Flavor Physics
Part 1

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References

Benjamin Grinstein

TASI-2013 Lectures on Flavor Physics

arXiv:1501.05283

Gino Isidori

Flavor physics and CP violation

arXiv:1302.0661

Yossi Nir

Flavour physics and CP violation

arXiv:1010.2666

Yuval Grossman

Introduction to flavor physics

arXiv:1006.3534

1. Flavor Physics in the Standard Model
   - Flavor Symmetry and Flavor Symmetry Breaking
   - The CKM Matrix
   - Meson Mixing and CP Asymmetries
   - The Standard Model Flavor Puzzle

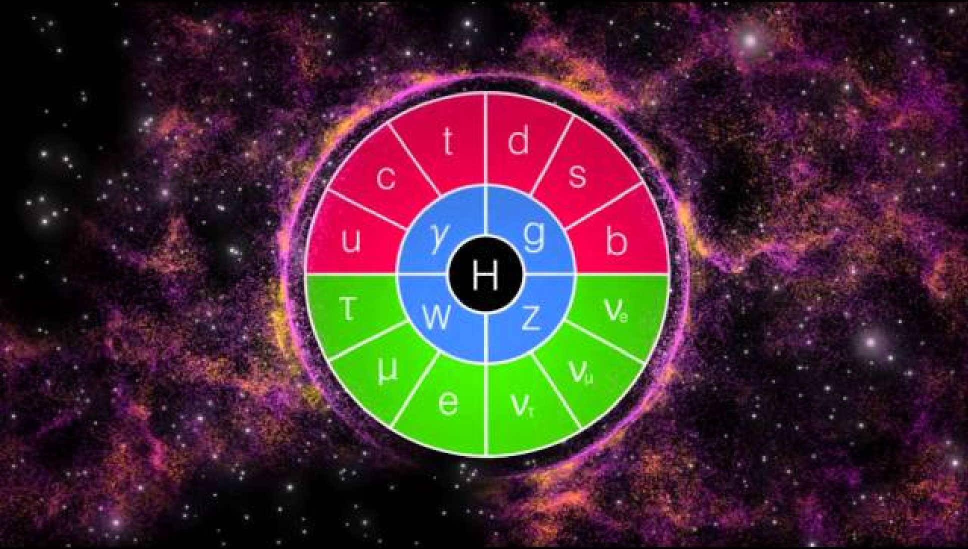
Outline of the Lectures

1. Flavor Physics in the Standard Model
   - Flavor Symmetry and Flavor Symmetry Breaking
   - The CKM Matrix
   - Meson Mixing and CP Asymmetries
   - The Standard Model Flavor Puzzle
2. Flavor Physics Beyond the Standard Model
   - The New Physics Flavor Puzzle
   - Flavor in BSM Models
   - Flavor Anomalies

Part 1

Flavor Physics in the Standard Model

The Standard Model of Particle Physics



particlefever.com

What distinguishes the three generations/flavors of quarks and leptons?

Enter Higgs

The Standard Model Lagrangian

$$\mathcal{L}_{\rm SM} \sim \Lambda^4 + \Lambda^2 H^2 + \lambda H^4$$
$$+ \bar{\Psi} \not{D} \Psi + (D_\mu H)^2 + (F_{\mu\nu})^2$$
$$+ Y H \bar{\Psi} \Psi + \frac{1}{\Lambda} (LH)^2 + \cdots$$

The Standard Model Lagrangian

Higgs potential

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The Standard Model Lagrangian

kinetic terms

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Yukawa couplings

The Standard Model Lagrangian

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neutrino masses

(see lecture by Walter Winter)

The Fermion Gauge Quantum Numbers

				$SU(3)$	$SU(2)$	$U(1)_Y$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1

3 replica of the basic fermion family

Flavor Symmetry of the Gauge Interactions

$$\bar{\Psi} \not{D} \Psi = \sum_{i=1}^{3} \bar{Q}_i \not{D} Q_i + \sum_{i=1}^{3} \bar{u}_i \not{D} u_i + \sum_{i=1}^{3} \bar{d}_i \not{D} d_i$$

$$+ \sum_{i=1}^{3} \bar{L}_i \not{D} L_i + \sum_{i=1}^{3} \bar{e}_i \not{D} e_i$$

gauge interactions are flavor universal

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gauge interactions are flavor universal

this part of the SM Lagrangian has a large $U(3)^5$ flavor symmetry

$$Q \to V_Q Q \ , \quad u \to V_u u \ , \quad d \to V_d d \ , \quad L \to V_L L \ , \quad e \to V_e e$$

Flavor Symmetry of the Gauge Interactions

The $U(3)^5$ flavor symmetry can be decomposed in the following way

$$U(3)^5 =$$

$$U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_D \times U(1)_E$$

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baryon number, lepton number, hypercharge

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RH down-quark number, RH lepton number

flavor mixing

Flavor Symmetry Breaking

the flavor symmetry is explicitly broken by the Yukawa couplings

$$Y\ H\bar{\Psi}\Psi = \sum_{i,j}(\hat{Y}_u)_{ij} H^c \bar{Q}_i u_j + \sum_{i,j}(\hat{Y}_d)_{ij} H \bar{Q}_i d_j + \sum_{i,j}(\hat{Y}_\ell)_{ij} H \bar{L}_i e_j \ + \ \text{h.c.}$$

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after electro-weak symmetry breaking we get fermion masses

$$\to \sum_{i,j} (\hat{m}_u)_{ij} \bar{u}_i^L u_j^R + \sum_{i,j} (\hat{m}_d)_{ij} \bar{d}_i^L d_j^R + \sum_{i,j} (\hat{m}_\ell)_{ij} \bar{e}_i^L e_j^R \ + \ \text{h.c.}$$

Going to Mass Eigenstates

Yukawa couplings and fermion masses are generic 3×3 matrices
(not necessarily symmetric, hermitian, ...)

$$\bar{u}^L \hat{m}_u u^R + \bar{d}^L \hat{m}_d d^R \ + \ \text{h.c.}$$

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mass matrices for the fermions can be diagonalized by bi-unitary transformations

$$u^L \to V_u^L u^L \;\;, \quad u^R \to V_u^R u^R \;\;, \quad d^L \to V_d^L d^L \;\;, \quad d^R \to V_d^R d^R$$

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$$(V_u^L)^\dagger (\hat{m}_u)(V_u^R) = \text{diag}(m_u, m_c, m_t)$$
$$(V_d^L)^\dagger (\hat{m}_d)(V_d^R) = \text{diag}(m_d, m_s, m_b)$$

V_u^L, V_u^R, V_d^L, and V_d^R are unitary matrices

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[Exercise: show that any matrix can be diagonalized by a bi-unitary transformation]

What happens to the gauge interactions in the mass eigenstate basis?

Lets start with the interactions of the W boson

$$\bar{\Psi} \not{D} \Psi \supset \frac{g_2}{\sqrt{2}} \left(\bar{u}_i^L \gamma^\mu d_i^L W_\mu^+ + \bar{d}_i^L \gamma^\mu u_i^L W_\mu^- \right)$$

The CKM Matrix

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$$\to \frac{g_2}{\sqrt{2}} \left(V_{kj} (\bar{u}_k^L \gamma^\mu d_j^L W_\mu^+) + V_{kj}^* (\bar{d}_j^L \gamma^\mu u_k^L W_\mu^-) \right)$$

$V = (V_u^L)^\dagger (V_d^L)$ is the Cabibbo-Kobayashi-Maskawa matrix

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The CKM matrix is unitary (product of 2 unitary matrices)

No Flavor Changing Neutral Currents at Tree Level

Lets look at the couplings of the photon

$$\bar{\Psi} \not{D} \Psi \supset \frac{2}{3} e \Big(\bar{u}_i^L \gamma^\mu u_i^L + \bar{u}_i^R \gamma^\mu u_i^R \Big) A_\mu - \frac{1}{3} e \Big(\bar{d}_i^L \gamma^\mu d_i^L + \bar{d}_i^R \gamma^\mu d_i^R \Big) A_\mu$$

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$$\to \quad \frac{2}{3} e \Big((V_u^L)^*_{ij} (V_u^L)_{ik} (\bar{u}_j^L \gamma^\mu u_k^L) + (V_u^R)^*_{ij} (V_u^R)_{ik} (\bar{u}_j^R \gamma^\mu u_k^R) \Big) A_\mu$$

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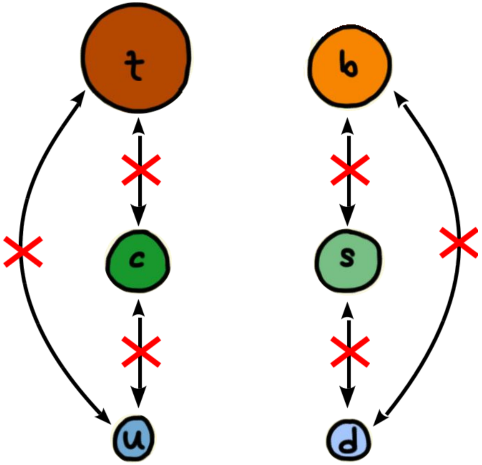
Completely analogous for gluon and Z couplings

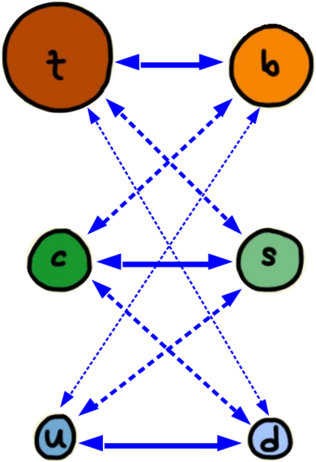
[Exercise: show that also the couplings of the Higgs are flavor diagonal]

$\rightarrow$ There are no Flavor Changing Neutral Currents (FCNCs) in the Standard Model at tree level

Flavor Transitions among Quark



no FCNCs at tree level

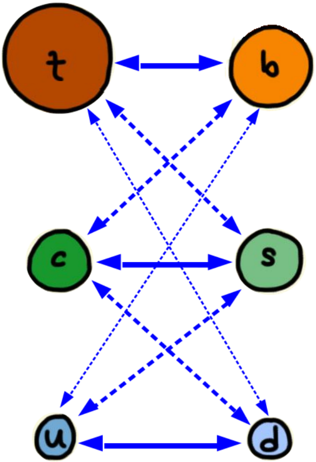


no FCNCs at tree level

transitions among the generations are mediated by the $W^{\pm}$ bosons and their relative strength is parametrized by the CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Flavor Transitions among Quark



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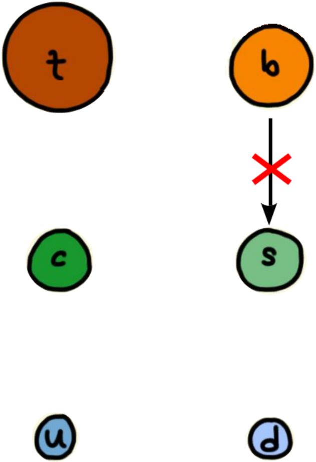
note that without the Higgs
$V = \mathbb{1}$

Flavor Changing Neutral Currents at Loop Level



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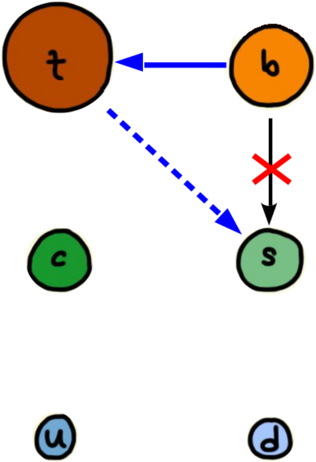

FCNCs can arise
at the loop level

they are suppressed
by loop factors

and small CKM elements

$\Delta F = 1$ processes

rare B decays: $B \to X_s \gamma$, $B \to K^{(*)} \ell^+ \ell^-$, $B_{s,d} \to \ell^+ \ell^-$, ...
rare Kaon decays: $K \to \pi \nu \bar{\nu}$, $K \to \pi \ell^+ \ell^-$, ...

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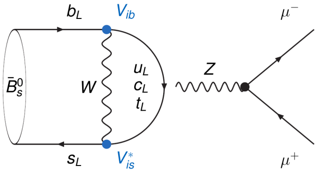
Example: $B_s \to \mu^+\mu^-$



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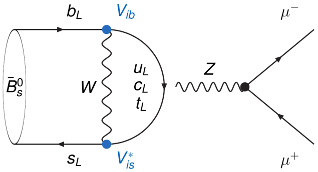

$$A \sim G_F \frac{g_2^2}{16\pi^2} \sum_{i=u,c,t} V_{is}^* V_{ib} F(m_i^2 / m_W^2)$$

$\Delta F = 1$ processes

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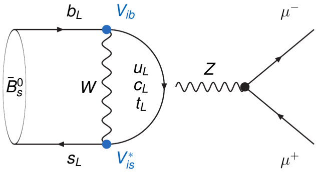

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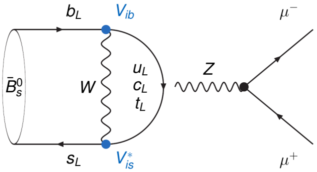

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$$= V_{ts}^* V_{tb}\big(F(m_t^2/m_W^2) - F(m_c^2/m_W^2)\big) + V_{us}^* V_{ub}\big(F(m_u^2/m_W^2) - F(m_c^2/m_W^2)\big)$$

Glashow-Iliopoulos-Maiani (GIM) mechanism:
FCNC amplitudes in the down-sector vanish for $m_t = m_c = m_u$

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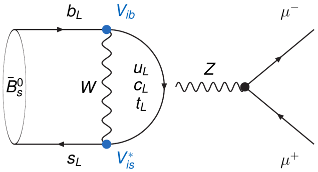

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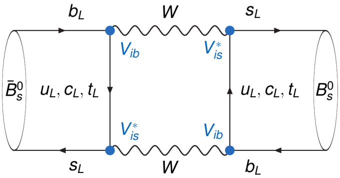
$$= V_{ts}^* V_{tb}(F(m_t^2/m_W^2) - F(m_c^2/m_W^2)) + V_{us}^* V_{ub}(F(m_u^2/m_W^2) - F(m_c^2/m_W^2))$$

$$\simeq V_{ts}^* V_{tb}(F(m_t^2/m_W^2) - F(0)) = V_{ts}^* V_{tb}\tilde{F}(m_t^2/m_W^2)$$

Glashow-Iliopoulos-Maiani (GIM) mechanism:
FCNC amplitudes in the down-sector vanish for $m_t = m_c = m_u$

top loop is often dominant

meson mixing: example $B_s \leftrightarrow \bar{B}_s$ oscillations



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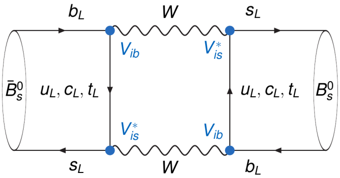

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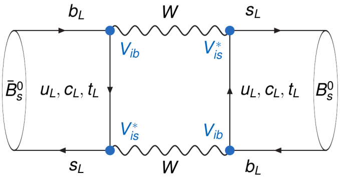

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unitary 3×3 matrix $\rightarrow$ 9 free parameters: 3 angles + 6 phases

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$\rightarrow$ CKM matrix is determined by 3 angles and 6-5=1 phase

Parametrization of the CKM Matrix

Standard Parametrization: product of 3 rotation matrices

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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(many equivalent parametrizations possible)

Wolfenstein Parametrization: introduce the parameters λ, A, ρ, η

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$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

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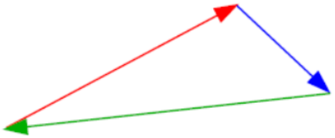
[Exercise: Demonstrate the above expansion of the CKM matrix]

Unitarity Triangles

The CKM matrix is unitary $\rightarrow$ relations between CKM elements

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

three complex numbers adding up to 0



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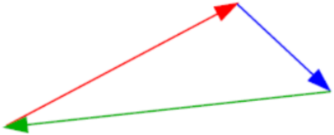

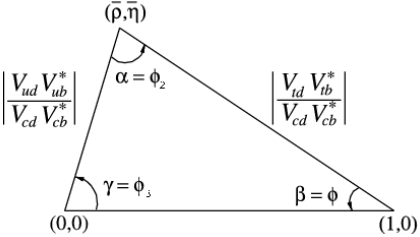

It is convenient to normalize one side to 1

$$\bar\rho + i\bar\eta = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$

$$\bar\rho = \rho(1 + O(\lambda^2)) \ , \quad \bar\eta = \eta(1 + O(\lambda^2))$$

Exercise: Show that

$$\alpha = \text{Arg}\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \quad , \qquad \beta = \text{Arg}\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma = \text{Arg}\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

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Lets proof this statement starting from the charged current interaction in the SM Lagrangian

$$\mathcal{L}_{\text{SM}} \supset \frac{g_2}{\sqrt{2}} V_{td} \bar{t}_L \gamma^\mu d_L W_\mu^+ + \frac{g_2}{\sqrt{2}} V_{td}^* \bar{d}_L \gamma^\mu t_L W_\mu^-$$

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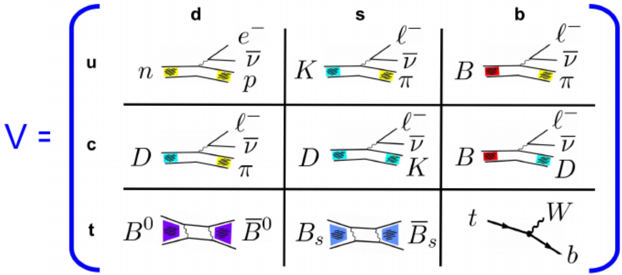
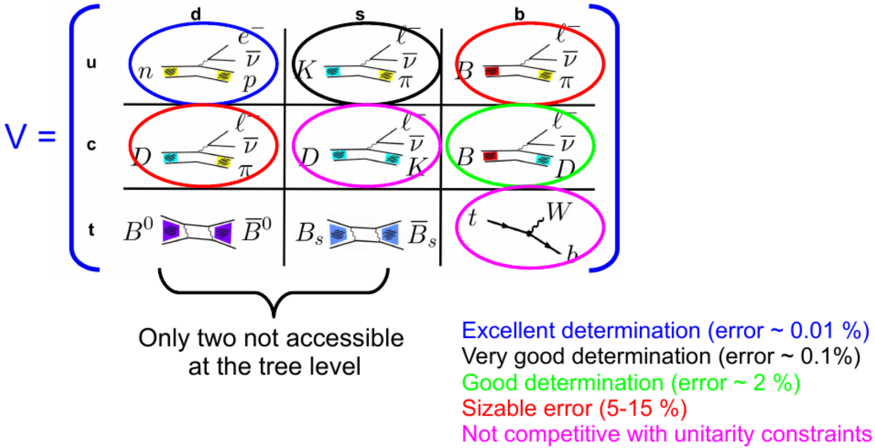
CP Violation $\Leftrightarrow V \neq V^* \Leftrightarrow \bar{\eta} \neq 0$
$\Leftrightarrow$ unitarity triangles do not collapse to a line

Measuring CKM Parameters





need to know meson decay constants and form factors from lattice
$\rightarrow$ lectures by Andreas Kronfeld

Neutral Meson Mixing

There are 4 neutral meson anti-meson systems in nature

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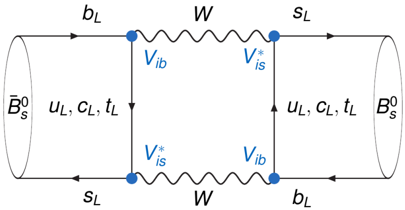
B_d and B_s mixing allow to access the CKM elements V_{td} and V_{ts}



Time Evolution of Neutral Meson Systems

$$i\partial_t \begin{pmatrix} B(t) \\ \bar{B}(t) \end{pmatrix} = \left(\hat{M} + \frac{i}{2}\hat{\Gamma} \right) \begin{pmatrix} B(t) \\ \bar{B}(t) \end{pmatrix}$$

mass matrix $\hat{M} = \hat{M}^\dagger = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}$, decay matrix $\hat{\Gamma} = \hat{\Gamma}^\dagger = \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$

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diagonalize the Hamiltonian

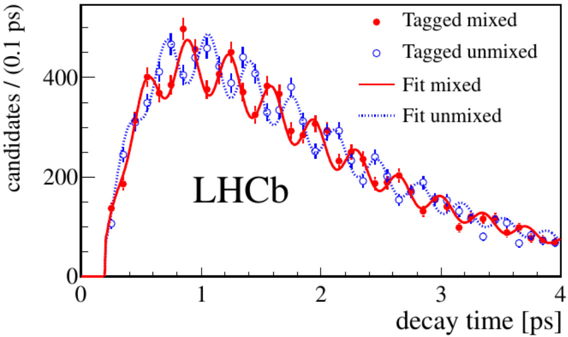
$$B_H = pB + q\bar{B} \ , \quad B_L = pB - q\bar{B} \ , \quad \left(\frac{q}{p} \right)^2 = \frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}$$

$$\Delta M_s = M_s^H - M_s^L \simeq 2|M_{12}^s| \propto |V_{ts}|^2$$

$$\Delta M_d = M_d^H - M_d^L \simeq 2|M_{12}^d| \propto |V_{td}|^2$$

need lattice input
to extract the CKM elements

Mixing Frequencies



$$\Gamma(B_s(t) \to D_s^+ \pi^-) \sim e^{-\Gamma_s t}\left(\cosh(\frac{\Delta\Gamma_s t}{2}) + \cos(\Delta M_s t)\right)$$

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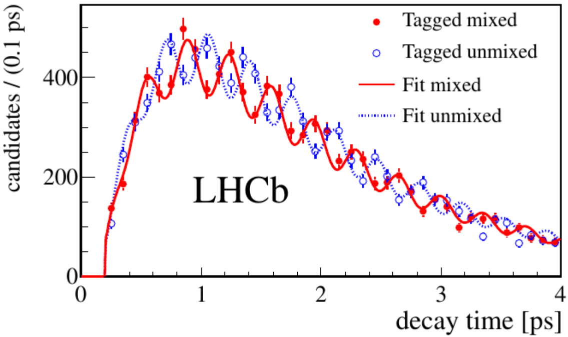

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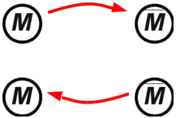
$$\Delta M_s = (17.757 \pm 0.021)/ps \;\;,\quad \Delta M_d = (0.5064 \pm 0.0019)/ps$$

(Heavy Flavor Averaging Group)

1) CP Violation in Mixing

probability to oscillate from meson to anti-meson $\neq$ probability to oscillate from anti-meson to meson



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2) CP Violation in the Decay

decay rate of a meson to a final state
$\neq$ decay rate of the anti-meson
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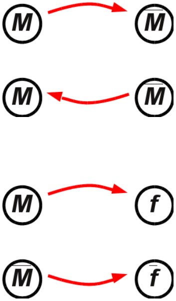

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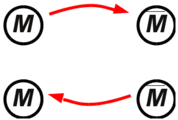

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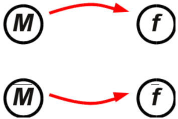
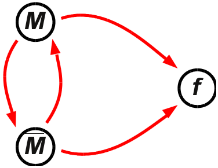


3) CP Violation in the Interference of Mixing and Decay

can occur in decays of
meson and anti-meson
to a common final state



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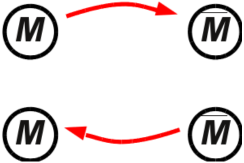
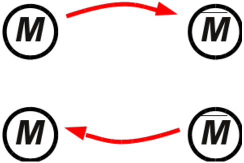


$$\left|M_{12} - i\frac{\Gamma_{12}}{2}\right| \neq \left|M_{12}^* - i\frac{\Gamma_{12}^*}{2}\right| \Leftrightarrow \left|\frac{q}{p}\right| \neq 1$$

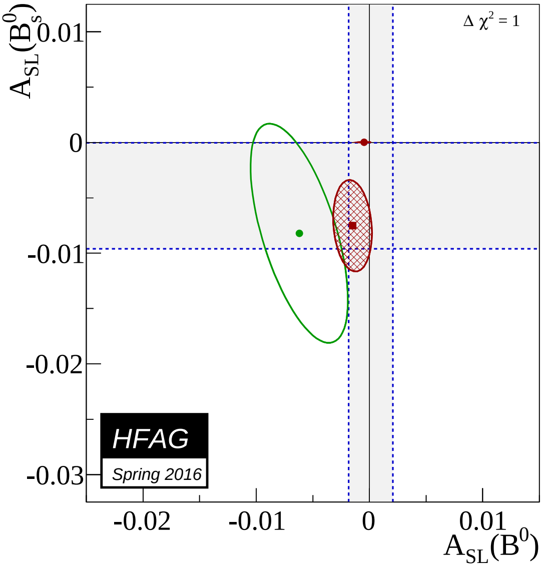


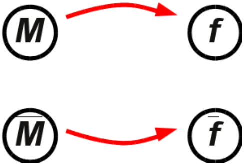
$$\left| M_{12} - i\frac{\Gamma_{12}}{2} \right| \neq \left| M_{12}^* - i\frac{\Gamma_{12}^*}{2} \right| \Leftrightarrow \left| \frac{q}{p} \right| \neq 1$$

Look at decays to “wrong sign” final states, e.g.

$$a_{\text{SL}}^d = \frac{\Gamma(\bar{B}_d(t) \to \ell^+ \nu X) - \Gamma(B_d(t) \to \ell^- \bar{\nu} X)}{\Gamma(\bar{B}_d(t) \to \ell^+ \nu X) + \Gamma(B_d(t) \to \ell^- \bar{\nu} X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4}$$

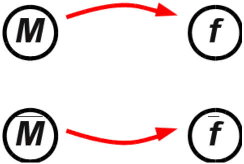
CP Violation in Mixing





$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| = \left|\frac{\sum_k A_k e^{i(\delta_k - \phi_k)}}{\sum_k A_k e^{i(\delta_k + \phi_k)}}\right| \neq 1$$

requires more than one interfering amplitude with different strong phase and weak phase



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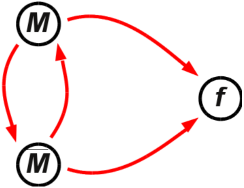
requires more than one interfering amplitude with different strong phase and weak phase

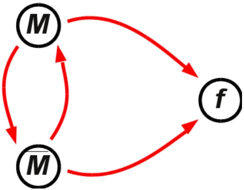
look e.g. at decays of charged B mesons:

$$a_{f^\pm} = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)} = \frac{1 - |\bar{A}_{f^-}/A_{f^+}|^2}{1 + |\bar{A}_{f^-}/A_{f^+}|^2}$$

CP Violation in the Interference



$$\text{Im}\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

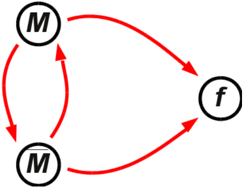


$$\text{Im}\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$$

look at time dependent CP asymmetries in decays to final CP eigenstates

$$a_{f_{CP}}(t) = \frac{\Gamma(\bar{B}(t) \to f_{CP}) - \Gamma(B(t) \to f_{CP})}{\Gamma(\bar{B}(t) \to f_{CP}) + \Gamma(B(t) \to f_{CP})} \simeq$$

CP Violation in the Interference



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(if $\Delta\Gamma \simeq 0$ and if there is no direct CP violation i.e. $|A_f| = |\bar{A}_f|$)

Golden channel to determine the angle β: $B_d \to J/\psi K_S$

CP Violation in the Interference

Golden channel to determine the angle β: $B_d \to J/\psi K_S$

$$A(B_d \to J/\psi K_S) \simeq A(\bar{B}_d \to J/\psi K_S) \simeq \text{ real },\quad \frac{q}{p} \simeq -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \simeq e^{-i2\beta}$$

$$\Rightarrow \quad a_{J/\phi K_S}(t) \;\simeq\; \sin(2\beta)\sin(\Delta M_d\, t)$$

CP Violation in the Interference

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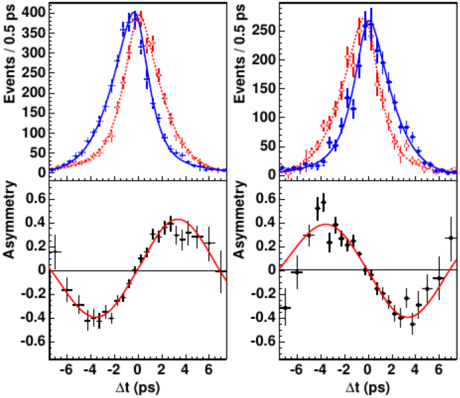


HFAG average

$$\sin(2\beta) = 0.691 \pm 0.017$$

Experimental Situation of the CKM Matrix

overall consistent
picture within
$O(10\%)$ uncertainties

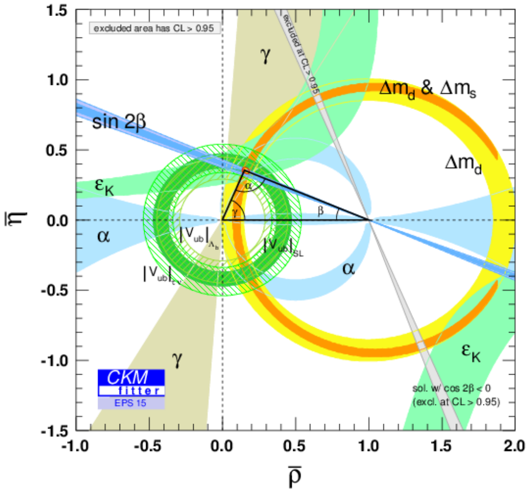
$$\lambda = 0.2254^{+0.0004}_{-0.0003}$$

$$A = 0.823^{+0.007}_{-0.014}$$

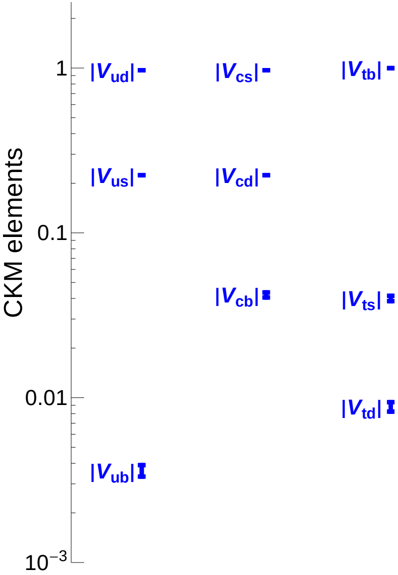
$$\bar{\rho} = 0.150^{+0.012}_{-0.006}$$

$$\bar{\eta} = 0.354^{+0.007}_{-0.008}$$

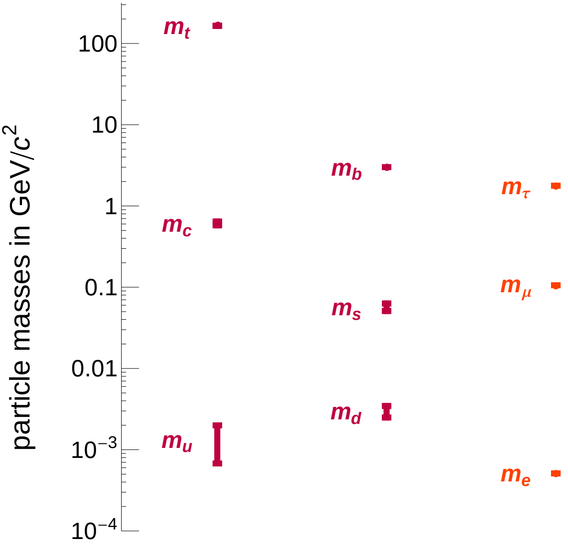
http://ckmfitter.in2p3.fr/
http://www.utfit.org/
http://latticeaverages.org/



Flavor Hierarchies



Flavor Hierarchies

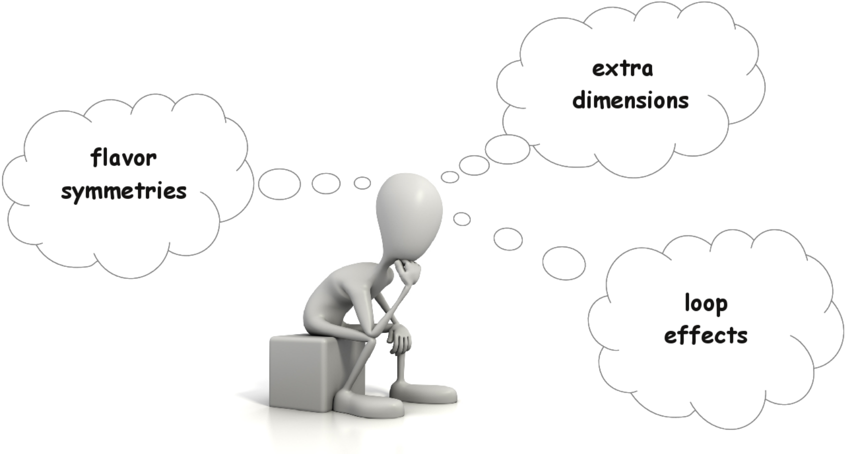


The Standard Model Flavor Puzzle

The Standard Model gives a reasonable description of all flavor transitions measured up to now, but it does not explain its mysteries

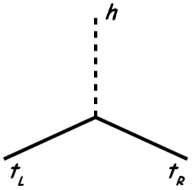
- Why are there three generations of quarks and leptons?
- What is the origin of the hierarchies in the fermion spectrum?
- What is the origin of the hierarchies in the quark mixing?

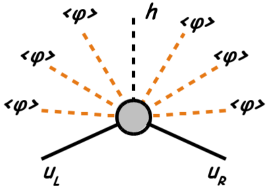
Addressing the SM Flavor Puzzle



(Froggatt, Nielsen ’79; ...)

fermion masses are forbidden by flavor symmetries
and arise only after spontaneous breaking of the symmetry





Simple U(1) model:

$Q(t_L)=Q(t_R)=0$
$Q(u_L)=-Q(u_R)=3$
$Q(h)=0$
$Q(\varphi)=-1$

$$h\bar{t}_R t_L$$

$$\frac{\varphi^6}{M^6} h\bar{u}_R u_L$$

Hierarchy from Symmetry

mass and mixing hierarchies given by powers of the "spurion" $\langle \varphi \rangle / M$.
in the example from the previous slide we have

$$\frac{m_u}{m_t} \sim \left(\frac{\langle \varphi \rangle}{M} \right)^6 \sim \epsilon^6$$

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in the example from the previous slide we have

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Exercise: Construct a U(1) model with the following hierarchies

$$m_u \sim \epsilon^6 \ , \quad m_c \sim \epsilon^3 \ , \quad m_t \sim 1$$

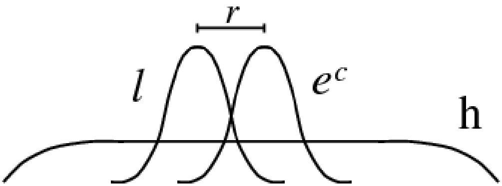
$$m_d \sim \epsilon^5 \ , \quad m_s \sim \epsilon^4 \ , \quad m_b \sim \epsilon^2$$

Which predictions does your model make for the CKM hierarchies?

Hierarchy without Symmetry: Geometry

(Arkani-Hamed, Schmaltz ’99; Grossman, Neubert ’99; ...)

fermions are localized at different positions in an extra dimension

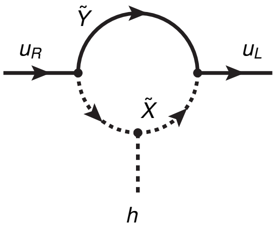


hierarchies from exponentially small wave-function overlap between left-handed and right-handed fermions and the Higgs

$$\frac{m_u}{m_t} \sim e^{-\Delta}$$

(Weinberg '72; ...)

light fermion masses arise only from quantum effects



light fermions do not couple to the higgs directly

couplings are loop-induced by flavor violating new particles

mass and mixing hierarchies from loop factors

$$\frac{m_u}{m_t} \sim \left(\frac{1}{16\pi^2}\right)^n$$

Summary of the First Lecture

- In the SM, all flavor violation is due to the Higgs
- At tree level there are no FCNCs; flavor violation occurs only in charged currents, parametrized by the CKM matrix
- FCNCs arise at the 1-loop level. Example: meson mixing; gives access to CKM elements V_{td} and V_{ts} and their phases
- Where do the hierarchies in the CKM elements and the fermion masses come from?