Flavor Physics from Lattice QCD: 2

Andreas S. Kronfeld
Fermilab & IAS TU München

Symmetry Breaking in Fundamental Interactions
September 19–23, 2016
Kloster Frauenwörth

IAS TUM Institute for Advanced Study

Ein Einheimischer unter den Rednern?

- Tante Renate suggests:
  - Räucherfisch von Familie Lex;
  - 1000-year-old linden trees;
  - visit the Inseltöpferei;
  - Kaffen und Kuchen bei den Nonnen;
- Kampenwand, of course.

Outline

- Lecture 1—Lattice Gauge Theory:
  - Origins;
  - Formalism;
  - Numerical methods.
- Lecture 2—Lattice QCD Results:
  - Semileptonic form factors & neutral-meson mixing:
    - CKM determination and search for non-SM FCNC;
  - Decay constants (leptonic decays).

Flavor Physics in Two Slides

Energy Scales

high energy

TeV New Physics $\mathscr{L} = ?$

Electroweak (Higgs, top, W, Z) $\mathscr{L} = \mathscr{L}_{\text{SM}}$

Bottom quark $\mathscr{L} = -\mathscr{H}_{\text{eff}}$

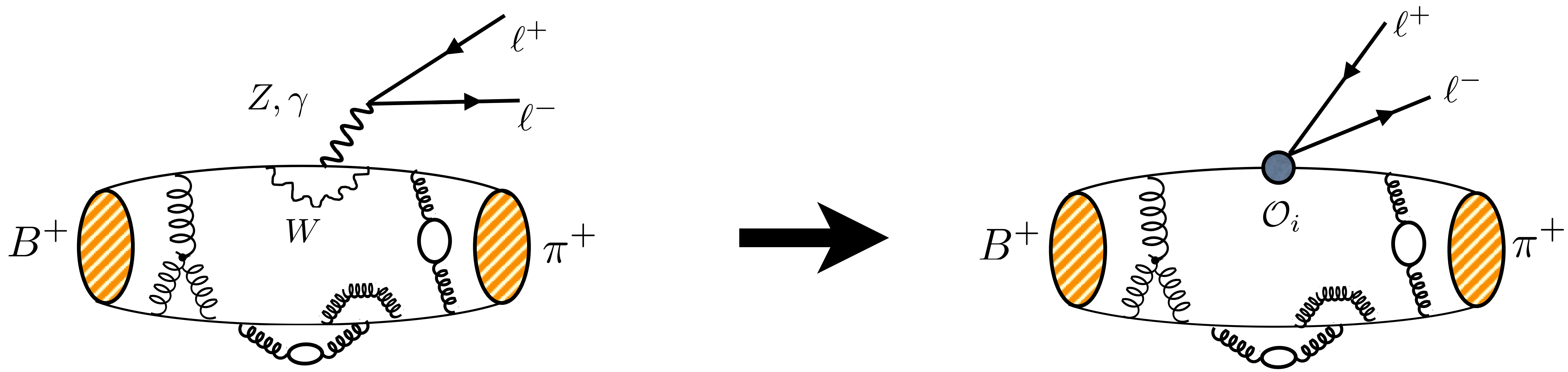
Hadronic physics: Λ_{QCD} Lattice QCD

low energy

Effective Hamiltonian

- Masses of W, Z, top, and Higgs are much greater than m_b:

$$\mathscr{L} = \mathscr{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathscr{C}_i(\alpha, \alpha_s, G_F, \sin^2\theta, m_\ell, m_q, V; \text{NP})\mathscr{L}_i[\ell, q, \gamma, g]$$



- Contributions of unknown massive particles lumped into $\mathscr{C}_i$.

Industrial Lattice QCD

Active Flavor Collaborations (n_f = 2+1, 2+1+1)

- HPQCD “High-Precision QCD”—Cornell, Glasgow, Ohio State, …
- Fermilab Lattice—Fermilab, Illinois, Syracuse, Granada, SAIC, Colorado
- MILC “MIMD Lattice Computation”—seven US universities + APS + …

} aka Fermilab/MILC or FNAL/MILC

- SWME “Seoul-Washington Matrix Element”

- RBC “RIKEN-BNL-Columbia”
- UKQCD—several UK universities, for flavor esp. Edinburgh & Southampton

} aka RBC/UKQCD

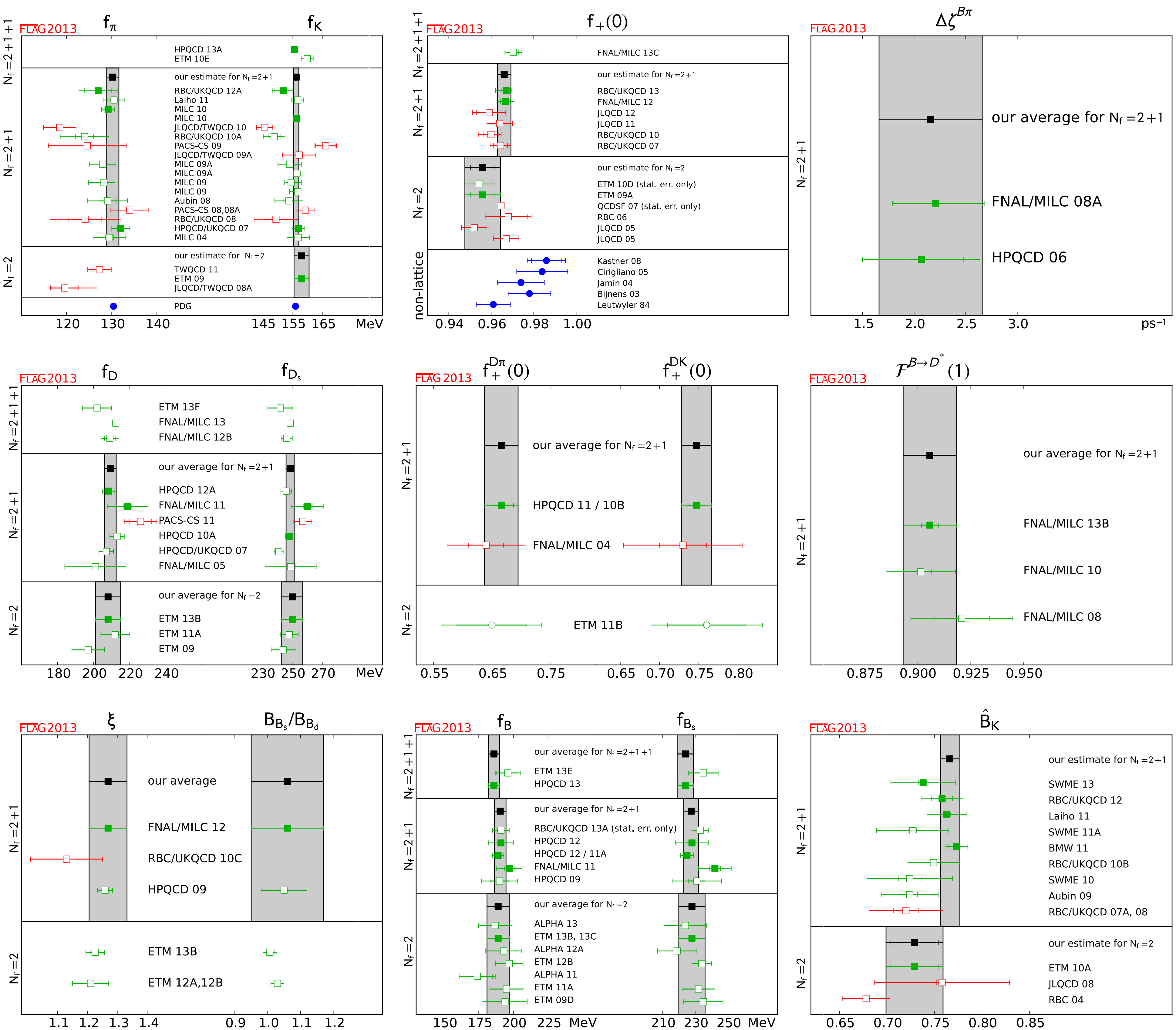
- JLQCD “Japan Lattice QCD”—KEK, Tsukuba, …

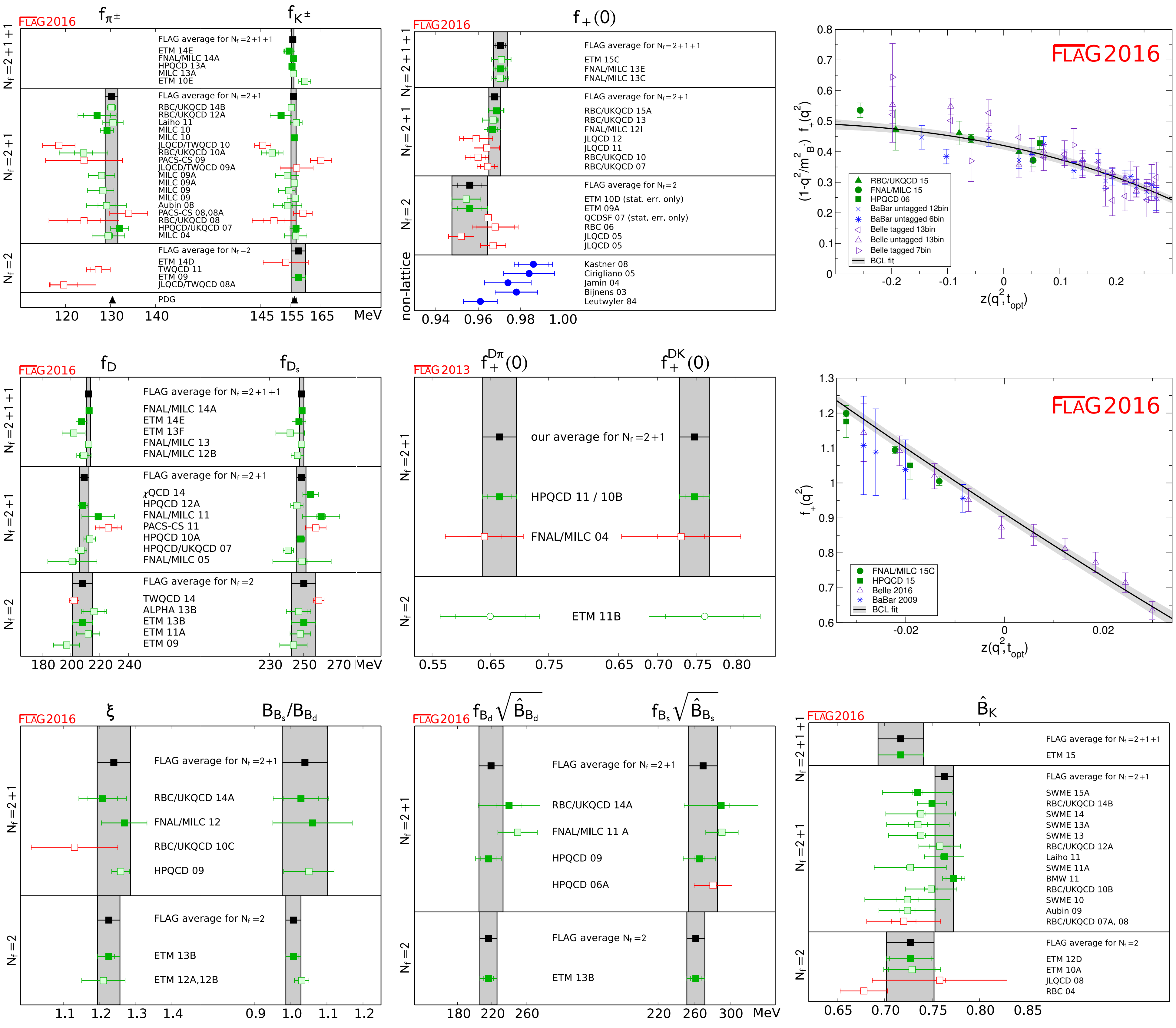
- PACS-CS (a computer name)—Tsukuba, KEK, …
- ETM “European Twisted Mass”—numerous EU institutions
- BMW—Budapest, Marseille, Wuppertal, Jülich, …

CKM and Lattice QCD

- Gold-plated quantities available to (over)determine CKM matrix:
  - zero or one hadron in initial and final state.
- Further such matrix elements for BSM flavor-changing neutral currents.
- Flavor Lattice Averaging Group FLAG 2016 Flavour Lattice Averaging Group:
  - self-selected group drawn from most major lattice QCD collaborations;
  - sets “discrete” quality criteria and produces averages of quantities germane to CKM;
  - compare & contrast to the PDG perhaps 30 years ago.

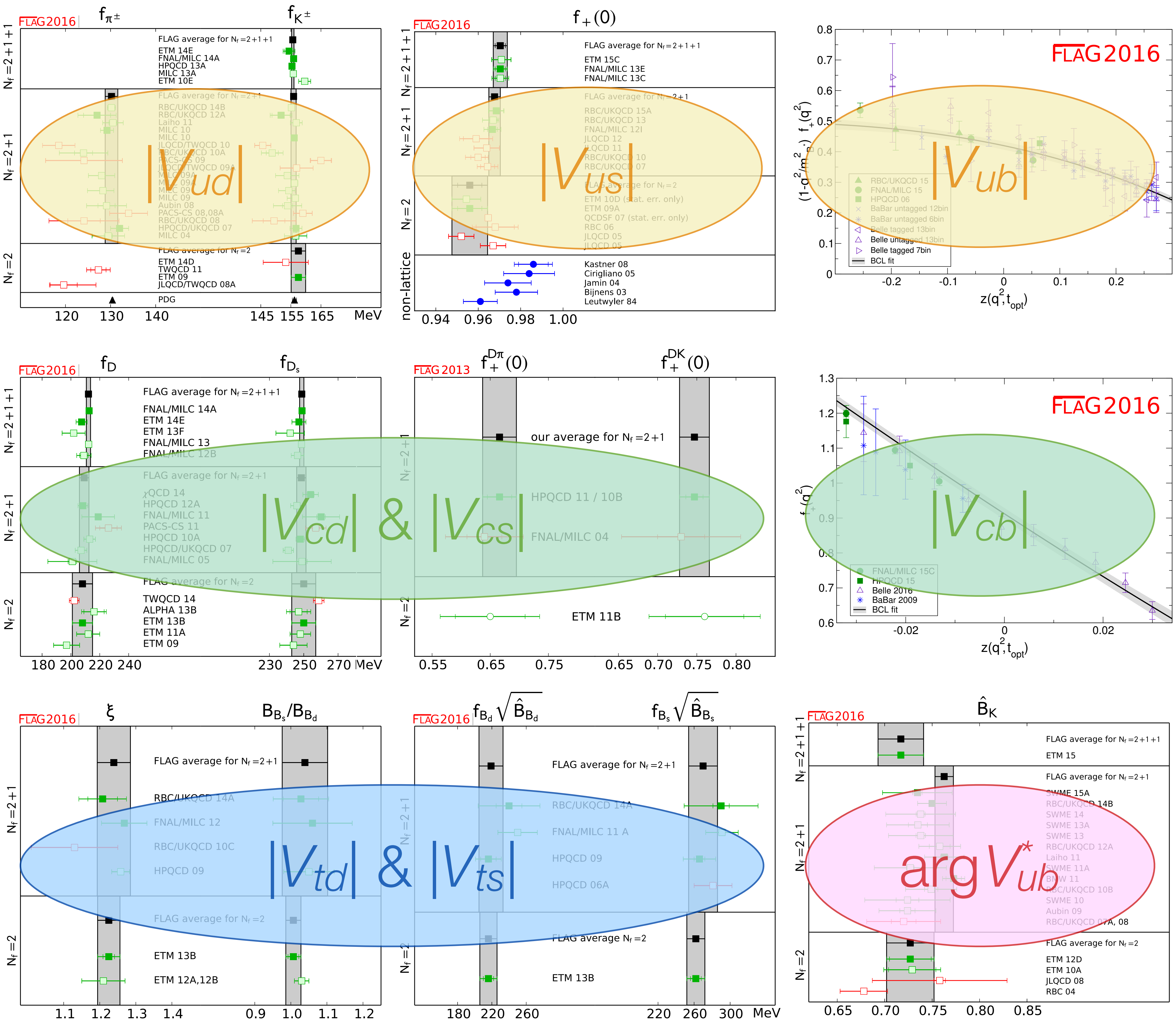
FLAG 2013
Flavour Lattice Averaging Group




FLAG2016
$f_{\pi^\pm}$
$f_{K^\pm}$
FLAG average for $N_f=2+1+1$
ETM 14E
FNAL/MILC 14A
HPQCD 13A
MILC 13A
ETM 10E
FLAG average for $N_f=2+1$
RBC/UKQCD 14B
RBC/UKQCD 12A
Laiho 11
MILC 10
MILC 10
JLQCD/TWQCD 10
RBC/UKQCD 10A
PACS-CS 09
JLQCD/TWQCD 09A
MILC 09A
MILC 09A
MILC 09
MILC 09
Aubin 08
PACS-CS 08,08A
RBC/UKQCD 08
HPQCD/UKQCD 07
MILC 04
FLAG average for $N_f=2$
ETM 14D
TWQCD 11
ETM 09
JLQCD/TWQCD 08A
PDG
120 130 140
145 155 165 MeV
$N_f=2+1+1$
$N_f=2+1$
$N_f=2$
FLAG2016
$f_+(0)$
FLAG average for $N_f=2+1+1$
ETM 15C
FNAL/MILC 13E
FNAL/MILC 13C
FLAG average for $N_f=2+1$
RBC/UKQCD 15A
RBC/UKQCD 13
FNAL/MILC 12I
JLQCD 12
JLQCD 11
RBC/UKQCD 10
RBC/UKQCD 07
FLAG average for $N_f=2$
ETM 10D (stat. err. only)
ETM 09A
QCDSF 07 (stat. err. only)
RBC 06
JLQCD 05
JLQCD 05
Kastner 08
Cirigliano 05
Jamin 04
Bijnens 03
Leutwyler 84
non-lattice
0.94 0.96 0.98 1.00
FLAG2016
$(1-q^2/m_{B^*}^2)\ f_+(q^2)$
$z(q^2,t_{opt})$
RBC/UKQCD 15
FNAL/MILC 15
HPQCD 06
BaBar untagged 12bin
BaBar untagged 6bin
Belle tagged 13bin
Belle untagged 13bin
Belle tagged 7bin
BCL fit
FLAG2016
f_D
f_{D_s}
FLAG average for $N_f=2+1+1$
FNAL/MILC 14A
ETM 14E
ETM 13F
FNAL/MILC 13
FNAL/MILC 12B
FLAG average for $N_f=2+1$
χQCD 14
HPQCD 12A
FNAL/MILC 11
PACS-CS 11
HPQCD 10A
HPQCD/UKQCD 07
FNAL/MILC 05
FLAG average for $N_f=2$
TWQCD 14
ALPHA 13B
ETM 13B
ETM 11A
ETM 09
180 200 220 240
230 250 270 MeV
FLAG2013
$f_+^{D\pi}(0)$
$f_+^{DK}(0)$
our average for $N_f=2+1$
HPQCD 11 / 10B
FNAL/MILC 04
ETM 11B
0.55 0.65 0.75
0.65 0.70 0.75 0.80
FLAG2016
$f_+(q^2)$
$z(q^2,t_{opt})$
FNAL/MILC 15C
HPQCD 15
Belle 2016
BaBar 2009
BCL fit
FLAG2016
ξ
B_{B_s}/B_{B_d}
FLAG average for $N_f=2+1$
RBC/UKQCD 14A
FNAL/MILC 12
RBC/UKQCD 10C
HPQCD 09
FLAG average for $N_f=2$
ETM 13B
ETM 12A,12B
1.1 1.2 1.3 1.4
0.9 1.0 1.1 1.2
FLAG2016
$f_{B_d}\sqrt{\hat{B}_{B_d}}$
$f_{B_s}\sqrt{\hat{B}_{B_s}}$
FLAG average for $N_f=2+1$
RBC/UKQCD 14A
FNAL/MILC 11 A
HPQCD 09
HPQCD 06A
FLAG average $N_f=2$
ETM 13B
180 220 260
220 260 300 MeV
FLAG2016
$\hat{B}_K$
FLAG average for $N_f=2+1+1$
ETM 15
FLAG average for $N_f=2+1$
SWME 15A
RBC/UKQCD 14B
SWME 14
SWME 13A
SWME 13
RBC/UKQCD 12A
Laiho 11
SWME 11A
BMW 11
RBC/UKQCD 10B
SWME 10
Aubin 09
RBC/UKQCD 07A, 08
FLAG average for $N_f=2$
ETM 12D
ETM 10A
JLQCD 08
RBC 04
0.65 0.70 0.75 0.80 0.85

FLAG 2016
Flavour Lattice Averaging Group

FLAG 2013
Flavour Lattice Averaging Group



Comments on Using FLAG Averages

- It's easy to take FLAG averages carelessly.
  - Bear in mind that some parts of FLAG are inevitably out of date.
  - Also please follow FLAG's advice to cite the underlying papers.
- Fair and proper citation may seem obvious,
  - but it still doesn't happen all the time.
  - "It is easier to count than it is to read": folks who pay for the computers count citations of papers, not whether a paper appeared in a review.
- Carelessness is interpreted as not caring if the calculations to stop.

Calculations in this lecture

- From Fermilab Lattice and MILC collaborations:
    - I know all (or almost all) the details;
    - can therefore give a better idea of all the steps.
    - for the quantities discussed, the most precise results and/or most thorough study of systematic effects.
- Use ensembles generated by the MILC collaboration, with either
    - 2+1 flavors of sea quark (the determinant) with asqtad staggered action;
    - or 2+1+1 flavors of sea quark with the HISQ action.

asqtad Ensembles: 2+1

MILC, arXiv:0903.3598

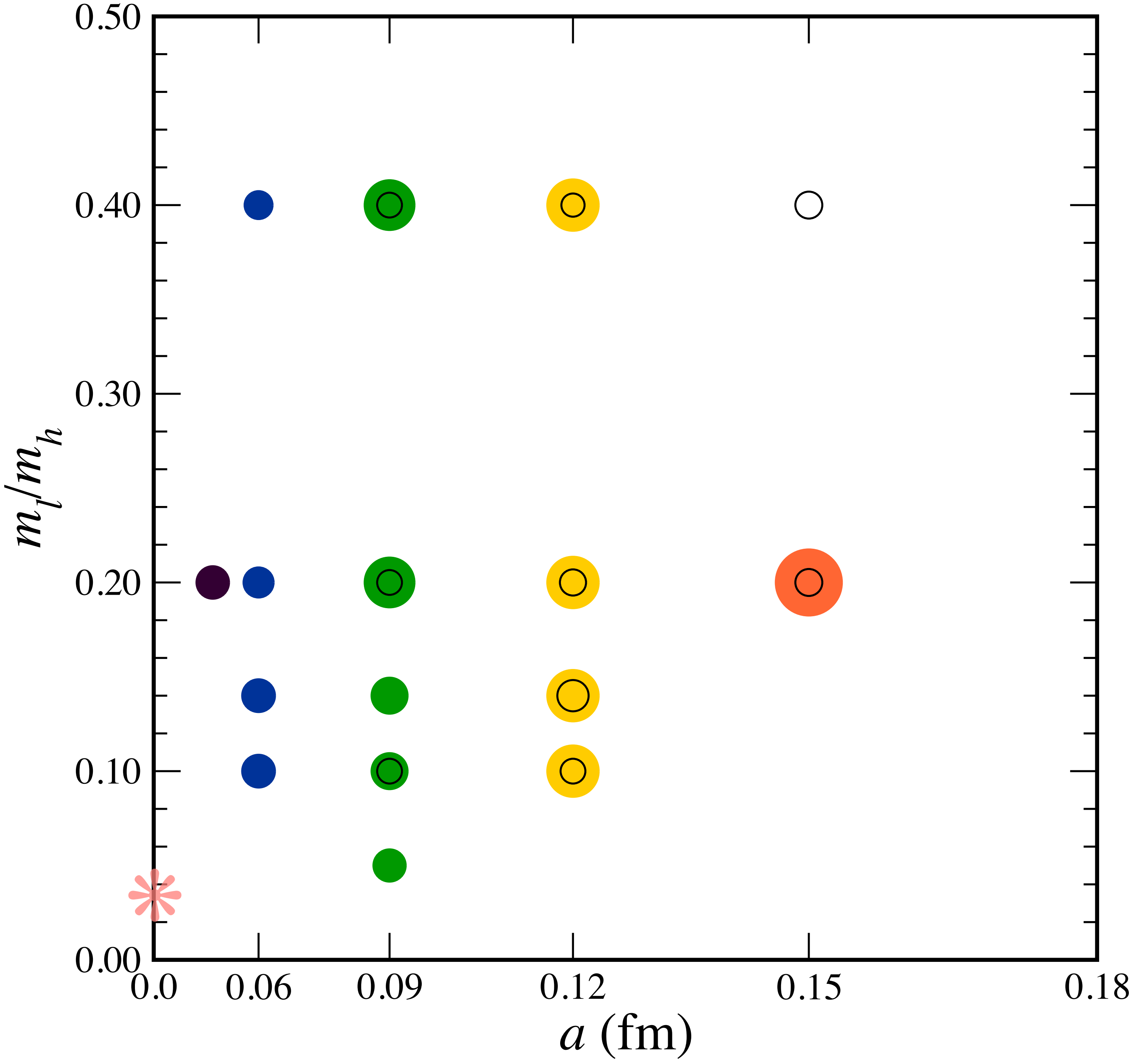
a (fm)	size	am'_l/am'_s	# confs	# sources
≈ 0.15	$16^3\times$ 48	0.0097/0.0484	628	24
≈ 0.12	$20^3\times$ 64	0.02/0.05	2052	4
≈ 0.12	$20^3\times$ 64	0.01/0.05	2256	4
≈ 0.12	$20^3\times$ 64	0.007/0.05	2108	4
≈ 0.12	$24^3\times$ 64	0.005/0.05	2096	4
≈ 0.09	$28^3\times$ 96	0.0124/0.031	1992	4
≈ 0.09	$28^3\times$ 96	0.0062/0.031	1928	4
≈ 0.09	$32^3\times$ 96	0.00465/0.031	984	4
≈ 0.09	$40^3\times$ 96	0.0031/0.031	1012	4
≈ 0.09	$64^3\times$ 96	0.00155/0.031	788	4
≈ 0.06	$48^3\times$144	0.0072/0.018	576	4
≈ 0.06	$48^3\times$144	0.0036/0.018	672	4
≈ 0.06	$56^3\times$144	0.0025/0.018	800	4
≈ 0.06	$64^3\times$144	0.0018/0.018	824	4
≈ 0.045	$64^3\times$192	0.0028/0.014	800	4

HISQ Ensembles: 2+1+1

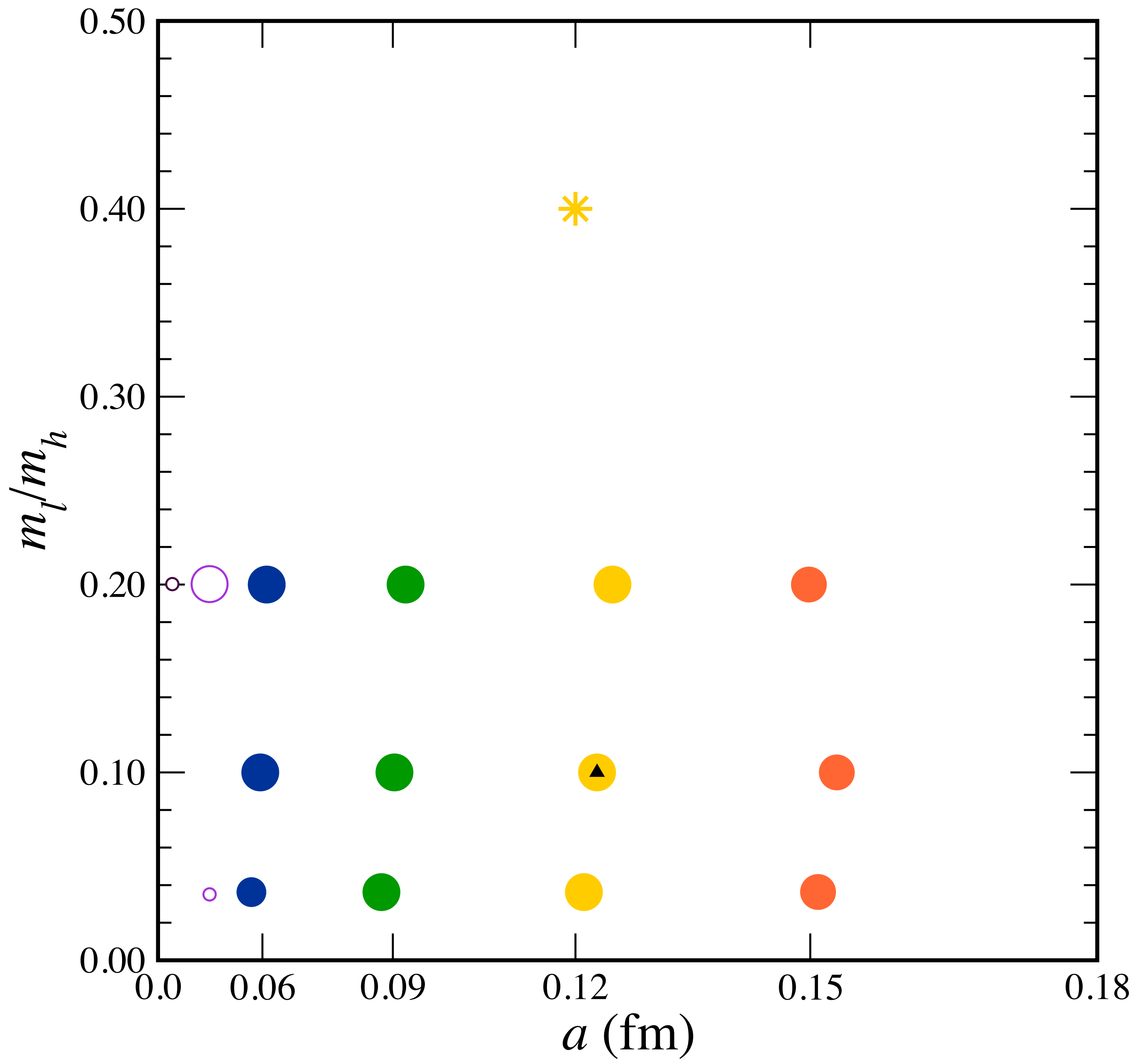
omitting some with $a \approx 0.12$ fm, newer with $a \approx 0.042, 0.03$ fm

a (fm)	size	$am'_l/am'_s/am'_c$	# confs	# sources	notes
≈ 0.15	$16^3\times$ 48	0.0130/0.065/0.838	1020	4	
≈ 0.15	$24^3\times$ 48	0.0064/0.064/0.828	1000	4	
≈ 0.15	$32^3\times$ 48	0.00235/0.0647/0.831	1000	4	
≈ 0.12	$24^3\times$ 64	0.0102/0.0509/0.635	1040	4	
≈ 0.12	$32^3\times$ 64	0.00507/0.0507/0.628	1000	4	3 sizes
≈ 0.12	$48^3\times$ 64	0.00184/0.0507/0.628	999	4	
≈ 0.12	$24^3\times$ 64	0.01275/0.01275/0.640	1020	4	$m'_s = m'_l$
≈ 0.12	$32^3\times$ 64	0.00507/0.0307/0.628	1020	4	$m'_s < m_s$
≈ 0.12	$32^3\times$ 64	0.00507/0.012675/0.628	1020	4	$m'_s \neq m_s$
≈ 0.09	$32^3\times$ 96	0.0074/0.037/0.440	1011	4	
≈ 0.09	$48^3\times$ 96	0.00363/0.0363/0.430	1000	4	
≈ 0.09	$64^3\times$ 96	0.0012/0.0363/0.432	1031	4	
≈ 0.06	$48^3\times$144	0.0048/0.024/0.286	1016	4	
≈ 0.06	$64^3\times$144	0.0024/0.024/0.286	1166	4	→1246
≈ 0.06	$96^3\times$192	0.0008/0.022/0.260	583	6	→ 701*

asqtad Ensembles: 2+1



HISQ Ensembles: 2+1+1



Neutral-Meson Mixing

arXiv:1602.03560

Effective Hamiltonian

- After integrating out heavy particles:

$$\mathscr{L} = \mathscr{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathscr{C}_i(\alpha, \alpha_s, G_F, \sin^2\theta, m_\ell, m_q, V; \text{NP})\mathscr{L}_i[\ell, q, \gamma, g]$$

- For $\Delta F = 2$ processes, discrete symmetries and Fierz rearrangement reduces the list of operators to 8 = 5 + 3:

$$\mathscr{O}_1 = \bar{b}\gamma^\mu L q\, \bar{b}\gamma^\mu L q \qquad \tilde{\mathscr{O}}_1 = \bar{b}\gamma^\mu R q\, \bar{b}\gamma^\mu R q$$

$$\mathscr{O}_2 = \bar{b} L q\, \bar{b} L q \qquad \tilde{\mathscr{O}}_2 = \bar{b} R q\, \bar{b} R q$$

$$\mathscr{O}_3 = \bar{b}^\alpha L q^\beta\, \bar{b}^\beta L q^\alpha \qquad \tilde{\mathscr{O}}_3 = \bar{b}^\alpha R q^\beta\, \bar{b}^\beta R q^\alpha$$

$$\mathscr{O}_4 = \bar{b} L q\, \bar{b} R q$$

$$\mathscr{O}_5 = \bar{b}^\alpha L q^\beta\, \bar{b}^\beta R q^\alpha$$

By parity in QCD: $\langle \bar{B}^0 | \mathscr{O}_i | B^0 \rangle = \langle \bar{B}^0 | \tilde{\mathscr{O}}_i | B^0 \rangle$

Effective Hamiltonian

- After integrating out heavy particles:

$$\mathscr{L} = \mathscr{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathscr{C}_i(\alpha, \alpha_s, G_F, \sin^2\theta, m_\ell, m_q, V; \text{NP})\mathscr{L}_i[\ell, q, \gamma, g]$$

- For $\Delta F = 2$ processes, discrete symmetries and Fierz rearrangement reduces the list of operators to 8 = 5 + 3:

$$\mathscr{O}_1 = \bar{b}\gamma^\mu Lq\,\bar{b}\gamma^\mu Lq \quad \text{SM}$$

$$\mathscr{O}_2 = \bar{b}Lq\,\bar{b}Lq$$

$$\mathscr{O}_3 = \bar{b}^\alpha Lq^\beta\,\bar{b}^\beta Lq^\alpha$$

$$\mathscr{O}_4 = \bar{b}Lq\,\bar{b}Rq$$

$$\mathscr{O}_5 = \bar{b}^\alpha Lq^\beta\,\bar{b}^\beta Rq^\alpha$$

$$\tilde{\mathscr{O}}_1 = \bar{b}\gamma^\mu Rq\,\bar{b}\gamma^\mu Rq$$

$$\tilde{\mathscr{O}}_2 = \bar{b}Rq\,\bar{b}Rq$$

$$\tilde{\mathscr{O}}_3 = \bar{b}^\alpha Rq^\beta\,\bar{b}^\beta Rq^\alpha$$

By parity in QCD: $\langle \bar{B}^0 | \mathscr{O}_i | B^0 \rangle = \langle \bar{B}^0 | \tilde{\mathscr{O}}_i | B^0 \rangle$

Effective Hamiltonian

- After integrating out heavy particles:

$$\mathscr{L} = \mathscr{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathscr{C}_i(\alpha, \alpha_s, G_F, \sin^2\theta, m_\ell, m_q, V; \text{NP}) \mathscr{L}_i[\ell, q, \gamma, g]$$

- For $\Delta F = 2$ processes, discrete symmetries and Fierz rearrangement reduces the list of operators to 8 = 5 + 3:

$\mathscr{O}_1 = \bar{b}\gamma^\mu L q \, \bar{b}\gamma^\mu L q$ SM		$\tilde{\mathscr{O}}_1 = \bar{b}\gamma^\mu R q \, \bar{b}\gamma^\mu R q$
$\mathscr{O}_2 = \bar{b} L q \, \bar{b} L q$		$\tilde{\mathscr{O}}_2 = \bar{b} R q \, \bar{b} R q$
$\mathscr{O}_3 = \bar{b}^\alpha L q^\beta \, \bar{b}^\beta L q^\alpha$	BSM	$\tilde{\mathscr{O}}_3 = \bar{b}^\alpha R q^\beta \, \bar{b}^\beta R q^\alpha$
$\mathscr{O}_4 = \bar{b} L q \, \bar{b} R q$		
$\mathscr{O}_5 = \bar{b}^\alpha L q^\beta \, \bar{b}^\beta R q^\alpha$		

By parity in QCD: $\langle \bar{B}^0 | \mathscr{O}_i | B^0 \rangle = \langle \bar{B}^0 | \tilde{\mathscr{O}}_i | B^0 \rangle$

Historical Baggage

- Usually, the mixing matrix elements are recast as "bag parameters":

$$\langle \bar{B}_q | \mathscr{O}_i | B_q \rangle = \left[\mathfrak{e}_i + \mathfrak{c}_i \left(\frac{M_{B_q}}{m_b + m_q} \right)^2 \right] M_{B_q}^2 f_{B_q}^2 B_{B_q}^{(i)}$$

  with certain fractions defined via the "vacuum saturation approximation", $\langle \bar{B}_q | \bar{b}\Gamma q\, \bar{b}\Gamma' q | B_q \rangle \sim \langle \bar{B}_q | \bar{b}\Gamma q | 0 \rangle \langle 0 | \bar{b}\Gamma' q | B_q \rangle$.

- The jargon "bag" dates back to the MIT bag model of hadrons.

- The "approximation" $B_B = 1$ varies with renormalization scheme/scale.

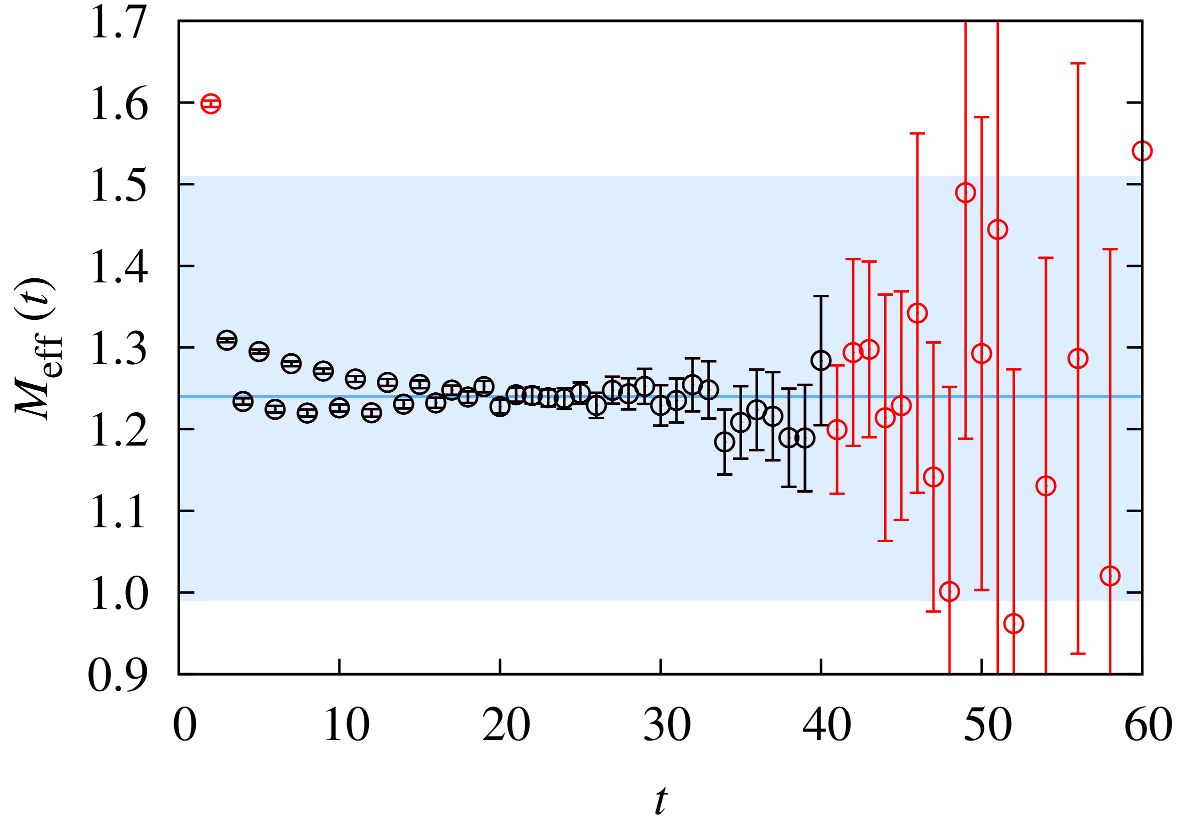
- Not "parameters" but outputs of a calculation.

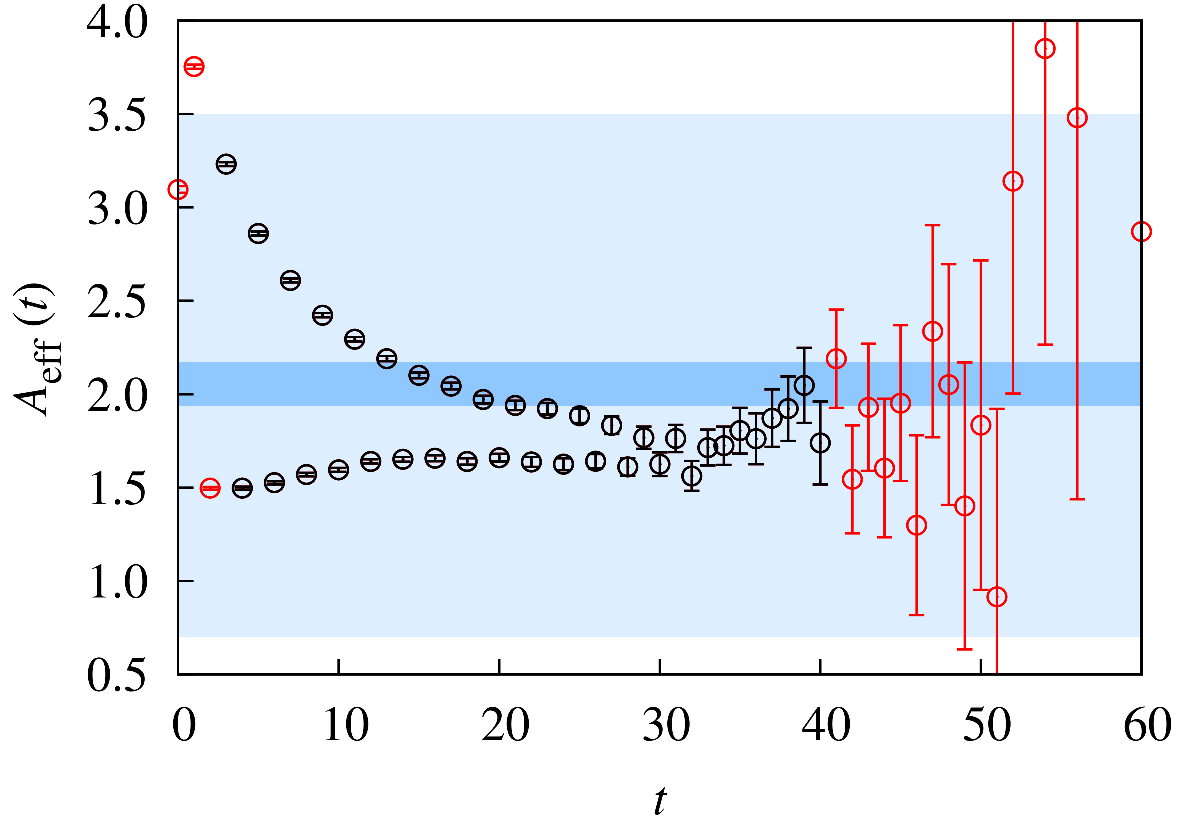
Two-Point Correlation Function

- Compare "effective" mass and amplitude to full fit

$$M_{\text{eff}}(t) = \cosh^{-1} \frac{C_B(t+1) + C_B(t-1)}{2C_B(t)}$$

$$C_B(t) = \langle B^\dagger(t) B^\dagger(0) \rangle \qquad A_{\text{eff}}(t) = C_B(t)\, e^{M_{\text{eff}} t}$$

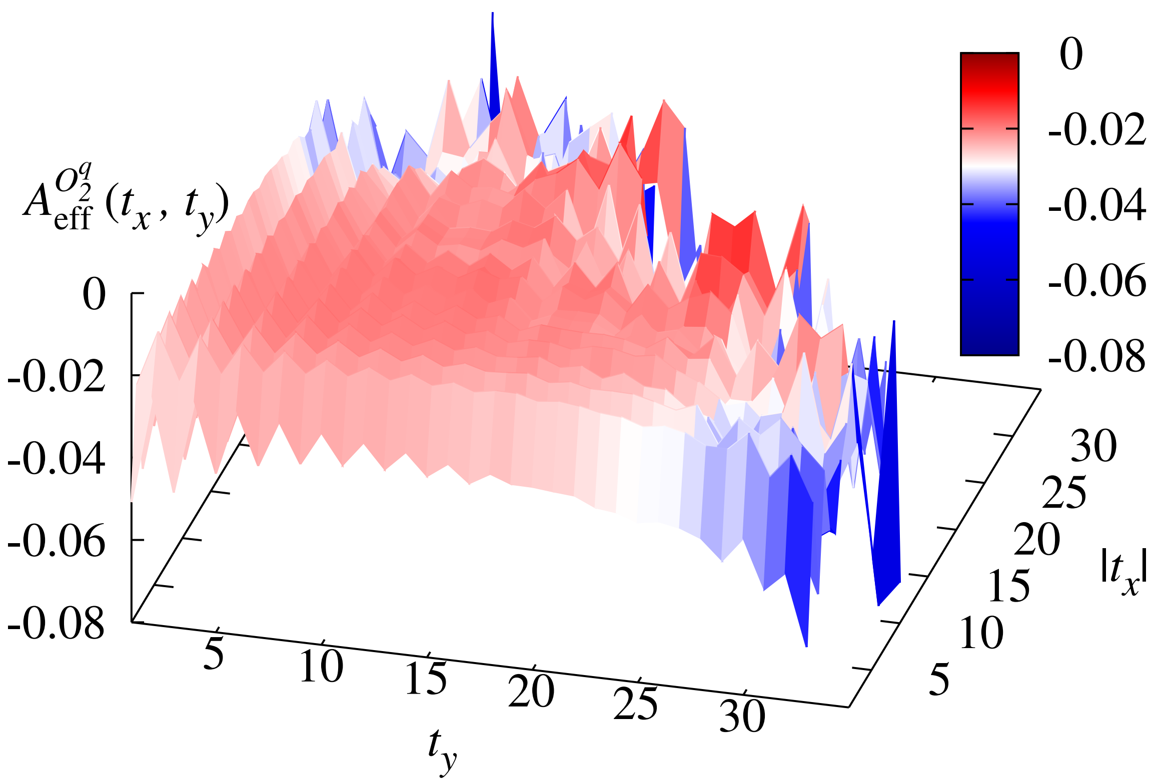


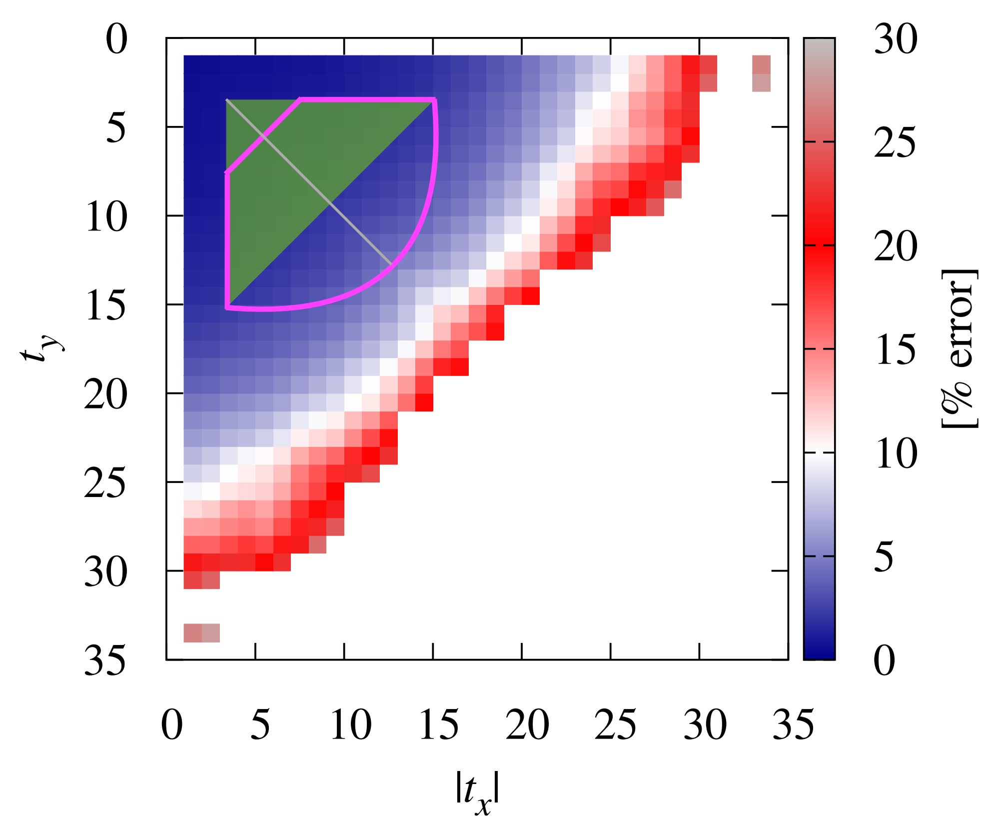


Three-Point Correlation Function

$$A_{\text{eff}}^{O_i}(t_x, t_y) = C_{O_i}(t_x, t_y)\, e^{M_{\text{eff}}(t_y + |t_x|)}$$

$$C_{O_i}(t_x, t_y) = \langle B^\dagger(t_y)\mathscr{O}_i(0)B^\dagger(t_x)\rangle$$





Matching to Continuum QCD

- At this stage, we have 510 data points for the five matrix elements.
- As in continuum QCD, they have to be renormalized, i.e., logarithmic dependence on the UV cutoff (π/a) must be removed.
- At the same time, we match to the renormalization schemes used in the literature: $\overline{\text{MS}}$-NDR-BBGLN or -BMU (evanescent operators) at $\mu = m_b$.
- Use a nonperturbative method for matching related to the vector current.
- Use one-loop (lattice & continuum) perturbation theory for the intrinsic four-quark terms: “mostly nonperturbative renormalization”.
- Moves individual data points < few %.

Chiral-Continuum Limit

- The renormalized 510 data points for the five matrix elements can now be
  - extrapolated to the physical quark mass: $m_l, m_q \to (m_u+m_d)/2$;
  - extrapolated to the continuum limit: $a \to 0$.
- Fit to a version of chiral perturbation theory (χPT) adapted to discretization effects in the pions: $B \leftrightarrow B^*\pi$.
- Augment with NNLO (and NNNLO) terms in χPT.
- Augment with heavy-quark discretization effects derived from HQET.

- Augment with two-loop and (three-loop) matching corrections.
- Augment with generic light-quark & gluon discretization effects.
- Input various auxiliary information: mass splittings, $B^*B\pi$ coupling, ….
- Minimize

$$\chi^2_{\text{aug}} = \sum_{\alpha,\beta} \left[F^{\text{base}} - Z\langle O\rangle\right]_\alpha \sigma^{-2}_{\alpha\beta} \left[F^{\text{base}} - Z\langle O\rangle\right]_\beta + \sum_m \frac{(P_m - \tilde{P}_m)^2}{\tilde{\sigma}^2_m}$$

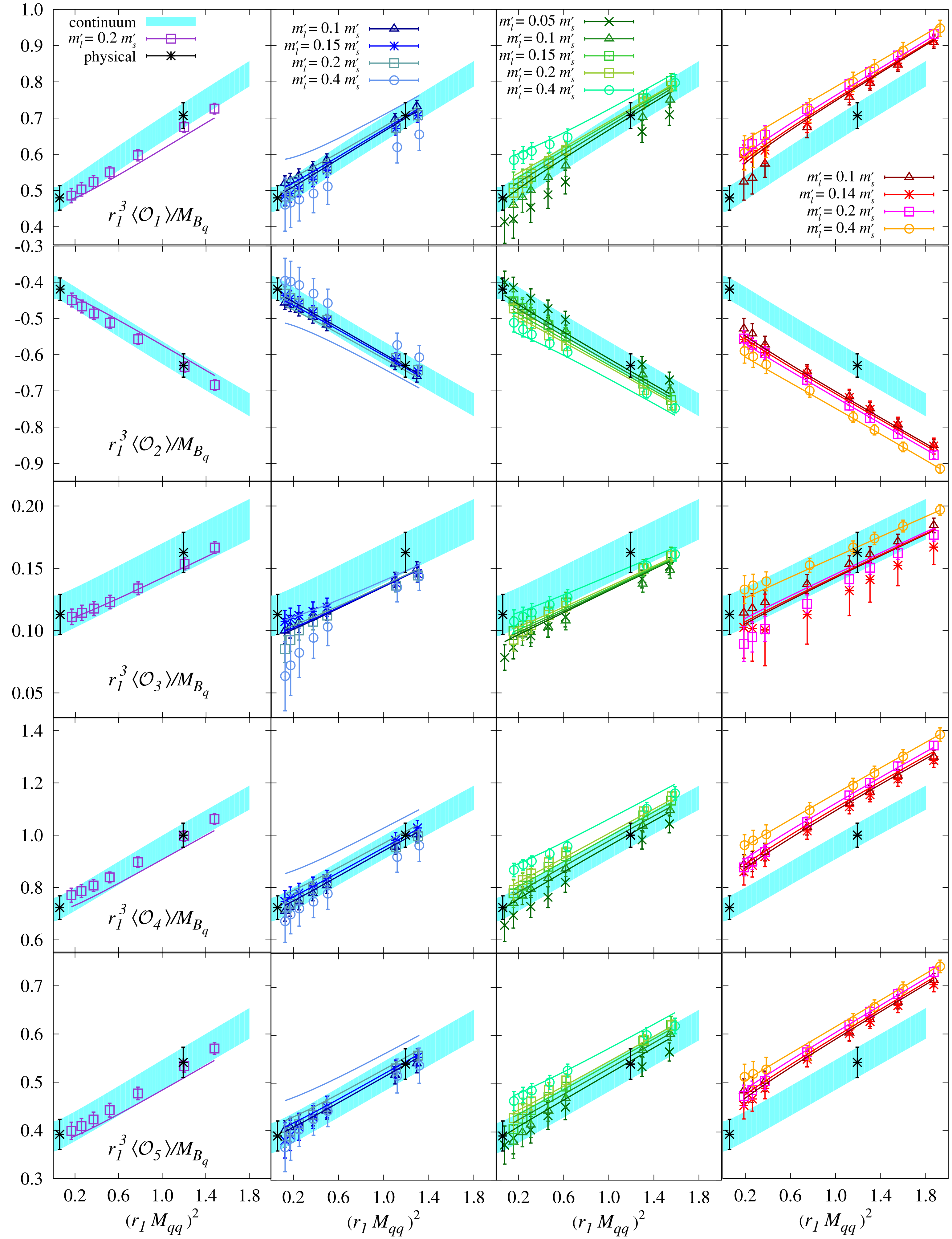
where α, β run over the 510 data points, and m runs over the ~100 fit parameters: $F^{\text{base}} = F^{\text{base}}(P_m)$.

- "Informative" priors would add ~1 to χ^2; ours are a bit looser than that.

- Data in r_1 units:

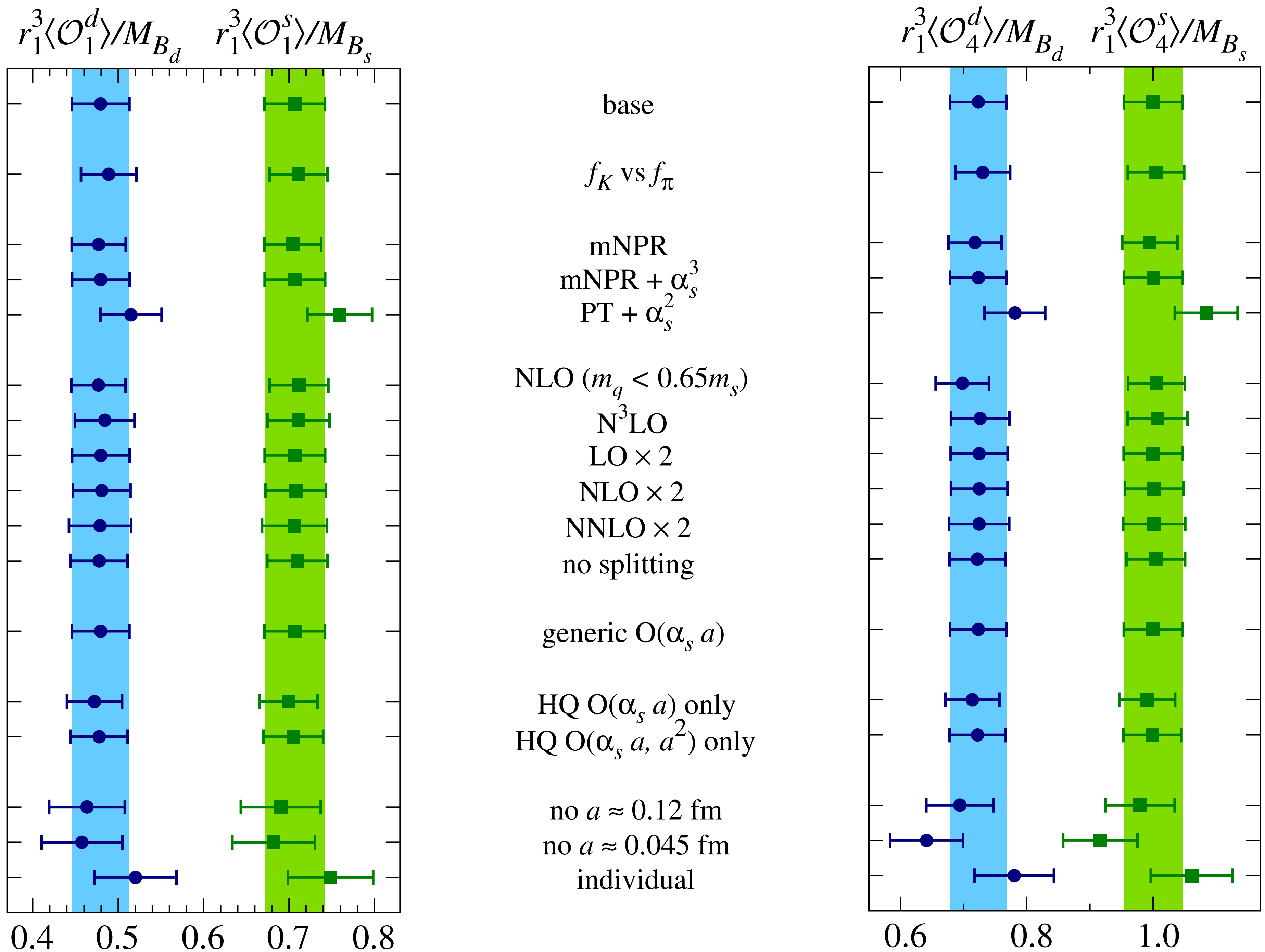
$$r_1^2 F(r_1) = 1$$

- χPT effects mix the 123 and 45 sets.
- Meson masses are the same for all 5.
- Thus, one fit to all 510 points.
- Reconstitute at $a = 0$ and $m_q = m_d, m_s$.
- $\chi^2_{\rm aug}/{\rm dof} = 137.7/510$



Stability

- f_K instead of f_π;
- different renorm'n;
- vary χPT data, priors;
- vary discretization effects included;
- "dumb" fits.



Basic Observables

	B_d–$\bar{B}_d$		B_s–$\bar{B}_s$	
	BMU	BBGLN	BMU	BBGLN
$f_{B_q}^2 B_{B_q}^{(1)}(\overline{m}_b)$	0.0347(27)(7)		0.0503(29)(10)	
$f_{B_q}^2 B_{B_q}^{(2)}(\overline{m}_b)$	0.0290(24)(6)	0.0305(25)(6)	0.0425(26)(9)	0.0451(27)(9)
$f_{B_q}^2 B_{B_q}^{(3)}(\overline{m}_b)$	0.0412(61)(8)	0.0409(60)(8)	0.0585(61)(12)	0.0581(60)(12)
$f_{B_q}^2 B_{B_q}^{(4)}(\overline{m}_b)$	0.0396(28)(8)		0.0540(30)(11)	
$f_{B_q}^2 B_{B_q}^{(5)}(\overline{m}_b)$	0.0366(31)(7)		0.0497(33)(10)	

$$f_{B_d}\sqrt{\hat{B}_{B_d}^{(1)}} = 229.4(9.0)(2.3)\ \text{MeV}$$
$$f_{B_s}\sqrt{\hat{B}_{B_s}^{(1)}} = 276.0(8.0)(2.8)\ \text{MeV}$$

$$\xi = 1.203(17)(6)$$

Error Budget

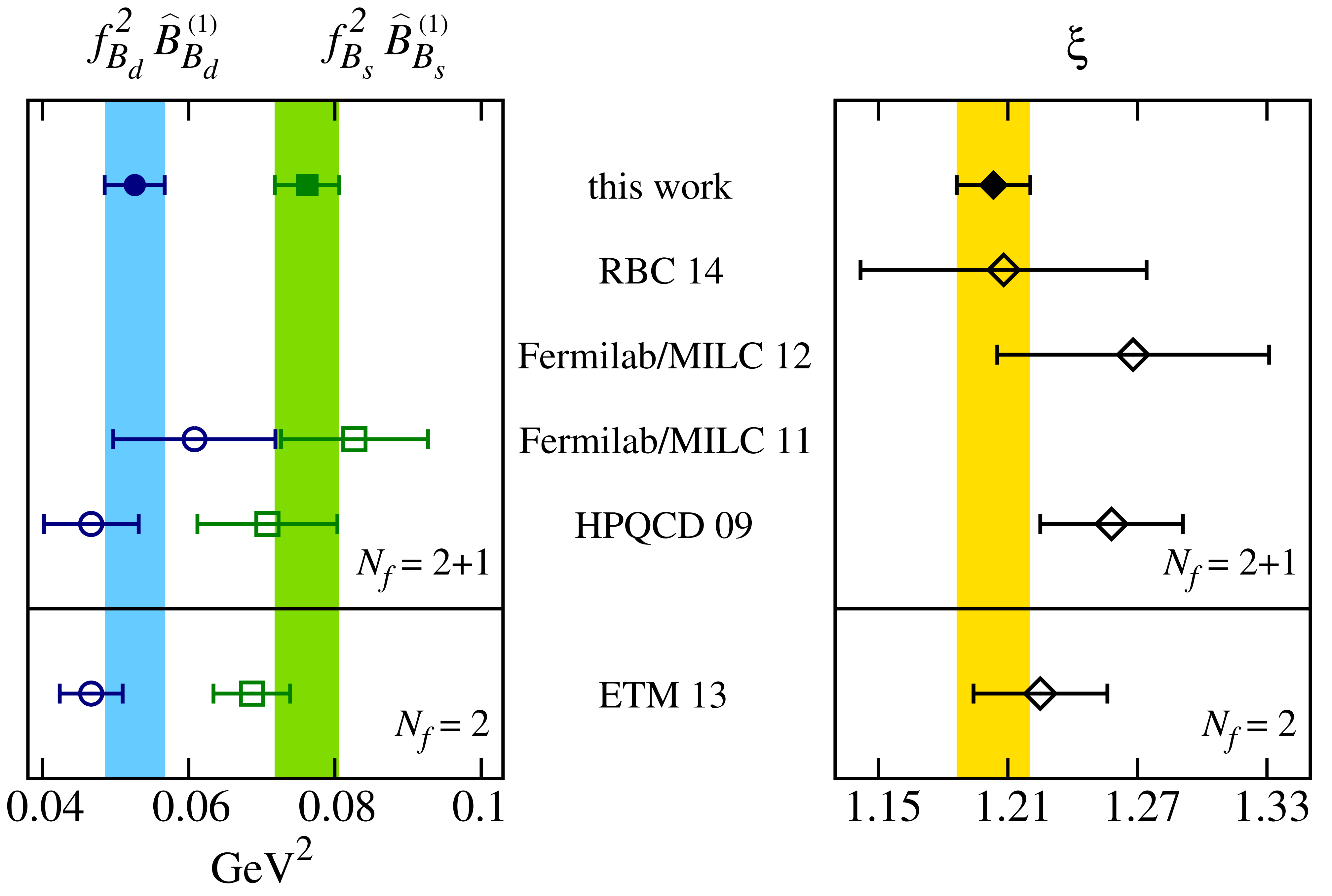
Table 1: Total error budget for matrix elements converted to physical units of GeV^3 and for the dimensionless ratio ξ. The error from isospin breaking, which is estimated to be negligible at our current level of precision is not shown. Entries are in percent.

	Fit total	FV	r_1/a	r_1	EM	Total	Charm sea
$\langle \mathcal{O}_1^d \rangle / M_{B_d}$	7.0	0.2	2.7	2.1	0.2	7.8	2.0
$\langle \mathcal{O}_2^d \rangle / M_{B_d}$	7.4	0.3	2.8	2.1	0.2	8.2	2.0
$\langle \mathcal{O}_3^d \rangle / M_{B_d}$	14.5	< 0.1	1.7	2.1	0.2	14.7	2.0
$\langle \mathcal{O}_4^d \rangle / M_{B_d}$	6.2	< 0.1	2.3	2.1	0.2	7.0	2.0
$\langle \mathcal{O}_5^d \rangle / M_{B_d}$	8.0	< 0.1	2.4	2.1	0.2	8.6	2.0
$\langle \mathcal{O}_1^s \rangle / M_{B_s}$	5.0	0.2	2.1	2.1	0.2	5.8	2.0
$\langle \mathcal{O}_2^s \rangle / M_{B_s}$	5.1	0.2	2.1	2.1	0.2	5.9	2.0
$\langle \mathcal{O}_3^s \rangle / M_{B_s}$	10.0	< 0.1	1.3	2.1	0.2	10.3	2.0
$\langle \mathcal{O}_4^s \rangle / M_{B_s}$	4.6	< 0.1	1.9	2.1	0.2	5.4	2.0
$\langle \mathcal{O}_5^s \rangle / M_{B_s}$	5.9	< 0.1	1.8	2.1	0.2	6.5	2.0
ξ	1.3	< 0.1	0.6	0	0.04	1.4	0.5

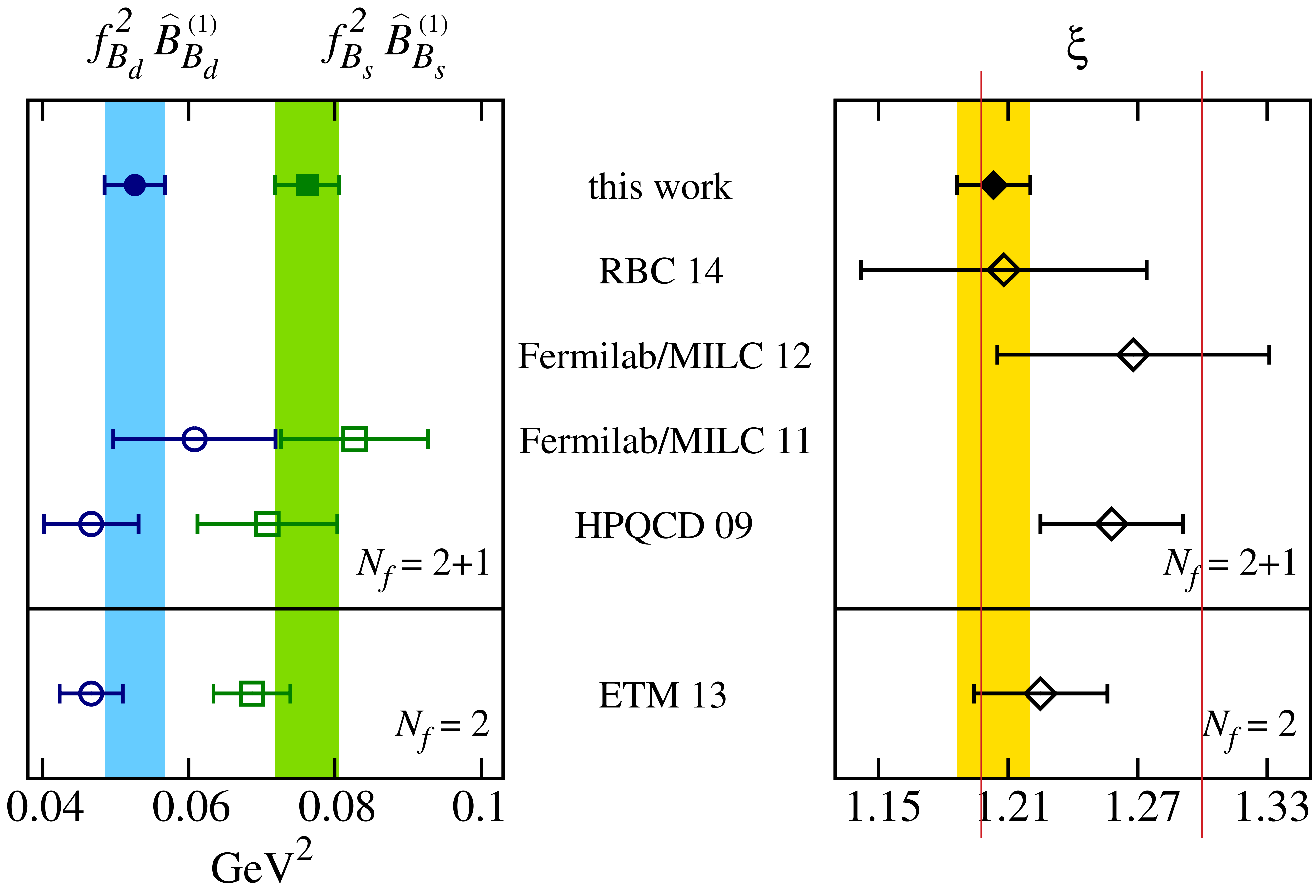
Correlation Matrix

	$f_{B_d}^2 B_{B_d}^{(1)}$	$f_{B_d}^2 B_{B_d}^{(2)}$	$f_{B_d}^2 B_{B_d}^{(3)}$	$f_{B_d}^2 B_{B_d}^{(4)}$	$f_{B_d}^2 B_{B_d}^{(5)}$	$f_{B_s}^2 B_{B_s}^{(1)}$	$f_{B_s}^2 B_{B_s}^{(2)}$	$f_{B_s}^2 B_{B_s}^{(3)}$	$f_{B_s}^2 B_{B_s}^{(4)}$	$f_{B_s}^2 B_{B_s}^{(5)}$
$f_{B_d}^2 B_{B_d}^{(1)}$	1	0.415	0.124	0.320	0.297	0.845	0.417	0.142	0.323	0.308
$f_{B_d}^2 B_{B_d}^{(2)}$		1	0.332	0.349	0.281	0.416	0.841	0.348	0.360	0.295
$f_{B_d}^2 B_{B_d}^{(3)}$			1	0.204	0.119	0.133	0.316	0.954	0.203	0.125
$f_{B_d}^2 B_{B_d}^{(4)}$				1	0.457	0.343	0.380	0.232	0.848	0.468
$f_{B_d}^2 B_{B_d}^{(5)}$					1	0.312	0.300	0.140	0.449	0.879
$f_{B_s}^2 B_{B_s}^{(1)}$						1	0.464	0.175	0.385	0.357
$f_{B_s}^2 B_{B_s}^{(2)}$							1	0.368	0.437	0.354
$f_{B_s}^2 B_{B_s}^{(3)}$								1	0.257	0.169
$f_{B_s}^2 B_{B_s}^{(4)}$									1	0.508
$f_{B_s}^2 B_{B_s}^{(5)}$										1

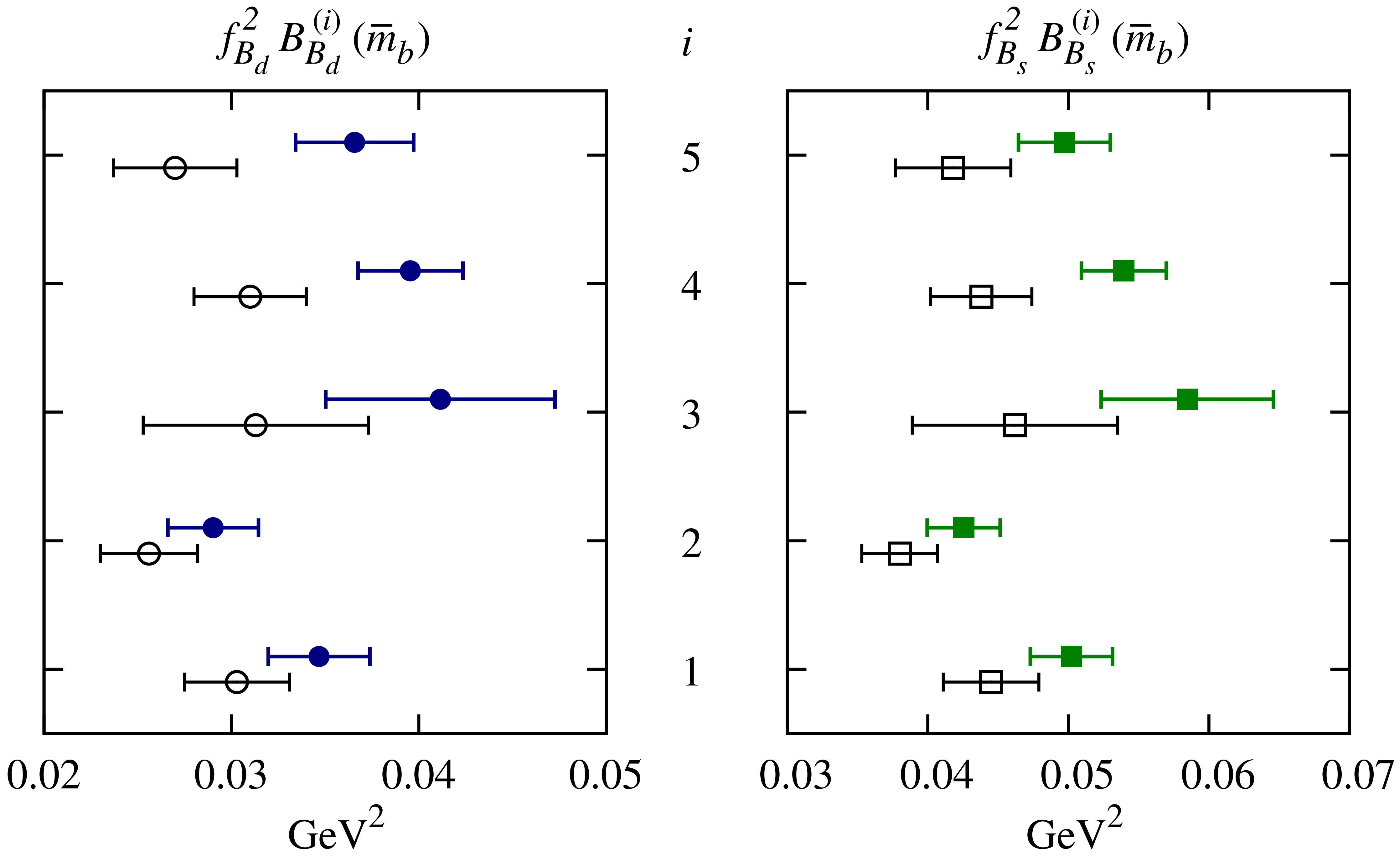
Comparison with Other Calculations 1



Comparison with Other Calculations 1



Comparison with ETM Calculation ($n_f = 2$)



Oscillation Frequencies

- New results for mixing in arXiv:1602.03560: 2–3 times more precise.

- Taking CKM from tree-only inputs (from CKMfitter):

$$\Delta M_d^{\mathrm{SM}} = 0.630(53)(42)(5)(13)\ \mathrm{ps}^{-1}$$

$$\Delta M_s^{\mathrm{SM}} = 19.6(1.2)(1.0)(0.2)(0.4)\ \mathrm{ps}^{-1}$$

$$\frac{\Delta M_d^{\mathrm{SM}}}{\Delta M_s^{\mathrm{SM}}} = 0.0321(10)(15)(0)(3)$$

- Contrast with the measured frequencies:

$$\Delta M_d^{\mathrm{expt}} = (0.5055 \pm 0.0020)\ \mathrm{ps}^{-1}$$

$$\Delta M_s^{\mathrm{expt}} = (17.757 \pm 0.021)\ \mathrm{ps}^{-1}$$

- These amount to discrepancies of 1.8σ, 1.1σ, and 2.0σ, respectively.

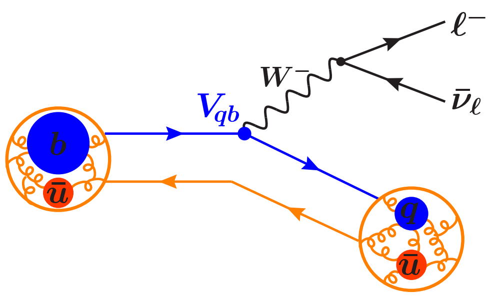
- Below, examine these tensions with those in other FCNC processes, casting each one as a “CKM determination”.

Semileptonic Decays

Processes

$\ell = \mu, e$

- CKM Determination
  - ✓ $B \to \pi \ell \nu$
  - ✓ $B \to D \ell \nu$
  - ✓ $B \to D^* \ell \nu$
  - ✓ $(\Lambda_b \to \Lambda_c / p \, \ell \nu)$
- New Physics Search
  - $B \to \pi \tau \nu$
  - $B \to D \tau \nu$
  - $B \to K \nu \bar{\nu}$
  - $B \to \pi \nu \bar{\nu}$
  - ✓ $B \to K \ell^+ \ell^-$
  - ✓ $B \to \pi \ell^+ \ell^-$



Matrix Elements and Form Factors

- Decompose amplitudes in form factors ($q = p - k = \ell + \nu$):

$$\langle \pi(k)|\bar{u}\gamma^\mu b|B(p)\rangle = \left(p^\mu + k^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right) f_+(q^2) + \frac{M_B^2 - M_\pi^2}{q^2} q^\mu f_0(q^2),$$

$$= \sqrt{2M_B}\left[p^\mu f_\parallel(q^2)/M_B + k_\perp^\mu f_\perp(q^2)\right]$$

$$\langle \pi(k)|\bar{u}\sigma^{\mu\nu} b|B(p)\rangle = -2i\frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2),$$

$$\langle \pi(k)|\bar{u}b|B(p)\rangle = \frac{M_B^2 - M_\pi^2}{m_b - m_u} f_0(q^2),$$

PCVC: same

- The kinematic variable $q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$.

Basic Formulas for $B \to \pi l \nu$

- Relevant term in effective Hamiltonian: $\mathscr{L}_i = \bar{b}\gamma^{\mu}(1-\gamma^5)u\,\bar{\nu}_{\ell}\gamma_{\mu}(1-\gamma^5)\ell$
- Differential decay rate:

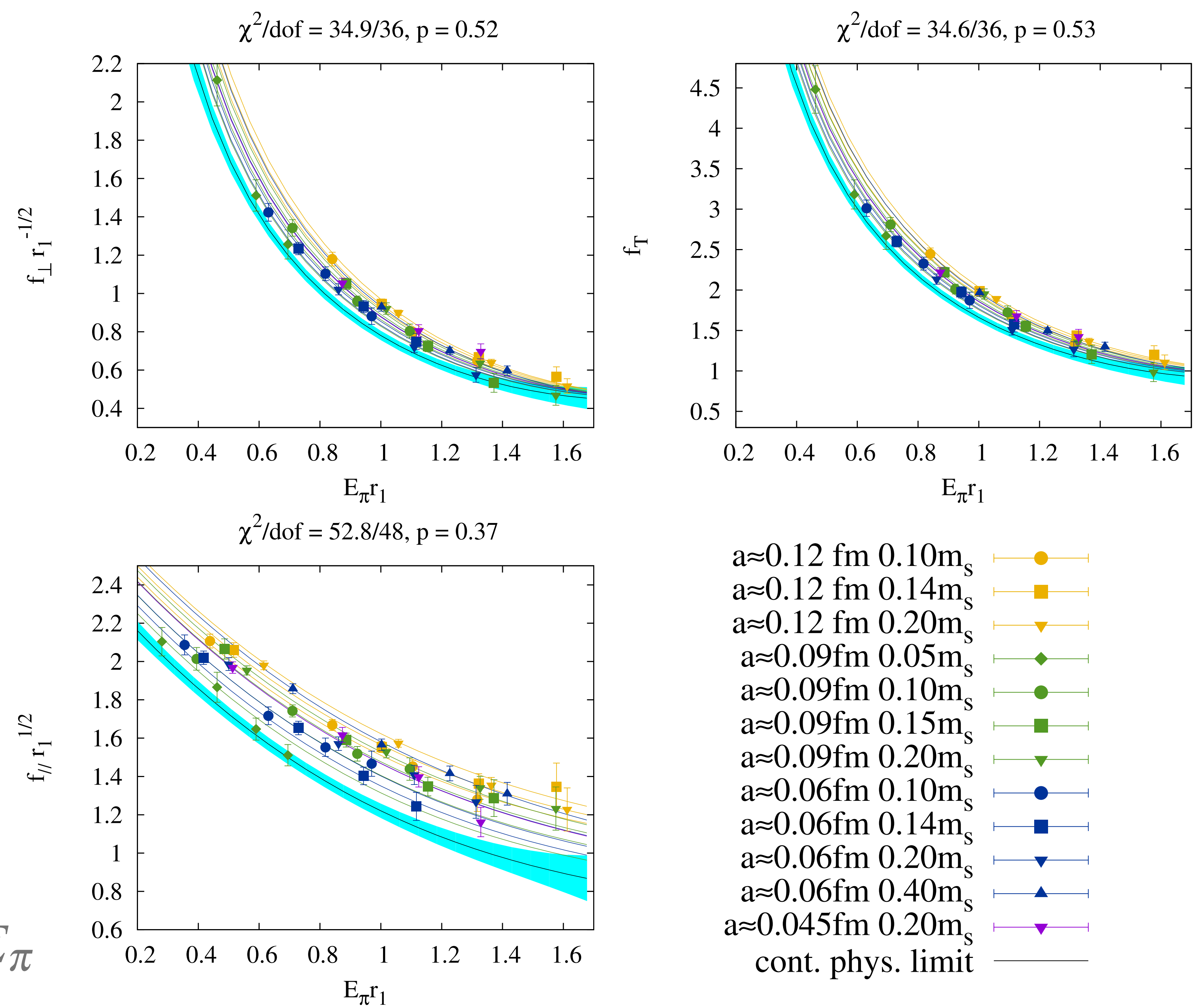
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}|V_{ub}|^2|\boldsymbol{k}|^3|f_+(q^2)|^2 + \mathrm{O}(m_\ell^2),$$

- Steps:
  - generate numerical data at several $\boldsymbol{k}$, m_l, a;
  - chiral continuum extrapolation;
  - extend to full kinematic range with z expansion.

Semileptonic $B \to \pi l \nu$ for $|V_{ub}|$

arXiv:1503.07839

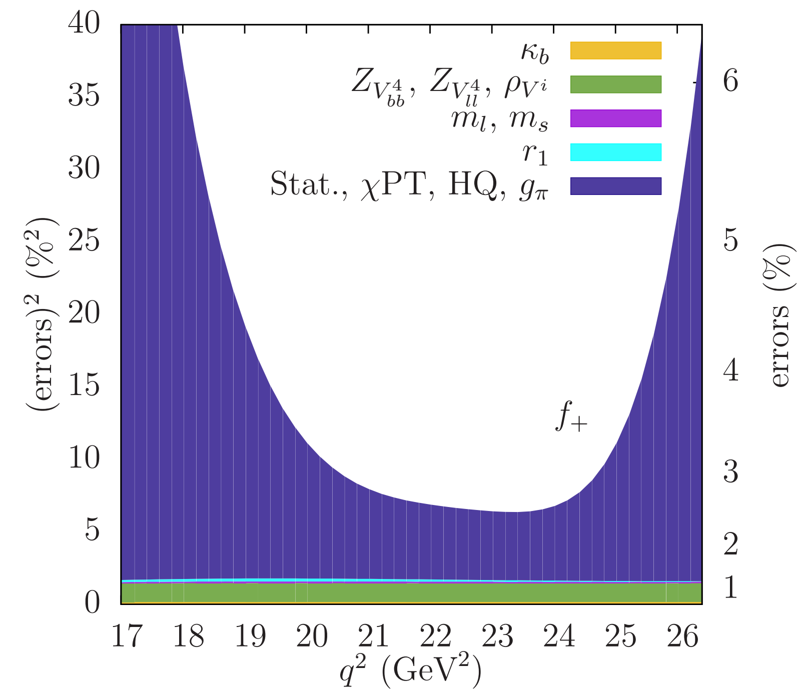
- Compute $f(\boldsymbol{k}, m_s, m_l, a)$
- Combine data with Symanzik EFT & χPT:
  - $m_l \to ½(m_u+m_d)$;
  - $a \to 0$.
- Limited range: $|\boldsymbol{k}|a \ll 1$.
- NB: $q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$

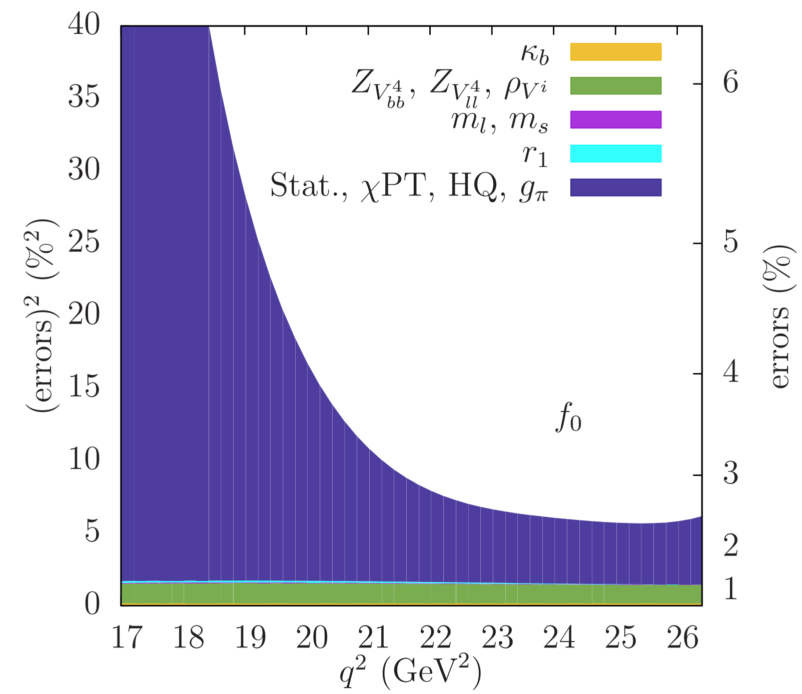


Error Charts

near region with lattice data

- The largest uncertainty, by far, comes from MC statistics, as amplified via the chiral-continuum extrapolation.
- Next (and independent of q^2) is matching from LGT to continuum QCD.
- Error on input parameters (m_l, m_s, $\varkappa_b$) & relative scale (r_1) disappear in quadrature sum.
- Challenge: extend reach to lower q^2, without being killed by the (amplified) statistical error.





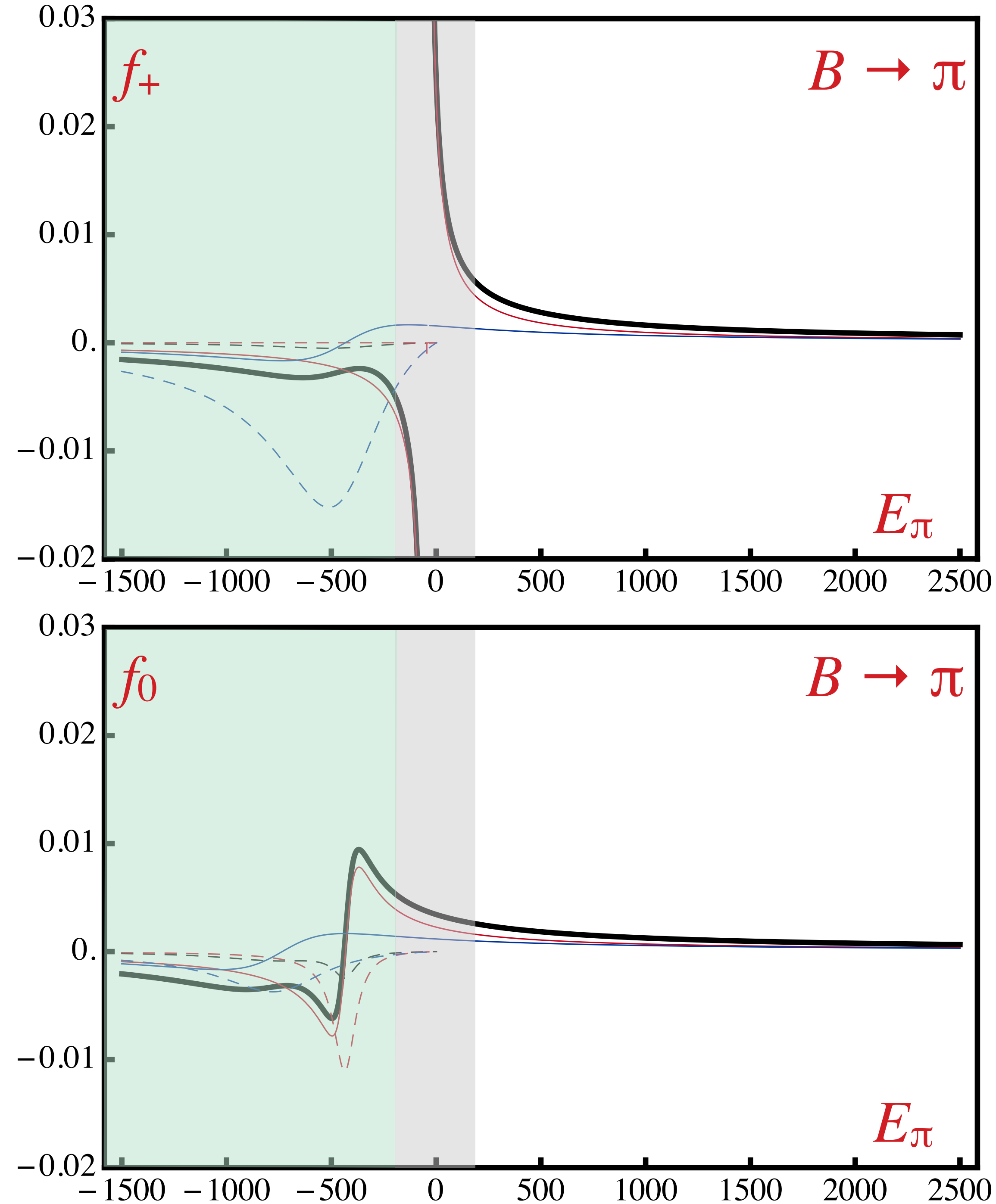
Analyticity and Unitarity

- The form factor is analytic in q^2 except a cut for $q^2 \geq (M_B + M_\pi)^2$ and (possibly) subthreshold poles $(M_B - M_\pi)^2 \leq q^2 < (M_B + M_\pi)^2$.

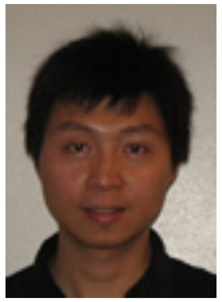
- With $t_+ = (M_B + M_\pi)^2$, set

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

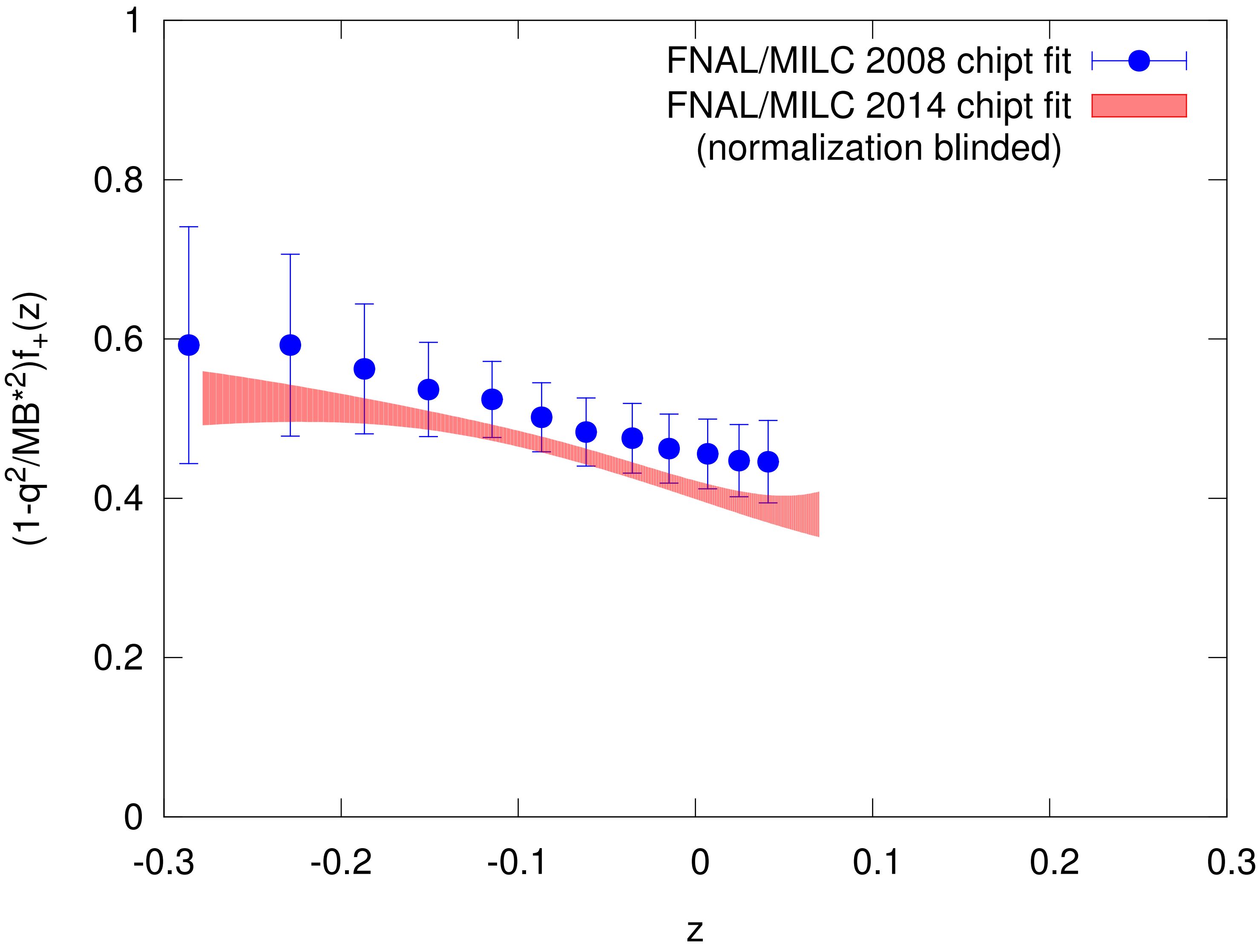
which maps cut to unit circle and semileptonic decay to real $|z| \leq 0.28$ for optimal t_0.



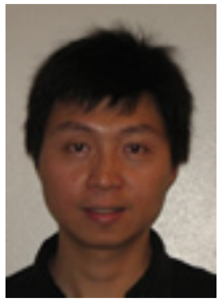
Determination of $|V_{ub}|$



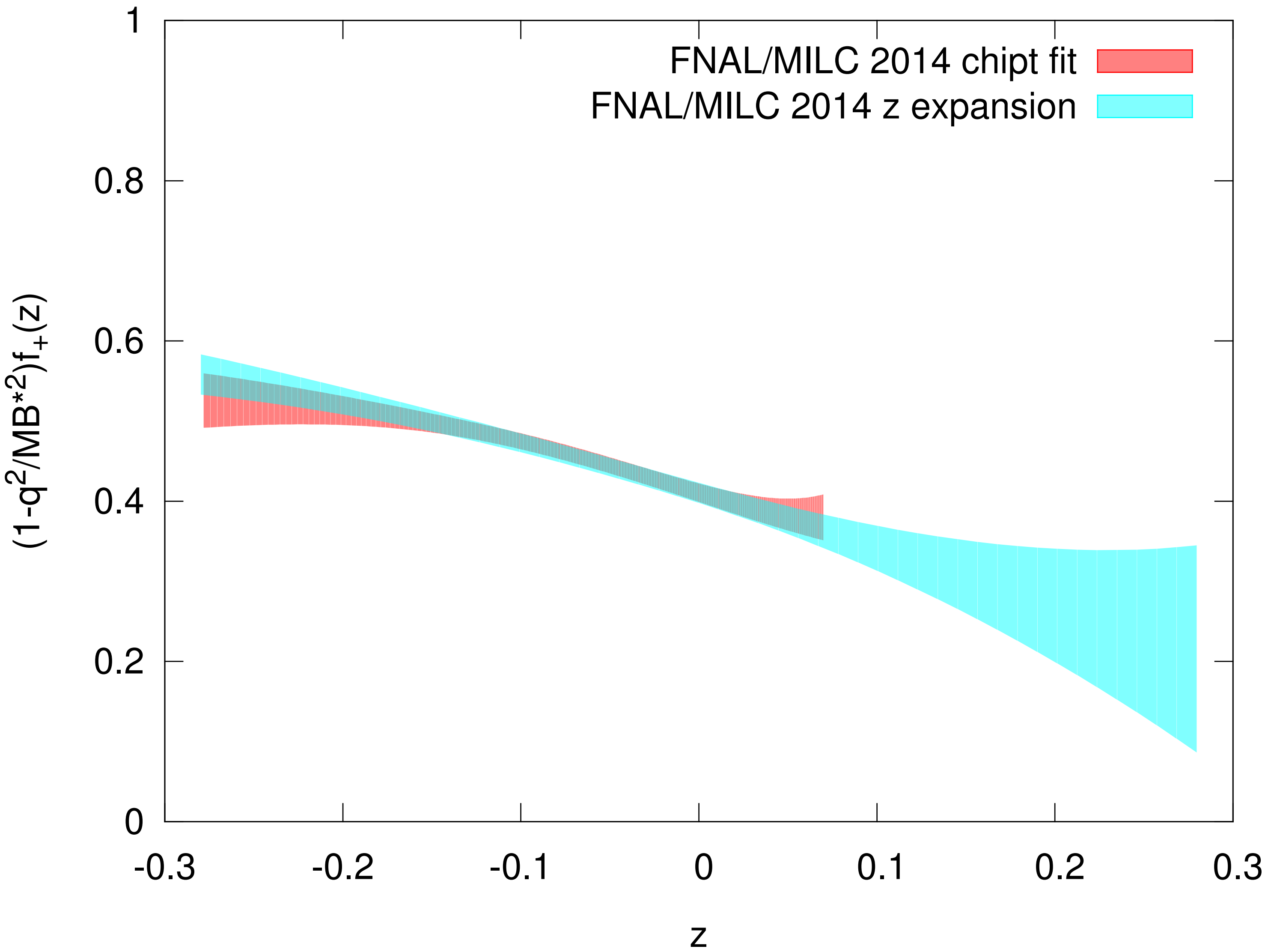
- Much more precise than 2008.
- **BLINDED PLOTS!!**
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$: 4.1%.



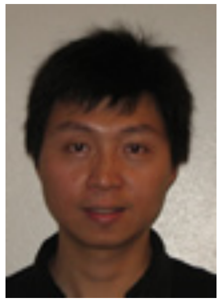
Determination of $|V_{ub}|$



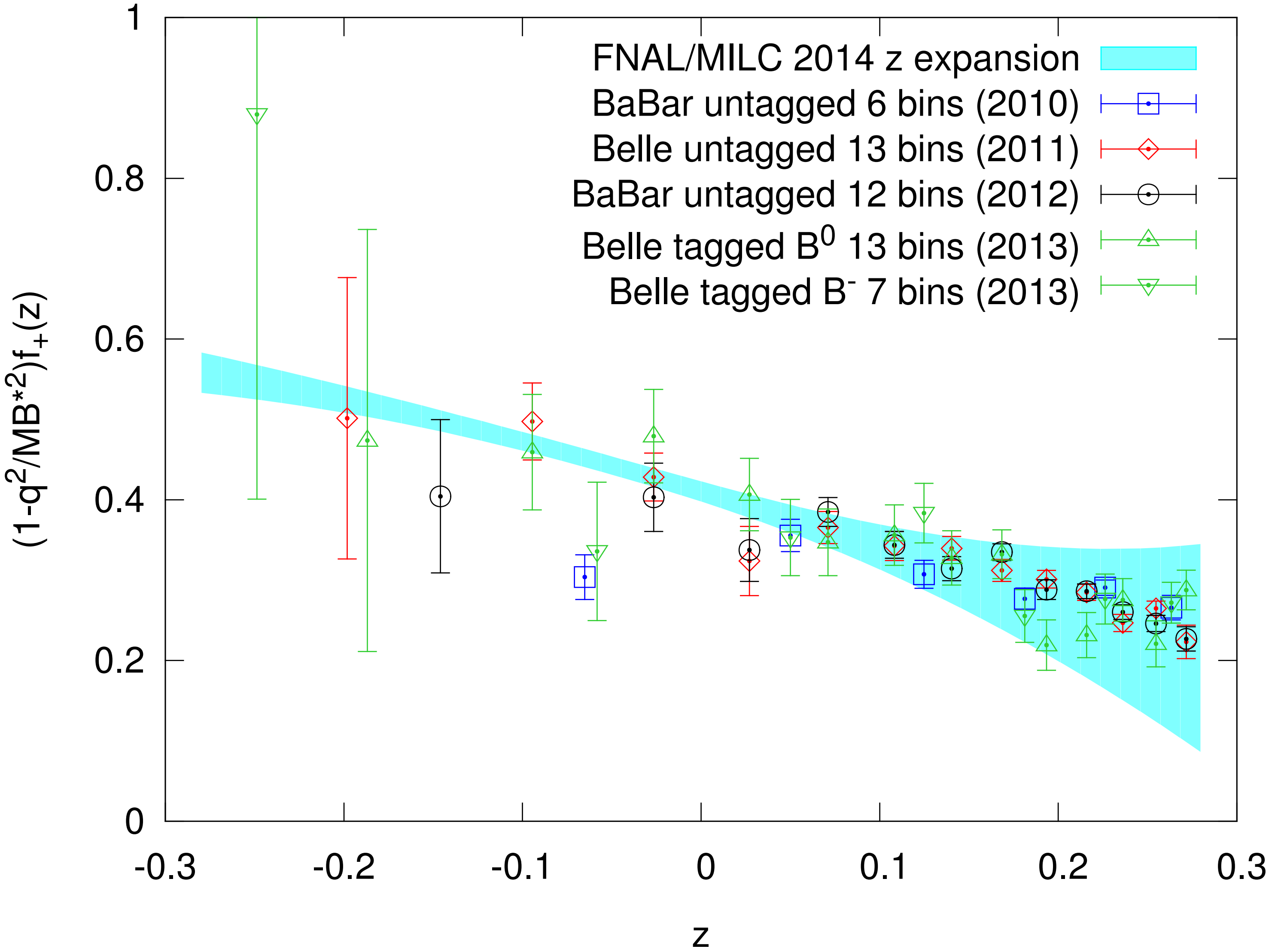
- Much more precise than 2008.
- **BLINDED PLOTS!!**
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$: 4.1%.



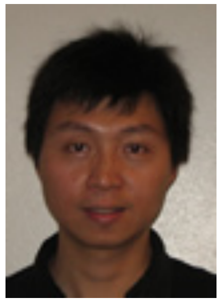
Determination of $|V_{ub}|$



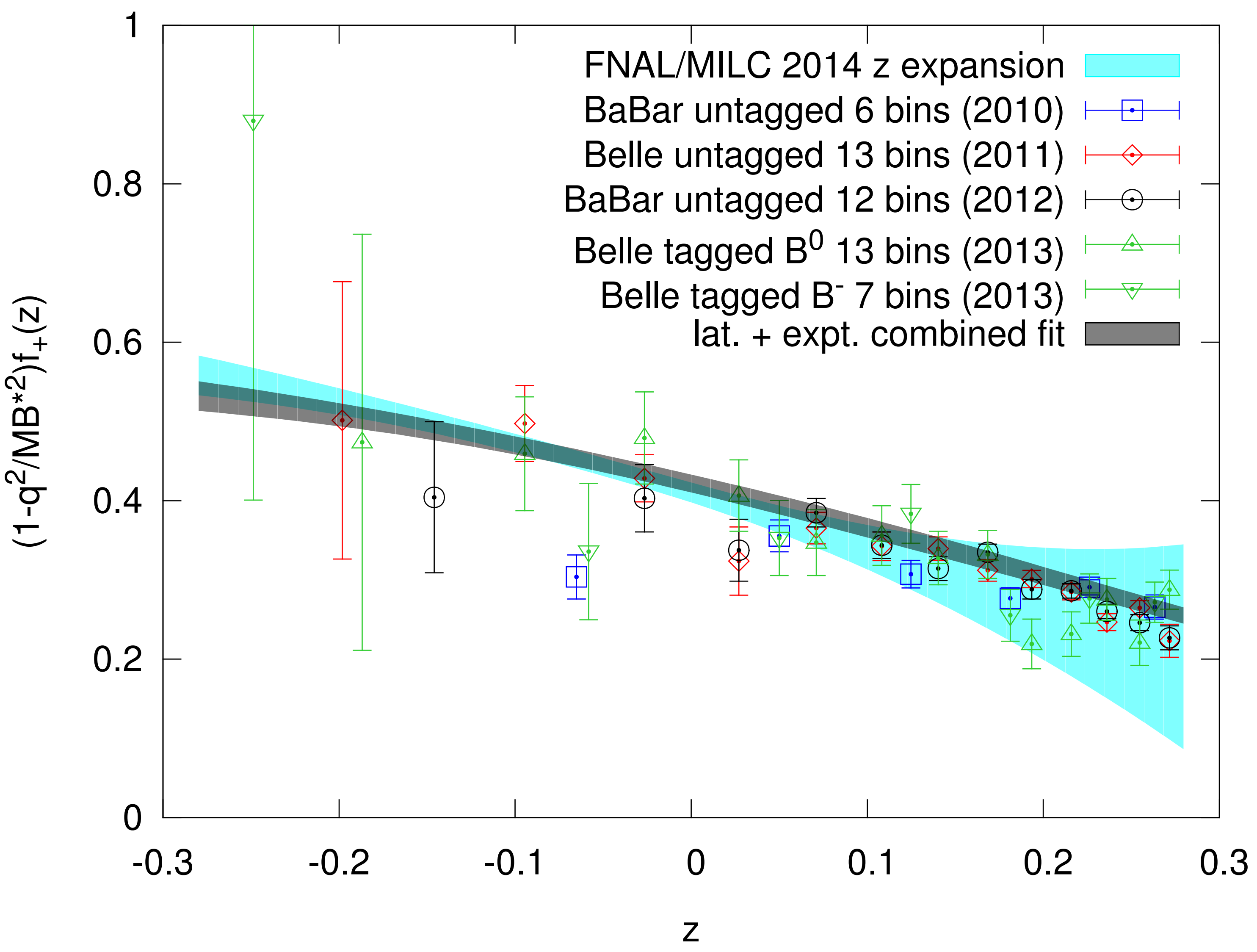
- Much more precise than 2008.
- **BLINDED PLOTS!!**
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$: 4.1%.



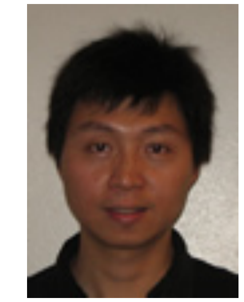
Determination of $|V_{ub}|$



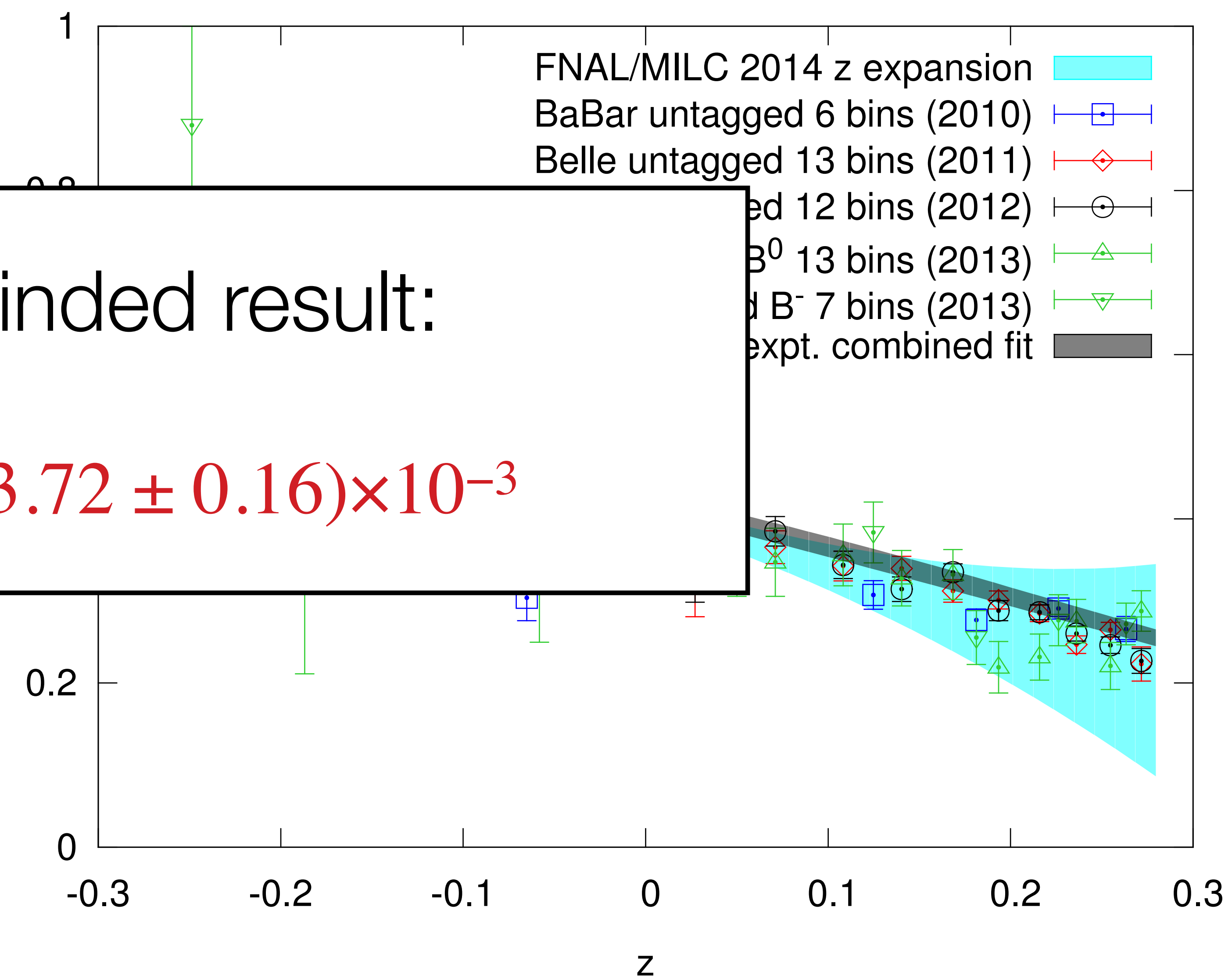
- Much more precise than 2008.
- **BLINDED PLOTS!!**
- z variable extends range.
- Functional fitting method.
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$: 4.1%.



Determination of $|V_{ub}|$



- Much more precise than 2008.
- **BLINDED PLOTS!!**
- z variable exte
- Functional fittin
- Relative norm'n yields $|V_{ub}|$.
- Total error on $|V_{ub}|$: 4.1%.



Unblinded result:

$|V_{ub}| = (3.72 \pm 0.16)\times 10^{-3}$

Reconstructing Form Factors

- For additional applications, the z expansion provides a useful summary.
- Formulas (Bourrely, Caprini, Lellouch, arXiv:0807.2722):

$$f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N_z - 1} b_n^+ \left[z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right]$$

$$f_0(z) = \sum_{n=0}^{N_z} b_n^0 z^n$$

$$t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

- Subthreshold 1^- pole in f_+; first 0^+ excitation (for f_0) is unstable.

- Coefficients and correlations:

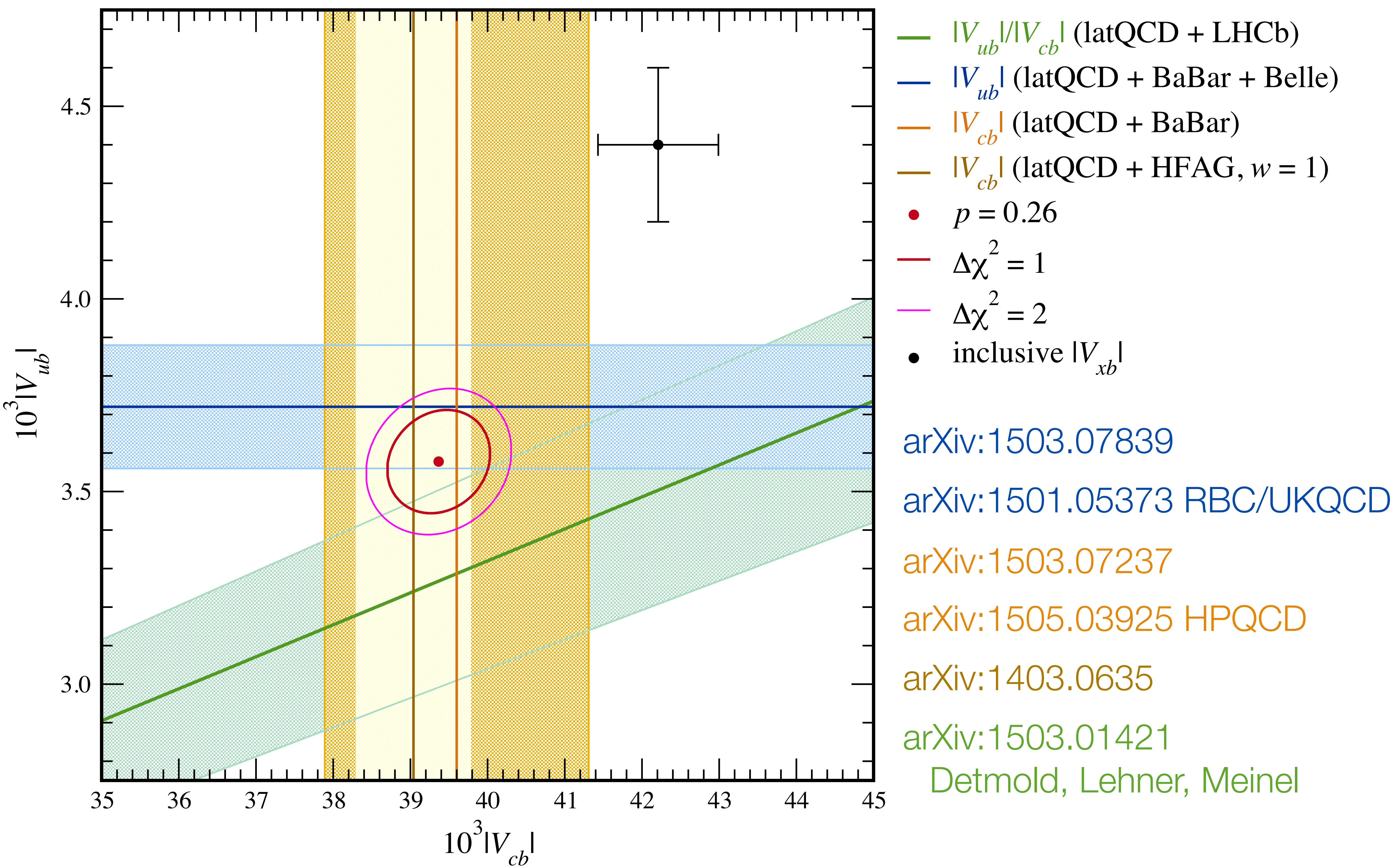
	b_0^+	b_1^+	b_2^+	b_3^+	b_0^0	b_1^0	b_2^0	b_3^0
	0.407(15)	−0.65(16)	−0.46(88)	0.4(1.3)	0.507(22)	−1.77(18)	1.27(81)	4.2(1.4)
b_0^+	1	0.451	0.161	0.102	0.331	0.346	0.292	0.216
b_1^+		1	0.757	0.665	0.430	0.817	0.854	0.699
b_2^+			1	0.988	0.482	0.847	0.951	0.795
b_3^+				1	0.484	0.833	0.913	0.714
b_0^0					1	0.447	0.359	0.189
b_1^0						1	0.827	0.500
b_2^0							1	0.838
b_3^0								1

- For your own work, just take this table and the formulas from the last slide and use the resulting form factors.

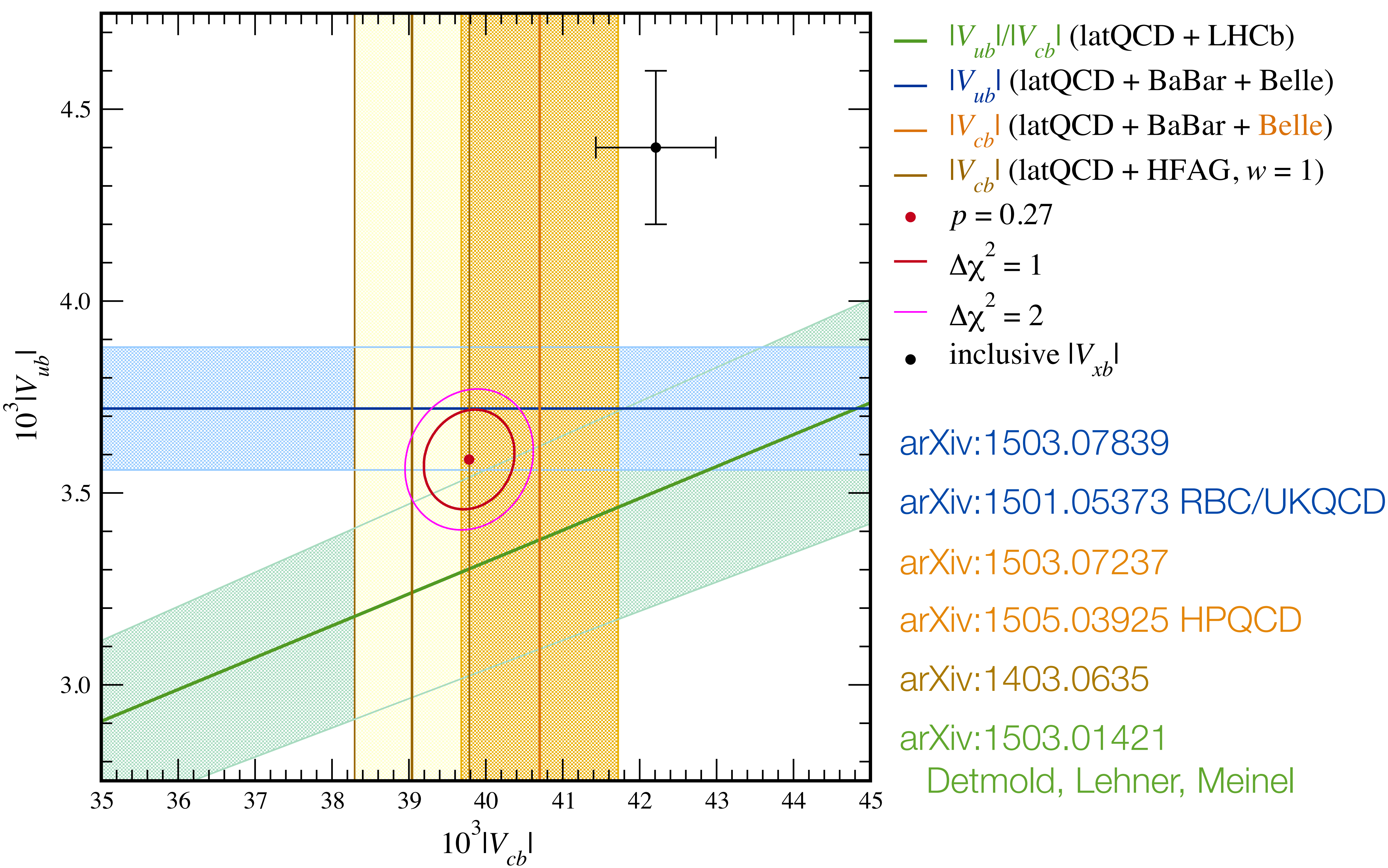
Semileptonic $B \to Dl\nu$ for $|V_{cb}|$

arXiv:1503.07237

- Similar strategy as above:
  - compute sequence of form-factor values;
  - chiral continuum extrapolation;
  - combined z-expansion fit to obtain $|V_{cb}|$.
- Differences:
  - HQET control of cutoff effects more central [hep-lat/0002008, hep-lat/0112044, hep-lat/0112045];
  - use Boyd, Grinstein, Lebed form of z expansion [hep-ph/9508211].

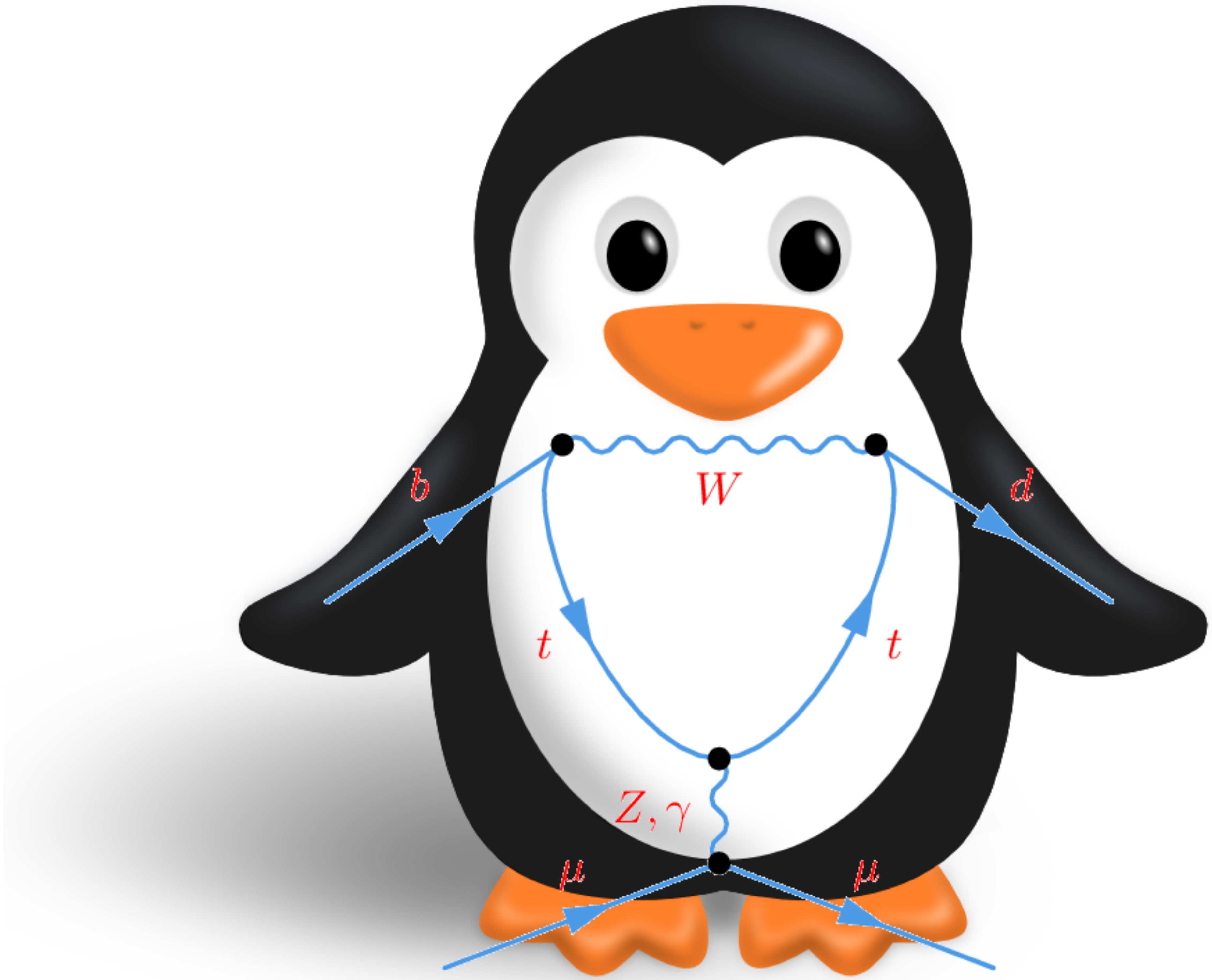


$|V_{ub}|/|V_{cb}|$ (latQCD + LHCb)
$|V_{ub}|$ (latQCD + BaBar + Belle)
$|V_{cb}|$ (latQCD + BaBar)
$|V_{cb}|$ (latQCD + HFAG, w = 1)
$p = 0.26$
$\Delta\chi^2 = 1$
$\Delta\chi^2 = 2$
inclusive $|V_{xb}|$
$10^3|V_{ub}|$
$10^3|V_{cb}|$
4.5
4.0
3.5
3.0
35
36
37
38
39
40
41
42
43
44
45
arXiv:1503.07839
arXiv:1501.05373 RBC/UKQCD
arXiv:1503.07237
arXiv:1505.03925 HPQCD
arXiv:1403.0635
arXiv:1503.01421
Detmold, Lehner, Meinel



4.5
4.0
3.5
3.0
$10^3|V_{ub}|$
35 36 37 38 39 40 41 42 43 44 45
$10^3|V_{cb}|$
$|V_{ub}|/|V_{cb}|$ (latQCD + LHCb)
$|V_{ub}|$ (latQCD + BaBar + Belle)
$|V_{cb}|$ (latQCD + BaBar + Belle)
$|V_{cb}|$ (latQCD + HFAG, w = 1)
$p = 0.27$
$\Delta\chi^2 = 1$
$\Delta\chi^2 = 2$
inclusive $|V_{xb}|$
arXiv:1503.07839
arXiv:1501.05373 RBC/UKQCD
arXiv:1503.07237
arXiv:1505.03925 HPQCD
arXiv:1403.0635
arXiv:1503.01421
Detmold, Lehner, Meinel

Penguins



graphic adapted by Daping Du

Basic Formulas for $B \to \pi l^+ l^-$, $K l^+ l^-$

cf., arXiv:1510.02349, Sec. 2 & Appendix B

- One-loop effective Hamiltonian contains many operators ($q = d, s$):

$$Q_1^u = (\bar{q}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L) \qquad Q_2^u = (\bar{q}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L)$$

$$Q_1 = (\bar{q}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L) \qquad Q_2 = (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

$$Q_3 = (\bar{q}_L \gamma_\mu b_L) \sum_{q'} (\bar{q}' \gamma^\mu q') \qquad Q_4 = (\bar{q}_L \gamma_\mu T^a b_L) \sum_{q'} (\bar{q}' \gamma^\mu T^a q')$$

$$Q_5 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q') \qquad Q_6 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q')$$

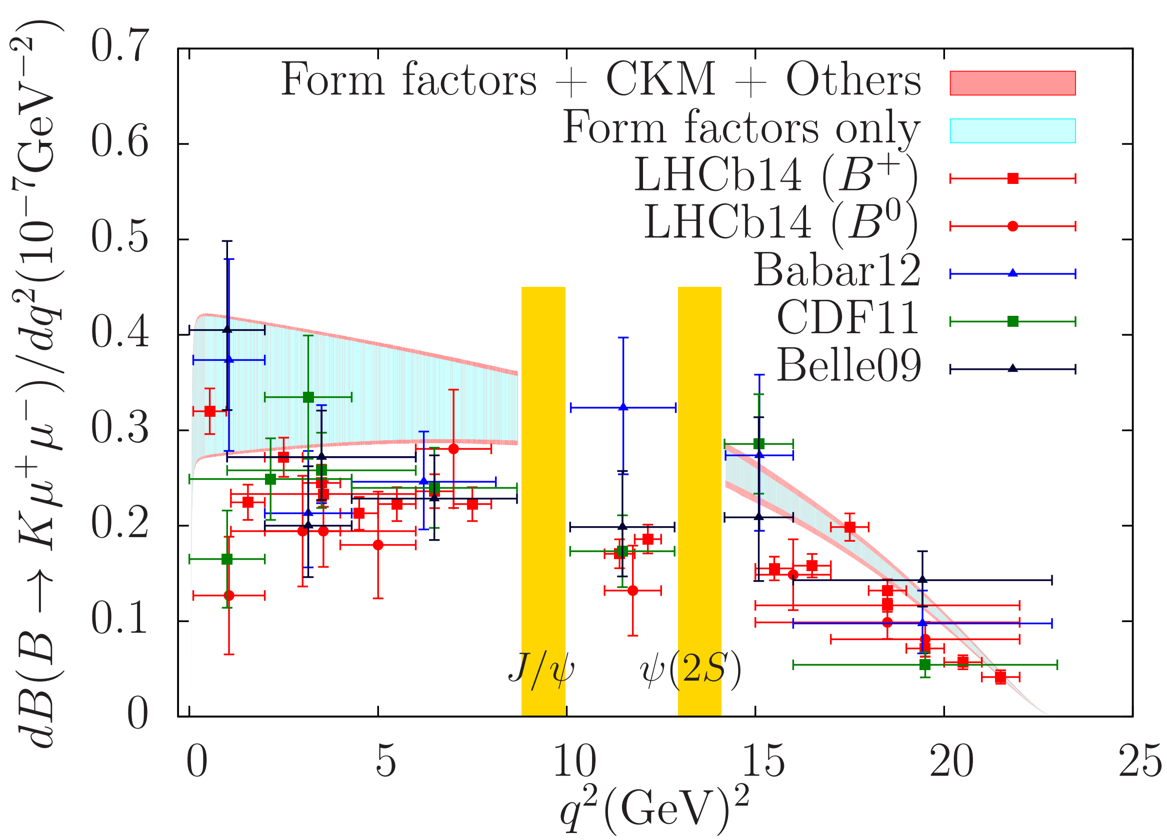
$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

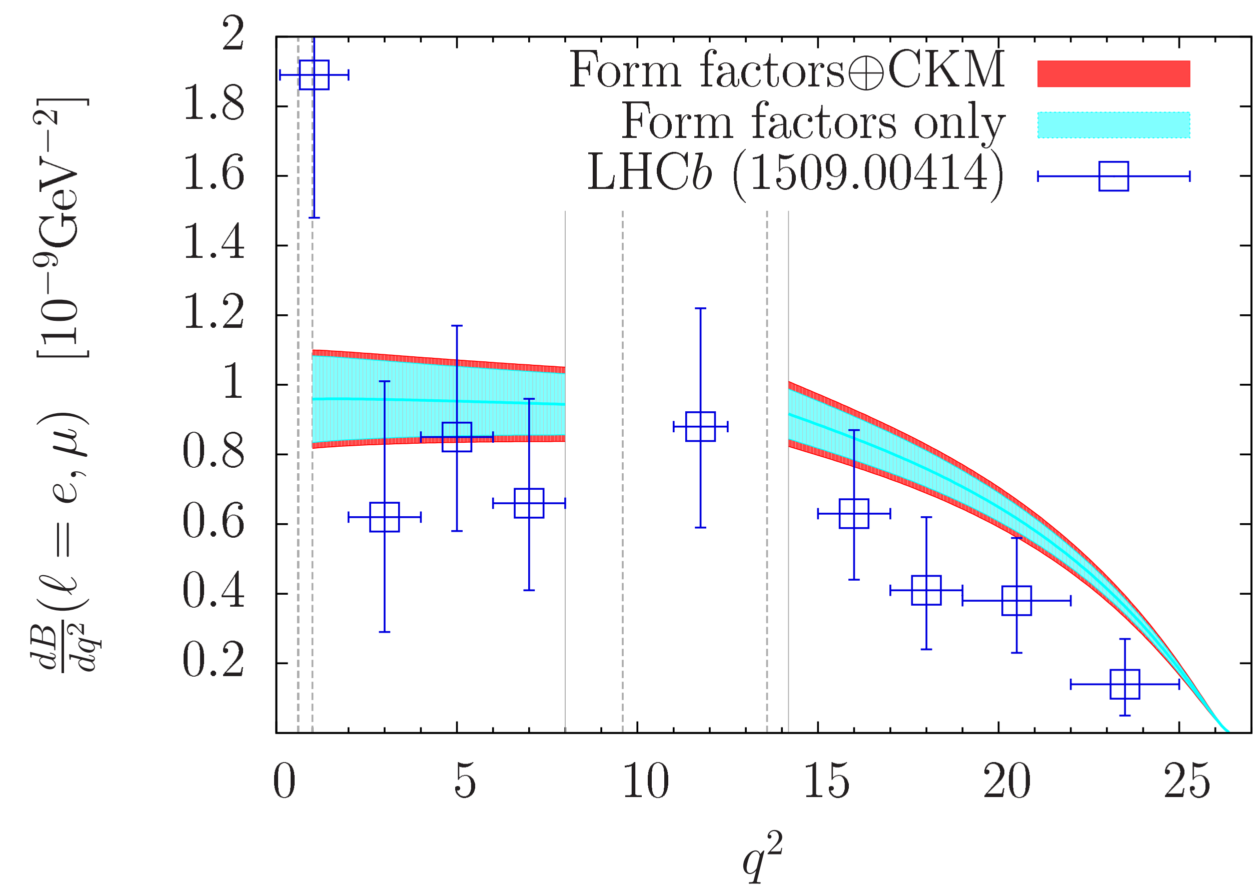
$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum_{\ell} (\bar{\ell} \gamma^\mu \ell) \qquad Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum_{\ell} (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- Matrix elements of Q_7, Q_9, Q_{10} yield form factors, including tensor f_T.

Kinematic Distributions

- Experimental data from LHCb [arXiv:1403.8044, arXiv:1509.00414] and earlier experiments; right plot's theory **preceded** measurement:

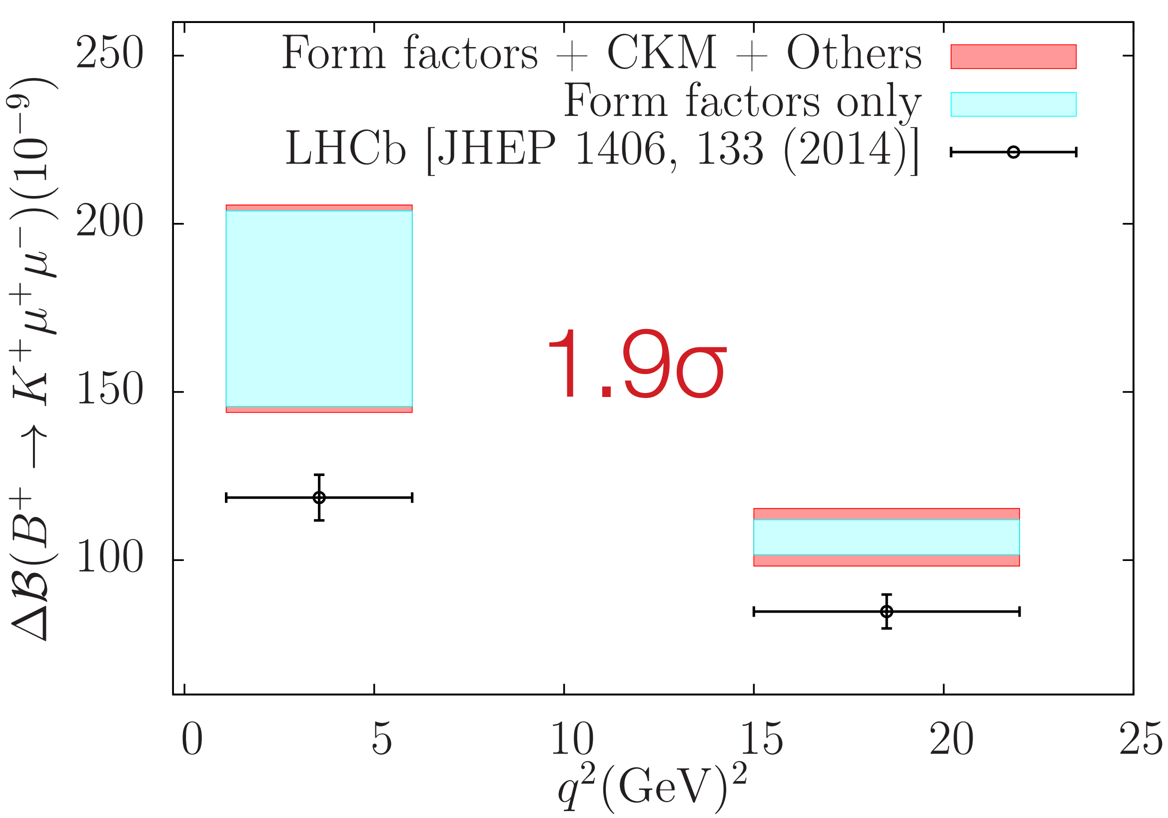


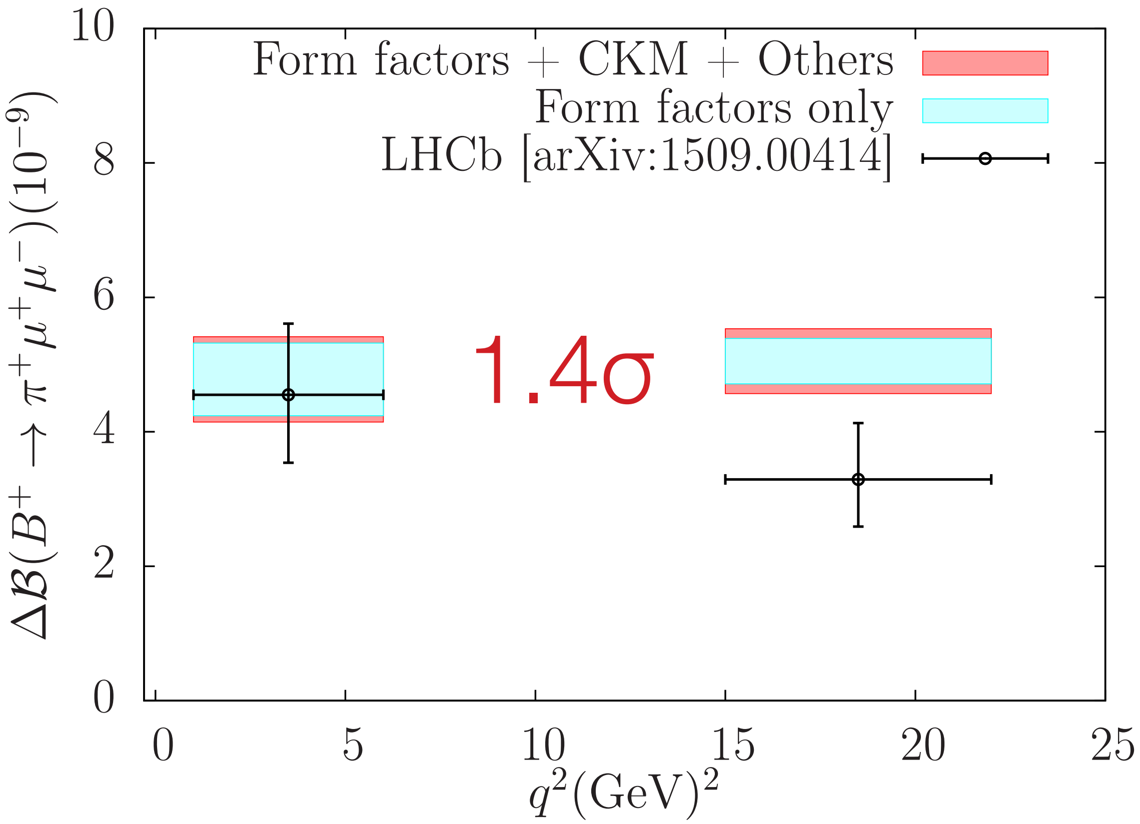


- LHCb wide bins: $q^2 \in [1\ \text{GeV}^2, 6\ \text{GeV}^2]$, and $q^2 \in [15\ \text{GeV}^2, 22\ \text{GeV}^2]$.

Kinematic Distributions

- Experimental data from LHCb [arXiv:1403.8044, arXiv:1509.00414] and earlier experiments; right plot's theory **preceded** measurement:





- LHCb wide bins: $q^2 \in [1\ \mathrm{GeV}^2, 6\ \mathrm{GeV}^2]$, and $q^2 \in [15\ \mathrm{GeV}^2, 22\ \mathrm{GeV}^2]$.

CKM: $|V_{td}|$ and $|V_{ts}|$

- Assume that there is no new physics buried in the Wilson coefficients.
- Then the combination of our calculations with experimental measurements yield the third row of the CKM matrix.
- We find

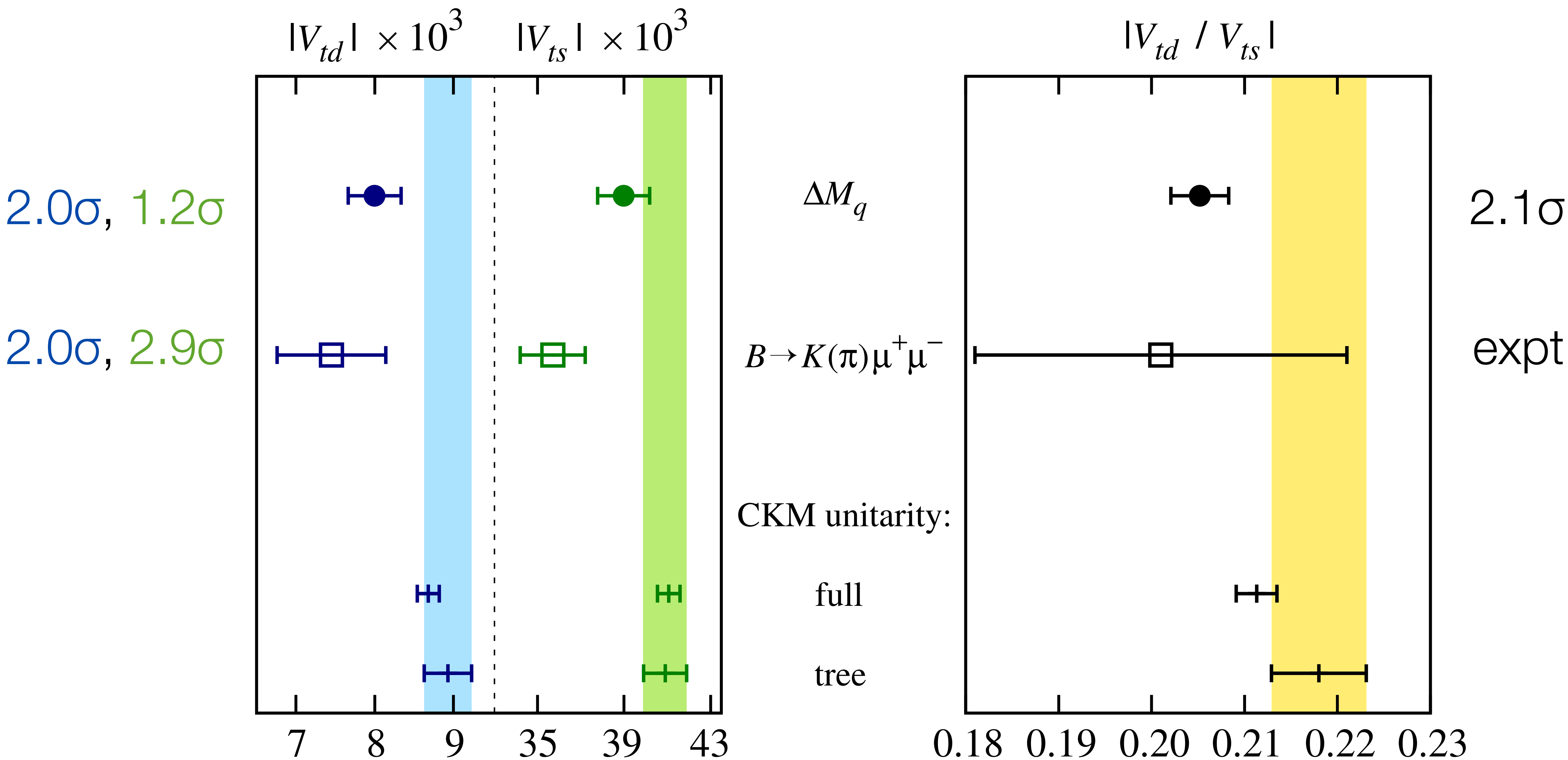
$$|V_{td}/V_{ts}| = 0.201(20)$$
$$|V_{tb}^* V_{td}| \times 10^3 = 7.45(69)$$
$$|V_{tb}^* V_{ts}| \times 10^3 = 35.7(1.5)$$

- The uncertainty here is commensurate with neutral *B*-meson mixing.
- The FCNC result for $|V_{ts}|$ is 1.6σ lower than the *B*-mixing result.

CKM Comparison

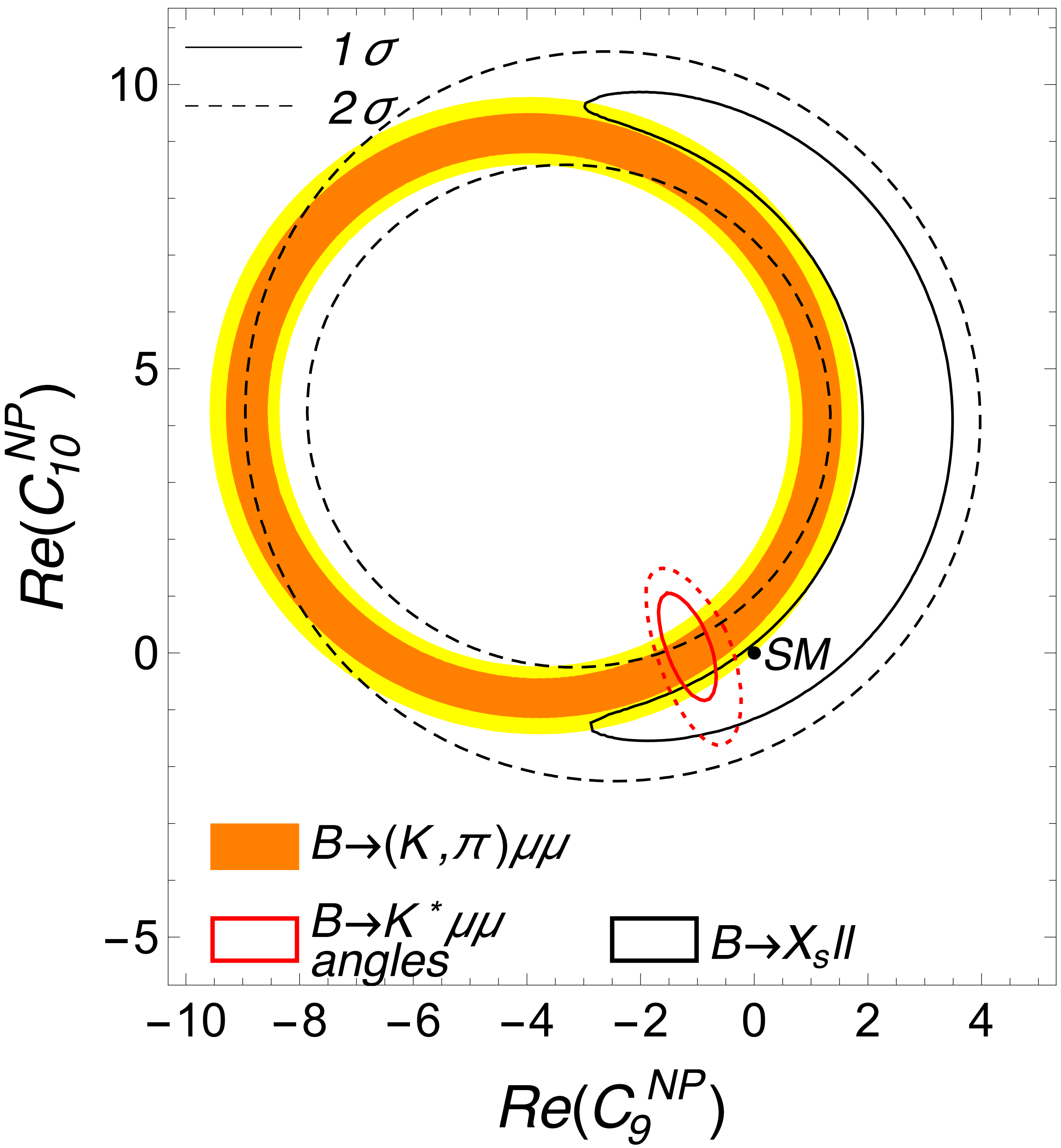
CKMFitter from S. Descotes-Genon
plots by C. Bouchard

- CKM from FCNC are lower than determinations from trees and unitarity.



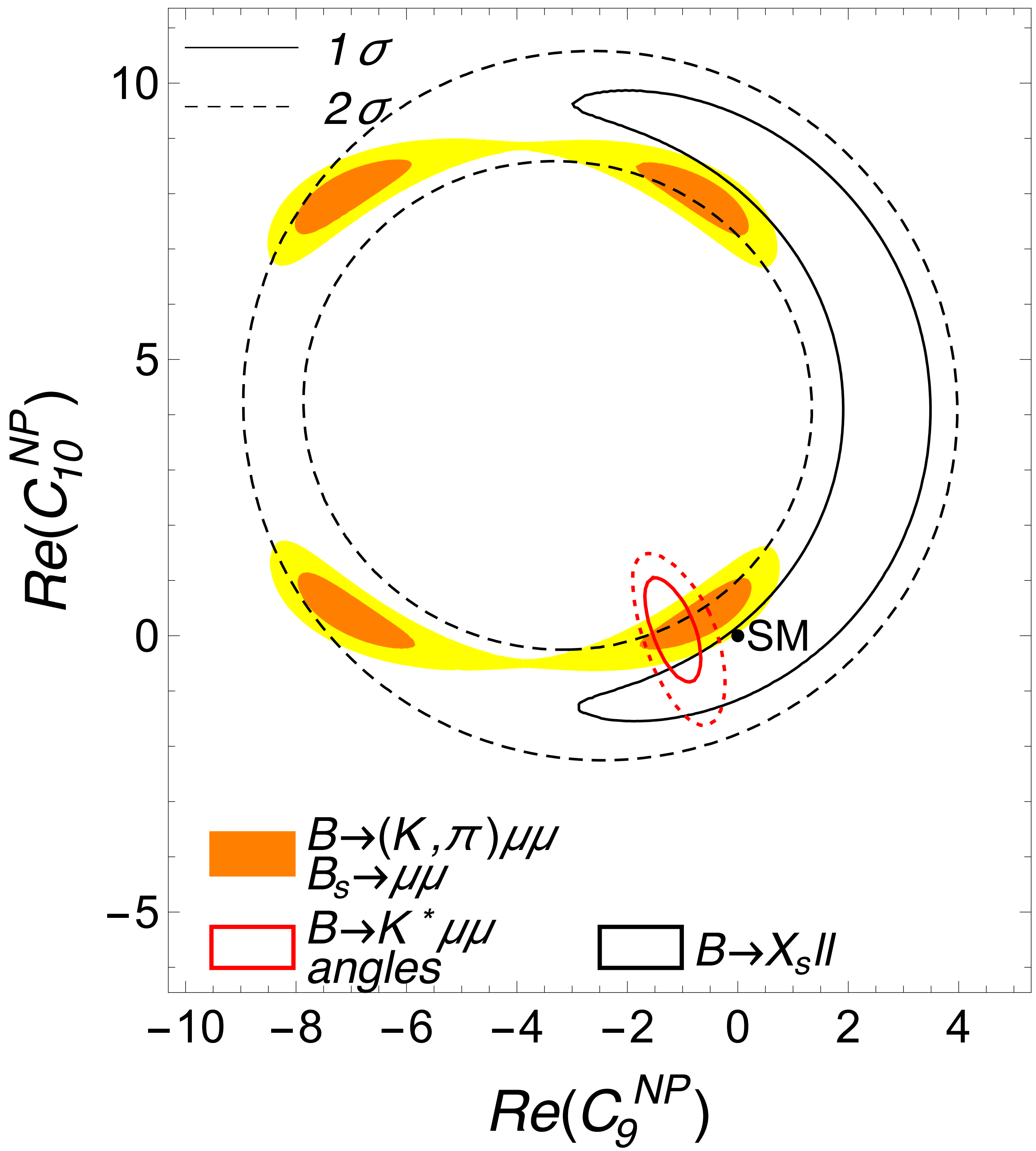
Wilson Coefficients

- Assuming no new physics is sad:
  - take the CKM matrix from a global fit;
  - determine best fit to Wilson coefficients C_9 and C_{10}.
- From the observables considered here, the SM is 2σ away from the best fit.
- Comparable but complementary to angular observables in $B \to K^* \mu\mu$.



Wilson Coefficients 2

- Add $B_s \to \mu\mu$, which also relies on lattice QCD—f_{B_s}.
- Favored region shrinks but only away from SM point.
- NB: assuming no new CPV and avoiding $b \to s\gamma$ constraints on C_7 and C_8.



Summary of this part

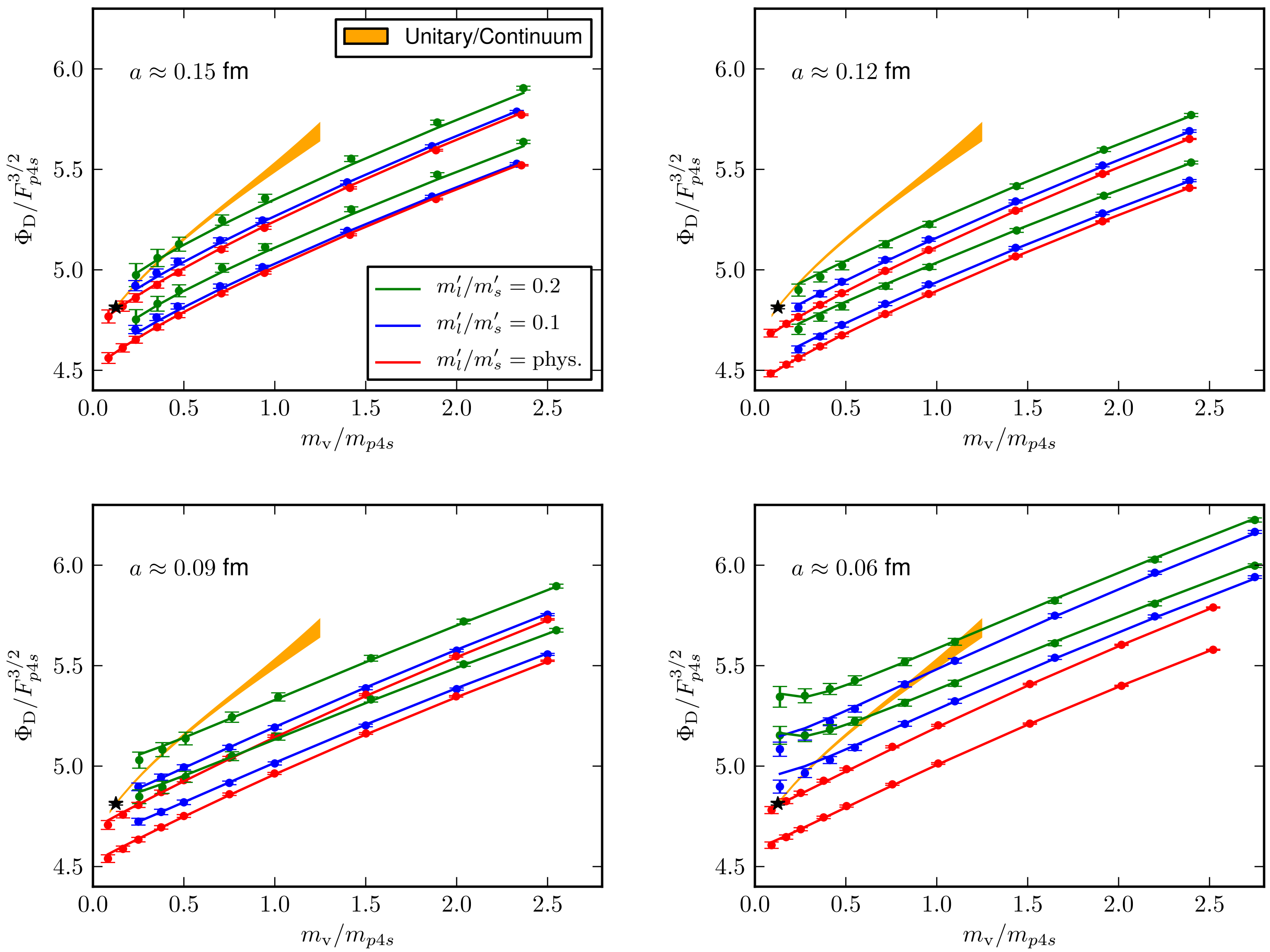
- The overarching take-home message:
  - we provide a convenient useful parametrization of the form factors, including correlations needed for joint fits, ratios, *etc*.;
  - the scope of application is not limited to what we’ve done;
  - just like collider physicists use CTEQ or MRSW parton densities, flavor physicists can use our (or other group’s) form factors.
- Future work, e.g., on MILC HISQ ensembles, will improve the precision (over the coming few years).

Leptonic K & D and Semileptonic K Decays with 2+1+1 Sea Quarks

skip

All Numerical Lattice Data at a Glance

- Simultaneous fit to all data, with EFT formula (Symanzik ⊗ χPT).



- Good fit: $\chi^2/\mathrm{dof} = 347/339$, with $p = 0.36$.
- Fits lines have a and m_l fixed, but vary m_v.
- Orange band (same in all panels) shows $a = 0$, $m_v = m_l$.

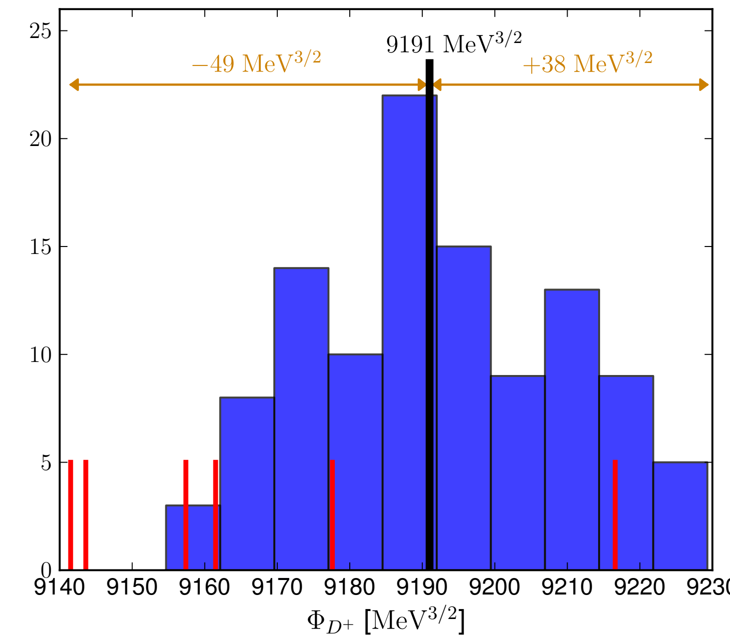
Combining Analyses

- Blue histograms show outcomes of χPT analyses:

$$f_{D^+} = 212.6 \pm 0.4_{\text{stat}} \, {}^{+0.9}_{-0.8}\big|_{a^2\,\text{extrap}} \pm 0.3_{\text{FV}} \pm 0.0_{\text{EM}} \pm 0.3_{f_\pi\,\text{PDG}}\ \text{MeV}$$

$$f_{D_s} = 249.0 \pm 0.3_{\text{stat}} \, {}^{+1.0}_{-0.9}\big|_{a^2\,\text{extrap}} \pm 0.2_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.4_{f_\pi\,\text{PDG}}\ \text{MeV}$$

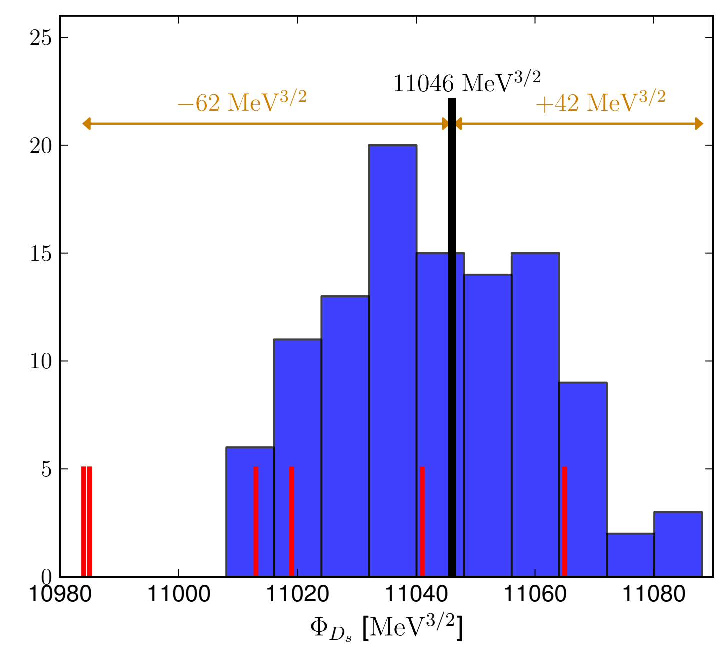
- Red bars show outcomes of the physical-point-only analysis.



- Inflate systematic errors to account for slightly wider spread in final result:

$$f_{D^+} = 212.6 \pm 0.4_{\text{stat}} \, {}^{+0.9}_{-1.1}\big|_{a^2\,\text{extrap}} \pm 0.3_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.3_{f_\pi\,\text{PDG}}\ \text{MeV}$$

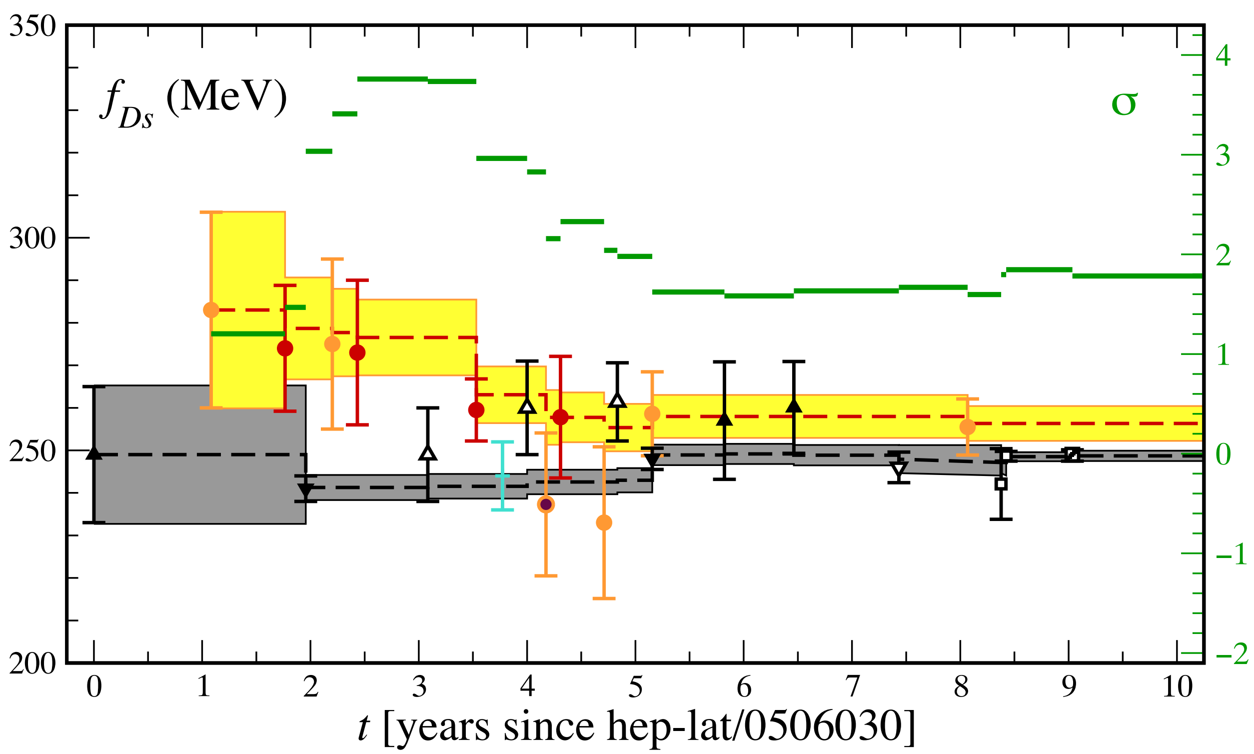
$$f_{D_s} = 249.0 \pm 0.3_{\text{stat}} \, {}^{+1.0}_{-1.4}\big|_{a^2\,\text{extrap}} \pm 0.2_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.4_{f_\pi\,\text{PDG}}\ \text{MeV}$$

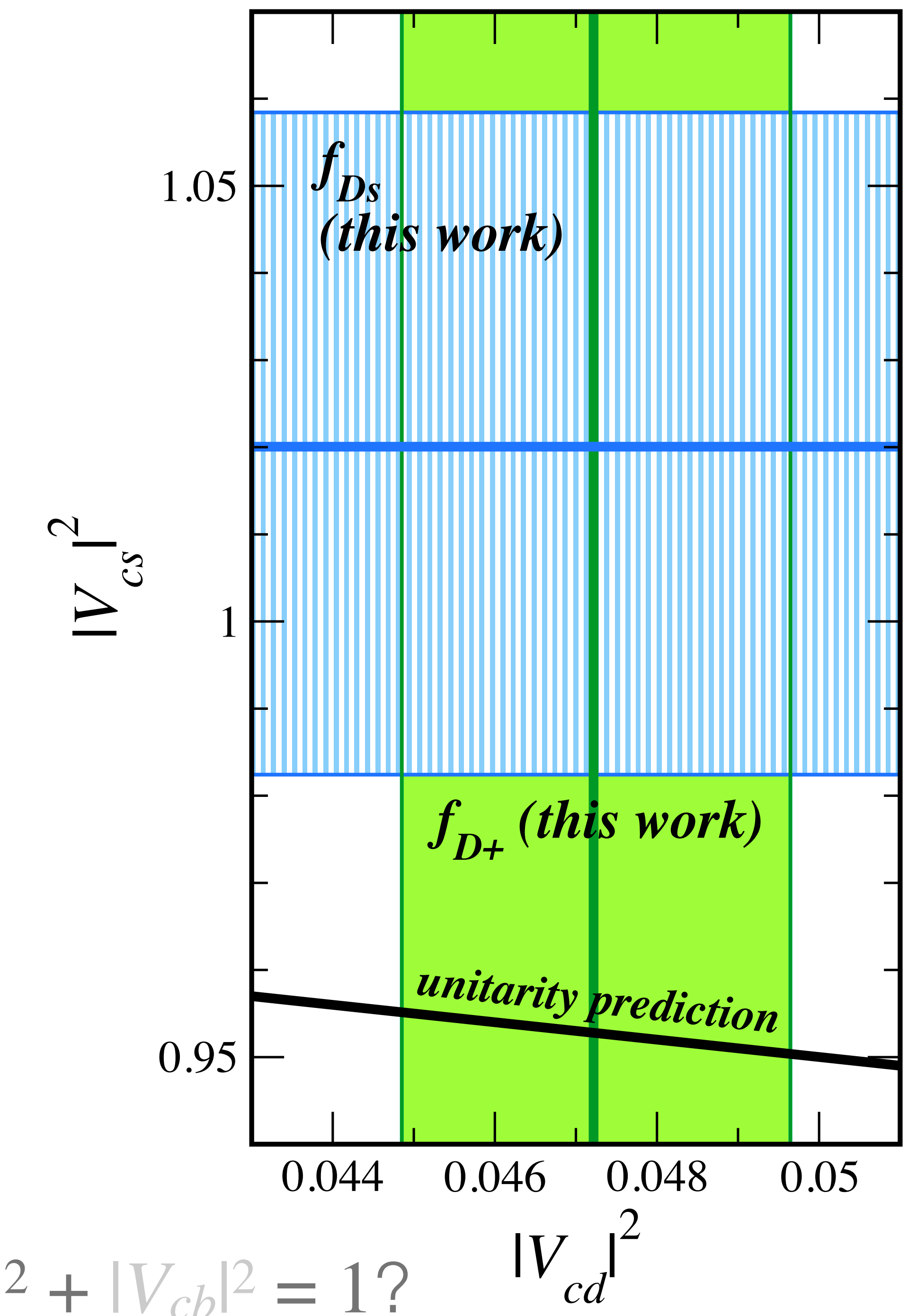


- Thus, the additional "non-physical" ensembles principally reduce the statistical error.

f_D and f_{D_s} or $|V_{cd}|$ and $|V_{cs}|$

- Taking $|V_{cs}|$ from CKM unitarity, expt yields f_{D_s} (or take $|V_{cd}|$ and get f_D).
- Here the QCD uncertainty is smaller than the experimental uncertainty:





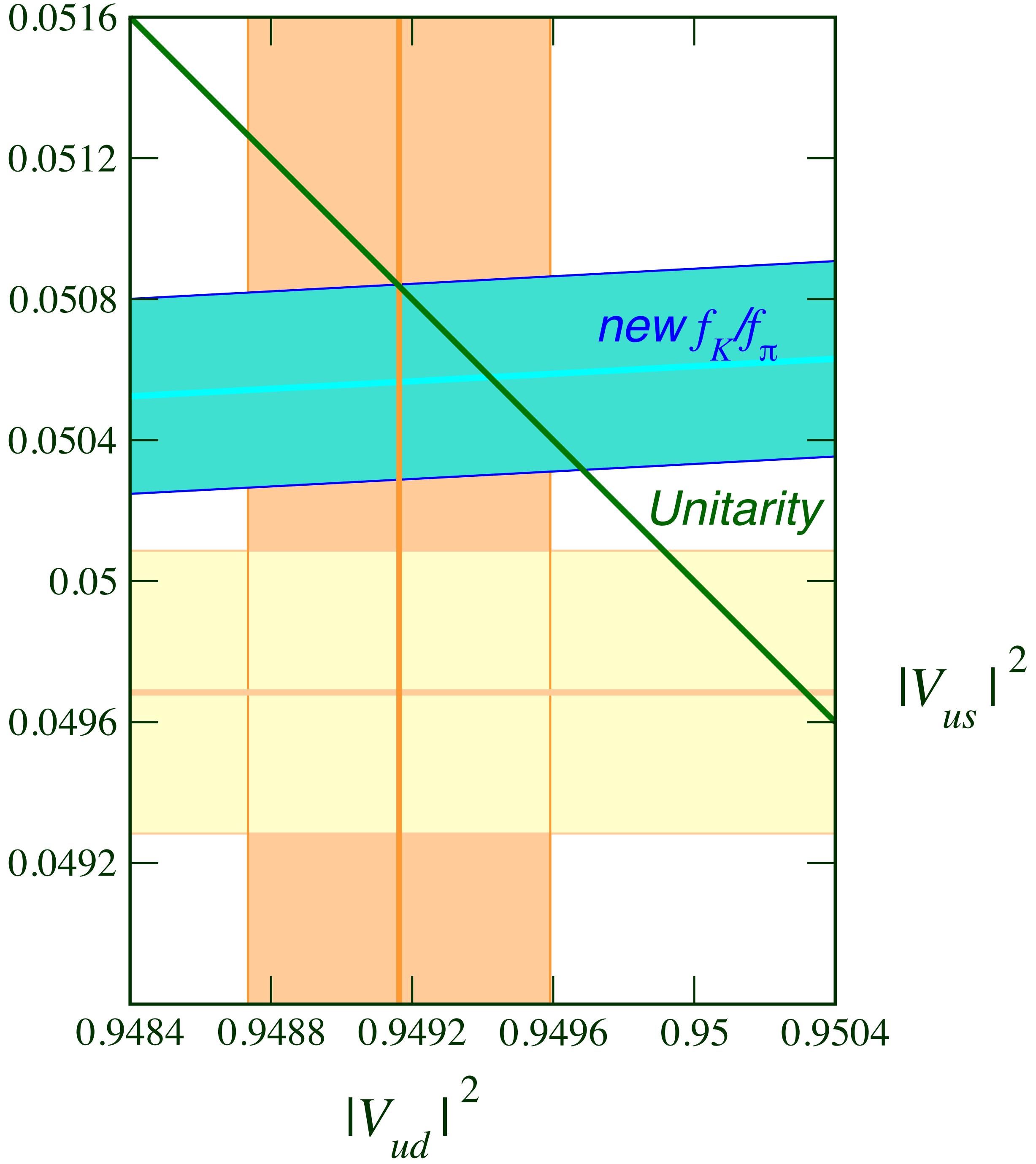
- Or determine CKM directly & test $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$?

First Row Unitarity Test

- Similar precision from f_K/f_π and semileptonic kaon decay.
- First-row unitarity test:
  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$?
- Now $|V_{us}|^2$ is as precise as $|V_{us}|^2$ (from nuclear physics).

First Row Unitarity Test

- Similar precision from f_K/f_π and semileptonic kaon decay.
- First-row unitarity test: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$?
- Now $|V_{us}|^2$ is as precise as $|V_{us}|^2$ (from nuclear physics).



Future Outlook

Three Frontiers

- High precision with mesons:
  - QCD error is (nearly) as small as QED and isospin breaking;
  - will require great attention to detail.
- Nucleons: many matrix elements needed for fundamental physics:
  - nucleon form factors for neutrino physics;
  - quark content of the nucleon for dark matter detection and $\mu N \to eN$;
  - (moments of) parton densities for LHC collisions;
  - electric dipole moments probing new sources of CP violation.

- New avenues in lattice QCD:
  - hadronic contributions to muon g–2: HVP, HLbL;
  - long-distance contributions, *e.g.*, ΔM_K;
  - multi-hadron final states, *e.g.*, *CP* violation in $K \to \pi\pi$.
- New avenues in lattice gauge theory (beyond QCD):
  - composite Higgs models;
  - nonperturbative supersymmetry (public code by David Schaich).
- And many topics in nuclear physics, *e.g.*, detailed hadron structure and QCD thermodynamics.

Community Tools

- Libraries of ensembles of lattice gauge fields:
  - Gauge Connection @ NERSC—ensembles now have a doi
  - International Lattice Data Grid
- Publicly available software:
  - USQCD “SciDAC” framework, with MILC, CPS, and Chroma APIs
    - has enabled junior researchers to start their own projects, even senior researchers who didn’t grow up with lattice gauge theory
  - openQCD (Lüscher).

Vielen herzlichen Dank!