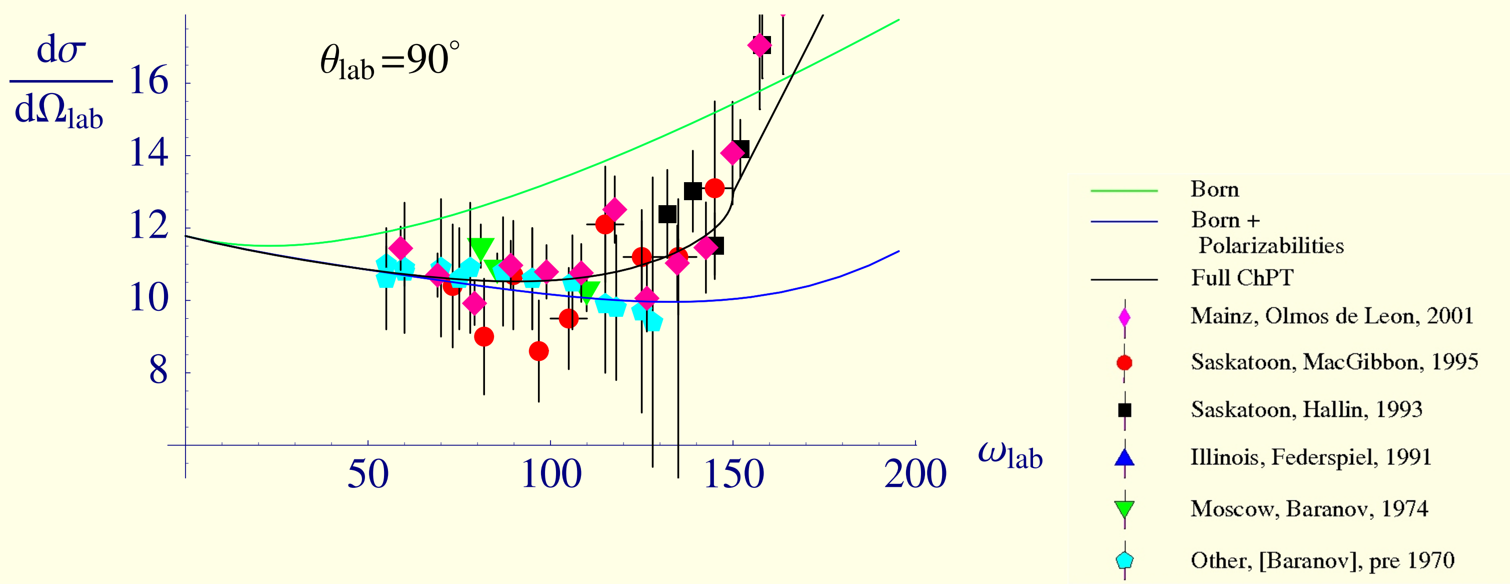
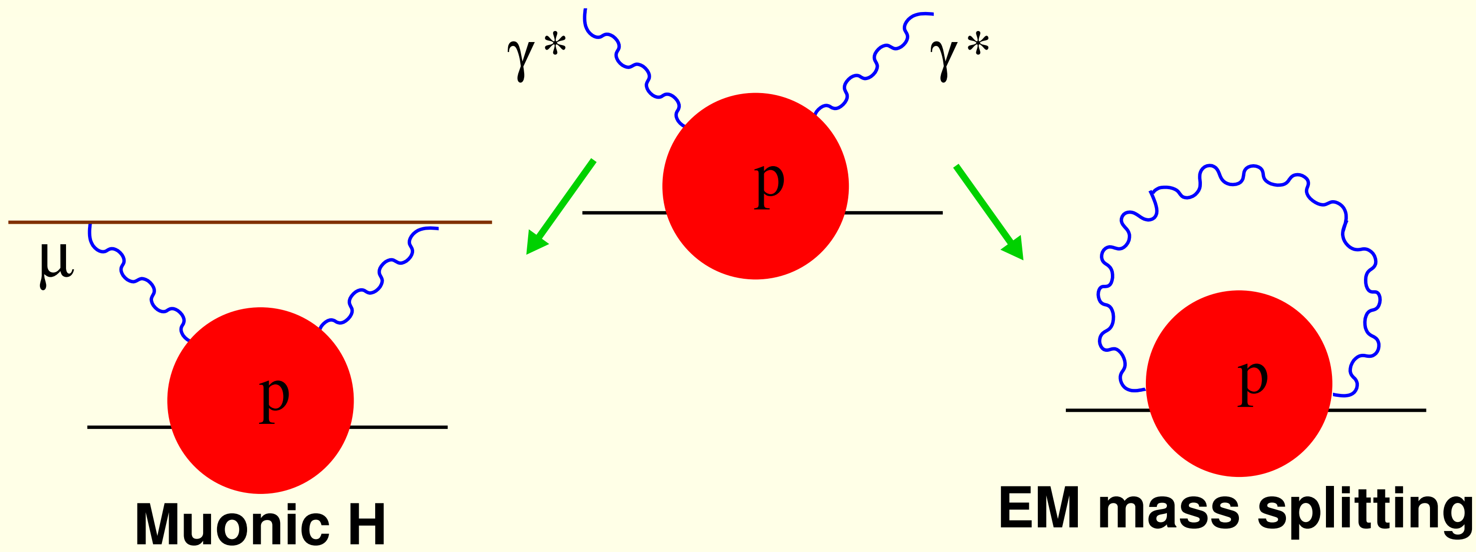
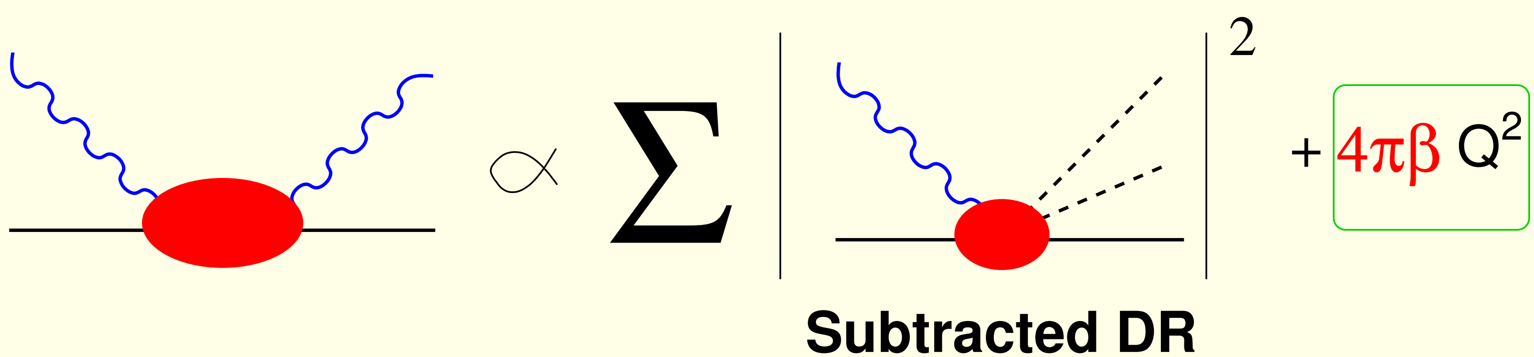


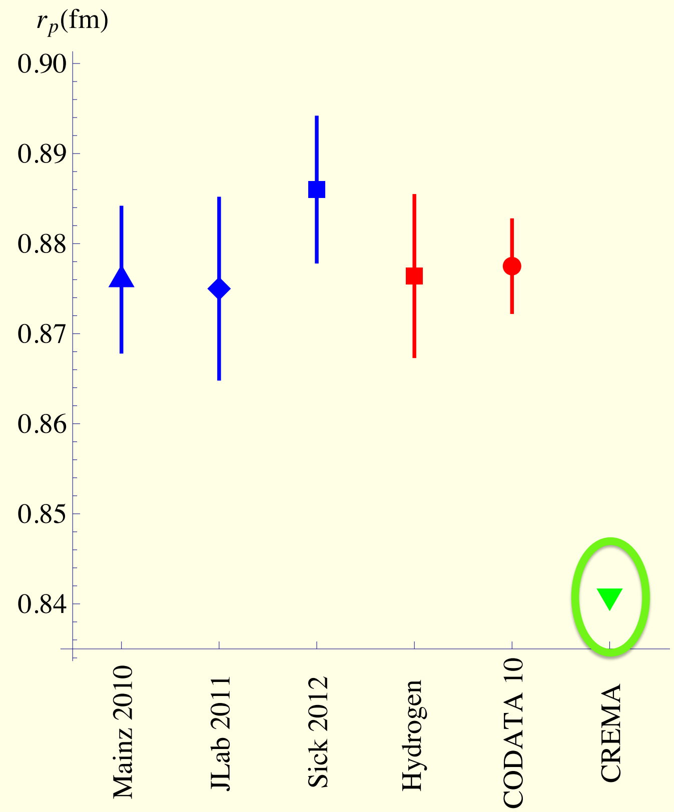
Why β matters





$$\overline{T}_1(\nu,Q^2) = -\nu^2 \int_{\nu_{th}^2}^{\infty} \frac{d\nu'^2}{\nu'^2}\,\frac{W_1(\nu',Q^2)}{\nu'^2-\nu^2} + 4\pi\beta\, Q^2 + O(Q^4)$$

Proton radius puzzle



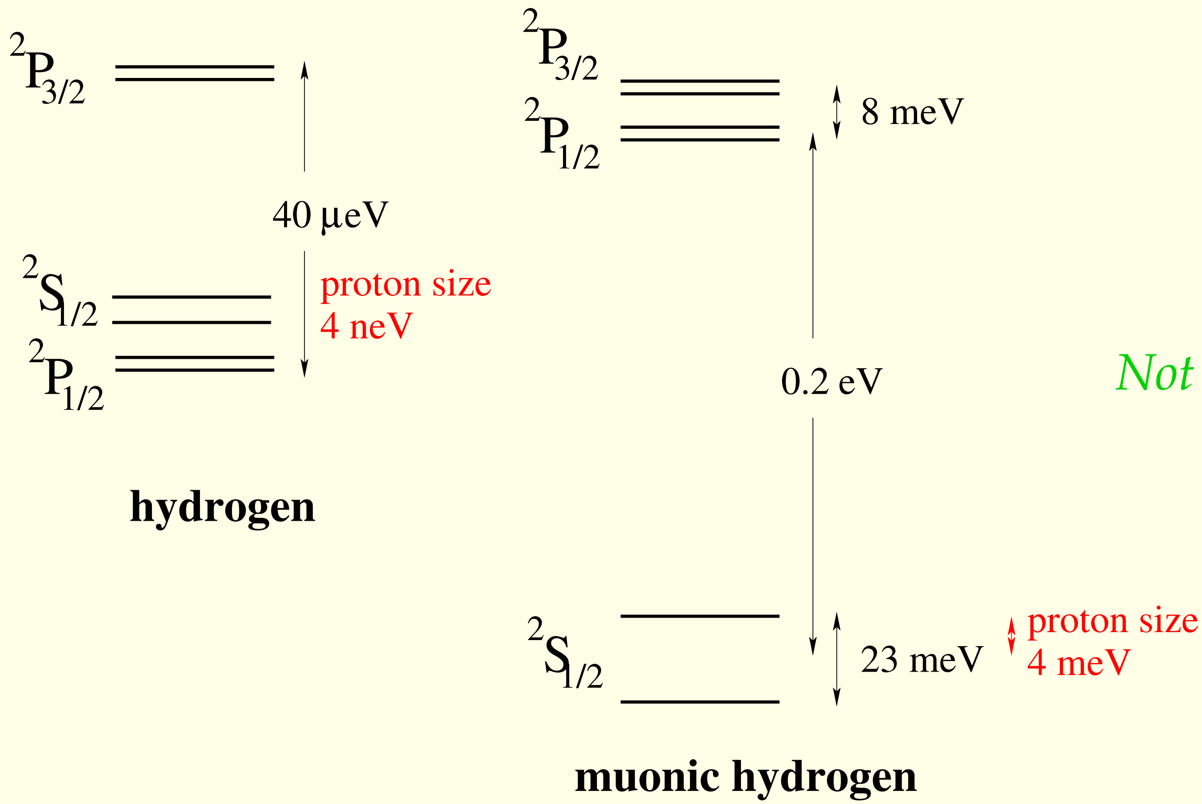
Hydrogen etc: $r_p = 0.8775(51)$ fm, CODATA 2010

Muonic hydrogen: $r_p = 0.84087 \pm 0.00039$ fm

Pohl et al, Nature **466**, 213 (2010)Antognini et al, Science **339** 417

7σ deviation!

Lamb shift



Not to scale!

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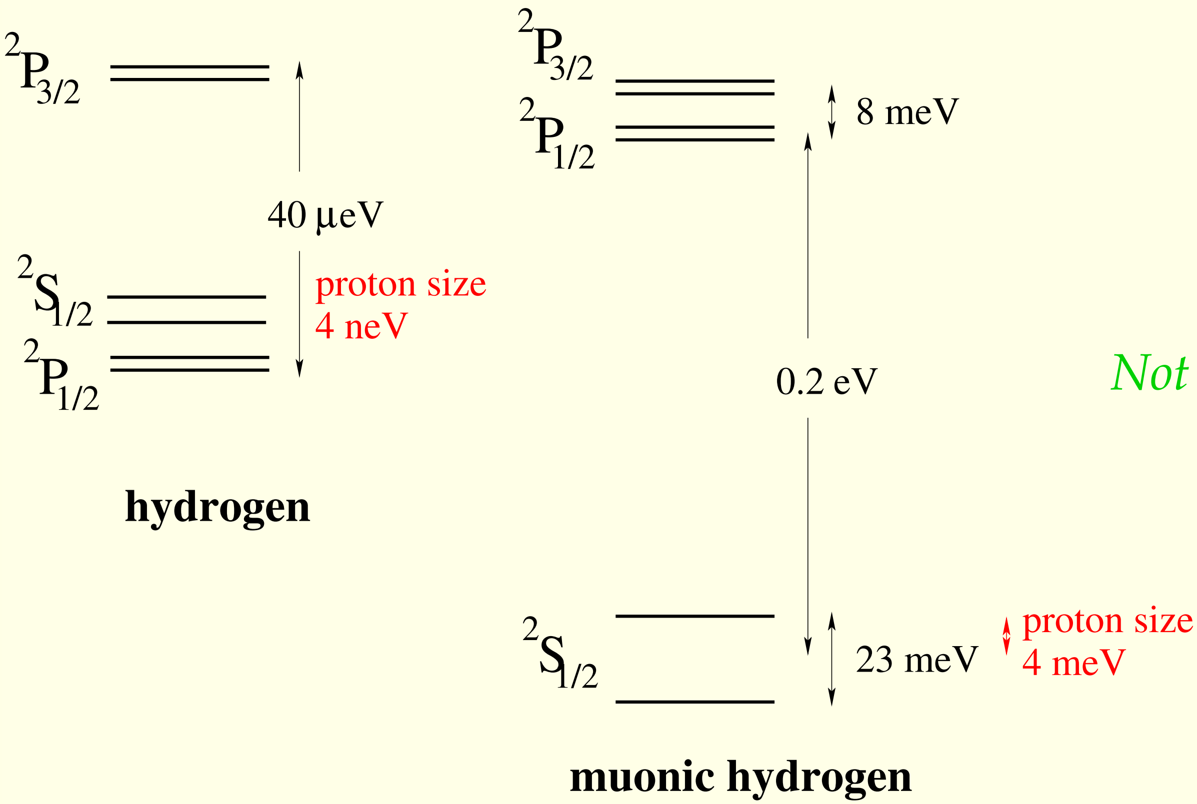

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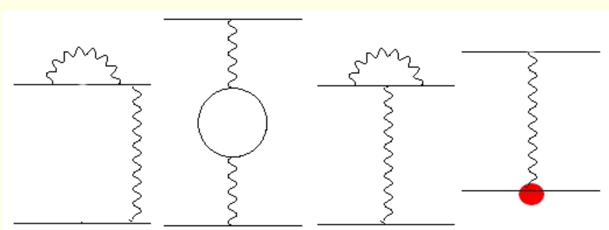

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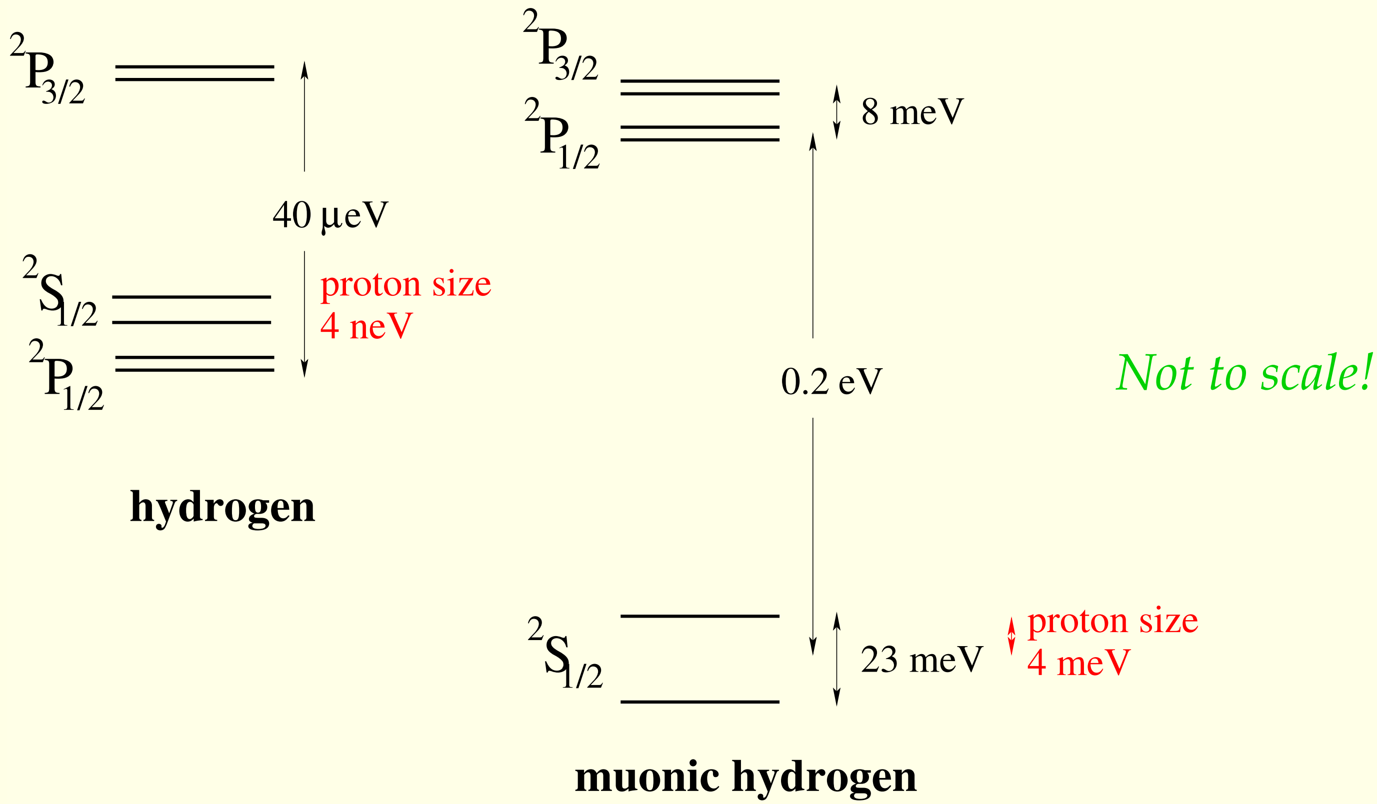
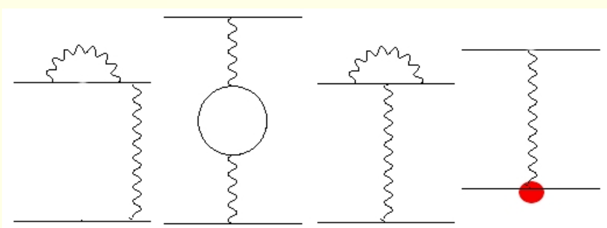
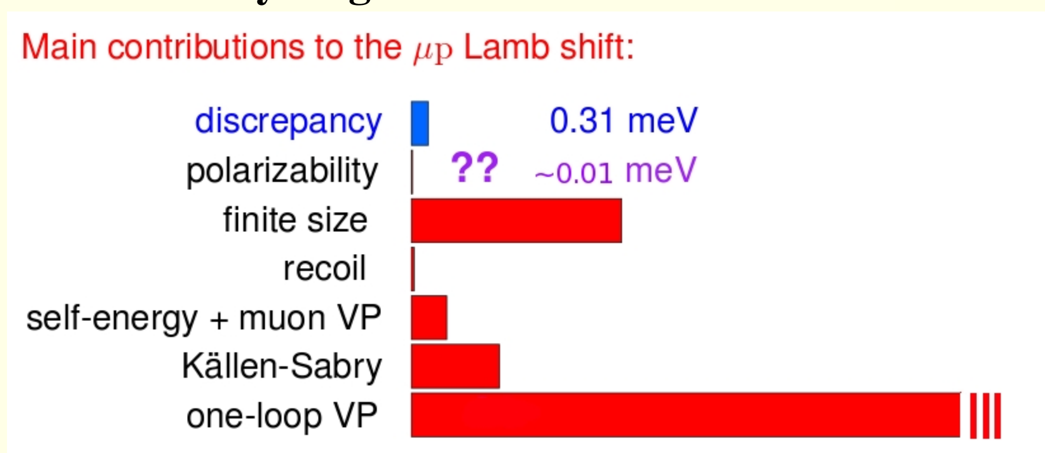
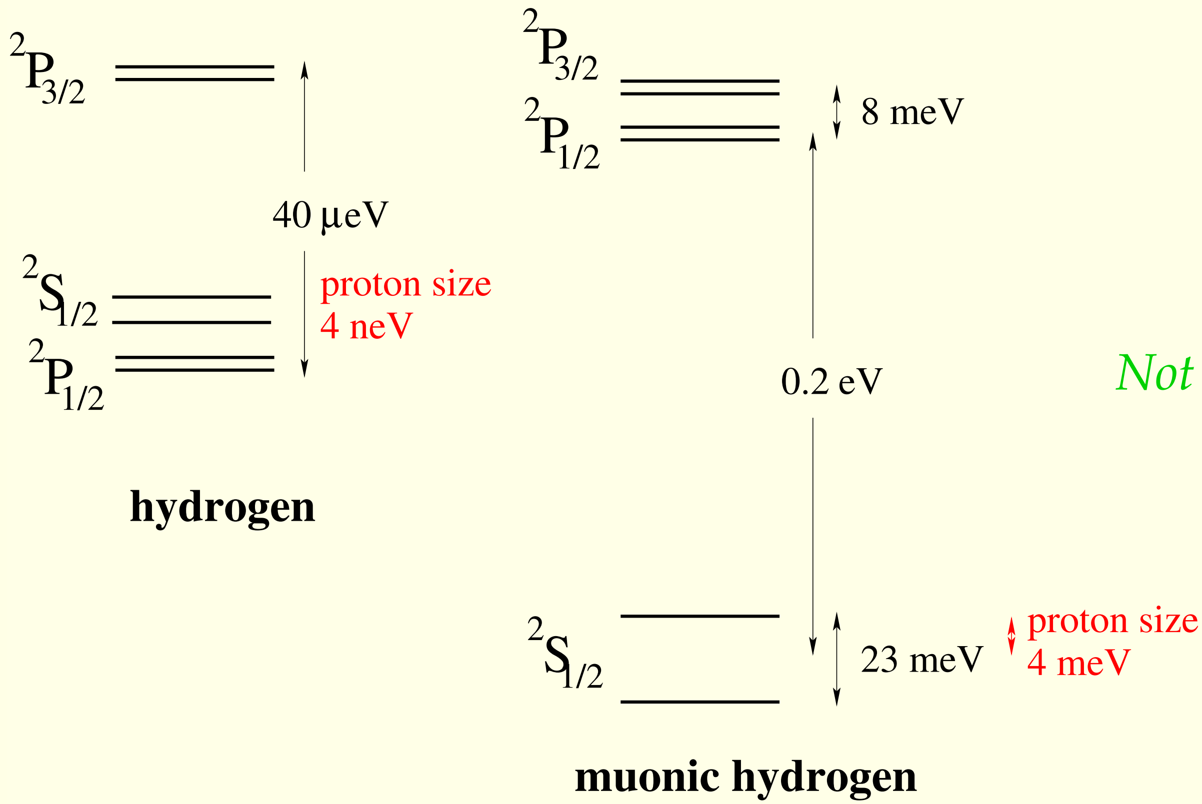

$^2P_{3/2}$
40 μeV
$^2S_{1/2}$
$^2P_{1/2}$
proton size
4 neV
hydrogen
$^2P_{3/2}$
$^2P_{1/2}$
8 meV
0.2 eV
$^2S_{1/2}$
23 meV
proton size
4 meV
muonic hydrogen
Not to scale!




Main contributions to the μp Lamb shift:
discrepancy
0.31 meV
polarizability
??
~0.01 meV
finite size
recoil
self-energy + muon VP
Källen-Sabry
one-loop VP

Lamb shift



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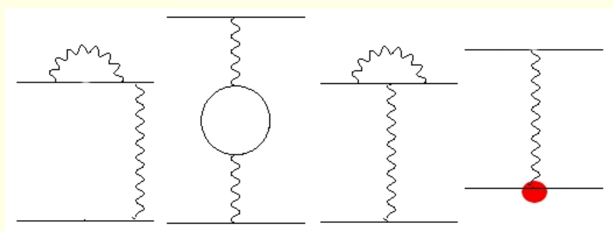
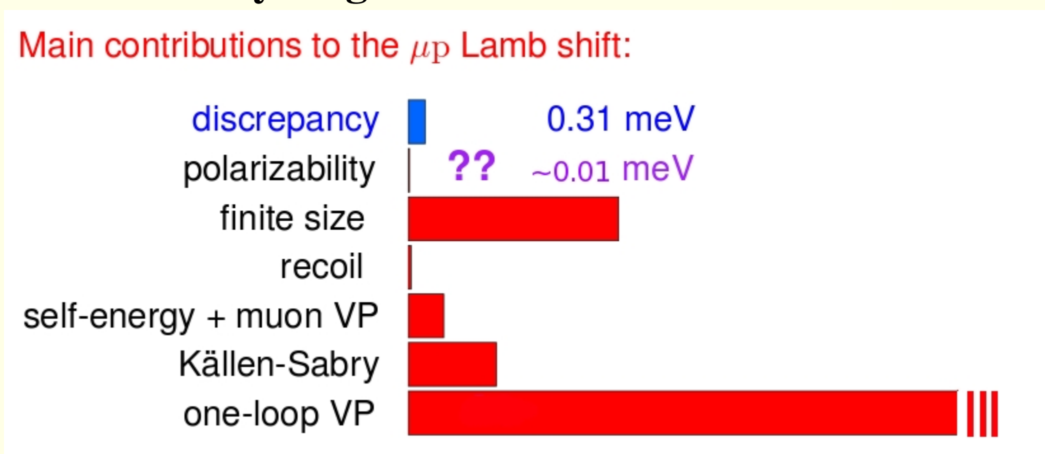


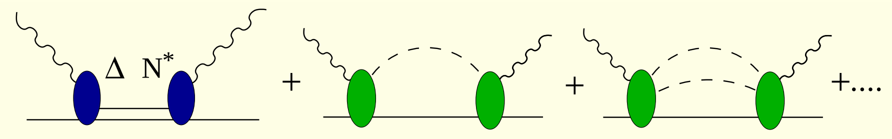


$\beta = 3.1 \pm 0.5 \implies$

$\Delta E_{\mathrm{pol}} = -0.0085(11)\,\mathrm{meV}$

M. Birse & JMcG, Eur. Phys. J. A **48** (2012) 120

At a hadronic level, we consider Compton scattering from the nucleon as probing its excitations and particularly its pionic cloud.



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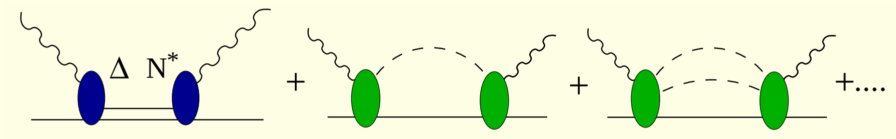
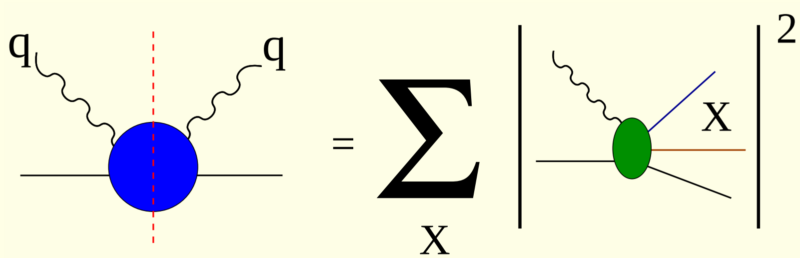


Optical theorem leads to sum rules for forward scattering



Baldin SR: $$\alpha+\beta = \frac{1}{2\pi^2}\int_{\omega_{th}}^{\infty}\frac{\sigma_{tot}(\omega)}{\omega^2}d\omega$$ and $$\gamma_0 = \frac{1}{4\pi^2}\int_{\omega_{th}}^{\infty}\frac{\sigma_{1/2}(\omega)-\sigma_{3/2}(\omega)}{\omega^3}d\omega$$

Both quite accurately evaluated for the proton:
$\alpha^{(p)}+\beta^{(p)} = (13.8\pm 0.4)\times 10^{-4}$ fm^3 Olmos de Léon *et al.* EPJA **10** 207 (2001);
$\gamma_0 = (-0.90\pm 0.08(\text{stat})\pm 0.11(\text{sys}))\times 10^{-4}$ fm^4 as byproduct of GDH expt. at MAMI and ELSA. Pasquini *et al.* Phys. Lett. B **687** 160 (2010)

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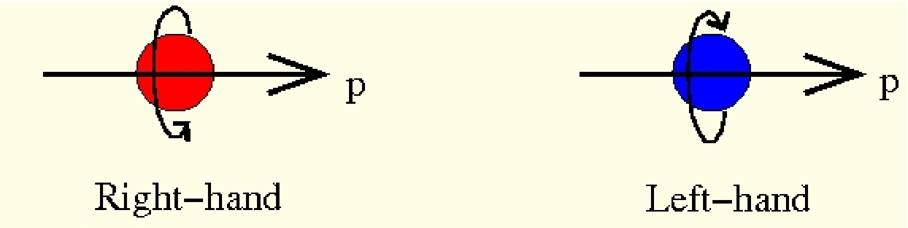
Both have difficulties with parameter-free predictions; both can be used to fit Compton scattering data and extract polarisabiltiies.

Chiral symmetry is an extension of isospin symmetry which is exact for massless quarks: we are free to redefine up and down for right- and left-handed quarks separately.



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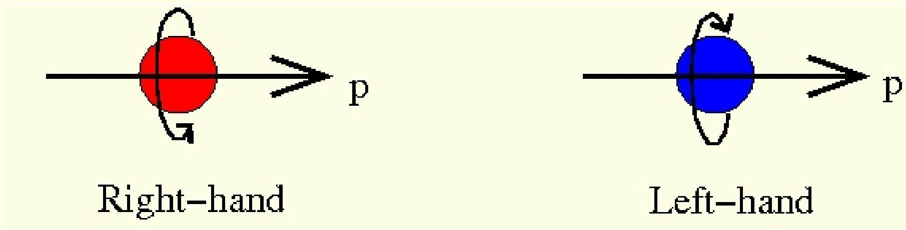

The symmetry is hidden – it is a symmetry of the QCD Lagrangian but not of the vacuum or hadron spectrum (isospin multiplets but no parity doublets).

This is the "Higgs mechanism" of QCD: hadrons get (almost all of) their mass from their interactions with the QCD vacuum; $\langle \overline{q}q \rangle \neq 0$.

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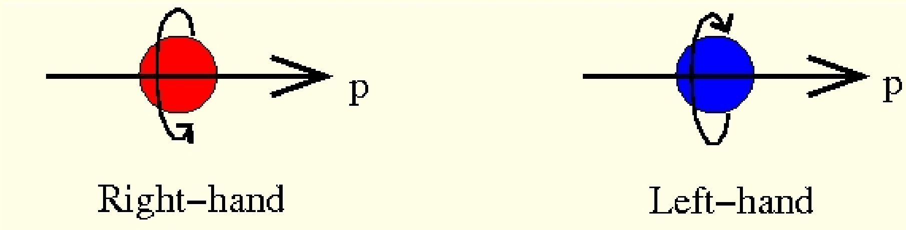

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The hidden symmetry shows up as a massless Goldstone bosons — the pion.

m_π is not quite zero because the quark masses also couple to the Higgs condensate (also contributes 5-10% of the mass of other hadrons)

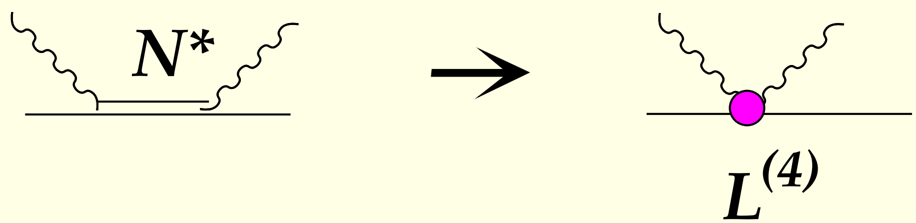
Effective field theory of QCD– relies on separation of scales

- pions are light ($m_\pi \ll m_\rho$)
- low-energy pions interact weakly with other matter ($L_{\pi NN} \propto \overline{N}\partial_\mu \pi N$).

Thus pion loops are suppressed by $\approx m_\pi^2/\Lambda^2$ where $\Lambda \approx m_\rho$. The Lagrangian contains infinitely many terms:

$$\mathcal{L} = \sum_n \mathcal{L}^{(n)}(c_i^{(n)})$$

Non-pionic nucleon structure shows up in low energy constants $c_i^{(n)}$, but is suppressed by power of momentum: $(k/\Lambda)^n$:



Calculations to nth order involve vertices from $\mathcal{L}^{(n)}$ and pion loops with vertices from $\mathcal{L}^{(n-2)}$; truncation errors are $\sim (k/\Lambda)^{(n+1)}$.

We include nucleons, pions and the Delta in our Lagrangian.

$$\mathcal{L}_{\pi N}^{(4),\mathrm{CT}} = 2\pi e^2 H^\dagger \left[\left(\delta\beta^{(s)} + \delta\beta^{(v)} \tau_3 \right) \left(\tfrac{1}{2} g_{\mu\nu} - v_\mu v_\nu \right) - \left(\delta\alpha^{(s)} + \delta\alpha^{(v)} \tau_3 \right) v_\mu v_\nu \right] F^{\mu\rho} F^\nu{}_\rho H.$$

Counterterms shift α and β at 4th order. Counterterms for spin pols at 5th order.

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$$\mathcal{L}_{\gamma N\Delta}^{\mathrm{PP},(2)} = \frac{3e}{2M_N(M_N+M_\Delta)} \left[\bar{\psi}(\mathrm{i} g_M \tilde{F}^{\mu\nu} - g_E\gamma_5 F^{\mu\nu})\partial_\mu \Psi^3_\nu - \bar{\Psi}^3_\nu \overleftarrow{\partial}_\mu (\mathrm{i} g_M \tilde{F}^{\mu\nu} - g_E \gamma_5 F^{\mu\nu})\psi \right],$$

χPT for Compton Scattering from the nucleon

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$\Delta \equiv M_\Delta - M_N \approx 271$ MeV is a rather small scale. Traditionally it is counted as $\Delta/\Lambda_\chi \sim m_\pi/\Lambda_\chi$ ("SSE"). But in Compton scattering the pion is clearly important at lower energies than the Delta.

Alternative: count $\quad \dfrac{m_\pi}{\Delta} \sim \dfrac{\Delta}{\Lambda_\chi} \quad \Rightarrow \quad \delta^2 \equiv \left(\dfrac{\Delta}{\Lambda_\chi}\right)^2 \sim \dfrac{m_\pi}{\Lambda_\chi}$

Then graphs with one Δ propagator are one order of δ higher than the corresponding nucleon graphs in low energy region.

Pascalutsa and Phillips, Phys. Rev. C67 (2003) 055202

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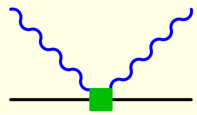
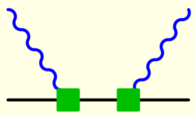
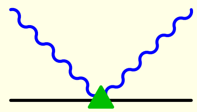
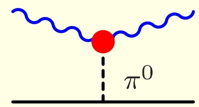
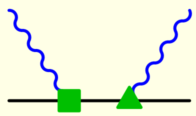
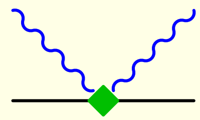
Different counting in resonance region; we work to at least NLO in both.

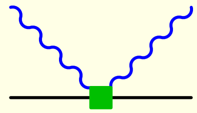
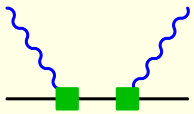
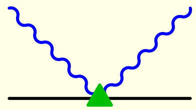
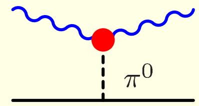
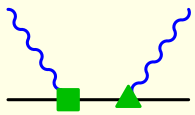
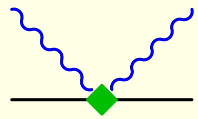
Tree graphs

Born terms give the Thomson term and spin-dependent LETs (ensured by gauge and Lorentz invariance)

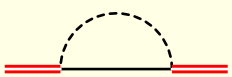
	contribution with typical size	$\omega \sim m_\pi$	$\omega \sim \Delta$
(i)		$e^2\delta^0$ (LO)	$e^2\delta^0$
(ii)	(a)  (b)  (c) 	$e^2\delta^2$	$e^2\delta^1$
(iii)	(a)  (b) 	$e^2\delta^4$	$e^2\delta^2$

Tree graphs

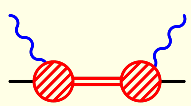
Born terms give the Thomson term and spin-dependent LETs (ensured by gauge and Lorentz invariance)

contribution with typical size		$\omega \sim m_\pi$	$\omega \sim \Delta$
(i)		$e^2\delta^0$ (LO)	$e^2\delta^0$
(ii)	(a)  (b)  (c) 	$e^2\delta^2$	$e^2\delta^1$
(iii)	(a)  (b) 	$e^2\delta^4$	$e^2\delta^2$

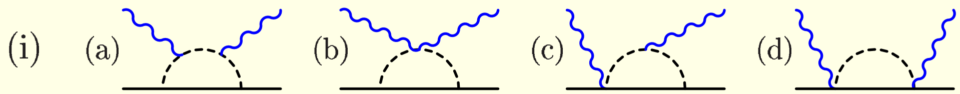
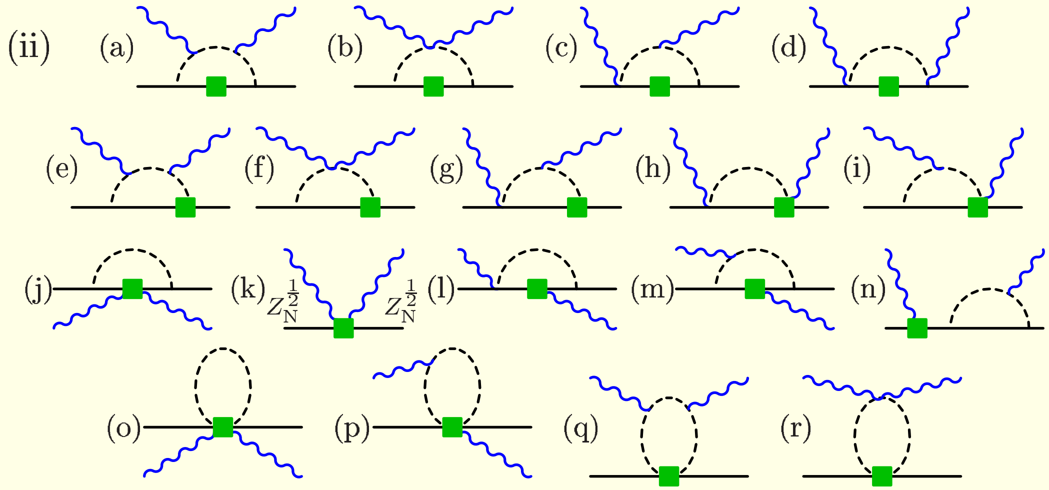
In resonance region Delta-pole graph dominates: width from resuming self-energy



$$\Rightarrow \quad S_\Delta \sim \frac{1}{\omega - (M_\Delta - M_N) + i\Gamma(\omega)}$$

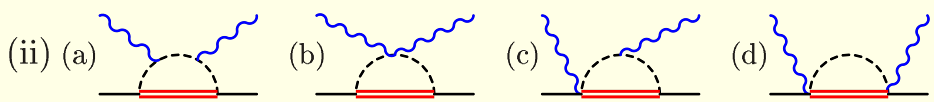
(i)		$e^2\delta^3$	$e^2\delta^{-1}$ (LO)

Loops

contribution with typical size	$\omega \sim m_\pi$	$\omega \sim \Delta$
	$e^2\delta^2$	$e^2\delta^1$
	$e^2\delta^4$	$e^2\delta^2$

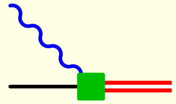
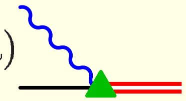
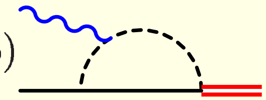
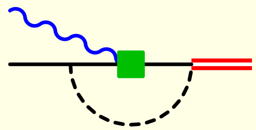
At 4th order we have $1/M$ corrections and c_i contributions

Delta loops are less important in low-energy region

	$e^2\delta^3$	$e^2\delta^1$

Important: predicts full energy-dependent amplitudes, not just polarisabilities

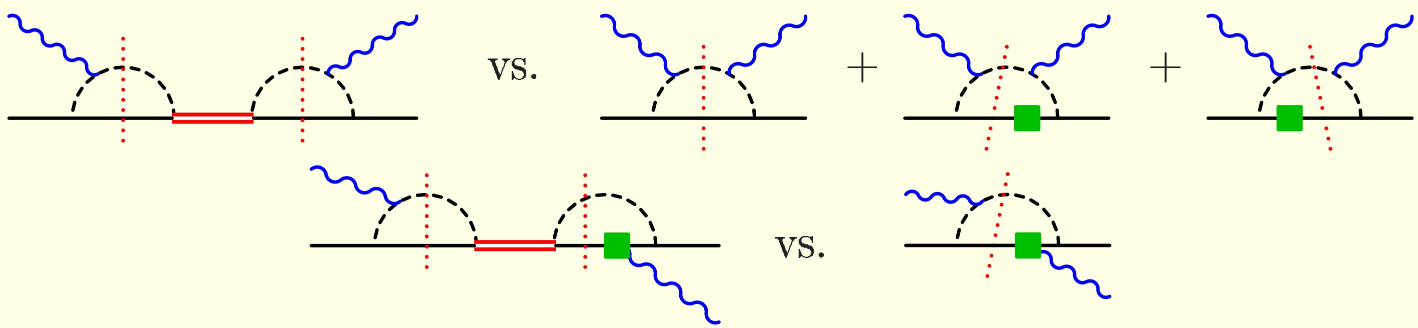
Running of γNΔ vertex

contribution with typical size		$\omega \sim m_\pi$	$\omega \sim \Delta$
(i)		$e\delta^2$	$e\delta^1$
(ii)	(a)  (b) 	$e\delta^4$	$e\delta^2$
(iii)		$e\delta^6$	$e\delta^3$

The inclusion of the imaginary part of running vertices satisfies Watson's theorem

- cancellation of $I = 3/2$ loops at resonance

(i) (ii)



Restricting to lowest photon angular momentum, but at finite photon energy, we can write the effective Hamiltonian

$$
\begin{aligned}
H_{eff} &= \frac{(\mathbf{p}-Q\mathbf{A})^2}{2m} + Q\phi - \frac{1}{2}4\pi\Big(\alpha_{E1}(\omega)\vec{E}^2 + \beta_{M1}(\omega)\vec{H}^2 \\
&+ \gamma_{E1E1}(\omega)\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1M1}(\omega)\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} \\
&- 2\gamma_{M1E2}(\omega)E_{ij}\sigma_i H_j + 2\gamma_{E1M2}(\omega)H_{ij}\sigma_i E_j\Big)
\end{aligned}
$$

with $\alpha \equiv \alpha_{E1}(0)$ etc

We can predict the full energy-dependence of the amplitudes, and only the value at the origin for α, β and γ_{M1M1} are fitted.

Multipoles

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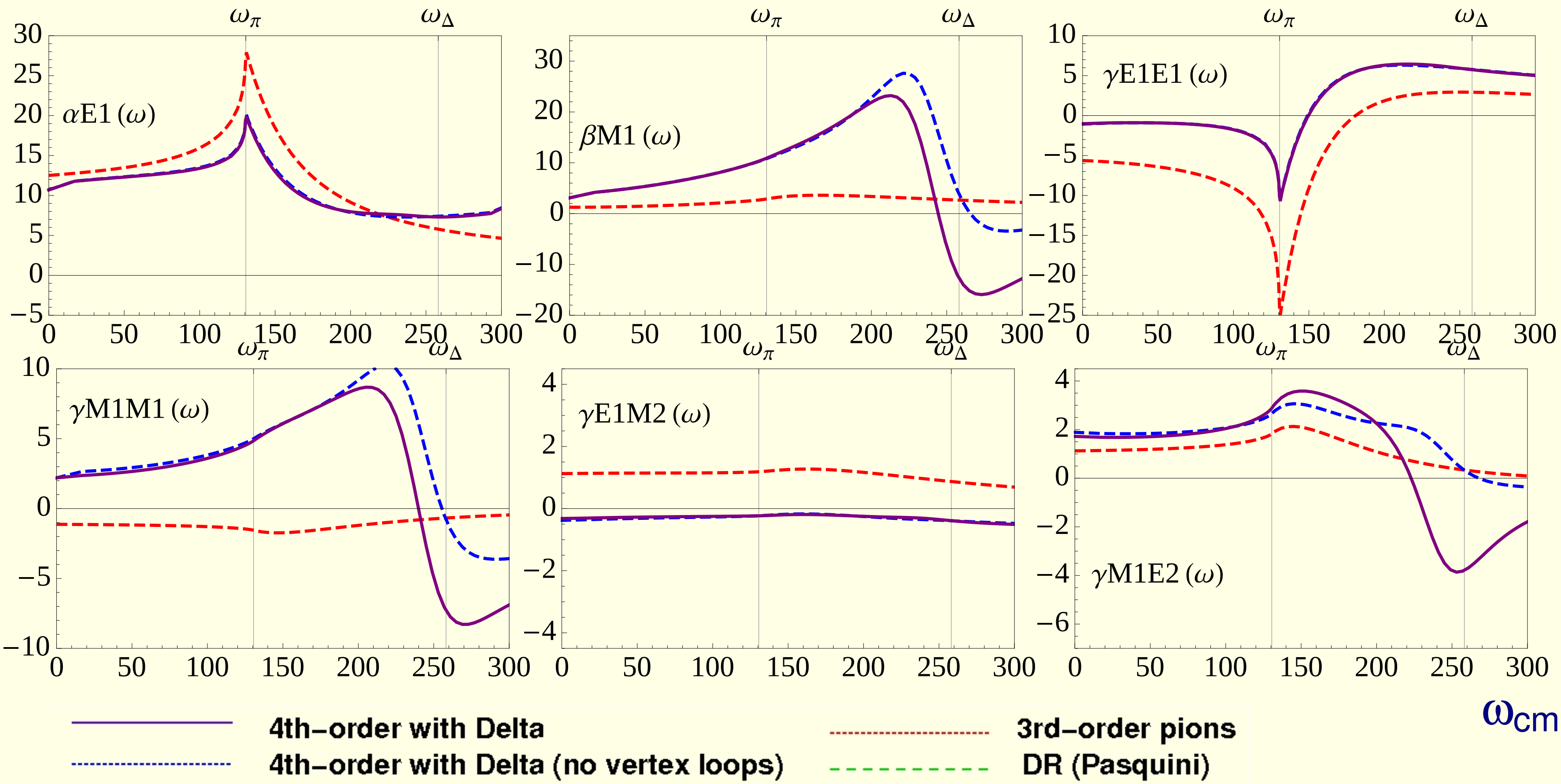

Note contribution of Delta, and also of the running of the γNΔ vertex.

JMcG *et al.* , in preparation

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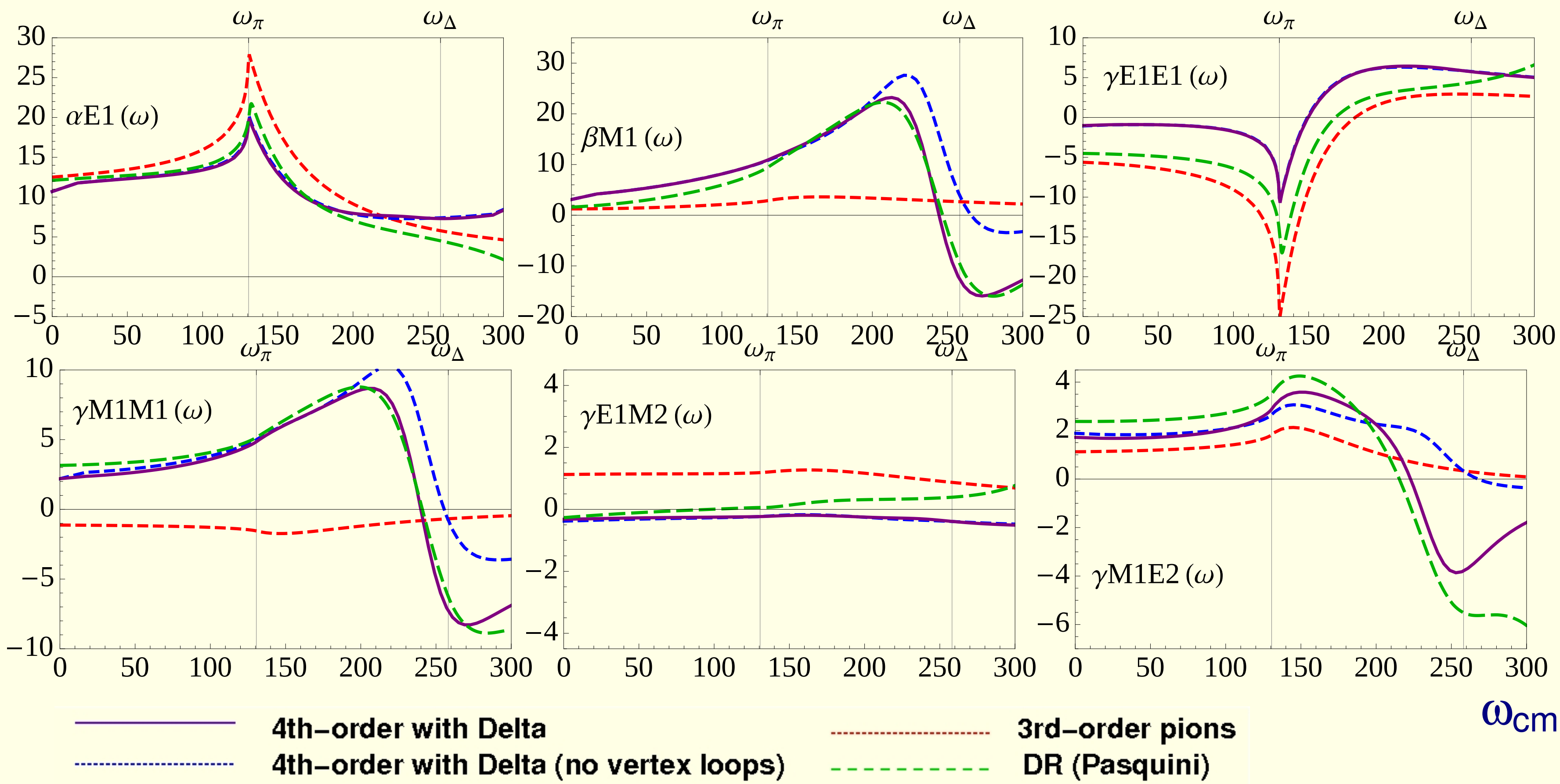
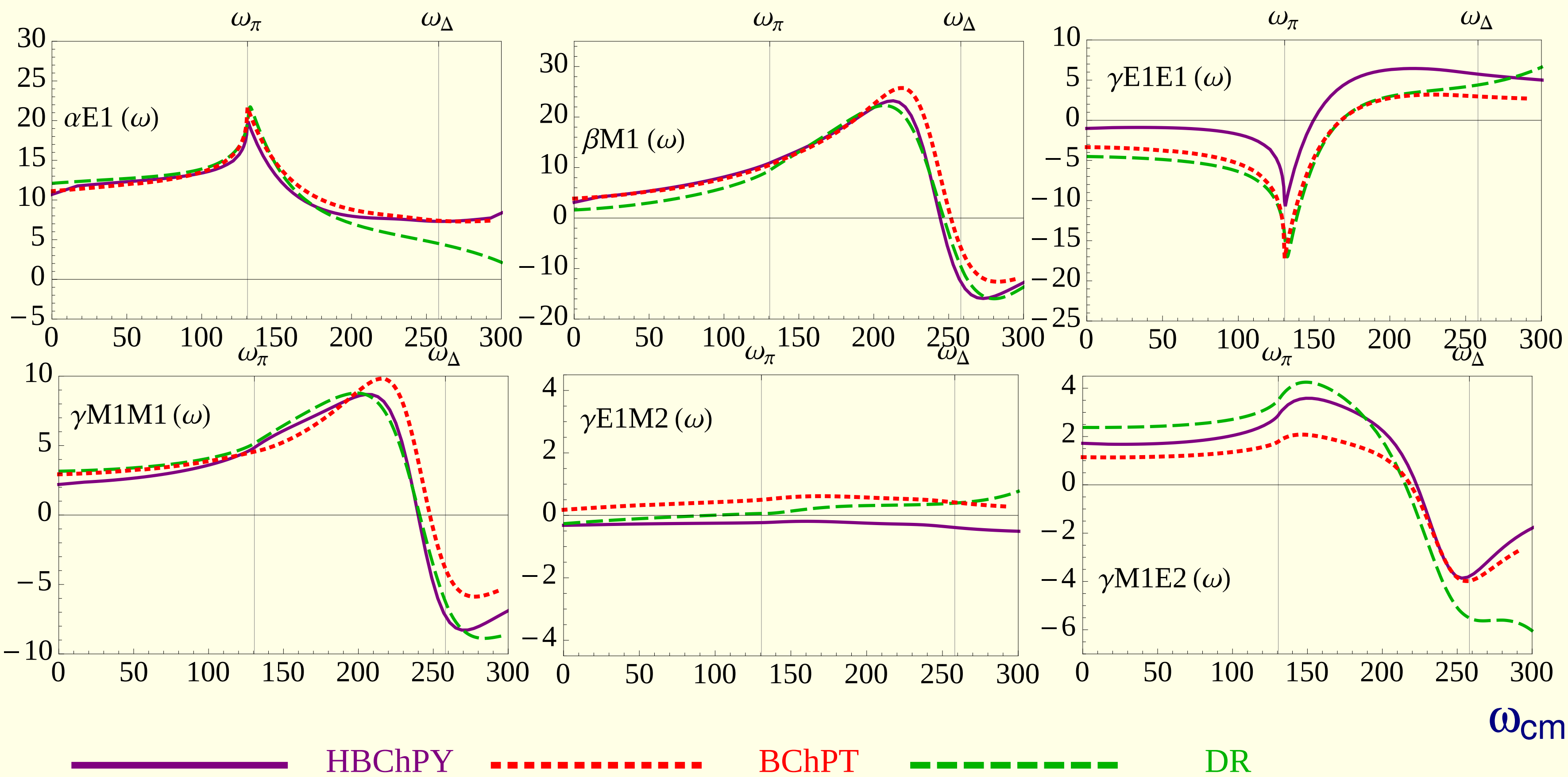


Note contribution of Delta, and also of the running of the $\gamma N\Delta$ vertex.

JMcG *et al.* , in preparation

Aside: Comparison of Multipoles

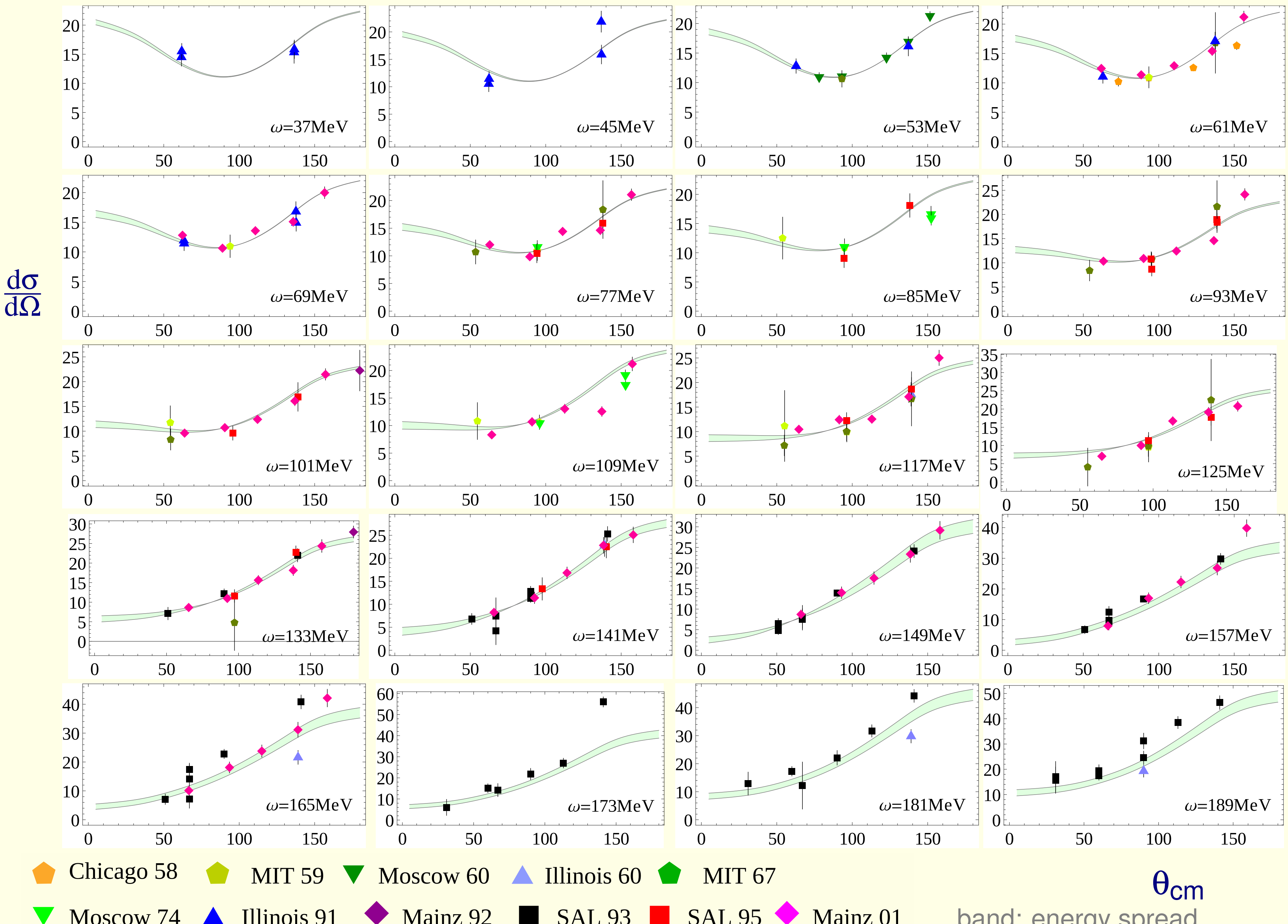
Different predictions do not fully agree on the physical origins of the polarisabilities.
But Chiral and DR predictions agree very well for the shape of the energy dependence of corresponding multipoles



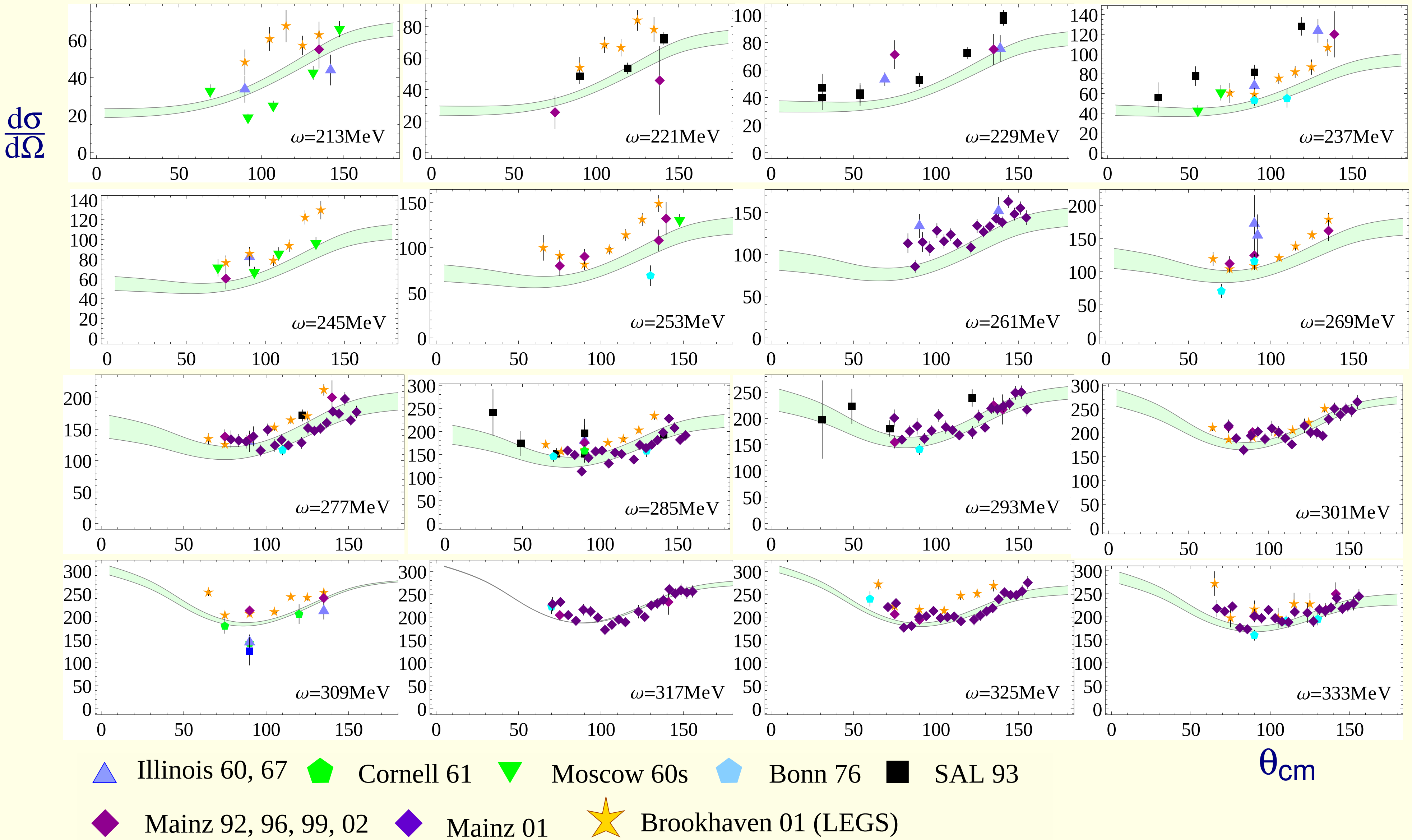
DR: Hildebrandt *et al.* , Eur. Phys. J. A **20** 293 (2004) Chiral: V Lensky *et al.* EPJC **75** 604 (2015)

Our strategy: Static polarisabilities best obtained from Compton scattering.

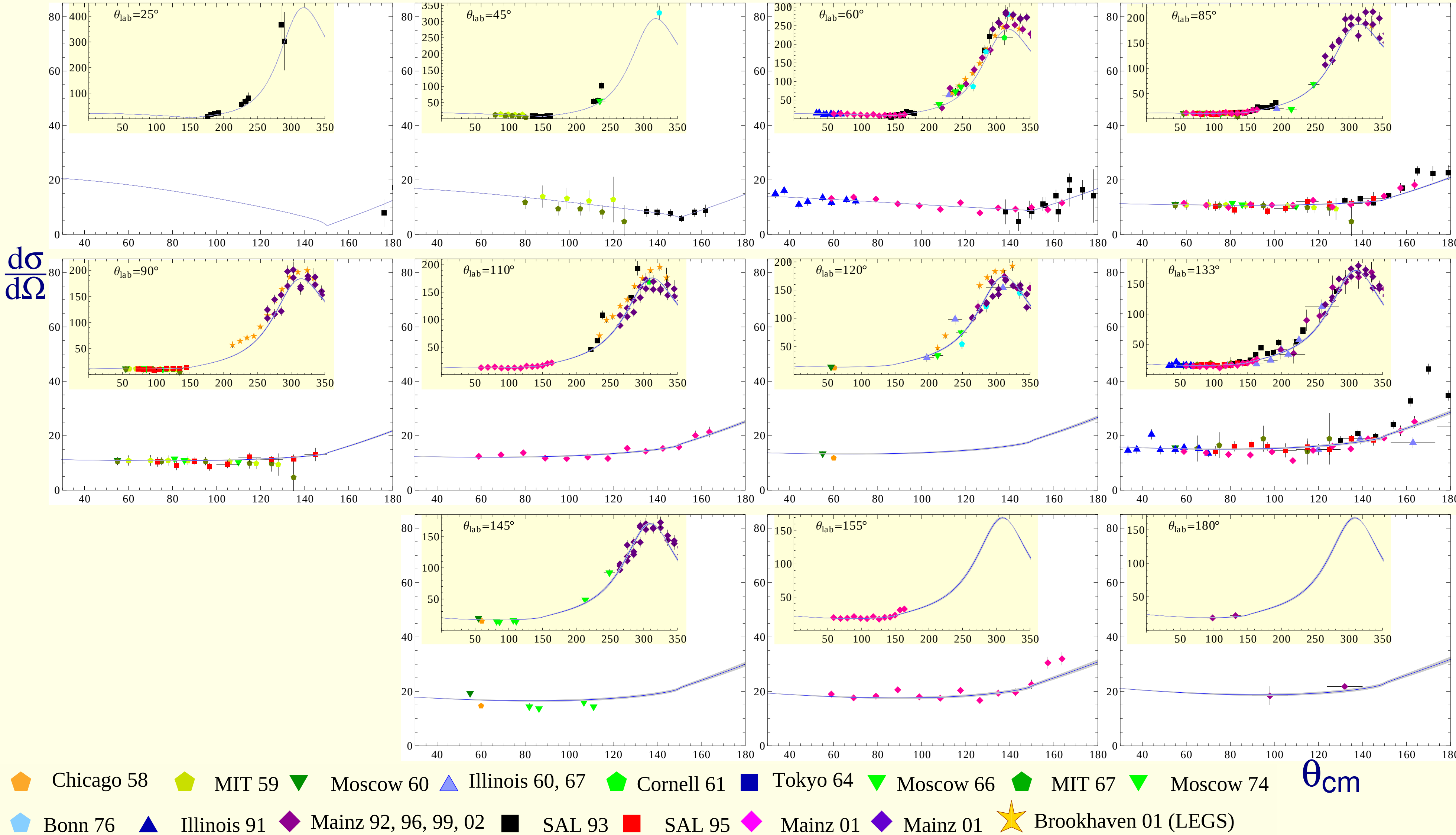
Fitting the proton data



band: energy spread



band gives spread of theory curve due to energy binning



Constraining $\alpha + \beta$ with Baldin Sum rule and fitting consistent data set up to 170 MeV:

$$\alpha_p = (10.65 \pm 0.35(\text{stat}) \pm 0.2(\text{Bald}) \pm 0.3(\text{theory})) \times 10^{-4}\,\text{fm}^3$$

$$\beta_p = (3.15 \mp 0.35(\text{stat}) \pm 0.2(\text{Bald}) \mp 0.3(\text{theory})) \times 10^{-4}\,\text{fm}^3$$

Comparison

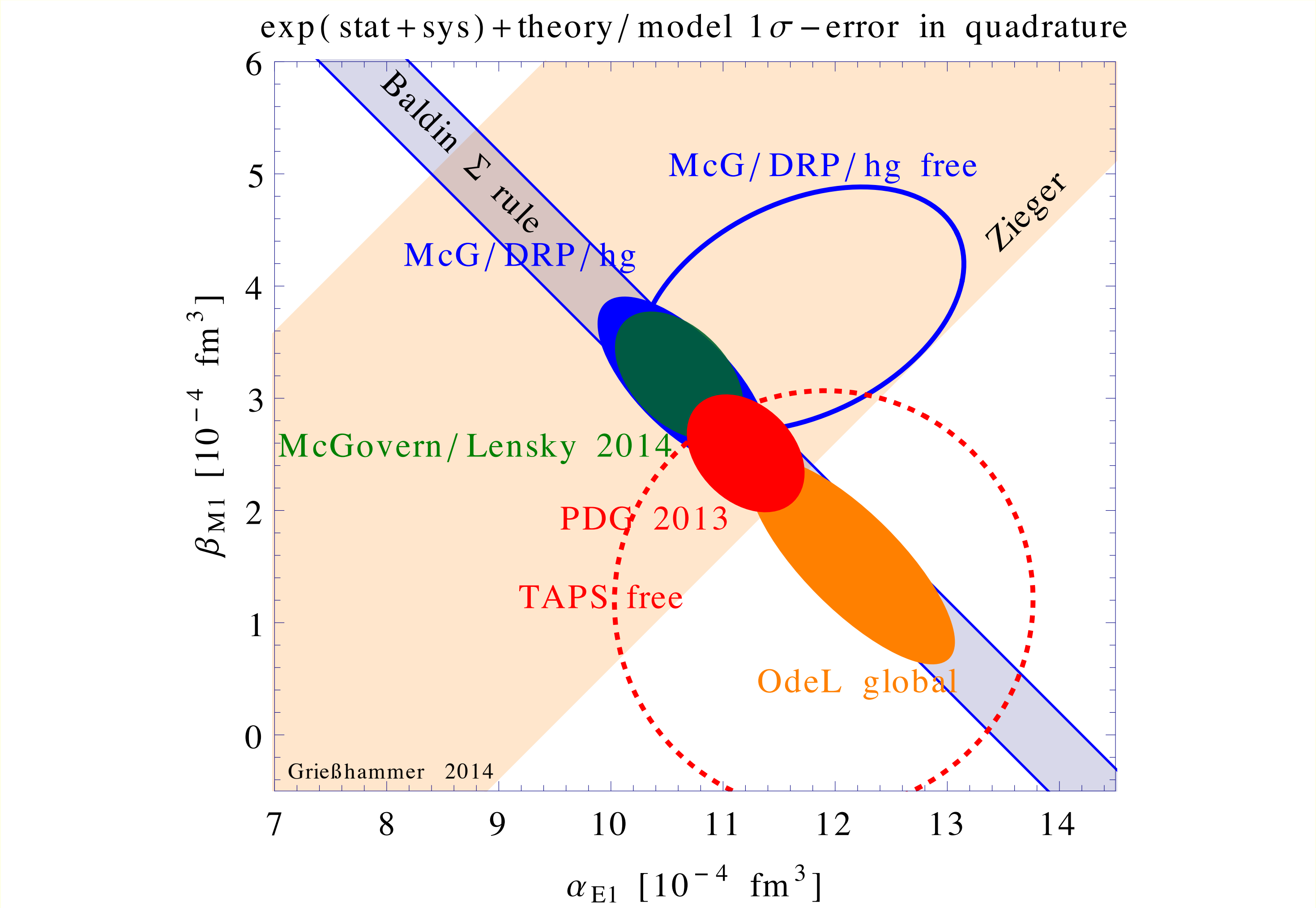
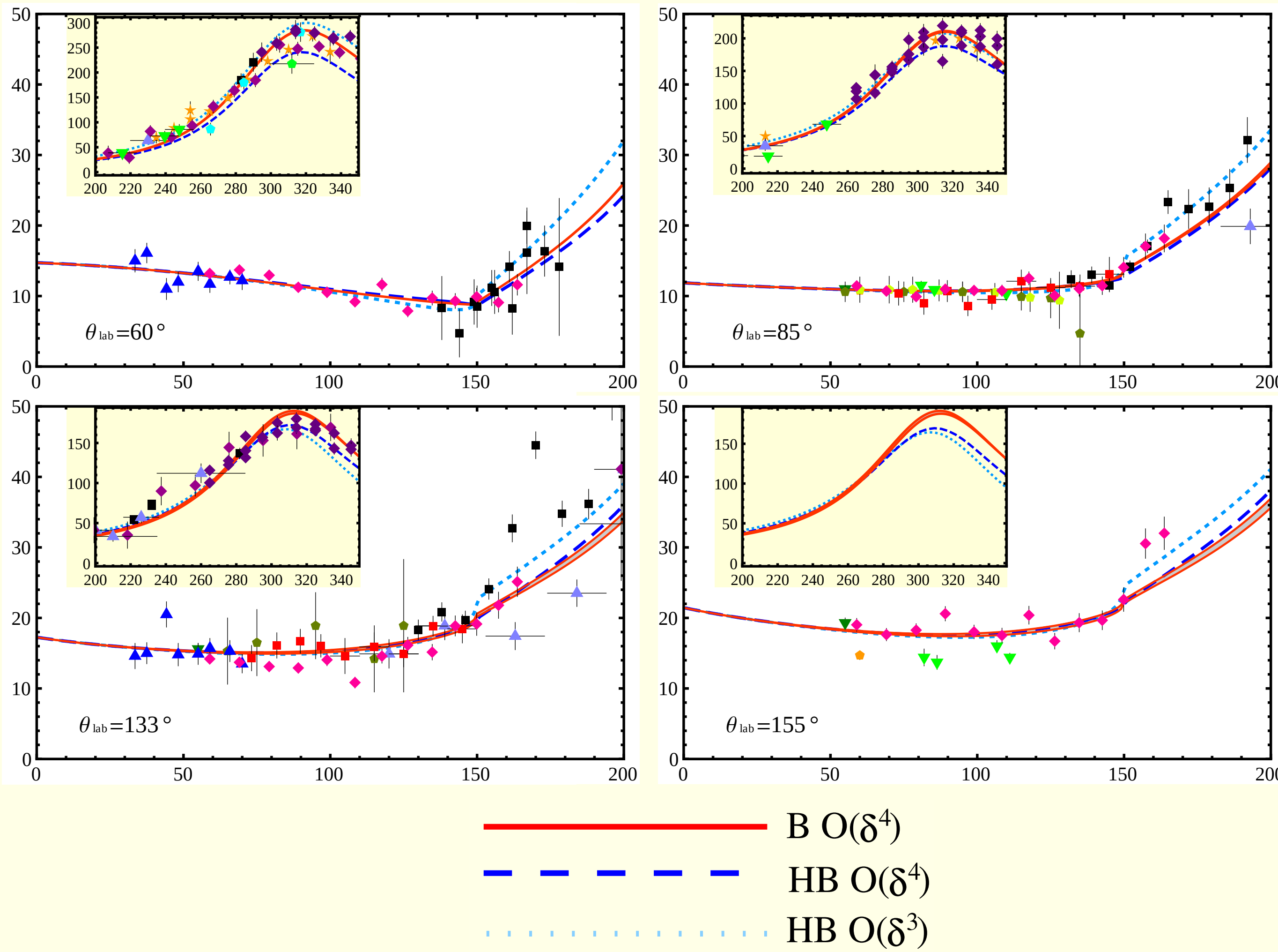


figure courtesy of H. Grießhammer

Checking in covariant framework (3rd order)



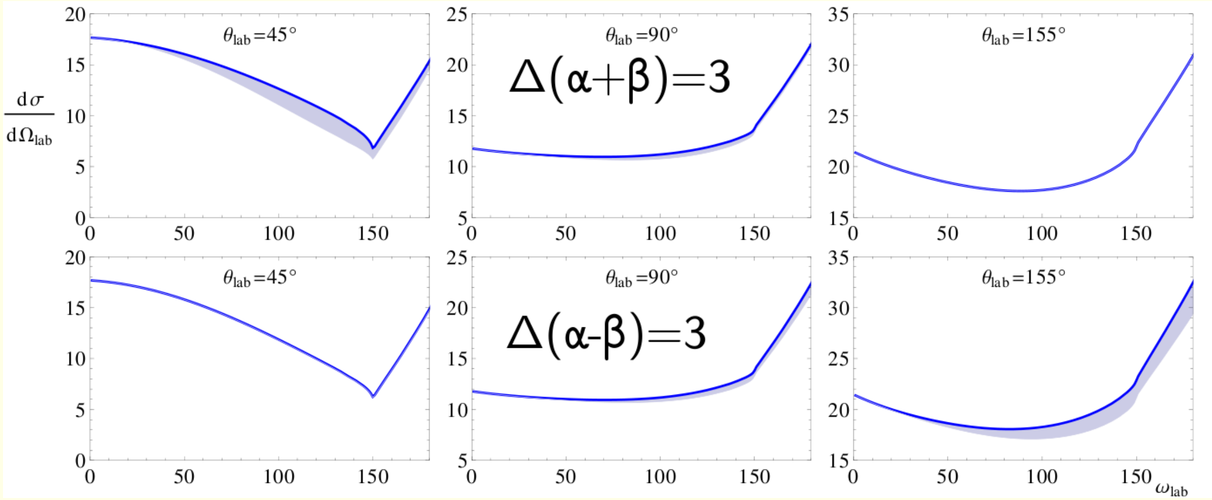
$$\alpha_p = (10.6 \pm 0.25(\text{stat}) \pm 0.2(\text{Bald}) \pm 0.4(\text{theory})) \times 10^{-4}\,\text{fm}^3$$
$$\beta_p = (3.2 \mp 0.25(\text{stat}) \pm 0.2(\text{Bald}) \pm 0.4(\text{theory})) \times 10^{-4}\,\text{fm}^3$$

V. Lensky & JMcG Phys. Rev. **C89** 032202 (2014) ; V. Lensky *et al.* Phys. Rev. **C86** 048201 (2012)

More data needed?

We fit to low-energy data (up to 164 MeV), but with constraints from the higher-energy data to ensure the Δ parameters are sensible.

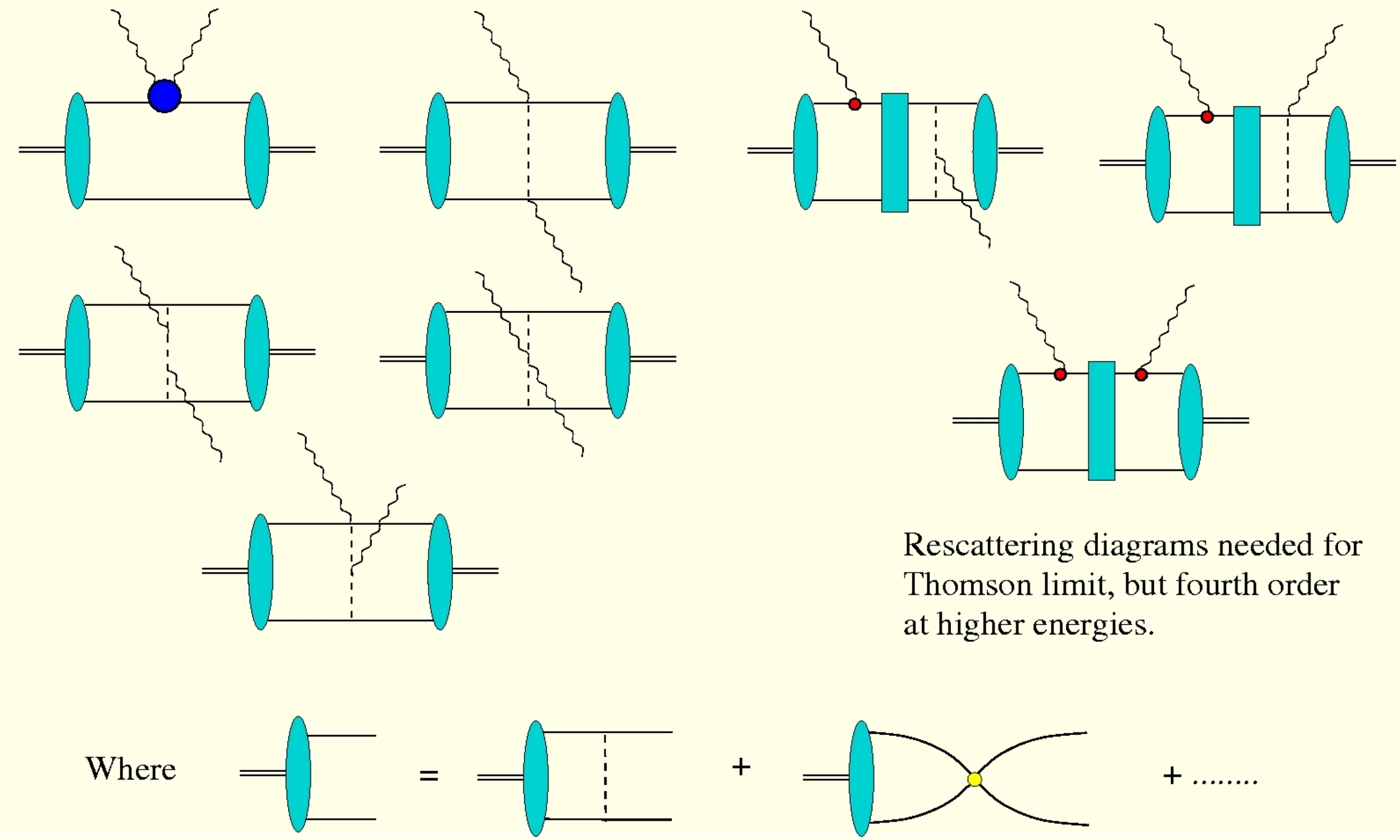
In spite of the amount of data, the sensitivity to the polarisabilities especially β is not very high. Magnetic response varies rapidly with energy and zero-energy value is only a small fraction of the total by 150 MeV.



What would help

- Better low-energy data! (Theorist's view...)
- More data in the region 160-250 MeV
- More data especially at forward angles
- Data for polarised scattering (beam and target)

Consistent treatment of one- and two-body diagrams



The Δ only enters in ● at this order.

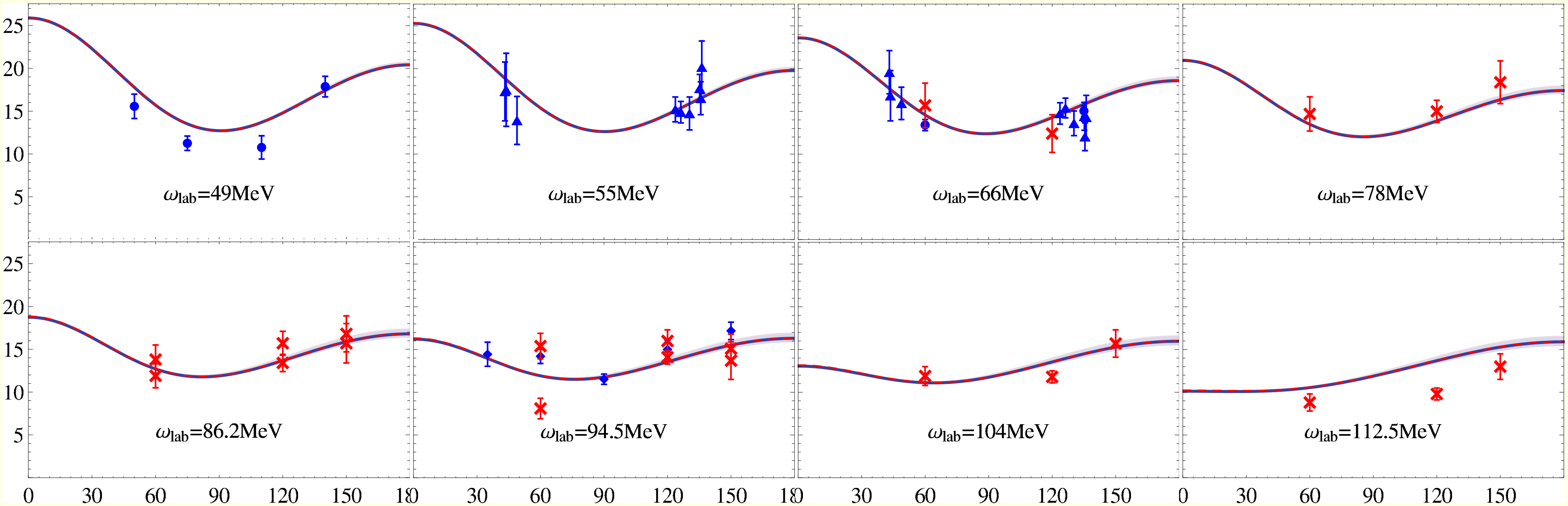
Ensuring correct Thomson limit for deuteron is important even at 50-60 MeV.

Extraction of isoscalar polarisabilities

So far only $O(Q^3)$; further work required to go above pion threshold.

Older data from Illinois ●, Saskatoon, ◆ and Lund ▲ (29 pts in total)

New data from Lund ×, 23 points. Myers *et al.* , Phys. Rev. Lett. **113,** 262506 (2014)



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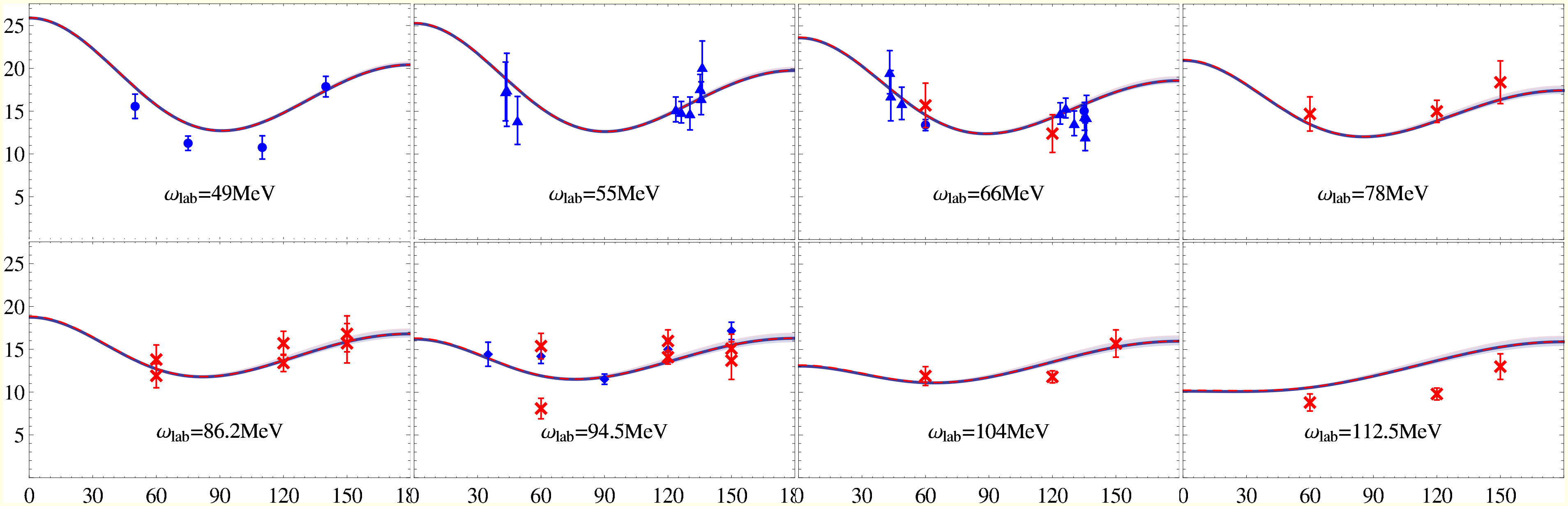


$$\alpha_s = 11.1 \pm 0.6(\text{stat}) \pm 0.2(\text{BSR}) \pm 0.8(\text{th})$$
$$\beta_s = 3.4 \mp 0.6(\text{stat}) \pm 0.2(\text{BSR}) \mp 0.8(\text{th}).$$

$$\alpha_n = 11.65 \pm 1.25(\text{stat}) \pm 0.2(\text{BSR}) \pm 0.8(\text{th})$$
$$\beta_n = 3.55 \mp 1.25(\text{stat}) \pm 0.2(\text{BSR}) \mp 0.8(\text{th})$$

Comparison

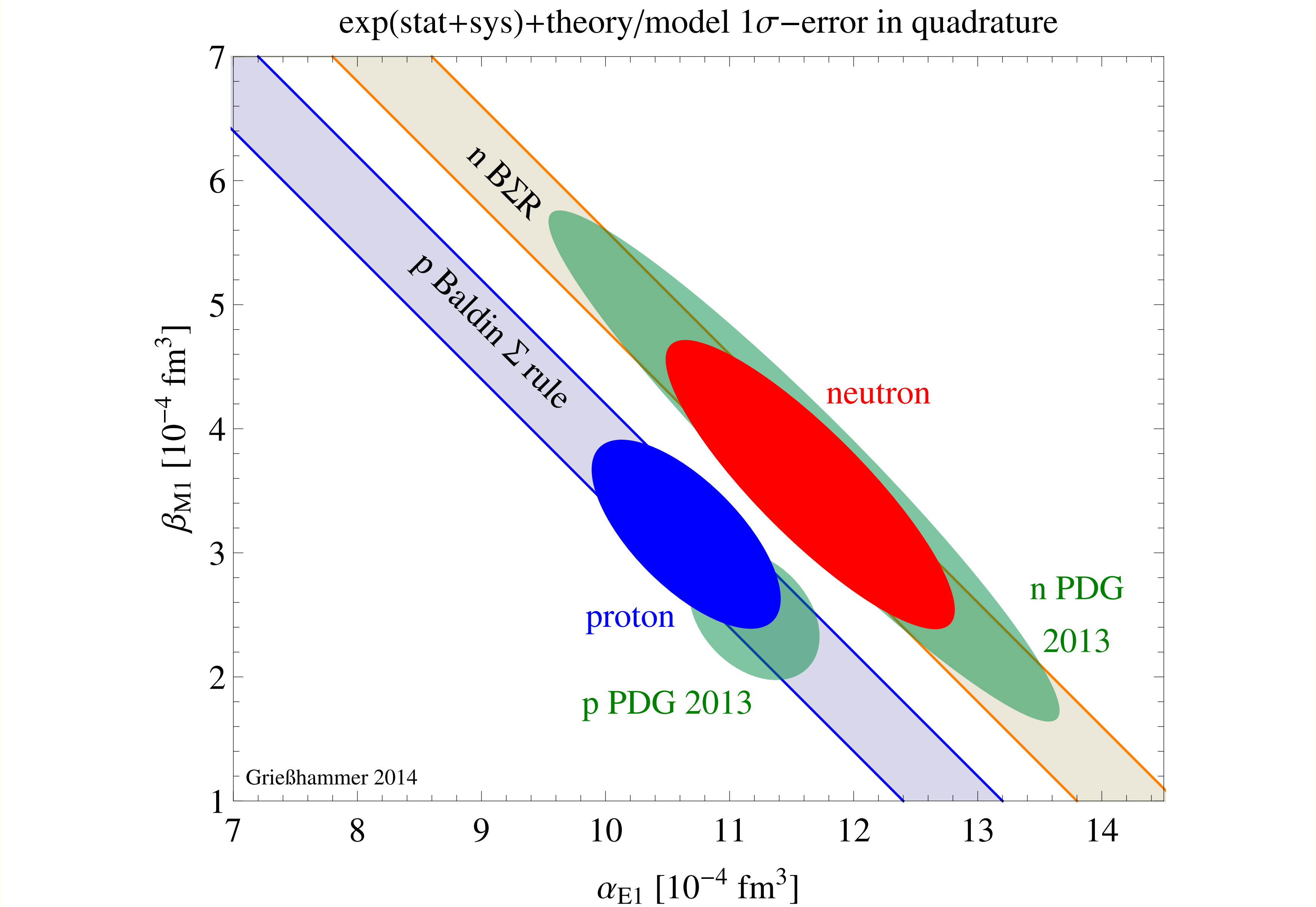
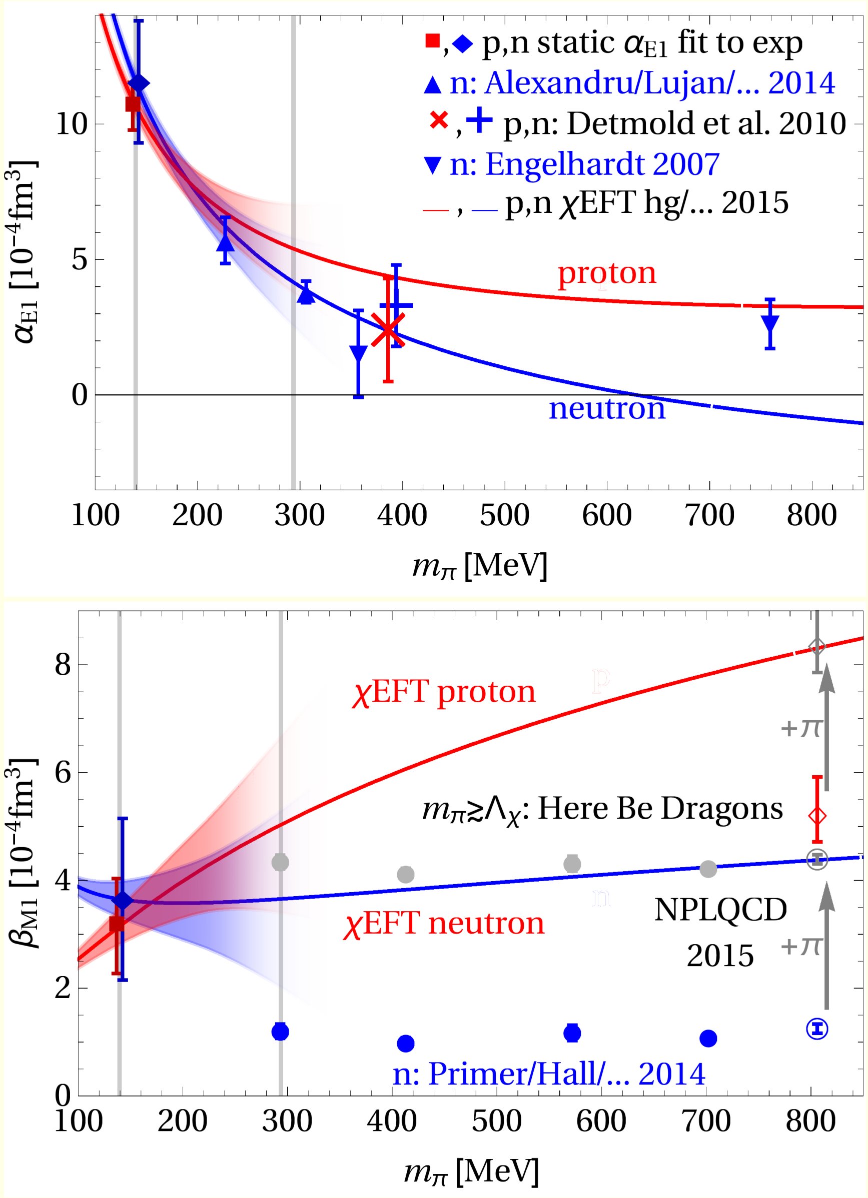


figure courtesy of H. Grießhammer

Lattice and chiral extrapolations



H. Grießhammer, JMcG, D. Phillips arXiv:1511.01952

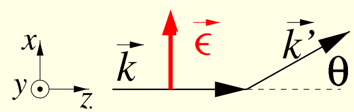
Spin-dependent Compton scattering

$$H_{\text{eff}} = \frac{(\mathbf{p}-Q\mathbf{A})^2}{2m} + Q\phi - \frac{(Q+\kappa)}{2m}\sigma\cdot H - \frac{1}{2}4\pi\Big(\alpha\vec{E}^2 + \beta\vec{H}^2$$
$$+ \gamma_{E1E1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1M1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} - 2\gamma_{M1E2}E_{ij}\sigma_i H_j + 2\gamma_{E1M2}H_{ij}\sigma_i E_j\Big)$$

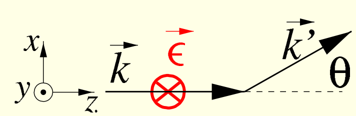
Spin-polarisabities have most influence if the beam or target or both are polarised.

Linearly polarised beam $\Sigma_3 = \dfrac{\sigma^{\parallel} - \sigma^{\perp}}{\sigma^{\parallel} + \sigma^{\perp}}$

$\left[\dfrac{d\sigma}{d\Omega}\right]^{\text{lin}}_x$:

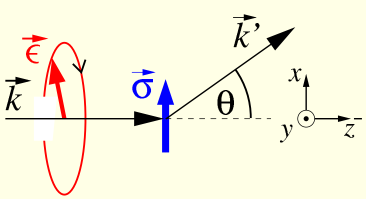


$\left[\dfrac{d\sigma}{d\Omega}\right]^{\text{lin}}_y$:

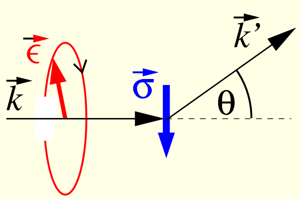


Circular beam, polarised target $\quad \Sigma_{2x} = \dfrac{\sigma^R_{\perp} - \sigma^L_{\perp}}{\sigma^R_{\perp} + \sigma^L_{\perp}} \quad \Sigma_{2z} = \dfrac{\sigma^R_{\parallel} - \sigma^L_{\parallel}}{\sigma^R_{\parallel} + \sigma^L_{\parallel}}$

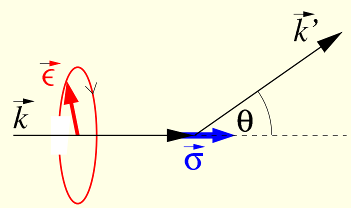
Δ^{circ}_x:



$-$



Δ^{circ}_z:



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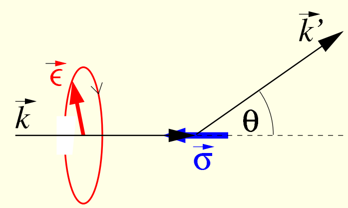
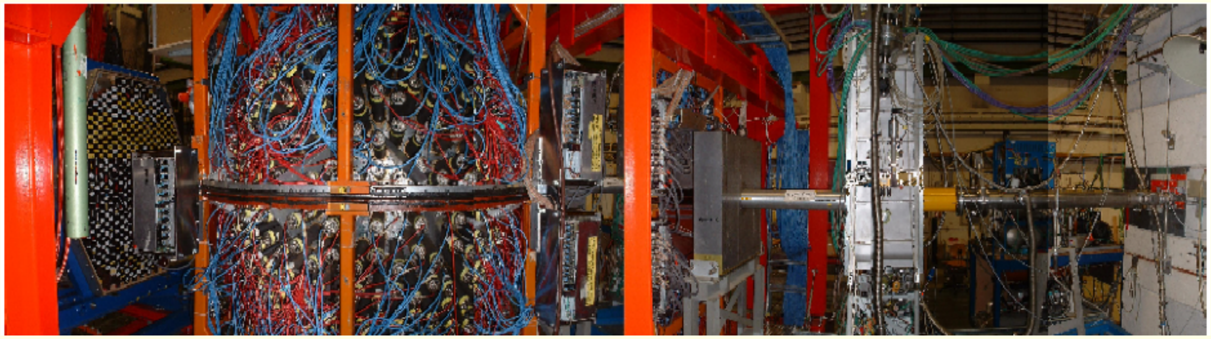
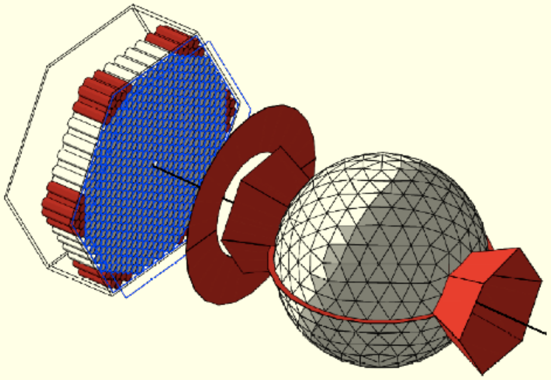


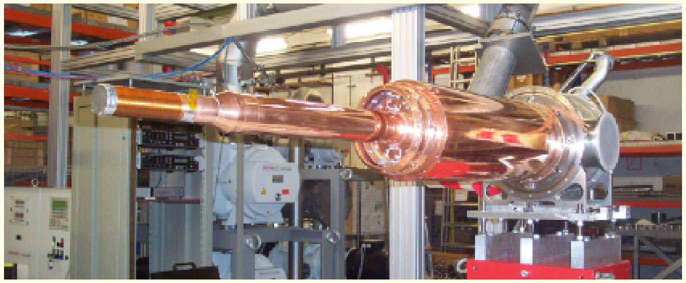
New programme at A2 experiment using Crystal Ball and TAPS detectors





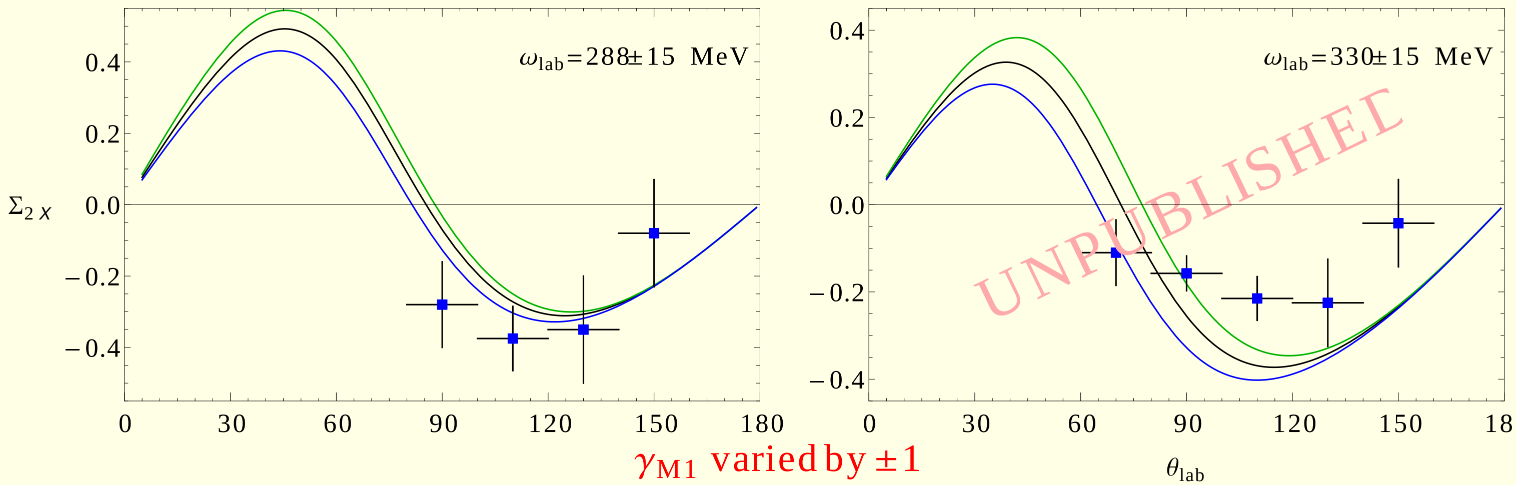
Large-acceptance detector

Tagged photon beam, circ. or lin. polarised or unpolarised,

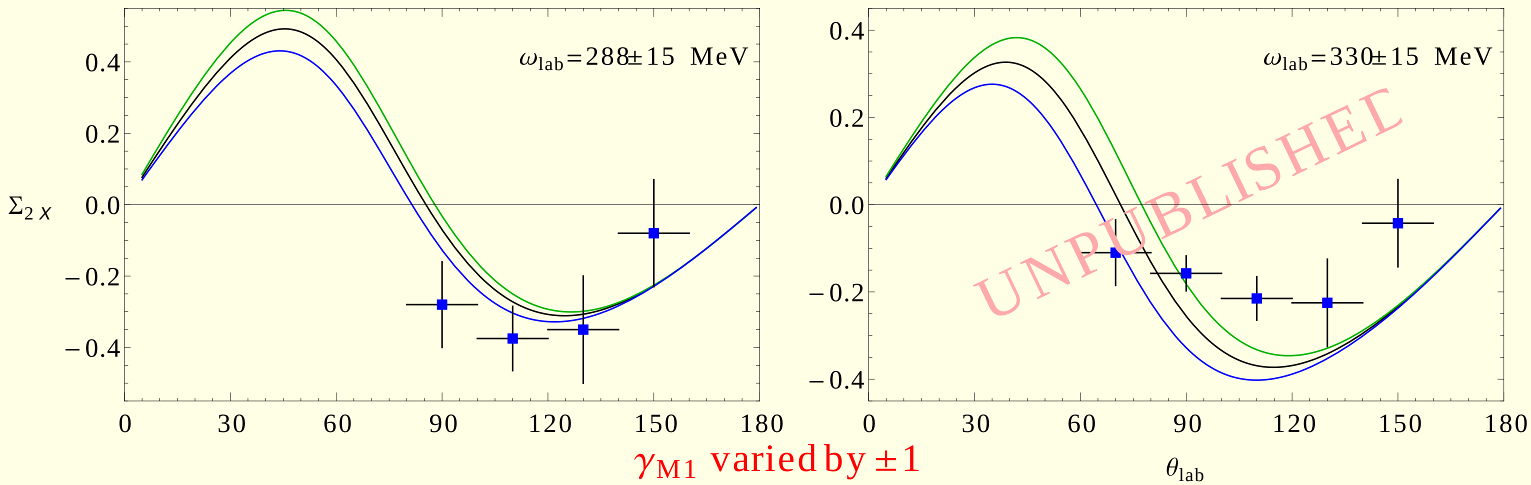


Unpolarised (liquid hydrogen)... or polarised (butanol) protons

Σ_{2x}: Target polarised perpendicular to reaction plane, RH or LH circularly polarised photons P. Martell, PhD thesis

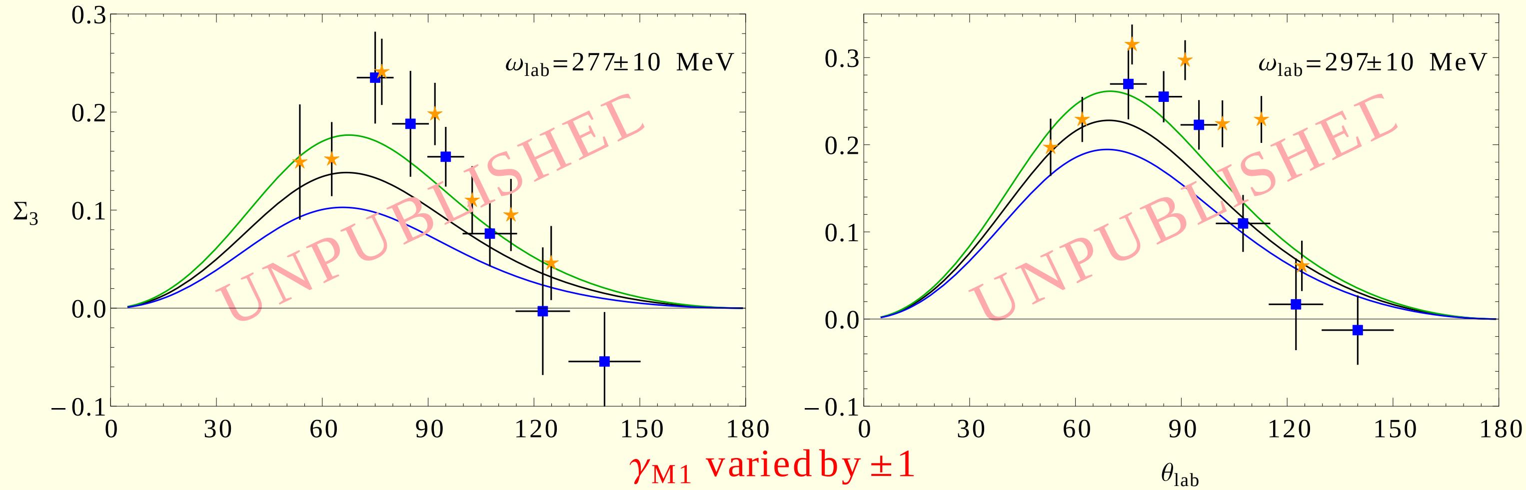


Σ_{2x}: Target polarised perpendicular to reaction plane, RH or LH circularly polarised photons P. Martell, PhD thesis



Σ_3: Unpolarised target, photons polarised in or perpendicular to reaction plane

■ C. Collicott, PhD thesis (LEGS data ☆)



Predictions and fits for proton polarisabilities

Chiral prediction (δ^3, BChPT, Lensky *et al.* , EPJC **75** 604 (2015)) and NLO (δ^4, HBChPT, JMcG *et al.* E. P. J. A **49** 12 (2013), Grießhammer *et al.* 1511.01952

	$\alpha+\beta$	$\alpha-\beta$	γ_0	γ_π
δ^3 B	15.1 ± 1.0	7.3 ± 1.0	-0.9 ± 1.4	$[-46.4]+7.2\pm1.7$
δ^4 HB	$13.8\pm0.4^*$	$7.5\pm0.7\pm0.6$	$-2.6\pm0.5_{\text{stat}}\pm0.6^*_{\text{th}}$	$[-46.4]+5.5\pm0.5_{\text{stat}}\pm1.8^*_{\text{th}}$
SR/DR	13.8 ± 0.4	10.7 ± 0.2	-0.9 ± 0.14	$[-46.4]+7.6\pm1.8$

DR: fixed-angle, Drechsel *et al.* Phys. Rep. **378** 99;

	γ_{E1E1}	γ_{M1M1}	γ_{E1M2}	γ_{M1E2}
δ^3 B	-3.3 ± 0.8	2.9 ± 1.5	0.2 ± 0.2	1.1 ± 0.3
δ^4 HB	-1.1 ± 1.9	$2.2\pm0.5_{\text{stat}}\pm0.6^*_{\text{th}}$	-0.4 ± 0.6	1.9 ± 0.5
DR	-3.85 ± 0.45	2.8 ± 0.1	-0.15 ± 0.15	2.0 ± 0.1
MAMI1	-3.5 ± 1.2	3.2 ± 0.9	-0.7 ± 1.2	2.0 ± 0.3
MAMI2	-5.0 ± 1.5	3.1 ± 0.9	1.7 ± 1.7	1.3 ± 0.4

DR: fixed-t, summarised in HG, JMcG, DP & GF Prog. Nucl. Part. Phys. **67** 841 (2012)

MAMI1: published extraction from MAMI Σ_{2x} and LEGS Σ_3 Martel

MAMI2: unpublished extraction from Σ_{2x} and Σ_3 Collicott

δ^4: theory errors from convergence. *: γ_{M1M1} from fit, otherwise $\gamma_{M1M1}=6.4$

Note errors mean different things in different lines; DR especially only reflect spread from two databases

Predictions and fits for neutron polarisabilities

Chiral prediction (δ^3, BChPT, Lensky *et al.* , EPJC **75** 604 (2015)) and NLO (δ^4, HBChPT, Grießhammer *et al.* 1511.01952

	$\alpha+\beta$	$\alpha-\beta$	γ_0	γ_π
δ^3 B	18.3 ± 4.1	9.1 ± 4.1	0 ± 1.4	$[46.4]+9.0\pm2.0$
δ^4 HB	15.2 ± 0.4	$8.1\pm2.5\pm0.8$	$0.5\pm0.5_{\text{stat}}\pm1.8^*_{\text{th}}$	$[46.4]+7.7\pm0.5_{\text{stat}}\pm1.8^*_{\text{th}}$
SR/DR	15.2 ± 0.4	11.5	-0.25	$[46.4]\pm13.35$

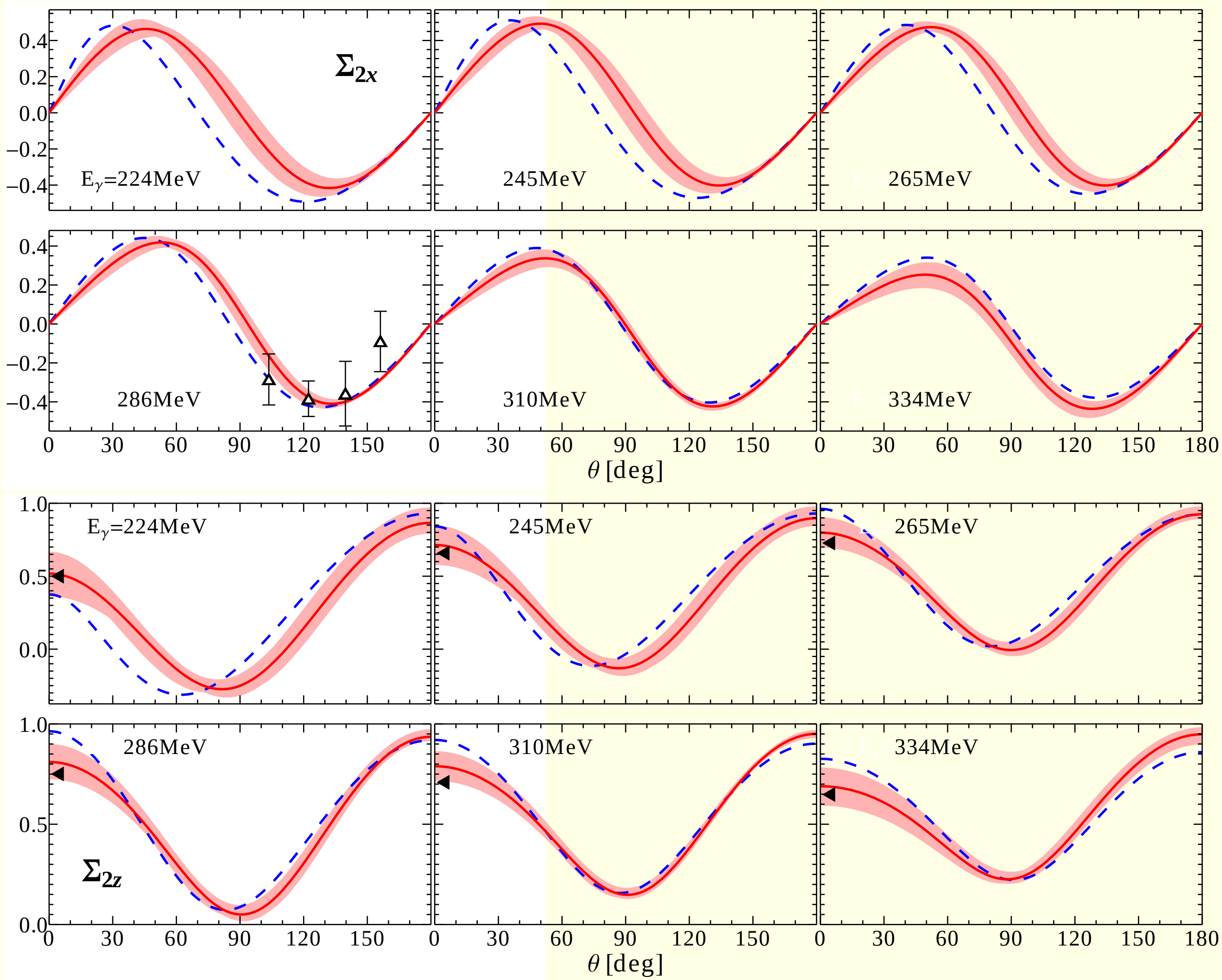
DR: fixed-t, Drechsel *et al.* Phys. Rep. **378** 99;

	γ_{E1E1}	γ_{M1M1}	γ_{E1M2}	γ_{M1E2}
δ^3 B	-4.7 ± 1.1	2.9 ± 1.5	0.2 ± 0.2	1.6 ± 0.4
δ^4	-4.0 ± 1.9	$1.3\pm0.5_{\text{stat}}\pm0.5^*_{\text{th}}$	-0.1 ± 0.6	2.4 ± 0.5
DR	-5.75 ± 0.15	3.8 ± 0.1	-0.8 ± 0.1	3.0 ± 0.1

DR: fixed-t, Holstein *et al.* , Babusci *et al.*

δ^4: theory errors as proton. *: including input from proton fit.

Chiral predictions for asymmetries



- - - - - - - - - : Born+Delta, ———————: $\mathcal{O}(\delta^3)$ BχPT

V. Lensky *et al.* , EPJC **75** 604 (2015)

But: uncertainties more than just polarisabilities

Σ_3: Varying theory details and $\gamma N\Delta$ coupling:

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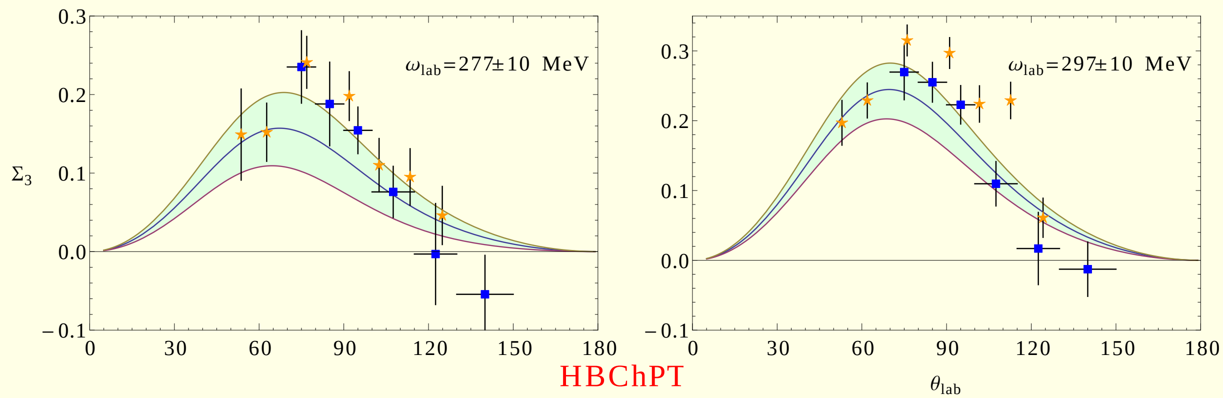
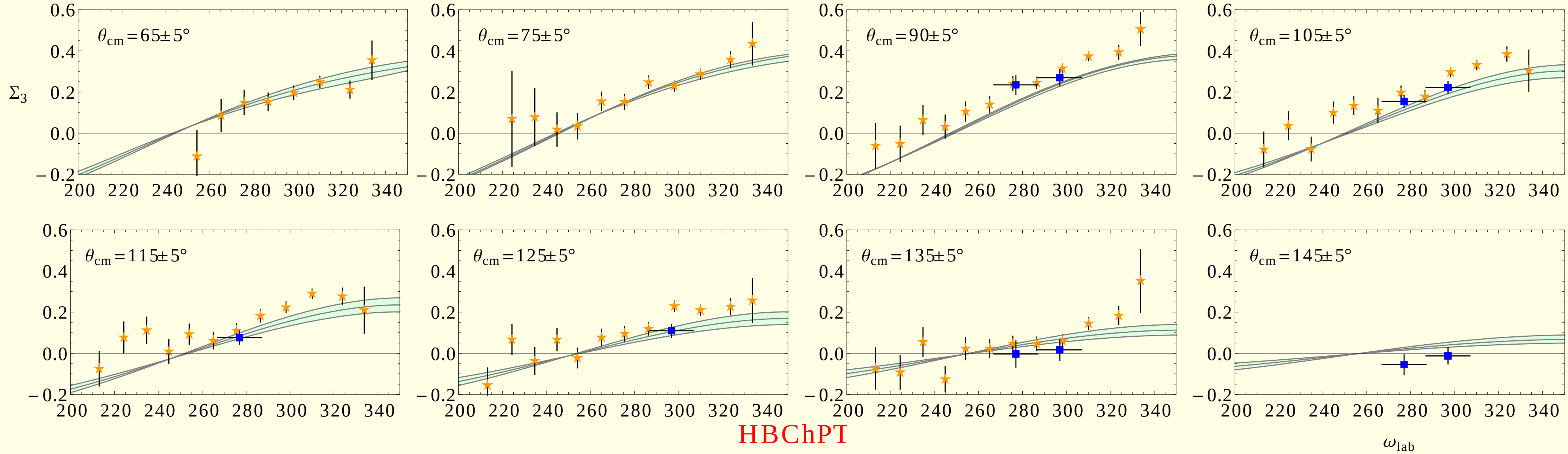
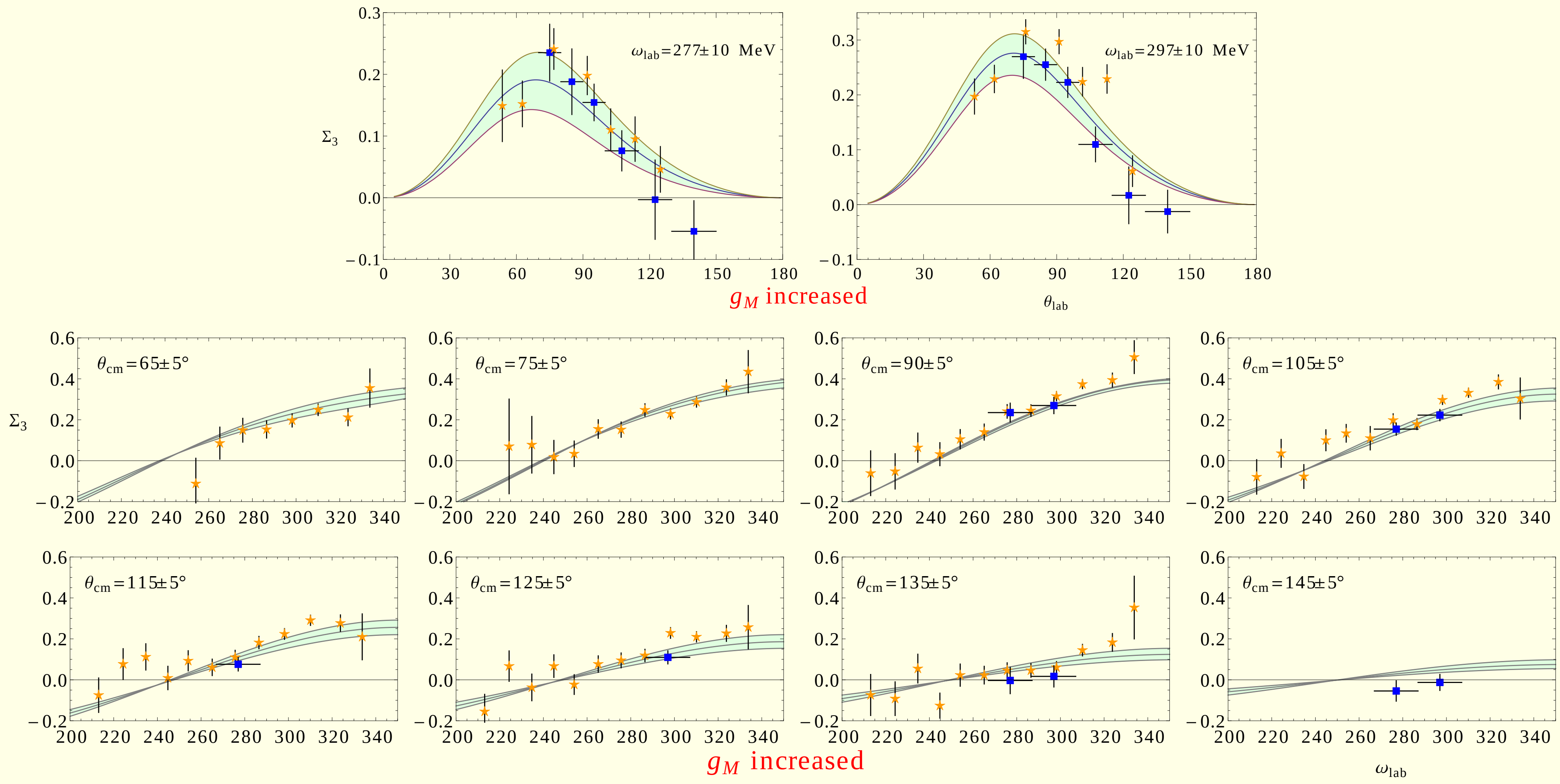




But: uncertainties more than just polarisabilities

Σ_3: Varying theory details and γNΔ coupling:



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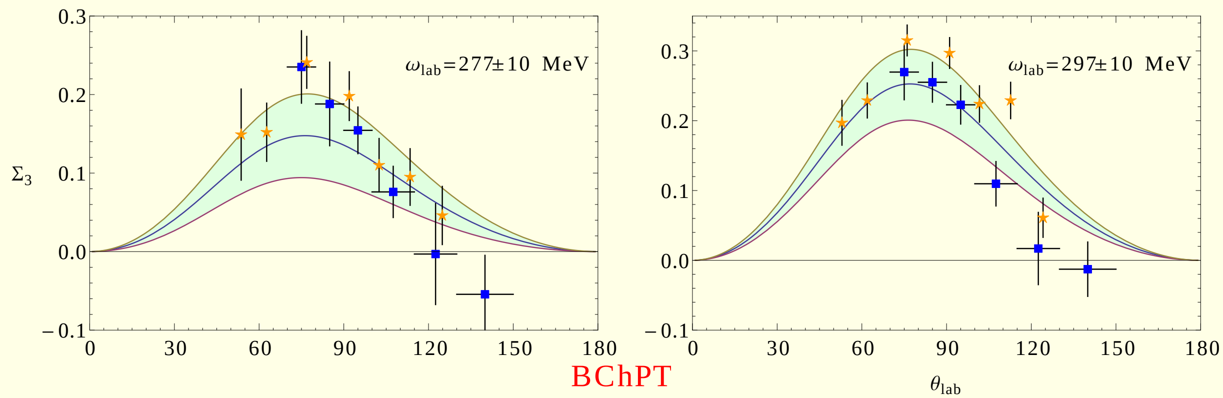
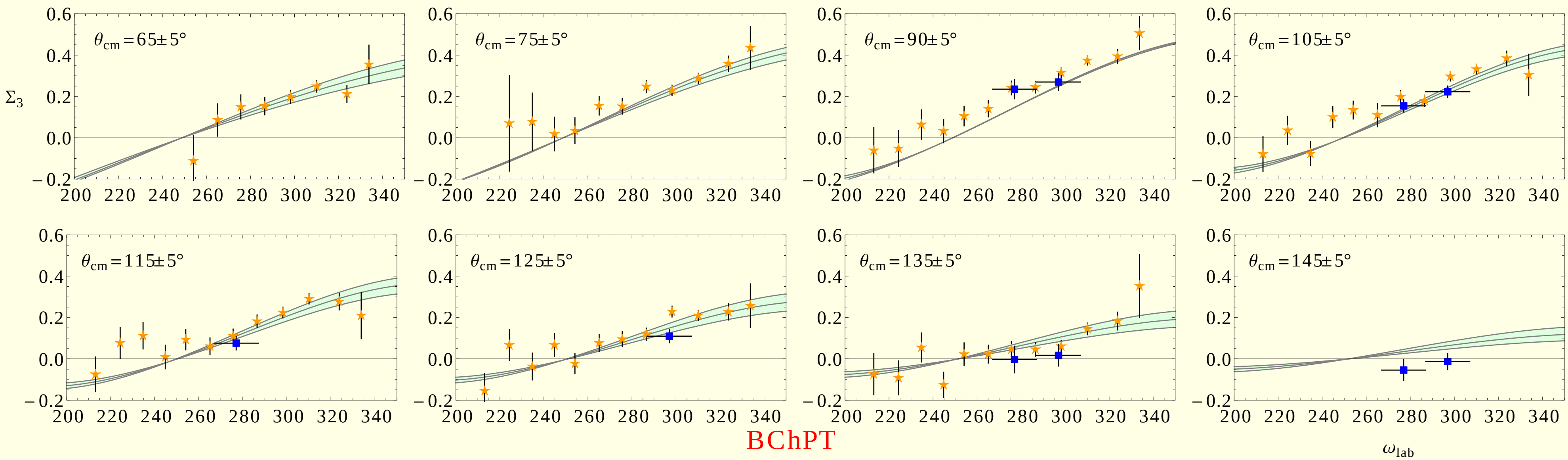


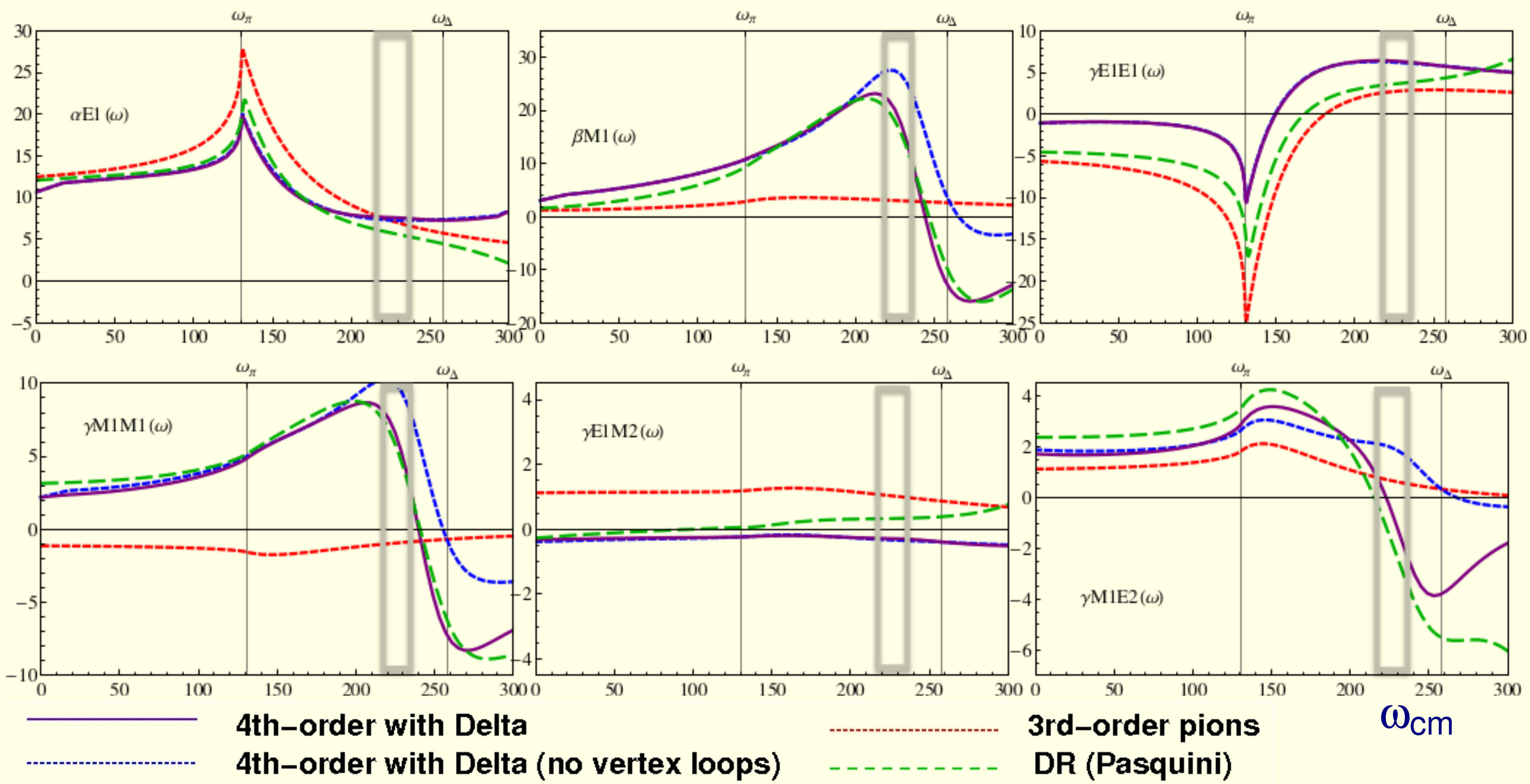


BChPT: V. Lensky *et al.* , EPJC **75** 604 (2015)

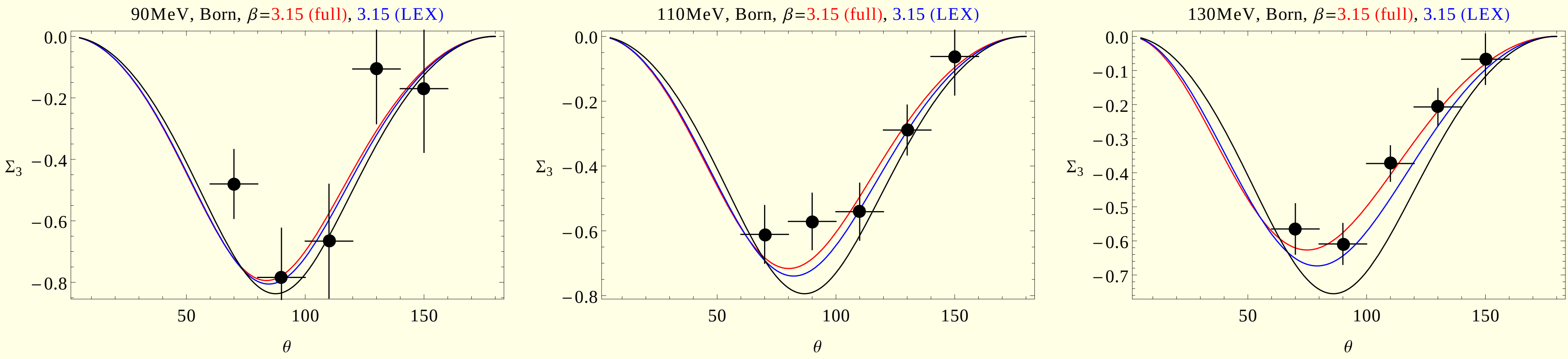
Multipoles again

MAMI data is taken well into the resonance region....

Not ideal for extracting zero-energy polarisabilities!



Some PRELIMINARY data on Σ_3 from MAMI V. Sokhoyan and E. Downie



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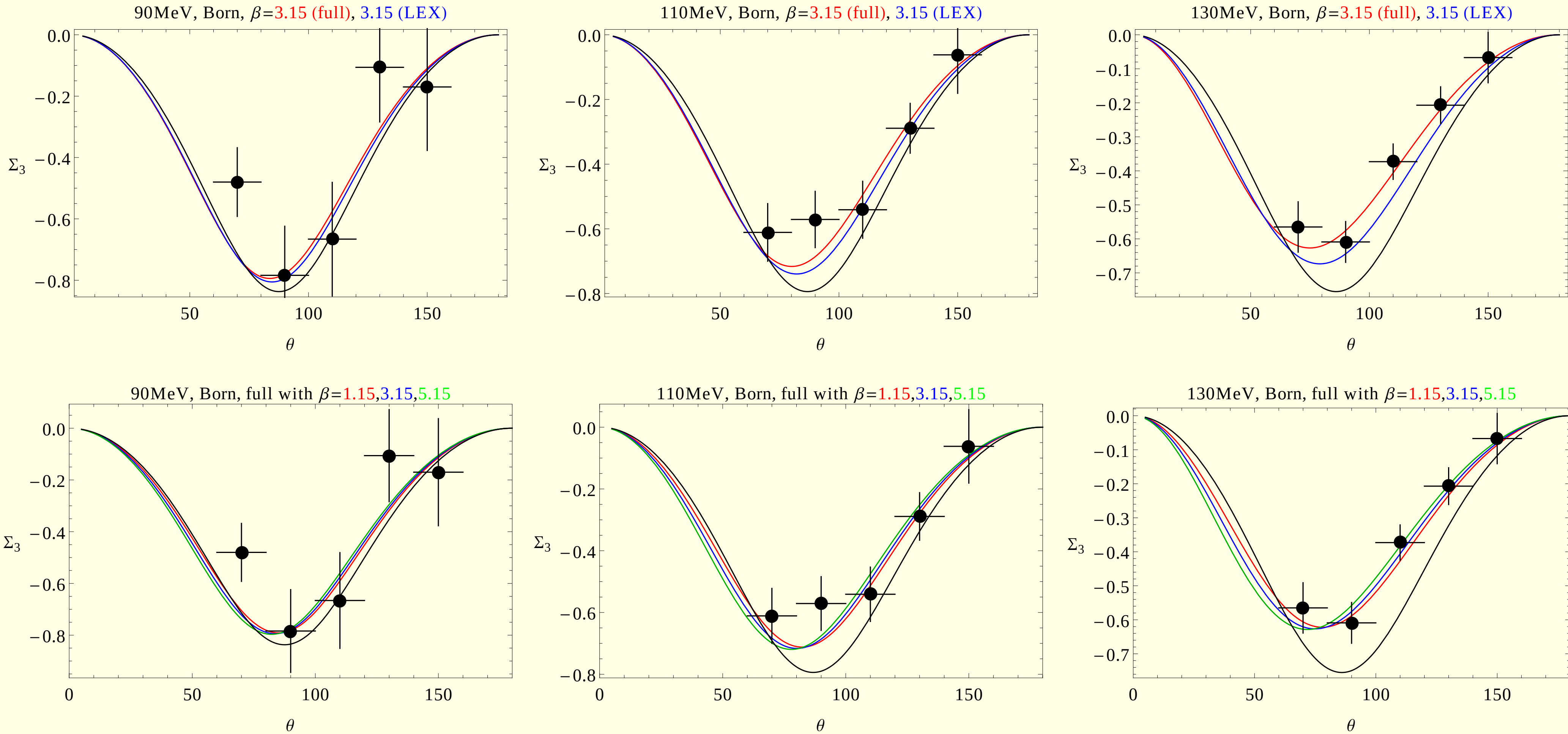

Lower energy experiments

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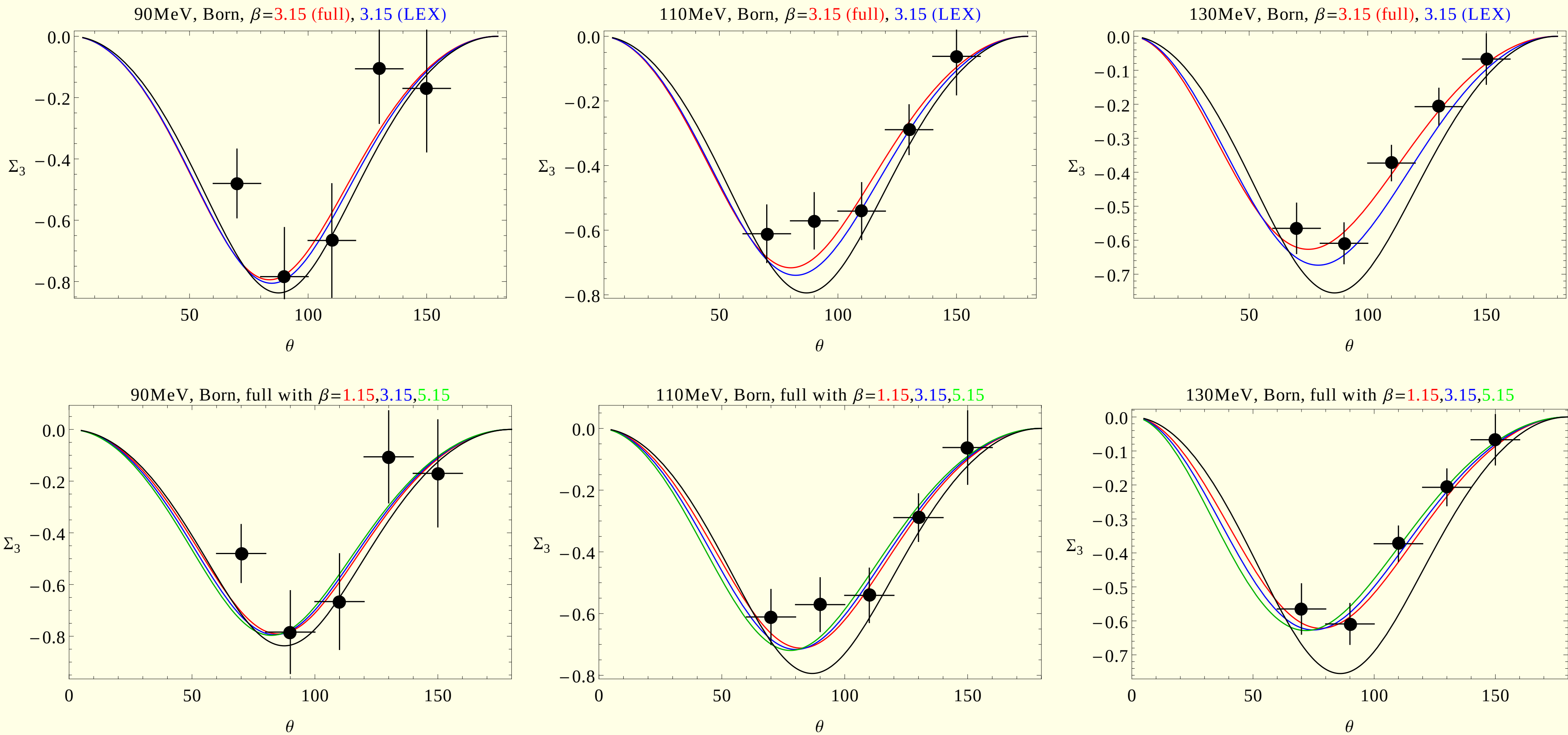
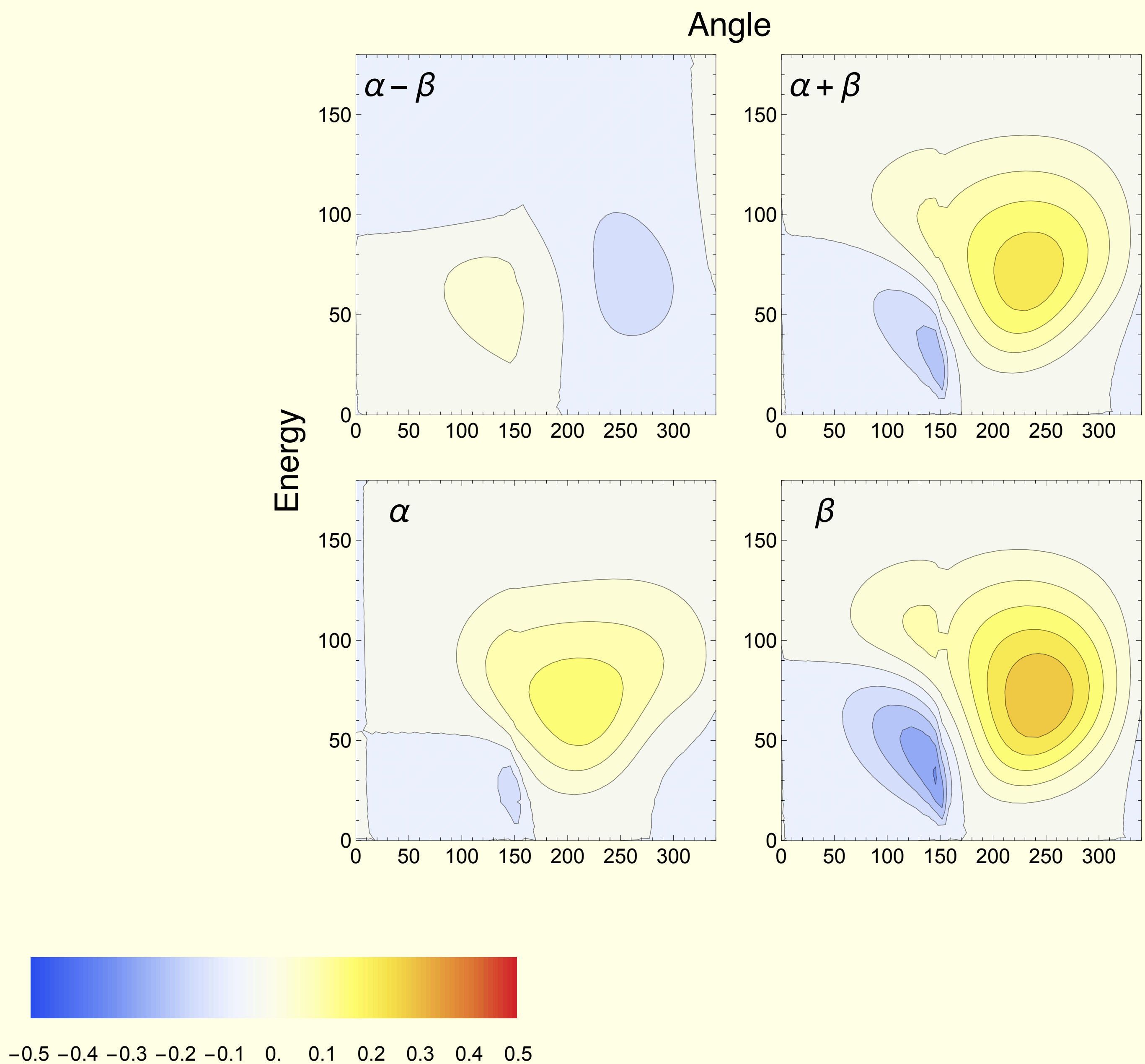


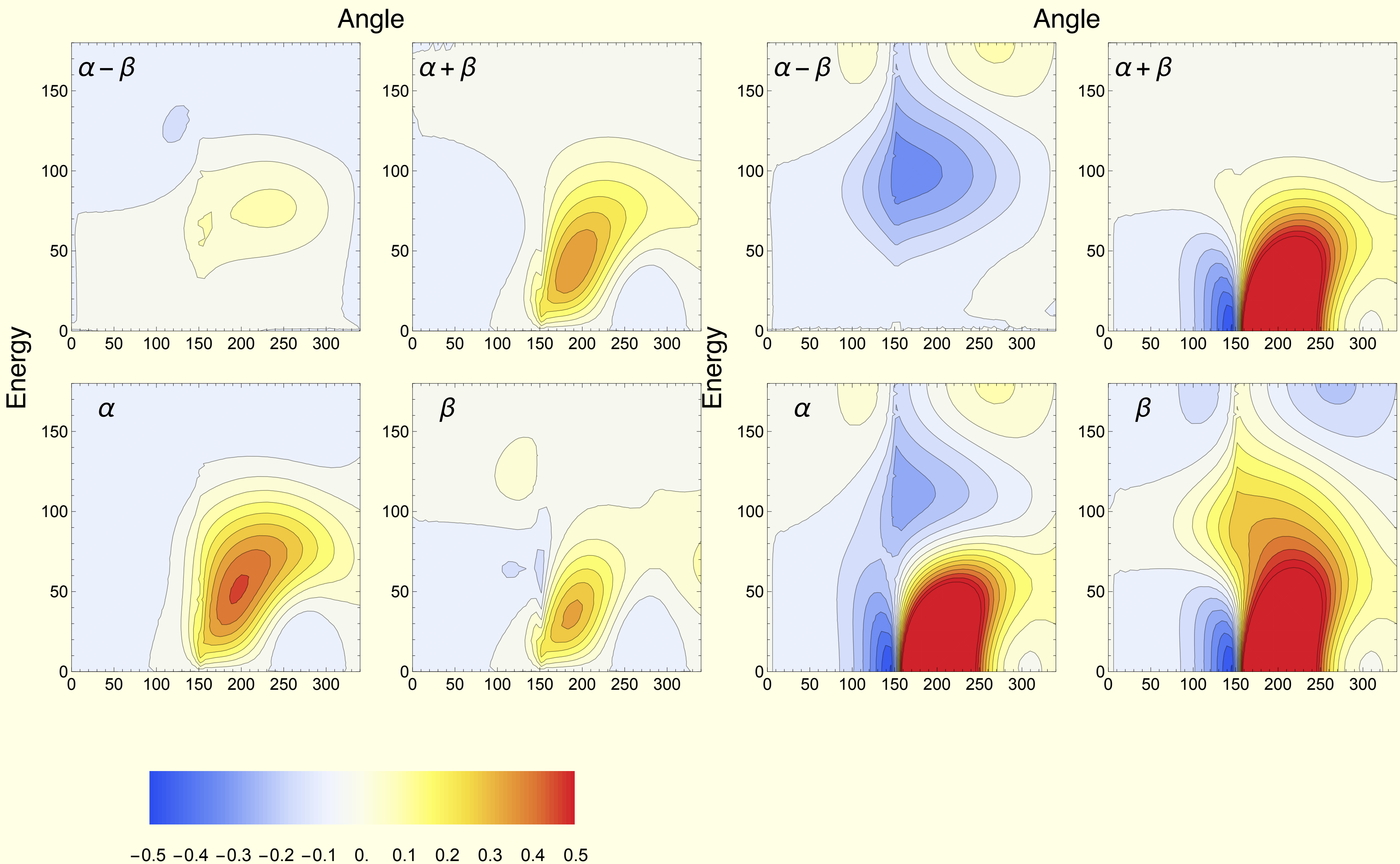
Experiments also planned at HIγS @TUNL

low energy–up to about 100 MeV currently, 150 MeV after upgrades.

Sensitivity studies: Σ_3 (PRELIM)



Sensitivity studies: Σ_{2x} and Σ_{2z} (PRELIM)



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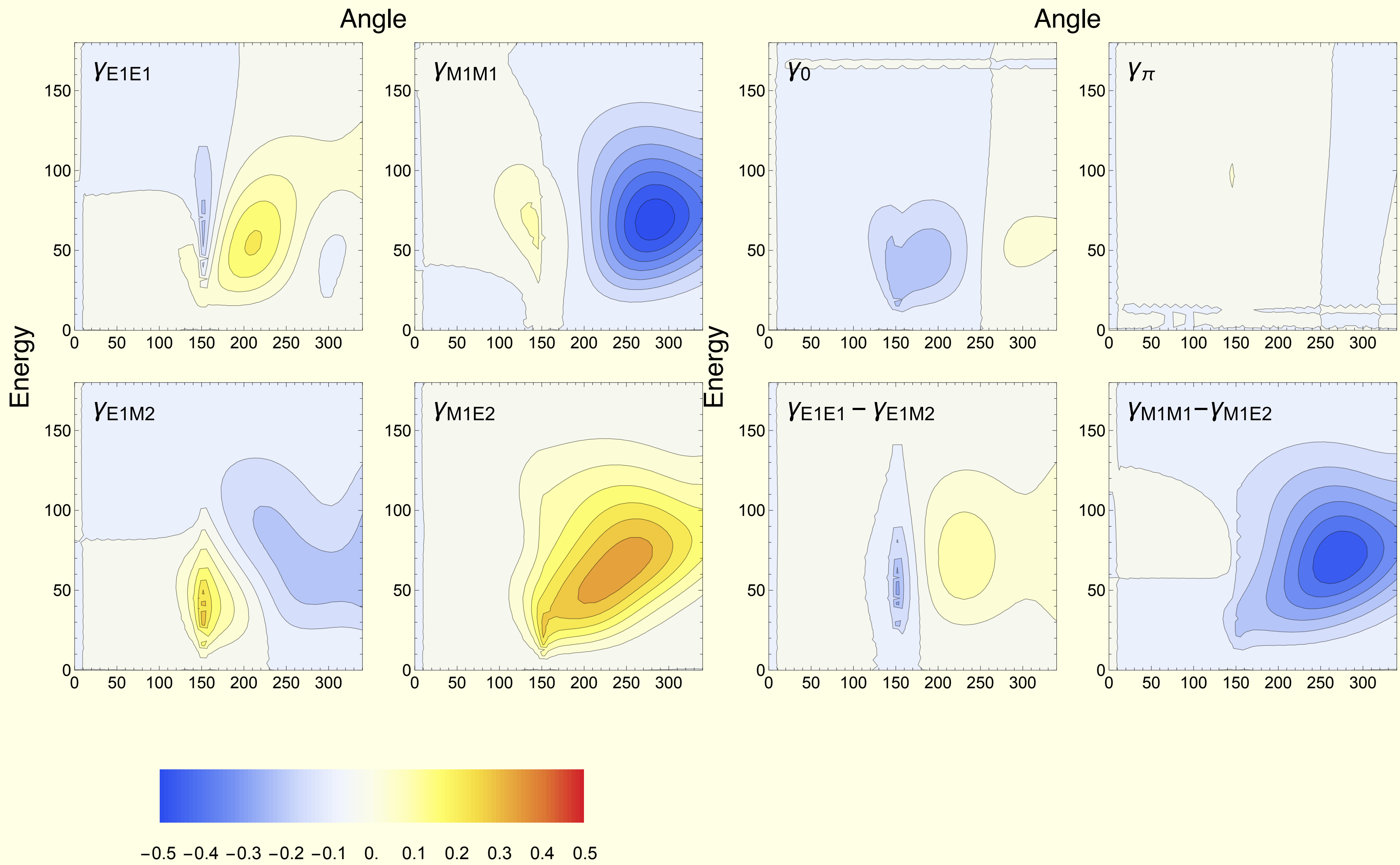

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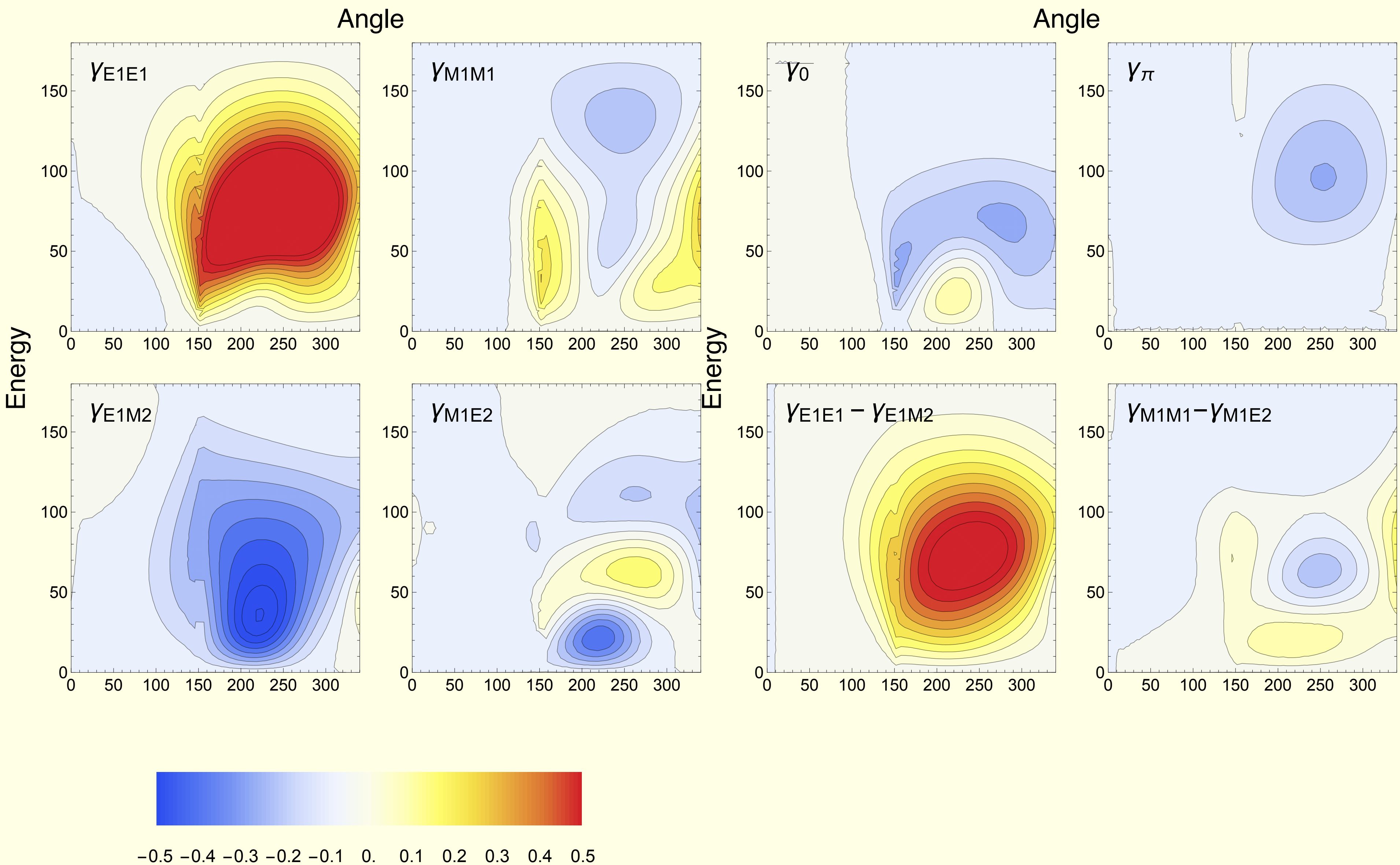
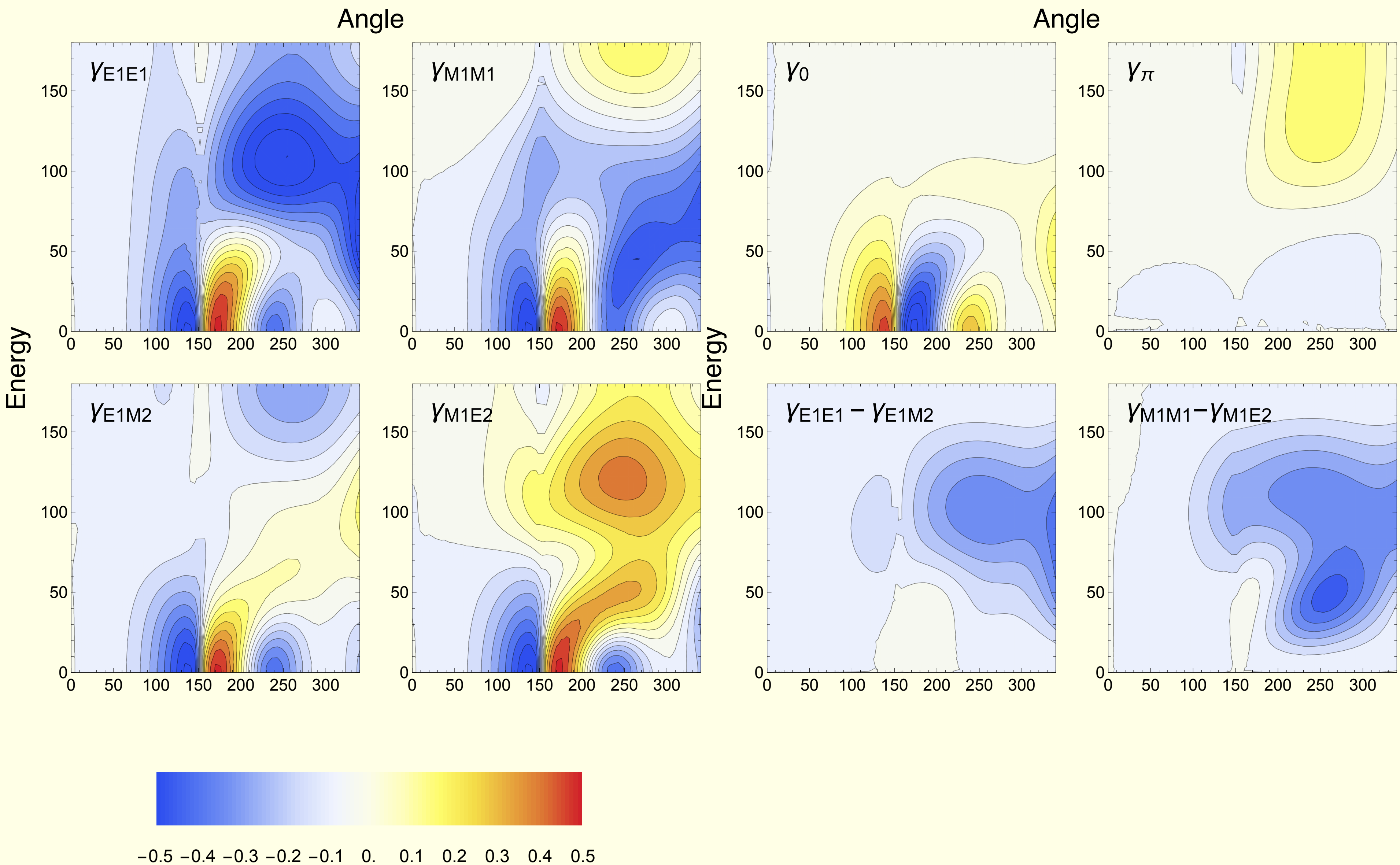
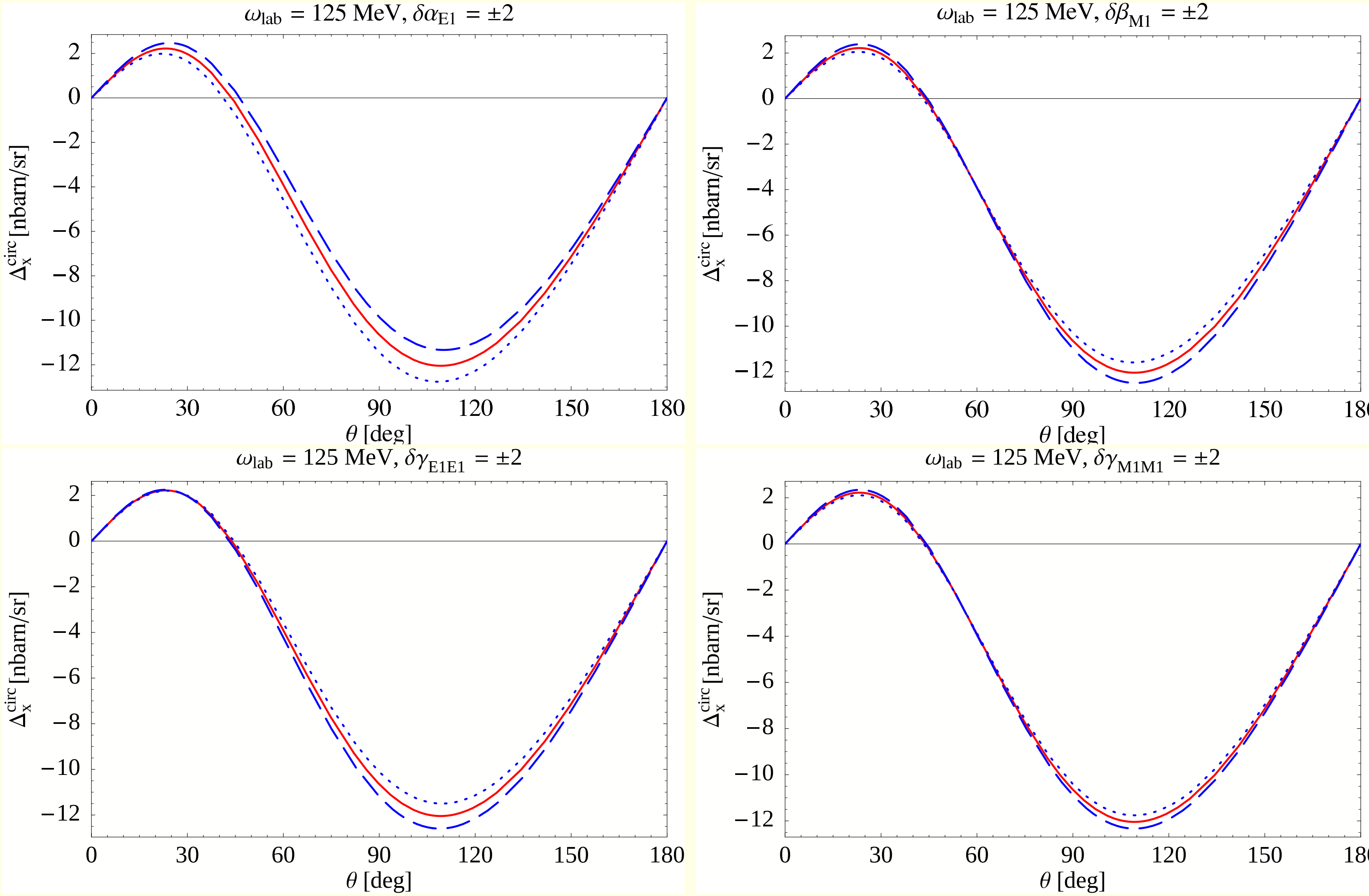


Sensitivity studies: Σ_{2z} (PRELIM)



Polarised scattering from deuterium

$$\Delta_x^{\text{circ}} = \frac{d\sigma}{d\Omega_{\uparrow\rightarrow}} - \frac{d\sigma}{d\Omega_{\uparrow\leftarrow}}$$

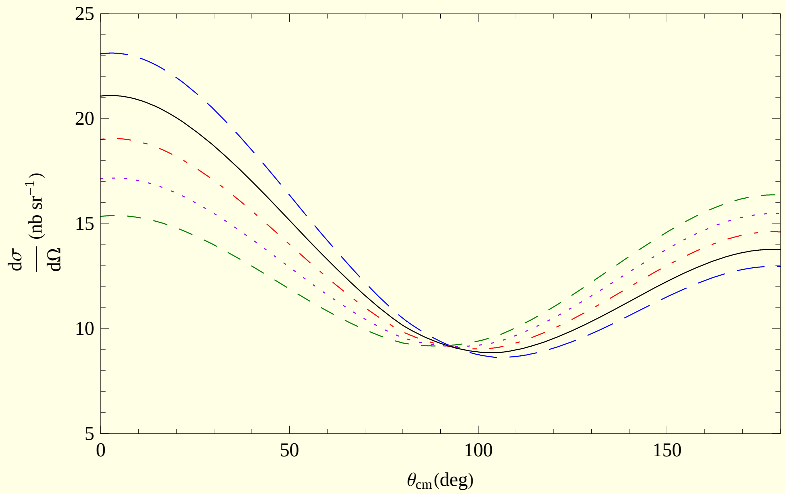


Δ included, 3rd order.

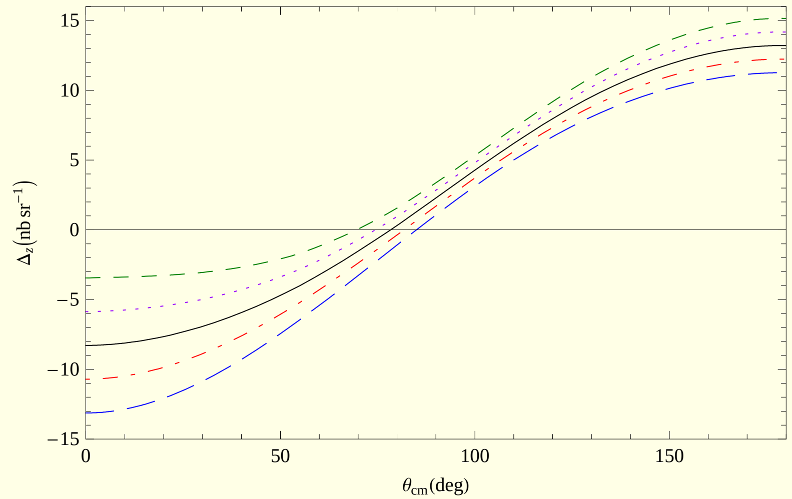
source: Griesshammer and Shukla Eur. Phys. J. A46:249, 2010

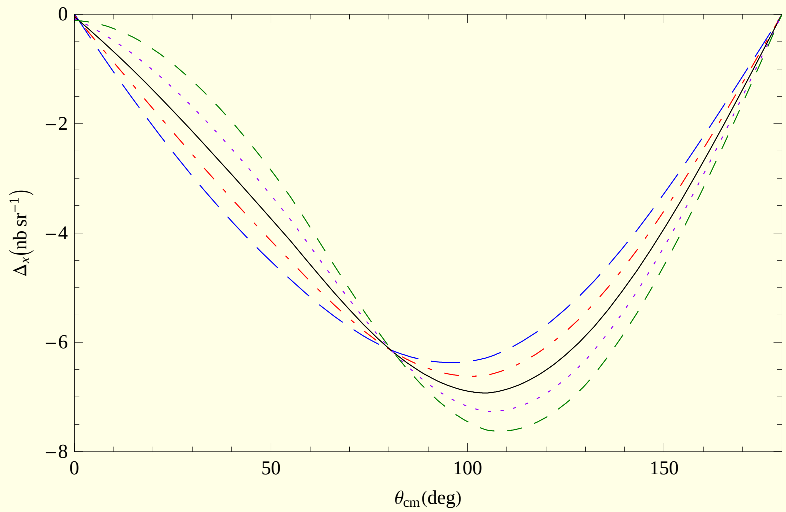
Polarised scattering from ^{3}He

Unpolarised, varying β_n

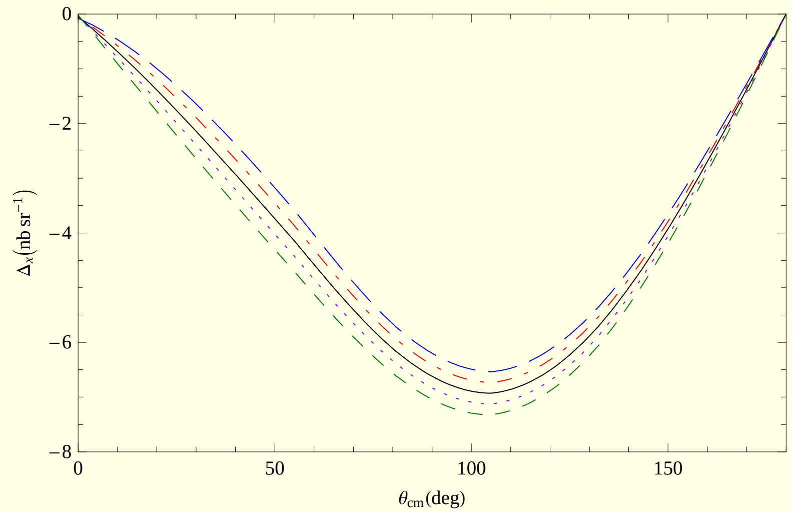


Δ_z, varying γ_{1n}





Δ_x, varying γ_{1n}



Δ_x, varying γ_{4n}

120 MeV, 3rd order, no Δ

source: Shukla, Nogga and Phillips Nucl. Phys. A 819 (2009) 98

Experimental programme at MAMI, MAXlab and HIγS

Future

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Polarised γp scattering at MAMI: data being analysed; 120 MeV data especially interesting for polarisabilities. Active target being developed for double-polarised experiments at low energies

Plans for ^{3}He

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HIγS up to about 100 MeV: approved experiments on polarised proton, deuteron and ^{3}He

Should soon know much more about the polarisabilities of the proton and neutron

Backup slides

Details of fit

Resonance region—very sensitive to magnetic $\gamma N\Delta$ coupling ($\sim g_M^4$). We iteratively fit g_M; value 10% lower than fit to photo production.

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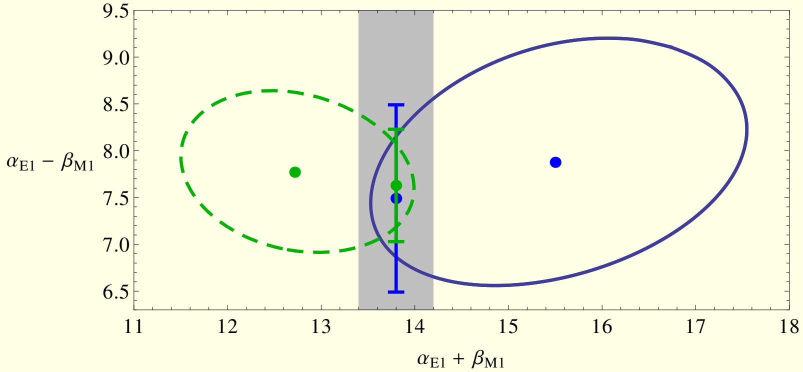
We FIT it to give $\gamma_{M1M1} = 2.2 \pm 0.5$(stat). Final fit good: χ^2=113.2 for 135 d.o.f.

4th-order statistical errors on $\alpha - \beta$ are larger than 3rd order.



Deduce theory error from convergence:

LO ($O(e^2\delta)$, BKM) $\alpha - \beta = 11.25$

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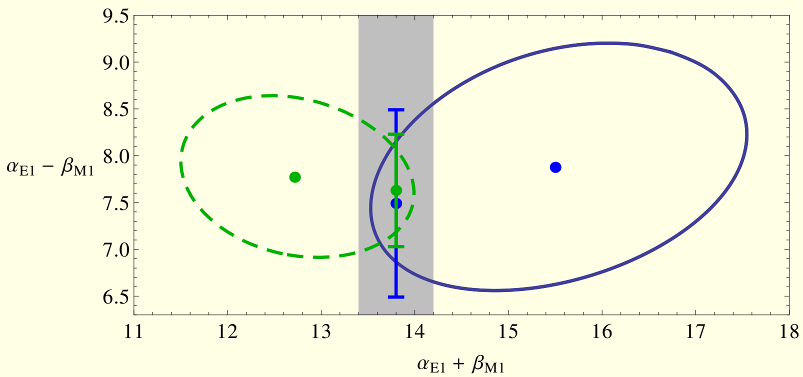

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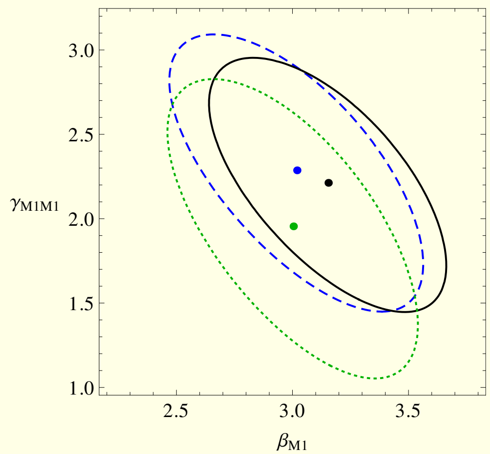

Also check sensitivity to data: need to be somewhat selective of old data sets to get a good χ^2, can't fit Hallin data above 150MeV.